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A Compilation of Published Research In Elementary Arithmetic Since 1931

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A COMPILATION OF PUBLISHED RESEARCH IN ELEMENTARY ARITHMETIC SINCE 1931

being

A thesis presented to the Graduate Faculty of the Fort Hays Kansas State College in partial fulfillment of the requirements for the Degree of Master of Science

by

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DEDICATION

This thesis is dedicated to my patient and faithful wife, Lorraine, and to my two children, Joyce Louise and Warren Charles, to my parents and my wife's mother, all of whom have made innumerable sacrifices to make my education possible.

ACKNOWLEDGMENTS

The writer of this thesis appreciates the patient, thoughtful guidance of Dr. Robert T. McGrath, Professor of Education, Fort Hays Kansas State College, and the timely suggestions of Dr. F. B. Streeter, Librarian and Secretary of the Graduate Council.

PREFACE

The writer of this thesis has always been interested in improvement of instruction in arithmetic. It is believed that the present investigation will be quite beneficial in showing what progress has been made in teaching methods and will serve to direct the teacher of elementary arithmetic toward a renewed interest in the best possible instructional procedures. This work consists of a survey of articles of published research in the field of elementary arithmetic. They have been classified under many headings to facilitate their use.

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CHAPTER I

INTRODUCTION

Many articles are published each year pertaining to elementary arithmetic. Some of these may very properly be designated "mere opinion" while others may very properly be classified as "research". Because of the large number of such articles appearing in print pertaining to some aspect of arithmetic, no doubt a service may be rendered to teachers of the subject by evaluating and classifying them as research or otherwise. From this situation grew the problem of this thesis, the title of which may be stated as follows: A Compilation of Published Research in Elementary Arithmetic Since 1931. Such a guide should be useful to teachers of arithmetic in classrooms wherever they may be.

Research has been defined in various ways by different investigators. Whitney defines research as a state of mind--a friendly, welcoming attitude toward change.[1] C. C. Crawford at the University of Southern California explains research after this fashion: Research is simply a refined technique of thinking, employing specialized tools, instruments and procedures in order to obtain a more adequate solution of a problem than would be possible under ordinary means. It starts with a problem, collects facts, analyzes these critically and reaches decisions based on the actual evidence. It involves

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1. Frederick Lamson Whitney, The Elements of Research (New York: Prentice Hall, Inc., 1942), p. 19.

original work rather than exercise of mere opinion. It evolves from genuine desire to know rather than a desire to prove something.[2] According to Webster, it is careful or critical inquiry or examination seeking facts or principles, diligent investigation in order to ascertain something.[3] Likewise, in the Dictionary of Education by Good, research is said to be of such a nature that others would get the same result.[4] But for the purpose of this study the term research will be used to mean a study which starts with a problem, collects facts, analyzes these critically and reaches decisions based on the obtained facts. It will usually consist of an experiment in which the sample is general enough that others would get the same results.

This assembly began with the year 1932 because December, 1931, marks the close of a similar survey by Guy T. Buswell and C. H. Judd, who published a monograph[5] in 1925 based on research in elementary arithmetic up to that time. Buswell issued seven annual supplements to the original monograph, bringing the total number of articles and books reviewed to 697. The final supplement included research published and reported during the entire year of 1931.

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2. Ibid., p. 20.

3. A. Merriam Webster, Webster's Collegiate Dictionary, 5th Edition, p. 847.

4. Carter V. Good, Dictionary of Education (New York: McGraw Hill Book Co., Inc., 1945), p. 346.

5. G. T. Buswell, Survey of Research in Elementary Arithmetic (Chicago: University of Chicago Press, 1925), 212 pp.

Since that time, Leo J. Brueckner, as well as Buswell, continues to report through the Elementary School Journal on selected references in elementary arithmetic, but does not distinguish between research articles and those of critical discussion which do not bear the earmarks of research. This fact is clearly set forth by a number of authorities in the field.[6] The number of non-research articles is not only increasing, but it is believed that many articles which have been considered research should not be classified as such. Buswell challenges the field to recapture the spirit of true research that we might progress in effective teaching of arithmetic.[7]

The problem is limited to scientific investigations reported in the field of elementary arithmetic since 1931. Algebra and geometry, even though sometimes taught in the elementary school, will not be included. While completeness is attempted, caution will be taken to include only those articles which in the opinion of the author can be classified as research.

The survey method was employed in securing the data. Since it was not possible to obtain and appraise every investigation in the field of elementary arithmetic over such a long period, some articles were excluded on the ground that the titles suggested they

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6. W. S. Monroe and M. D. Engelhart, A Critical Summary of Research Relating to the Teaching of Arithmetic (University of Illinois, College of Education, Bureau of Educational Research. Bul. No. 58. Urbana: University of Illinois, 1931), 115 pp.

7. G. T. Buswell, "Outlook for Research in Arithmetic," Elementary School Journal, XLVII (January, 1947), 243.

were not research. The selected references published in the Elementary School Journal were helpful in discovering many of the citations in this study. Also, the references given in the Education Index include articles of research together with numerous articles which did not measure up to the desired standard. Other sources and bibliographies also containing reports were used to complete this survey.

CHAPTER II

RESEARCH PERTAINING TO ARITHMETICAL FUNDAMENTALS

More research has been conducted pertaining to the fundamental operations than any other phase of arithmetic. Not only does the study of the four fundamental operations--addition, subtraction, multiplication and division--make up a large part of the elementary arithmetic curricula, but it is the basis for all other phases of arithmetic. It is for this reason that it is placed first in this compilation.

Concepts of Pre-School Children

Children learn quite a few quantitative ideas before they begin formal study of arithmetic. As these ideas are very important in determining later progress, it is highly necessary that the primary teacher be familiar with the amount and nature of the child's understanding of the number system. The following studies reveal many astounding facts relative to concepts of pre-school children.

Carper, Doris, "Seeing Numbers as Groups in Primary-Grade Arithmetic," Elementary School Journal, XLIII (November, 1942), 166-70.

Reports a study of the comparative effectiveness of counting and grouping in the development of certain mathematical concepts.

Grant, Albert, "An Analysis of Number Knowledge of First Grade Pupils According to Levels of Intelligence." See Correlation with Intelligence.

Koenker, R. H., "Arithmetic Readiness at the Kindergarten Level," Journal of Educational Research, XLII (November,

1948), 218-23.

Value of arithmetic readiness program together with a study of child interest in activities based on their needs and experiences.

MacLactchy, Josephine, "Number Abilities of First-Grade Children," Childhood Education, XI (May, 1935), 344-7.

Presents evidence on number experiences of pre-school children.

Mott, Sina M., "Number Concepts of Small Children," The Mathematics Teacher, XXXVIII (November, 1945), 291-301.

Reports data gathered through individual interviews regarding forty-four pupils of kindergarten age.

Stotlar, Carolyn, "Arithmetic Concepts of Pre-School Children," Elementary School Journal, XLVI (February, 1946), 342-5.

Experiment testing knowledge of pre-school children finding wide range of skills.

Addition and Subtraction

Addition and subtraction are very closely related and are often taught together. It has also been convenient to study the problems of addition and subtraction together in a number of cases. The work in addition and subtraction of fractions presents problems of a special nature and is taken up in a separate section.

Berlund-Gray, Gungorg, and Robert V. Young, "The Effect of Process Sequence on the Interpretation of Two-Step Problems in Arithmetic." See Relative Difficulty and Errors.

Brownell, W. A., "Borrowing in Subtraction," Journal of Educational Research, XXXIII (February, 1940), 415-24.

A review of a larger study in third grade arithmetic.

Brownell, W. A., "An Experiment on Borrowing in Third-Grade Arithmetic," Journal of Educational Research,

XLI (November, 1947), 161-71.

Experiment reveals that there are merits to various methods, but consideration of the value of the instruction is at stake.

Brownell, W. A., "Study of Learning in One Phase of Arithmetic." See Problem Analysis.

Grossnickle, F. E., "The Effectiveness of Checking Subtraction by Addition," Elementary School Journal, XXXVIII (February, 1938), 436-41.

Checking should not be taught with the process, but after the pupil understands its value.

MacLatchy, Josephine H., "Another Measure of Difficulty of Addition Combinations," Educational Research Bulletin, XII (March 8, 1933), 57-61.

Familiarity with the combinations upon entering school is a factor in learning.

Smith, Henry Lester, and Merrill T. Eaton, "An Analysis of Efficiency in Addition," Twenty-Fourth Annual Conference on Educational Measurements, (Bulletin of the School of Education, Indiana University, Vol. XII, No. 4). Bloomington, Indiana: School of Education, Indiana University, 1937, pp. 52-78.

Reports an inverstigation with college students with various conditions of drill in addition.

Wheeler, Lester R., "A Comparative Study of the Difficulty of the 100 Addition Combination," Pedagogical Seminary and Journal of Genetic Psychology, LIV (June, 1939), 295-312.

Extensive correlations and difficulty ratings expressed, but recommends further study.

Wilburn, D. Banks, "A Method of Self-instruction for Learning the Thirty-Six Addition Combinations with Sums from Eleven to Eighteen and Their Corresponding Subtraction Facts in Grade II," Mathematics Teacher, XXXVIII (October, 1945), 246-51.

A study of the achievement of sixty-four pupils with a method of self-instruction for learning the addition combinations.

Wilson, Guy M., "For 100 Per Cent Subraction, What Method? A New Approach," Journal of Educational Research, XXVII (March, 1934), 503-8.

Shows evidence of many methods of teaching subtraction, but recommends that we stick to one method to eliminate confusion of students.

Fractions

Under the heading of fractions is listed research the type of research studies pertaining to decimals, ratios and percentages as well as those pertaining to common fractions. It is believed that this arrangement will facilitate the use of this compilation of research in analyzing the best methods of classroom instruction.

Becker, Angeline L., "Remedial Work in the Addition of Common Fractions," California Journal of Elementary Education, IX (August, 1940), 43-47.

Gives data on types of errors in fractions in a sixth-grade class, and suggests four methods of improving teaching of fractions.

Guiler, W. S., "Difficulties Encountered in Percentage by College Freshmen," Journal of Educational Research, XL (October, 1946), 81-95.

Difficulties in percentage encountered by adults may be an indication of poor teaching.

Guiler, W. S., "Difficulties Encountered by College Freshmen in Decimals," Journal of Educational Research, XL (September, 1946), 1-13.

Analyzes difficulties and errors in decimals.

Guiler, W. S., "Difficulties Encountered by College Freshmen in Fractions," Journal of Educational Research, XXXIX (October, 1945), 101-115.

Large itemized list of difficulties shown by experiment and tells where more emphasis needs to be placed.

Guiler, W. S., "Difficulties in Decimals Encountered by Ninth-Grade Pupils," Elementary School Journal, XLVI (March, 1946), 384-93.

Shows errors and difficulties that the school should investigate.

Guiler, W. S., "Difficulties in Percentage Encountered by Ninth-Grade Pupils," Elementary School Journal, XLVI (June, 1946), 563-73.

Evidence is presented showing that present methods of learning fractions lacks retention.

Johnson, J. T., "Are Ratios Fractions?" Elementary School Journal, XLVIII (March, 1948), 374-8.

Studies four concepts of fractions, presenting teaching methods on the subject.

Johnson, J. T., "Can Concepts in Elementary Mathematics be Developed?" See Problem Concepts.

Johnson, J. T., "Decimal Usage in the Occupational World," Journal of Educational Research, XXXVIII (April, 1945), 633-8.

We need to abandon continued drill on some decimals, but at the same time appreciate their existence.

Souder, Hugh Cowan, "The Construction and Evaluation of Certain Readiness Tests." See Testing Readiness.

Wilson, Guy M., and Charles O. Dalrymple, "Useful Fractions," Journal of Educational Research, XXX (January, 1937), 341-7.

A study of frequency of occurrence of fractions.

Multiplication

Only those research studies dealing with the teaching of the multiplication facts are considred in this section. In view of the fact that multiplication is a definite part of the long division process, it is not necessary to study multiplication as such alone.

Brownell, W. A., "Rate, Accuracy and Process in Learning," See Problem Analysis.

Burge, Lafton V., "Types of Errors and Questionable Habits of Work in Multiplication," Elementary School Journal, XXXIII (November, 1932), 185-94.

Considers sixty-eight types of errors in multiplication made by 2,577 pupils in grades four, five and six.

Ruch, G. M., "Relative Difficulty of the One Hundred Multiplication Facts with Special Reference to Textbook Construction," Elementary School Journal, XXXII (January, 1932), 369-77.

The difficulties encountered in the learning process are different from those in the final process.

Division

Division is much more complex than any of the other three basic operations. It not only involves the use of multiplication and subtraction, which are memory processes, but in long division it is necessary to estimate the quotient in advance. Since setting the decimal point is also a part of the division process, the division of decimals is given consideration in this section.

Brueckner, L. J., and Harvey O. Melbye, "Relative Difficulty of Types of Examples in Division with Two-Figure Divisors," Journal of Educational Research, XXXIII (February, 1940), 401-14.

A good article on relative difficulty of examples in grades 5B and 6A.

Dahle, Casper O., "The Verbal Thought and Overt Behavior in Children During Their Learning of Long Division." See Problem Analysis.

Distad, H. W., "An Analysis of the Drill Provisions in Division of Decimals in Ten Arithmetic Series." See Curriculum, Selection of Textbooks.

Grossnickle, F. E., "Classification of the Estimations in Two Methods of Finding the Quotient in Long Division," Elementary School Journal, XXXII (April, 1932), 595-604.

A detailed classification and analysis of estimates of quotients in long division.

Grossnickle, F. E., "Comparison of Achievement of Pupils Who Are Good and Poor in Learning with a Two-Figure Divisor," Journal of Educational Research, XXXIV (January, 1941), 346-51.

A study of ninety-four children shows no great difference in the two groups either in native ability or in eventual achievement.

Grossnickle, F. E., "Constancy of Error in Learning Division with a Two-Figure Divisor," Journal of Educational Research, XXXIII (November, 1939), 189-196.

A study based on 231 pupils of grade four indicating the fact that error is not constant, but invites further investigation.

Grossnickle, F. E., "Cues in Division Problems Given in Nine Representative Textbooks in Arithmetic," Elementary School Journal, XXXIII (February, 1933), 452-61.

Listings of cues in division problems are presented from textbooks for grades three to six.

Grossnickle, F. E., "Division Facts and Their Use in Estimation of Quotient with Two-Figure Divisor," Elementary School Journal, XLV (June, 1945), 569-74.

Study shows the importance of the division facts for use in estimating the quotient.

Grossnickle, F. E., "Estimating the Quotient by Two Methods in Division with a Three-Figure Divisor," Elementary School Journal, XXXIX (January, 1939), 352-56.

A comparison of the relative accuracy of the apparent and the increase-by-one method.

Grossnickle, F. E., "An Experiment with a One-Figure Divisor in Short and Long Division," Elementary School Journal, XXXIV (March and April, 1934), 496-506, 590-99.

An experiment which suggests that only the long division

form should be taught.

Grossnickle, F. E., "An Experiment with Two Methods of Estimation of the Quotient," Elementary School Journal, XXXVII (May, 1937), 668-77.

There was no appreciative difference in the results of the two methods used in this experiment.

Grossnickle, F. E., "How a Method of Scoring a Test in Division Affects the Score," Elementary School Journal, XL (January, 1940), 366-70.

Shows that a score for correct estimate is also necessary for instructional purposes.

Grossnickle, F. E., "Kinds of Errors in Division of Decimals and Their Constancy," Journal of Educational Research, XXXVII (October, 1943), 110-117.

Identifies twenty-one different kinds of errors, some of which are constant.

Grossnickle, F. E., "Methods of Estimating the Quotient in Long Division Used by Teacher-Training Students," Elementary School Journal, XXXV (February, 1935), 448-53.

Investigation shows that some methods are much preferred over others.

Grossnickle, F. E., "Practice Material in the Estimation of the Quotient in Long Division Found in Current Arithmetic Workbooks," Journal of Educational Research, XXVIII (May, 1935), 668-88.

Extensive study of examples in workbooks shows need for improvement.

Grossnickle, F. E., "Practice Material in the Estimation of the Quotient in Long Division Found in Current Textbooks," Elementary School Journal, XXXIII (October, 1932), 130-41.

Survey study of examples in textbooks.

Grossnickle, F. E., "Some Factors Affecting a Test Score in Division of Decimals." See Test Scoring.

Grossnickle, F. E., "Types of Errors in Division of Decimals," Elementary School Journal, XLII (November,

1941), 184-94.

A study of two hundred pupils in grades six to nine, showing common errors and how to avoid them.

Holland, Henrietta, "Difficulties Involved in Long Division and Some Suggestions for Teaching the Process," Elementary School Journal, XLII (April, 1942), 585-96.

Analysis of thirteen sources of difficulty in the long division process with valuable suggestions for the benefit of the learner.

Koenker, Robert H., "Certain Characteristic Differences Between Excellent and Poor Achievers in Two-Figure Division." See Correlation with Intelligence.

Morton, R. L., "Estimating Quotient Figures When Dividing by Two-Place Numbers," Elementary School Journal, XLVIII (November, 1947), 141-48.

Suggested methods for estimating. Recommends rounding numbers for greater meaning.

Olander, Herbert T., and Preston E. Sharp, "Long Division Versus Short Division," Journal of Educational Research, XXVI (September, 1932), 6-11.

Experiment with pupils in grades four to twelve showed greater accuracy by three-fourths of the students when long division was the standard method.

Osburn, Worth J., "Levels of Difficulty in Long Division," Elementary School Journal, XLVI (April, 1946), 441-47.

A detailed study survey of examples in long division.

Unrich, Louis E., Sr., "A Study of the Process of Transformation with Special Emphasis on its Application to the Division of Decimal Numbers," The Mathematics Teacher, XL (November, 1947), 345-54.

Examination of transformations from one unit to another.

Computation, Drill and Review

The question often arises as to the optimum amount of drill and review. A great deal of research is being conducted relative to

these important aspects of instruction. No doubt this was brought on in part by the need of the Armed Forces for speedy learning or review of many fundamental operations.

Bemis, Eaton O., and W. C. Frow, "Remedial Arithmetic After Two Years," Journal of Educational Research, XXXV (February, 1942), 445-52.

A follow up study which indicates that remedial work is of little value in many cases.

Boss, Mabel E., "Arithmetic, Now and Then," School and Society, LI (March 23, 1940), 391-92.

A comparison of scores on the Cleveland Survey Arithmetic Test given in St. Louis in 1916, and again in the year 1938.

Braverman, Benjamin, "Does a Year's Exposure to Algebra Improve a Pupil's Ability in Arithmetic?" The Mathematics Teacher, XXXII (November, 1939), 301-12.

A study suggesting that algebra helps improve the arithmetical ability of high school pupils.

Brownell, W. A., and Charlotte B. Chazel, "The Effects of Premature Drill in Third-Grade Arithmetic," Journal of Educational Research, XXIX (September, 1935), 17-28.

Shows that drill does not guarantee child growth for several given reasons.

Christoffersen, B. O., and W. A. Wittich, "Navy Statistics and Mathematics Retention - A Challenge to the School," School and Society, LVI (November 21, 1942), 502-04.

Discusses deficiencies in arithmetic according to tests in Navy. Proposes refresher course in high school.

Davis, Robert A., and Edward J. Rood, "Remembering and Forgetting Arithmetical Abilities," Journal of Educational Psychology, XXXVIII (April, 1947), 216-22.

Reports the results of successive usage of an arithmetic test during grades seven and eight.

Foran, T. G., "Drill in Arithmetic," Catholic Educational Review, XXXI (April, 1933), 232-43.

A summary of investigations relating to drill as well as a discussion of the use of drill.

Grossnickle, F. E., "Practice Material in Estimation of the Quotient in Long Division Found in Current Textbooks." See Division.

Grossnickle, F. E., "Practice Material in the Estimation of the Quotient in Long Division Found in Current Arithmetic Workbooks." See Division.

Guiler, W. S., and H. B. Hoffman, "Effect of Different Types of Mathematics Courses on Computational Ability," Educational Administration and Supervision, XXIX (November, 1943), 449-56.

Some courses contribute more than others. Recommends remedial work as part of algebra.

Harding, Lowry W., and Inez P. Bryant, "An Experimental Comparison of Drill and Direct Experience in Arithmetic Learning in a Fourth-Grade," Journal of Educational Research, XXXVII (January, 1944), 321-27.

Reports the results of an experiment with various instructional procedures.

Umrich, Louis E., Sr., "A Study of the Process of Transformations with Emphasis on its Application to the Division of Decimal Numbers." See Division.

Wilson, G. M., "Basic Considerations for Profitable Research in Arithmetic," Journal of Educational Research, XXXVIII (October, 1944), 119-23.

A discussion of errors as a basis for research and an evaluation of a few research papers.

Wilson, G. M., and Gertrude L. Hanley, "For Basic Drill in Arithmetic - What Norm or Average is Satisfactory?" See Test Scoring.

Zyve, Claire, "An Experimental Study of the Teaching of Arithmetic Combinations," Educational Method, XII (October, 1932), 16-18.

Gives results of test comparing the effects of blackboard and lantern-slide presentation of arithmetic combinations.

Errors and Relative Difficulty

Extensive studies have been made concerning the many types of errors in the fundamental processes and the implication with regard to teaching arithmetic in the elementary school. Along with this is the relative difficulty of the various processes and how their order in a given situation affects the difficulty.

Berlund-Gray, Gunborg, "Difficulty of the Arithmetic Processes," Elementary School Journal, XL (November, 1939), 198-203.

The position of a process has a decided influence on the difficulty of the entire problem.

Berlund-Gray, Gunborg, and Robert V. Young, "The Effect of Process Sequence on the Interpretation of Two-Step Problems in Arithmetic," Journal of Educational Research, XXXIV (September, 1940), 21-29.

Subtraction-addition is more difficult than addition-subtraction.

Brueckner, Leo J., and Gerold S. Laumann, "The Measurement of Accuracy of Judgments of the Difficulty of Arithmetic Problems," Educational Method, XII (March, 1933), 338-45.

Describes with reliability a technique to determine relative difficulty of problems.

Brueckner, Leo J., and John H. Snyder, "Constancy of Errors to Basic Facts in the Fundamental Operations in Arithmetic," Journal of Educational Research, XXXII (January, 1939), 336-44.

Errors are more constant in division than in other operations. Important for diagnosis.

Grossnickle, F. E., and John H. Snyder, "Constancy of Errors to Basic Facts in the Fundamental Operations in Arithmetic," Journal of Educational Research, XXXII (January, 1939), 336-44.

Shows how the two types of errors (chance and constant) are related to various operations as well as to maturity.

Guiler, W. S., "Computational Errors Made by Teachers of Arithmetic," Elementary School Journal, XXXIII (September, 1932), 51-58.

Analyzed the errors made by a group of teachers on a survey test in computational arithmetic.

Guiler, W. S., "Difficulties in Decimals Encountered by Ninth-Grade Pupils," Elementary School Journal, XLVI (March, 1946), 384-93.

A study to show where more stress needs to be placed.

Guiler, W. S., "Difficulties in Fractions Encountered by Ninth-Grade Pupils," Elementary School Journal, XLVI (November, 1945), 146-56.

Borrowing and changing to common denominators proved to be major difficulties.

Henderson, Kenneth B., "Weaknesses in Arithmetic-Teaching," Elementary School Journal, XLVI (June, 1946), 579-81.

A brief article based on arithmetical weaknesses found in the Army Air Forces. Accuracy was the first glaring weakness.

Knight, F. B., "A Report of Four Studies in Arithmetic." See Problem Concept Development.

CHAPTER III

PROBLEM SOLVING AND THE PSYCHOLOGY OF LEARNING

The analysis and solution of written problems is without question one of the more difficult parts of the arithmetic program. This chapter deals with the very nature of problem solving and then a later chapter will be given to methods of instruction which will have a bearing both on the fundamental processes and the solution of written problems.

Psychology and Analysis of Problem Solving

Real problem solving is not a trial and error procedure; rather, it is a process based on analysis. For this reason it is necessary for both the teacher and the student to fully understand the basic operations involved in the solution of the problem. There are many factors which contribute to the success of the student in problem solving.

Brownell, W. A., "Rate, Accuracy and Process in Learning," Journal of Educational Psychology, XXXV (September, 1944), 321.

A good article on educational psychology based on tests and interviews pertaining to learning multiplication combinations.

Brownell, W. A., "Study of Learning in One Phase of Arithmetic," Journal of Genetic Psychology, XXIV (April, 1941), 457-65.

Drawing on data of previous research, this article shows the contribution of the crutch in borrowing in subtraction.

Buswell, G. T., "Types of Learning and General Conditions Affecting Learning," Review of Educational Research, IX (June, 1939), 274-84.

A review of research on the learning process.

Dahle, Casper O., "The Verbal Thought and Overt Behavior in Children During Their Learning of Long Division," Journal of Experimental Education, IX (September, 1940), 1-8.

An elaborate study of the learning process with an implication in arithmetic.

Dexter, C. E., "Analysis of Written Problems in a Recent Arithmetic Series." See Evaluation of Textbooks.

Johnson, Donald M., "A Modern Account of Problem Solving," Psychological Bulletin, XLI (April, 1944), 201-27.

Identification and description of the essential processes of problem solving.

Johnson, Harry C., "Problem-Solving in Arithmetic: A Review of the Literature," Elementary School Journal, XLIV (March and April, 1944), 396-403, 476-82.

An important summary of the various methods of problem solving.

Johnson, J. T., "On the Nature of Problem-Solving in Arithmetic," Journal of Educational Research, XLIII (October, 1949), 110-15.

A study of the various factors contributing to problem-solving ability.

Mitchell, Claude, "Problem Analysis and Problem-Solving in Arithmetic," Elementary School Journal, XXXII (February, 1932), 464-66.

Study of improvement of 117 pupils in grades seven and eight when problems were rearranged.

Monroe, W. W., and Max D. Engelhart, "The Effectiveness of Systematic Instruction in Reading Verbal Problems in Arithmetic," Elementary School Journal, XXXIII (January, 1933), 377-81.

Experiment in fifth grade to determine the value of specific training in reading verbal problems.

Schaaf, William L., "A Realistic Approach to Problem-Solving in Arithmetic," Elementary School Journal, XLVI (May, 1946), 494-7.

Identifies nine aspects of problem-solving ability.

Sutherland, Ethel, "One-Step Problem Patterns and Their Relation to Problem Solving in Elementary Arithmetic," Teachers College Record, XLIX (April, 1948), 492-93.

An analysis of 15,000 verbal problems in terms of the thought-patterns involved.

Willey, Roy D., "Arithmetical Processes Needed by Children," Elementary School Journal, XLII (March, 1942), 524-27.

A study to determine which of the fundamental processes were needed most in solving functional problems.

Development of Concepts

The development of concepts is an essential part of the problem solving technique. It consists of the original knowledge of quantative data together with formation of mental pictures of the essential elements in the problem at hand.

Bramhall, Edwin W., "An Experimental Study of Two Types of Arithmetic Problems," Journal of Experimental Education, VIII (September, 1939), 36-38.

There is no significant difference between the effectiveness of the conventional type and the imaginative type problem.

Friedman, K. C., "Time Concepts of Elementary School Children," Elementary School Journal, XLIV (February, 1944), 337-42.

An extensive study showing that time problems should be based on the out-of-school experience of the children.

Gunderson, Agnes G., "Number Concepts Held by Seven-Year-Olds." See Readiness Tests.

Hall, Jack V., "Solving Verbal Arithmetic Problems Without Pencil and Paper," Elementary School Journal, XLVIII (December, 1947), 212-17.

Shows great need for oral work at various levels of difficulty.

Johnson, J. T., "Can Concepts in Elementary Mathematics Be Developed?" School Science and Mathematics, XLIV (February, 1944), 146-54.

Common fractions should be limited to halves, fourths, and eighths and increase emphasis on decimals.

Knight, F. B., "A Report of Four Studies in Arithmetic," Journal of Educational Research, XXX (January, 1937), 325-40.

Experimental study of the relative perceptual value of concrete number situations in primary grades.

Van Engen, H., "An Aspect of Meaning in Arithmetic," Elementary School Journal, XLVI (January, 1946), 272-77.

Central ideas in teaching problem-solving must be developed.

White, Helen M., "Does Experience in the Situation Involved Affect the Solving of a Problem?" Education, LIV (April, 1934), 451-55.

A study based on one thousand sixth-grade children. Shows value of experience.

Improvement and Generalization

The pupil's ability to generalize plays a very important part in the successful solution of problems. Along with this the student needs to learn to use various aids and devices to help him in problem-solving. Practice in these will no doubt bring improvement in the solution of written problems.

Brownell, W. A., "Two Kinds of Learning in Arithmetic," Journal of Educational Research, XXXI (May, 1938), 656-64.

Mastery of arithmetic calls for learning by both repetition and insight. Shows need for understanding aids to learning.

Conner, William L., "What Materials Are Most Useful to Children in Learning to Solve Problems?" Educational Method, XIII (April, 1943), 369-70.

Results on Stone Reasoning Test show that best problem-solving material comes from children's own environment.

Conner, William L., and Gertrude C. Hawking, "What Materials Are Most Useful to Children in Learning to Solve Problems?" Educational Method, XVI (October, 1936), 21-29.

An experiment to discover a means of improving the problem-solving ability of upper-grade pupils.

Dickey, John W., "The Value of Estimating Answers to Arithmetic Problems and Examples," Elementary School Journal, XXXV (September, 1934), 24-31.

Experiment shows practice in estimating improves results.

Ebert, Reuben S., "Generalization Abilities in Mathematics," Journal of Educational Research, XXXIX (May, 1946), 671-81.

A condensed article reporting reactions to fifty-four generalizations in arithmetic.

Hall, Jack V., "Oral Aids to Problem-Solving," Elementary School Journal, XLIII (December, 1942), 220-24.

Grades five and six reports analyzed for rapid, average and slow pupils.

Hansen, C. W., "Factors Associated with Successful Achievement in Problem Solving in Sixth-Grade Arithmetic," Journal of Educational Research, XXXVIII (October, 1944), 111-18.

Lists many factors and discusses the age range in which they are the most pronounced.

Hendrix, Gertrude, "A New Clue to Transfer of Training," Elementary School Journal, XLVIII (December, 1947), 197-208.

A review of the nature of generalization and the need for more study in this field.

Karstens, H., "Effective Guidance in Problem-Solving," Mathematics Teacher, XXXIX (April, 1946), 172-75.

Shows that guidance in problem-solving is a requisite.

Klugman, Samuel F., "Cooperative Versus Individual Efficiency in Problem Solving," Journal of Educational Psychology, XXXV (February, 1944), 91-100.

Children working together were more successful in problem solving than were control children working alone.

CHAPTER IV

CORRELATION WITH OTHER ABILITIES

Ability in arithmetic is closely associated with other knowledge and abilities. For instance, a child who comes to school with experience in number concepts is able to do much better work than the child who has had limited experience in numbers. Likewise, general intelligence is a primary factor in determining ability to do good quality work in arithmetic.

General Intelligence

For efficient instructional and scoring purposes, it is desirable to correlate the material with the general intelligence of the students. There is also a close relationship between this concept and readiness testing. Students learn best with material of optimum difficulty, so it is necessary for efficient instruction to understand the general ability of the students.

Grant, Albert, "An Analysis of Number Knowledge of First-Grade Pupils According to Levels of Intelligence," Journal of Experimental Education, VII (September, 1938), 63-66.

The responses of a large number of first-grade pupils in a test of number knowledge.

Grossnickle, F. E., "Comparison of Achievement of Pupils Who Are Good and Poor in Learning with a Two-Figure Divisor." See Division.

Koenker, Robert H., "Certain Characteristic Differences Between Excellent and Poor Achievers in Two-Figure Division," Journal of Educational Research, XXXV (April, 1942), 578-86.

Analysis of responses of 180 pupils on situations in division.

Stevens, B. A., "Problem Solving in Arithmetic," Journal of Educational Research, XXV (April-May, 1932), 253-60.

Considers the interrelations of ability in silent reading, power in the fundamentals, problem solving, and general intelligence.

Vocabulary of Arithmetic

This unit deals with the vocabulary of elementary arithmetic used by pupils and teachers. It shows that there is a positive correlation between vocabulary or experience and ability to understand and solve arithmetic problems.

Brueckner, Leo J., and Harvey O. Melbye, "Relative Difficulty of Types of Examples in Division with Two-Figure Divisors," Journal of Educational Research, XXXIII (February, 1940), 401-14.

An extensive attempt to determine relative difficulty with recognition of previous studies.

Ellsworth, Elmer E., "Number Experiences of 390 Children from Grades III-VI in an Urban Area," Education, LXI (April, 1941), 485-87.

Classified list of seventeen arithmetical functions in the daily lives of children.

Foran, T. G., "The Reading of Problems in Arithmetic," Catholic Educational Review, XXXI (December, 1933), 601-12.

Report of difficulty of arithmetical terms.

Gorman, Frank H., "The Arithmetic Vocabulary of the Elementary School Teacher," Elementary School Journal, XXXVIII (January, 1938), 373-79.

A comparison of arithmetical vocabularies of students and teachers.

Johnson, H. C., "The Effect of Instruction in Mathematical Vocabulary upon Problem-Solving in Arithmetic; Abstract of Doctor's Dissertation," Journal of Educational Research, XXXVIII (October, 1944), 97-110.

Value of vocabulary toward the improvement of problem-solving ability set forth.

O'Rourke, Everett V., and Cyrus D. Mead, "Vocabulary Difficulties of Five Textbooks in Third-Grade Arithmetic," Elementary School Journal, XLI (May, 1941), 683-91.

A study of vocabulary difficulties in certain textbooks.

Pressey, L. C., and M. K. Elam, "The Fundamental Vocabulary of Elementary-School Arithmetic," Elementary School Journal, XXXII (September, 1932), 46-50.

A study of the social usefulness of the arithmetic vocabulary in four investigations.

Russel, Ned M., "Arithmetical Concepts of Children," Journal of Educational Research, XXIX (May, 1936), 647-63.

Experiment shows important evolution of concepts at various age levels, pre-school and primary.

Willey, Roy DeVerl, "Vocabulary of Arithmetic in Elementary Grades," Elementary English Review, XIX (February, 1942), 64-66.

Arithmetic terms used in seventy-seven classrooms in California with frequencies of each work.

Wilson, Dorothy W., "Teaching Denominate Numbers and Measures," Educational Method, XVI (January, 1937), 177-81.

Gives data showing knowledge of various units of measure. Based on a group of 2,819 children and adults.

Arithmetic in Reading Material

Another important aspect of instruction is the correlation with other formal subjects such as reading. In this way opportunity

is provided for learning arithmetic in a more natural situation. Several studies have been conducted to determine how much opportunity for learning arithmetic concepts are provided in the reading material of elementary school children.

Gunderson, Agnes G., "Nature and Amount of Arithmetic in Readers for Grades I and II," Elementary School Journal, XXXVI (March, 1936), 527-40.

A study of arithmetical vocabulary in reading textbooks. Shows need for more correlation.

Semmelmeyer, M., "Extensional Methods in Dealing with Abstractions in Reading," Elementary School Journal, L (September, 1949), 28-36.

Meaning in arithmetic suggests actual significance of work and also the immediate social setting. Reading is very important in solving arithmetic problems.

Stemens, B. A., "Problem Solving in Arithmetic." See General Intelligence.

Treacy, J. P., "Relationship of Reading Skills to the Ability to Solve Arithmetic Problems," Journal of Educational Research, XXXVIII (October, 1944), 86-96.

A very good research article on the need for reading skills.

Wheeler, Lester R., and Viola D., "An Experimental Study in Learning to Read Numerals," The Mathematics Teacher, XXXIII (January, 1940), 25-31.

An experiment showing why children can learn to read numbers very quickly in the first grade.

Woody, Clifford, "Nature and Amount of Arithmetic in Types of Reading Material for the Elementary Schools," Educational Outlook, VI (May, 1932), 199-217.

Reviews related studies and presents the results of analyses of books and materials assigned in Grades III-VIII.

CHAPTER V

METHODS OF INSTRUCTION

There are a number of outstanding methods of teaching certain aspects of arithmetic, each of which has its own merit. Several methods have been studied recently in an effort to improve instruction. A review of the various methods of the past with their weaknesses together with an understanding of the prevailing methods of today should be of considerable worth to any teacher of mathematics.

Analysis of Teaching Methods

The many methods of instruction are compared in this section together with some methods that cannot be classified in the few major divisions following. This also includes some of the history of methods of instruction.

Brownell, W. A., "Frontiers of Educational Research in Arithmetic." See Weaknesses in Teaching Methods.

Brownell, W. A., "The Place of Meaning in the Teaching of Arithmetic," Elementary School Journal, XLVII (January, 1947), 256-65.

A study of the development of meaning in arithmetic and its value to the child.

Dickey, John W., "Arithmetic and Gestalt Psychology," Elementary School Journal, XXXIX (September, 1938), 46-53.

An interpretation of the significance of Gestalt psychology for the teaching of arithmetic.

Dickey, John W., "Readiness for Arithmetic," Elementary School Journal, XL (April, 1940), 592-98.

A presentation of the research on readiness in the light of the methods of instruction.

Knipp, Minnie B., "An Investigation of Experimental Studies Which Compare Methods of Teaching Arithmetic," Journal of Experimental Education, XIII (September, 1944), 23-30.

An analysis of sixty-four experiments reported between 1911 and 1940 pertaining to methods of teaching arithmetic.

LeBaron, W. A., "Study of Teachers Opinions in Methods of Teaching Arithmetic in the Elementary School," Journal of Educational Research, XLIII (September, 1949), 1-9.

Opinions of teachers about fifty percent in agreement with research and judgment of experts. A valuable study for determining need for study in workshops and similar undertakings.

Spitzer, Herbert F., "Abacus in the Teaching of Arithmetic," Elementary School Journal, XLII (February, 1942), 448-51.

The abacus is useful in exploring number meanings and was an indispenable part of the equipment of arithmeticians for over a thousand years.

Willey, Roy D., "Functional Arithmetic, 1893-1940; A Review of Typical Theoretical Discussion and the Theory to Which It Has Led," Journal of Educational Psychology, XXXIII (February, 1942), 105-17.

Shows decline of drill and the rise of arithmetic as a social study.

Self Instruction

A number of aids and devices have been used to make it possible for the student to learn certain habits in arithmetic without the constant aid of an instructor. Among these are special rules for

learning the fundamental combinations in addition and subtraction. Workbooks offer another opportunity for the student to work under his own power.

Andreen, Earl P., "A Study of Workbooks in Arithmetic," Journal of Educational Research, XXXII (October, 1938), 108-22.

An analysis of seventy-three workbooks in arithmetic followed by an extensive bibliography.

Brownell, W. A., "The Place of 'Crutches' in Instruction," Elementary School Journal, XXXIV (April, 1934), 607-19.

A timely article on the use of "crutches". Points out both values and dangers in the use of such devices.

Grossnickle, F. E., "Practice Material in the Estimation of the Quotient in Long Division Found in Current Arithmetic Workbooks." See Division.

Wilburn, D. Banks, "A Method of Self-Instruction for Learning the Easier Addition and Subtraction Combinations in Grade I," Elementary School Journal, XLII (January, 1942), 371-80.

A limited experiment with seventy-two pupils in grade one. Shows value of a simple method of self-instruction.

Social Utility Method of Instruction

It is commonly believed that students are more interested in problems taken from their own environment than they are in those which seem to have no meaning to them. However, it is not easily determined just what constitutes meaningfulness to children. At the same time there must not only be meaning but motivation. For this reason a few experiments have been conducted to determine how much social arithmetic is desirable.

Bramhall, Edwin W., "An Experimental Study of Two Types of Arithmetic Problems." See Development of Concepts.

Brownell, W. A., "When Is Arithmetic Meaningful?" Journal of Educational Research, XXXVIII (March, 1945), 481-98.

A presentation of the meaning theory in arithmetic, followed by an answer to some objections to teaching for meaning.

Brueckner, Leo J., "Social Problems as a Basis for a Vitalized Arithmetic Curriculum," Journal of Experimental Education, I (June, 1933), 320-22.

Presents a list of problems from the field of social science to be used in courses of study in arithmetic.

Conner, W. L., and G. C. Hawkins, "What Materials Are Most Useful to Children in Learning to Solve Problems?" Educational Method, XVI (October, 1936), 21-29.

Problems that children select are most useful.

Gorman, Frank H., "An Experiment in Integrating Seventh and Eighth Grade Science and Mathematics," Science Education, XXVII (December, 1943), 130-34.

A comparative study of effectiveness of teaching integrated mathematics and science. Little difference noted.

Grossnickle, F. E., "Concepts in Social Arithmetic for the Eighth-Grade Level," Journal of Educational Research, XXX (March, 1937), 475-88.

Examination of business concepts used in thirteen textbook series. These concepts were not adequately provided for.

MacLatchy, Josephine H., and Eugene Hummel, "Arithmetic with Understanding," Educational Research Bulletin, XXI (November 10, 1942), 227-38, 246.

A lower grade experiment in isolating number experiences and giving them preference over social uses.

Riess, Anita, "The Meaning of the 'Meaningful Teaching of Arithmetic'," Elementary School Journal, XLV (September, 1944), 23-32.

Very good to increase the child's insight into the problem.

Wahlstrom, Ebba L., "The Computational Arithmetic of Social Experiences of Third-Grade Children," Journal of Educational Research, XXX (October, 1936), 175-83.

Indicates that the arithmetic now taught in grade three exceeds the child's needs for addition and that subtraction covers most of the needs at this age.

Willey, Roy DeVerl, "A Study of the Use of Arithmetic in the Elementary Schools of Santa Clara County, California," Journal of Educational Research, XXXVI (January, 1943), 353-65.

Attempt to improve teaching by making it interesting and putting it in life situations.

Willey, Roy D., "Use of Arithmetic in the Out-of-School Life of Children," The Mathematics Teacher, XXXV (January, 1942), 23-28.

A story of the evolution of arithmetic in the out-of-school life of children beginning with the work of Binet in 1890.

Wilson, Guy M., "Criteria of the Written Problem in Arithmetic," Education, LIV (April, 1934), 457-60.

A good article showing the absurdity of many written problems.

Wilson, Guy M., "The Social Utility Theory as Applied to Arithmetic: Its Research Basis and Some of Its Implications," Journal of Educational Research, XLI (January, 1948), 321-37.

Social utility theory defended as a method of teaching arithmetic.

Instruction Through Co-curricular Activities

Just as there is evidence to support the social utility method of instruction, there is justification for teaching some aspects of arithmetic through the activity program. It is believed that a good teacher must reap the good from all these methods as they apply to the situation at hand.

Harap, Henry, and Charlotte E. Mapes, "The Learning of Decimals in an Arithmetic Activity Program," Journal of Educational Research, XXIX (May, 1936), 686-93.

A continuation of a very favorable experiment on learning decimals in socially real situations.

Harap, Henry, and Charlotte E. Mapes, "The Learning of Fundamentals in an Arithmetic Activity Program," Elementary School Journal, XXXIV (March, 1934), 515-25.

Presents the results of a study showing that fundamentals in fractions are efficiently learned when they arise in activity units in a random order without further practice.

Passehl, George, "Teaching Arithmetic Through Activity Units," Peabody Journal of Education, XXVII No. 3 (November, 1949), 148-52.

A classroom study showing that children can learn the fundamental operations in an informal atmosphere such as the activity program.

Wheeler, Lester R., "A Comparative Study of the Difficulty of Learning the Multiplication Combinations," Pedagogical Seminary and Journal of Genetic Psychology, LIX (September, 1941), 189-206.

Study of 342 children in the third grade who learned multiplication by playing a game.

Weakness in Teaching Methods

The above studies have all been of a positive nature and classified accordingly, but in this section entitled "weaknesses" an attempt has been made to include those studies which emphasize the low qualities of some methods. In this way it is possible to present a better view of the facts pertaining to methods of instruction.

Grossnickle, F. E., "Illustrations in Arithmetic Textbooks," Elementary School Journal, XLVII (October,

1946), 84-92.

There is need for more functional pictures with more problems accompanying each illustration.

Guiler, W. S., and Vernon Edwards, "An Experimental Study of Methods of Instruction in Computational Arithmetic," Elementary School Journal, XLIII (February, 1943), 353-60.

A controlled experiment of 412 pupils in the upper grades showing abilities in which they are weak. Individualized group instruction is more effective than conventional group instruction.

Henderson, Kenneth B., "Weaknesses in Arithmetic Teaching." See Computation, Drill and Review.

MacLatchy, Josephine H., "Seeing and Understanding in Number," Elementary School Journal, XLV (November, 1944), 144-52.

Shows that early deficiency in number continues unless pupils are definitely taught the facts and the processes of number.

Swenson, Esther J., "Difficulty Ratings of Addition Facts as Related to the Learning Method," Journal of Educational Research, XXXVIII (October, 1944), 81-85.

The method of teaching has a very important part in the learning process.

Tyler, Ralph W., "What the Schools Can Learn from the Training Programs of the Armed Forces," Elementary School Journal, XLV (May, 1945), 495-502.

Points out that the schools need to have clearer objectives for the students.

Wilson, Dorothy W., "Teaching Denominate Numbers and Measures," Educational Method, XVI (January, 1937), 177-81.

Shows that usual method is time wasted.

CHAPTER VI

CURRICULUM AND COURSES OF STUDY

Under the above title are gathered together those reports of research which deal with the evolution of the textbook showing the trends of emphasis and the best arrangement of materials. In earlier chapters materials and methods have been stressed, but in this chapter the emphasis is on content and arrangement to facilitate instruction in the classroom.

Trends on Emphasis in Textbooks

Through the years and in different schools of thought, emphasis has been placed on different parts of the arithmetic curricula. As these trends are carefully studied, we should arrive at a reasonable balance of stress on various topics. These studies cover a long period of time, in some cases going back as far as the textbooks of 1790, so the errors of the past should be quite evident. With this continued follow up, the trend should be more easily seen.

Metter, Harry L., "Trends in the Emphasis on Various Topics in Arithmetic Since 1860," Elementary School Journal, XXXIV (June, 1934), 767-75.

Lists items receiving more or less attention in fifty-five textbooks between 1860 and 1933.

Nyberg, Joseph A., "Arithmetic Now and Ten Years Ago," Journal of Business Education, XXV No. 5 (January, 1950), 15-16.

An interesting piece of classroom research showing the

need for a practical arithmetic course for those students not going beyond high school.

Smith, Henry L., and Merrill T. Eaton, "Analysis of Arithmetic Textbooks (First Period--1790 to 1820)," Bulletin of the School of Education, Vol. 18, No. 1, Bloomington, Indiana: Indiana University, 1942, 56 pp.

(Second Period--1821-1850, and Third Period--1851-1880), Vol. 18, No. 6, 1942, 112 pp.

(Fourth Period--1881-1910), Vol. 19, No. 4, 1943, 62 pp.

(Fifth Period--1911-1940), Vol. 19, No. 6, 1943, 45 pp.

Smith, Henry L., Merrill T. Eaton, and Kathleen Dogdale, "One Hundred Fifty Years of Arithmetic Textbooks," Bulletin of the School of Education, Indiana University, Vol. XXI, No. 1, Bloomington, Indiana: Bureau of Cooperative Research and Field Service, School of Education, Indiana University, 1945, 154 pp.

The best available factual data for a history of the development of arithmetic textbooks in America during the past 150 years. The present publication is culmination of a series of previous investigations.

Wheat, H. G., "Change and Trends in Arithmetic Since 1910," Elementary School Journal, XLVII (November, 1946), 134-44.

A good discussion of the trend toward "meaning" for the child.

Yorke, Gertrude C., "Three Studies on the Effect of Compulsory Metric Usage," Journal of Educational Research, XXXVII (January, 1944), 343-51.

Review of three studies, one her own, on the extent of usage of metric system in countries where it is legally compulsory.

Grouping and Grade Placement of Materials

Another important aspect of teaching arithmetic consists of grouping the material in such a way that it is easily understood. Some processes being much more complex than others,

curriculum designers and textbook writers should arrange the material according to the maturity of the pupils. As the responsibility falls back on the teacher to arrange the material in the best possible order, teachers will find this contribution vital to improved instruction.

Brownell, W. A., "Readiness and the Arithmetic Curriculum," Elementary School Journal, XXXVIII (January, 1938), 344-54.

Describes three sources of research supported by a lengthy bibliography.

Brueckner, L. J., "Typical Research Relating the Curriculum to Child Development," Elementary School Journal, XL (January, 1940), 348-65.

A study of the kinds of research on child development as applied to curriculum building.

Johnson, J. T., "Evaluation of Research on Gradation in the Field of Arithmetic," Journal of Educational Research, XXXVII (November, 1943), 163-73.

A review and a very extensive discussion of some of the results of research on grade placement in arithmetic.

Osborne, Raymond W., and Harry O. Gillet, "Mental Age Is Important Factor in Teaching Arithmetic," Nations Schools, XII (July, 1933), 19-24.

Shows optimum placement of topics.

Raths, Louis E., "The Grade-Placement of Addition and Subtraction of Fractions," Educational Research Bulletin, XI (January 20, 1932), 29-38.

Determined mental age at which seventy-five percent of children mastered certain processes.

Ruch, G. M., "Relative Difficulty of the One Hundred Multiplication Facts with Special Reference to Textbook Construction," See Multiplication.

Ulrich, Robert P., "Grade Placement of Computational Topics in Arithmetic," Elementary School Journal, XLII (September, 1941), 50-59.

Topics of several textbooks classified.

Washburne, Carlton, "One Reason Children Fail in Arithmetic: A Committee of Seven Investigation," Progressive Education, IX (March, 1932), 215-223.

Investigation shows failure is due to introducing material before children are ready for it.

Washburne, Carlton, "The Teaching of Linear Measure, Square Measure and Time," Elementary School Journal, XXXIX (March, 1939), 509-23.

An attempt to determine the mental-age level at which pupils may understand certain topics in arithmetic.

Wetherington, Julia, "Grade Placement of the Content of Arithmetic," Elementary School Journal, XXXVII (September, 1936), 41-46.

An analysis of ten courses of study.

Evaluation of Textbooks

Textbooks are often scrutinized in an effort to determine the books containing the best qualities. On the other hand, it may be desirable to analyze a given series of textbooks to determine to just what extent various functions are taken care of without reference to comparison with other books. The studies which follow are therefore of varying nature, but all bear upon an analysis or evaluation of textbooks.

Andreen, Earl P., "A Study of Workbooks in Arithmetic," See Self Instruction.

Brueckner, Leo J., and James A. Irving, "A Technique for Comparing the Difficulty of Problems in Textbooks

in Arithmetic," Elementary School Journal, XXXIII (December, 1932), 283-85.

Gives a measuring device for textbook committees on relative difficulty of problems.

Dexter, C. E., "Analysis of Written Problems in a Recent Arithmetic Series," Education, LXV (April, 1945), 488-90.

Experiment showing that many problems in textbooks are pointless.

Distad, H. W., "An Analysis of the Drill Provisions in Division of Decimals in Ten Arithmetic Series," Journal of Educational Research, XXVII (March, 1934), 509-23.

Analysis of ten series of texts published between 1925 and 1930. Gives relative amount of space given to each type of problem.

Grossnickle, F. E., "Concepts in Social Arithmetic for the Eighth-Grade Level." See Social Utility Method of Instruction.

Grossnickle, F. E., "Cues in Division Problems Given in Nine Representative Textbooks in Arithmetic." See Division.

Grossnickle, F. E., "Illustrations in Arithmetic Textbooks." See Weakness in Teaching Methods.

Gunderson, Agnes G., "Measurement in Arithmetic Textbooks for Grade Three," The Mathematics Teacher, XXXV (March, 1942), 117-21.

Analysis of twelve textbooks for space given to measurement.

Jaffee, Philip, "Technique for Evaluating Arithmetic Courses of Study," Journal of Experimental Education, V (June, 1937), 351-55.

Statements supported by research for setting up program.

James, Grace E., "A Study of the Isolated Problems of Four Arithmetic Test Series," Educational Method, XVI (January, 1937), 198-99.

Shows that many problems have little permanent value.

O'Rourke, Everett V., and Cyrus D. Mead, "Vocabulary Difficulties of Five Textbooks in Third-Grade Arithmetic." See Vocabulary of Arithmetic.

Stadtlander, Elizabeth, "Arithmetic Theories, A State Course of Study and Textbooks," Elementary School Journal, XLI (February, 1941), 438-53.

Analyzes five textbooks in relation to four prevailing theories of instruction.

CHAPTER VII

TESTING AND TESTING TECHNIQUES

Testing is being given much more consideration in the schools today than formerly. We have come a long way with the testing program, but much remains to be accomplished in this field. A wise teacher will want not only to experiment for finding better methods, but will want also to keep up with the results of research in this important program. It is believed that the citations following will be of great help to teachers in improving testing techniques.

Readiness Tests

Testing for readiness is one of the newer concepts in the testing program. It consists of determining the relative difficulty of various items and then correlating this with the mental and educational age of the pupils. Preliminary work in this field is already done by the textbook writers and courses of study, but with increased individual differences and other variable factors it is always necessary for teachers to consider readiness testing. Suggestions are presented here for devising and using readiness tests.

Brueckner, L. J., "The Development of Readiness Tests in Arithmetic," Journal of Educational Research, XXXIV (September, 1940), 15-20.

Illustrates procedures employed in devising and validating a readiness test.

Brueckner, L. J., "Development and Validation of an Arithmetic Readiness Test," Journal of Educational Research,

XL (March, 1947), 496-502.

Detailed analysis of a good readiness test.

Dickey, John W., "Readiness for Arithmetic." See Analysis of Teaching Methods.

Gunderson, Agnes G., "Number Concepts Held by Seven-Year-Olds," The Mathematics Teacher, XXXIII (January, 1940), 18-24.

A study of the concepts which children recall. Useful in determining readiness.

Koenker, R. H., "Arithmetic Readiness at the Kindergarten Level," See Concepts of Pre-School Children.

Olander, Herbert, and others, "Predicting Arithmetic Achievement," Journal of Educational Research, XLIII (September, 1949), 66-73.

Examination of several types of readiness tests together with the relationship between predicting success in arithmetic as compared with other subjects.

Souder, Hugh Cowan, "The Construction and Evaluation of Certain Readiness Tests in Common Fractions," Journal of Educational Research, XXXVII (October, 1943), 127-34.

Readiness tests proven to be valuable in instruction.

Wittich, Walter A., "A Number-Readiness Test," School Executive, LXI (March, 1942), 11-13.

Describes test for first grade for determining readiness for items.

Wulfing, Gretchen, "Maturation as a Factor in Learning," California Journal of Elementary Education, IV (February, 1936), 148-64.

First half deals with reading and last half with arithmetic readiness. Review of studies and bibliography.

Methods of Scoring

Scoring is certainly an important part of the total testing program. It has been shown that the method of scoring makes a

decided difference in the results to be obtained. Unless the score is as nearly accurate as possible and we can interpret it, our testing program is worthless. For this reason experiments have been conducted to help make the score more meaningful.

Grossnickle, F. E., "How a Method of Scoring a Test in Division Affects the Score." See Division.

Grossnickle, F. E., "Some Factors Affecting a Test Score in Division of Decimals," Journal of Educational Research, XXXVII (January, 1944), 338-42.

Test items should be arranged and relatively easy.

Wilson, G. M., and Gertrude L. Hanley, "For Basic Drill in Arithmetic, What Norm or Average is Satisfactory?" The Mathematics Teacher, XXXII (April, 1939), 175-78.

Drill work must be perfect in order to be of value to the student. Norms are unreliable for this work.

Measuring Understandings

This phase of measurement is quite new and is as yet not fully developed. It would certainly be wonderful if teachers of mathematics could be able to determine to what degree pupils understood and appreciated certain ideas in arithmetic. Oftentimes accuracy and other factors come between understanding and the test score.

Spache, G., "Test of Abilities in Arithmetic Reasoning," Elementary School Journal, XLVII (April, 1947), 442-45.

Analysis of a test for measuring the child's ability to reason. Correlation is positive between reasoning and other mathematical skills.

Sueltz, B. A., "Measuring the Newer Aspects of Functional Arithmetic," Elementary School Journal, XLVII (February, 1947), 323-30.

Improving the program in arithmetic by showing that old formal tests do not show the child's appreciation for the subject.

Reliability of Tests

Too often teachers give tests and assume that the results are infallible. While there are a large number of standardized tests on the market, it is often more practicable to give a good teacher-made test. Teachers then should be familiar with methods of determining validity and reliability of their tests.

Beattie, Louise, "Standardized Tests in Arithmetic," Educational Method, XVI (January, 1937), 175-76.

An analysis of several good arithmetic tests.

Brueckner, L. J., "Persistence of Error as a Factor in Diagnosis." See Remedial Measures.

Individual Differences and Achievement

Arithmetic is a subject in which individual differences is most pronounced. If children fail to see a given process, they are from that time on retarded. This makes it challenging to a teacher of arithmetic to see that the needs and abilities of all the pupils are cared for.

Cruickshank, W. M., "Arithmetic Ability of Mentally Retarded Children," Journal of Educational Research, XLII (November and December, 1948), 161-70, 279-88.

I. "Ability to Differentiate Extraneous Materials from Needed Arithmetic Facts." Retarded students of a given mental age are inferior in problem solving to normal children.

II. "Understanding Arithmetic Processes." Shows that mentally retarded children of a given mental age are

not consistent in their solutions.

Eaton, M. T., "Survey of the Achievement in Arithmetic of 11,348 Sixth-Grade Pupils in 486 Schools in Indiana," Indiana University School of Education Bulletin, XX (March, 1944), 1-62.

Extensive tabulation of results of Stanford Intermediate Arithmetic Test when given to about a quarter of the sixth-grade enrollment of Indiana.

Engelhart, Max D., "The Relative Contribution of Certain Factors to Individual Differences in Arithmetical Problem Solving Ability," Journal of Experimental Education, I (September, 1932), 19-27.

A good article of ability of varying kinds.

Long, Louis D., and Livingston Welch, "The Development of the Ability to Discriminate and Match Numbers," Pedagogical Seminary and Journal of Genetic Psychology, LIX (December, 1941), 377-87.

An experiment showing that ability to discriminate and match numbers increases with age.

Olander, Herbert, and others, "Predicting Arithmetic Achievement." See Readiness Tests.

Wilson, G. M., "A New Incidence of Learning for the Fundamentals of Arithmetic," Journal of Educational Research, XXXIII (February, 1940), 425-33.

Distribution of scores on the Wilson Addition Process Test.

Diagnostic Testing

Among the many kinds of testing is diagnostic testing, which consists of giving a test that is so especially designed that one is capable of discovering specific weaknesses not determined by ordinary means.

Brueckner, Leo J., "Persistence of Error as a Factor in Diagnosis," Education, LVI (November, 1935), 140-44.

Shows that pupils do not make the same errors repeatedly on the same test; therefore, one test is usually unreliable.

Brueckner, Leo J., and Mabel J. Hawkinson, "The Optimum Order of Arrangement of Items in a Diagnostic Test," Elementary School Journal, XXXIV (January, 1934), 351-57.

Tests should consist of four or more items of each type arranged in rows.

Brueckner, Leo J., and Mary Elwell, "Reliability of Diagnosis of Error in Multiplication of Fractions," Journal of Educational Research, XXVI (November, 1932), 175-85.

Shows that at least three examples of each type is necessary for reliable diagnosis.

Olander, Herbert T., "The Need for Diagnosis Testing," Elementary School Journal, XXXIII (June, 1933), 736-45.

Discusses effect on the score by method employed in problem solving.

Shores, J. H., "Why Children Dislike Arithmetic," Educational Administration and Supervision, XXXII (September, 1946), 357-63.

Review of previous research on this important question, with recommendations.

Williams, Claude L., and Ruth L. Whitaker, "Diagnosis of Arithmetic Difficulties," Elementary School Journal, XXXVII (April, 1937), 592-600.

A general report of individual diagnosis tests given in the upper grades.

Remedial Measures

After difficulties are located, it is necessary to take remedial measures to correct the deficiency. Sometimes that work comes in the form of review and drill in the fundamentals while on

other occasions it is special training which seems to lead to more indirect results. There is a difference of opinion as to the value of this work. No doubt this is an area in which more work needs to be done in order to show appreciable results.

Bemis, Eaton O., and W. C. Trow, "Remedial Arithmetic After Two Years," Journal of Educational Research, XXXV (February, 1942), 443-52.

Reports that remedial work is of less value than usually supposed.

Blair, Glenn Myers, "Remedial Arithmetic in Senior High School," The Mathematics Teacher, XXXVI (December, 1943), 346-50.

Discusses the subject on the basis of a previous research study in 166 high schools.

Christoffersen, R. O., and W. A. Wittich, "Navy Statistics and Mathematics Retention," See Computation, Review and Drill.

Guiler, W. S., and H. B. Hoffman, "Improving Computational Habits of Ninth Grade Pupils," Educational Administration and Supervision, XXIX (September, 1943), 345-56.

Experiment on remedial needs of a large number of students.

Jacqueline, Sister Mary, O. P., "An Experiment in Remedial Teaching in Arithmetic," Elementary School Journal, XLI (June, 1941), 748-55.

Remedial work should be interesting and very beneficial.

Randall, Joseph H., "Corrective Arithmetic in Junior High School," Educational Method, XVI (January, 1937), 182-85.

Results of experiment in remedial arithmetic in the junior high school.

CHAPTER VIII

GENERAL RESEARCH

This chapter contains reports of research which are either general in scope so that they could not be rightfully classified under any special topic in the field of arithmetic as well as those dealing with research itself. Many of the titles listed below are those of the nature of this compilation except that they cover only a very limited period of time or a special aspect of teaching arithmetic. A few of these studies proved useful to the author for the purpose of securing information about other research articles which were unavailable for review for this compilation. Several bibliographies contain a large number of titles which have been classified as quite important even though they do not fall into the category of true research. Should the reader be interested in some of those titles, they may be easily obtained from the following reports.

Brownell, William, "Frontiers of Educational Research in Arithmetic," Journal of Educational Research, XL (January, 1947), 373-80.

A history of research with list of problems and needs yet to be studied.

Brownell, William A., "Teaching of Mathematics in Grades I Through VI," Review of Educational Research, XV (October, 1945), 276-88.

Classified review of eighty-two studies of the previous three years.

Brownell, W. A., and F. E. Grossnickle, "Teaching of Mathematics in Grades I Through VI," Review of Educational Research, XII (October, 1942), 386-404.

A review of 142 articles and books of research and otherwise.

Brueckner, Leo J., and others, "Arithmetic," Review of Educational Research, VII (December, 1937), 453-63.

A review of research over a three year period.

Brueckner, Leo J., and others, "Special Methods and Psychology of the Elementary School Subjects: Arithmetic," Review of Educational Research, VII (December, 1937), 453-63; Arithmetic-545-47.

Fifty titles in arithmetic considered over a three year period.

Buswell, Guy T., or Leo J. Brueckner, "Selected References on Elementary School Instruction: III, Arithmetic," Elementary School Journal, XXXIV--L (November, 1933-49).

This annual report, usually by Buswell, carries a good bibliography of research and critical discussion related to instruction. While this is a very select list, the emphasis is on material to improve instruction and is not all true research.

Buswell, Guy T., "Outlook for Research in Arithmetic," Elementary School Journal, XLVII (January, 1947), 243-53.

Shows that much inferior material is coming out and challenges the field to recapture the spirit of true research.

Douglas, H. R., and others, "Mathematics," R view of Educational Research, II (February, 1932), 7-20.

A review of a few valuable studies in mathematics, grades seven to twelve.

Grossnickle, Foster E., "Still More Ado About Zero," Elementary School Journal, XXXIII (January, 1933), 358-64.

An analysis of previous articles by Dickey and Wheat regarding zero.

Knipy, M. B., "An Investigation of Experimental Studies which Compare Methods of Teaching Arithmetic," See Methods.

Moser, Harold E., "Teaching of Mathematics in Grades VII and VIII," Review of Educational Research, XV (October, 1945), 298-300.

A bibliography and review of twelve studies.

Schribner, Margaret, "A Summary of Arithmetic Research," Educational Administration and Supervision, XXXIV (October, 1948), 368-72. Part 2 of "Undergraduates Interpret Educational Research," by Lois Ada Wilson, 365-76.

A number of ideas are covered, ranging from order of difficulty of fundamentals to curriculum and social utility methods.

Wilson, Guy M., "Bibliography Written Problems in Arithmetic," Education, LIV (April, 1934), 480-83.

Unannotated bibliography of eighty-five references.

Woody, Clifford, "Nature and Amount of Arithmetic in Types of Reading Material for the Elementary Schools." See Arithmetic in Reading Materials.

Woody, Clifford, "Arithmetic," Review of Educational Research, V (February, 1935), 14-30.

The literature published from October 1, 1931 to July 1, 1934 is reviewed.

CHAPTER IX

CONCLUDING SUMMARY AND RECOMMENDATIONS

The number of articles appearing in print each year pertaining to arithmetic is increasing rapidly, but in spite of this fact, the number of these articles which bear the earmarks of research is by no means increasing. It has been discovered in reviewing numerous articles for this compilation that almost without exception the majority of the titles listed in the bibliographies bear dates prior to 1930. This fact indicates that fewer contributions of significant worth are appearing as well as the fact that the few studies which otherwise might be of real value are unnoticed amid the large volume of material of lesser importance.

It is believed that this present compilation should serve the purpose of keeping before the reader those experiments and findings which are new and of value to all concerned. While a detailed analysis of each piece of research is left to be accomplished by the coming generation, it is firmly believed that this present preliminary classification is of utmost importance to teachers and others interested in securing the best information regarding certain aspects of arithmetic. Since a large majority of the experiments covered in this study were conducted in the classroom situation, the results lend themselves most readily to classroom teaching.

It is the contention of the writer that compilations of this nature in other fields of endeavor should likewise be made. It is also believed that future study in the field of elementary arith-

metic would be justified. While completeness is striven for, it is many times impossible for the examiner to obtain and review all the articles which appear in less well known periodicals. For this reason the writer welcomes further research along these lines. At the same time, future study could keep the index current.

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A survey of research in arithmetic from early time to the end of the year, 1925, with complete bibliography.

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A dictionary of concepts pertaining to educational research and education in general.

Monroe, W. S., and M. D. Engelhart, A Critical Summary of Research Relating to the Teaching of Arithmetic. Urbana: University of Illinois, 1931. 115 pp.

Another survey of research in arithmetic following the one by Buswell and similar to it except that the emphasis is on teaching the subject.

Whitney, Frederick Lawson, The Elements of Research. New York: Prentice Hall, Inc., 1942. 497 pp.

A technical guide for research activity.

Periodicals

Buswell, G. T., "Outlook for Research in Arithmetic," Elementary School Journal, XLVII (January, 1947), 243-53.

Views the problems of research and challenges the field to recapture the spirit of true research.

California Journal of Elementary Education, Vol. 1 (1932) - Vol. 18 (1950). Sacremento: California Elementary School Principals Association, 1932-1950.

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