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A SIMPLE APPROACH TO DETERMINING THE SUPER-EFFICIENT INVESTMENT PORTFOLIO

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This paper presents a simple approach to Modern Portfolio Theory that makes the process more understandable and accessible to students. The methodology is a five-step process that begins with the calculation of mean returns, excess returns, betas, unsystematic risk, and excess returns over beta and then systematically ranks a set of funds to determine a super-efficient optimal portfolio. Data from the TIAA-CREF family of funds was employed in this study but the analysis can be applied to any distinct set of mutual funds. This linear optimization methodology, based on the Elton, Gruber, Brown and Goetzmann (2003) methodology, is a straightforward tool that can be used to teach students the underlying constructs of modern portfolio theory because it enables the students to learn by performing the analysis themselves. This research will also benefit mutual fund investors because it can be widely applied to help investors make better asset allocation decisions.

Since Markowitz first originated the concept of portfolio investing in the late 1950s, portfolio theory has exploded. Although Markowitz's mean-variance (MV) optimization is conceptually intuitive, the process is computationally complex. Despite its complexities, mean-variance optimization is a fundamental building block of portfolio theory, and it is important for students to understand both the theory and process behind portfolio optimization. As more sophisticated software and more powerful computer resources become available, the nature of optimization continues to increase in complexity. Professional financial consultants, risk analysts, and academics continue to push the frontiers of portfolio optimization knowledge and theory, causing the process to become even more complex from the point of view of a student learning the material for the first time.

Optimization techniques are difficult to simplify and teach in undergraduate finance and investments courses, which often means that finance students are not exposed to problems that require MV optimization. This paper illustrates how an existing simplified linear approach to optimization can be adapted to teach the constructs of fund selection and portfolio optimization using an integrated framework. The intent is to clearly explain the interrelationship of the three unifying constructs in portfolio optimization (the Markowitz Portfolio Model (MPM), Modern Portfolio Theory (MPT), and the Capital Asset Pricing Model (CAPM)) so that students of finance can better understand them in the context of a risk and expected return market environment. The unique feature of this study is the development of a simple mathematical process for evaluating both fund valuation and portfolio optimization simultaneously. Since the CAPM is a linear process, its risk and return relations can be established in linear association with each other, and it can be used to demonstrate fund evaluation due to its simplicity. Although Sharpe (1972) simplified Markowitz's MV optimization by using the

assumption of a risk-free security combined with the Sharpe Ratio maximization methodology, complexity is still inherent in the process, which presents a learning constraint to students.

To reduce complexity, the optimization construct can be demonstrated by utilizing the equivalent mean-beta computed optimal portfolio that Elton, Gruber, Brown and Goetzmann (2003) developed and explained. This single-index model methodology is particularly interesting because it enables the creation of the same optimal portfolio as the Sharpe 1972 methodology with minimal learning constraints. Elton et al (2003) demonstrate an optimal procedure for portfolio selection as an alternative method for the complexity of forecasting the covariance structure of returns. First, they present a ranking criterion to rank securities selected for the optimal portfolio and then discuss employing the ranking methodology to form the optimal portfolio. In addition, they present a demonstration of the linear process to use in forming the optimal portfolio. This method eliminates the need for non-linear optimization.

This paper replicates the demonstration from Elton et al (2003) using selected fund data in a format that is accessible to students and has the potential to enhance classroom learning though hands-on application. The unique contribution of this paper is the validation of the Elton et al (2003) methodology with the standard Sharpe (1972) maximization optimization technique, which is the benchmark for portfolio optimization.

PORTFOLIO THEORY

The Markowitz Portfolio Model (MPM), capital market theory (CMT), and fund pricing are the three sub-constructs that comprise MPT. Markowitz (1952) built his portfolio model on several assumptions with respect to investor behavior: (1) investors consider alternative investments based on a log-normal probability distribution of returns, (2) investors maximize one-period expected utility and their

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utility curves demonstrate diminishing marginal wealth, (3) investors estimate portfolio risk on the basis of variability or returns, (4) investors make investment decisions based solely on risk and expected return, (5) and for a given level of risk, investors prefer higher returns.

The MPM operates in risk-expected return space. The expected rate of return for a portfolio of investments is simply the weighted average of the respective rates of returns of the individual investments in the portfolio, and the Markowitz portfolio standard deviation is computed as a function of the weighted averages of the individual variances squared plus the weighted covariances between the funds in the portfolio. Equation 1 is known as the non-linear quadratic function Markowitz (1952) standard deviation:

$$\sigma_P^2 = \sum_{i=1}^{N}\left(X_i^2\sigma_i^2\right) + \sum_{i=1}^{N}\sum_{j=1: k \neq i}^{N}\left(X_i X_j \sigma_{ij}\right) \qquad (1)$$

where, X_i = the weight of variable i or j, σ_{ij} = the covariance of variable i with j, and σ_i^2 = the variance of variable i.

a. The Efficient Frontier. The efficient frontier is the combination of funds that has the maximum return with the minimum level of risk. The concave efficient frontier function is derived from the quadratic equation for the standard deviation as reported in Equation (1). An infinite number of weighted portfolios lie beneath this frontier. In risk-expected return space, the portfolios on the frontier dominate those below the frontier. The frontier is defined starting with a minimum variance portfolio at the bottom left of the arc and ending with a maximum return portfolio at the top right of the arc. The dominant or tangent portfolio lies between the two end points of the arc and has the attribute of maximum return for a minimum level of risk. Investor utility theory suggests that all investors would select this tangent portfolio because of this characteristic. MPT is a natural extension of this construct. All portfolios on this frontier are efficient but the tangent portfolio is "super efficient" because it provides the investor with the greatest return for the minimum amount of risk.

b. Modern Portfolio Theory. MPT extends the MPM by including the following additional assumptions concerning investors: (1) investors are Markowitz efficient and prefer target points on the efficient frontier, i.e., they can select unique levels of risks, (2) investors prefer the tangent portfolio, (3) investors can borrow at the risk-free rate, and (4) the capital markets are in equilibrium. Introduction of the risk-free asset is an important distinction of MPT. The risk-free asset is unique in that it (1) has no risk, (2) does not correlate with risky assets, and (3) lies on the expected return axis in risk and expected return space. The covariance of the risk-free asset with others in a portfolio of risky assets is zero. The standard deviation of a portfolio that combines the risk-free asset with risky assets is in linear proportion

with that of the risky asset portfolio. Given the addition of the risk-free asset, the ray, reported in Figure 1, can be drawn from the risk-free asset to the efficient frontier due to the linearity imposed by the condition that the risk-free asset has a zero standard deviation. When this ray is drawn tangent to, or extends to the frontier, it touches the super efficient portfolio, which again dominates all others in risk-expected return space. Both risk and expected return increase in a linear fashion along this ray, which becomes know as the capital allocation line (CAL). When the tangent portfolio is the market portfolio, it becomes the capital market line (CML), which becomes the new frontier. The CAL and CML are both linear models, and their expected returns are derived by adding the standard deviation of the risky portfolio times the market risk market premium of the risky portfolio to the risk-free asset return.

c. Capital Asset Pricing Model. A natural extension of the CML is the security market line (SML), introduced by Sharpe (1972), which values the expected return of individual funds in relationship to those that make up the market portfolio of all known funds. By extension, substituting the standard deviation of the combined portfolio of funds with the covariance of the individual fund with the market portfolio, the CAPM can evaluate these funds relative to the market portfolio. The expected return is the risk-free return plus a market risk premium multiplied by the covariance factor (beta). The actual return can be compared to the expected return to determine if the fund's risk-adjusted return is undervalued, properly valued, or overvalued relative to the market portfolio.

Optimization is a rare economic phenomenon that exists only when a portfolio is established in risk-expected return space so that the optimal fund mix results in efficient portfolio returns, or a portfolio that provides the highest return for a given amount of risk. Capital allocation across different fund classes is a key investor decision. Although theoretically appealing, the Markowitz methodology, which is quadratic in nature, presents computational constraints that make it difficult to apply in practice. Markowitz's critical insight was the realization that the co-movement of funds with each other is more important than individual fund characteristics when forming a portfolio of funds.

Elton, Gruber, and Padberg (1976) attempted to operationalize MPT, which although meant to be a practical tool, had primarily developed as a normative, theoretical construct. They suggested that the implementation difficulties for portfolio managers were caused by the following: (1) estimating the correlation matrices, (2) the time and costs in generating efficient quadratic portfolios, and (3) the necessity of understanding the risk-return tradeoffs expressed as co-variances and standard deviations.

This paper provides a practical application of the single-index linear model proposed by Elton, Gruber and Padberg (1977) that satisfies the original conditional boundaries of the Markowitz model. This single-index model uses the

relationship between a single market index and a single security in determining valuations and portfolio efficiency. Sharpe (1994) confirmed the ability of his Sharpe ratio to identify the optimal portfolio out of a population of random portfolios. The ability to evaluate multiple capital classes with speed and ease is an especially beneficial characteristic of this methodology. The empirical intent of this study is to determine the optimal portfolio combination of a designated portfolio of funds utilizing the portfolio optimization constructs discussed above and derived from MPT.

METHODOLOGY

Current Practitioner Limitations

A search of current software that allows finance student to perform optimization techniques returned a limited number of cost effective portfolio optimization services. The methodology proposed in this paper seeks to overcome these limitations by following the Elton, et al (2003) linear methodology to provide a user-friendly algebraic algorithm combined with an easily adaptable Microsoft Excel spreadsheet application. The intent is to have the student download monthly fund closing prices from TIAA-CREF.org or another mutual fund provider and monthly risk-free rates and market index closing prices from a source such as finance.yahoo.com.

The method presented in this paper does not require quadratic (Jackson and Staunton, 1999) or simplex optimization engines to obtain optimal convergence. Selection of the portfolio with the maximum Sharpe Ratio inherently accomplishes the same objective as the optimization technique if the portfolio holder chooses the portfolio with maximum return and minimum risk. Furthermore, this method allows for the inclusion of several funds in the analysis, which extends the two-asset example developed for classroom use by Arnold, Nail and Nixon (2006).

Fund Family Selection

The TIAA-CREF family of funds was selected for this evaluation. TIAA-CREF is a major provider of defined contribution retirement plans to the academic community. Rugh (2003) reports that TIAA-CREF is the largest pension provider in the U.S., managing $300 billion in total assets for more than two million individuals. The plans, which are provided through the employer of the respective investor, are referred to as retirement (or group retirement) annuity contracts. Investment amounts are dependent on contractual agreements between the employee and employer and are specified in the contract. Typically, contributions are made on a tax-deferred basis, which means that pre-tax dollars fund the accounts, but the employee pays taxes on withdrawals. The information was obtained from the TIAA-CREF website, www.tiaa-cref.org/performance/retirement/data/index.html. (Note: this web location is subject to being moved).

Simple Techniques for Determining an Optimal Portfolio

This model will use the simple techniques of Elton, et al (2003) to determine an optimal portfolio. Implicit in this process is the identification of undervalued funds according to the parameters of Sharpe's 1972 single-index CAPM. The identification of these funds is inherent in the Elton, et al. (1976) methodology, which makes it very clear why a fund does or does not enter into an optimal portfolio. The Elton methodology calculates mean returns, excess returns, betas, unsystematic risk, and excess returns over beta and then systematically ranks the included securities to determine an optimal portfolio. The elimination of overvalued funds and the optimization of undervalued funds are inherent in this ranking process. Thus, the CAPM is the benchmark for fund inclusion.

Review of Fund Valuation using the CAPM Methodology

The valuation process as determined by the CAPM is implicit in the Elton, et al (2003) optimization process. The CAPM is defined as $E(R) = \overline{R}_f + \beta_i(\overline{R}_m - \overline{R}_f)$, where the expected return equals to the average risk free return plus the product of the beta of fund i and the difference in the average market return minus the average risk free return. Here, fund valuation is simply determined by two factors: (1) the expected or market demanded return and (2) the realized return of the fund. If the realized return is greater (less) than the expected return, then the fund is underpriced (overpriced). If the two returns are equal, then the fund is properly priced.

Formation of the Optimal Portfolio

The portfolio formation proceeds by constructing a ranking system using a single computed number to determine the funds that should be included in the optimal portfolio. This process utilizes the assumption that the single-index model accurately describes the co-movement between these funds, and this description can be derived using a single number (beta) as the unit of measurement. Thus, the desirability of any fund is directly related to its excess return to beta (Treynor) ratio. The excess returns are measured by the difference between the expected return on the fund and the risk-free rate of interest, as measured by the 13-week Treasury bill. This ratio measures the additional return on the fund (beyond that offered by a risk-free asset) per unit of nondiversifiable risk. The form of this ratio makes it easy interpret and has led to its wide acceptance by investors as an explanation of the relationship between risk and reward. This is known as the Treynor measure (1965) and is illustrated in Equation 2:

$$\frac{\left(\overline{R}_i - \overline{R}_f\right)}{\beta_i} \tag{2}$$

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where, $\overline{R}_i$ = the expected return on fund i, $\overline{R}_f$ = the return on a risk-free fund, and β_i = the expected change in the rate of return on fund i associated with a 1% change in the market index rate of return.

This excess return to beta ratio (Ratio) ranking represents the desirability of any fund in a portfolio. Therefore, if one fund with a certain Ratio is included in an optimal portfolio, all others with a higher ratio would also be included. Conversely, if funds with higher Ratios are excluded, then funds with lower Ratios are excluded. Since the single-index model is assumed to represent the covariance structure of the funds for this study, funds will be included or excluded according to this Ratio. The quantity of funds selected will depend on a unique cut-off rate such that funds with higher ratios are included and those with lower Ratios are excluded.

Rules for Portfolio Inclusion

The following rules or constraints were used to determine which funds to include in the optimal portfolio: (1) determine the ratio for each fund in the fund family and rank from highest to lowest and (2) establish the optimal portfolio that consists of investing in all funds for which the ratio is greater than a particular cut-off point. This is done by ranking the securities in ascending order by the Ratio. Once ranked, these values are then compared to the cut-off rate, which is determined by inspection. Those funds with values equal to and above the cut-off rate are included and those less than the cut-off rate are excluded from the calculation process. Once the included securities are determined, an optimal portfolio mix of these funds is determined. The determination of the cutoff point for fund inclusion versus exclusion is described as part of the five-step process in the following section.

PRACTICUM

To illustrate the methodology, the five-step process (including sub-steps) used to construct an optimal TIAA-CREF variable annuity retirement fund portfolio is explained in this section. The TIAA-CREF funds include the TIAA Real Estate fund and the CREF Global, Social, Stock, Equity, Growth, Inflation-linked Bond, Money Market, and Bond Fund. The Russell 3000 (^RUA) was selected as the market index, and the Chicago Board Options Exchange (CBOE) index (^IRX), as the 13-Week Treasury-Bill. TIAA-CREF used the Russell 3000 as its benchmark non dividend-yielding index, and the TIAA-CREF funds do not produce dividends. The ^IRX Treasury-Bill Index was selected because it reports actual annual discount rates, which makes the conversion to monthly returns straightforward. Explanation of the five steps in the process follows:

Step 1

The 61-beginning of the month closing prices from December 2001 to December 2006 for ^RUA and ^IRX indices were collected from finance.yahoo.com and the TIAA-CREF retirement fund family data from www.tiaa-cref.org. The data was uploaded into Microsoft Excel and 60 months of lognormal returns for ^RUA and for each TIAA-CREF funds were computed using Equation 3:

$$\ln\left(\frac{S_t}{S_{t-1}}\right)*100 \tag{3}$$

where, S_t is the fund close price at time t and S_{t-1} is the price at t-1. The average risk-free rate, $\overline{R}_f$, was computed by converting annual discount rates to monthly discount rates by dividing each annual value by 12 and then averaging the monthly rates to determine $\overline{R}_f$ using Equation (4):

$$\overline{R}_f = \frac{\sum_{f=1}^{N} R_f \big/ 12}{N} \tag{4}$$

which is the average expected return on the risk-free fund where R_f = the average discount rate of the 13-week U.S. Treasury-Bill using the same date range of the funds.

Step 2

The average mean returns ($\overline{R}_i$) for each fund i and the market index, $\overline{R}_m$ are computed from the ln returns using Equation 5:

$$\overline{R}_{i,m} = \frac{\sum_{i=1}^{N} R_{i,m}}{N} \tag{5}$$

where $\overline{R}_i$ = the expected return on fund i and R_m = the average expected return on the market index m, ^RUA, and N = number of observations. These values were computed and reported in Range B3:B11 in Table 1.

b. Covariances of each fund with the market index, σ_{ei}^2, are computed as $\sigma_{ei}^2 = \sigma_i^2 - \beta_i^2 \sigma_m^2$, which is the variance of fund i minus the product of the beta squared of the fund and the variance of the market index. Note: Here, σ_{ei}^2 is considered unsystematic risk. (See Elton et al (2003) pp. 134-35 for an explanation and formal proof for the

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computation of σ_{ei}^2). Covariances for each variable with the Russell 3000 market index were computed and reported in the Range C3:C11 in Table 1.

c. Beta, $\beta_i = \dfrac{\sigma_{ei}^2}{\sigma_m^2}$, or systematic risk, is computed as

the covariance of the fund with the market divided by the variance of the market. The beta for each fund was computed and reported in the Range D3:D11 in Table 1.

d. Market Premium, M_P, is computed as $(\overline{R}_i - \overline{R}_f)$,

where $\overline{R}_i$ = the average return on fund i minus the average risk-free rate of return. These values were computed and reported in the Range E3:E11 in Table 1.

e. The Capital Asset Pricing Model (CAPM), R_{CAPM}, is

computed as $\overline{R}_f + M_P * \beta_i$, the average return of the risk-free asset plus the product of the market premium and the beta of asset i. These values were computed and reported in the Range F3:F11 in Table 1.

f. Market index variance, σ_M^2, is computed as in Equation 6:

$$\sigma_M^2 = \left(\frac{\sum_{i=1}^{N}(\overline{R}_i - \overline{R}_m)^2}{N} \right) \qquad (6)$$

where m = population mean of a market index and $\overline{R}_i$ = ith value of return R, and $\sum_{i=1}^{N}(R_i - R_m)^2$ = summation of all the squared differences between the R_i values and m. The computed value of 12.946 is reported in Table 1.

Step 3

a. X_1 is computed as $X_1 = \sigma_{ei}^2$, unsystematic risk, or the variance of a fund's movement that is not associated with the movement of the market index. These values are reported in the Range G3:G11 in Table 1.

b. X_2 is computed as $X_2 = \dfrac{\overline{R}_i - \overline{R}_f}{\beta_i}$, the market

premium of fund i divided beta. These values are reported in the Range H3:H11 in Table 1.

Step 4

This step begins by sorting X_2 in descending order so that the maximum row value is the starting number of the column. Then, the following steps are taken:

a. X_3 is computed as $X_3 = \dfrac{(\overline{R}_i - \overline{R}_f)\beta_i}{\sigma_{ei}^2}$, the market

premium multiplied by the beta of fund i divided by X_1. These values are reported in the Range I3:I7 in Table 1.

b. X_4 is computed as $X_4 = \dfrac{\beta_i^2}{\sigma_{ei}^2}$, the beta of fund i

squared, divided by X_1. These values are reported in the Range J3:J7 in Table 1.

c. X_5 is computed as $X_5 = \sum_{j=1}^{i} \dfrac{(\overline{R}_i - \overline{R}_f)\beta_i}{\sigma_{ej}^2}$, the sum of

the product of the market premium and the beta of fund i divided by the covariance of the fund with the market index. This function is cumulative where it begins with the initial row value and then cumulates subsequent values of X_3. These values are reported in the Range K3:K7 in Table 1.

d. X_6 is computed as $\sum_{j=1}^{i} \dfrac{\beta_i^2}{\sigma_{ej}^2}$, which is the cumulative

sum of X_4 from $j = 1$ to i. This function is cumulative where it begins with the initial row value and then cumulates subsequent values of X_4. These values are reported in the Range L3:L7 in Table 1.

e. Compute C*:

1. Calculate C_i, which takes the form as presented in Equation 7:

$$C_i = \frac{\sigma_m^2 \sum_{j=1}^{i} \dfrac{(\overline{R}_j - \overline{R}_f)\beta_j}{\sigma_{ej}^2}}{1 + \sigma_m^2 \sum_{j=1}^{i}\left(\dfrac{\beta_j^2}{\sigma_{ej}^2} \right)} \qquad (7)$$

where σ_m^2 is the variance of the market index and σ_{ej}^2 is the unsystematic risk. This function is the product of the variance of the market index and the sum of X_5 divided by one plus the product of σ_m^2 and X_6. These values are reported in the Range M3:M7 in Table 1.

2. Determine C*, which is computed by evaluating column M for the fund with the highest C_i value. The data in column M has already been ordered using the rules for portfolio inclusion as described previously. C* is the designated break point for fund inclusion in the optimal portfolio. Funds that were equal to and above C* in column M were retained, and funds that were below C* were eliminated from the portfolio. For this set of data, the highest fund value, C*, was 0.688. The C_i values are reported in Table 1.

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Step 5 – Construct Optimal Portfolio

Compute Z_i and X_i. After C^* is determined and the funds for retention in the optimal portfolio are identified, the investment percentage of the retained funds is calculated by computing Z_i as shown in Equation 8:

$$Z_i = \frac{\beta_i}{\sigma_{ei}^2}\left(\frac{(\overline{R}_i - \overline{R}_f)}{\beta_i} - C^*\right) \tag{8}$$

where Z_i is the product of the beta of fund i divided by X_1 multiplied by the difference between X_2 and C^*, which is the highest C_i value. The individual Z_i values are reported in the Range N3:N7 and then summed to Z_i, which is reported in cell N8 in Table 1.

Identification of the Tangent Portfolio. X_i, the percentage to invest in each fund, is computed using Equation 9:

$$X_i = \frac{Z_i}{\sum_{included} Z_j} \tag{9}$$

where each Z_i, for $i = 1$ to n, is divided by Zj, the sum of the individual Z_i values. Equation (9) determines the relative investment in each fund, where the weights on the individual funds must sum to one to ensure 100% investment.

It is important to note that this analysis relies on the assumption that the past variance-covariance structure of these funds will continue in the future. The X_i values are reported in the Range O3:O7 in Table 1, while the sum of the X_i's is reported in cell O8. To illustrate these calculations, Z_{i1} was computed as $2.548 = .009/(0.237*(64.201-0.688))$ and reported in Cell N3. The value X_{i1} is computed as $0.946 = 2.548/2.694$ and reported in Cell O3 in Table 1. As suggested by Elton, et al (2003), this solution will be identical to the results achieved by quadratic programming.

Table 1: Practicum Results

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1		Step 2.a.	Step 2.c.	Step 2.d.	Step 2.e.	Step 2. f.	Step 3.a.	Step 3.b.	Step 4. a.	Step 4. b.	Step 4. c.	Step 4. d.	Step 4. e.1.	Step 5. a.	Step 5. b.
2	Variable	$\overline{R}_i$	σ_{ei}^2	β_i	M_p	R_{CAPM}	X_1	X_3	X_i	X_4	X_5	X_6	C_i	Z_i	X_i
3	TIAA Real Estate	0.812	0.123	0.009	0.610	0.205	0.237	64.201	0.024	0.000	0.024	0.000	0.315	2.548	94.58%
4	CREF Global	0.733	2.574	0.199	0.530	0.259	10.844	2.666	0.010	0.004	0.034	0.004	0.420	0.036	1.35%
5	CREF Social	0.524	2.094	0.162	0.322	0.248	3.333	1.990	0.016	0.008	0.050	0.012	0.559	0.063	2.35%
6	CREF Stock	0.665	3.035	0.234	0.462	0.269	9.833	1.971	0.011	0.006	0.061	0.017	0.642	0.031	1.14%
7	CREF Equity	0.542	3.528	0.273	0.340	0.280	9.487	1.247	0.010	0.008	0.071	0.025	0.688	0.016	0.60%
8	CREF Growth	0.090	4.778	0.369	-0.113	0.307	12.046	-0.306	Step 4. e. 2.				0.688	2.694	100.00%
9	CREF IL-Bond	0.550	-1.296	-0.100	0.347	0.174	3.127	-3.468							
10	CREF Money	0.180	0.020	0.002	-0.023	0.203	0.016	-14.712							
11	CREF Bond	0.408	-0.166	-0.013	0.205	0.199	1.260	-15.982							

Table 1 Notes:

(1) Table 1 provides the accumulation of the empirical results of the practicum exercise. It begins with Step 2 by calculating average returns for each fund and ends with Step 5 by computing the optimal portfolio weights for the selected funds of the TIAA-CREF fund.

(2) Step 2 consists of the following:

Step 2.a., the calculation of the average market (0.485) and risk-free returns (0.203), and the average returns of the funds of the TIAA-CREF funds, $\overline{R}_i$;

Step 2.b., the calculation of the market variance (12.946);

Step 2.c., calculation of the covariances of the funds with the market index, σ_{ei}^2;

Step 2.d., calculations of betas, β_i; and

Step 2.e., calculation of market premiums, M_p; and

Step 2.f., calculation of the CAPMs.

Next, Step 3 consists of the following:

Step 3.a., the calculation of X_1 and

Step 3.b., the calculation of X_2.

Next, Step 4 consists of the following:

Step 4.a., the calculation of X_3,

Step 4.b., the calculation of X_4,

Step 4.c., the calculation of X_5,

Step 4.d., the calculation of X_6,

Step 4.e.1., the calculation of C_i, and

Step 4.e.2., the calculation of the maximum value of C_i.

Next, Step 5 consists of the following:

Step 5.a., the calculation of Z_i and

Step 5.b., the calculation of X_i, which includes the summing of the Z_i's.

(3) TIAA IL-Bond is the TIAA inflation-linked bond fund.

The identification of the super-efficient or optimal portfolio mix for the TIAA-CREF funds is shown in Figure 1, which is integrated with the efficient frontier. The fund weights of the super efficient portfolio, using data from December 2001 to December 2006, are as follows: TIAA Real Estate: 94.58 percent, CREF Social: 2.35 percent, CREF Global: 1.35 percent, CREF Stock: 1.14 percent, and CREF Equity: 0.60 percent, for a total of 100 percent. Dates

for the first set of weights Fixed. We used data from August 2003 to August 2008 to validate the stability of the model and we obtained similar results. The weights using the 2003-2008 data are the following: TIAA Real Estate: 94.88 percent, CREF Social: 2.13 percent, CREF Global: 1.34 percent, CREF Stock: 1.11 percent, and CREF Equity: 0.53 percent, for a total of 100 percent.

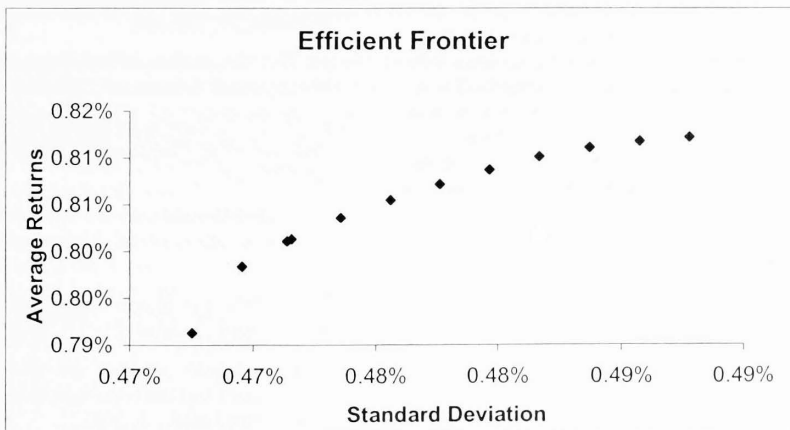## Figure 1: Efficient Frontier

Figure 1: Efficient Frontier

Figure 1 Notes:
(1) This is the efficient frontier with the tangent portfolio shown.
(2) The percentages are the portfolio weighted average monthly returns on the Y-axis and the Markowitz (1952) derived standard deviations on the X-axis as calculated using the single-index linear model.
(3) The ray from the risk-free asset to the super efficient portfolio is not shown due to scaling issues. The average risk-free return is 0.203%.

(4) This Excel based super efficient portfolio weighted the TIAA Real Estate 94.58%, the CREF Social 2.35%, CREF Global 1.35%, CREF Stock 1.14%, and CREF Equity 0.60%, for a total of 100%.

To validate that this optimal mix is indeed the super efficient portfolio, the single-model non-linear optimization technique was performed, and the results were identical to those reported in Table 1. The optimization algorithm is reported in Appendix A.

SUMMARY AND CONCLUSIONS

This paper provides straightforward computational results for an efficient TIAA-CREF portfolio optimal mix of funds using the Elton, etc. al., (2003) linear optimization method. Using the constructs of MPM, MPT, and CAPM, a teaching tool that enables the seamless integration of these constructs is provided. The paper includes a general blended discussion of these three constructs, briefly highlights current research in MV optimization and fund valuation methods, values funds from a selected fund family, establishes a portfolio mix of these undervalued funds, linearly optimizes this portfolio, and reports the findings. A non-linear optimization algorithm that validates the linear model, which is a unique contribution to the field of investment literature, is also included. In addition to the initial optimization results, the portfolio was re-optimized using current data, and the re-optimization yielded similar results, which suggests model stabilization.

The intent of this paper was to outline a model that operationalizes a simplistic approach to MPT. This methodology is a five-step process that includes calculating mean returns, excess returns, betas, unsystematic risk, excess returns over beta, and then systematically ranking the TIAA-CREF family of funds to determine a super-efficient optimal portfolio. The elimination of overvalued funds and the optimization of undervalued funds is inherent in this ranking process. This linear optimization methodology is an easy-to-use linear tool that can be used to model the underlying MPT constructs. While this optimization process is based entirely on historical data, students can replicate this study using projected data sets. When using historical data, it is important to keep in mind that past performance may not be indicative of future results.

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APPENDIX A

The Elton, etc. (1976) methodology was operationalized using the following optimization algorithm or Sharpe Ratio (1966) calculated as in Equation 10:

$$\theta_{Max} = \frac{\overline{R}_P - \overline{R}_f}{\sigma_P} \qquad (10)$$

where,

$$\overline{R}_P = \alpha_P + \beta_P * \overline{R}_M ,$$

$$\alpha_P = \sum_{i=1}^{N} w_i \alpha_i' , \text{ and}$$

$$\beta_P = \sum_{i=1}^{N} w_i \beta_i'$$

and where,

$$\sigma_P = \sqrt{\sigma_P^2} = \sqrt{\sigma_M^2 * \beta_P^2 + \sigma_P'^2} ,$$

$$\beta_P = \sum_{i=1}^{N} w_i \beta_i' ,$$

$$\sigma_P^2 = \sum_{i=1}^{N} w_i \sigma_i'^2 ,$$

$$\beta_i' = w_i * \beta_i , \text{ and}$$

$$\sigma'^2_i = w_i * \sigma_i^2 .$$

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