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Bridging the Mathematics Achievement Gap in Struggling Urban Schools

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Academic Leadership Journal

Despite massive and costly efforts nation-wide, the majority of high-poverty schools –especially those with predominantly minority populations – are failing their students. The majority of failing schools are in urban areas. The Council of the Great City Schools, a coalition of 65 of the nation’s largest urban public school systems, conducted a national study to determine the impact of school reform on urban schools. Its findings show that urban school achievement is far below the national average in math. Only 10.7% of these school systems are at or near state averages in math (Duvall, 2004).

As a result of school reform efforts, many school districts report that gains have been made in students’ math scores in the elementary years. But America’s high-poverty middle and high schools remain in crisis. Beyond the elementary years, students in the nation’s high-poverty schools are failing. For example, despite years of school reform, math achievement in Detroit has declined in the last five years. For example, 25% of Detroit’s high school students scored proficiently on statewide math tests in 2004 – as compared with only 16% in the most recent reports. (www.schoolmatters.com). At the end of high school, Hispanic students’ reading and mathematical skills are comparable to those of white 13 year olds (Keller and Garcia, 2006). And in many areas, the achievement gap between black and white students has actually widened in recent years (Loveless, 2007).

Learning to Learn Math

There is promise in a new mathematics curriculum developed by Marcia Heiman, the originator of Learning to Learn[®] (LTL), a proven learning improvement system (Heiman, 1987). Initially developed and used for educationally/economically disadvantaged college students, LTL was the only program that the U.S. Department of Education’s Joint Dissemination Review Panel (JDRP) or the Program Effectiveness Panel (PEP) ever found to have valid studies showing significant, long-term impacts on (1) student achievement across all subject matter areas and (2) retention through graduation for educationally/economically disadvantaged college students (U. S. Department of Education, 1995). Studies showed significant impacts of LTL on all measures when treatment groups were compared with control groups (grade point average: $p < .05$; number of credits completed: $p < .018$; retention through graduation: $p < .001$).

In recent years LTL strategies have been field-tested in high-poverty schools at the middle and high school levels. The Learning to Learn Math curriculum, available for grades 6 –9, is a product of that work. LTL Math uses many of the same core strategies for learning math that were shown to be effective in the earlier study. However, it has taken us years to effectively adapt the LTL strategies to high-poverty, low-achieving public schools. In struggling public schools, we needed to find solutions to both lower skill levels and low student motivation. Our challenge in bringing LTL to public schools has been to adapt effective learning strategies to classrooms in struggling schools. For this reason, this curriculum includes classroom management and reward systems designed to keep students on task in class.

As data from our field research studies show, we have successfully made this transition: We have

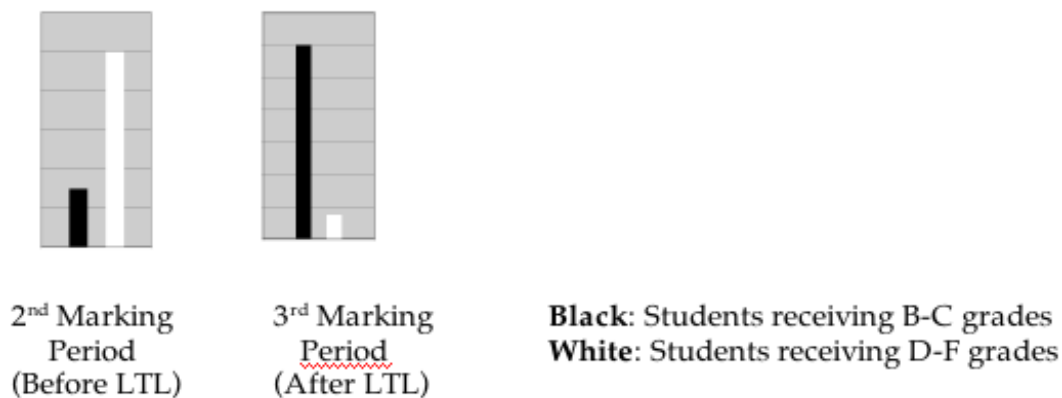
learned how to bring LTL Math to struggling public schools.

LTL Math's Impacts on Students in Struggling Schools

In all of our research pilots, we have used the group as its own control. Most of our pilots have been of a small sample size (30-50 students). Our largest pilot, with 240 students, was in Benton Harbor, MI. Results are strong, and – unlike other attempts at math instructional improvement – with LTL Math we have found no differences in the math achievement of students of different races.

Here are some examples of the impacts of Learning to Learn Math.

Boston, MA Learning to Learn Algebra was first piloted with a class of high behavior-problem, transient 9th graders at Boston's Brighton High School in spring, 2004. Most of the students in the pilot had failed every Algebra test since September. (Many of these students had also failed district-wide math tests in the 6th and 8th grades.) Here are the results of the project's first six weeks:



Since these students had been failing Algebra since the beginning of the school year, they started LTL Algebra with the first skills in the Algebra 1 curriculum – on March 8, 2004. By June 8, these students scored as well on Boston's district-wide Algebra 1 final exam as did students at their school who had earned B and C grades since September.

These students – who had experienced years of math failure – continued to do well in subsequent math classes. The students' Guidance Counselor reported that success with LTL Algebra changed the students' self-perception, and their work in other classes improved. Nearly half of these students – identified as high-risk for school drop-out when LTL Algebra was introduced in 2004 – are now completing their junior year of college.

Benton Harbor , Michigan

In Benton Harbor, previously-failing students succeeded in Algebra with this new curriculum. Like many high-poverty cities nationally, student failure rates in Benton Harbor are high – especially in math. In 2007, only 5% of the high school students in Benton Harbor scored proficiently on the state mathematics tests. Yet in 2008-09, the 240 ninth graders at the MLK Freshman Academy experienced high levels of achievement with Learning to Learn Algebra.

When they started LTL Algebra in the first week of December, 2008, none of these students were able to multiply minus signs accurately. By Mid-March, 2009 – within three months of starting LTL Algebra – all of these students were graphing linear equations with 100% accuracy – creating and solving their own complex problems, and accurately translating their math into their own words. (Examples of this work are included in the Appendix of this paper)

Broward County, FL

In the spring of 2008, an LTL Math pilot was conducted with the lowest-achieving Level 1 & 2 sixth graders at Deerfield Beach Middle School in Broward County, Florida. Three weeks after beginning LTL Math 6, these students took a district-wide benchmark test in math. 60% of the students performed significantly better on this test than they did on a similar state-wide test given before LTL.

Lawrence, MA

The Health & Human Services High School in Lawrence, Massachusetts is a struggling school in the state's lowest-performing school district. The school has begun using Learning to Learn Algebra with 9th graders who had failed the state's 8th-grade Math MCAS. With LTL Algebra, all but two of these students scored A – C grades on the first district-wide test.

Other LTL Math Pilots

We have conducted numerous small LTL Math pilots during the past five years, all of them in struggling urban schools. Some of the cities where we piloted LTL Math include: Pittsburgh, PA, Detroit, MI, Rochester, NY, Baton Rouge, LA, Fall River, MA, Jackson, MI, Bridgeport, CT, and Fresno, CA.

Our first aim in adapting LTL for public school use was to increase on-task behavior during classroom hours. We found that our 100% Wall Chart had a major impact on increasing on-task behavior, as seen in these emails received from teachers at Harding High School in Bridgeport, CT. Harding High has the highest rate of school violence in Bridgeport, and only 4.5% of the students perform proficiently in math on state-wide tests.

“I've applied the LTL to my 7th period class and they were motivated. Once I put the 1st 100% up, everyone else rushed to get their recognition.”

– David Wallace, Algebra 1 teacher

“I must admit that [LTL] has motivated 95% of my students to be consistent in their work.” – Garfield Pilliner, Algebra 1 teacher

“Students are really speeding through the book. Some of the students are competing against each other to have the most 100's on the wall chart, keeping track of each other, etc. Several students have taken the first test and have done quite well.”

– Brad Charbonneau, Algebra 1 teacher

Process Research

In our field research studies we have used the group as its own control. Our research goal has been to determine if this curriculum is effective in authentic educational settings, and to make immediate changes (from data-based feedback) where needed to ensure student success. Field research studies are a form of process research, where the group is used as its own control, and the researcher continually looks for feedback, modifying the intervention where needed. Accurate formative evaluation is essential. Using the group as its own control is central: If progress is not empirically demonstrated with the target students, immediate changes are made based on formative evaluation data. We feel strongly that kind of process research is required in order to develop effective curricula for students with a long history of academic failure.

Theoretical Foundation

Our over-riding theory of change is an integrated system of instructional materials and practices derived from both cognitive psychology and behavioral learning theory.

There are two theoretical underpinnings of this work: The work of cognitive psychologists, principally Benjamin Bloom, Arthur Whimbey, and Jack Lochhead; and behaviorists, principally B. F. Skinner and Fred Keller.

Mastery Learning

While the concept of mastery learning was used in some American schools as early as the 1920s, the practice was not sustained. Skinner's programmed instruction brought back the practice in the 1950s – and to some extent this practice is sustained today through individualized instruction in computer-based programs. Bloom (Bloom, 1968) is now widely regarded as the principal theoretician of mastery learning – especially when this practice is implemented in classroom settings, where programmed instruction is not used. Bloom argued that mastery learning, used correctly, can bring 95% of students to the learning level previously reached by only 5% of students. In addition, he wrote that mastery learning has side benefits that may be as important as the information learned: That is, when previously-struggling students begin to learn well, they are more motivated to learn –so they engage in more effort, more energetically, across all learning tasks (Fehlen, 1976). Bloom maintained that the greatest impact of mastery learning would be in math and science, since these fields have structured and sequential knowledge bases (Guskey & Gates, 1986).

Keller's use of Personalized System of Instruction (PSI) combined mastery learning with reinforcement theory (Fox, 2001). The components of PSI are as follows: (1) mastery learning is required because course knowledge is cumulative; self-pacing is needed since students learn at different rates; (2) lecturing is not seen as practical because it blocks students from progressing at their own pace; (3) proctoring is needed in order to provide students with immediate feedback, additional, individualized instruction, and social reinforcement. Typically, students in a PSI course individually work through small units in a study guide or textbook. Students receive assessment when they complete a given unit – and individual students do not proceed to the next unit until they demonstrate mastery of the current unit.

Mastery Learning and Learning to Learn Math

Mastery Learning is a central component of LTL Math. The PSI system is reflected in nearly all aspects of LTL Math classroom instruction: (1) The teacher serves mainly as a facilitator; (2) Students move to

the next skill set only after completing a given skill set with 100% accuracy; and (3) Students work at their own pace, taking tests when they are ready to do so.

Math-Word Translation

During the 1980s, Marcia Heiman had frequent and extensive contact with Arthur Whimbey and Jack Lochhead, two cognitive psychologists who shared Heiman's interest in bridging the achievement gap for low-income, primarily minority students. Whimbey (1975) had conducted research building on Bloom's early work (Bloom, 1960), demonstrating the efficacy of verbalizing non-verbal problem-solving tasks in raising performance levels of students who scored poorly on aptitude tests. Lochhead, trained as a physicist, was working independently with under-achieving students at the University of Massachusetts-Amherst. He developed a methodology where students talked aloud the process of solving math-based physics problems. Whimbey and Lochhead then collaborated on an instructional text designed to help students improve their problem-solving skills in math-based curricula (Whimbey and Lochhead, 1985). Whimbey and Lochhead theorized that the impact of their intervention was due to the dyadic exchange of pair problem-solving, where pairs of students alternated as critical listener and problem-solver. However, Heiman was convinced that the learning gains resulting from Whimbey and Lochhead's work stemmed from the verbalization of math steps –that students understood math with clarity once they could explain it in their own words. Heiman was working with individual students in a program for first-generation college, largely minority students at Boston College. She found that freshmen who were initially failing Calculus were able to earn A's and B's as a final course grade if they systematically translated their math into words shortly after their first (failing) hourly exam. (Heiman, 1988).

It is this single-subject research that led to the development of Learning to Learn Math.

LTL Math in the Classroom

Mastery Learning, Math-Verbal Translation, & Reinforcement System

1. This intervention consists of (1) curricular materials (Learning to Learn Math, grades 6-9) that meet state standards of all of target schools and (2) a set of metacognitive strategies and reinforcement practices that inform instruction on a daily basis and can be externally measured on a daily basis.
2. In LTL Math, the math is broken down into small skill sets. Every math step is translated into words. Further, students use the textbook's examples as models to create and solve their own problems, then translate every step of their math into words. Verbalizing math allows students to have an ongoing internal dialogue with information they learn – a core learning practice that is not available if the information is presented in a language (math) that the student neither speaks nor understands. Success in math-verbal translation provides students with a strategy for learning math. As shown in our Brighton High School pilot, we have some evidence that this practice may have a long-term impact on math learning.
3. Since our target population is also deficient in reading, except for required mathematics terms, this curriculum is written at the fourth-grade reading level.
4. The student does not proceed to the next skill set until he/she creates, solves, and "translates" math

problems with 100% accuracy; most students achieve accuracy on the first trial, which usually takes one class period.¹ Each student's work is entered into a composition book provided by the project. These composition books are the students' math notebooks. By the end of the school year, each LTL student creates and solves at least two hundred pages worth of math-verbal problems in his/her math notebook.

5. A wall chart is posted. When a student completes a skill set with 100% accuracy, 100% is posted on the wall chart next to his name, providing immediate reinforcement. As a result of the posted wall charts, ongoing formative evaluation is integral to this curriculum.

6. Weekly team games provide group reinforcement for individual student success – as well as reinforcement of specific-skill learning.

7. Students who accurately complete their work early become Student Mentors, helping to check other students' work.

Additional Aspects of LTL Math

- No repetitive drill practice. Repetition is a continuous part of this curriculum – but not the meaningless, drill-practice repetition found in traditional math classrooms. In this curriculum, practice is part of the inherent sequential structure of math: When a student uses any part of these sub-skills, he/she must provide an accurate verbal translation of each math step.

- Frequent word problems. Each sub-topic, sequentially taught, culminates in related word problems. The sub-skills provide essential scaffolding for the word problems, which students learn to break up into component parts and re-build with their own words, step by step.

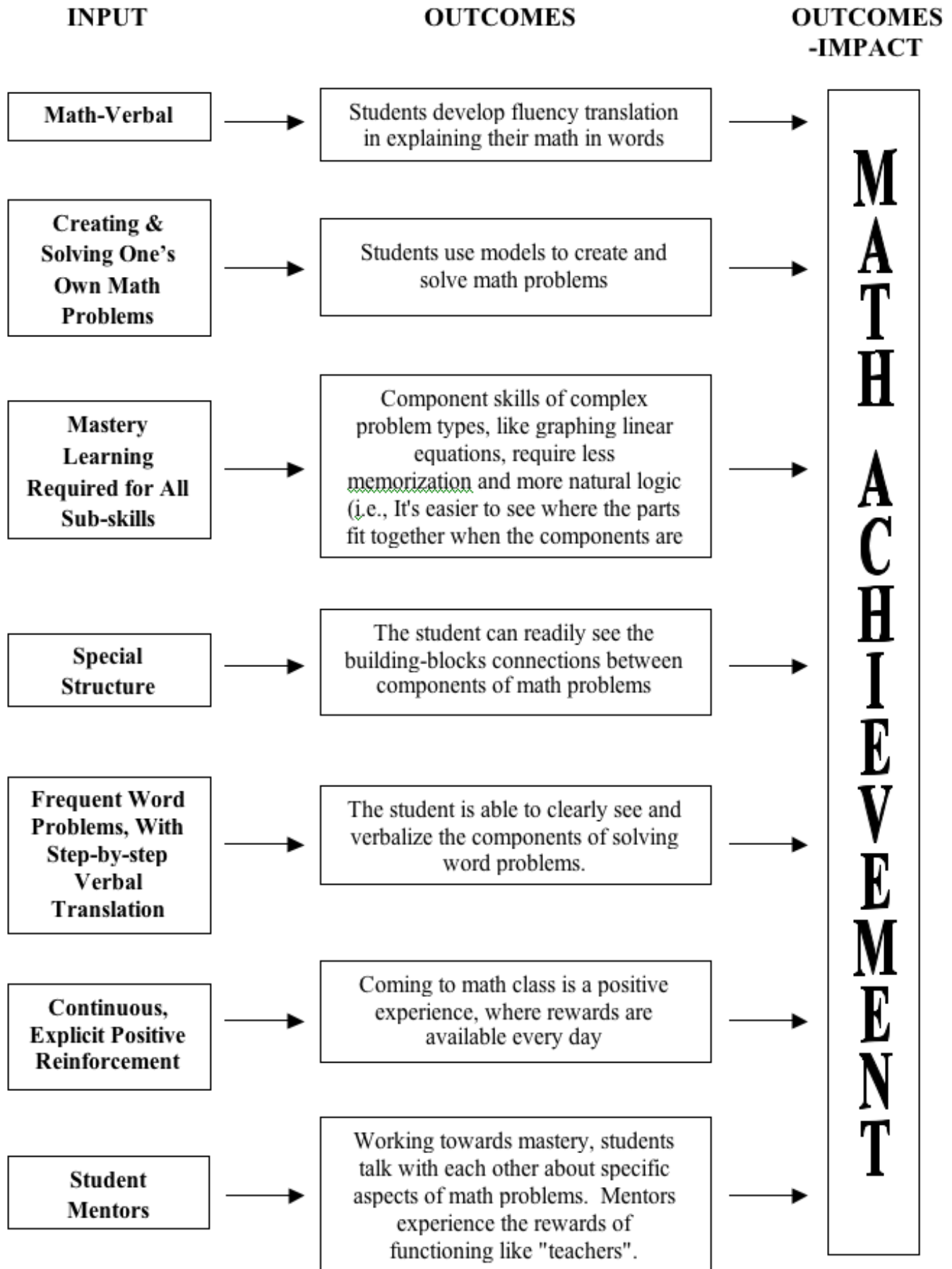
- Mastery learning and appropriate step-size. Mastery learning is essential in basic skills development. Since math is cumulative, a passing grade is not sufficient for success in mathematics. In a sequentially-structured, building-blocks discipline like math, if students acquire only 70% mastery of one math skill, and only 70% of a second math skill, they will be unable to perform later problems requiring an accurate combination of the first two skills.

- Sequential structure. LTL Math is entirely sequential. Along with the focus on mastery learning and math-verbal “translation”, this structure allows the student to follow the logic of math naturally, much as he would the steps of a complex video game. By contrast, traditional math instruction in the United States is often not sequential and does not call for mastery learning. Instead, there is “spiraling”, a frequent repetition of previously-introduced concepts.

¹ If this process is to be effective, instructional goals must be attainable by all students: In a math, one cannot expect 100% accuracy if the learner has unaddressed sub-skill deficits. In the first field research trials, the curriculum was continuously revised, based on students' ability to complete each skill set with 100% accuracy. If one student could not perform a given skill set with 100% accuracy, that skill set was revised and re-presented to the student. By 2007-08, materials had been refined such that not one student in a field-test pilot was “stuck” – unable to achieve 100% accuracy in a skill set.

The Logic Model below shows the overall structure and function of this curriculum:

LOGIC MODEL



Current “Cutting Edge” Attempts at Bridging the Achievement Gap in Mathematics – And Why They Do Not Solve the Problem

Other Attempts at Math-Verbal Translation

Many teachers have already heard about the importance of math-verbal translation. In countless professional training seminars, teachers have been asked to encourage students to explain math problems in their own words.

Math-verbal translation exercises have been included in other (non-LTL) math curricula designed for struggling inner-city students. For example, until recently, most high schools in Chicago had second-hour classes for students needing additional help with 9th grade math. The district-assigned curriculum required the use of materials where students were asked to construct manipulable representations of math formulas and also translate their math into words. Consulting at one of these schools in 2005, Marcia Heiman attended these classes in one struggling school, and examined a semester’s worth of student work in the students’ daily workbooks. As evidenced by this student output, using this (non- LTL Math) curriculum, only one student in 30 was able to translate his/her math into words.

The failure of that attempt to teach students math-verbal translation is reflected in students’ math scores in Chicago. Thousands of students used this curriculum. Yet in Chicago, the percentage of high school students achieving in math has remained at 29% since 2003; and only 17% of the city’s African American high school students achieve math proficiency, as compared with 63% of their white peers.²

That is, more is needed than teachers’ hearing about math-verbal translation : An effective curriculum assuring students’ ability to perform this activity is needed. An example of the impact of LTL Math on students’ ability to use math-verbal translation may be seen in the Benton Harbor Student Samples, in the Appendix of this paper.

Tutoring Projects

Across the country, there are a variety of costly interventions based on subject-matter tutoring in math – none of which effectively bridge the achievement gap in mathematics. For example, the University of California-San Diego has recently established a multi-purpose Community Technology Center serving Gompers Secondary School in the San Diego Unified School District. Made possible by a \$280,000 grant awarded to UCSD by the U.S. Department of Education, the project includes live video online tutoring by UCSD undergraduates, and tutoring and mentoring in computer engineering by Qualcomm employee volunteers. Whether online or live, tutoring is manpower-intensive, costly, and is not an effective model for improving the learning of hundreds of thousands of failing students. Further, content-area tutoring does not produce independent learning.

² By contrast, in every LTL Math pilot, there has been no difference between the academic performance of students of color and Caucasian students. Learning success does not derive from the concept of math-verbal translation, but rather in the details of instructional design.

The Algebra Project (TAP)

The Algebra Project (TAP) is a learning improvement system that attempts to improve student performance in mathematical reasoning. TAP provides professional training to middle school teachers in an “experientially based curricular process, and inquiry based, cooperative strategies for classroom management and teaching.” (www.algebra.org, 2009) This project is implemented in at least 30 schools across the nation, in both classrooms and after-school programs.

TAP focuses on helping middle school students understand some basic concepts in Algebra through experientially-based modules, like a unit on . Designed for sixth graders, this module focuses on ratio and proportion in the context of making lemonade concentrate of various strengths . However, unlike LTL Math , TAP does not attempt to teach rigorous mathematics skills, like solving two-step linear equations or finding square roots; and it does not teach the 9th grade Algebra 1 curriculum – which includes procedures such as graphing linear equations, using substitution in systems of equations, and multiplying polynomials.

Everyday Math

Everyday Math is another program that attempts to help students succeed in Algebra by introducing them to a number of real-life experiences that reflect simple Algebraic concepts. For example, there is a lesson in which students create and analyze mystery line plots. In this lesson, “Students draw line plots to represent data about themselves. Then they determine which of their ‘mystery plots’ represents which information. They also identify landmarks of the data – minimum, maximum, median, mode, and range – for each line plot.” (Tripatlas.com, 2008). This is an initially teacher-centered lesson where students later work together in small groups to create a visual representation – a “real-life” example – of an idea in math.

The central idea underlying is that once Algebra is “meaningful” to students, they will be ready to learn it. However, this curriculum presents only real-life analogies to math. Unlike LTL Math, Everyday Math does not actually teach the complex math problem-solving and the tightly sequential, cumulative skills that comprise Pre-Algebra and Algebra. And there is no evidence that Everyday Math impacts students’ achievement in middle- or high-school math.

Singapore Math

Singapore Math is based on textbooks from the national curriculum of [Singapore](#). Attention was called to these materials after a 2003 study showed Singapore at the top of the world in 4th and 8th grade mathematics.

Singapore Math calls for:

- frequent use of word problems and strategies for solving them, rather than repetitive drill;
- the use of bar-models in teaching problem-solving (a form of pre-algebra) rather than the trial-and-error methods found in most standard U.S. math textbooks; and
- an approach that builds upon succeeding levels, and assumes that what was taught need not be

taught again. By contrast, the typical U.S. curriculum uses a “spiral” method of teaching that revisits at each level, so that each school year begins with a review of [place value](#).

Singapore Math was piloted in four schools in the Montgomery County Public School system. Results were mixed, and three of the four pilot schools soon dropped the program. Alan Ginsburg, director of the policy and program studies service at the U.S. Department of Education, examined outcomes data at these four sites in the U.S. where the Singapore approach had been adopted. Only two of those sites achieved results superior to control groups, and those two sites received additional staff development (Leinhard and Ginsburg, 2007). The most frequent criticism of this program is the extensive teacher training it requires. The Singapore textbooks contain no narrative explanation of how a procedure or concept works; instead, there are problems and questions accompanied by pictures that provide hints about what is going on. This methodology requires continual inventive instruction on the part of the teacher. As a result, many teachers find it complex and difficult to teach. In addition, the MCPS schools that tested Singapore Math, while ethnically diverse, serve middle-class, suburban students. In fact, there is no evidence that Singapore Math is effective in bridging the achievement gap in high-poverty schools, where our nation’s largest skill deficits are found.

Carnegie Learning, Inc. is an [American](#) publisher of [math](#) curriculum for [middle school](#), [high school](#), and post-secondary students. The company uses a blended approach, with a textbook and software (called The [Cognitive Tutor](#)) for each subject.

The Cognitive Tutor curriculum is based on the [ACT-R](#) (Adaptive Control of Thought–Rational) theory of learning, memory and performance. ACT-R’s most important assumption is that human knowledge can be divided into two kinds of representations: [declarative](#) and [procedural](#). Declarative memory is the aspect of human [memory](#) that stores [facts](#). Procedural memory is the long-term [memory](#) of [skills](#) and procedures.

In practice, in the Cognitive Tutor, an ACT-R-based computer program attempts to “guess” the difficulties that students may have and provides focused help. In layman’s language, the Cognitive Tutor program identifies a student’s math skill deficits and provides practice in those areas until the student develops proficiency. That is, the Cognitive Tutor provides individualized drill practice.

There are two major deficits in the approach offered by the Cognitive Tutor. First, it does not teach students how to learn math. It provides no generic strategies for learning math that help students acquire later math skills – skills beyond a given Cognitive Tutor lesson. Second – and most importantly, the Cognitive Tutor does not affect students’ scores on math exams. A report delivered to Congress by the U.S. Department of Education reported that math and reading software produce no better test results than conventional teaching methods (Trotter, 2007). That is, they do not bridge the achievement gap found in high-poverty schools.

In summary, Learning to Learn Math is the only curriculum with a strong promise of bridging the mathematics achievement gap in high-poverty middle and high schools.

Appendix: Benton Harbor Student Samples

Benton Harbor, MI, is a high-poverty (92%), high-minority (97%) city whose students are the lowest-performing in Michigan: In 2007 only 5% of their high school students earned proficiency on state math

tests. When the 9th graders started Learning to Learn Algebra in fall 2008, most of the students could accurately multiply minus signs. Within three months, nearly all 240 of the Benton Harbor 9th graders were graphing linear equations and translating their math into words with 100% accuracy. By then, every student's math notebook contained at least 70 pages of math/word problems like those below.

In the student sample below, a student used the "Point Slope Formula" to graph a linear equation. This is an example of the curriculum's central learning activity, where every day students create new problems, solve them, and translate their math into words.

Student Sample 1: Using The Point-Slope Formula

The math here has many "hidden" complexities. For example, when the student wrote "DP", he was writing an abbreviation for himself that he'd used the Distributive Property, where he multiplied the coefficient, 2, times both numbers in the parentheses.

Skill Set 66

Math	Words
$(0, 3) = \text{point}$	given
$2 = \text{slope}$	
$y - y_1 = m(x - x_1)$	Point-Slope Formula
$y - 3 = 2(x - 0)$	Substitute
$y - 3 = 2(x - 0)$	Given
$y - 3 = 2x - 0$	Simplify right side by multiplying (DP)
$y - 3 + 3 = 2x - 0 + 3$	Add 3 to both sides to solve for y.
$y = 2x + 3$	Slope-Intercept form

Student Sample 2: Finding the Slope of the Line

- Students were asked to:
- Find the slope of the line that passes through $(2, -3)$ and $(2, 3)$.
 - Label the rise (Δy) and the run (Δx) on your graph.
 - Show all math steps and translate your math into words.

Skill Mastery 5.02 100%

$x \mid 4$
 $y \mid -3$
 $y \mid 3$

Graph showing a vertical line passing through $(2, 3)$ and $(2, -3)$. Rise = 6, Run = 0. $\frac{6}{0} = \text{undefined}$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope Formula
 $= \frac{3 - (-3)}{2 - 2}$ Substitute coordinates into formula
 $= \frac{6}{0}$ Simplify
 $= \text{undefined}$ can't divide by zero

This student wrote that the slope is "undefined" because you can't divide by zero. So she's explaining what she's doing. Also, it's moving to see a in a math problem – right next to the 100%.

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