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A study of the n-body problem in celestial mechanics for college students

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A STUDY OF THE N-BODY PROBLEM
IN CELESTIAL MECHANICS FOR
COLLEGE STUDENTS

by
John Arsu-Swanzy

A Thesis

Submitted in partial fulfillment of the requirement of the
Master of Arts Degree in the Graduate Division
of Rowan University in Mathematics
Education 1997

Approved by: ______________________
John Sooy

Date Approved: May 5, 1997

DEDICATION

Dedicated to the memory of my paternal grandmother,

MAGA DAχOkε SODZI (1879-1983)
OF AŋLOGA, Eveland

who knew the value of
education without
herself being
lettered.

A VISIONARY WHO LIVED TO BE 103 YEARS OLD.

ABSTRACT

John Atsu-Swanzy, A Study of the N-Body Problem in Celestial Mechanics for College Students, 1997, J. Sooy, Mathematics Education.

The purpose of the study is to investigate simple solutions of the many-body problem otherwise known as the n-body problem. The study focuses on elementary solutions of the n-body problem that can be understood by undergraduate students and college preparatory students of applied mathematics.

Historical origins of the problems were traced to the ancient Egyptians Babylonians, and Greeks. Further development and interest dated back to the time of Copernicus, Galileo, Kepler, and finally to Newton who proposed its modern form.

Analytical and numerical solutions of specific n-body problems were solved to demonstrate solvability of certain type of n-body problems. Analytical solutions for velocities of the masses were calculated. Numerical methods written in the QB computer language generate solutions of specific n-body problems. Two- and three-body numerical solutions were solved to demonstrate solvability by writing a computer algorithm using the Euler or Runge-Kutta method. The numerical solution displays the trajectories of the masses in graphics and the behavior the masses are shown. No formula has been developed for determining general solutions of n-body problems in this research.

In conclusion, there are simple solutions for certain n-body problems. The subject can be studied at the undergraduate and college preparatory level.

MINI ABSTRACT

John Atsu-Swanzy, A Study of the N-Body Problem in Celestial Mechanics for College Students, 1997, J. Sooy, Mathematics Education.

The primary purpose of this study was to generate interest in the n-body problem at the undergraduate level. Simple solutions of specific n-body problems were provided. Numerical and analytical solutions were presented at a level that the undergraduate and the college preparatory students can comprehend. The study concluded that there are simple solutions of the n-body problem that the undergraduate and college prep students can understand.

ACKNOWLEDGMENTS

I would like to thank my advisor, Dr. John Sooy, for his guidance, advice, and general help throughout the writing of this thesis. My special thanks go to Dr. Thomas J. Osler, whose assistance, encouragement, and support throughout my graduate study is unparalleled. He has become my mentor, friend, neighbor, role model, and general advisor.

There are others who supported me in several ways. My rector, Reverend Dr. Ralph Firneno, who gave me special permission to leave school early. He gave me moral, financial, and spiritual support. I thank the SMA and SVD missionaries who guided me in the early years of my education for the special kindness and support they bestowed on us all. I thank Mrs. Joan Cioffi of Rowan University for her support in various ways to me and my family. I thank Ms. Mary Lou Papa, my supervisor, for her support and encouragement. I am eternally grateful to Lory and Tom Cicalese for typing the scripts and offering support and encouragement. I am solely responsible for any shortcomings of this thesis.

Finally, I thank my wife, Gina, and my children Yvonne, Dela, and Edem for their patience for bearing with me being away from them in the evening to pursue higher education. Those whom I have not mentioned; I greatly appreciate your contributions. I simply say to all of you, "Akpε Na Mi Kata."

TABLE OF CONTENTS

CHAPTER 1

Introduction to the Study

Introduction

This chapter introduces the n-body problem and the difficulty of finding simple solutions to the problem. The chapter includes background information, the statement of the problem, the significance of the study, limitations of the study, definitions of terms, and procedures for implementing the study.

Background Information

The n-body problem is not a new topic in mathematics. Although the question was first posed by Isaac Newton in 1687, the problem is older than Stonehenge. It assumed its modern form when Newton (1687) proposed this problem in his philosophical essays of *Philosophiae Naturalis Principia Mathematica*. Mathematicians of all centuries since Newton's days have been preoccupied with finding solutions to the n-body problem. Different methods of solutions have been presented at mathematical forums, but to my knowledge, no collection of simple solutions has yet been presented. The n-body problem has many applications in today's space exploration. For example, computer-generated solutions of trajectories of celestial bodies and satellites' motions are common applications of the n-body problem.

The study of how celestial bodies move under gravitational forces is an old problem for ancient and modern man. This subject area traces its origins to the earliest reaches of mankind. It is very easy to preempt that the study of the many bodies, referred to as n-body problem, is the "world's oldest profession." "If it isn't the oldest, then most surely it is the second oldest" (Saari 1992).

Statement of the Problem

The purpose of the study is to present the case of simple solutions of the n-body problem. These simple solutions are for the understanding of the college undergraduate students and upperclassmen in high schools in mathematics and physics programs.

Significance of the Study

The researcher has recently examined several texts and research articles on the n-body problem. This examination indicates there are no simple solutions of the n-body problem for high school and undergraduate students. Almost all relevant researches on the n-body problem are written for an advanced audience. The simple solutions of the n-body problem for the undergraduate students will generate early interest in the n-body problem. Therefore, there exists a need for simple analytical and numerical solutions of the n-body problem.

Limitations of the Study

The study has considerable limitations among which:

- There is scarcity of relevant research and literature that addresses the n-body problem at this particular level;
- there is no single textbook devoted solely to simple solutions of the n-body problem;
- the researcher has observed that there is a lack of awareness among high school teachers about the existence of the n-body problem;
- the scope of the study is limited to undergraduate mathematics and physics major students;
- the students must have as a prerequisite a physics course that gives them considerable knowledge of Kepler's Laws of Gravitational Motion and Newton's Law of Motion.
- The solution is limited to cases of mass of bodies; n = {2, 3, 4, 5, 6 . . . , n} with a fixed body at the center or not at the center.

Definition of Terms

Acceleration: the rate of change of velocity "v" with respect to time "t" (dv/tv = a). For a particle moving along a curved path, the velocity is directed along the tangent to the path (James and James, 1959).

Angular Acceleration (α): the time rate of change of angular velocity (ω), $\alpha = d\omega/dt$.

Angular Velocity: the rate of change of the angle between a fixed line and the line joining the moving particle to a fixed point. It is centripetal, normal, and tangential components of acceleration (James and James).

Centripetal Force: the force which restrains a body, in motion, from going in a straight line. It is directed towards the center of curvature (James and James).

Centrifugal Force: the force which a mass m, constraining to move in a path, exerts on the constraint in a direction along the radius of curvature. It is equal and opposite to centripetal force. When the path is a circle of radius r, the magnitude of this force is $r\omega^2 = v^2/r$ (James and James).

Universal Constant of Gravity: $G = 6.67 \times 10^{-11}$ Nm^2kg^{-2} (James and James)

Law of Universal Gravitation: the law of attraction. Formulated by Newton in accordance with which two particles of masses M and m interact so that the force of attraction is proportional to the product of the masses and varies inversely as the square of the distance between the particles. In symbols, $F = GMm/r^2$ where r is the distance between the particles and G is the universal constant of gravitation (James and James).

Procedures

The first phase is to read and analyze related research and literature on the n-body problem. This will involve searching the on-line libraries, the Rowan University Library, and libraries of colleges in close proximity of the researcher, including the Universities of Pennsylvania, Delaware, Temple, Drexel, Rutgers, and Princeton.

The second phase is to discuss with Dr. T.J. Osler, a professor of applied mathematics at Rowan University, on the research outcomes and work with him on simple solutions to the specific n-body problems.

CHAPTER 2

Review of Related Literature and Research

Introduction

The primary purpose of this chapter is to introduce relevant research and literature to support the study. There is very little published research to the n-body problem that is relevant to the undergraduate or academically excelled high school student. However, there is adequate relevant research and literature that is suitable for advanced course work on the n-body problem. There are good textbooks on celestial mechanics which appropriately treat the subject of gravitational motions of heavenly bodies. These books are cited in the relevant literature section of this chapter. The chapter also introduces the historical development of astronomy from the times of ancient Egyptians, Babylonians, and the Greeks with their preoccupation of movement of heavenly bodies. During the Renaissance and the Age of Reason periods, scholars like Copernicus, Galileo, Kepler, Newton, and others were also able to put together a scientific explanation of the movements of the heavenly bodies instead of some of the superstitious explanations given by earlier ancient astronomers.

Historical Background

It is difficult to credit a particular group or person for the early astronomical discoveries. Probably, these discoveries were made in stages and rediscovered and then spread slowly. According to Rogers (1960), urban civilizations developed in several great river valleys 5,000 or more years ago. Much applied science had already been discovered a few thousand years before this time. Artificial irrigation of crops by canals and ditches, the plow, sailboat, and wheeled vehicles; use of animals for power; production, use of copper, bricks, glazes; and finally, a solar calendar; writing; a number system; and the use of bronze had been developed too by the ancient Babylonians, Egyptians, Sumerians, and Chaldeans.

By 2000 B.C., there were towns flourishing with extensive trade. They had excellent commerical arithmetic that was almost algebra. They could solve problems leading to quadratic, even cubic, equations. The value of $\sqrt{2}$ was accurately known, but π was taken to be roughly 3. They used similar triangles and knew Pythagoras' rule. They had good weights and measure, sundials and water-clocks. Near the equator, the sun's path did not provide a good working basis for the calendar, so the moon was much easier as the basis for the calendar. The early Babylonians based their calendar on new moons but had to reduce that into a solar calendar of seasons for agriculture and seasonal religious ceremonies. Careful observations of the moon and the sun were required. A careful mathematical system for predicting the motions of the sun and the moon was developed. Belief in omens took a prophetic turn, and astrology took a stronghold on the people (Rogers, 1960).

The ancient Egyptians were the forerunners for astronomical studies. They engaged in fewer wars and devoted more time to spiritual and intellectual developments. The ancient Egyptians lived peacefully and with more "friendly" gods. Their gods did not encourage wars but devoted their priestly class to mathematics, astronomy, and astrology. Ancient Egyptian mathematicians served on magic and commerce, recorded corn stalls, divided property, and built an exact pyramid. Egyptian astronomy was simpler than Babylonians' astronomy. They had an efficient solar year of twelve months of thirty days each plus five extra days; so they paid less attention to eclipses of the moon and the planets. Two thousand years before Christ was born, they recorded accurate planetary observations (Rogers, 1960).

Next and closely related to the ancient Egyptians were the Greek city-states. Scholars and priests travelled between the two lands exchanging knowledge. The city of Alexandria in Egypt was named after Alexander the Great of Greece (Rogers, 1960). About some 3,000 years ago, Greek civilization began to evolve. It produced mathematicians, scientists, and philosophers who made such important advances. Thales (600 B.C.) was a founder of Greek science and philosophy. He collected geometrical knowledge perhaps from the Egyptians and began to reduce geometry to a system of principles and deductions; that was the beginning of science that Euclid later was to bring into fruition. Euclid set forth an explanation of the universe in his book *Elements* (Euclid~323 B.C.). After this period, not much activity on celestial mechanics had been recorded until the time of the Renaissance. The earlier advances made by Thales, Ptolemy, and Aristotle became the prevailing views on celestial motions. The Renaissance, which was at its peak in the seventeenth century, spread all over what is known today as western Europe. It brought in many advances in scientific, technological, and economic leadership of the English Channel. Scholars began to pay less attention to what was already written and place more reliance on their own observations. This period was characterized by an eagerness to experiment and to determine how things happen. The appearance of William Gibler's *De Magnete* in 1600, the first treatise on physical science, to Newton's *Optiks* in 1704 brought in a new awakening in the spirit of inquiry. In between the *De Magnete* and the *Optiks* came Kepler's theory on planetary motions. Kepler built on earlier works of Tycho Brahe (1546-1601) and refuted the prevailing Aristotelian concept of "ideal circular motions" and pushed forward an explanation for elliptical orbits. Kepler then formulated the Laws of Terrestrial Motion in 1619. The period 1637 to 1687 was regarded as the fountainhead of modern mathematics. The first date, 1637, alludes to the publication of Rene Decartes' *La Geometric* and the second, 1687, to Newton's *Principia Mathematica*. The two works had a considerable influence on mathematical thoughts of the period and influenced problem solving in mathematics. Prior to this, the mathematician-physicist-astronomer Galileo Galilei (1564-1642) laid a permanent foundation for modern science. He was credited with the invention of the telescope for observation of heavenly bodies. He observed four satellites revolving around the planet Jupiter. This was a dramatic disproof of the existing Aristotelian notion of the earth as the center of all astronomical motions. His observations were published in a twenty-nine page booklet entitled *Sidereus Nuncius* (The Staffy Messenger). This was the beginning of the recognition of the existence of unknown stars, the Milky Way, and the rugged surface of the moon. Galileo's discoveries were so startling that some professors of his time refused to look into his telescope for fear of seeing in it things that would discredit the infallibility of Aristotle, Ptolemy, and the Church. Galileo's publications of Copernican views made him an enemy of the church and his position at Padua, a stronghold of Aristotelianism, untenable. The Aristotelian conception of the universe which was elaborated by Ptolemy placed the earth at the center of the universe. At increasing distances from it came nine crystalline and concentric spheres. The first seven carried the sun, the moon, and the five known planets, and the fixed stars were attached to the eighth one, often call the "firmament." On the outside lay the ninth sphere, known as the "premium mobile" and representing the Prime Mover or God. Beyond this was no matter, no space, nothing at all. It makes the universe finite, one contained within the premium mobile. From the standpoint of Aristotle, the earth was the main body in the universe, and everything else existed for its sake and the sake of its inhabitants. In the new cosmology produced by Nicolaus Copernicus (1473-1543), the sun changed places with the earth, the sun became the central body, and the earth merely one of several planets revolving around the sun. It was Galileo who advocated the Copernican view and was tried by the Inquisition and imprisoned by the church because his teachings were against the authority of the church. Galileo was given a papal apology posthumously in 1992 by Pope John Paul II. Johannes Kepler (1571-1630) was taught the Copernican theory of the universe secretly by Michael Masslin, a professor of mathematics at the University of Tubingen in southern Germany. Kepler published his astronomical observations in the *Mysterium Cosmogaphicum* (The Mystery of the Universe) in 1596. Kepler was sent packing out of town after this publication. His book caught the attention of the Danish astronomer Tycho Brahe (1546-1601) who employed him as his assistant. Kepler was a brilliant mathematician but a poor observer; and Tycho Brahe was a brilliant observer but a poor mathematician. The two became a formidable pair and worked together to produce the most sophisticated table of celestial motions. After Tycho Brahe's death, Kepler continued to work on the data and developed his three planetary laws of motion with his observational data based on Mars and used that data to generalize for the motion of other planets in his book *Astronomica Nova* in 1609. Kepler's celebrated Laws of Planetary Motion are:

1. The planets move in elliptical orbits with the sun as the focus.
2. Each planet moves around its orbit, not uniformly, but in such a way that a straight line drawn from the sun to the planet sweeps out equal areas in equal time intervals.
3. The squares of the times required for any two planets to make complete orbits about the sun is proportional to the cubes of their mean distances from the sun.

His laws overturned the existing Aristotelian cosmology and physics. The question of what held the planets together was not yet explained. This task fell to Isaac Newton (1642-1727). Young Newton went to Cambridge University as a poor student who helped provide domestic sevices to other students in order to finance his own education. During the plague, Newton went back to his native village of Woolsthorpe in Lincolnshire, England. One night during this period in his life, he was sitting in the family farm when he saw an apple falling. He looked up and saw the moon and quickly thought about the connection between the moon and the apple as bodies in space. He began to wonder about what kept the moon from falling to the ground. He questioned himself: "Why did apples fall straight down to the earth's surface, rather than askance? What if the apple had started from higher altitude—probably a mile, a hundred miles, or as high as the moon—would it still have fallen to the earth?" (Guillen, 1995). Newton came to the realization that the moon had a tug on the Earth as the Earth had a tug on the moon. The Aristotelian conception was the moon existed in heaven and was incorruptible as a heavenly body. The universe had two domains according to Aristotelian philosophers: the earthly corruptible and the heavenly incorruptible. Newton's speculation then became heretical. How could a corrupt Earth have gravitational influence on an incorruptible heaven? That was the kind of statement that sent Galileo and Copernicus to the Inquisition. He realized that if the moon felt the Earth's tug, it would fall to the ground like the apple. He conjectured by using Huygen's centrifugal force equation that the moon pulled away from the Earth and the Earth pulled away from the moon with the same force, and that kept the moon in orbit. This was a significant revolutionary thinking that Newton came to understand. About a millenia earlier, Kepler had discovered the three laws of planetary motions. If T stands for the time the planet takes to complete one revolution and d stood for the planet's distance from the sun, then Kepler's discovery world would be written as: T^2 = constant times d^3, the result of his painstaking observations and calculations Kepler did with Braho. In plain English, the square of a planet's year always equaled some multiple of the cube of the planet's distance from the sun. That is, planets close to the sun had short years, and those far had long years. Newton built up on this statement to propound his own gravitational laws. He reckoned that if the moon did not fall, then the Earth's gravitational force was being opposed by the moon's own centrifugal force. Newton realized that the moon's centrifugal force depended first on the mass "m" of the moon, second on the distance between the moon and the Earth "d," and the third on the times "T" it took the moon to make a complete journey, normally called one Earth year. Using Kepler's equation,

$$T^2 = Cd^3 \qquad 2\text{-}1$$

Newton substituted the right-hand side of the equation into Huygen's Centrifugal Force equation.

$$\text{Moon's Centrifugal Force} = Cmd/T^2$$

which is the mass of the moon "m" times "d" the distance of the moon from the Earth times a constant which was later determined to be Newton's constant of Universal Gravity (G) and divided by the square of the time "T." The centrifugal force for any orbiting object became:

$$\text{Centrifugal Force} = \frac{Cmd}{T^2} \qquad 2\text{-}2$$

But Kepler had argued earlier that the planets whirled around the sun in elliptical orbits and that they obeyed the law of motion given as:

$$T^2 = Cd^3$$

where T is the time, and d is the distance from the sun. This simple law became very useful and the cornerstone for Newton to launch his new mathematical revolution. Newton summarized this as the Centrifugal Force of the Moon, which is the product of a constant and the mass of the moon divided by the square of the distance between the moon and the Earth.

$$F = Cm/d^2$$

The moon's Centrifugal Force equals the product of the mass of the moon and the distance between the moon and the Earth divided by the distance cubed.

Substituting the right-hand side of equation 2-1 into equarion 2-2 gave the new equation for Newton as:

$$\text{Moon's Centrifugal Force} = Cm/d^2$$

Newton summarized this as the centrifugal force of the moon as the product of a constant and the moon's mass divided by the square of the distance between the moon and the inertial frame of reference.

$$F = Cm/d^2$$

He concluded that if there were a cosmic stand-off between the moon and the Earth, then the Earth's Gravitational Force (EGF) would equal the Moon's Centrifugal Force (MCF).

$$EGF = MCF = \frac{Cm}{d^2}$$

That is, the Earth's gravitational pull weakened the farther away the moon was from the Earth. It weakened inversely with the square of the distance, smaller and smaller force resulted by dividing by bigger and bigger d^2.

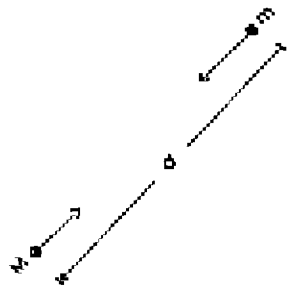


Figure 2-1

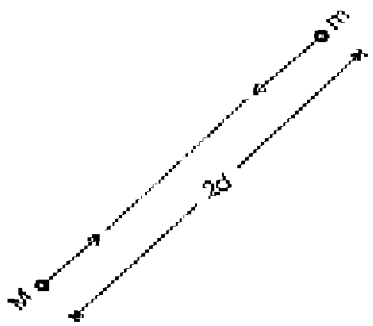


Figure 2-2

Newton came up with the conclusion that if two particles exert gravitational force on each other, then the force equation would be the product of the masses of the two bodies times a constant divided by the square of the distance between the two bodies, written simply as $F = \frac{CMm}{d^2}$.

If M and m remain unchanged but the distance between the two doubled, then,

$$F = \frac{CMm}{(2d)^2} = \frac{CMm}{4d^2}$$

which is one-fourth the force on the original particles. This directly confirmed his observations that the further away the particles are from each other, the weaker the centrifugal force. This theory was in complete agreement with earlier experimental results about intensity of light: diminishes as an object moves further away from the source of light. Newton's reduction of the concept into particle physics changed how we look at celestial motions. He concluded that the Earth's gravity did not belong exclusively to the Earth; that all particles of matter felt a force of attraction between them.

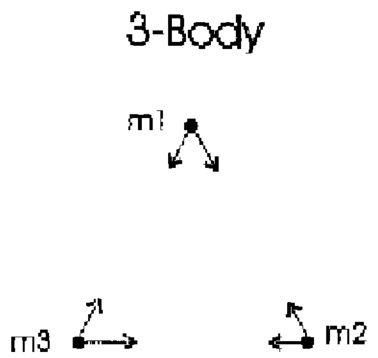


Figure 2-3

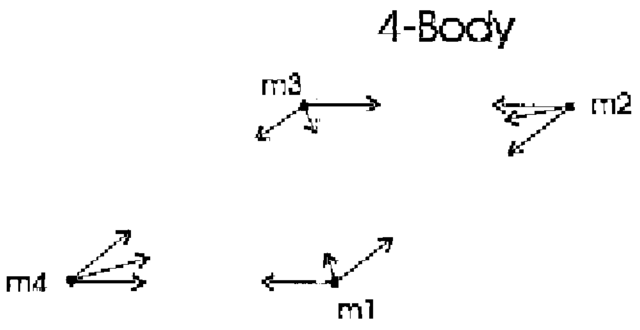


Figure 2-4

Newton then represented the mass of the Earth by M and the mass of a body by m. The revised conception did not change the perfect equation of the Earth's Gravitational Force.

$$\text{Earth's Gravitational Force} = \frac{GMm}{d^2}$$

This means, between the Earth and massive objects close to it, the force of attraction was very strong and irresistable; between the Earth and tiny objects far away, the force was quite weak. In short, the Earth and any other object were attracted to one another with a force whose strength depended on the distance between their centers, their two masses, and some constant number. Later scientific experiments gave a very accurate value of the constant of proportionality of the force. This value is called Newton's gravitational constant represented by G.

The new equation is now:

$$\text{Earth's Gravitational Force} = \frac{GMm}{d^2}$$

In the most general terms, Newton's equation expressed the gravitational force between any two objects; the letters M and m could stand for the mass of the moon and Jupiter, or a comet and the sun, or any pair of bodies; if two bodies are involved, we then have a two-body problem. In short, Newton concluded that gravity was the force that glues objects together everywhere in the universe. With all of these achievements, Newton was regularly bullied by Robert Hooke, a member of the Royal Society and later, its president. Hooke was very jealous of Newton's depth of knowledge and constantly opposed his papers. Newton was afraid to face rejection, possibly the result of a childhood trauma that always haunted him. Edmund Halley admired Newton's works and gave him encouragement to publish his papers. Newton published his findings in the *Principia* and waited after the death of Hooke to publish the *Optiks*, the work that he wrote on the light spectrum and his newly invented telescope which Hooke criticized with hate and jealousy. By developing the construct of particle physics of celestial motions, Newton then became the proposal of the n-body problem. His preoccupation with the apple and the moon in his family farm in his village revolutionized how we look at celestial motions. If one considers the Earth and the moon, then we have the two-body problem. If one adds the sun, the moon, and the Earth, we have the three-body problem; so the list can continue into infinite bodies.

It was not surprising, therefore, that in 1969, many people thought the idea of going to the moon was impossible. Some were skeptical for technical reasons. How could we transport ourselves to something that was a quarter of a million miles away, let alone land on it and return safely? Others were doubtful for religious reasons. The Earth's gravity might extend into the heavenly realm, but earthlings themselves would never do so. They would never plant their dirty feet on the moon or any other heavenly body. The doubters notwithstanding, the United States had pressed ahead in response to President John F. Kennedy's 1961 State of the Union Challenge. The United States, under the leadership of the National Aeronautics and Space Administration (NASA), formed a "think tank" on landing a man on the moon. NASA was racing to beat the Russians in space explorations. NASA was trying also to fill a visceral desire first articulated by the astronomer Johannes Kepler in his book *Somnium* (meaning "The Dream"), history's first work of science fiction. Published posthumously in 1634, *Somnium* had described a boy journeying to the moon with the supernatural aid of a friendly demon, conjured up by the boy's witch of a mother. This story was unbelievable but had affected other writers like the Frenchman Jules Verne (1865.) In his novel, *From the Earth to the Moon,* Jules Verne wrote how three men made a long journey inside a huge aluminum bullet fired from a 900-foot-long cast-iron cannon located in Tampa, Florida. A century later, NASA sent three men to the moon travelling inside what amounted to a giant titanium bullet fired from a launch pad in Cape Canaveral, Florida, one-hundred miles directly east of Tampa. The astronauts rode in a liquid-fueled rocket, the Saturn V. It sounded so detached from Newton's work of gravitational equations, yet Newton's equation played a crucial role in man's mission to the moon. Using Newton's equation, astronomers over the years had calculated the moon's orbit so precisely that NASA engineers were now able to know exactly where their lunar target would be at any moment in time. By calculating the rate at which the Earth's gravity diminished at any point along the way to the moon, NASA also had been able to determine what rocket size was needed for the job. It was to give the rockets a 5% boost that NASA had chosen to launch them from Cape Canaveral, which was closer to the equator than any other place in the U.S. There, close to the equator, the effect of the Earth's spinning was felt more than anywhere else in the country. The rockets were whipped around with the greatest centrifugal force at the equator, because the equator was far from the Earth's axis. NASA took full advantage of the earthly boost to find an answer to a three-body problem of the Earth, moon, and spaceship once it was rocketed into motion. The best one could do was to approximate answers with the aid of computers by application of Newton's equation to the landing of the spaceship on the moon. The giant rocket inclined slowly upward against the unrelenting force that had held us captive on the earth. Somewhere in the clouds, it spins like a bullet travelling at 25,000 miles per hour. Once out of the Earth's gravitational field, it started speeding up without the use of excessive fuel.

Newton's prediciton came true. The dream of Johannes Kepler (*Somnium*) came true. Newton's perfection of the Kepler equation and his dream of the attraction between the moon and the apple had come true when Neil Armstrong, the astronaut of Saturn V, intoned, "One giant leap for mankind."

Historical development here just shows ancient and modern man's interest in celestial mechanics. It is not a new field of study, but rather, we are revisiting the oldest profession of planetary studies. The history here again shows clearly how science grows and how scientific theory evolves over the centuries. It is this gradual observation and collection of data that created a body of knowledge that defines what we call the solar system today. The solar system is considered as the sun, moon, and the Earth, which are the large visible planets to the naked eye, and other planets that are not easily visible to the naked eye. A further treatment of the historical perspective will shed some light on the relationship between scientific discovery, social environment, and other branches of philosophy. This chapter does not set out to achieve all that. This I leave out for the inquiring mind to pursue for joy or for the rigors of intellectual discourse. The historical discourse here is to establish how small steps over many years culminated in a giant leap. Man's first visit to the moon was not formulated, planned, and executed solely in the 1960s. Rather, this was an idea that was shaped over the years by numerous significant but minor achievements. That is what this section sets out to achieve. It is not a historical account of those who made it happen. "No scientific victory was ever won by sheer numbers or by the mass of projectiles. Each was won by a series of efforts, the humblest of which was deliberate to a degree" (Sarton, 1931).

Today's space exploration confirms what the ancient astronomers had recorded. They recorded the movement of the planets and the stars. It has been documented that a few bright stars do change their positions and move so unevenly compared with the sun, moon, and the rest, that they are called planets, meaning "wanderers." These planets look like bright stars with less twinkling, wandering across the sky in tracks of their own near the elliptic path (Burton, 1995). They follow the general backward movement of the sun and the moon through the constellations of the zodiac, but at different speeds and with occasional reverse motions. The zodiac belt includes the sun's yearly path and the moon's monthly, and the wandering paths of all the planets. In modern terms, the orbits of the earth, moon, and other planets all lie in the same plane. Five wandering planets were known to early astronomers in addition to the sun and the moon which were counted with them. These are Mercury, Venus, Jupiter, Mars, and Saturn. Mercury and Venus are bright "stars" which never wander far from the sun but move to and fro in front or behind it. They are seen only near dawn or sunset. Mercury is small and keeps close to the sun, so it is difficult to locate. Venus is a great bright lamp in the evening or morning. It was called the "evening star" and the "morning star" by the early astronomers who did not realize it was the same planet they were locating twice. Mars is a reddish "star" which wanders in a looped track around the zodiac path, taking about two Earth years for a complete trip. Jupiter is a very bright "star" wandering slowly around the elliptic once in a dozen years. Saturn is a bright "star" wandering slowly around the elliptic once in about thirty years. Jupiter and Saturn make many loops in their track—one loop in each of our Earth years (Figure 2-5). When one of the outer planets, which are Mars, Jupiter, or Saturn, makes a loop along its path, it "crawls" slower and slower eastward among the stars, comes to a stop, then crawls in reverse direction westward for a while, then comes to a stop. It then crawls eastward again like the sun and the moon. This movement is known as the epicycloid, which comes from the Greek word for outercircle (Figure 2-6). This strange motion of the outer planets excited much wonder and superstition among the ancient astronomers. We can explain this movement today by demonstrating with two circles—one big and other smaller. When a large wheel W spins steadily around a fixed axle, at some point A on its rim, there is an axle carrying a small wheel w, which spins much faster than the big wheel W. The point P on the rim of the smaller circle traces an epicycloid.

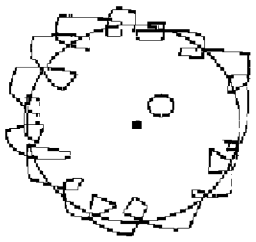


Figure 2-5

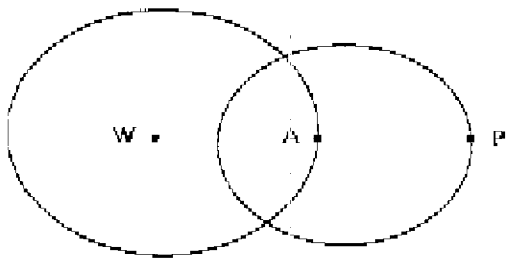


Figure 2-6

Review of Related Research

There are two informative articles from the *Mathematical Intelligencer* written by Florin Diacu (1993 and 1996) on the n-body topic in two different volumes of the journal. Another excellent source of research article is the *American Mathematical Monthly* with articles written by Donald Saari (1990). Saari (1990) in his article, "A Visit to the N-Body Prolblem Via Complex Analysis," presented a solution suitable for an advanced audience. His solutions, like the others, are not adequate for easy comprehension for the undergraduate student but have offered tremendous insight into how to solve the n-body problem by analytical methods. Saari's introduction of elementary complex analysis helped to shed light on how to understand the advanced texts on n-body problems. Jeff Xia (1988), a former student of Saari, also wrote on the n-body problem but restricted his solution to the 5-body problem in his doctoral thesis. Xia showed how bodies in pairs moved in highly eccentric orbits parallel with the x-y plane. He further proved Paul Painleve's (1987) conjecture for the case of 5-body problem. The conjecture stated simply that "for $n \geq 4$, solutions of the n-body problem admit solutions with noncollision singularities." Painleve had proved the case for $n \geq 3$ using differential equations as the method of solution. Painleve in 1887, as a young graduate student, suspected that one particle could oscillate between two others in a three-body motion without colliding but becoming closer and closer at each close encounter. Many of Painleve's contemporaries tried to find examples of solutions with pseudocollisions, but no one succeeded. Diacu (1993), in his article entitled "Painleve Conjecture," wrote on the lost years of the n-body problem since its formal proposal by Newton in the *Principia*. Diacu reviewed the case when Gosta Mittag-Leffler, the Editor-in-Chief of *Acta Mathematics*, was to coordinate a team that would find a solution of the n-body problem as a birthday present to King Oscar II of Norway and Sweden on his sixtieth birthday on January 21, 1889. The questions were partly formulated by Karl Weierstrass who was on the committee. The original proposal was in both German and French, but an English translation by Daniel Goroff (1993) reads: "Given a system of arbitrarily many mass points that attract each other according to Newton's laws, under the assumption that no two points ever collide, try to find a representation of the coordinates of each point as a series in a variable that is some known function of time and for all of whose values the series converges uniformly." There was no better solution than the one produced by Henry Poincaré out of twelve others submitted. His solution was later found out to be full of mistakes after he was awarded the prize. According to Diacu (1996), it was reported in volume 7, 1886/87 of *Acta Mathematica* that a solution was required for the 3-body problem in power-series. Poincaré later published his solution in volume 12 of the *Acta Mathematica* in 1890. His contributions were remarkable for the understanding of dynamics equations called today Hamiltonian Systems for the many new ideas he brought into mathematics and mechanics. More than a century earlier, in 1710, John Bernoulli provided a solution to the 2-body problem normally called Kepler Problem in honor of Kepler for his excellent works on planetary motions. Diacu (1996), writing on the theme "The Solution of the N-Body Problem," commented on earlier attempts made by prominent mathematicians who for more than a century after Bernoulli had solved the 2-body problem, tried to find solutions for n≥3-body problem. Interest in the problem grew towards the end of the 1800s, and attention was paid to celestial mechanics more than ever before. Another interesting work on the n-body problem was from Luitzen Brouver, the editor of the Dutch journal *Mathematische Annale*. In 1913, as the chief editor, he rejected all solutions to the problem using reductio ad absurdum, a method of mathematical proof that assumes the opposite of the result to be true and proceeds to show that it is incorrect, and that the opposite of the original assumption is true. His rejection brought conflict between proponents of Formalism and Intuitionism, the main schools of mathematical-philosophy at the beginning of the twentieth century. The quest for a perfect solution for the n-body problem created challenging rival camps for the good of mathematics. On one hand was the Intuitionist led by Brouver, the chief editor of the influential Dutch journal *Mathematische Annalen*, and his opponents were led by a German, Hilbert, and his school of Formalism. These were the two main schools of mathematical philosophers at the beginning of our century. In this regard, the German was wrong to assert that all theorems can be deduced by logical steps. In 1913, when Brouver was launching his attack on Formalism, he was unaware of the solution provided by a Finn, Karl Sundman. Sundman (1912) published his solution after he received some of his own earlier works and built up on the works of the Italian mathematician Giulio Bisoncini (1906). Sundman provided a series solution to the 3-body problem and showed that the series converges for those values when angular momentum is zero. Sundman's method failed to apply to the case of n>3. In 1991, a Chinese student, Quidong Don Wang (1991), provided a convergence power series solution of the n-body problem. He omitted only the case of solutions leading to singularities—collision in particular. Paradoxically, Sundman's and Wang's solutions provide very slow convergence for insignificantly short intervals of time. At first it looks like a solution was provided, but to sum up millions of terms to determine the motion of the particle for insufficiently short intervals of time makes the work unusable. In 1984, Joe Gerver, from Rutgers University in New Brunswick, New Jersey, proposed a solution for a planar 5-body problem in which the particle escapes to infinity in finite time. Gerver did not give a complete proof of his assumption but provided support for the existence of such a problem. Later, using radial symmetry, Gerver obtained a solution for his planar case by proving his previous heuristic example. His is the first confirmation of Painleve's conjecture using a planar solution. Six months prior to Gerver's solution, Xia had successfully submitted a solution to the Painleve's conjecture for a 5-body problem. Xia and Gerver differed in their approaches in the solutions. In his unpublished manual on the n-body problem, Osler (1996) systematically introduced how to set up both numerical and some analytical solutions of the n-body problem. His numerical solution with computer-aided programs demonstrated a visual image of the n-body problem. If anything, his method of solutions has made the conceptualization more meaningful. I lean towards his style and method. All these researches are relevant but rather difficult for the beginning scholar of the n-body problem.

Review of Related Literature

Forrest R. Moulton (1970) treated celestial mechanics with increasing difficulty of progression in his book *Introduction to Celestial Mechanics*. Moulton has collaborated on earlier works of Hill, Poincaré, and Darwin to present a strong case for planetary motions. This text, despite its high-level presentation, has sophistication that, if well-understood by the student of applied mathematics, will surely advance the body of knowledge on the n-body problem. The book has treated the 3-body problem in detail using differential equations as the method of approach. Eric Rogers (1960), in his *Physics for the Inquiring Mind,* gave a fundamental account of celestial motions. He gave an excellent historical presentation on planetary motions. He treated the subject from the times of ancient Egyptian astronomers, the Babylonians, the Greeks, and then to the present day view on astronomy. He traced the development from the Aristotelian view of movement of heavenly bodies to the Copernican heliocentric view on the ecliptic system of the universe and recounted the gradual progression of contributions made by Ptolemy, Copernicus, Tycho Brahe, Johannes Kepler, Galileo Galilei, and to Isaac Newton, who proposed the n-body problem. Burton (1995) in his book, *History of Mathematics,* gave a good historical perspective on how the ancient scholars put together the body of knowledge on astronomy and how the various theories were gradually developed into the physical laws we use today to solve celestial motion problems. Thomson (1986) demonstrated in his book, *Introduction to Space Dynamics,* how to set up dynamical equations of particle dynamics in orbits. This book helps the challenging scholar to capture the golden heights of celestial motions. In his *Five Equations That Changed the World,* Gullen (1995) wrote an excellent treatise on the history of the five most known equations in mathematics. He looked at the achievements of Newton and his Universal Law of Gravity, Daniel Bernoulli and his Law of Hydropressure, Michael Faraday and his Law of Electromagnetic Induction, Rudolf Clausius and the Second Law of Thermodynamics, and Albert Einstein and the Theory of Spacial Relativity. As we approach the millennia, more scientific discoveries will be made. New mathematical equations will be discovered to keep the dream alive. The future belongs to the young students of today and those yet to be born.

CHAPTER 3

Procedures

Introduction

The purpose of this chapter is to explain the procedures the researcher used to write on the n-body problem. The topics discussed in this chapter include how relevant research material was gathered, how the questions were selected for solution, and the design of related computer programs.

Relevant Research Materials

The researcher selected articles on the n-body problem after a library search at Rowan University' Savitz Library, the Universities of Delaware, Princeton, Pennsylvania, and Rutgers. These library searches did not produce any information on simple solutions of the n-body problem. The research yielded articles on the n-body problem suitable for advanced graduate work and post-doctoral studies. Some of these were doctoral dissertations reproduced for publications in professional journals like the *American Mathematical Monthly* or in the *Mathematical Intelligencer*, and textbooks already mentioned in chapter two. The Internet was used as a resource center to seek help from the general readership. Encouragement, as well as discouragement, were offered by users who read the note the researcher posted on the n-body problem on the Internet.

Selection of N-Body Problems

The researcher received assistance in designing QB programs that solved numerical solutions for the n-body problem from Dr. Tom Osler of Rowan University. The number of bodies is unlimited in the use of computer programs developed for numerical solutions. However, the number of bodies involved in the analytical solutions were limited. It is the analytical solution type that is commonly referred to as "no solutions exist for the n-body problem." There are solutions for n-body problems with numerical methods. It is inappropriate to say there are no solutions. The method of analytical solution was limited to Newtonian mechanics. Introduction of differential equations and polar equations were not included in the solutions, since the audience of this study is the undergraduate and academically-excelled high school students.

Construction of Related Computer Programs

The researcher wrote two computer programs to be used in this study whose purpose was to introduce students to computer-generated numerical solutions. The two programs were designed as to help students gain an understanding of:

1. Euler Method of Numerical Integration for a two-body problem.
2. The Runge-Kutta Method of Numerical Integration for a two-body problem.
3. The Euler Method of Numerical Solution of a three-body problem.

4. The Runge-Kutta Method of Numerical Solution of a three-body problem.

5. The Euler Method of Numerical Integration of an n-body problem.

6. The Runge-Kutta Method of Numerical Integration of an n-body problem.

The computer programs numbers 5, 6, and 7 above on the applications of the Euler, Runge-Kutta, and the Central Force problems with the parameter n can be manipulated to generate different questions of the n-body problem. These different equations have different solutions. Whenever n is changed, the solution will be different from the previous one, and the graphical displays are different. All of these programs were written in QuickBasic because this is a computer language which has good graphic capabilities and is commonly available on almost every IBM-compatible computer.

The researcher developed the Euler and the Runge-Kutta methods so that students can see the efficiency and the effectiveness of the different numerical methods. Another reason in using these methods is to keep the algorithm as simple as possible so that their basic structure could be easily understood without any prior computer programming knowledge. All of the QB programs used by the researcher for the numerical solutions were constructed on a Quest 486 DX 330-megahertz IBM-compatible computer.

CHAPTER 4

Analysis of Data

Introduction

This chapter describes the solution of selected n-body problems by analytical and numerical methods. The chapter shows solutions of specific n-body problems in Qbasic. There are graphical illustrations of analytical solutions and visual displays of numerical solutions on the computer.

Specifically, the chapter introduces analytical and numerical solutions of two-, three-, four-, six-, and eight-body problems. A numerical solution of the n-body problem is also presented by Euler and Runge-Kutta methods.

Newtonian Mechanics of Objects in Gravitational Orbits

If mass M rotates in a circle of radius r with velocity V, its acceleration is:

$$a = \frac{V^2}{r}$$

The acceleration points towards the circle's center.

By Newton's second law,

Force = Mass·Acceleration

$$F = m \cdot \frac{V^2}{r} \qquad 4\text{-}1$$

Newton's Law of Universal Gravitation for two masses m and M, distance r, the force of attraction between them is:

$$F = \frac{GMm}{r^2} \qquad 4\text{-}2$$

m •— - - - - —• M
--- r ---

G is gravitational constant. Equation 4-1 equals 4-2.

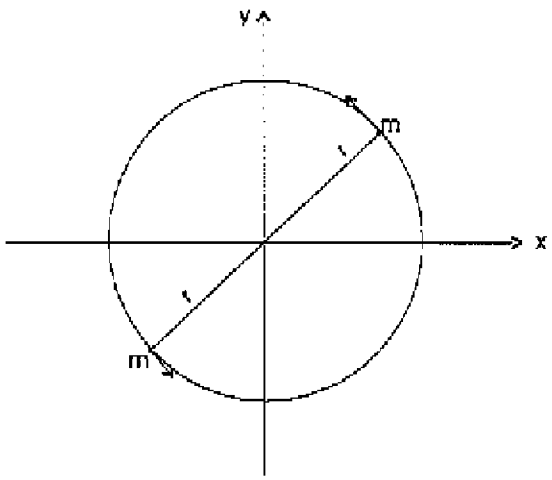
$$\frac{mV^2}{r} = \frac{Gmm}{r^2}$$

$$\frac{V^2}{r} = \frac{GM}{r^2}$$

Analytical Solution of a Two-Body Problem–Equal Masses

The problem involves finding the velocity of two equal masses, m rotating in a circle of radius, r.

Fig. 4-1



Let r be the distance from the center to the mass.
Let 2r be the distance between the masses.
Let v be the velocity of the masses.
By Newton's Law of Universal Gravitation.

$$\frac{v^2}{r} = \frac{GM}{(2r)^2}$$

$$\frac{v^2}{r} = \frac{GM}{4r^2}$$

$$v^2 = \frac{GM}{4r}$$

$$v = \frac{1}{2}\sqrt{\frac{Gm}{r}}$$

is the velocity of two equal bodies rotating in a circle from a center.

Analytical Solution of Two-Body Problem–Unequal Masses

The problem involves finding analytic solution of the velocities of two unequal masses M and m rotating in circles around a center without any mass at the center.

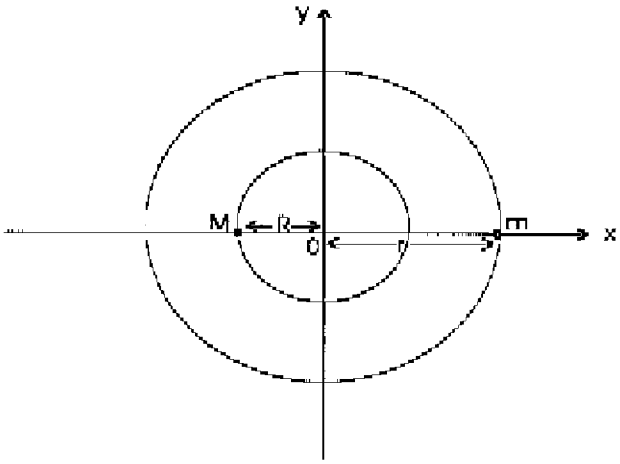


Fig. 4-2

The origin 0 is the center of gravity. So,

$$MR = mr \qquad (1)$$

By Newton's Laws:

$$F = \frac{GMm}{r^2} = \frac{v^2 m}{r}$$

$$\frac{v^2}{r} = \frac{GM}{(R + r)^2} \qquad (2)$$

From (1)

$$R = \frac{mr}{M}$$

$$\frac{v^2}{r} = \frac{GM}{({}^{mr}/_{M} + r)^2}$$

$$\frac{v^2}{r} = \frac{GM^3}{r^2(M + m)^2}$$

$$v^2 = \frac{GM^3}{r(M + m)^2}$$

$$v = \frac{M}{(M + m)}\sqrt{\frac{GM}{r}}$$

For the other mass M,

$$V = \frac{m}{(M + m)}\sqrt{\frac{Gm}{R}}$$

Since $\frac{M}{r} = \frac{m}{R}$, the both have the same angular velocity. We do not expect them to have equal velocities. The angular velocity $\omega = \frac{v}{r} = \frac{V}{R}$.

Analytic Solution of a Two-Body Problem With a Third Body Fixed at Center

This problem involves finding the velocities of two unequal masses M and m rotating in a circle and a third mass fixed at the center.

Can a third mass M_o is placed at the origin (CG) of two unequal masses in a circular motion and the motion still remain circular?

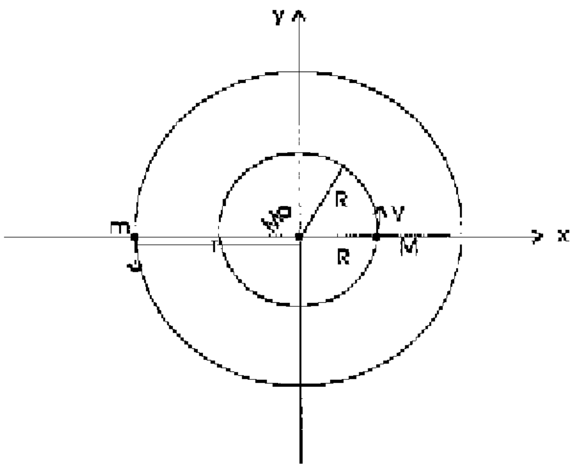


Fig. 4-3

$$F = \frac{GMm}{r^2} \qquad \text{Also, } F = \frac{v^2 m}{r}$$

$$RM = rm \qquad (1)$$

$$\frac{v^2}{r} = \frac{GM}{(r+R)^2} + \frac{GMo}{r^2} \qquad (2)$$

$$\frac{V^2}{R} = \frac{Gm}{(r+R)^2} + \frac{GMo}{R^2} \qquad (3)$$

Using equations (2) nd (3), we can calculate v and V. But is the angular velocity the same?

(4) $\frac{v^2}{r^2} = \frac{GM}{r(R+r)^2} + \frac{GM_0}{r^3}$ multiplying (2) by $^1/_r$

From (3), we obtain:

(5) $\frac{V^2}{R^2} = \frac{Gm}{R(R+r)^2} + \frac{GMo}{R^3}$ multiplying (3) by $^1/_R$

From (1), we obtain:

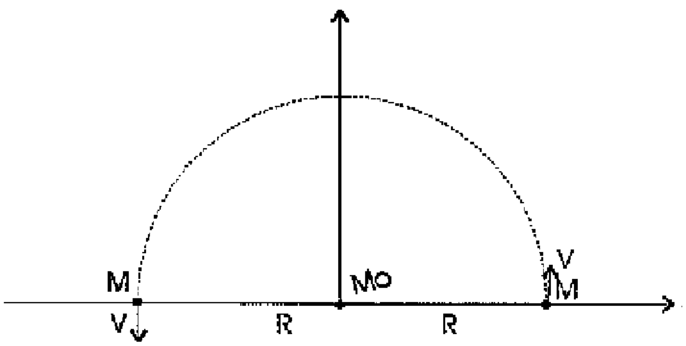
$\frac{m}{R} = \frac{M}{r}$ we substitute into (5) to obtain:

(6) $\frac{V^2}{R^2} = \frac{GM}{r(R+r)^2} + \frac{GMo}{R^3}$

Comparing (4) and (6), the first terms are equal, but the second terms GMo/r^3 and GMo/R^3 are not equal.

The answer is No. To enable the system to work, we must have m = M and r = R.

Fig. 4-4



Then,

$\frac{v^2}{R} = \frac{GM}{(2R)^2} + \frac{GMo}{R^2}$

$\frac{V^2}{R} = \frac{GM}{4R^2} + \frac{GMo}{R^2}$

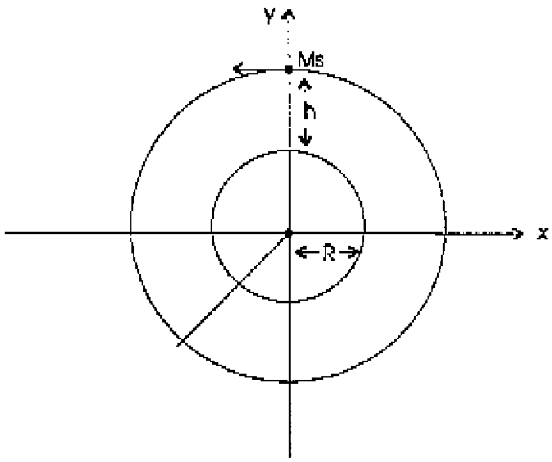
$$V^2 = \frac{GM}{4R} + \frac{GMo}{R}$$

$$V = \frac{1}{2}\sqrt{\frac{(G(M + 4Mo)}{R}}$$ is the required velocity.

Analytic Solution of a Satellite Orbiting the Earth Problem

This problem involves finding the velocity of a satellite in a circular orbit at an altitude of 1,000 kilometers from the surface of the Earth, assuming that the Earth is a homogeneous spherical body.

Fig. 4-5



Let R_e be the equatorial radius of the Earth and h the altitude of the satellite above the Earth.

The distance from the center of the earth to the satellite is $R_e + h$.

$$R_e + h = (6378136 \pm 1)m + 1000 \times 1000m$$

$$R_e + h = 7378136 \pm 1)m$$

F_g is the gravitational force acting on the satellite.

(1) $F_g = \frac{GM_E M_S}{d^2}$ where d is the distance between the center of the Earth and the satellite, M_E is the mass of the earth, M_S is the mass of the satellite.

F_g is the centripetal force acting on the satellite.

(2) $F_c = M_S d\omega^2$ and ω is the angular velocity.

Equation (1) and (2)

$$\frac{GM_E M_S}{d^2} = M_S d\omega^2$$

$$\frac{GM_E}{d^2} = \omega^2 d$$

$$\omega^2 = \frac{GM_E}{d^3}$$

$$\omega = \frac{1}{d}\sqrt{\frac{GM_E}{d}}$$

Let n mean motion in orbital mechanics equal ω. But nd is the velocity of motion.

$$\omega = \frac{1}{d}\sqrt{\frac{GM_E}{d}}$$

$$n = \frac{1}{d}\sqrt{\frac{GM_E}{d}}$$

$V_c = nd = \sqrt{\frac{GM_E}{d}}$ is the velocity of the satellite

$$V_c = \sqrt{\frac{G \times 5\text{-}9742 \times 10^{24}}{7378136}}$$

$$V_c = \sqrt{\frac{6.672 \times 10^{-11} \times 5.9742 \times 10^{24})}{7378136}}$$

$V_a = 19964934,9$ m/s^{-1} of the satellite.

Analytic Solution of a Four-Body Problem

This problem involves finding the velocity of four equal masses m rotating in a circle with radius r.

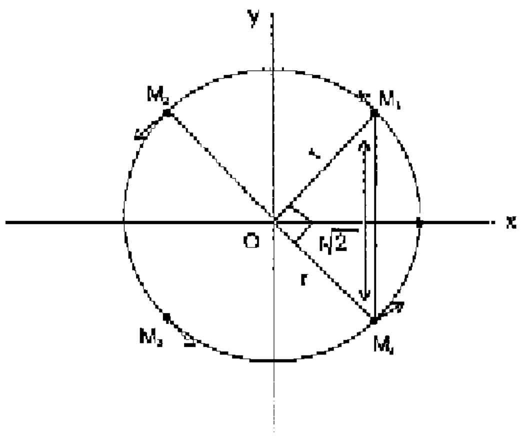


Fig. 4-6

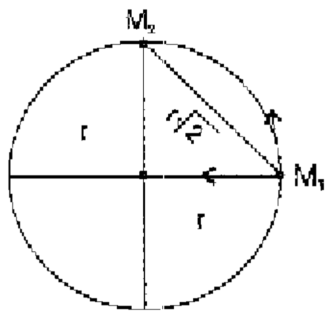


Fig. 4-7

Let r be the distance from the center of the circle to the mass.

The distance between any two diametrically opposite masses is 2r.

The distance between any two closest masses on the circle is $\sqrt{2}r$.

M_1 has forces FCos ~~θ~~ horizontally FSin~~θ~~ vertically acting on it. Where ~~θ~~ = $^{\pi}/_{4}$. Forces acting on M_1 are M_1 relative to M_2, M_2 relative to M_3, M_1 relative to M_4. The forces acting on M_2 and M_4 are equal; $M_1 = M_2 = M_3 = M_4$.

By Newtonian Mechanics, $F = \frac{GMm}{r^2} = \frac{mV^2}{r}$

$$\frac{V^2}{r} = \frac{GM}{(2r)^2} + \frac{GM}{(\sqrt{2}r)^2} \operatorname{Cos} {}^{\pi}/_4 + \frac{GM}{(\sqrt{2}r)^2} \operatorname{Cos} {}^{\pi}/_4$$

$$\frac{V^2}{r} = \frac{GM}{4r^2} + \frac{GM}{2r^2}\frac{1}{\sqrt{2}} + \frac{GM}{2r^2}\frac{1}{\sqrt{2}}$$

$$\frac{V^2}{r} = \frac{GM}{4r^2} + \frac{2GM}{2r^2}\frac{1}{\sqrt{2}}$$

$$\frac{V^2}{r} = \frac{GM}{4r^2} + \frac{GM}{\sqrt{2}r^2}$$

$$\frac{V^2}{r} = \frac{GM}{4r^2} + \frac{\sqrt{2}GM}{2r^2}$$

$$\frac{V^2}{r} = \frac{GM}{4r^2}\left(1 + 2\sqrt{2}\right)$$

$$V^2 = \frac{GM}{4r}(2\sqrt{2} + 1)$$

$$V = \frac{1}{2}\sqrt{\frac{GM}{r}(2\sqrt{2} + 1)}$$ is the velocity of the mass.

Analytic Solution of a Six-Body Problem

This problem requires finding the velocity of six equal masses m rotating in a circular orbit with a mass M at the center of gravity.

Fig. 4-8

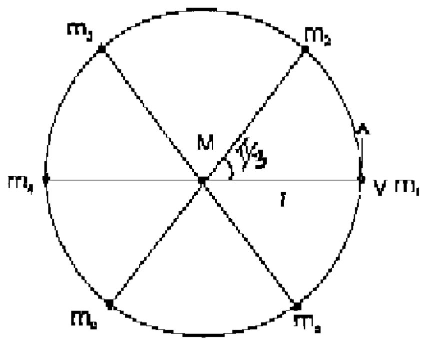


Fig. 4-9

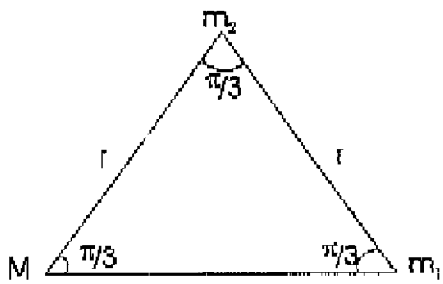
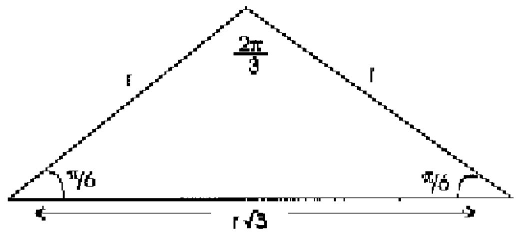


Fig. 4-10



The distance between m_1 and $m_4 = 2r$.

The distance between m_1 and m_2 equals the distance between m_1 and m_6.

The distance between m_1 and m_3 equals the distance between m_1 and m_5.

$m_1 = m_2 = m_3 = m_4 = m_5 = m_6$.

The sum of the forces of attraction between M_1 and M_2, M_3, M_4, M_5, and M_6 and M are respectively.

$$\frac{V^2}{r} = \frac{Gm_2}{r^2}\,\text{Cos}\,\frac{\pi}{3} + \frac{GM_6}{r^2}\,\text{Cos}\,\frac{\pi}{3} + \frac{GM}{(2r)^2} + \frac{GM_3}{3r^2}\,\text{Cos}\,\frac{\pi}{6} + \frac{Gm_5}{3r^2}\,\text{Cos}\,\frac{\pi}{6} + \frac{GM}{r^2}$$

But $m_1 = m_2 = m_3 = m_4 = m_5 = m_6 = m$.

$$\frac{V^2}{r} = \frac{Gm}{r^2}\,\frac{1}{2} + \frac{Gm}{r^2}\,\frac{1}{2} + \frac{Gm}{4r^2} + \frac{Gm\sqrt{3}}{3r^2 2} + \frac{Gm}{3r^2}\,\frac{\sqrt{3}}{2} + \frac{GM}{r^2}$$

$$\frac{V^2}{r} = \frac{5Gm}{4r^2} + \frac{Gm\sqrt{3}}{3r^2} + \frac{GM}{r^2}$$

$$\frac{V^2}{r} = \frac{15Gm}{12r^2} + \frac{4\sqrt{3}Gm}{12r^2} + \frac{12GM}{12r^2}$$

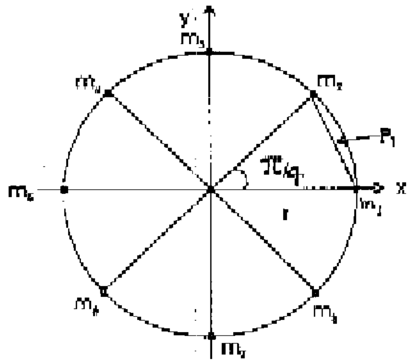
$$V = \frac{G}{12r}\left((15 + 4\sqrt{3})m + 12M\right)$$

$$V = \sqrt{\frac{G}{12r}\left[(15 + 4\sqrt{3})m + 12M\right]}$$ is the required velocity.

Analytic Solution of an Eight-Body Problem

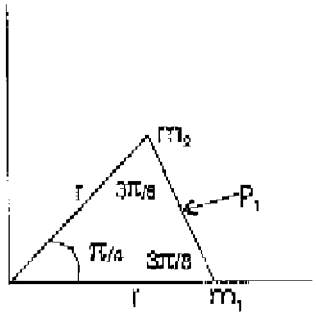
This problem involves finding the velocity of eight equal masses m rotating in a circle with radius r.

Fig. 4-11



P_1 is the distance between m_2 and m_1.

Fig. 4-12



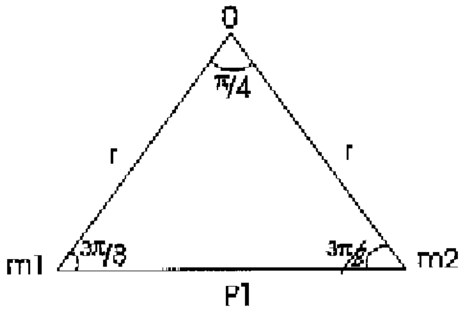


Fig. 4-13

$$P_1^2 = r^2 + r^2 - 2r.r.\ \mathrm{Cos}\ {}^{\pi}/_4$$

$$P_1^2 = 2r^2 - 2r^2\ \mathrm{Cos}\ {}^{\pi}/_4$$

$$P_1^2 = 2r^2\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$P_1^2 = 2r^2\left(\frac{\sqrt{2} - 1}{\sqrt{2}}\right)$$

$$P_1^2 = r^2\sqrt{2}(\sqrt{2} - 1)$$

$$P_1^2 = r^2(2 - \sqrt{2})$$

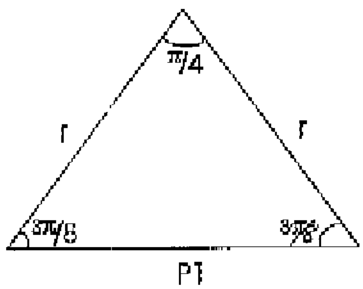


Fig. 4-14

The distance between m_1 and m_3 is $r\sqrt{2}$.

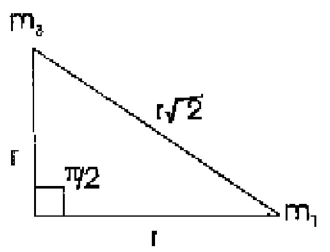


Fig. 4-15

The distance between m_1 and m_4 is P_2.

$$P_2^2 = r^2 + r^2 - 2r.r.\ Cos\ ^{3\pi}/_4$$

$$P_2^2 = 2r^2 - 2r.r.\ Cos\ ^{3\pi}/_4$$

$$P_2^2 = 2r^2\ (1 - Cos\ ^{3\pi}/_4)$$

$$P_2^2 = 2r^2(1 + \frac{1}{\sqrt{2}})$$

$$P_2^2 = 2r^2\frac{(2 + \sqrt{2})}{2}$$

$$P_2^2 = r^2(\sqrt{2} + 2)$$

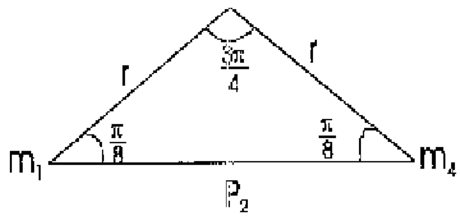


Fig. 4-16

Where, the acceleration of M_1 due to M_2 = acceleration of M_1 due to M_8

the acceleration of M_1 due to M_4 = acceleration of M_1 due to M_6

the acceleration of M_1 due to M_3 = acceleration of M_1 due to M_7.

Acceleration of mass = acceleration of M_2 and M_8 + acceleration of M_4 and M_6 + acceleration of M_3 and M_7 + acceleration of M_5.

$$\frac{V_2}{r} = \frac{2Gm}{P_1^2} \text{Cos } 3\pi/8 + \frac{2Gm}{P_2^2} \text{Cos } \pi/8 + \frac{2Gm}{(r\sqrt{2})^2} \text{Cos } \pi/4 + \frac{Gm}{(2r)^2}$$

$$\frac{V^2}{r} = \frac{2Gm \text{ Cos } 3\pi/8}{r^2(2-\sqrt{2})} + \frac{2Gm \text{ Cos } \pi/8}{r^2(2+\sqrt{2})} + \frac{2Gm}{2r^2}\frac{1}{\sqrt{2}} + \frac{Gm}{4r^2}$$

$$\frac{V^2}{r} = \frac{2Gm}{(2-\sqrt{2})r^2} \text{Cos } 3\pi/8 + \frac{2Gm \text{ Cos } \pi/8}{(2+\sqrt{2})r^2} + \frac{1}{\sqrt{2}}\frac{Gm}{r^2} + \frac{Gm}{4r^2}$$

But, $\text{Cos } 3\pi/8 = \text{Cos } (2\pi/8 + \pi/8)$

$$\text{Cos } 3\pi/_8 = \text{Cos } \pi/4 \text{ Cos } \pi/8 - \text{Sin } \pi/4 \text{ Sin } \pi/8$$

$$\text{Cos } 3\pi/8 = \frac{1}{\sqrt{2}}\text{Cos } \pi/8 - \frac{1}{\sqrt{2}} \text{Sin } \pi/8$$

$$\text{Cos } 3\pi/8 = \frac{1}{\sqrt{2}} (\text{Cos } \pi/8 - \text{Sin } \pi/8)$$

Also, $\text{Cos } \pi/8 = \sqrt{\frac{1 + \text{Cos } \pi/4}{2}}$ And, $\text{Sin } \frac{\pi}{8} = \sqrt{\frac{1 - \text{Cos } \pi/4}{2}} = \sqrt{\frac{1 - ^1/_{\sqrt{2}}}{2}}$

$$\text{Cos } \pi/8 = \frac{1}{2}\sqrt{(2+\sqrt{2})}, \quad \text{Sin } \pi/8 = \frac{1}{2}\sqrt{(2-\sqrt{2})}$$

$$\text{Cos } 3\pi/8 = \frac{1}{\sqrt{2}}\left[\frac{1}{2}\sqrt{(2+\sqrt{2})} - \frac{1}{2}\sqrt{(2-\sqrt{2})}\right]$$

$$\text{Cos } 3\pi/8 = \frac{1}{2\sqrt{2}}\left[\sqrt{(2+\sqrt{2})} - \sqrt{(2-\sqrt{2})}\right]$$

$$\cos 3\pi/8 = \frac{\sqrt{2}}{4}\left[\sqrt{(2+\sqrt{2})} - \sqrt{(2-\sqrt{2})}\right]$$

Therefore,

$$\frac{V^2}{r} = \frac{Gm}{r^2}\left[\frac{1+2\sqrt{2}}{4} + \frac{\sqrt{2}}{2}\left(\frac{\sqrt{2+\sqrt{2}} - \sqrt{2-\sqrt{2}}}{(2-\sqrt{2})}\right) + \frac{\sqrt{2+\sqrt{2}}}{2+\sqrt{2}}\right]$$

$$\frac{V^2}{r} = \frac{Gm}{4r^2}\left[1 + 2\sqrt{2} + \sqrt{2}(2+\sqrt{2})^{3/2} - 2\sqrt{2+\sqrt{2}} + 2\sqrt{2}\right]$$

$$\frac{V^2}{r} = \frac{Gm}{4r^2}\left[1 + 4\sqrt{2} + \sqrt{2+\sqrt{2}}\,(\sqrt{2}(2+\sqrt{2}) - 2)\right]$$

$$\frac{V^2}{r} = \frac{Gm}{4r^2}\left[1 + 4\sqrt{2} + 2\sqrt{2}\sqrt{2+\sqrt{2}}\right]$$

$$\frac{V^2}{r} = \frac{Gm}{4r^2}\left[1 + 2\sqrt{2}(2 + \sqrt{2+\sqrt{2}}\right]$$ is the acceleration.

$$V^2 = \frac{Gm}{4r}\left[1 + 2\sqrt{2}(2 + \sqrt{2+\sqrt{2}}\right]$$

$$V = \tfrac{1}{2}\sqrt{{}^{Gm}/_{r}\,(1 + 2\sqrt{2}(2 + \sqrt{2+\sqrt{2}})}$$ is the required velocity.

Numerical Solution for a Two-Body Problem Using Euler Method

This program generates a numerical solution of a two-body problem using the Euler method. The number of masses, the coordinates of the masses, and the initial velocities are given.

```
N = 2              'N = number of bodies
                      X       Y      VX      VY      M
100   DATA            1,      0,     0,      1,      1
```

```
110    DATA         -1,         0,      0,        -1,        1

120    FOR I = 1 TO N
130    READ X(I), Y(I), VX(I), VY(I), M(I)
140    NEXT I

150    G = 1                ' Universal gravitational constant
160    T = 0: DT = .001     ' Initial time and increment

       ' Establish screen parameters

200    SCREEN 12            ' VGA resolution
210    RAD = 3              ' Screen RADius

       ' x distances times 1.33333 so that circles look circular

220    WINDOW (-1.33333 * RAD, RAD)-(1.33333 * RAD, -RAD)
230    CLS

       ' Calculate next position
300    PSET (0, 0)          ' Mark center of coordinates

310    WHILE KEY$ = ""   ' Loop until key pressed
       FOR I = 1 TO N

       ' Find AX(I) and AY(I) components of acceleration on I-th mass
       AX(I) = 0: AY(I) = 0
       FOR J = 1 TO N
          IF J <> I THEN
              DELTAX(J) = X(J) - X(I)
              DELTAY(J) = Y(J) - Y(I)
              R(J) = (DELTAX(J) ^ 2 + DELTAY(J) ^ 2) ^ .5
              AX(I) = AX(I) + G * M(J) * DELTAX(J) / R(J) ^ (P + 1)
              AY(I) = AY(I) + G * M(J) * DELTAY(J) / R(J) ^ (P + 1)
          END IF
       NEXT J

       'FIND dVX, dVY, dX, and dY on the left of our interval
       DVX(I) = AX(I) * DT
       DVY(I) = AY(I) * DT
       DX(I) = VX(I) * DT
       DY(I) = VY(I) * DT
```

```
        VX(I) = VX(I) + DVX(I)]
        VY(I) = VY(I) + DVY(I)
        X(I) = X(I) + DX(I)
        Y(I) = Y(I) + DY(I)

410     PSET (X(I), Y(I))      ' Plot position on screen
        T = T + DT

        LOCATE 1, 1: PRINT USING "Time : ###.### "; T

420     KEYS = INKEY$          ' See if key is pressed to stop program
        NEXT I
430     WEND                   ' Start while loop again to calculate
                               ' next position
        STOP
```

Numerical Solution of a Two-Body Problem of the Runge-Kutta Method

This program generates the solution of a two-body problem using the Runge-Kutta method. The number of masses, coordinates of the masses in the x-y planes, and the initial velocities are already determined.

```
G = 1       ' Universal gravitational constant
P = 2       ' Central force = G M m / r^P
N = 2       ' N = number of bodies

                        X        Y      VX      VY      M
100     DATA            1,       0,     0,      .55,    1
110     DATA           -1,       0,     0,     -.55,    1

120     FOR I = 1 TO N
130     READ X(I), Y(I), VX(I), VY(I), M(I)
140     NEXT I

150     T = 0: DT = .01        ' Initial time and increment

        ' Establish screen parameters

200     SCREEN 12              ' VGA resolution
```

```
210   RAD = 3                ' SCREEN RADius

      ' x distances times 1.33333 so that circles look circular

220   WINDOW (-1.33333 * RAD, RAD)-(1.33333 * RAD, -RAD)
230   CLS

      ' Calculate next position
300   PSET (0, 0)            ' Mark center of coordinates

310   WHILE KEY$ = ""   ' Loop until key pressed
320   FOR I = 1 TO N

      ' Find AX(I) and AY(I) components of acceleration on I-th mass
400   AX(I) = 0: AY(I) = 0
410   FOR J = 1 TO N
420     IF J <> I THEN
430         DELTAX(J) = X(J) - X(I)
440         DELTAY(J) = Y(J) - Y(I)
450         R(J) = (DELTAX(J) ^ 2 + DELTAY(J) ^ 2) ^ .5
460         AX(I) = AX(I) + G * M(J) * DELTAX(J) / R(J) ^ (P + 1)
470         AY(I) = AY(I) + G * M(J) * DELTAY(J) / R(J) ^ (P + 1)
480     END IF
490   NEXT J

      FIND dVX, dVY, dX, and dY on the left of our interval
500   DVX(I) = AX(I) * DT
510   DVY(I) = AY(I) * DT
520   DX(I) = VX(I) * DT
530   DY(I) = VY(I) * DT

      ' Estimate VXR = VX on right side of our interval, VYR = etc
600   VXR(I) = VX(I) + DVX(I)
610   VYR(I) = VY(I) + DVY(I)
620   XR(I) = X(I) + DX(I)
630   YR(I) = Y(I) + DY(I)

640   NEXT I

700   FOR I = 1 TO N

800   AXR(I) = 0: AYR(I) = 0
```

```
810    FOR J = 1 TO N
820      IF J <> THEN
830          DELTAXR(J) = XR(J) - XR(I)
840          DELTAYR(J) = YR(J) - YR(I)
850          RR(J) = (DELTAXR(J) ∧ 2 + DELTAYR(J) ∧ 2) ∧ .5
860          AXR(I) = AXR(I) + G * M(J) * DELTAXR(J) / RR(J) ∧ (P + 1)
870          AYR(I) = AYR(I) + G * M(J) * DELTAYR(J) / RR(J) ∧ (P + 1)
880      END IF
890    NEXT J

900    DVXR(I) = AXR(I) * DT
910    DVYR(I) = AYR(I) * DT
920    DXR(I) = VXR(I) * DT
930    DYR(I) = VYR(I) * DT

950    DVXA(I) = (DVX(I) + DVXR(I)) / 2
960    DVYA(I) = (DVY(I) + DVYR(I)) / 2                  ' etc
970    DXA(I) = (DX(I) + DXR(I)) / 2
980    DYA(I) = (DY(I) + DYR(I)) / 2

1000   VX(I) = VX(I) + DVXA(I)    ' new VX =old VX + dVX average
1010   VY(I) = VY(I) + DVYA(I)    ' etc
1020   X(I) = X(I) + DXA(I)
1030   Y(I) = Y(I) + DYA(I)

1100   PSET (X(I), Y(I)), I + 1        ' Plot position on screen
1110   T = T + DT
1120   ' Locate 1, 1: PRINT USING "Time : ###.### "; T

1130   KEY$ = INKEY$     ' See if key is pressed to stop program

1140   NEXT I

1150   WEND              ' Start while loop again to calculate
                         ' next position
2000   STOP
```

Numerical Solution of a Three-Body Problem Using the Euler Method

This program illustrates the numerical solution of a three-body problem using the Euler method. The number of masses, initial velocities, and the positions of the masses are predetermined in the program.

```
N = 3            ' N = number of bodies

                    X        Y      VX      VY      M
100   DATA          1,       0,     0,      1,      1
110   DATA         -1,       0,     0,     -1,      1
115   DATA          0,       1,     0,      1,      1
120   FOR I = 1 TO N
130   READ X(I), Y(I), VX(I), VY(I), M(I)
140   NEXT I

150   G = 1               ' Universal gravitational constant
160   T = 0: DT = .001    ' Initial time and increment

      ' Establish screen parameters

200   SCREEN 12           ' VGA resolution
210   RAD = 3             ' Screen RADius

      ' x distances times 1.33333 so that circles look circular
220   WINDOW (-1.33333 * RAD, RAD)-(1.33333 * RAD, -RAD)
230   CLS

      'Calculate next position
300   PSET (0,0)          ' Mark center of coordinates

310   WHILE KEY$ = ""              ' Loop until key pressed
         FOR I = 1 TO N

      ' Find AX(I) and AY(I) components of acceleration on I-th mass
         AX(I) = 0: AY(I) = 0
         FOR J = 1 TO N
            IF J <> I THEN
               DELTAX(J) = X(J) - X(I)
               DELTAY(J) = Y(J) - Y(I)
```

```
            R(J) = (DELTAX(J) ∧ 2) ∧ .5
            AX(I) = AX(I) + G * M(J) * DELTAX(J) / R(J) ∧ (P + 1)
            AY(I) = AY(I) + G * M(J) * DELTAY(J) / R(J) ∧ (P + 1)
        END IF
    NEXT J

    ' Find dVX, dVYm dX, and dY on the left or our interval
    DVX(I) = AX(I) * DT
    DVY(I) = AY(I) * DT
    DX(I) = VX(I) * DT
    DY(I) = VY(I) * DT

    VX(I) = VX(I) + DVX(I)
    VY(I) = VY(I) + DVY(I)
    X(I) = X(I) + DX(I)
    Y(I) = Y(I) + DY(I)

410 PSET (X(I), Y(I))            ' Plot position on screen
    T = T + DT
    LOCATE 1, 1: PRINT USING "Time : ###.### "; T

420 KEY$ = INKEY$ ' See if key is pressed to stop program

    NEXT I
430 WEND              ' Start while loop again to calculate
                      ' next position
    STOP
```

Numerical Solution of a Three-Body Problem

This program illustrates the numerical solution of a three-body problem using the Runge-Kutta method. The number of masses, the initial velocities, and the coordinates of the masses are predetermined.

```
P = 2
N = 3                 ' N = number of bodies
```

```
                     X          Y      VX       VY      M
100    DATA        -1.5,       -2,     0,       0,      4
110    DATA         1.5,       -2,     0,       0,      5
115    DATE         1.5,        2,     0,       0,      3

120    FOR I = 1 TO N
130    READ X(I), Y(I), VX(I), VY(I), M(I)
140    NEXT I

150    G = 1                    ' Universal gravitational constant
160    T = 0: DT = .000001      ' Initial time and increment

       ' Establish screen parameters

200    SCREEN 12                ' VGA resolution
210    RAD = 6                  ' Screen RADius

       ' x distances times 1.33333 so that circles look circular
220    WINDOW (-1.33333 * RAD, RAD)-(1.33333 * RAD, -RAD)
230    CLS

       ' Calculate next position
300    PSET (0, 0)              'Mark center of coordinates

310    WHILE KEY$ = ""          ' Loop until key pressed
320    FOR I = 1 TO N

       ' Find AX(I) and AY(I) components of acceleration on I-th mass
400    AX(I) = 0: AY(I) = 0
410    FOR J = 1 TO N
420       IF J <> I THEN
430          DELTAX(J) = X(J) - X(I)
440          DELTAY(J) = Y(J) - Y(I)
450          R(J) = (DELTAX(J) ^ 2 + DELTAY(J) ^ 2) ^ .5
460          AX(I) = AX(I) + G & M(J) * DELTAX(J) / R(J) ^ (P + 1)
470          AY(I) = AY(I) + G & M(J) * DELTAY(J) / R(J) ^ (P + 1)
480       END IF
490    NEXT J

       ' Find dVX, dVY, dX, and dY on the left of our interval
500    DVX(I) = AX(I) * DT
510    DVY(I) = AY(I) * DT
```

```
520   DX(I) = VX(I) * DT
530   DY(I) = VY(I) * DT

      ' Estimate VXR = VX on right side of our interval, VYR = etc
600   VXR(I) = VX(I) + DVX(I)
610   VYX(I) = VY(I) + DVY(I)
620   XR(I) = X(I) + DX(I)
630   YR(I) = Y(I) + DY(I)
640   NEXT I

700   FOR I = 1 TO N

800   AXR(I) = 0: AYR(I) = 0
810   FOR J = 1 TO N
820      IF J <> I THEN
830         DELTAXR(J) = XR(J) - XR(I)
840         DELTAYR(J) = YR(J) - YR(I)
850         RR(J) = (DELTAXR(J) ^ 2 + DELTAYR(J) ^ 2) ^ .5
860         AXR(I) = AXR(I) + G * M(J)J * DELTAXR(J) / RR(J) ^ (P + 1)
870         AYR(I) = AYR(I) + G * M(J)J * DELTAYR(J) / RR(J) ^ (P + 1)
880      END IF
890   NEXT J

900   DVXR(I) = AXR(I) * DT
910   DVYR(I) = AYR(I) * DT
920   DXR(I) = VXR(I) * DT
930   DYR(I) = VYR(I) * DT

950   DVXA(I) = (DVX(I) + DVXR(I)) / 2
960   DVXY(I) = (DVY(I) + DVYR(I)) / 2        ' etc
970   DXA(I) = (DX(I) + DXR(I)) / 2
980   DYA(I) = (DYH(I) + DYR(I)) / 2

1000  VX(I) = VX(I) + DVXA(I)    ' new VX = old VX + dVX average
1010  VY(I) = VY(I) + DVYA(I)    ' etc
1020  X(I) = X(I) + DXA(I)
1030  Y(I) = Y(I) + DYA(I)

1100  PSET (X(S), Y(I)), I + 1       ' Plot position on screen
1110  T = T + DT
1120  ' Locate 1, 1: PRINT USING "Time : ###.### "; T
```

```
1130  KEY$ = INKEYS     ' See if key is pressed to stop program

1140  NEXT I

1150  WEND              ' Start while loop again to calculate
                        ' next position
2000  STOP
```

Numerical Solution of the N-Body Problem Using the Euler Method

This program illustrates the numerical solution of the n-body problem using the Euler method. There are two masses given in the program which makes it look like a two-body problem. As many bodies can be added and each with its required parameters. This, then, is the general numerical solution of the n-body problem.

```
N = n          ' N = number of bodies

                  X       Y     VX     VY     M
100   DATA        1,      0,    0,     1,     1
110   DATA       -1,      0,    0,    -1,     1

120   FOR I = 1 TO N
130   READ X(I), Y(I), VX(I), VY(I), M(I)
140   NEXT I

150   G = 1              ' Universal gravitational constant
160   T = 0: DT = .001   ' Initial time and increment

   ' Establish screen parameters

200   SCREEN 12          ' VGA resolution
210   RAD = 3            ' Screen RADius

      ' x distances times 1.33333 so that circles look circular
220   WINDOW (-1.33333 * RAD, RAD)-(1.33333 * RAD, -RAD)
230   CLS
```

```
        ' Calculate next position
300     PSET (0, 0)         ' Mark center of coordinates

310     WHILE KEY$ = ""              ' Loop until key pressed
        FOR I = 1 TO N

        ' Find AX(I) and AY(I) components of acceleration on I-th mass
        AX(I) = 0: AY(I) = 0
        FOR J = 1 TO N
            IF J <> I THEN
                DELTAX(J) = X(J) - X(I)
                DELTAY(J) = Y(J) - Y(I)
                R(J) = (DELTAX(J) ^ 2 + DELTAY(J) ^ 2) ^ .5
                AX(I) = AX(I) + G * M(J) DELTAX(J) / R(J) ^ (P + 1)
                AY(I) = AY(I) + G * M(J) DELTAY(J) / R(J) ^ (P + 1)
            END IF
        NEXT J

        ' Find dVX, dVY, dX, and dY on the left of our interval
        DVX(I) = AX(I) * DT
        DVY(I) = AY(I) * DT
        DX(I) = VX(I) * DT
        DY(I) = VY(I) * DT

        VX(I) = VX(I) + DVX(I)
        VY(I) = VY(I) + DVY(I)
        X(I) = X(I) + DX(I)
        Y(I) = Y(I) + DY(I)

410     PSET (X(I), Y(I))             ' Plot postion on screen
        T = T + DT
        LOCATE 1, 1: PRINT USING "Time : ###.### "; T

420     KEY$ = INKEY$ ' See if key is pressed to stop program

        NEXT I
430     WEND                ' Start while loop again to calculate
                            ' next position
        STOP
```

Numerical Solution of the N-Body Problem Using the Runge-Kutta Method

This program illustrates the general solution of the n-body problem using the Runge-Kutta method. Six specific masses are stated in the program. This is just to illustrate how masses can be included in the program. To generate solutions, masses must be included in the program with required parameters stated. More masses can be added, and those already stated can be changed completely or partly to generate the desired n-body problem.

```
70     G = 6.67259E-11  ' Universal gravitational constant
80     P = 2            ' Central force = G M m / r^P
90     N = 6            ' N = number of bodies

                        X            Y          VX           VY          M
100    DATA      2.5E+10,          0,          0,    .155E+41,      1E+30
110    DATA     -2.5E+10,          0,          0,  -1.155E+$-1,         51
       DATA            0,          0,          0,            0,      1E+24
       DATA        .5E+11          0,          0,        4.8E+4   .33E+24
       DATA       1E+11,           0,          0,       3.5E+4,      5E+24
       DATA            0,   1.5E+11,       -3E+4             0,      6E+24

120    FOR I = 1 TO N
130    READ X(I), Y(I), VX(I), VY(I), M(I)
140    NEXT I

150    T = 0: DT = 360       ' Initial time and increment

                             ' Establish screen parameters

200    SCREEN 12             ' VGA resolution
210    RAD = 5E+10           ' Screen RADius

       ' x distances times 1.33333 so that circles look circular
220    WINDOW (_1.33333 * RAD, RAD)-(1.33333 * RAD, -RAD)
230    CLS

       ' Calculate next position
```

```
300   PSET (0, 0)          ' Mark center of coordinates

310   WHILE KEY$ = ""     ' Loop until key pressed

      ' Find AX(I) and AY(I) components of acceleration on I-th mass
400   AX(I) = 0: AY(I) = 0
410   FOR J = 1 TO N
420      IF J <> I THEN
430         DELTAX(J) = X(J) - X(I)
440         DELTAY(J) = Y(J) - Y(I)
450         R(J) = DELTAX(J) ^ 2 + DEL:TAY(J) ^ 2) ^ .5
460         AX(I) = AX(J) + G * M(J) * DELTAX(J) / R(J) ^ (P + 1)
470         AY(I) = AY(J) + G * M(J) * DELTAY(J) / R(J) ^ (P + 1)
480      END IF
490   NEXT J

      ' Find dVX, dVY, dX, and dY on the left of our interval
500   DVX(I) = AX(I) * DT
510   DVY(I) = AY(I) * DT
520   DX(I) = VX(I) * DT
530   DY(I) = VY(I) * DT

      ' Estimate VXR = VX on right side of our interval, VYR = etc
600   VXR(I) = VX(I) + DVX(I)
610   VYR(I) = VY(I) + DVY(I)
620   XR(I) = X(I) + DX(I)
630   YR(I) = Y(I) + DY(I)

640   NEXT I

700   FOR I = 1 TO N

800   AXR(I) = 0: AYR(I) = 0
810   FOR J = 1 TO N
820      IF J <> I THEN
830      DELTAX(J) = X(J) - X(I)
840      DELTAY(J) = Y(J) - Y(I)
850      R(J) = (DELTAX(J) ^ 2 + DELTAY(J) ^ 2) ^ .5
860      AX(J) = AX(I) + G * M(J) DELTAX(J) / R(J) ^ (P + 1)
870      AY(J) = AY(I) + G * M(J) DELTAY(J) / R(J) ^ (P + 1)
880      END IF
890   NEXT J
```

```
900    DVXR(I) = AXR(I) * DT
910    DVYR(I) = AYR(I) * DT
920    DXR(I) = VXR(I) * DT
930    DYR(I) = VYR(I) * DT

950    DVXA(I) = (DVX(I) + DVXR(I)) / 2
960    DVXY(I) = (DVY(I) + DVYR(I)) / 2          ' etc
970    DXA(I) = (DX(I) + DXR(I)) / 2
980    DYA(I) = (DYH(I) + DYR(I)) / 2

1000   VX(I) = VX(I) + DVXA(I)    ' new VX = old VX + dVX average
1010   VY(I) = VY(I) + DVYA(I)    ' etc
1020   X(I) = X(I) + DXA(I)
1030   Y(I) = Y(I) + DYA(I)

1100   PSET (X(S), Y(I)), I + 1        ' Plot position on screen
1110   T = T + DT
1120   ' Locate 1, 1: PRINT USING "Time : ###.### "; T

1130   KEY$ = INKEY$       ' See if key is pressed to stop program

1140   NEXT I

1150   WEND                ' Start while loop again to calculate
                           ' next position
2000   STOP
```

CHAPTER 5

Summary of Findings, Conclusions, and Recommendations

Introduction

This chapter summarizes the content of the n-body problem solved by the researcher. Conclusions on the introduction of the n-body problems in the college preparatory and undergraduate curriculum is discussed. The researcher concludes this chapter with recommendations concerning the development of a syllabus that will integrate mathematics and physics as a course to be offered.

Summary of Findings

The researcher has solved specific n-body problems that are appropriate at the undergraduate level. The researcher has found that topics suitable for undergraduate and college preparatory students are:

1. Analytic solution of a two-body (two equal masses) problem rotating in a circle.
2. Analytic solution of a two-body problem with a third body not in motion but bigger than the two equal masses in motion.
3. Analytic solution of a two-body problem involving two unequal masses.

4. Analytic solution of a four-body problem. Four masses of equal masses rotating in a circle.
5. Analytic solution of a six-body problem with a seventh mass of different size at the center.
6. Analytic solution of an eight-body problem.
7. The numerical solution of a two-body problem by Euler's method.
8. The numerical solution of a two-body problem by Runge-Kutta.
9. The numerical solution of a three-body problem by the Euler method.
10. The numerical solution of a three-body problem by the Runge-Kutta method.
11. The numerical solution of an n-body problem by the Euler method.
12. The numerical solution of an n-body problem by the Runge-Kutta method.

Conclusion

Based on the solutions provided at the elementary level, a course in the n-body problem for undergraduate and college preparatory students can be developed.

Recommendations

Applied mathematics is not popular in the high school mathematics curriculum. The introduction of a course in physics with mathematics will generate interest in applied mathematics. This course could be taught to college preparatory juniors and seniors throughout the college undergraduate curriculum. The success of this program will gradually create a pool of future applied mathematicians who can devote time to the n-body problem.

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