12-31-2004

Three-dimensional shape characterization for particle aggregates using multiple projective representations

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by

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A Thesis Submitted to the

Graduate Faculty in Partial Fulfillment of the

Requirements for the Degree of

MASTER OF SCIENCE

Department: Electrical and Computer Engineering
Major: Engineering (Electrical Engineering)

Approved: Members of the Committee

In Charge of Major Work

For the Major Department

For the College

Rowan University
Glassboro, New Jersey
2004

ABSTRACT

Shape descriptors are used extensively in computer vision/automated recognition applications such as fingerprint matching, robotics, character recognition, etc. The conventional two-dimensional shape descriptors used in these applications do not readily lend themselves to compact representations in three dimensions. The situation is even more challenging when one attempts to numerically describe the three-dimensional shapes of a mixture of objects such as in an aggregate mix.

The goal of this study is the design, development and validation of automated image processing algorithms that can estimate three-dimensional shape-descriptors for particle aggregates. The thesis demonstrates that a single set of numbers representing a composite three-dimensional shape can be used to characterize all the varying three-dimensional shapes of similar particles in an aggregate mix. The composite shape is obtained by subdividing the problem into a judicious combination of simple techniques – two-dimensional shape description using Fourier and/or invariant moment descriptors, feature extraction using principal component analysis, statistical modeling and projective reconstruction.

The algorithms developed in this thesis are applied for describing the three-dimensional shapes of particle aggregates in sand mixes. Geomaterial response such as shear strength is significantly affected by particle shape – and a numerical description of shape allows for calculation of functional characteristics using other previously established models. Results demonstrating the consistency, separability and uniqueness of the three-dimensional shape descriptor algorithms are presented.

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ACKNOWLEDGEMENTS

First and foremost I would like to express my appreciation for the guidance given to me by my thesis advisor, Dr. Shreekanth Mandayam. He has offered a considerable amount of his time to assist me in every step of my research. Next I would like to thank Dr. Beena Sukumaran, whose patience allowed me to ask innumerable questions about the area of study I knew least about. Without their help and faith in me, I would never have received this opportunity. I would also like to note special thanks to Dr. Robi Polikar for committing his time through the revisions of my thesis.

In addition to my advisors, I would like to thank my family and friends for supporting me throughout my graduate studies. Pursuing my master's degree has not been easy inside or outside of classroom. They have helped me tremendously along the arduous path that they knew I wanted to follow. Most of all I would like to thank my fiancée Melissa Ross, whose understanding allowed me to spend the time I needed on my research.

My friends and colleagues from the graduate ECE program have also assisted me in my research, particularly, Michael Kim and Scott Papson who both helped me with some of the implementation. All of the other students helped to make the entire experience an enjoyable one. Finally, I would also like to thank the National Science Foundation, Division of Civil and Mechanical Systems, Geomechanics and Geotechnical Systems Program, Award #0324437 for supporting this work.

CHAPTER 1: INTRODUCTION

Describing purely the shape of an object regardless of its size can sometimes be a trivial task. A simple object such as a circle or a square may be quickly identified and recognized by the use of a single word. If the shape needs to be depicted later, the word describing the shape is the only information necessary to replicate it. Oftentimes in the real world shapes are not as simple as squares and circles. There are only so many words to describe shapes with increasingly more sides, until they simply become n-sided polygons. In addition to this, the shapes are not always equilateral and equiangular; therefore lengths and angles of all sides must be recorded in order to recreate a complicated shape. When shapes are seen as irregular objects with no specific length or angle pattern, words are no longer adequate for describing their boundary.

The best solution is to use a set of numbers to capture the behavior of the edge of an object. This could be something as simple as finding the length and angle of each portion of the shape, or something more complex, which may relate many features together using fewer numbers. Numbers offer a quantitative way to examine shape boundaries and allow computers to evaluate the data. Computers can be used to process large amounts of data retrieved from many images to make an automated process for analyzing shapes. The numbers obtained by characterizing a shape can be used to spot generic trends or patterns that relate values to physical geometric properties. This could be helpful, for example, when trying to determine the roundness or angularity of a shape. Numbers describing shapes with smooth curves may show a trend towards one end of the scale, while shapes with sharp angles and bends may be at the other. Similar shapes will have similar characterizing numbers and therefore can be distinguished from dissimilar shapes through the use of algorithms that exploit these numbers.

1.1 Common Applications for Shape Characterization

Shape characterization is used in many fields of research and has an extensive list of application areas. Describing objects using numbers plays a significant role in computer vision for recognition and classification. Character recognition, face recognition and automated fingerprint recognition are made possible through the use of shape description. Figure 1.1 illustrates a fingerprint matching application that makes use of shape descriptors [1]. The figure uses invariant moments to distinguish between two similar fingerprints and two different ones.

Figure 1.1: Example of shape characterization to identify fingerprint matches.

In some recognition cases, a direct comparison with another image may not be needed as shown in the fingerprint example above. In character recognition a database of descriptor calculations could be stored for each letter or number. In this way any new character presented to the algorithm could be compared to the descriptor values of each character in the database and match itself to the most similar. By describing each possible character in terms of numbers instead of as a reference image, computer resources including processor power and system memory can be saved [2]. When dealing with a large database of objects, this can be extremely helpful. In Figure 1.2 below an example of a database of descriptors being used to classify a new object is shown for character recognition. For simplicity the database only has descriptors for two characters; **a** and **b**.

Character	Descriptors	
a →	Phi 1:	1.0292
	Phi 2:	2.5359
	Phi 3:	8.917
	Phi 4:	14.1381
	Phi 5:	29.2098
	Phi 6:	15.4456
	Phi 7:	29.1866

Descriptor Database

	a	**b**
Phi 1:	1.058	1.2377
Phi 2:	2.664	3.403
Phi 3:	9.4284	7.8057
Phi 4:	14.2453	13.702
Phi 5:	29.8432	27.6783
Phi 6:	16.0222	16.1285
Phi 7:	29.4245	28.2324

Figure 1.2: A character being analyzed and compared to a previously calculated database.

1.2 Motivation

The need for automated shape description conventionally arises in two-dimensional imaging applications related to computer vision. There has not been a significant need for numerical descriptors for three-dimensional shapes – and conventional two-dimensional shape descriptors do not readily lend themselves to compact representations in three dimensions. The situation is even more challenging when one attempts to numerically describe the three-dimensional shapes of a mixture of objects.

The application that is the focus of this thesis is describing the three-dimensional shapes of particle aggregates in a sand mix, which is of considerable importance to Civil and Environmental engineers. The shape characteristics of sand grains in a particular soil have a large impact on the geotechnical properties of the entire mixture. The size, shape, and way sand particles interlock with one another are all factors affecting a soils behavior when loads are applied. There are three major categories that affect the stress-strain behavior of soils; inherent particle characteristics, geologic and environmental [3]. Figure 1.3 shows these categories and gives examples of each.

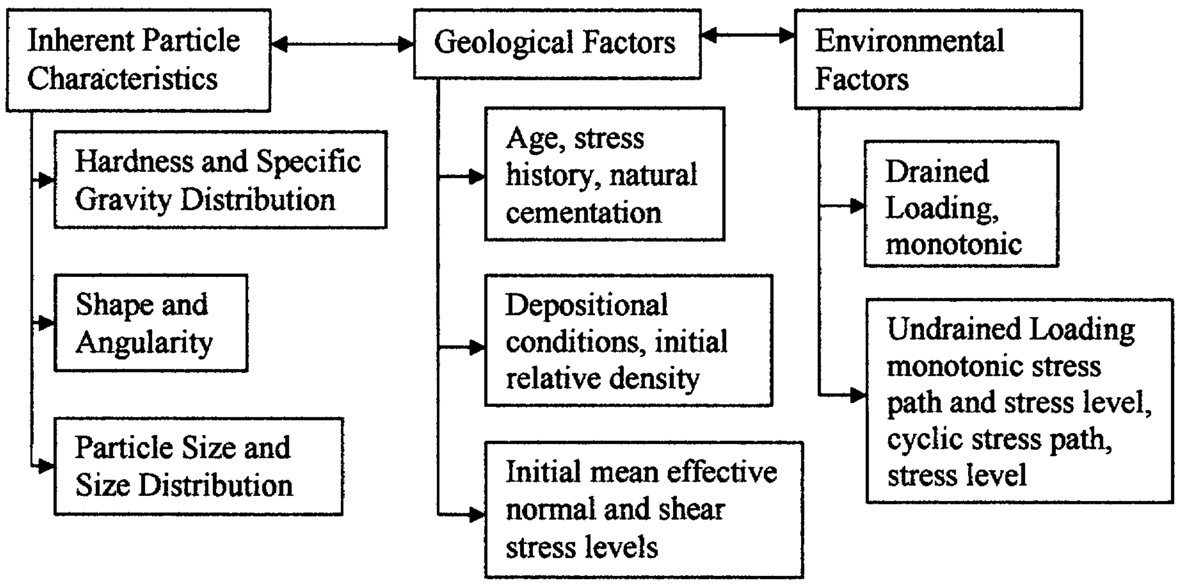


Figure 1.3: Factors affecting the stress-strain behavior of soil [3].

Standard techniques exist through which most of these factors can be quantified for further examination. For instance, particle size and size distribution can be calculated using a sieve analysis. This is when a sample of the soil is placed on a mesh screen and then is sifted down into another screen with a smaller mesh. This process then continues and the mass retained in each sieve is recorded. Specific gravity distribution also has well documented techniques for calculating by measuring the displacement of water. The shape and angularity of a particle is the only inherent particle characteristic that still needs an effective algorithm to quantify.

The shape of sand particles affects the particle to particle interaction which is known as the friction angle between aggregates in the mix. These friction angles determine the strength of a sand and are extremely important when trying to understand the properties of natural soils.

Compaction of the soil with its minimum and maximum void ratios, a measurement of the space between the particles in a mix, are also greatly dependant on the shape of the particles [3]. For instance, more rounded sands will typically have less compaction and lower yield strength than a mix of soil with more jagged particles. In this way, shape information can be more important than other inherent particle characteristics, and yet a definitive method does not exist for calculating this feature.

The quantification of shape and angularity will enable the inter-relationship between shape and shear strength to be determined. A qualitative understanding of the relationship between shear strength and shape already exists; with a quantification of shape parameters a more quantitative relationship can be obtained. Reconstruction of the particles will allow more realistic computer models using the discrete element method to be developed. These models can then be used to observe inter-particle contact forces and microstructure effects on shear strength, which will enable better constitutive models to be developed.

The problem with these methods lies in the difficulty of finding valid data that provide three-dimensional description of aggregates in a mix. Two-dimensional models have been developed using optical microscopy observations; but they are not very accurate and are only reliable for charting behavior trends [4]. In order to develop a more realistic model that closely replicates actual tests, a three-dimensional model is essential. However, the information required for developing three-dimensional models is difficult to obtain, since most existing methods require expensive equipment and large computational resources. Figure 1.4 shows an X-ray tomographic reconstruction of a single Melt Sand particle – the digitization and reconstruction process for this experiment took approximately 2 hours.

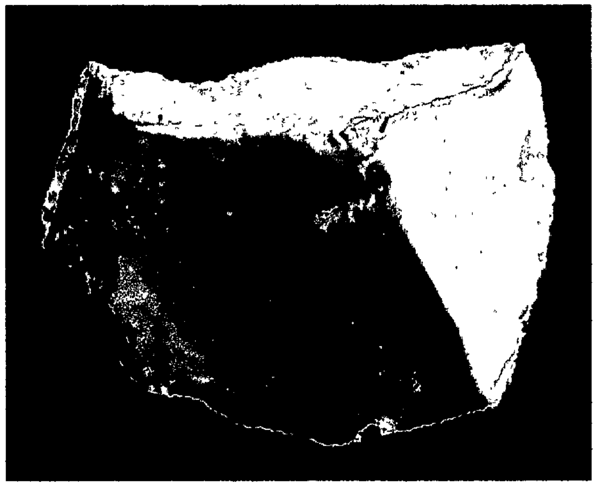


Figure 1.4: X-ray tomography of a Melt Sand particle.

The concept of finding shape numbers to describe three-dimensional shapes is not trivial. Instead of discretizing a flat, continuous shape, which would only require an x and y coordinate; a z coordinate, is also necessary. By finding all three coordinates, the shape would be described in layers, where a set of x and y coordinates would be needed for every different value of z. This would dramatically increase the number of points required to analyze the boundary of a shape. Since a significant number of particles from a mix would need to be observed, each with varying three-dimensional coordinates, this technique of direct three-dimensional characterization is not efficient. As will be discussed in this thesis, finding a two-dimensional approach for characterizing the shapes of three-dimensional particles in a mix will be rapid, computationally efficient and parsimonious. The imaging techniques required for developing the method will also be inexpensive (optical microscope and digital camera). Furthermore, such a technique would allow for the use of the extensive work already done in the area of two-dimensional shape description.

In addition to finding descriptors to characterize different shapes of sand, the technique must also be able to reconstruct a three-dimensional object from this data, in order to be useful for the discrete element model. The reconstruction procedure must be able to combine a set of two-dimensional projections to estimate a three-dimensional particle. Developing a method as easy to implement as a two-dimensional characterization algorithm, but with the ability to obtain the accuracy of a three-dimensional computer model, would be extremely helpful in soil analysis.

1.3 Objectives, Scope and Organization of Thesis

The goal of this thesis is the design, development and validation of automated image processing algorithms that can estimate three-dimensional shape-descriptors for particle aggregates. The specific research objectives are:

1. The design and development of automated algorithms that can estimate three-dimensional shape-descriptors for particle aggregates using a statistical combination of two-dimensional shape-descriptors from multiple two-dimensional projections.
2. The demonstration of consistency, separability and uniqueness of the three-dimensional shape-descriptor algorithm by exercising the method on a set of sand particle mixes.
3. Preliminary efforts towards the demonstration of the algorithm's ability to accurately and repeatably construct composite three-dimensional shapes from multiple two-dimensional shape-descriptors.

The three-dimensional shape characterization technique developed in this thesis is exercised on 5 different aggregate mixes. A database containing a library of approximately 200-300 two-dimensional digital images for each aggregate mixture was constructed for this purpose. In addition, a set of X-ray tomographic reconstruction images of a single Melt Sand particle was obtained for qualitative validation of the ability of the algorithm to reconstruct three-dimensional shapes from two-dimensional projections.

This thesis is organized as follows. Chapter 1 establishes the problems associated with three-dimensional shape characterization and describes the specific application for geomaterial aggregates. Chapter 2 provides a survey of literature on previously established methods for two- and three- dimensional shape characterization. Chapter 3 describes the proposed three-dimensional shape characterization technique that is developed in this thesis. Methods proposed for "reconstructing" a composite three-dimensional shape from two-dimensional projections are also discussed. The experimental set up for imaging geomaterial aggregates is described in Chapter 4. Results obtained by exercising the proposed characterization and "reconstruction" techniques are provided. The thesis concludes in Chapter 5 by making observations about the effectiveness of the algorithms developed and explores avenues for further improvement in research results.

1.4 Expected Contributions

This thesis expects to demonstrate that a single set of numbers representing a composite three-dimensional shape can be used to characterize all the varying three-dimensional shapes of similar particles in an aggregate mix. This composite shape will be obtained by numerically describing the two-dimensional shapes of a set of two-dimensional images of the individual particles in the soil mixture. It is expected that the individual two-dimensional images can be used as projective representations for arriving at the composite three-dimensional shape descriptors for all the particles in the aggregate mix.

CHAPTER 2: BACKGROUND

All techniques used for shape description must contain four fundamental qualities in order to be effective [5].

Uniqueness – An algorithm's ability to distinguish between two different shapes. A set of numbers for a particular shape should be unique to only that shape.

Parsimony – The set of numbers found for a particular shape should be as small as possible. The fewer numbers used for description, the less susceptible the method is to noise.

Independent – Each descriptor should be independent of the next. One descriptor should not be based on the outcome of another.

Invariant – The descriptors should not be dependant upon the orientation of the shape. Similar shapes should have similar descriptors even if they are rotated, translated, or scaled versions of one another.

Although generally a good method will possess all of these qualities, special applications may need to identify orientation as well as shape. In such cases, the invariance quality is not desired and need not be included. All of the two dimensional techniques discussed in this section attempt to possess all the major qualities including invariance, and wherever possible, three additional qualities that make them more useful [5]. These qualities are listed below.

Reconstruction – Allows a shape to be reconstructed from its descriptors. This can be an extremely useful technique for compression.

Interpretation – This is the amount of physical relationship between the descriptor and the actual shape.

Automatic Collection – The algorithm's ability to automatically collect and analyze data. Removes human error and makes processing quicker.

2.1 Previous Work

Table 2.1: Summary of previous techniques used to describe shape.

Proponents	Method	Explanation	Application
Wentworth [6]	Elongation and Flatness, Roundness of sharp corners	One of the first to characterize form and roundness. Opened the field for many of the subsequent studies	Used a variety of sand types including, Conglomerate, Breccia, and Sandstone
Wadell [6]	Sphericity	First method developed to measure the sphericity of a particle to characterize its form	Wadell attempted to quantify the shape of quartz particles
Sebestyn and Benson [5]	"unrolling" a closed outline	The concept of creating a 1-D function from a 2-D boundary. Introduced by Benson into the field of geology.	Benson introduced this concept to geology using a paleontology application
Ehrlich and Weinberg [7]	Radius Expansion	Introduced Fourier analysis for radius expansion into sedimentology.	Used a range of particles from smooth to very angular
Medalia [5]	Equivalent Ellipses	Fits an ellipse to have similar properties to the actual shape. Does not need outline.	Tested on carbon black aggregates for both 2-D and 3-D
Davis and Dexter [5]	Chord to Perimeter	Measures chord lengths between various points along an outline.	Measured irregularities of many soils
Zahn and Roskies [5]	Angular Bend	Zahn and Roskies discretized an outline into a series of straight lines and angles	Developed method using arbitrary closed curved shapes.
Garboczi, Martys, Saleh, and Livingston [8, 9]	Spherical Harmonics	A process similar to 3-D Fourier analysis, and requires 3-D information.	Applied to aggregates used in concrete captured using X-Rays
Sukumaran and Ashmawy [10]	Shape and Angularity Factor	Compares shapes to circles and measures their deviation. Uses a mean and standard deviation of many particles to compare mixes.	Algorithms applied to various types including Michigan Dune, Daytona Beach and a few kinds of Ottawa.

Previous work done in the field of shape description, primarily for two-dimensions, is summarized in Table 2.1. The next section will explain the two-dimensional techniques mentioned in Table 2.1, where only images from an optical microscope are necessary. Two of the methods from this section are implemented in the algorithms developed later in this thesis. The section following the two-dimensional techniques provides an explanation of two currently used techniques for obtaining three-dimensional shape descriptors using three-dimensional data.

2.2 Two Dimensional Shape Description Techniques

There are two types of shape description categories, boundary and planar surface techniques. Boundary techniques are only concerned with the actual boundary of the object and usually require "unrolling" the boundary to become a one-dimensional function. Planar surface techniques deal with the entire image and have to take special care to maintain orientation invariance [5]. Examples of both methods are shown in the following sections.

2.2.1 Boundary Techniques

This category can be broken down into two parts, one using Fourier analysis and the other using distributional approaches. For Fourier analysis a periodic function must be obtained from the boundary. This method allows reconstruction, through the use of the inverse Fourier Transform, and compression by removing the higher frequency values that hold some of the fine detail. As long as the low frequency values, which hold the general shape information, are kept, Fourier analysis can be a parsimonious and effective technique that offers reconstruction. The general concept of capturing the prominent low frequencies, while eliminating the negligible higher frequencies of the Fourier Transform is shown in Figure 2.1.

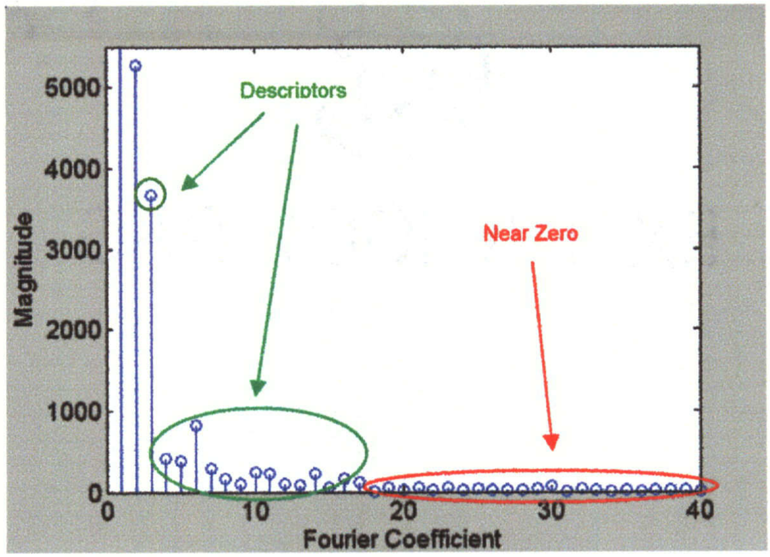


Figure 2.1: Illustration of Fourier analysis descriptors.

Distributional approaches do not allow reconstruction, but are often easier to make invariant to orientation, since they do not care about the sequence of the boundary. Also the distributional approaches are usually more statistically friendly, and when attempting to find statistical similarities in three-dimensional objects, could prove more useful. The methods of "unrolling" the boundary and turning it into a function often can be used for both approaches [5]. The rest of this section will describe the boundary description methods and then offer possible Fourier or distributional analyses that could be done with them.

2.2.1.1 Radius Expansion

One way to describe the boundary of an object is to use a method called radius expansion. The purpose of this method is to find the centroid of an object and move around the border at specified angles and calculate its distance to the border [5, 13]. The distance can be calculated in polar coordinates, first at zero degrees, and then continually checked at certain degree intervals all around the border. The number of degrees between each point observed decides the resolution of this technique.

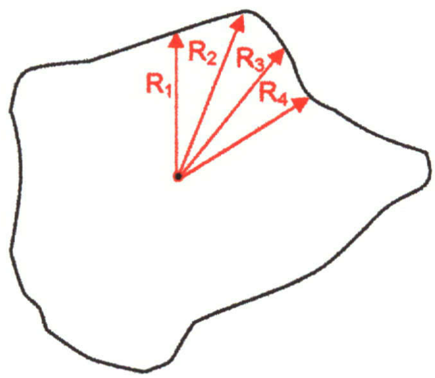


Figure 2.2: Example of first four points observed using radius expansion.

Once the points are all obtained, a periodic function can be created and analyzed. The major flaw in this technique is the possibility of multi-valued functions, where there are two possible amplitudes at a specific degree [7]. This is shown in Figure 2.3.

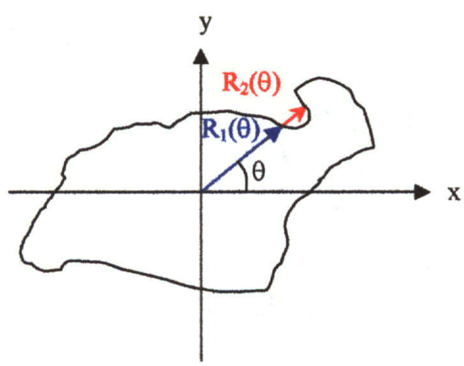


Figure 2.3: Multi-value example of radius expansion.

Fourier series analysis can be performed effectively on this technique, as well as, a distributional approach of finding a type of radius histogram of the shape. A distribution can be created that keeps track of the number of times particular radius ranges occur, but will not observe the angle they take place at and therefore will not allow reconstruction. A major flaw in this method is that even two dissimilar shapes can have similar distributions, such as a star and a kidney shape. Even though they are visually very different, they both have a large number of close and far amplitudes and could appear as the same object when only looking at its radius distribution.

2.2.1.2 Angular Bend

Angular bend is another method that can be used by both Fourier and distribution analysis. This technique draws a line between each discrete point of a boundary and calculates the angle at which each must be displaced to move to the next point. Since the angle from point to point is recorded reconstruction is possible with this method when using the Fourier series. The problem is that it cannot be compressed by removing the higher frequencies due to the fact that all errors are cumulative in the reconstruction. Each point relies on the accuracy of the last and often times when the Fourier series is truncated the boundary will cross itself or not connect at the end. The distributional approach finds a histogram of slopes, but similar to the radius distribution cannot be used for reconstruction, since the sequence of the slopes is not recorded [5]. Figure 2.4 illustrates this technique in detail.

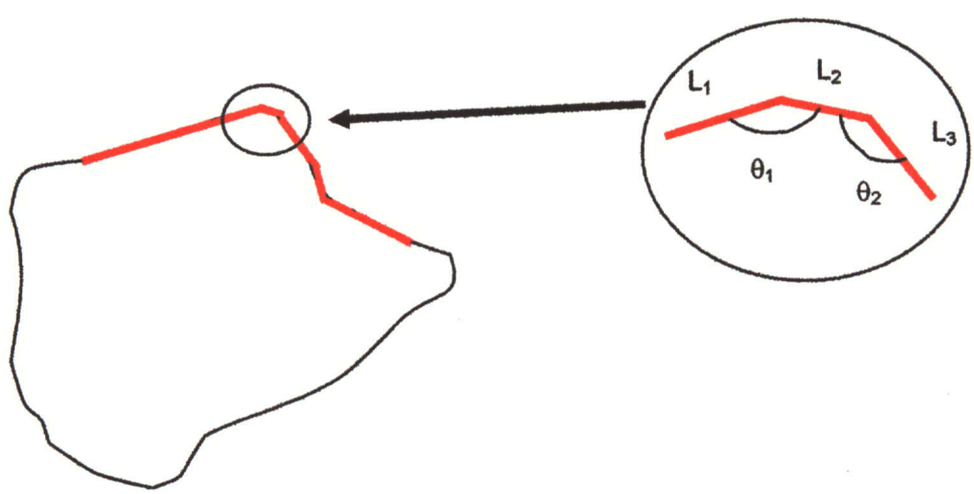


Figure 2.4: Example of angular bend.

2.2.1.3 Complex Coordinates

The last method discussed for Fourier analysis uses complex values to identify the boundary. This technique arbitrarily chooses a starting point on the boundary and then follows the shape around the edge recording all of the points. These values are stored as x and y coordinates and can be combined into one variable by making them complex with real and imaginary parts as shown in the equation $x + jy$. This new equation forms a periodic function describing the boundary of the object and can be analyzed using the Fourier Transform. The greatest advantage of this technique over the others is that the function decays more rapidly in the Fourier domain and allows for the best compression, while maintaining good reconstruction. This technique appears to be the most promising of all the boundary methods, when considering the qualities of an effective shape description method discussed earlier in this section [7].

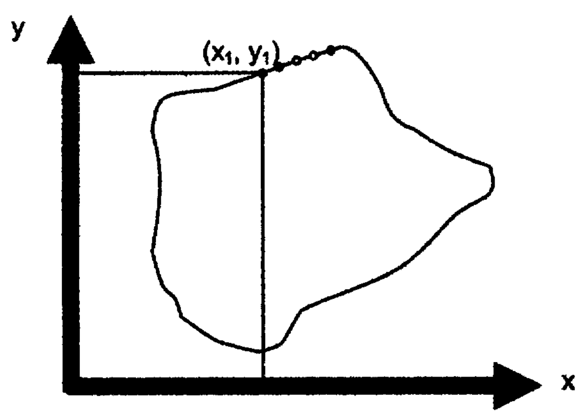


Figure 2.5: Example of complex coordinate boundary method.

2.2.1.4 Chord to Perimeter

The chord to perimeter method can only be made a distribution and cannot be used in conjunction with a Fourier series analysis. The objective of this technique is to compare the shape to that of a circle. This is done by calculating the distance between two points along the boundary, as well as, the distance of the perimeter that it encases. An example of this is shown in Figure 2.6, where the red line is the calculated distance between the two points and the bottom line represents the perimeter length from one point to the other.

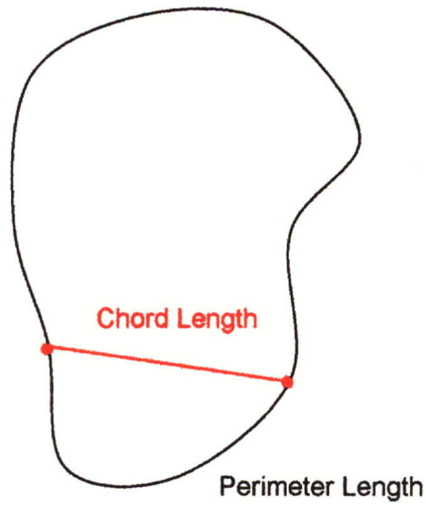


Figure 2.6: Example of chord to perimeter.

From these measurements, a ratio can be calculated by taking the perimeter covered between the two points and dividing by the total perimeter. This determines the irregularity of the boundary. Small ratios are used to measure small irregularities and as the ratio reaches one they begin to measure large irregularities. When these values are compared with those obtained from a circle, an asphericity spectrum can be created. The asphericity spectrum is simply a way to measure how similar a shape is to a circle. One limitation to this is that the objects being examined must be fairly round for the method to work properly, or else unusable results will be obtained [5].

2.2.2 Planar Surface

This category of techniques is useful for avoiding the need to locate the boundary of an object, since they use the entire image when processing. Often times these methods can be used to identify texture as well as shape, which when trying to classify objects could be an extremely useful extra feature. The major problem with these methods is that the location of an object in a picture could affect its calculations. In most shape description applications this could be a detrimental flaw and must be corrected in order to design effective shape description algorithms.

2.2.2.1 Equivalent Ellipses

This technique attempts to reduce a complicated shape into an ellipse that describes it. This is done by calculating the moments of inertia and principle axes of the object to create an equivalent ellipse. Two factors are extracted from these ellipses; anisometry and bulkiness. Anisometry is simply the ratio of the long to short axis and bulkiness is the ratio of the area between the original object and its ellipse [5]. One advantage of this method is that it is easily interpreted to physical characteristics of the shape.

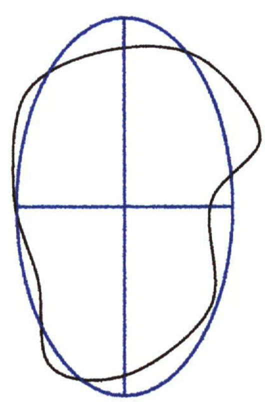


Figure 2.7: An object and its equivalent ellipse.

2.2.2.2 Two-Dimensional Invariant Moments

The last shape characterization method discussed uses a combination of two-dimensional moments. Statistical values such as mean, variance, and higher order moments can be used to make statistically well behaved descriptors [14, 15]. Similar shapes should have similar moment calculations and therefore can be used for characterization. As mentioned before, techniques such as two-dimensional moments, which deal with the entire image, are prone to errors through scale and rotation changes. This problem was addressed in a paper written by M. K. Hu [6]. He proposed using a combination of moments to create a set of invariant moments, which can characterize any image using only seven numbers. The general equation for a two-dimensional moment of a continuous function is given as:

$$m_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^p y^q f(x,y) dx dy \tag{2.1}$$

where p and q represent the order of the x and y moments respectively. These moments can be centralized by subtracting out the means, and these central moments can be written as:

$$\mu_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x-\bar{x})^p (y-\bar{y})^q f(x,y) dx dy \tag{2.2}$$

where p and q represent the order of the x and y moments respectively. These equations are for continuous functions and are not useful for images, which are discrete. The equations can simply be changed for images by summing the values over all the pixels instead of calculating the function integrals. The new formula is shown below.

$$\mu_{pq} = \sum_x \sum_y (x-\bar{x})^p (y-\bar{y})^q f(x,y) \tag{2.3}$$

where p and q represent the order of the x and y moments respectively. The $f(x,y)$ refers to the image's gray level value of the pixel at each x and y. This equation shows how every pixel in the image is used in the calculation and then they are summed to obtain the central moment. These moments can be normalized by dividing by the zero moments raised to the power of gamma as defined below.

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{\gamma}} \tag{2.4}$$

where,

$$\gamma = \frac{p+q}{2} + 1 \tag{2.5}$$

The use of these normalized moments lead to the creation of Hu's invariant moments. The seven invariant moments are only shown below; a complete derivation can be found in a paper written by Hu [16].

$$\phi_1 = \eta_{20} + \eta_{02} \tag{2.6}$$

$$\phi_2 = (\eta_{20} + \eta_{02})^2 + 4\eta_{11}^2 \tag{2.7}$$

$$\phi_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2 \tag{2.8}$$

$$\phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2 \tag{2.9}$$

$$\begin{aligned}\phi_5 = {} & (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})\left[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2\right] \\ & + (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03})\left[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2\right]\end{aligned} \tag{2.10}$$

$$\phi_6 = (\eta_{20} - \eta_{02})\left[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2\right] + 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03}) \tag{2.11}$$

$$\begin{aligned}\phi_7 = {} & (3\eta_{21} - \eta_{03})(\eta_{03} + \eta_{21})\left[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2\right] \\ & + (3\eta_{12} - \eta_{30})(\eta_{21} + \eta_{03})\left[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2\right]\end{aligned} \tag{2.12}$$

The most significant advantages of this method are its ease of implementation and its small number of descriptors. This technique only has seven descriptors, in comparison with Fourier analysis, which usually needs at least ten and oftentimes more. Moments are a fairly robust, easy-to-understand technique for describing shapes.

2.3 Three Dimensional Shape Characterization Techniques

Most algorithms for describing three dimensional shapes require the acquisition of the three dimensional objects. In this section two previously used three-dimensional description methods are presented and it is assumed that the coordinates of the objects being analyzed have already been obtained using a three dimensional imaging system. The most commonly used method to capture such objects is X-ray computed tomography. The above mentioned assumption is not a trivial one and oftentimes acquiring models via tomography can have a great number of problems in itself. Cost and resolution of a system, as well as, the time a reconstruction algorithm takes to build an object are all factors that must be considered and vary depending on the application. The two techniques being examined are spherical harmonics and three-dimensional invariant moments. Further discussions on the usefulness and efficiency of the algorithms presented will be noted at the end of this section.

2.3.1 Spherical Harmonics

Spherical harmonics express a shape in a more useful mathematical form [8, 9]. The ability to characterize an object as a set of values can be extremely useful in models that oftentimes use only spheres or ellipsoids to represent actual three dimensional shapes. As mentioned earlier, assuming a three dimensional object has already been obtained; this technique needs to locate the object using what is known as a "burning" algorithm by separating the background from the object. The particles are stored in a three dimensional matrix, where each voxel (three dimensional pixel) is represented by either a zero, for the background, or a one, for the object. The algorithm begins by searching the matrix until a one, indicating the object, is discovered and then find all the adjacent voxels that are also labeled as ones. All matrix values that are found to contain the object are stored as x, y, and z coordinates. In this way the entire object can be captured as a sequence of coordinates.

The next task needed to be performed is to find the location of a common center point. The centroid can be used for this and since the coordinates have already been obtained, this can simply be done by finding the average x, y, and z coordinates by adding up the location values in each axis and dividing by the total number of points. This center point does not need to be the centroid and can be chosen arbitrarily, but must remain consistent for all particles.

With the center point calculated, the characterization of the boundary shape can be performed. From the center point to the surface of the aggregate distances are measured at specific angle intervals. Two angles are necessary to obtain adequate three dimensional data, therefore both ϕ, ranging from 0 to 2π, and θ, ranging from 0 to π is used. Once all ϕ angles are obtained, θ is incremented and all of the ϕ angles are recalculated. When this is complete, a $r(\theta,\phi)$ function is created which can be used for further analysis. The equation for spherical components is then:

$$r(\theta,\phi)=\sum_{n=0}^{\infty}\sum_{m=-n}^{n}a(n,m)Y_n^m(\theta,\phi) \tag{2.13}$$

where $Y_n^m(\theta,\phi)$ is a spherical harmonic function of order (n,m) and $a(n,m)$ is a numerical coefficient. Orders for n are typically taken up to 20 or 30 for efficient characterization [8, 9].

2.3.2 Three-Dimensional Invariant Moments

This technique is an extension of the two-dimensional invariant moments, which are described in the next section of two-dimensional shape descriptors. A brief overview of three-dimensional moments is given here to portray the concept of this technique. The equation for a three-dimensional moment is given by,

$$m_{pqr} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x_1^p x_2^q x_3^r \rho(x_1, x_2, x_3) dx_1 dx_2 dx_3 \quad (2.14)$$

where p, q, and r signify the order of the moment and $\rho(x_1, x_2, x_3)$ represents the density of the object. The density function is assumed to be piecewise and continuous making it bounded [11, 12]. The equation above can be converted to a central moment, by subtracting out the centroid of the coordinates shown in the two equations below.

$$\mu_{pqr} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} (x_1 - \bar{x}_1)^p (x_2 - \bar{x}_2)^q (x_3 - \bar{x}_3)^r \rho(x_1, x_2, x_3) dx_1 dx_2 dx_3 \quad (2.15)$$

where,

$$\bar{x}_1 = \frac{m_{100}}{m_{000}}, \bar{x}_2 = \frac{m_{010}}{m_{000}}, \bar{x}_3 = \frac{m_{001}}{m_{000}} \quad (2.16)$$

Finally the equation is normalized using the following equation,

$$\eta_{pqr} = \frac{\mu_{pqr}}{(\mu_{000})^{\frac{p+q+r+3}{3}}} \quad (2.17)$$

This equation can be used to generate moments of a three-dimensional object. The major problem with this technique is the computation time required to analyze the moments. When moments become higher in order, the equations can become very complex making their implementation computationally expensive [11, 12].

The three-dimensional shape description techniques discussed all require the three-dimensional data of the particle, which can be obtained through X-ray tomography. Not only do they each rely on expensive equipment to do their analysis, but they also need significant processor power, in order to achieve results in a reasonable amount of time. Even if such technology is available, the specific nature of analyzing each individual grain of sand, which are innately different, such analysis may not be necessary. The generalization acquired by using two-dimensional descriptors could actually yield more effective results, by using an estimation based on statistics. Since no two sand particles are exactly alike, a statistical technique is preferable.

2.4 Principal Component Analysis

The general concept of PCA is to exploit patterns in a set of data in way that highlights the similarities and differences. When a dataset has many dimensions, PCA allows the most important components of the data to be isolated, which can reduce the number of dimensions with little loss of information. In this way, high dimensional data can be visualized in three-dimensions when plotted in what is known as PC-space.

PCA uses the variance of a dataset to identify the most important information. The technique assumes most of the classifying information lies along the axis with the most variance. The first principle component is the axis with the most variance and each subsequent component is calculated based on the axis with the next highest variance [17]. This is done by finding the mean vector, the mean of all instances about each descriptor, and covariance matrix of the data. The covariance matrix is calculated with the variance of the features along the main diagonal and the covariance between each pair of variables in the other matrix positions. Eigenvalues and eigenvectors are then computed and sorted by decreasing eigenvalues. The principal components are the projection of the data along these eigenvectors with the largest values being the most significant and oftentimes the smaller ones only contributing "noise". This allows the major principal components to be extracted, therefore reducing dimensionality and increasing separability.

Figure 2.8 illustrates how PCA works in two dimensions. In the figure two axes are chosen, which maximize the variance without reducing the dimensions. Later in this thesis PCA is applied to Fourier descriptor data to reduce the coefficients to three-dimensions. Only the first or major three axes would be chosen to represent the data, which originally has an extremely large number of axes. The specific application of PCA used in this thesis is discussed more in Chapter 4.

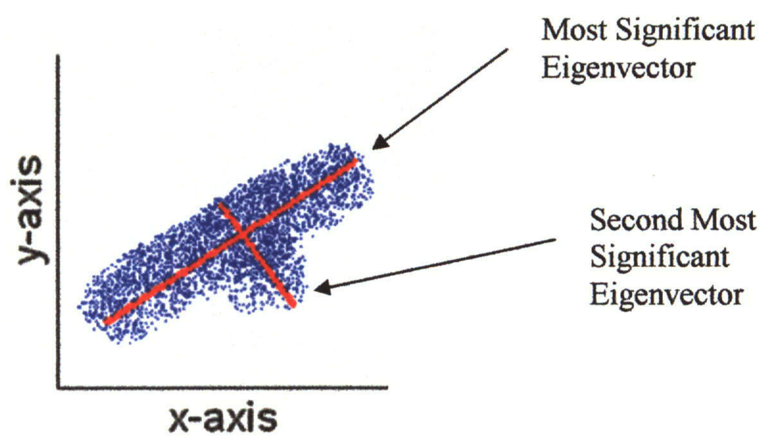


Figure 2.8: Example of PCA in two dimensions.

This chapter summarized some of the more popular methods for obtaining shape descriptors. Since this thesis focuses around finding a two-dimensional approach for acquiring three-dimensional shape descriptors, complex coordinate Fourier analysis and invariant moments were chosen to be implemented. Both a boundary and a planar surface method were chosen to attain two different processes with varying advantages of calculating descriptors. The Fourier method will allow reconstruction and the moments will offer a procedure, which does not require unrolling the boundary. The following chapters will discuss how these characterization techniques were applied in the overall procedure.

CHAPTER 3: APPROACH

As discussed in the previous chapters, finding a relatively simple method of describing three-dimensional shapes of soil aggregates would prove to be useful. Designing an algorithm, which will allow particles to be characterized for use in a discrete element model, would assist in increasing speed and accuracy of currently used models. It will also enhance the understanding of the influence of shape on shear strength. Implementing a method that uses more common equipment such as an optical microscope and digital camera will allow three-dimensional shape characterization to be available for a wide range of applications. Even a database of shape numbers calculated for common soil mixtures could be created for ubiquitous use. This chapter presents a possible approach for solving this problem. The information offered is broken down into four sections. The first will discuss the overall approach of the proposed technique and will lay the groundwork for detail in the later sections. The second and third sections will elaborate more on each two-dimensional description method being applied in the overall approach. The final section illustrates possible methods for reconstruction and validation of the procedure. At the end of the chapter is a summary of the approach. All the results for the techniques proposed in this chapter will be presented in the next chapter.

3.1 Overall Approach for the Proposed Technique

The general research approach taken in this thesis consists of two major parts. The first deals with obtaining effective shape descriptors that consistently characterize samples of sand through the use of two-dimensional projections of different particles within the mix. This will show how a set of similar three-dimensional particles can be described using a simpler two-dimensional technique. The second major part attempts to validate this procedure by reconstructing a three-dimensional object from the two-dimensional projections. When two-dimensional projections are taken from various angles of the newly created three-dimensional object, the descriptors should be similar to the corresponding mix.

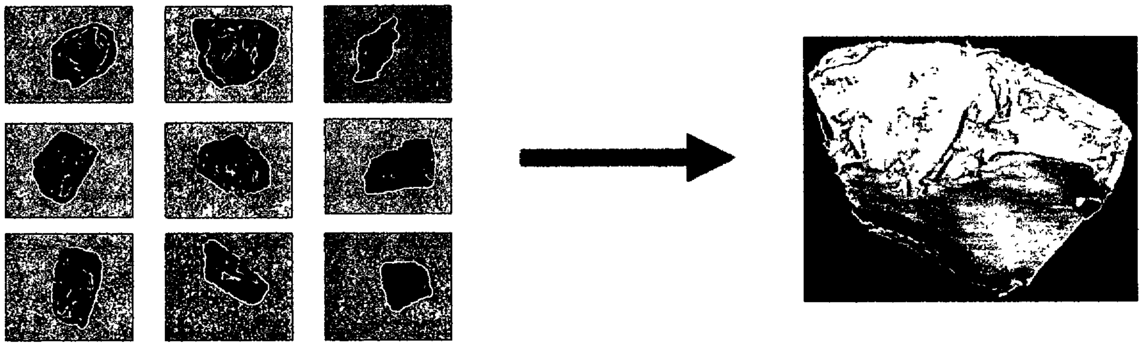


Figure 3.1: Premise for using 2-D projections to obtain a 3-D particle.

Once the technique has been proven, the descriptors themselves could be used to create its own projections of the mix and then applied to a reconstruction method in order to produce many three-dimensional models. In this way, an entire dataset of three-dimensional particles could be fabricated, that would fit the same descriptors as the actual mix. This allows a particular type of sand to be modeled on a computer without the need of having to use three-dimensional scans of thousands of particles to achieve an accurate representation of the soil. The dataset could then be applied to a discrete element modeling software program, which would allow many tests to be performed on the sand mixes that accurately represent real particles.

Figure 3.2 shows the overall approach of the proposed technique. The left side of the figure illustrates the process of finding three-dimensional descriptors by taking the statistical mean and variance of two-dimensional descriptors from a database of projections obtained from a mix of sand particles. The right side of the figure shows the validation and reconstruction portion of the procedure. A set of two-dimensional descriptors can be generated from the three-

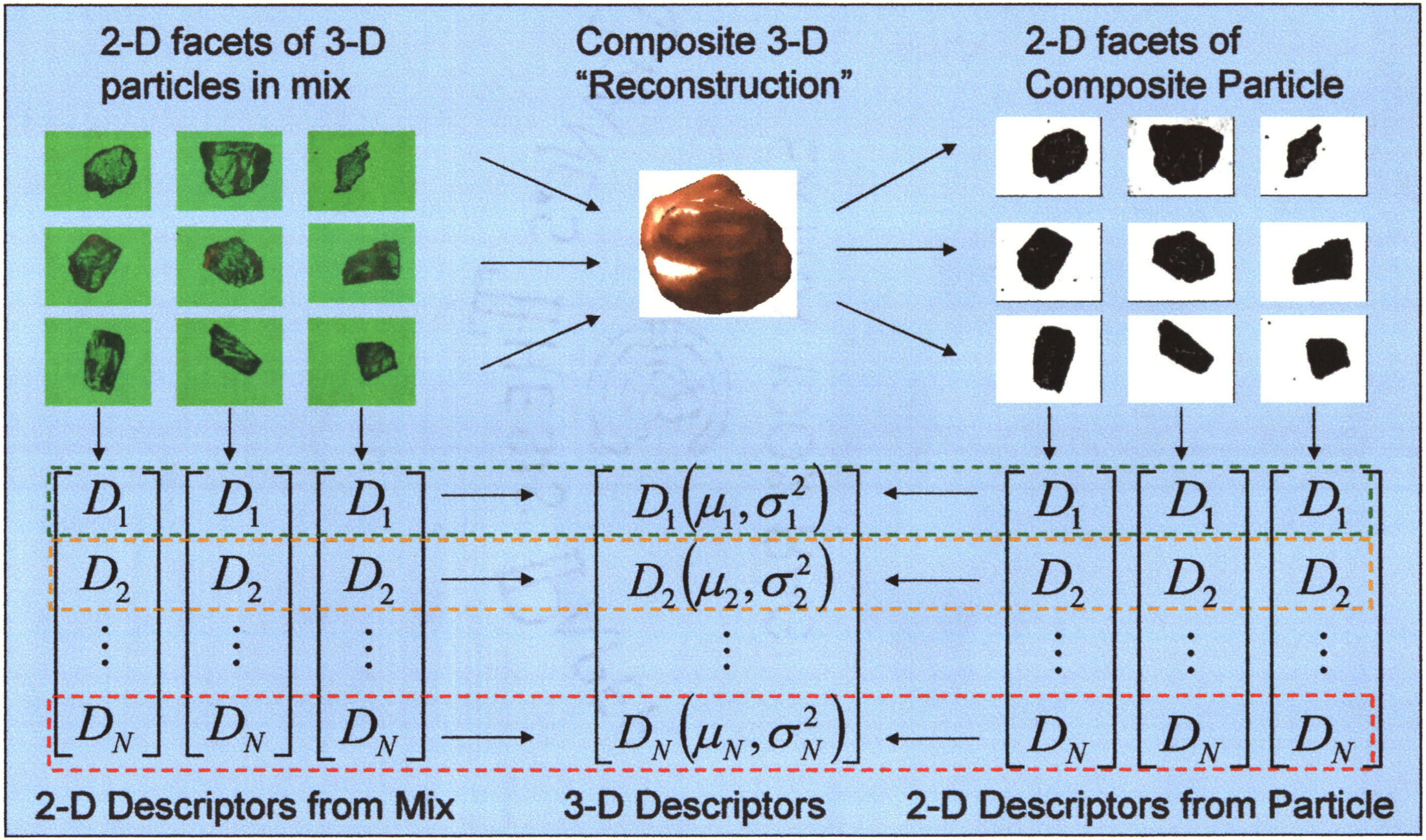


Figure 3.2: Overall approach for proposed technique.

dimensional characterization values and projections can be reconstructed. These projections can be used to build a three-dimensional object that is a representative particle of the mix.

In order for the presented procedure to be successful, some minor assumptions must be made. Every particle that is observed under the optical microscope depicts a different angle of the composite particle. If a large number of particles are used, eventually a sufficient number of particles should be able to represent every facet of the composite particle. In addition to this, all particles in a given sample set should have similar shapes. This regularity of the individual sand types is the basis for the premise.

The two-dimensional techniques used to calculate the descriptors are Fourier descriptors and invariant moment descriptors. Fourier descriptors allow the reconstruction and validation shown on the right side of the figure. Invariant moments are not reversible, but do offer an additional assurance, that the soils can be separated using the two-dimensional projections. The following two sections give more detail on each two-dimensional description procedure.

3.2 Three-Dimensional Aggregate Shape Description using Fourier Descriptors

One popular method for characterizing two-dimensional shapes is by using Fourier analysis. As discussed in the background chapter, many methods facilitate the use of the Fourier transform in order to describe the data. This thesis implements a method of expressing the two-dimensional projection in terms of complex coordinates. A two-dimensional outline can be converted to a one-dimensional function by tracing the boundary to obtain a collection of the x and y coordinates along it. An algorithm can be written to capture this one-dimensional function and can be plotted from the resulting x and y coordinates by plotting them as complex combination of $x + jy$.

Since the first and last points are the same, the function is known to be periodic and therefore suitable for Fourier analysis. In this way a two-dimensional picture of a sand particle can be reduced to a simple one-dimensional function, which contains all the necessary shape information for shape characterization. By transforming the one-dimensional function into the Fourier domain, the frequencies common among the border of the sand particle will be seen. This distinction can exploit differences in the outlines of particles in a mix. For example, a mix of jagged particles will contain higher frequencies than a mix of rounder particles. Figure 3.3 depicts the process of converting a two-dimensional projection into a one-dimensional periodic function.

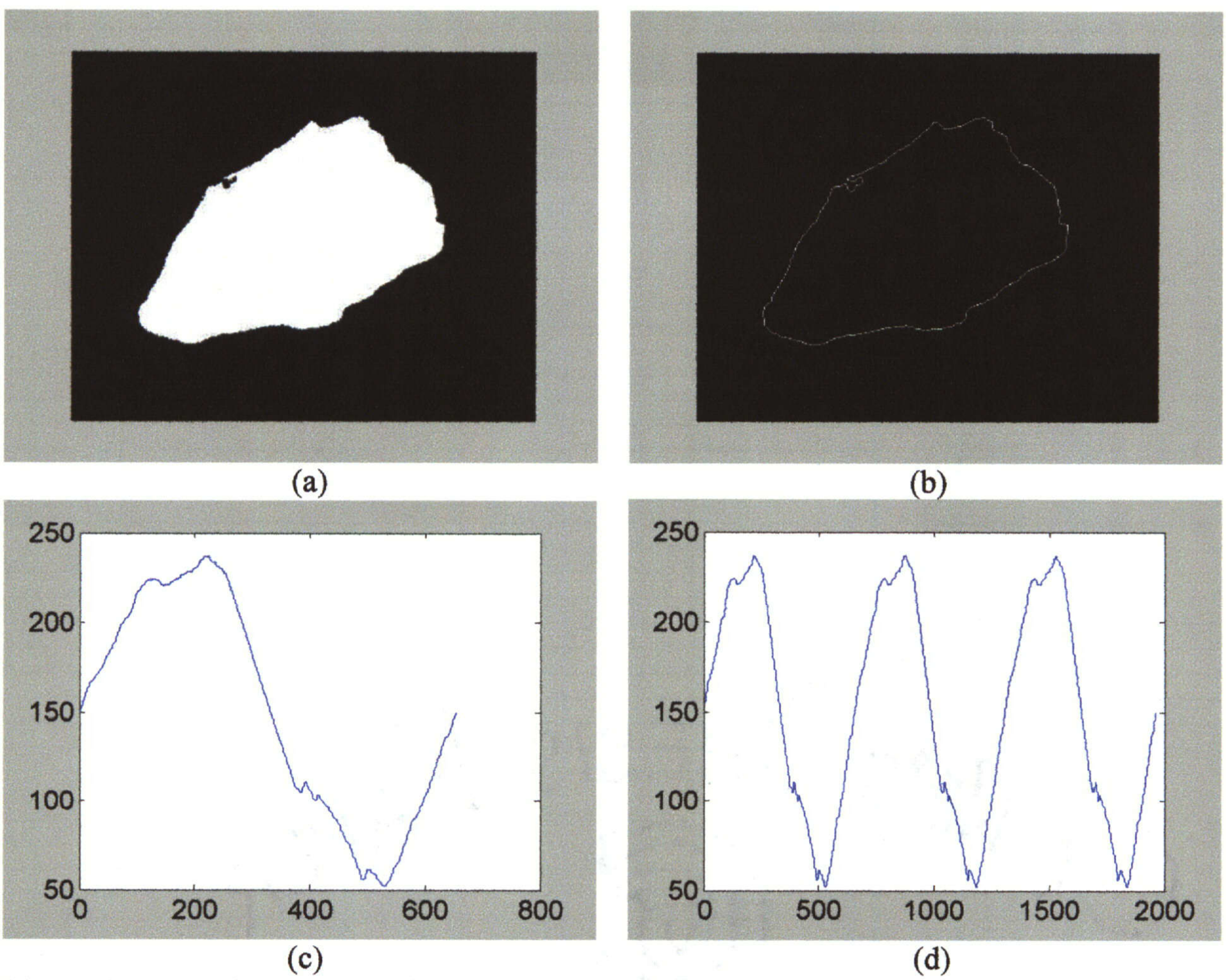


Figure 3.3: (a) Binary image of sand particle, (b) outline of particle, (c) 1-D function of particle, and (d) plot showing periodic nature of 1-D function.

3.2.1 Normalization of the One-Dimensional Function

Every particle examined will have a different number of points in the function, as well as, different Fourier amplitudes depending on the coordinate locations. In order for the Fourier transform to be useful, each one-dimensional function must be normalized so shape is the only factor considered. Without normalization, the size of the particle will affect the Fourier transform and may have different coefficients. Two particles with the same shape, even if they are different size, must have the same descriptors for the shape characterization method to be useful.

In order to normalize the functions, they can first all be resampled to the same number of points. This deals with finding the common coefficients between each particle. After being resampled, all coefficients match up and each frequency descriptor can be compared to an identical frequency in another. With the number of points in a particle no longer an issue, the amplitude adjustments must be made so size is not a factor in the shape descriptor values. This can simply be done by varying the original one-dimensional functions between -1 and 1. The absolute value of the largest number can be divided through the entire function to create a new function where all of the values are between -1 and 1. In this way, two identical particles of different sizes can still have the exact same one-dimensional functions. Results showing the normalization process will be described in the following chapter.

3.2.2 Using the Fourier Transform to Obtain Descriptors

With all of the data prepared as normalized one-dimensional functions, the Fourier analysis can begin in order to obtain the shape descriptors. Once the Fourier transform is taken, each coefficient can be considered a descriptor. The number of descriptors depends on the number of points the one-dimensional functions are resampled to. It is not beneficial to use all the descriptors, because many of them are close to zero, which makes them very sensitive to subtle changes.

The ideal set of descriptors to use lie in the middle of first half. The lower descriptors are used for identifying general shape and are similar even among very different particles; where as the higher descriptors are for identifying fine detail, which can be different even among very similar particles. For these reasons it is beneficial to use the middle descriptors to characterize different sand particle shapes.

Even after narrowing down the descriptors to use, there will still be a great number of Fourier coefficients left. In this thesis, in order to better visualize the description and classification process, principal component analysis (PCA) is used to reduce the number of descriptors to only three values. PCA can take any number of values and find the most efficient way to represent them as a smaller, more manageable number of values. It must be noted that PCA is used to classify and observe the differences between the sand mixes, but the original values obtained through the Fourier transform need to be used when attempting to reconstruct the particles. Reconstruction can not take place directly from the values received from the PCA. Later sections will discuss the reconstructions in more detail.

3.3 Three-Dimensional Aggregate Shape Description using Invariant Moment Descriptors

Two-dimensional invariant moments is a well established technique for comparing shapes. Moments can capture the similarities and differences among various shapes. In this thesis invariant moments are implemented to discover if diverse soil mixes are separable when using a two-dimensional characterization technique that does not need the outline. For Fourier analysis the outline of the particle must be acquired before the one-dimensional function can be created and the Fourier transform can be applied. Two-dimensional moments do not require finding the outline and can calculate descriptors from the binary images alone. This makes calculations quicker and offers an alternative solution to classifying soil mixtures by shape.

Invariant moments require much less preparation to implement than Fourier analysis. Since the method is already invariant to rotation, scale, and translation, no normalization of the projections is necessary. As discussed in detail in the background chapter, there are 7 invariant moment descriptors. This is a much smaller number of descriptors needed to classify shape using Fourier descriptors, making it a more parsimonious solution. The drawback of invariant moments is the inability to reconstruct a shape based on the 7 calculated descriptors. Fourier descriptors are needed for the reconstruction and validation attempts performed in this thesis.

Unlike Fourier analysis, PCA is not necessary when using invariant moments. Among the 7 moments calculated, some will not be as accurate at describing shape as others. By using PCA to reduce all 7 axes to only 3 axes may give worse results than simply choosing 3 of the descriptors. It could be used to realign the 3 axes for better separation without reducing the dimensions, but still may not be necessary. In this thesis, PCA was not used for the invariant moment descriptors.

3.4 Reconstruction and Validation

One of the objectives of performing shape description is to be able to input the information into a discrete element model. This could be achieved if three-dimensional particles could be created from the projections, which have the same statistics as the real soil mixture. Also by reconstructing these three-dimensional models, the overall concept of using two-dimensional projections to characterize three-dimensional shapes can be validated. The next few subsections discuss how more projections could be generated from the three-dimensional descriptors and how these projections can be used to construct three-dimensional particles.

3.4.1 Constructing Two-Dimensional Projections from the Descriptors

A database of two-dimensional projections already exists for each sand mix, but the quantity of projections is limited. The goal is to use the descriptors, which are based on the existing dataset, to randomly generate particles that share similar descriptors. Since each three-dimensional descriptor has both a mean and a variance associated with it, creating two-dimensional images becomes a matter of statistics. A random number can be generated to represent each descriptor and can then be multiplied by the standard deviation and have its mean added to it. This will create a set of descriptors, which will possess the same statistical characteristics of original dataset. These new descriptors can be applied in reverse to create two-dimensional projections.

Only the Fourier descriptors will work for reconstructing two-dimensional images, since the FFT is a reversible transform. The invariant moments cannot be used for reconstruction from the descriptors and are only helpful in this thesis to prove the separability of different sand mixes. With the new randomly generated Fourier descriptors, the inverse Fourier transform can be used to create a particle from the descriptors. By varying the descriptors an entire dataset of projection plots can be developed using the inverse Fourier transform. The plots of these projections can be converted into images by using the coordinates to establish an outline and using binary operations such as dilate and fill to complete the projection. The following chapter will illustrate the capability of reconstructing the random projections from the descriptors.

3.4.2 Three-Dimensional Object Construction from Two-Dimensional Projections

The final task to complete the reverse process is to construct a synthetic representative particle from the projections. This can be done from the original database of images taken from the optical microscope or from random projections generated using the procedure described above. By using different projections to assemble multiple three-dimensional particles, a variety of varying particles can be created. All of the particles will have similar characteristics, but will be unique. In this way a realistic model can be established of an actual sand mix. The following subsections present three techniques for three-dimensional object construction from two-dimensional projections.

3.4.2.1 Extrusion Reconstruction Method

This technique uses extrusion to give an artificial thickness to the projections used in the reconstruction. The extrusion will convert a two-dimensional projection into a three-dimensional object. Combining several of these extruded projections together will result in a particle-like object, which contains some elements of all projections used. Figure 3.4 shows the extrusion process.

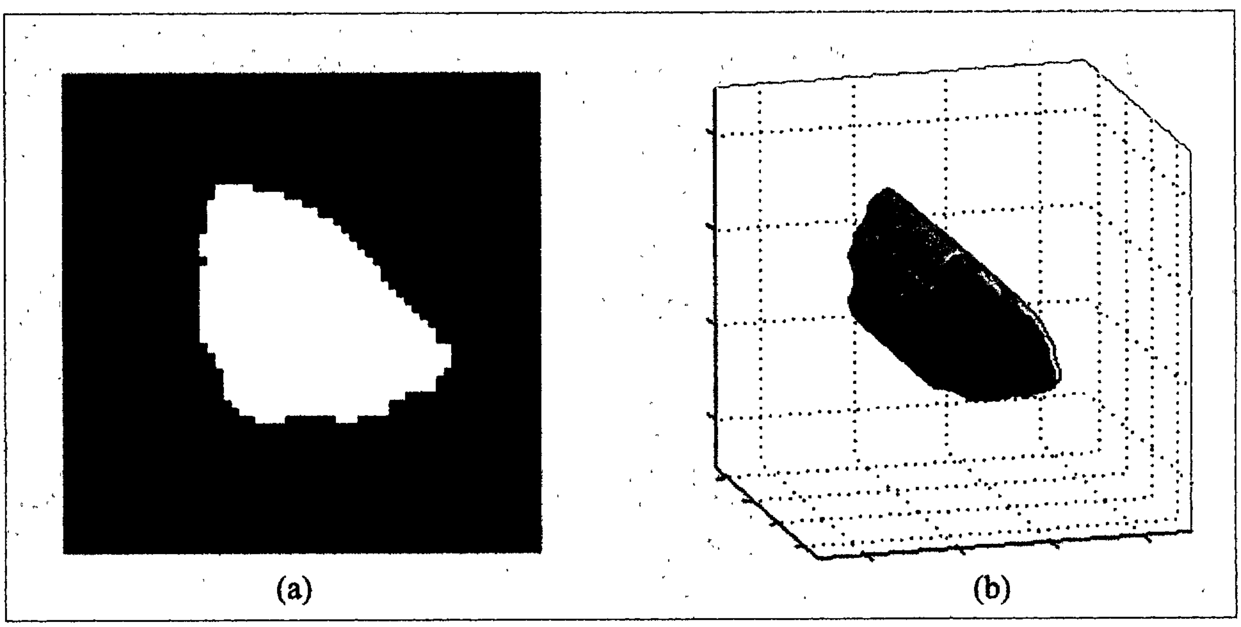
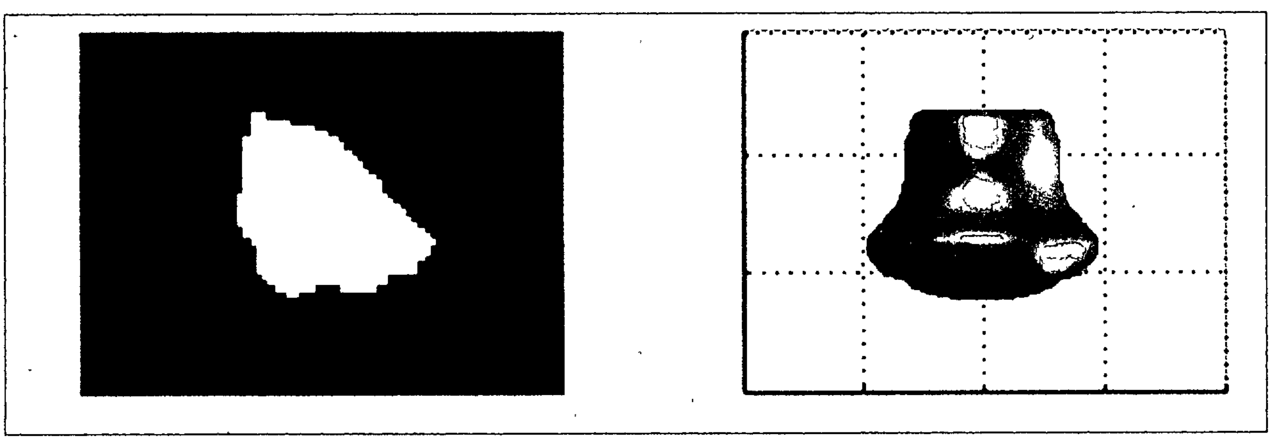


Figure 3.4: (a) Original projection and (b) extruded into 3-D space.

Different projections can be extruded along particular planes such as the x, y, and z planes. By placing the projections on different planes the result will cause them to intersect. A combination of these intersections could be used to find a particle. The common points shared between the projections could be used or even average points that are shared by most of the objects but not necessarily all of them. A good technique would be to apply many particles at a variety of angles to obtain the three-dimensional object. Results of this technique are shown in the following chapter.

3.4.2.2 Three-Dimensional Rotation Reconstruction Method

This method rotates the projection into three-dimensional space. The concept behind this approach lies in the fact that the orientation of each projection is unknown; therefore the rotation of the projection will allow it to represent all sides of the three-dimensional particle. By rotating a variety of these projections and averaging them together, a representative reconstruction of all of the two-dimensional images can be obtained. Figure 3.5 illustrates the basis for this method.



(a) (b)
Figure 3.5: (a) Original projection and (b) rotated into 3-D Space.

The rotation could be done to a multitude of particles and averaged together to produce a three-dimensional particle. The different rotations could be done about different axes; not just the y-axis, which is shown in Figure 3.5. The various results obtained for this method can be found in the next chapter.

3.4.2.3 Tomographic Reconstruction Method

Tomographic reconstruction uses a procedure where each projection is treated as a flat object in three-dimensional space. This means each two-dimensional image is placed into three-dimensional space and rotated at different angles. The easiest way to describe this technique is through the following two figures.

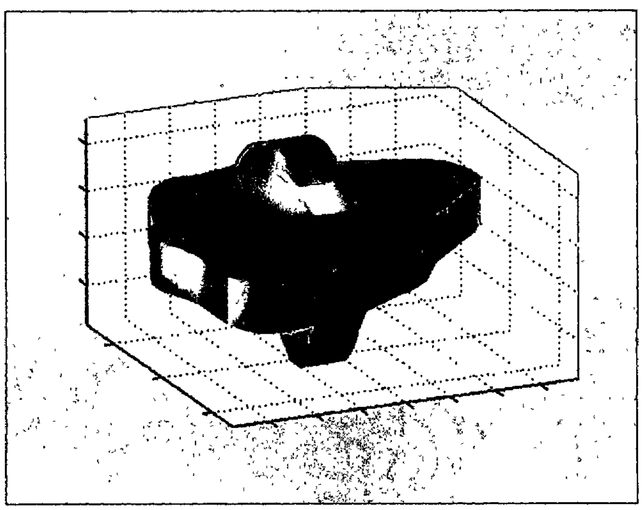


Figure 3.6: Two projections inserted into 3-D space at a 90° angle of each other.

In Figure 3.6 and 3.7 there are only two projections being used and are places at 90 degree angels of each other. This can be elaborated to as many projections at as many angles as necessary to represent the sand particle. Also, the projections can be rotated around other axes as

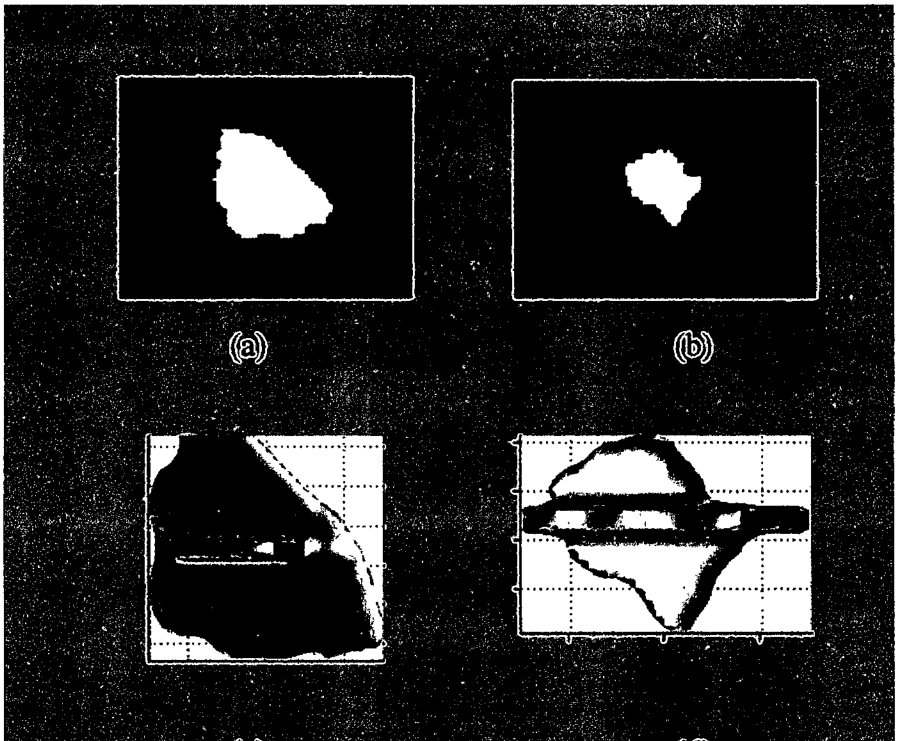


Figure 3.7: (a, b) Two images used to create Figure 3.6 and (c, d) alternative perspectives of Figure 3.6 to show how the images intersect.

well and blended together. Once all of the two-dimensional images are placed in the three-dimensional space, the entire object can be smoothed and with enough angles can look like an actual sand particle. This technique is the most promising of the three reconstruction procedures. The results displaying the effectiveness of this reconstruction method are in the next chapter.

3.5 Summary of Approach

This chapter details the approach taken to solve the shape characterization problem for soil aggregates. The approach discussed offers the ease of a two-dimensional implementation matched with the accuracy obtained from a three-dimensional description method. This will allow soil mixtures to be analyzed without the necessity of scanning individual grains of sand into a three-dimensional imaging system. The technique gives unique, parsimonious, and invariant descriptors, which permit reconstruction and automatic collection. The following chapter will present results displaying the successfulness of the approach at classifying and reconstructing three-dimensional aggregates from two-dimensional projections.

CHAPTER 4: RESULTS

This section will show the developments made to obtain the desired results and each step taken to achieve them. First, the experimental setup and preparation algorithms will be shown to explain how the two-dimensional images of the sand particles were converted into shape profiles of the aggregates. Results from the normalization methods used to isolate the shape from size will demonstrate the necessity of normalization. Classification results will be presented, which show the uniqueness of the descriptors for both Fourier analysis and invariant moments. The last section of this chapter will discuss the feasibility of reconstructing three-dimensional objects from the two-dimensional projections, as well as, reconstructing more two-dimensional projections from the descriptors themselves.

4.1 Experimental Setup

Before any shape description can take place a certain amount of preparation is required. Each soil mixture was scattered under an optical microscope and two-dimensional pictures were taken of various particles in the mix. Since the particles are three-dimensional, false contours are created inside the flat two-dimensional projections. All that is needed is a binary image, where black is the background and white is the object. In order for the two-dimensional shape techniques to work effectively the projections must be filled as well with the entire inside of the outline being white. This section explains detail on the collection of the sand projections and the preprocessing done to the original images.

4.1.1 Laboratory Setup

As previously mentioned, the main objective is to calculate shape descriptors, which describe three-dimensional shapes using two-dimensional projections. The first thing needed to start this process was the two-dimensional projections of the various sand mixes. This was done on an ordinary optical microscope, the Nikon Eclipse TS-100, which is shown in Figure 4.1.

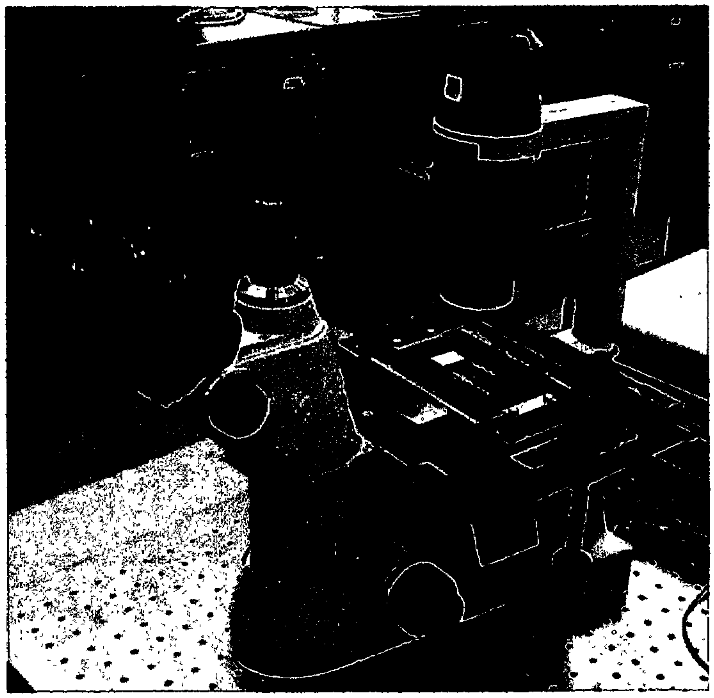


Figure 4.1: Nikon Eclipse TS-100 optical microscope used for data collection.

These projections were simply two-dimensional digital images of different particles within each mix. For each type of sand mix a large number of images were taken and used to generate the average descriptors for each sample set. Some typical images used are shown in Figure 4.2. The sands that we used included; #1 Dry Sand, Melt Sand, Daytona Beach Sand, Michigan Dune Sand, and Glass Beads. The #1 Dry and the Melt Sand are very similar sands that have many of the same properties, where Michigan Dune and Daytona Beach Sand are unique and have different tested parameters from the other mixes. The glass bead was used in the analysis as a standard reference in order to help show the algorithms effectiveness in identifying similar shapes.

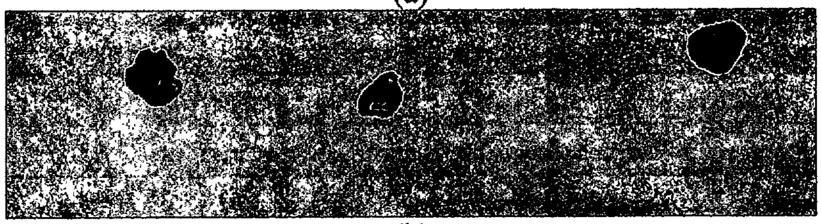
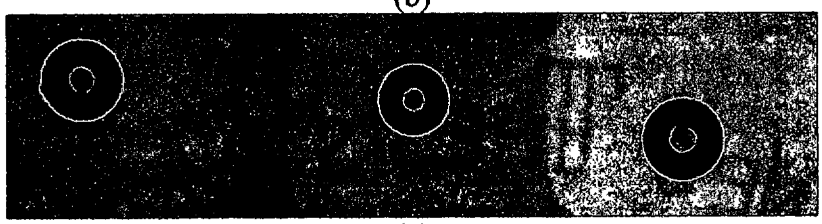

(a)

(b)

(c)

Figure 4.2: Example of sand images used for the analysis: (a) #1 Dry Sand, (b) Daytona Beach Sand, and (c) Glass Beads.

The following sections illustrate the effectiveness of the shape descriptors used in this thesis. All descriptors are modeled using a Normal distribution and the mean and variance for the individual descriptors in a mix are calculated.

All of the characterization results were obtained using 382 #1 Dry Sand, 391 Melt Sand, 299 Daytona Beach Sand, 193 Michigan Dune Sand, and 25 Glass Bead particles. The glass beads did not require as many samples since the variance between projections is extremely low.

4.1.2 Image Pre-Processing

Before any analysis can begin on the two-dimensional projections, some preparation must be performed on the images. The focus of each picture of sand taken from the digital camera is the outline of the particle itself. Therefore code must be written to take the original RGB image and convert it a one-dimensional periodic function. Each image must be subjected to a series of procedures to remove any extraneous objects, so the sand particle of interest is the only object left in the image. This is mostly done by converting the image to binary and using simplistic binary operations to perform further processing.

In Figure 4.3, the original raw image file taken directly from the microscope is shown. This image was taken with a green filter, which reduced glare and light shining through the particle. As can be seen in the image, the actual particle of interest is surrounded by sand fragments and even another particle can be seen touching the left border of the image. Before this sand particle can be of any use in discovering shape descriptors, the image must be cleaned up and the particle itself made the focus of the image.

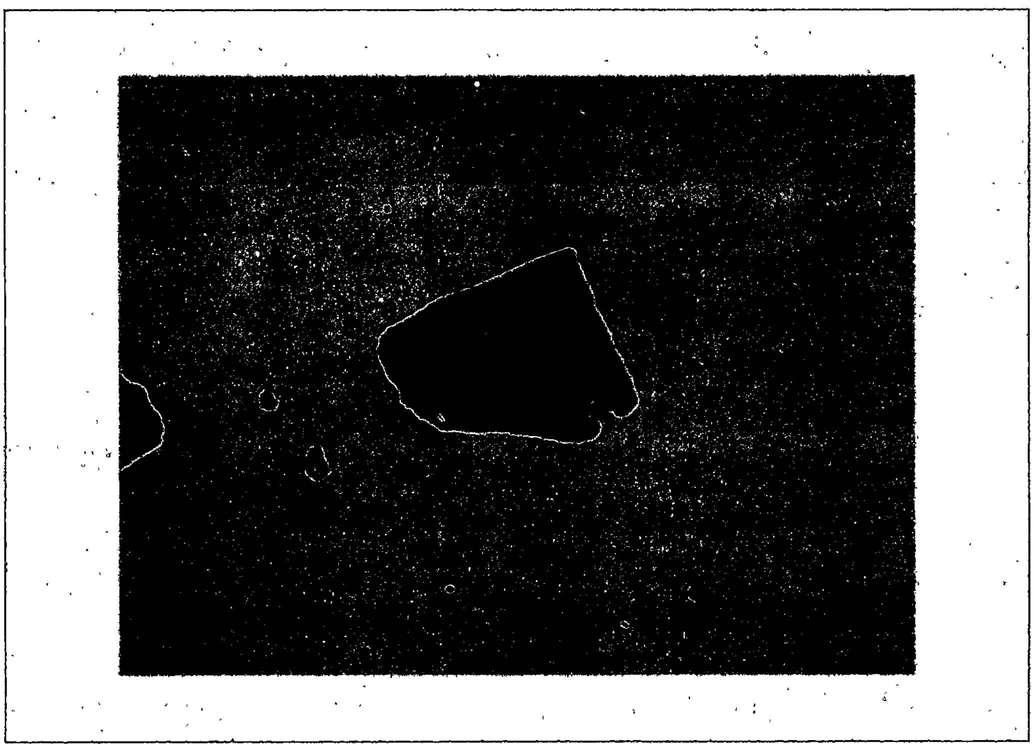
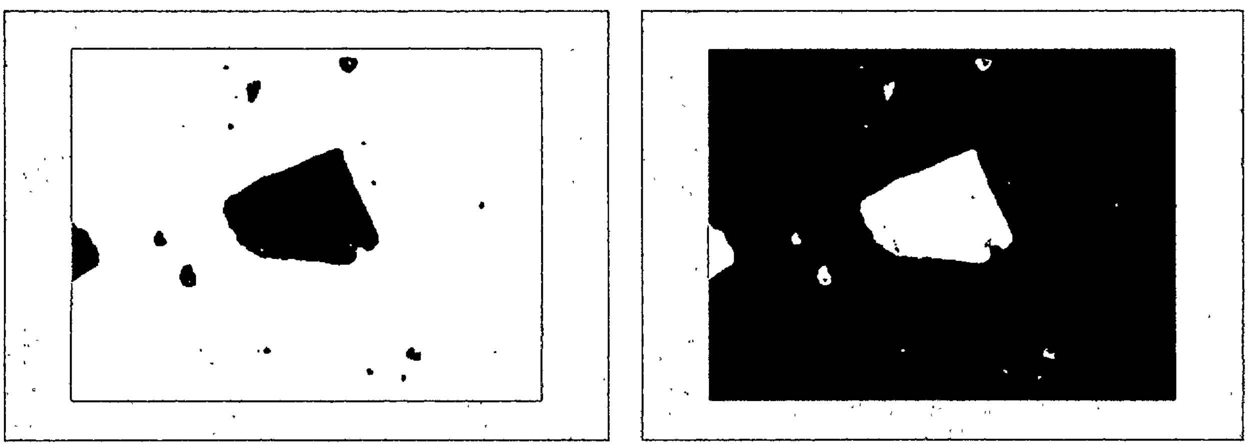


Figure 4.3: Original raw image taken from microscope.

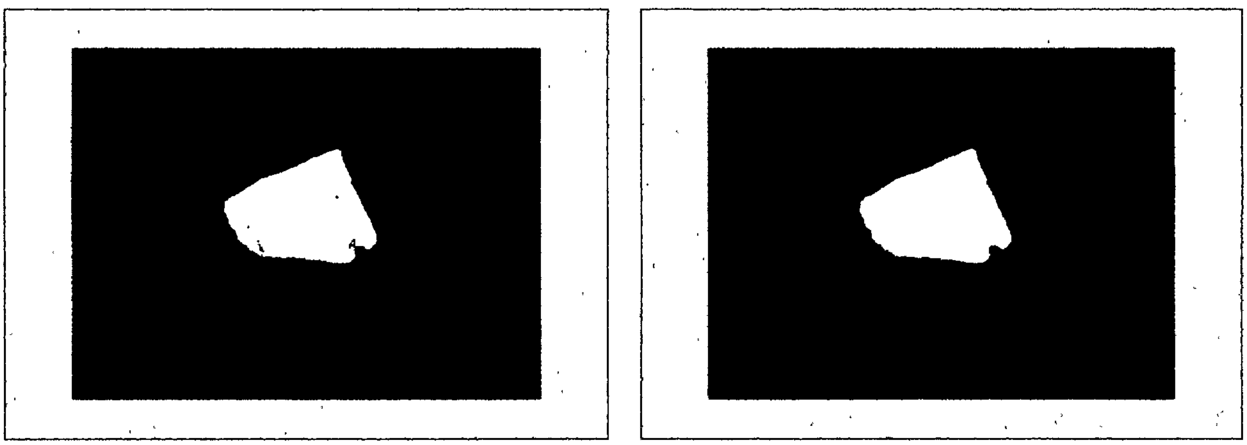
Since only the shape is needed from the image, a silhouette of object can be obtained, which will remove some of the irrelevant information, such as texture. The conversion to binary will make all of the lighter colors equal to one and the darker colors equal to zero. The problem with this is that the object of interest becomes black, while the background turns white. In MATLAB most binary operations are performed assuming the white pixels are important and the black pixels are not. For this reason, the image was inverted to make the particle white and the background black, which made further MATLAB processing easier. Figure 4.4 shows the converted binary image and the inversion of the image used throughout the rest of the processing.



(a) (b)

Figure 4.4: (a) Original binary image and (b) inverted binary image for processing.

With the entire image now being represented by zeros and ones, the removal of insignificant objects in the image, becomes much easier. The particle touching the left border can be easily removed in MATLAB using the `imclearborder` function, which eliminates any connected pixels touching the border. The tiny crushed sand pieces lying all around the main particle can be cleaned up, by finding all of the connected pixels in image and removing them all except the largest one. In every case, the object of interest should be the largest object in the image and everything else should be taken away. The last thing needed to be done to the binary image is to fill the holes appearing inside the object. Since eventually edge detection must be performed on the image, filling the holes will prevent false or irrelevant edges from being detected. This is easily done using `imfill`, which changes to white any black pixels completely surrounded by white pixels. Figure 4.5 below displays the clean binary image and the image after being filled.



(a) (b)

Figure 4.5: (a) Binary image after removal of extraneous objects and (b) with holes filled.

The last procedure performed on the images is the centering of the particle of interest. This can be done in various ways, but the method best suiting this application was to center the object about its centroid. Centroid centering the object placed a majority of the particle in the center regardless of its shape, unlike x and y limit techniques, which could cause the center of mass to be weighted heavily toward one side. For the Fourier analysis and invariant moments, this may have not been necessary, however later for attempted reconstructions centering the particle was essential. Figure 4.6 is the final clean and centered image.

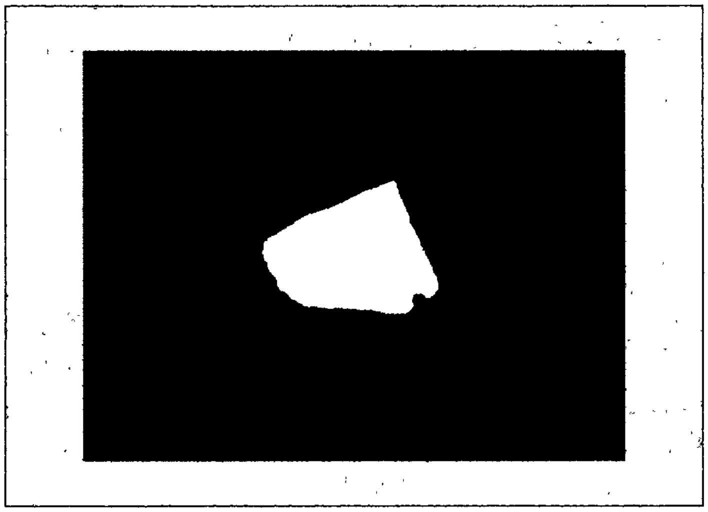


Figure 4.6: Final processed image.

4.2 Results of Shape Characterization using Fourier Descriptors

When trying to find the Fourier descriptors of shape, the outline of that particle must be transformed into a periodic one-dimensional function. In order to obtain the outline, the edge detection algorithm can be used in MATLAB to receive the particles boundary. The figure below shows the edge detected image of a particle of sand.

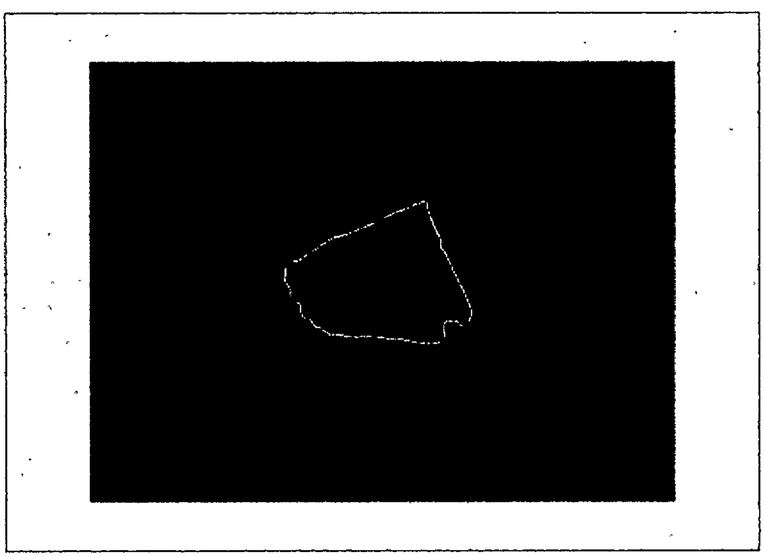


Figure 4.7: Edge detected image of a sand particle.

Obtaining the one-dimensional function from the outline can be achieved by choosing a starting point on the border and following around the edge. The function must be created from pixels in sequential order to attain the periodicity required for Fourier analysis. The starting point was arbitrarily chosen as the pixel directly to the right of the centroid. The boundary was traced by discovering m-connected pixels from the starting point. Each pixel was recorded before moving on to the next connected pixel, until the starting point was reached again. Since the tracing algorithm works based on connectivity, the preprocessing used to create the unbroken boundary is extremely important. With the entire set of x and y values obtained, the function can be plotted as shown in Figure 4.8.

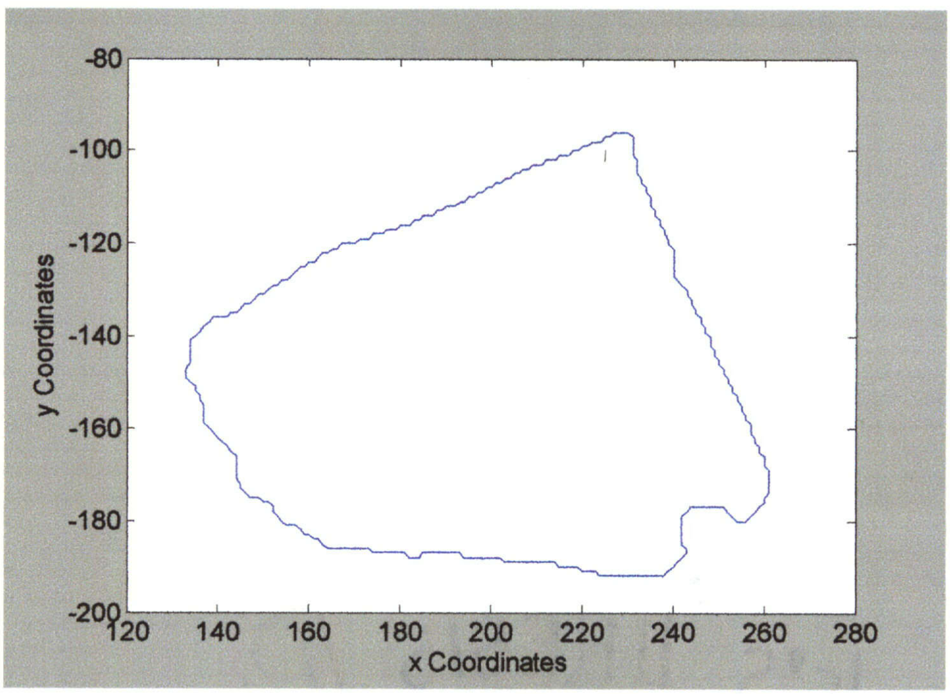


Figure 4.8: Plot of x and y coordinates obtained by tracing the edge.

The one-dimensional function formed by adding $x + j \times y$ is the periodic signal needed to complete the Fourier analysis. Figure 4.9 shows a plot of this function, where the x axis simply

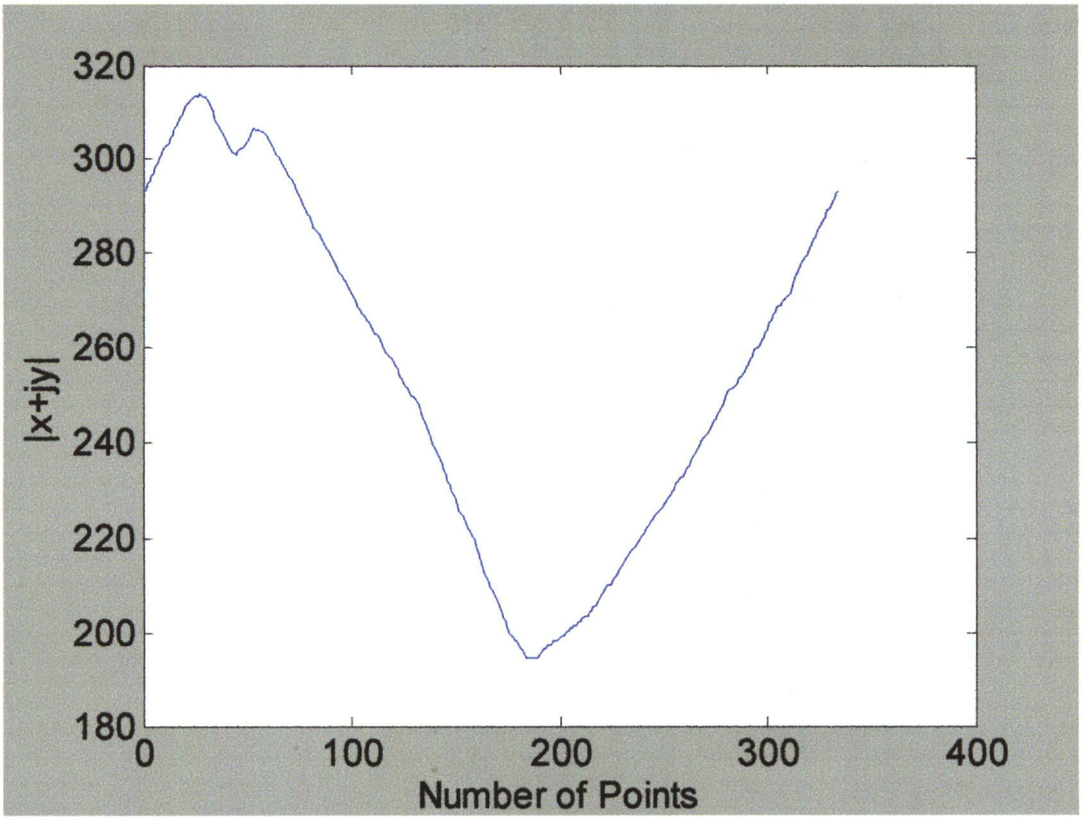


Figure 4.9: One-dimensional function representing the shape profile.

represents the number of points located along the boundary. The Fourier descriptors can be taken from this function, through the use of the Fast Fourier Transform (FFT). The descriptors are simply equal to the coefficients obtained when performing the transform. The figure below is a plot of the first 40 FFT coefficients of the function shown in Figure 4.10.

The inverse FFT can be used to reconstruct the original image from its FFT coefficients as shown in Figure 4.11. The next section will show how the higher coefficients, which are extremely small, can be set to zero and still maintain good reconstruction. This will prove useful later, when trying to compare similar shapes.

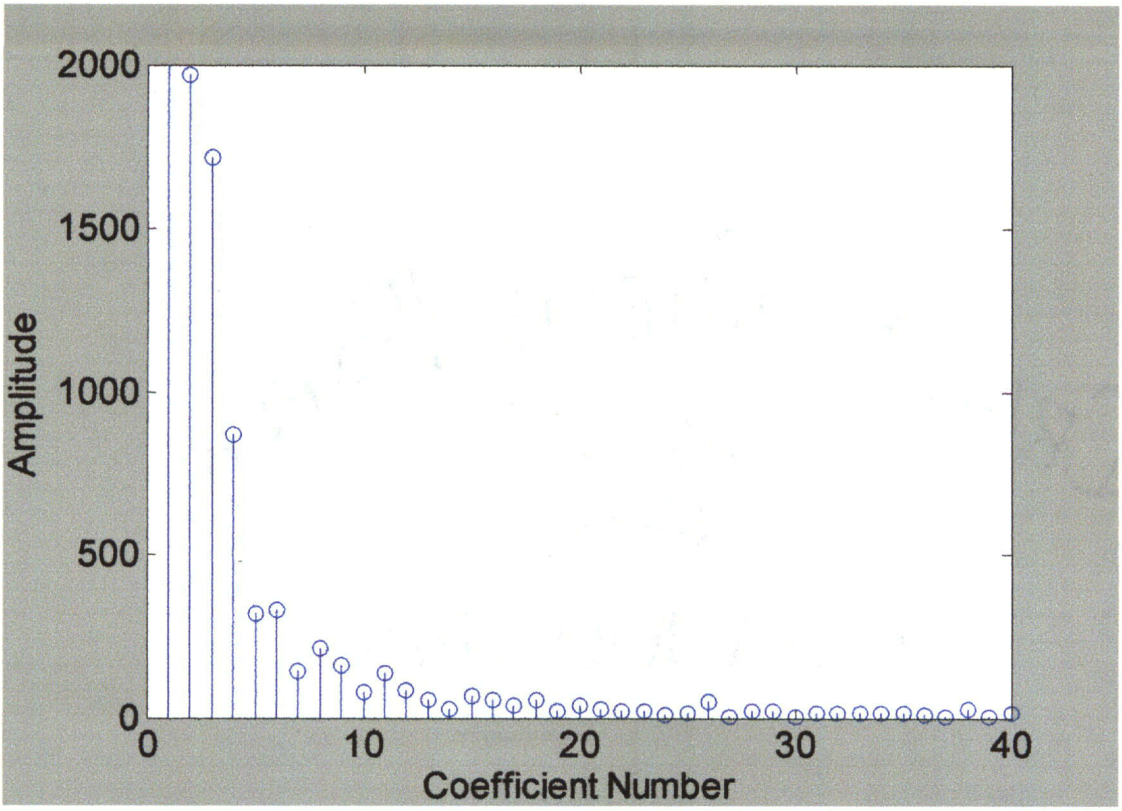


Figure 4.10: Fast Fourier transform of the one-dimensional function.

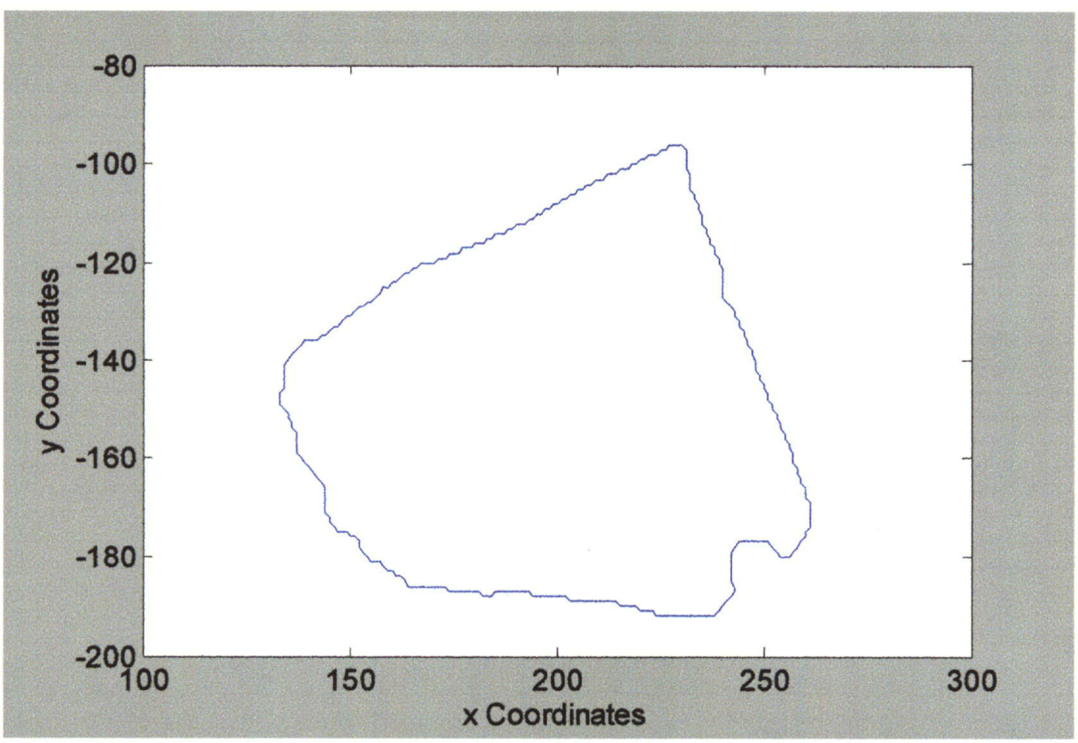


Figure 4.11: Reconstruction of the original shape using the inverse FFT.

4.2.1 Reconstruction using Fewer Coefficients

This section proves that not all the Fourier descriptors are needed to describe the shape of a sand particle. By showing how fewer coefficients can be used to obtain similar reconstructions, it proves fewer descriptors can be used to still adequately describe the shape. Also the higher coefficients have small near zero values, which makes them very sensitive to noise. Anomalies in the surface of the sand particle may lead undesired values at higher frequencies that may not necessarily describe the shape. The lower frequencies are, in general describe the general shape of the particle. Figure 4.12 shows an FFT where only twenty descriptors were used.

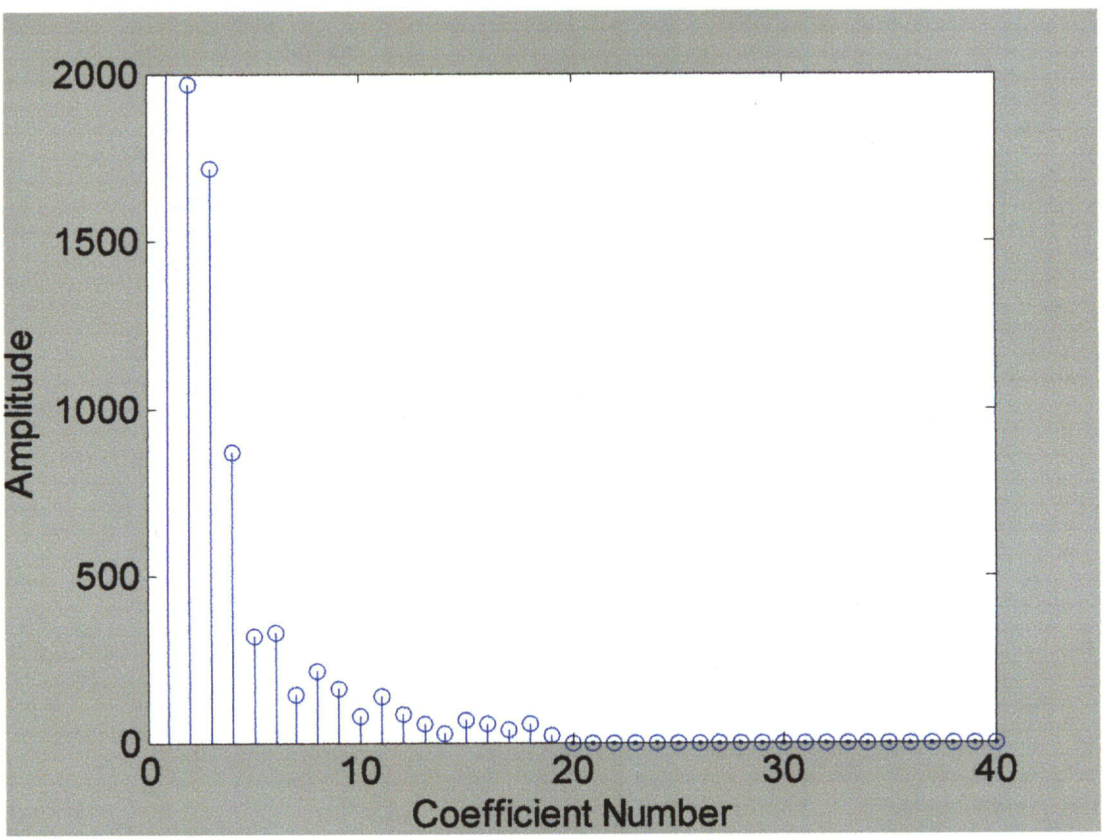


Figure 4.12: FFT with coefficients higher than 20 set to zero.

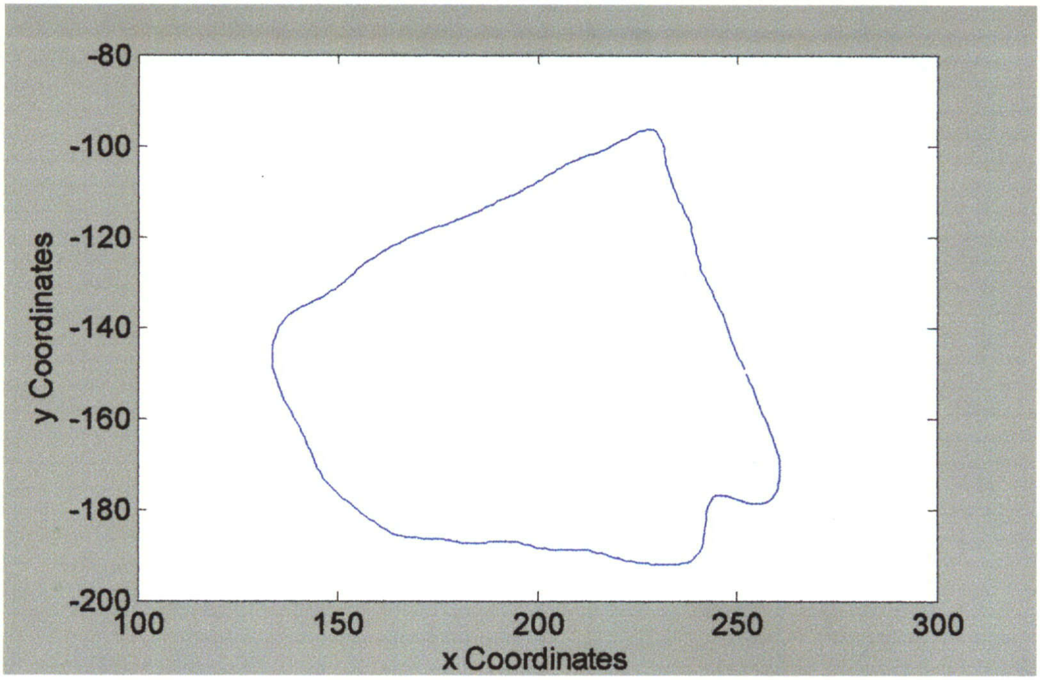
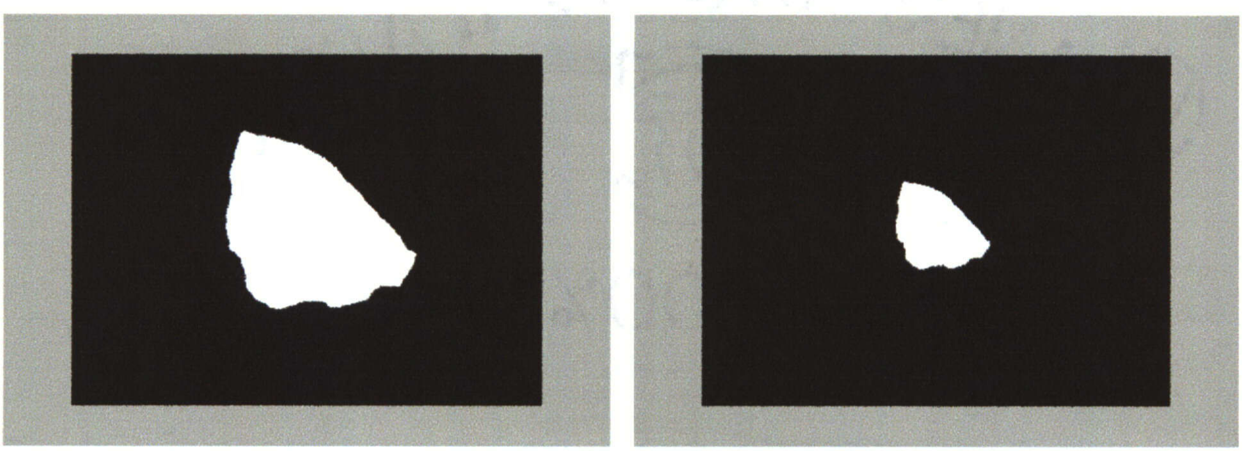


Figure 4.13: Reconstruction of sand particle with 20 descriptors.

4.2.2 Normalization

As discussed in the previous chapter, it is important to isolate shape in the descriptors. The functions must be normalized in order not to be influenced by size characteristics. This study is meant to be based on shape alone.



(a) (b)

Figure 4.14: (a) Original image and (b) half-sized image.

The following set of results shows the normalization process, with the original image in the left column and the same particle at half the size in the right. Figure 4.15 (a) and (b) show a plot of the x values obtained by tracing the boundary of the function. As can be seen, they are similar in behavior, but at different amplitude values. Also the two plots have a different number of points, with the smaller one having about half as many as the larger particle. Figure 4.15 (c) and (d), illustrate that the FFT of these two functions are different. The mean squared error between the two FFT plots is 7.6×10^{7}. Since the two shapes are the same, the FFTs need to be identical for this type of method to be effective.

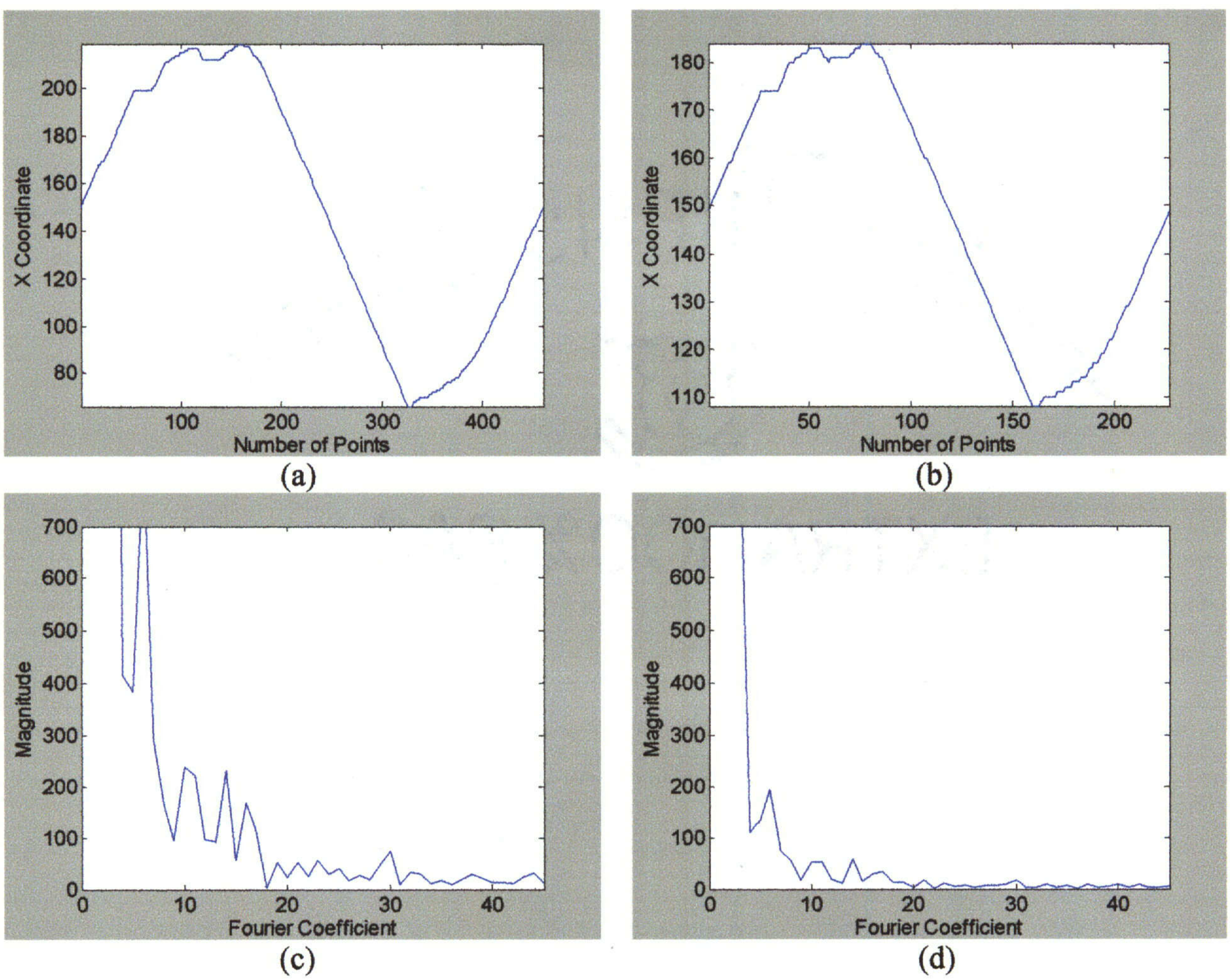


Figure 4.15: Plot of x coordinate values (a) original and (b) half-sized; FFT (c) original and (d) half sized.

The inconsistency in the FFT plots originates in the coordinate plots. If these plots are normalized to the same scale, the two should have similar FFTs. The first problem to fix is the amplitudes of the coordinate functions. This can be done by varying them between -1 and 1 as shown in Figure 4.16. After this process the two FFTs become more similar, but are still not quite the same. The mean squared error between the FFT plots is reduced to 28.7.

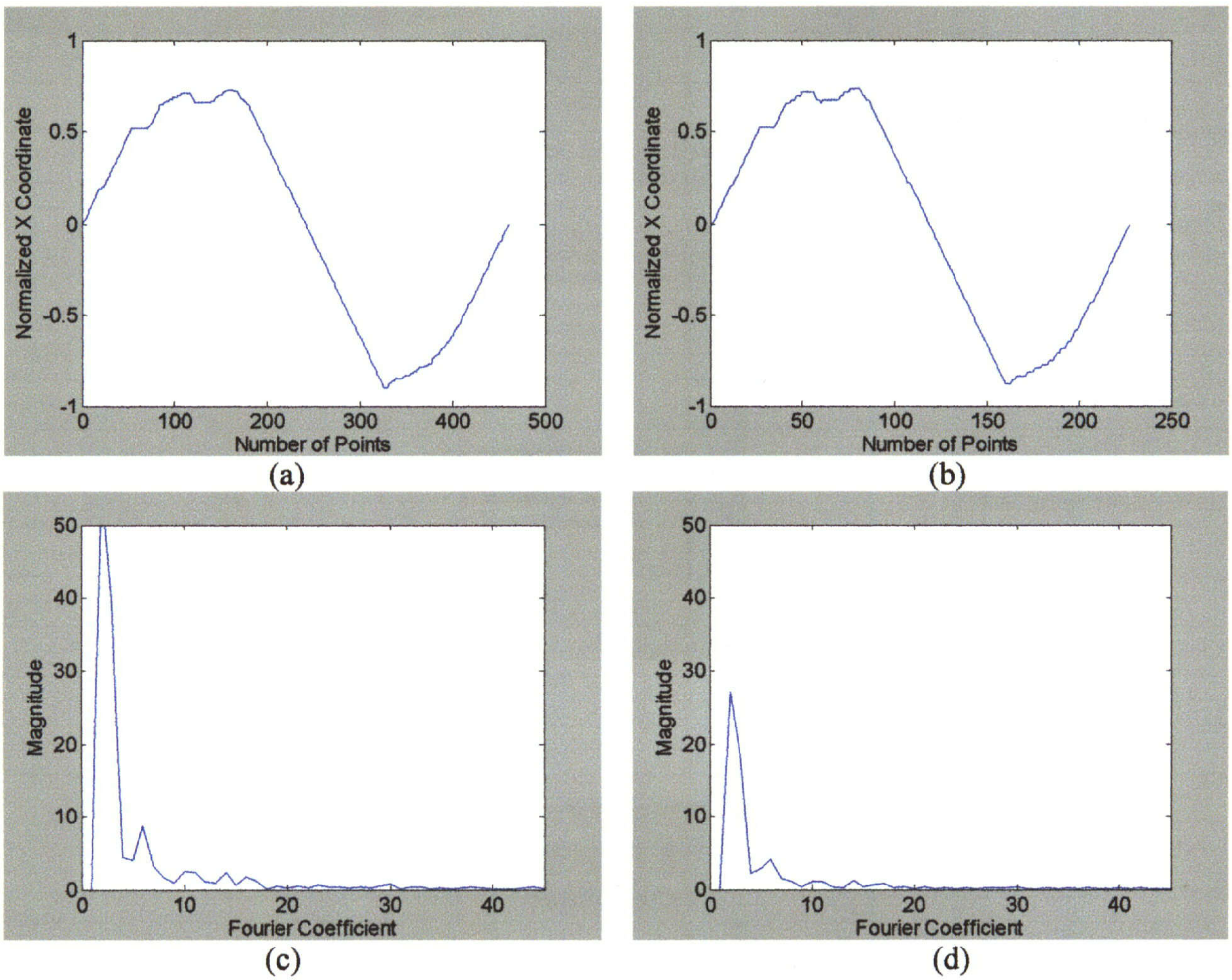
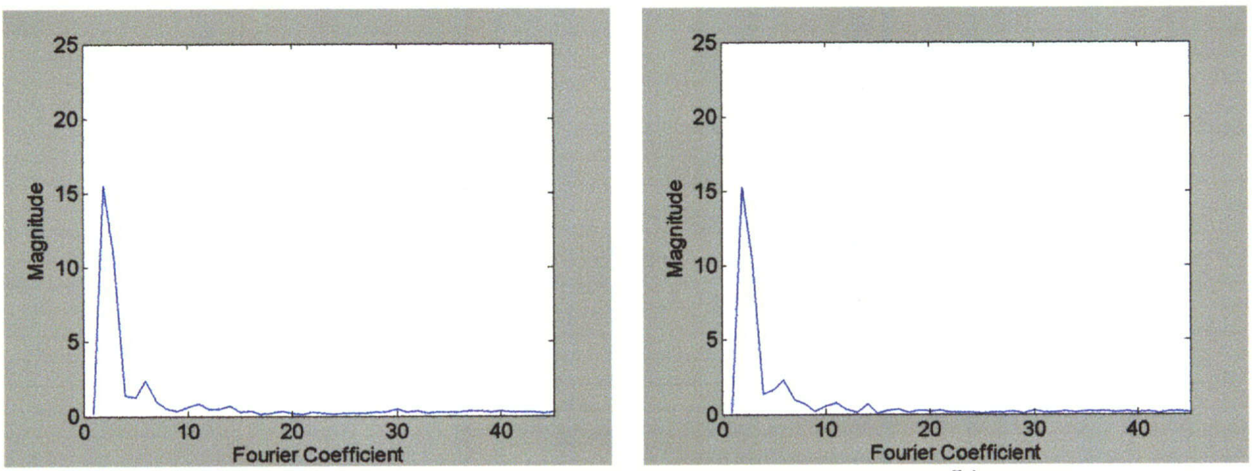


Figure 4.16: Plot of x coordinate values after normalization (a) original and (b) half sized; FFT after normalization (c) original and (d) half sized.

The plot can now be resampled to the same number of points. This should make them the exactly same function in both behavior and scaling. With both plots having similar scaling, the only factor, which will affect the FFT, is the actual shape. Therefore, as shown in Figure 4.17, two particles with similar shape will obtain similar descriptors from the FFT's. The mean squared error between the FFT plots after normalization and resampling is .044, which is dramatically lower than the original error.



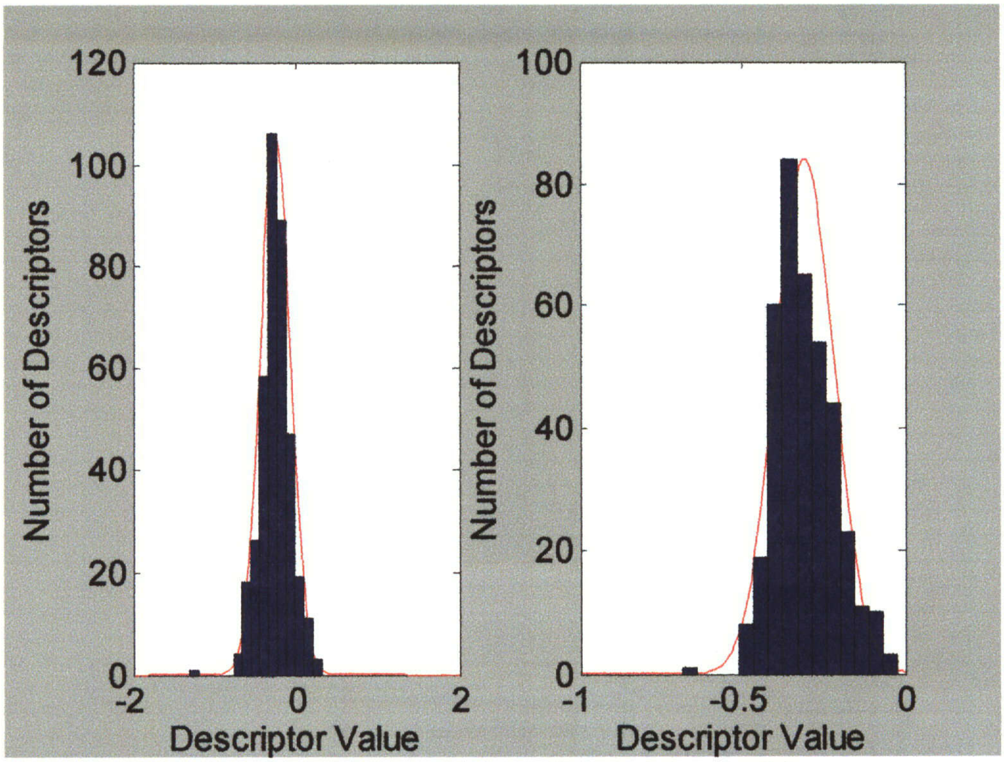
(a) (b)

Figure 4.17: FFT after normalization and resampling (a) original and (b) half-sized.

In the following results all of the one-dimensional functions representing the two-dimensional projections were normalized and resampled to 128 points. As shown previously, this eliminates the size dependence on the descriptors and ensures all of the FFT's are directly relatable. This means for example, the first coefficient corresponds to the same descriptor in all of the two-dimensional projections in the database.

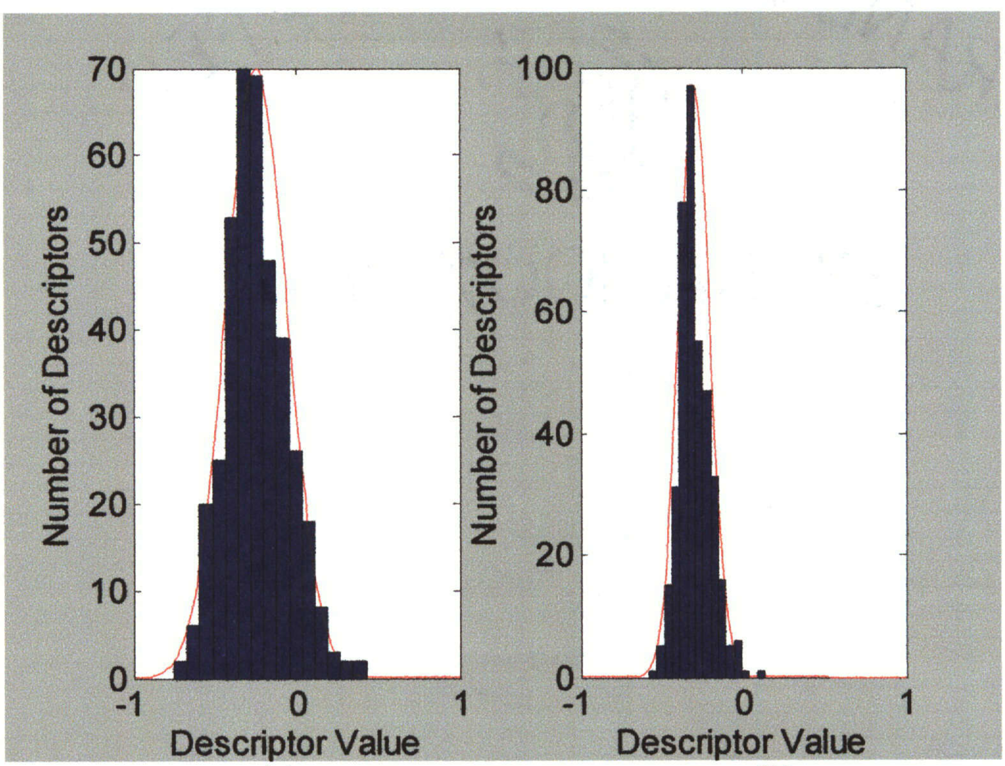
4.2.3 Statistics of Fourier Shape Descriptors

After the descriptors have been calculated, their distributions can be plotted. The following figures show the histograms of a few of the descriptors used to generate the Fourier analysis characterization results. Not all the descriptors are shown; only the 4th and 23rd of the 26 descriptors used in the later results for each mix are displayed. This allows a statistical look at both a low and high descriptor.



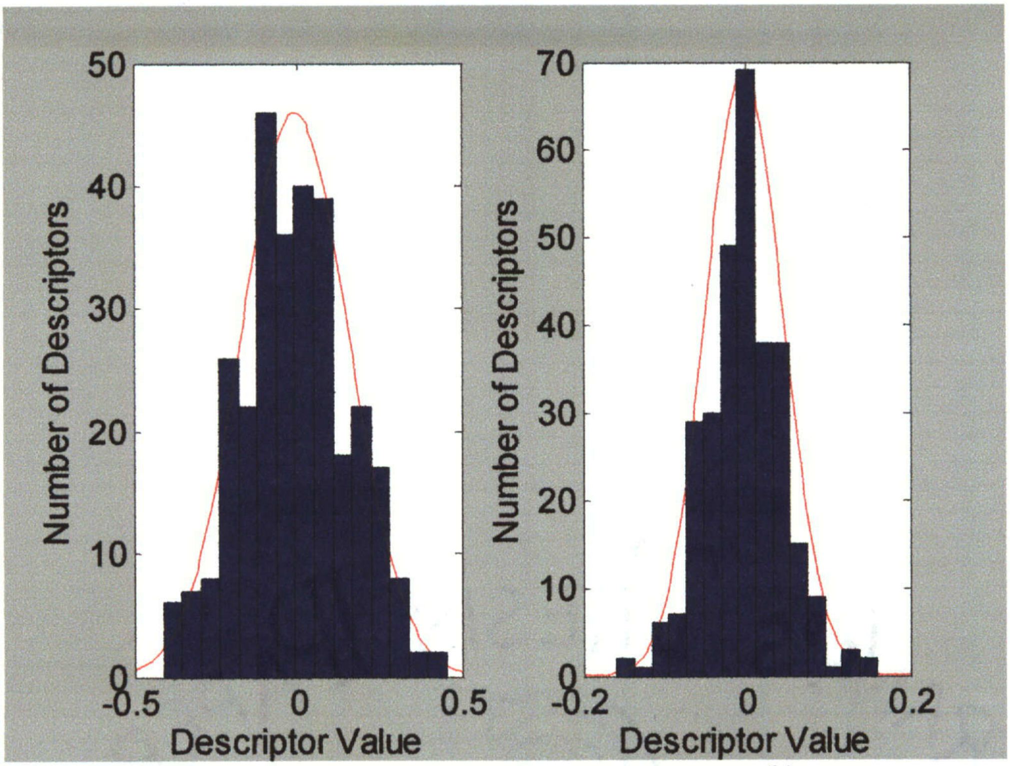
(a) (b)

Figure 4.18: #1 Dry Sand histogram of Fourier descriptors (a) 4^{th} and (b) 23^{rd}.



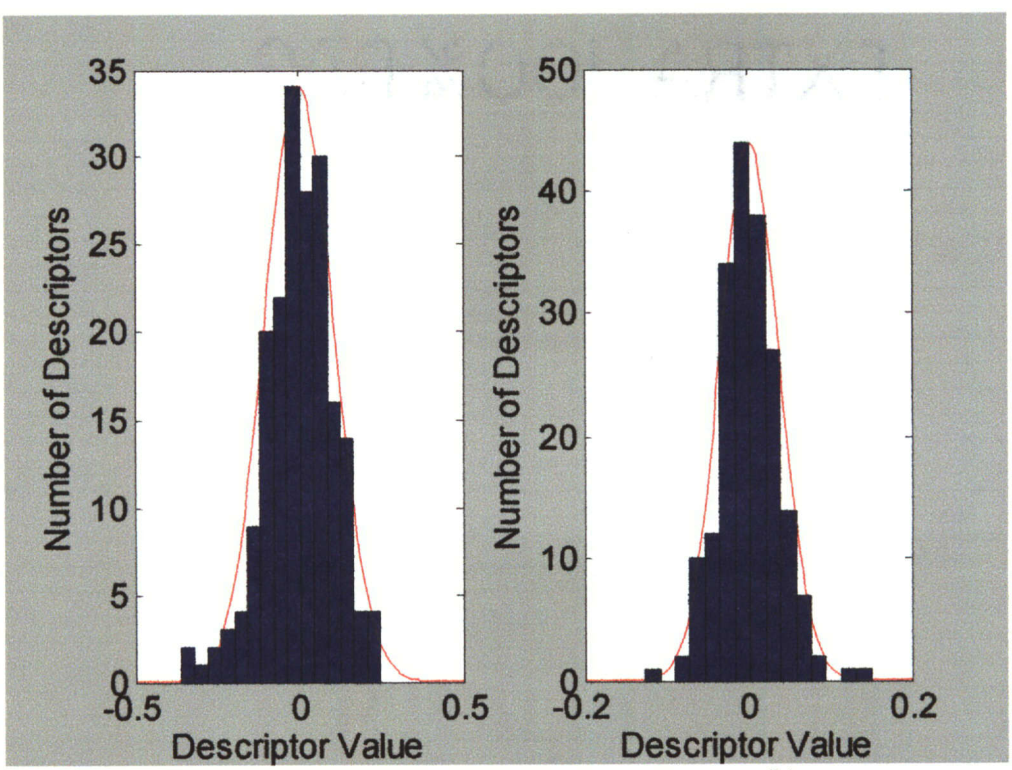
(a) (b)

Figure 4.19: Melt Sand histogram of Fourier descriptors (a) 4^{th} and (b) 23^{rd}.



(a) (b)

Figure 4.20: Daytona Beach Sand histogram of Fourier descriptors (a) 4th and (b) 23rd.



(a) (b)

Figure 4.21: Michigan Dune Sand histogram of Fourier descriptors (a) 4th and (b) 23rd.

4.2.4 Classification Results using Fourier Descriptors

Figure 4.22 was created as a way of visualizing the data in three dimensions, however it is important to understand it is not necessary to only use three dimensions. In fact, 26 descriptors, or features, were actually used and PCA was applied to reduce them to three dimensions. More or possibly fewer dimensions could be used to help classify this data.

For every projection, 128 descriptors were obtained and then each descriptor was averaged over all of the projections. This provided information about the average descriptor for each type of sand. As discussed in the previous chapter, only the middle of the first half of the FFT, where most of the distinctive information lies, was used to classify the soil. Of these values, 26 descriptors were chosen to attempt the classification of the different sand types. In the figures below, ellipsoids are used to represent the data after being reduced to three dimensions using PCA.

The center of each ellipsoid is determined by the averages of the three descriptors over all of the samples. Once each descriptor is averaged, three numbers are obtained and used as the x, y, and z coordinate for the center of the ellipsoid. The radius is determined by the variance of descriptors. With the variance calculated throughout all of the samples for the three descriptors x, y, and z radii are established. Figure 4.22 illustrates graphically the construction of the ellipsoids.

Figure 4.23 has no axis labels, because they are irrelevant and have no physical meaning. Since PCA was implemented to reduce the data from 26 to 3 dimensions, the values of the axes do not directly relate to the descriptor values. The plot by itself however gives no real insight on how separable the different sands are, because it offers no valid reference. The space between the different ellipsoids could be very small and may only appear separable. A method to test the

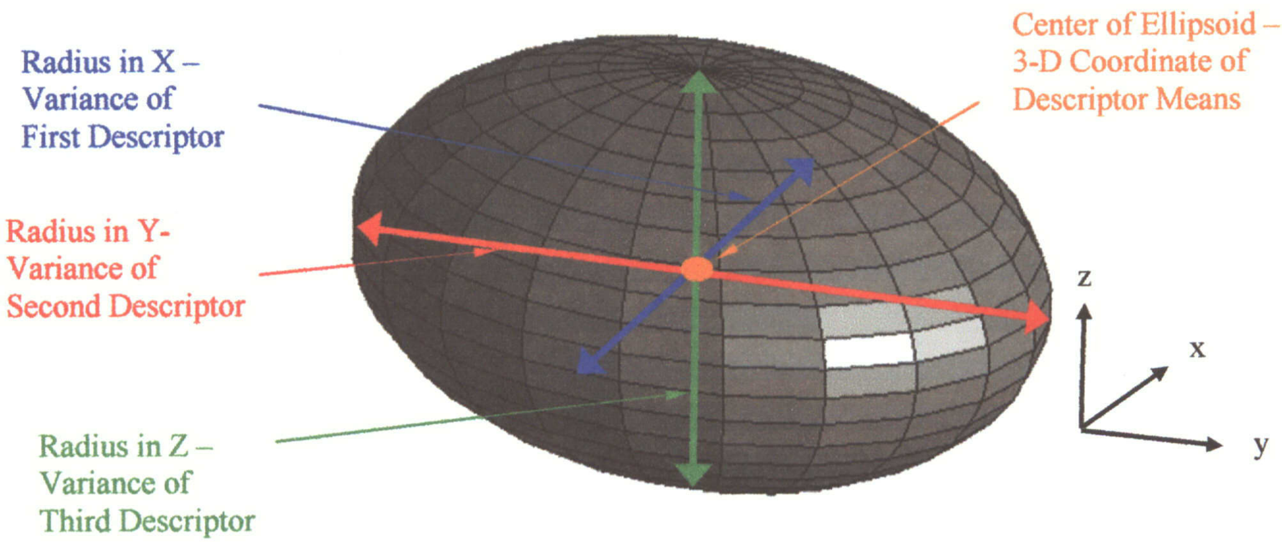


Figure 4.22: Ellipsoid model for depicting results.

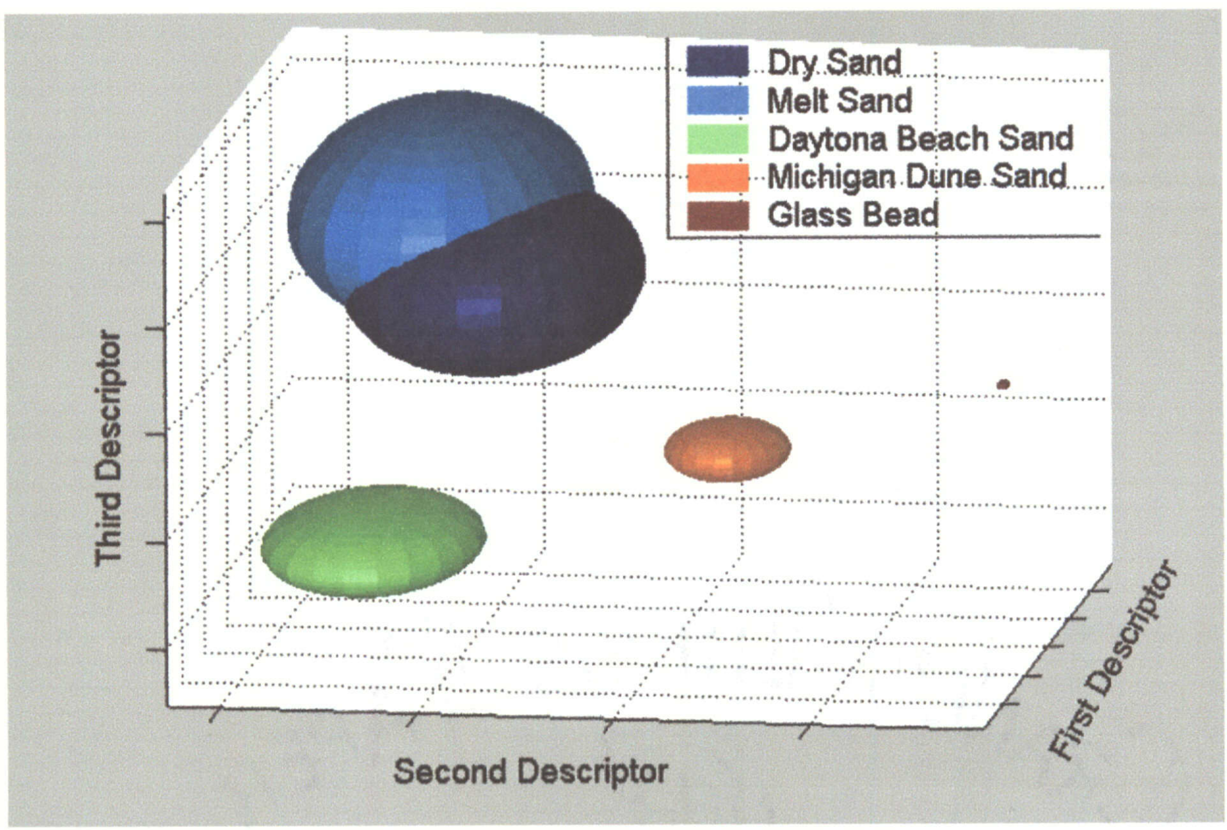


Figure 4.23: Ellipsoid plot showing separation of sand particles using Fourier descriptors.

separability would be to introduce more samples of each sand mix and plot them separately. This was simulated by splitting the main datasets in half to make two smaller data sets of each sand mix. In order to ensure consistency throughout the sets, the samples are randomly chosen to be placed in either the first or second set, instead of simply using the first half as one set and the second as another. The following figure shows these results.

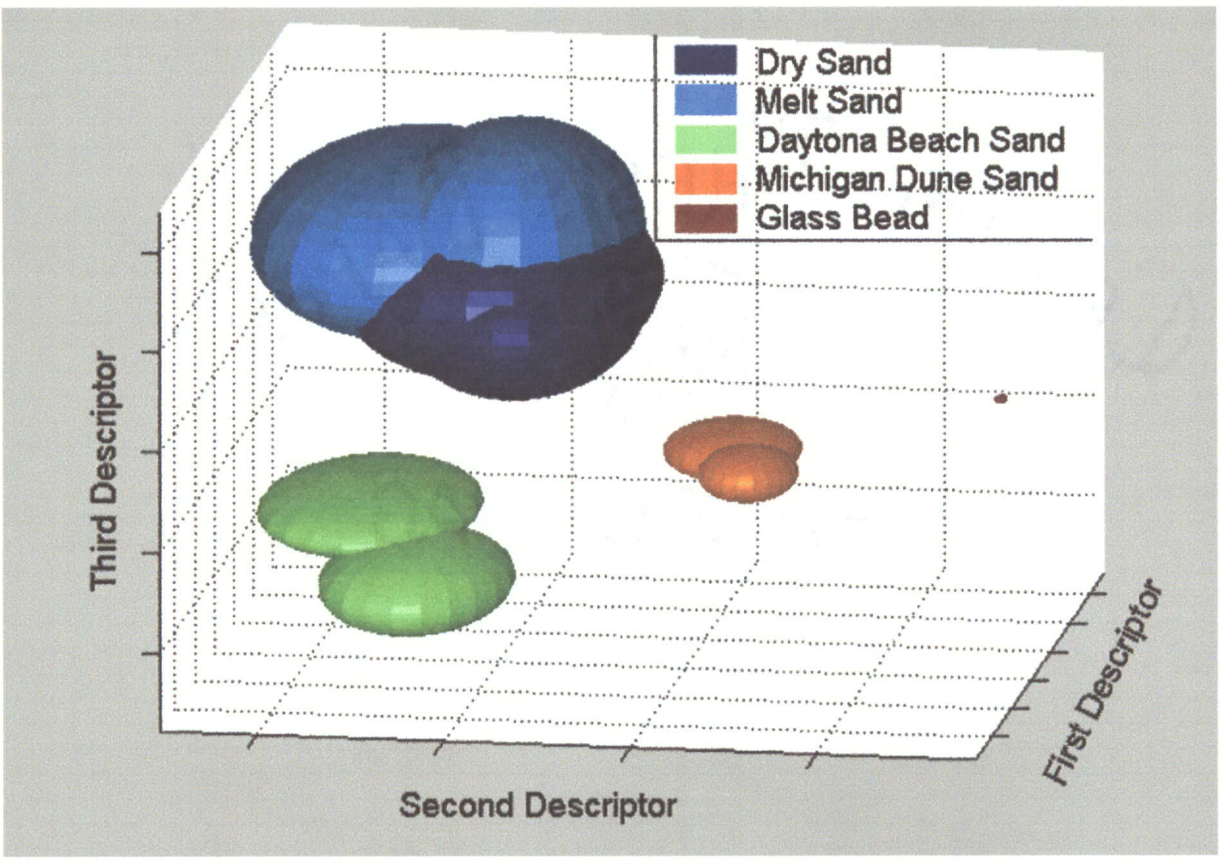


Figure 4.24: Ellipsoid plot using two datasets for each sand mix using Fourier descriptors.

Figure 4.24 shows, if an unknown dataset was placed into the algorithm with the known samples it would most likely be able to be classified. The similar shapes are grouped close to one another proving the effectiveness of the shape descriptors. The final chapter will discuss more on the impacts of these results.

4.3 Results of Shape Characterization using Invariant Moment Descriptors

Invariant moments are designed to identify similar shapes, regardless of its size or orientation. In order to ensure the invariant moment algorithm works properly, tests were run on shapes alone and the descriptors were compared. The following results show two examples of sand projections. The first two shapes are the same; however one is a rotated and scaled version of the other. The second example shows to dissimilar shapes. The results are documented in tables corresponding to the invariant moment.

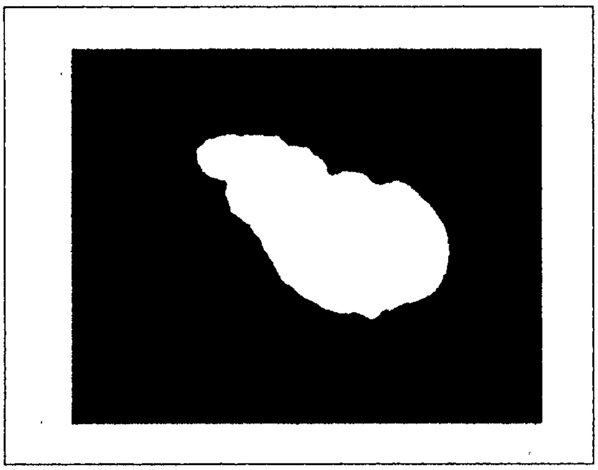
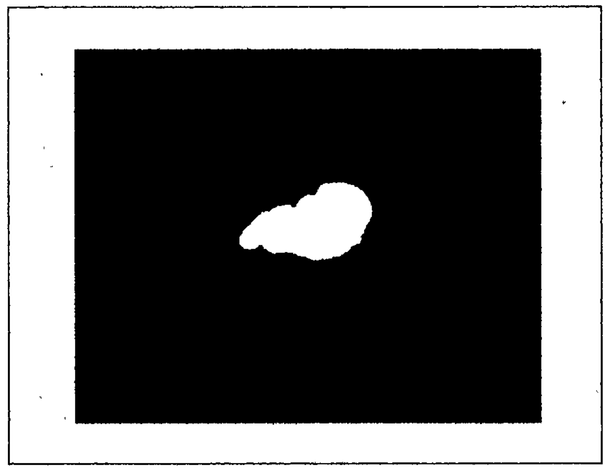



Figure 4.25: Two identical shapes at different orientations and scales.

Table 4.1: Comparison between Invariant Moments of Images in Figure 4.25

Invariant Moments	Image 1	Image 2	% Difference
ϕ_1	7.1164	7.1176	0.0168
ϕ_2	15.2953	15.3027	0.0483
ϕ_3	12.4116	12.1704	1.9434
ϕ_4	25.1942	25.2073	0.0516
ϕ_5	50.3498	49.0710	2.5398
ϕ_6	32.9973	33.0157	0.0557
ϕ_7	50.7640	50.7842	0.0398

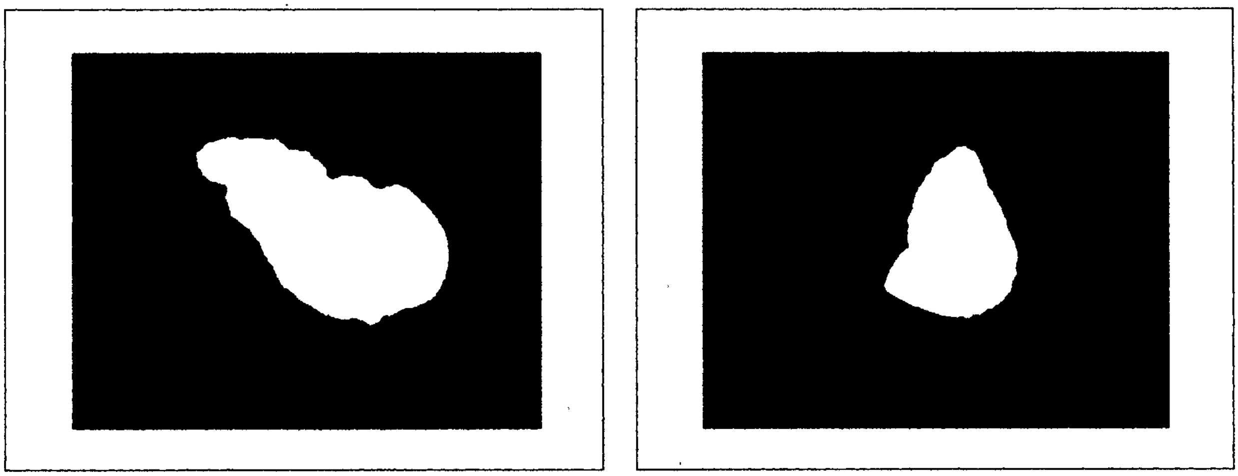


Figure 4.26: Two dissimilar shapes.

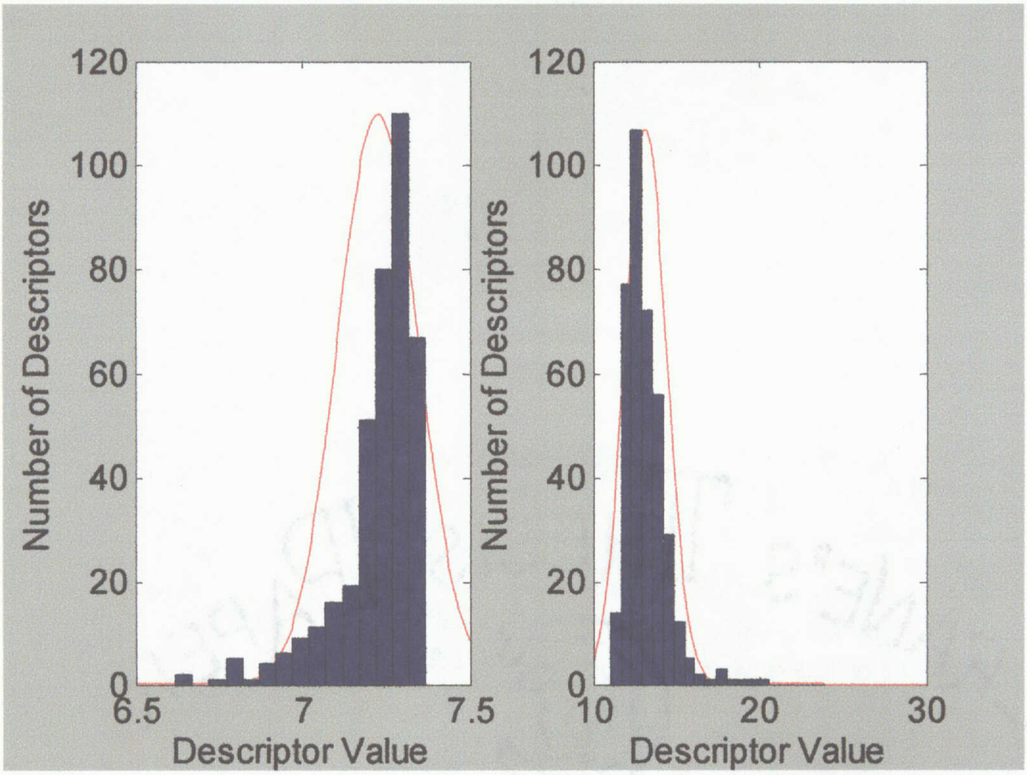
Table 4.2: Comparison between Invariant Moments of Images in Figure 4.26

Invariant Moments	Image 1	Image 2	% Difference
ϕ_1	7.1164	7.2694	2.1494
ϕ_2	15.2953	16.7749	9.6736
ϕ_3	12.4116	17.4857	40.8822
ϕ_4	25.1942	26.9251	6.8702
ϕ_5	50.3498	52.5509	4.3717
ϕ_6	32.9973	35.5863	7.8463
ϕ_7	50.7640	52.5528	3.5237

These results clearly illustrate the effectiveness of invariant moments in identifying similar shapes. Table 4.1 has a mean squared error of 0.242 between the two images and the highest percent difference is only about 2.5%. Table 4.2 on the other hand had a mean squared error of 6.529 between the images and the highest percent difference was over 40%. This proves the moment technique works in detecting dissimilar images and can be applied as shape descriptors.

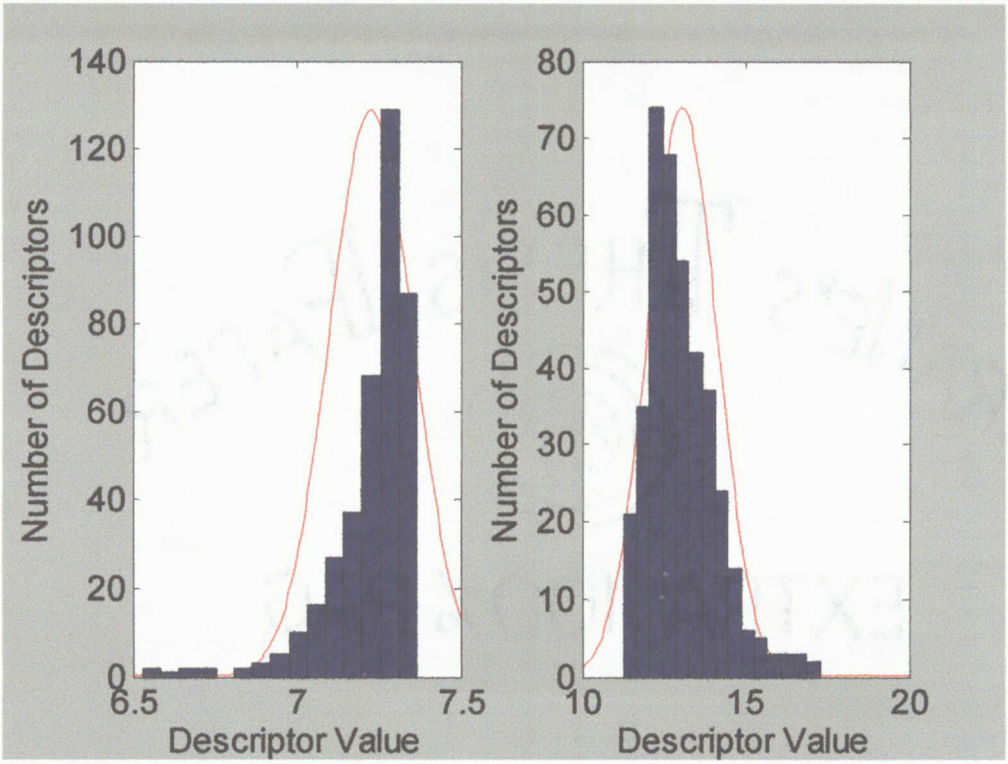
4.3.1 Statistics of Invariant Moment Shape Descriptors

The invariant moment histograms are also helpful to plot, since again a mean and a variance is used to classify the sand mixes. For invariant moments however, the process is not used in reverse to obtain reconstructions. This takes away some of the significance of the histograms, because random descriptors do not need to be generated from the statistics of the real descriptors. Well-behaved statistics are much more important for the reversible Fourier descriptors. The following figures display the histograms of the 1st and 3rd invariant moments.



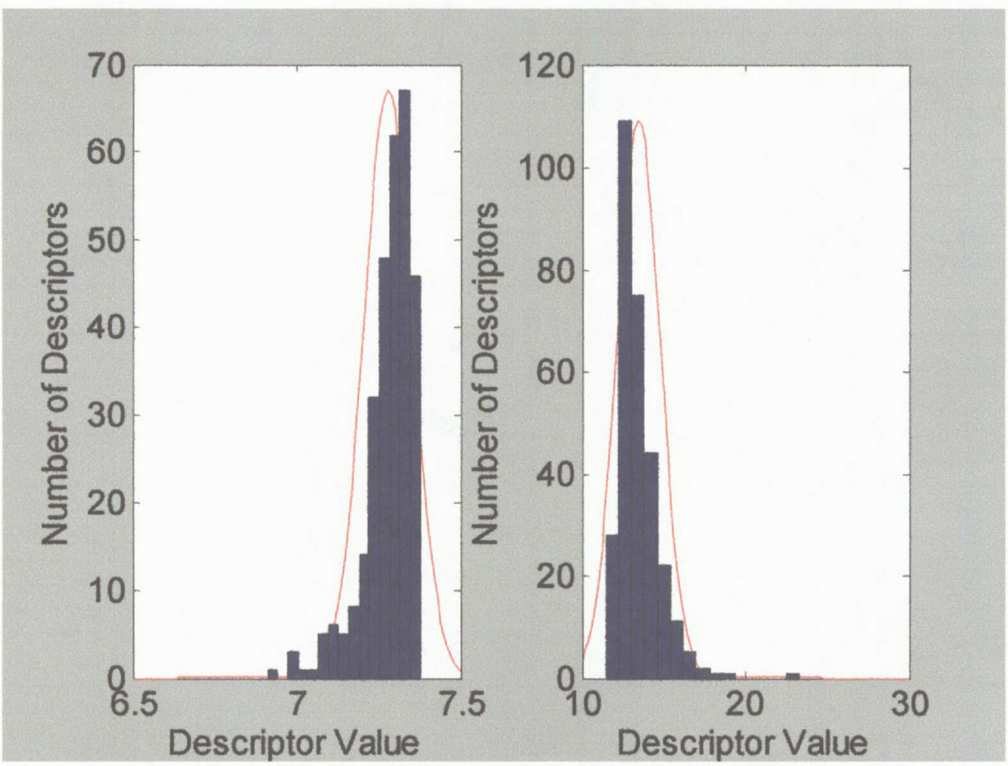
(a) (b)

Figure 4.27: #1 Dry Sand histogram of invariant moments (a) 1st and (b) 3rd.



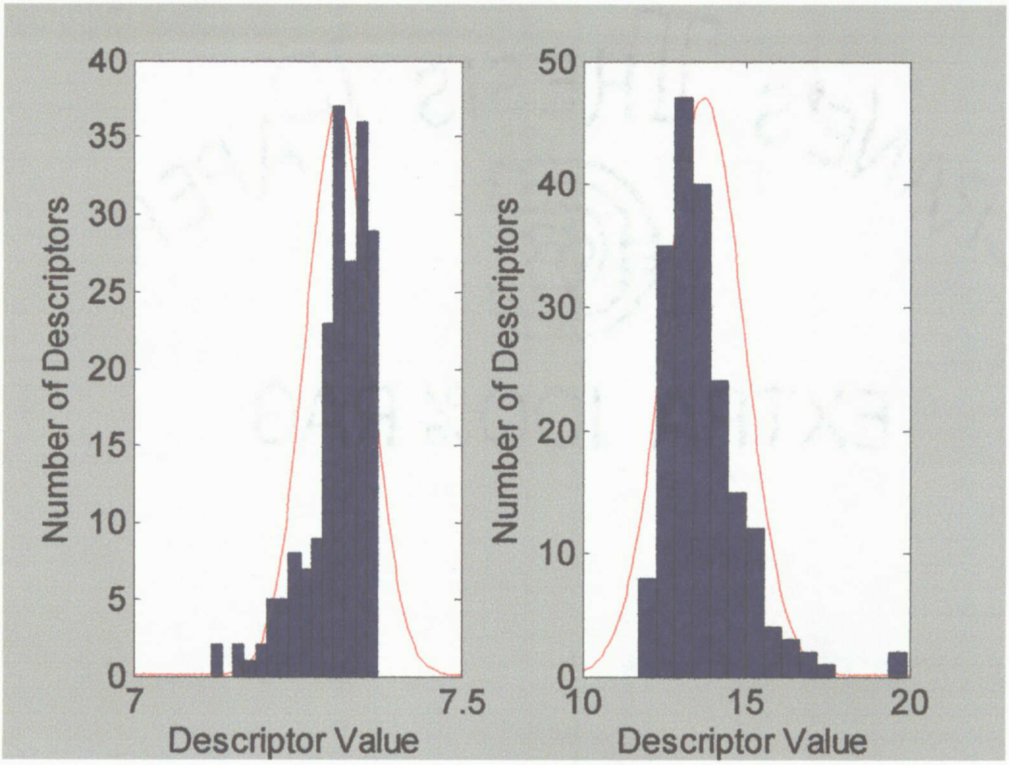
(a) (b)

Figure 4.28: Melt Sand histogram of invariant moments (a) 1st and (b) 3rd.



(a) (b)

Figure 4.29: Daytona Beach Sand histogram of invariant moments (a) 1st and (b) 3rd.



(a) (b)

Figure 4.30: Michigan Dune Sand histogram of invariant moments (a) 1st and (b) 3rd.

4.3.2 Classification Results using Invariant Moment Descriptors

The next set of results displays the use of invariant moments on the same database of particle projections used for the Fourier descriptors in Figures 4.23 and 4.24. Again for visualization purposes, only the three moments were plotted to give three dimensional ellipsoids as explained in Figure 4.22. The average ϕ_1, ϕ_2, and ϕ_3 values calculated throughout all the projections were used as x, y, and z coordinates respectively for the center of each ellipsoid. The variances of the moments were used as the radii for each ellipsoid. Figure 4.31 plots the results.

Similar to the Fourier descriptors, it is more meaningful to plot two sets of each sand mix to better demonstrate the effectiveness of the descriptors. Each dataset was randomly divided into two sets and plotted again in Figure 4.32.

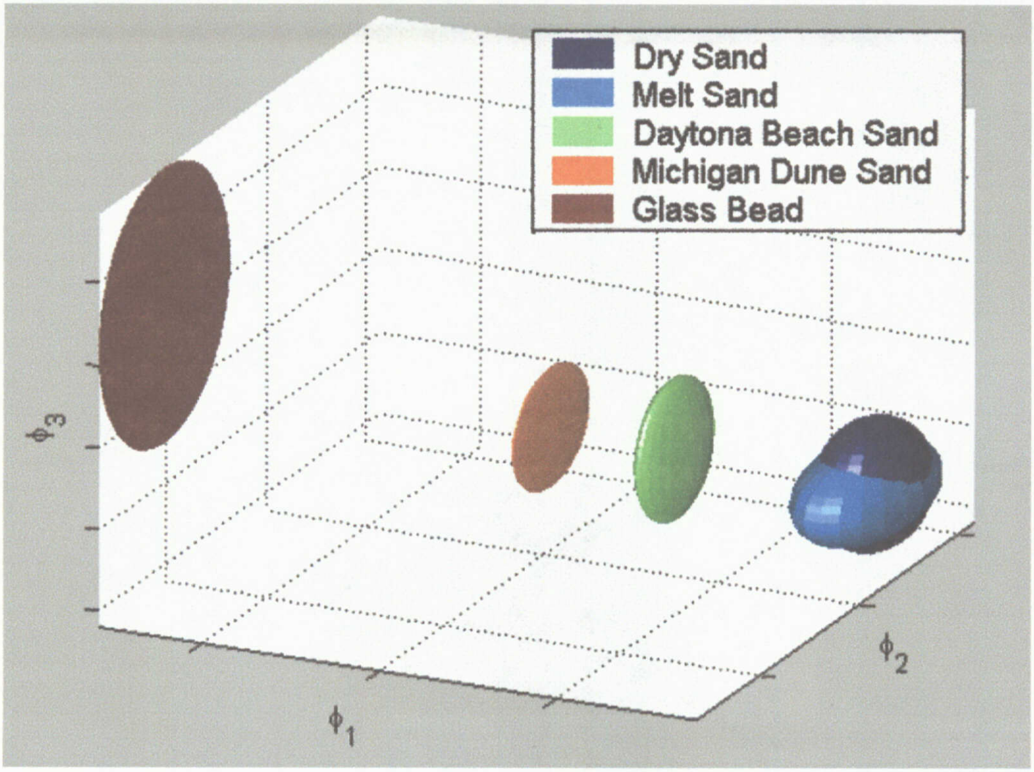


Figure 4.31: Ellipsoid plot showing separation of sand particles using invariant moments.

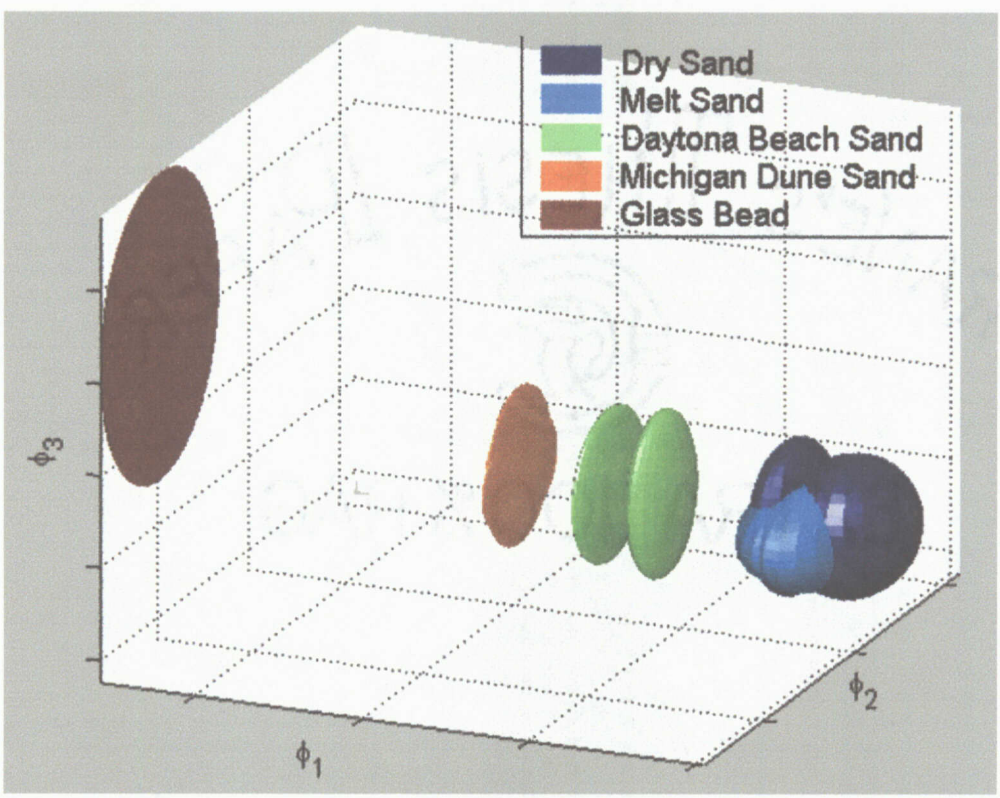


Figure 4.32: Ellipsoid plot using two datasets for each sand mix using invariant moments.

These results show the difficulty in separating the Dry and Melt Sand, which have similar shapes. The most notable conclusion attained from the invariant moment results is that the first moment is the only effective descriptor. For the most part the results are separable, but only the first descriptor is needed for the separation. Other than the glass bead, all four sand mixes lie on the same plane for the second and the third moments. This means the same separable results could have been obtained in one dimension using only the first descriptor.

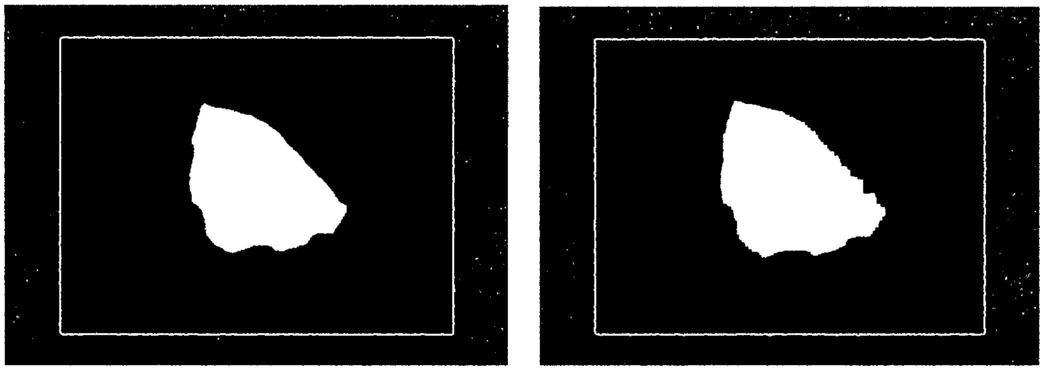
Although the invariant moment results do not appear to be as effective as the Fourier descriptors, they offer some validation for describing shapes using two-dimensional projections. It is encouraging that both results offer similar findings by being able to separate most particle mixes, which vary in shape. The next step is to try and reconstruct a three-dimensional object using the two-dimensional projections, in order to validate that the two-dimensional projections are a realistic representation of the actual three-dimensional shape.

4.4 Reconstruction and Validation

This section gives the results for three possible reconstruction methods, which can be used to validate the approach and analyze three-dimensional particles. A comparison between the methods is discussed at the end.

4.4.1 Two Dimensional Reconstructions from Descriptors

The ability to generate random projections is necessary to create models of actual sand particles without actually scanning every particle. By finding the average descriptors of the mix, an infinite number of projections can be created, with the same statistical characteristics as the original projections. Below is a comparison of two original images and their reconstructed projections. This reinforces the effectiveness of the reconstruction.



(a) (b)

Figure 4.33: Example of FFT reconstruction effectiveness; (a) original and (b) reconstructed.

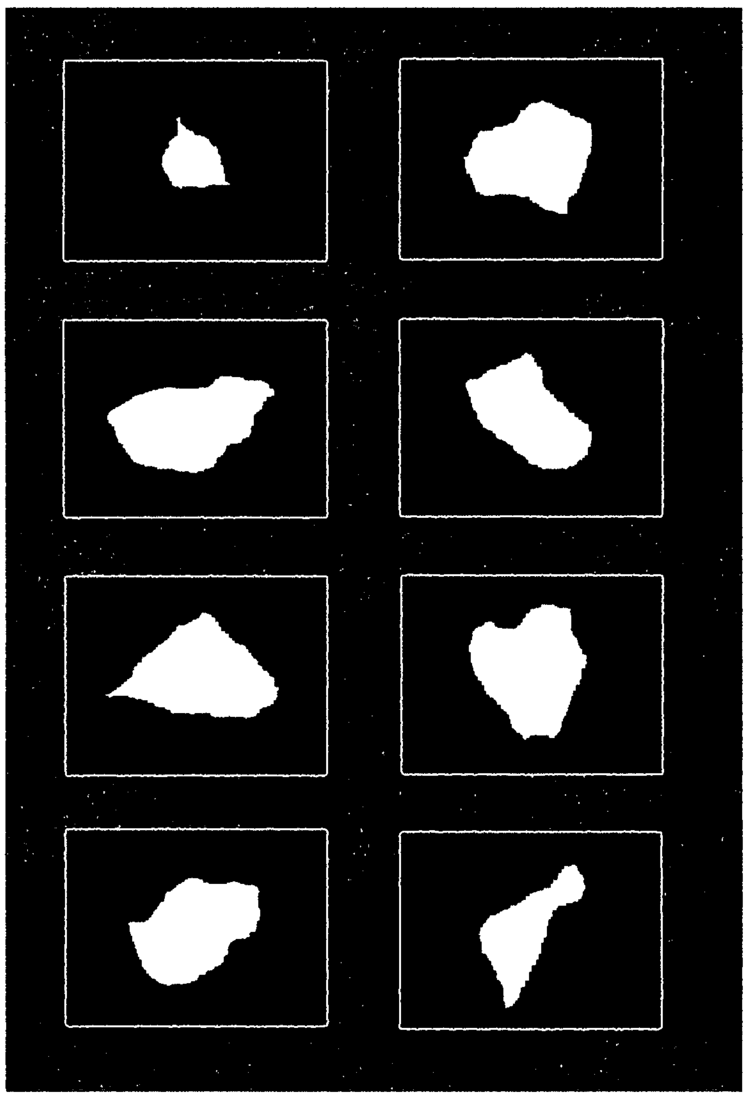


Figure 4.34: Randomly generated reconstructions.

To prove the effectiveness of these particles an ellipsoid plot was also created for a database of generated projections. The plot illustrates the difference between the original and synthetic databases. Since the statistics should be generally the same, little difference is expected from the ellipsoids. Figure 4.35 is a plot of the ellipsoids from the generated projections.

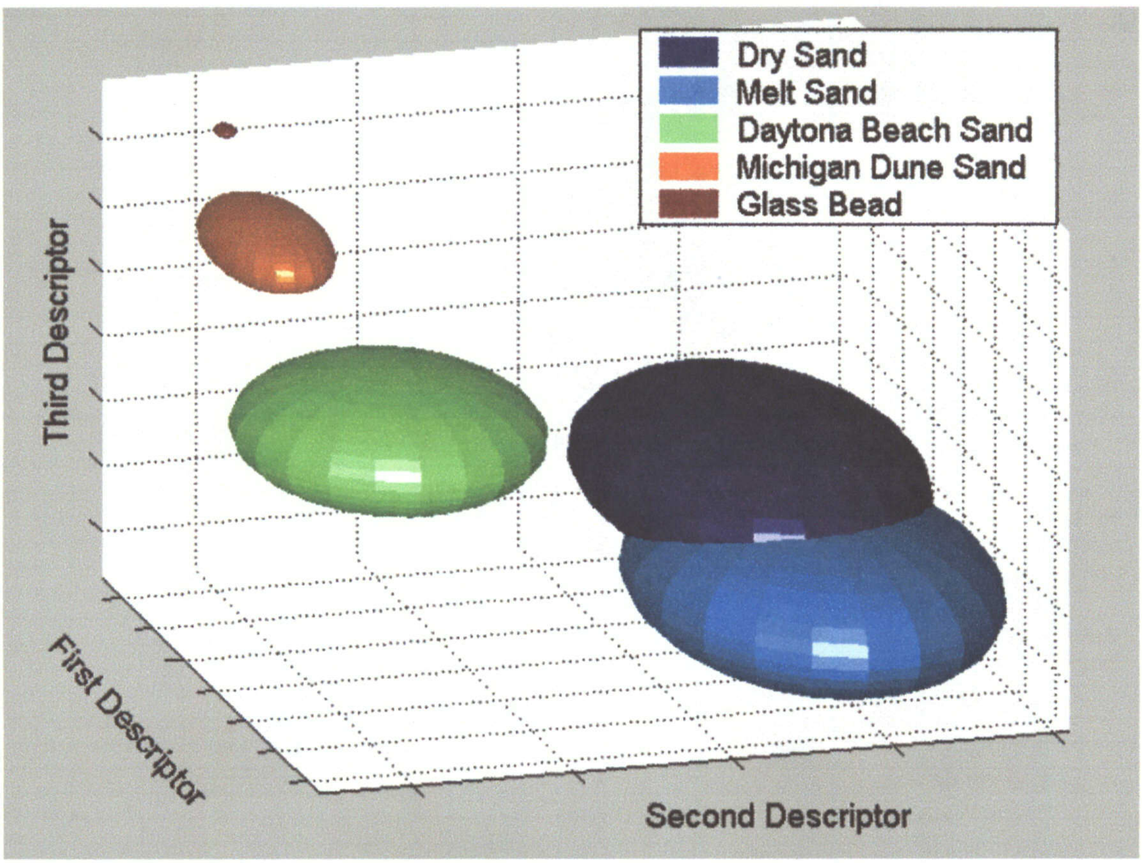


Figure 4.35: Ellipsoid plot of descriptor generated projections using Fourier descriptors.

Figure 4.36 compares the synthetic projections to the original database. Ellipsoids with a one in the middle represent the original dataset, while ellipsoids with a two in the middle represent the generated projection sets.

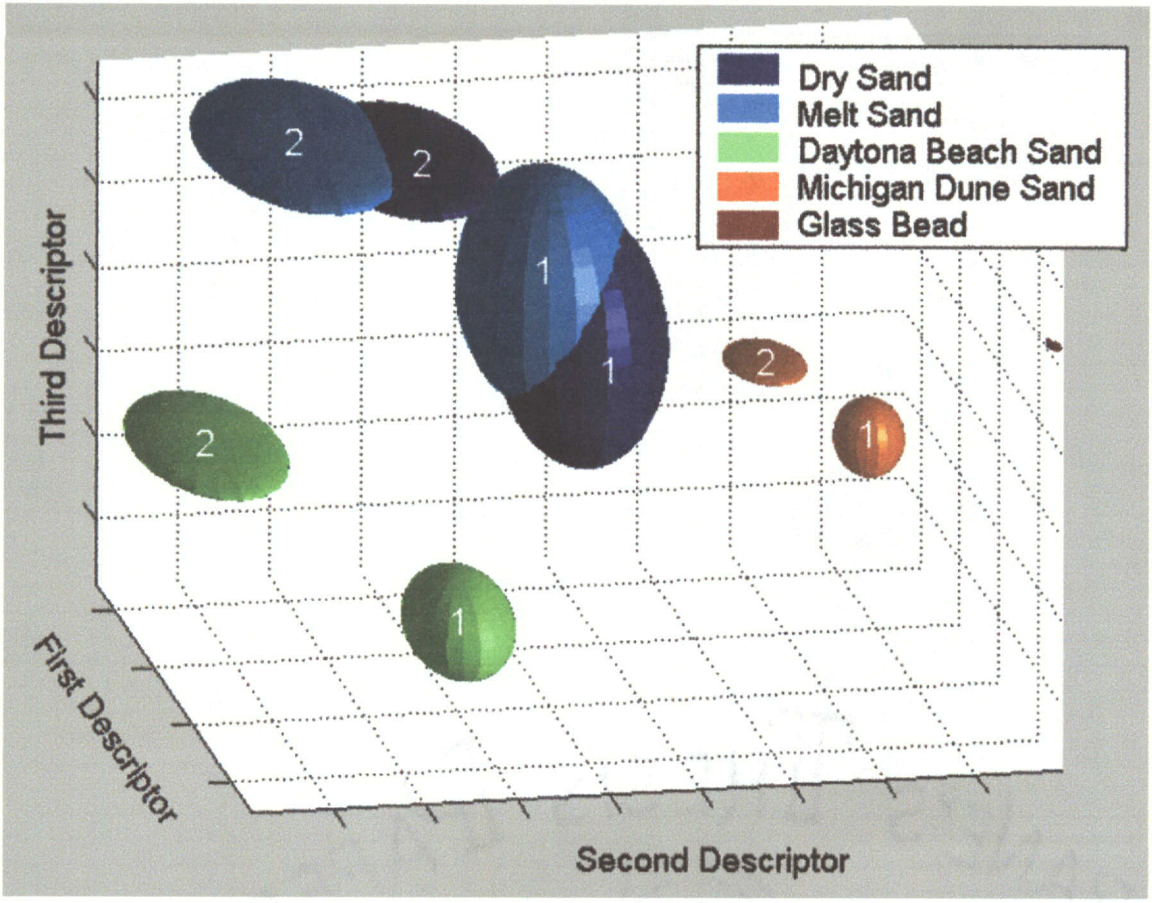


Figure 4.36: Comparison plot between (1) original dataset and (2) generated projections.

4.4.2 Three-Dimensional Reconstruction from Two-Dimensional Projections

Three different techniques for performing this task were attempted with varying degrees of success. This section will show the results of the three different techniques, starting with most easily implemented and ending with the most promising method. The finished three-dimensional representation of the particle will be shown first and then ellipsoid plots will be created to evaluate the algorithms effectiveness. All algorithms were tested using projections from the original #1 Dry Sand database.

4.4.2.1 Extrusion Reconstruction Method

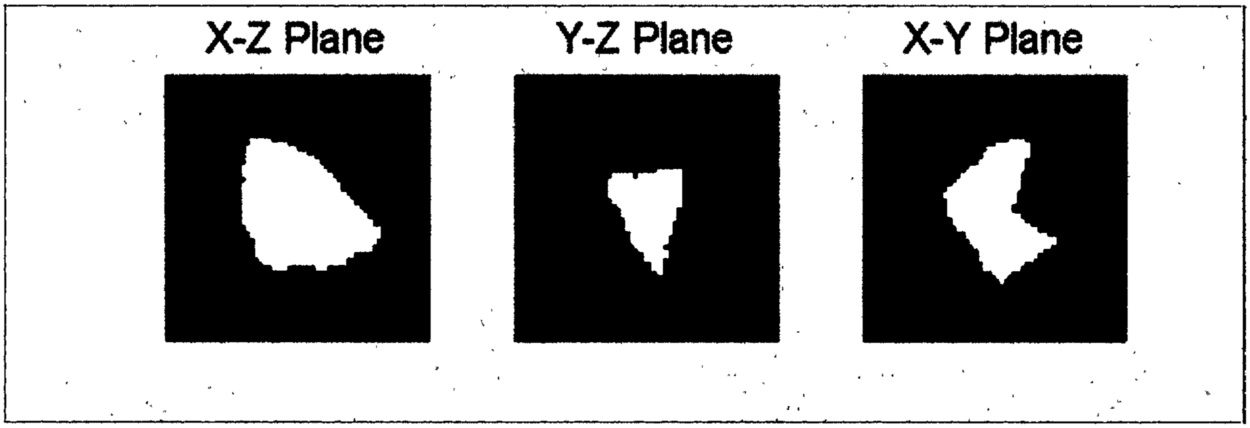


Figure 4.37: Projections used for reconstruction with Extrusion Method.

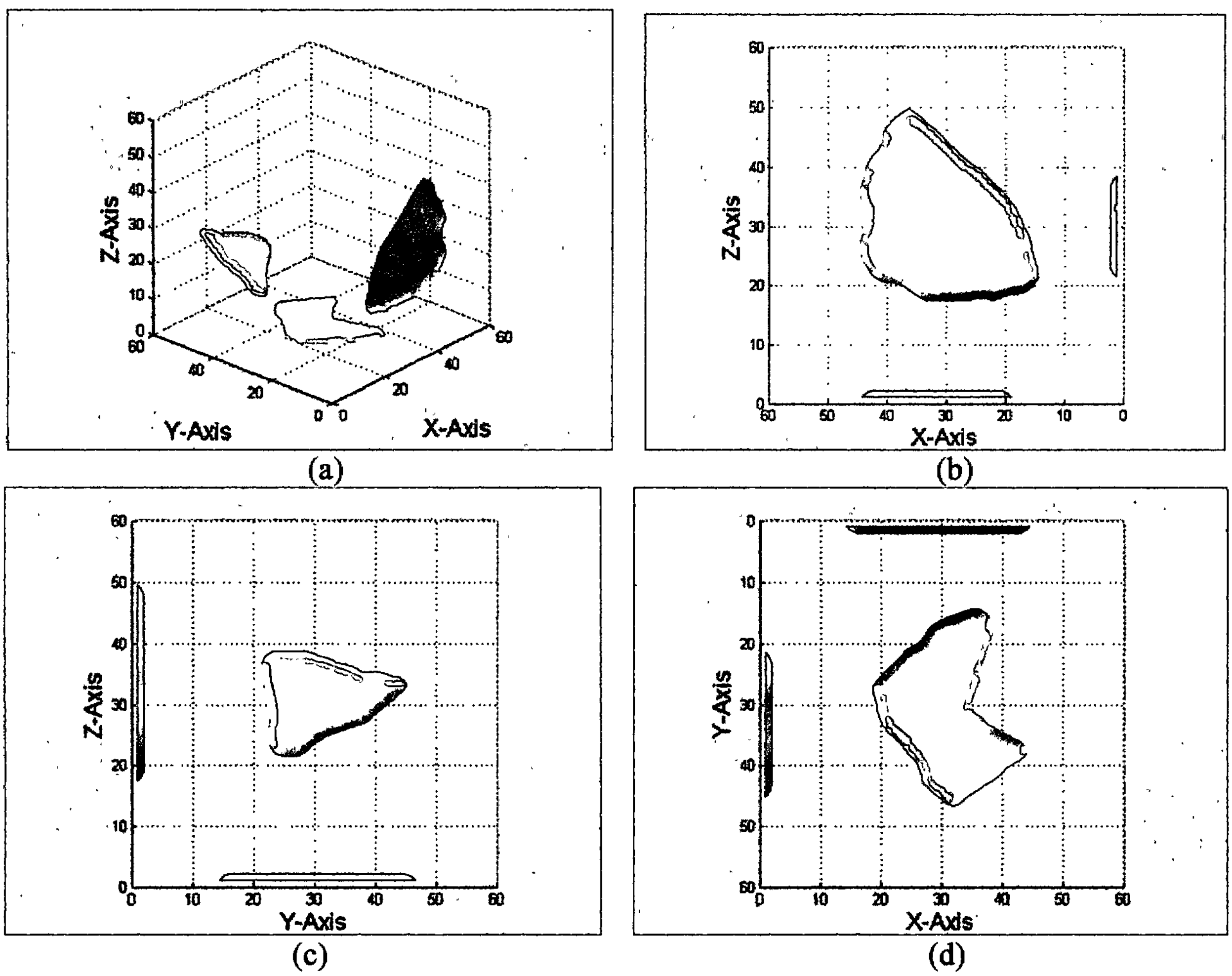


Figure 4.38: (a) View of projection placement prior to extrusion; (b, c, d) different angles of top left image to emphasize each projection used.

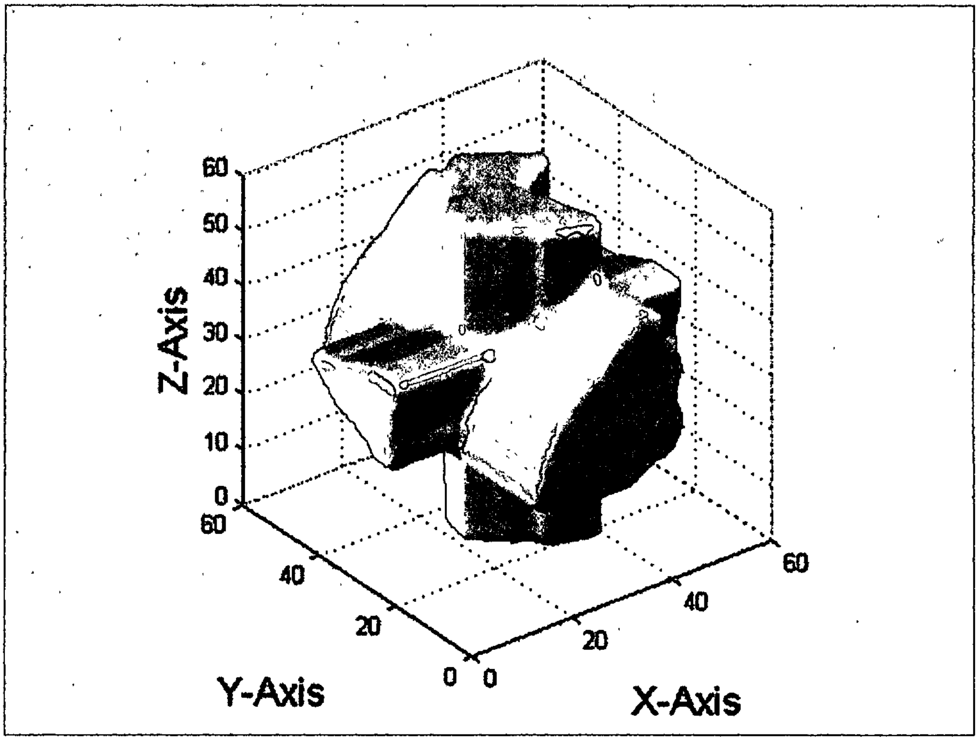


Figure 4.39: Projections after each one has been extruded into 3-D space.

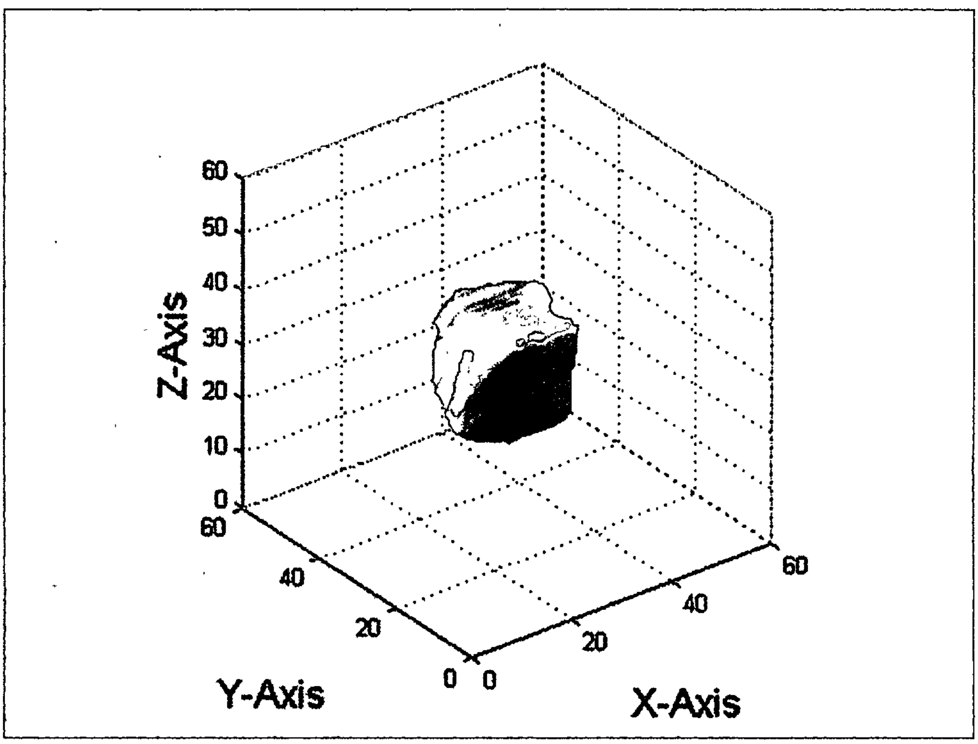


Figure 4.40: Final reconstruction of Dry Sand particle using values shared by most of the projections.

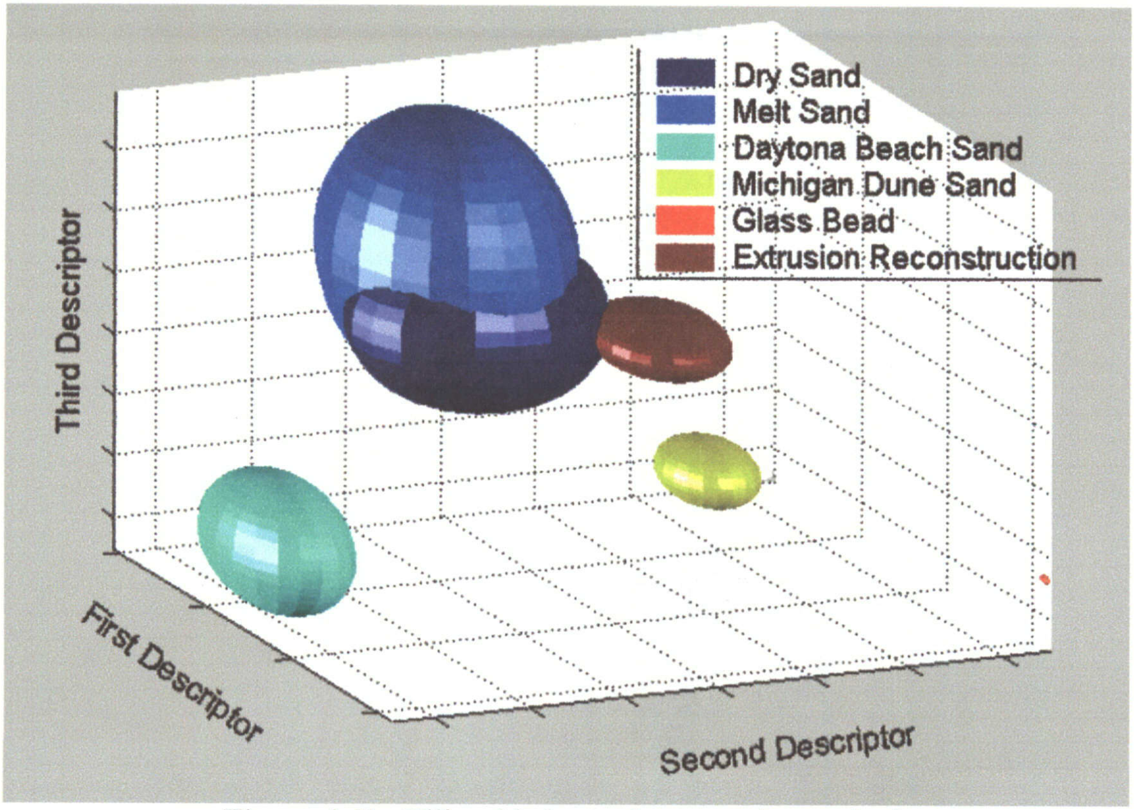


Figure 4.41: Ellipsoid plot testing extrusion method.

The results for this method only use three images extruded along three perpendicular axes. Figure 4.37 shows the three projections used and Figures 4.38 and 4.39 illustrate how they are placed into three-dimensional space and extruded. The final reconstructed particle in Figure 4.40 is a representation of where at least two of the three projections intersect. Figure 4.41 is a comparison between the original database and projections obtained from the three-dimensional object in Figure 4.40. More than three images and axes could be used to develop a more sophisticated model.

4.4.2.2 Rotate into Three-Dimensional Space Method

Rather than stretching the projections, this technique rotates the objects into three-dimensional space. This causes the result to be more of a rounded shape when generating a reconstructed particle from points shared by a majority of the projections. Figure 4.42 displays a combination of a reconstructed particle about two perpendicular axes. Figure 4.43 compares these results to the original database.

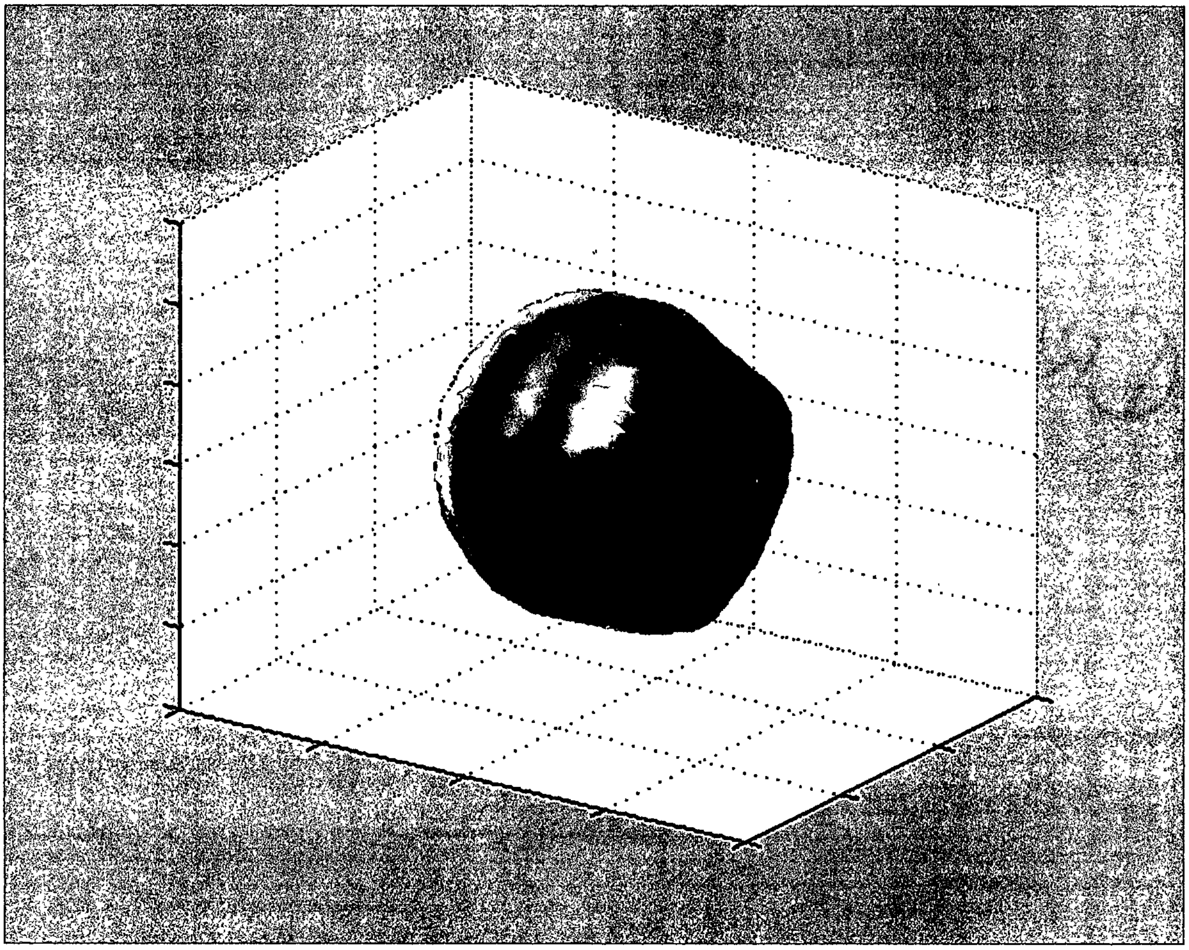


Figure 4.42: Reconstruction of Dry Sand particle using rotation into 3-D method by rotating about 2 axes.

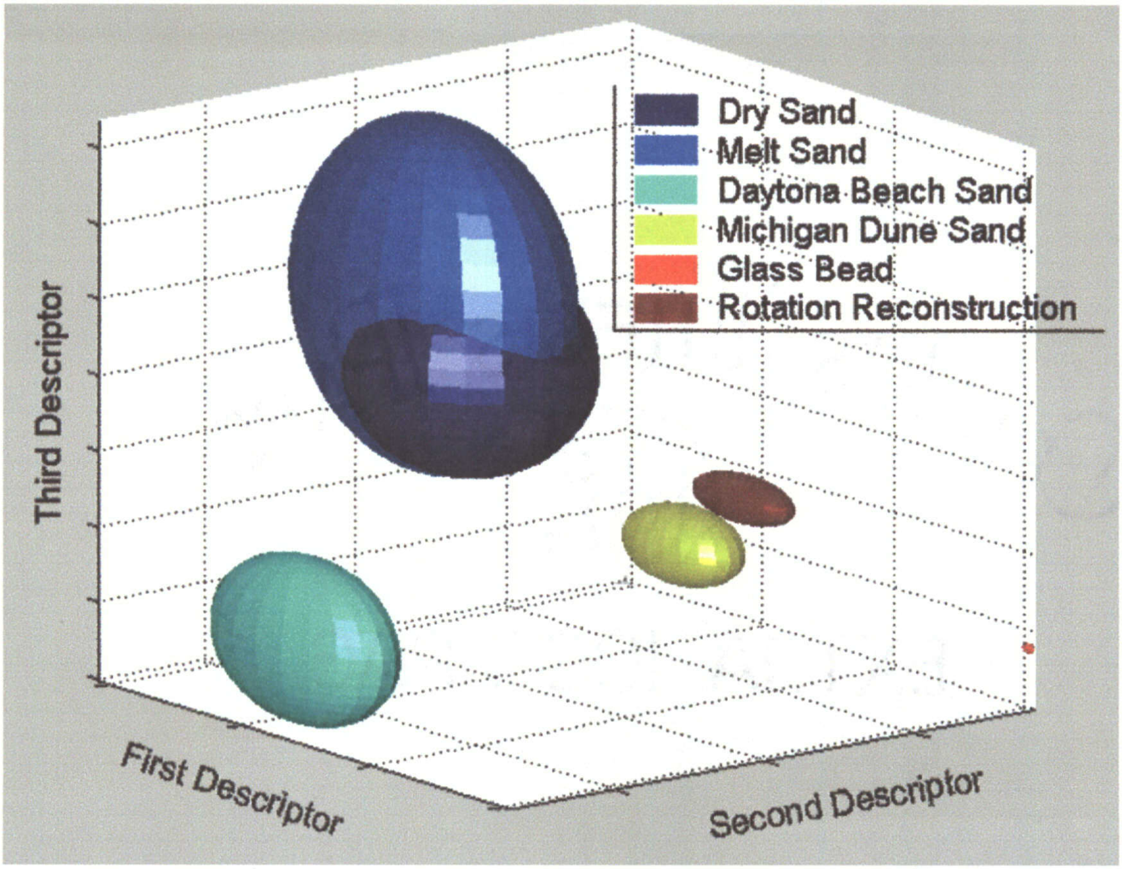


Figure 4.43: Ellipsoid plot testing rotation into 3-D space method.

4.4.2.3 Tomographic Method

With each projection being placed into three-dimensional space, rotated and combined at the images centroid, this is the most promising reconstruction method. The reconstruction shown in Figure 4.44 and 4.45 use 180 projections at one degree intervals. The first is rotated around the X-axis and the second around the Y-axis. These can be used alone or could possibly be combined and averaged together. Results on the comparisons with the original databases are shown in Figure 4.46 for the X-axis rotation.

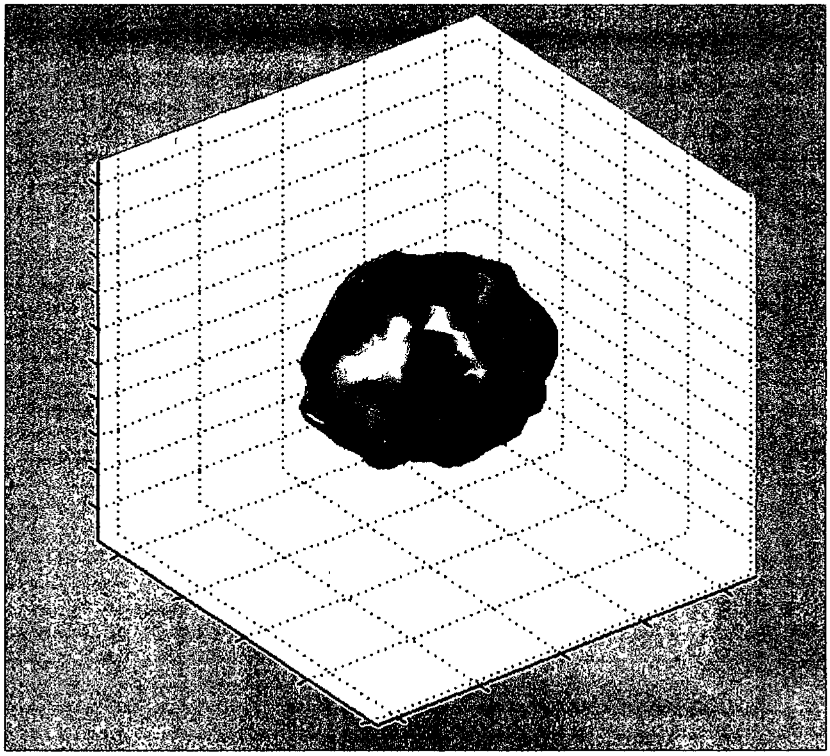


Figure 4.44: Tomographic reconstruction of Dry Sand particle with projection rotation only about the X-axis.

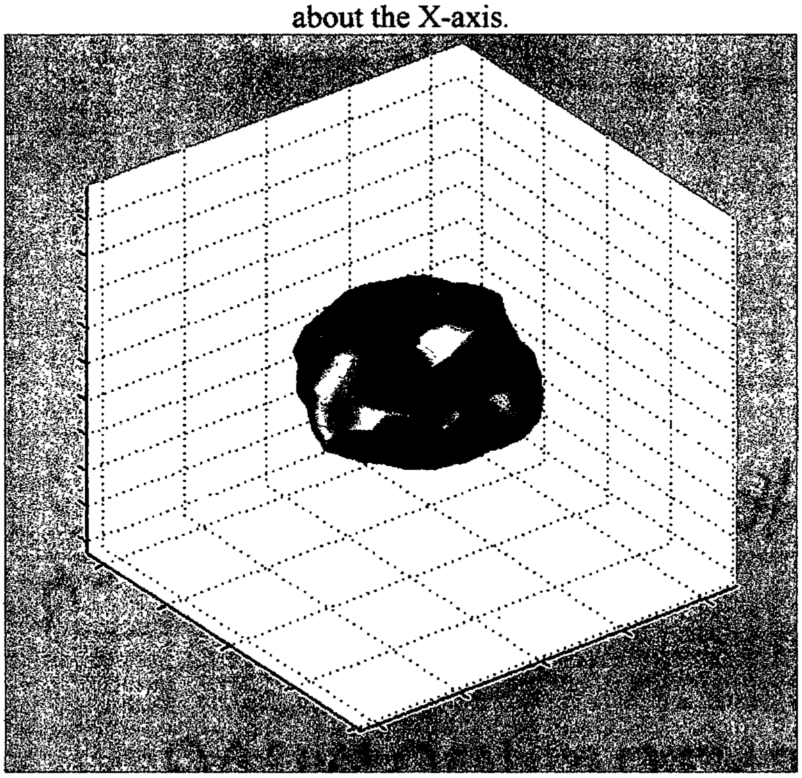


Figure 4.45: Tomographic reconstruction of Dry Sand particle with projection rotation only about the Y-axis.

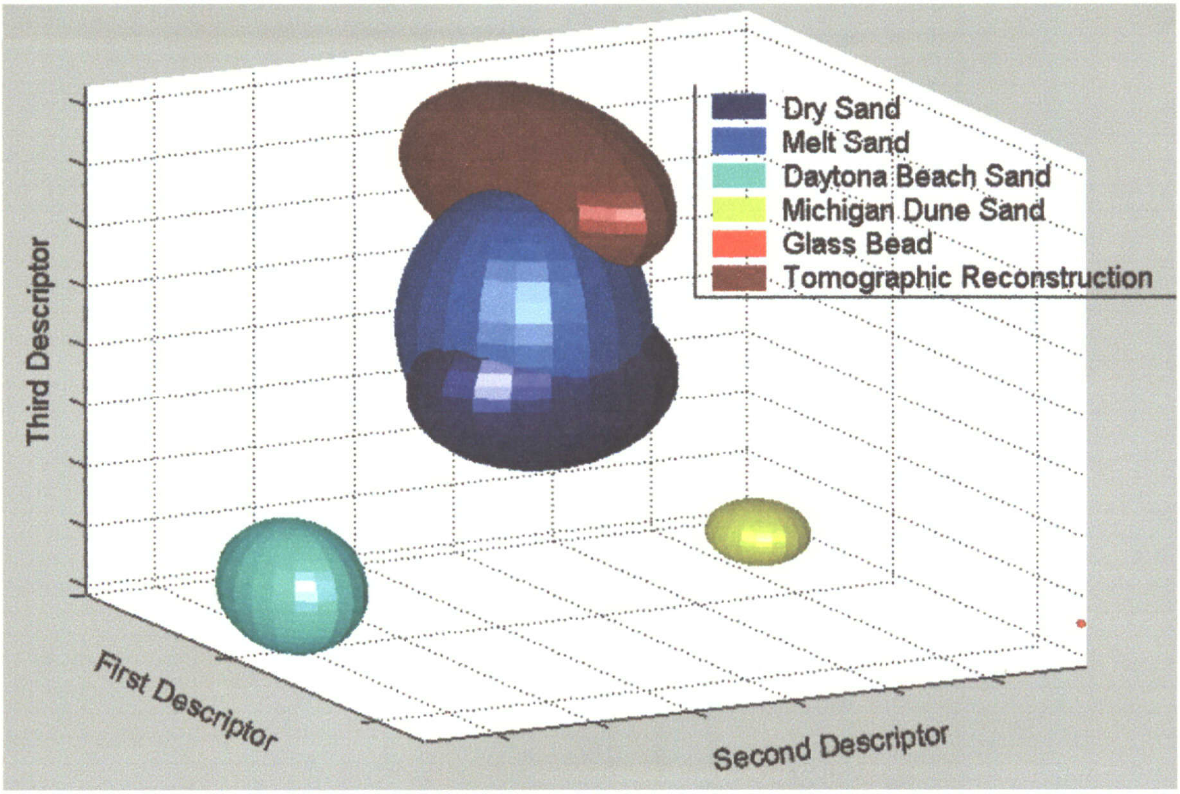


Figure 4.46: Ellipsoid plot testing tomographic method about X-axis only

Table 4.3 shows the inter-ellipsoid distance between each reconstruction ellipsoid to all of the ellipsoids of the various soil mixtures. The values illustrate the proximity of each of reconstruction method in relationship to the other ellipsoids. The actual values cannot be compared from one method to another. Only within each method are the distances relevant.

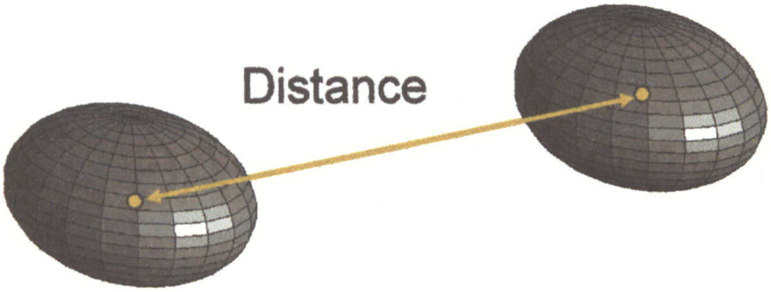


Figure 4.47: Illustration of inter-ellipsoid distance.

Table 4.3: Results of reconstructed particle ellipsoids compared with original ellipsoids

Reconstruction Method	Inter-ellipsoid Distance			
	Dry	Melt	Daytona Beach	Michigan Dune
Extrusion	.1505	.1899	.4778	.2225
3-D Rotation	.2937	.3549	.4798	.0800
Tomographic	.2976	.2163	.6619	.5556

4.5 Discussion of Results

The statistical two-dimensional approach to solving the three-dimensional problem shows a lot of promise based on the results of this section. The statistics of the descriptors illustrated in Figures 4.18-4.21 and 4.27-4.30 prove that the values obtained for the projections follow closely to a Normal Gaussian distribution. Determining all the descriptors follow a normal distribution ensure there is regularity to the particle descriptors, which makes them vary in a predictable manner around the mean. This gives confidence to the concept of using the mean and variance of the projections to calculate the three-dimensional descriptors.

Classification of the various sand mixes using Fourier analysis depicted by the ellipsoid plots in Figures 4.23 and 4.24 appear to perform as predicted. The dissimilar shapes were easily separated, while the #1 Dry and Melt Sand, which have similar shapes, were not as successful. This shows the effectiveness of the algorithm at distinguishing between very different shapes, but indicates an area of the procedure that could be improved. Since the objective is to characterize shape that influences soil mixture properties however, #1 Dry and Melt Sand may not need to be separated by this approach. Most of the physical behavior of the sand mixes is also similar and therefore, they could be classified together.

Invariant moment descriptors also successfully separated the collection of soil mixtures, but only seem to use the first moment. By looking at the ellipsoid plots in Figures 4.31 and 4.32, it can be seen that they are only able to be classified along the first descriptor. The other descriptors are no help to the result. This does prove however that the three-dimensional particles of the sand mixes can be classified using only two-dimensional methods on their two-dimensional projections. This reinforces the results discovered using Fourier analysis.

The final results generated, dealt with the feasibility of using different techniques to create three-dimensional particles from the two-dimensional projections. In Figure 4.34 randomly generated projections of the sand mixes from their descriptors is shown. The ellipsoid plots of 4.35 and 4.36 show their ability to be separated and how they match up with the original projections. These figures illustrate how it can be possible to construct a database of new projections from the three-dimensional descriptors. The projections can be used to reconstruct three-dimensional particles.

The three-dimensional reconstruction algorithms are presented with preliminary results. The results for the extrusion method shown in Figure 4.41 demonstrate the possible effectiveness of this approach, but more work needs to be done to cover more than three axes. The second method is not likely to be a successful technique for reconstruction. The results shown in Figure 4.43 plot the synthetic particle closer to Michigan Dune than to #1 Dry Sand. All the projections rotated and averaged together give a final particle that is too round, because of the smoothness of the rotations. Of the three methods reviewed for reconstructing a representative particle, the tomographic method shows the most promise. The final results compared in Table 4.3 are similar with those of the extrusion method, but the tomographic technique is more appropriate. The extrusion method only uses 3 of the projections to generate the model, where the tomographic method uses 180. This means the tomographic method incorporates a better estimate of the statistics of the aggregate mix than the extrusion method. More work can be done on both these methods to determine the more efficient technique. The next chapter discusses this and other recommendations for future work. The chapter also summarizes the objectives met and the contributions made.

CHAPTER 5: CONCLUSIONS

Unlike shape description in two-dimensions, the three-dimensional counterpart is more complex both in terms of approach and implementation. The situation is even more challenging when it comes to describing the three-dimensional shapes of particles in aggregate mixes such as small grains of sand. This thesis has attempted to address this challenge by subdividing the problem into a judicious combination of simple techniques – two-dimensional shape description, feature extraction, statistical modeling and projective reconstruction. This is a powerful concept which will enable experts to model entire sets of three-dimensional sand mixes with only a few set of numbers. These numbers are even easy to obtain for new mixes of sand with very little equipment; an optical microscope and a digital camera.

Currently researchers model three-dimensional sand particles using a series of overlapping spheres. It is very difficult to even know how to use the spheres to model a collection of sand particles, since there could be millions of grains of sand in a single test. If three-dimensional models could be created, which share similar characteristics to actual sand projections in a mix, then real data from the sand particles could easily be implemented in computer models. This will greatly increase the effectiveness of computer tests at modeling real stress-strain behavior in soils.

5.1 Summary of Accomplishments

The principal accomplishments in this research work are:

1. The design and development of automated algorithms that can estimate three-dimensional shape-descriptors for particle aggregates using a statistical combination of two-dimensional shape-descriptors from multiple two-dimensional projections. In particular, the use of Fourier descriptors and invariant moments were investigated for this purpose.

The corresponding two-dimensional descriptors from multiple particles were modeled as normal probability distributions.

2. A database containing a library of approximately 200-300 two-dimensional digital images for 5 aggregate mixtures was constructed. The three-dimensional shape-descriptor algorithm was exercised each of these aggregate mixes. Principal component analysis was performed to extract features from the two-dimensional descriptors and an ellipsoid model was used to demonstrate the consistency, separability and uniqueness of the algorithm.
3. Preliminary efforts were made to demonstrate the algorithm's ability to accurately and repeatably construct composite three-dimensional shapes from multiple two-dimensional shape-descriptors. Three methods were investigated – extrusion, rotation and tomographic reconstruction.

5.2 Conclusions

Three-dimensional shapes are complex and require sophisticated equipment to characterize properly. The concept of applying two-dimensional techniques to estimate the solution of a three-dimensional problem, offers a powerful and simplistic method of describing shape. In this thesis, an automated algorithm was designed to calculate three-dimensional shape descriptors using a combination of two-dimensional shape descriptors from multiple two-dimensional projections. The algorithm could effectively separate mixes of sand with different shapes to an acceptable degree of success. Based on previously used methods for obtaining three-dimensional shape descriptors, the procedure performed in this thesis is simple and requires little equipment.

In addition to proving its ability to distinguish differently shaped sand particles from one another, the developed algorithm allows for random three-dimensional reconstructions to be generated from the projections. As discussed earlier, this allows for the creation of an entire database of sand mix particles, which actually models the real aggregates. This reconstruction also helps validate the three-dimensional descriptors by proving that a three-dimensional object can be characterized by its two-dimensional projections.

These accomplishments make an important contribution to three-dimensional shape characterization. This thesis explores the possibility of using simple methods to achieve a complicated goal. The work done in this thesis is only the first step in completing a fully functional and effective shape descriptor and reconstruction algorithm. This work proves the usefulness and feasibility of a technique using two-dimensional projections to estimate three-dimensional descriptors and to construct more projections from those descriptors. More research and development, which could be explored to further this work is shown in the next section.

5.3 Recommendations for Future Work

As with almost any work, there is room for improvement of the algorithms in this thesis and for entirely new algorithms to be designed. This section will discuss a few of the advancements, which can be made directly following on the work already been done.

The first task completed in this thesis was the creation of an algorithm to obtain three-dimensional descriptors and to show the separability between differently shaped sand mixes. This technique could be tried different ways by using different numbers of Fourier coefficients or more importantly discovering the most descriptive coefficients. More or less than the twenty-six descriptors used in this thesis may be proven to produce a greater separability. In addition to this, PCA was used to reduce the dimensionality to three, which may not be necessary. Better results may be obtained from separating at higher dimensions rather than reducing them. These improvements are minor and attempt to achieve better results than techniques, which are already fairly effective.

A large improvement can be discovered by more work being done in reconstructing three-dimensional objects from two-dimensional projections. In this thesis, three techniques were presented for the purpose of advancement in this area. Any of the methods performed may offer an effective way of reconstructing sand particles from a mix. Significant work can be done on any of them, such as providing more angles or projections, which could offer better results than the preliminary ones obtained at the end of Chapter 4.

Shape description of particles in a sand mix can be extremely useful in stress-stain soil behavior. With the work done in this thesis and future work accomplished to perfect a reconstruction algorithm, the computer modeling of soil mixes can be greatly improved. This can increase the accuracy of computer models, which will decrease the necessity to perform the tests on real mixes. The computer models are more preferable, because they allow quick changes, take less time to run, and can be incorporated into full-scale models. Three-dimensional shape characterization of soil particles can assist in understanding how shape affects soil behavior and in the future might be used to create stronger materials, which will waste fewer resources.

The algorithms developed in this thesis have the potential for other applications where three-dimensional shape characterization for aggregate mixes is important. For example, the adherence properties of toner particles in photocopying and laser printing applications are critically dependent on their shape characteristics [18]. Another application may exist in industrial products that consist of powders. These materials often depend on shape and size distribution in both processing and storage, including properties such as flow rate of the particles [19]. The methods developed in this research work show considerable promise for addressing these diverse applications in future.

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