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William H. Widen
University of Miami School of Law, wwiden@law.miami.edu

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FORCING ANALOGIES IN LAW: BADIOU, SET THEORY, AND MODELS

*William H. Widen**

INTRODUCTION

Why think about set theory as part of legal studies? In particular, is there value in reading works by Alain Badiou, such as *Being and Event*,[1] as speaking to law? This is the general question considered by this essay. By "law" I mean to designate juridical law, unless otherwise indicated.

Being and Event makes many claims and distinctions. I focus on a few: first, the claim that equates mathematics with ontology; second, the distinction made among the one, the multiple, and multiplicities. In this regard, Badiou observes that traditional western metaphysics conceived of a multiple as dependent on the concept of the one.[2] He identifies the work of mathematician Paul Cohen as providing a conceptual framework for thinking about multiplicities independent of reliance on the concept of the one (i.e. without the need for a prior concept of individuals).[3] Cohen's innovation was to introduce the technique of "forcing" to prove that the traditional axioms of set theory are consistent with the negation of the continuum hypothesis.[4] Previously, Kurt Godel had proved that the traditional axioms of set theory are consistent with the truth of the continuum hypothesis.[5] Together, the results of Godel

* Professor of Law, University of Miami School of Law, Coral Gables, Florida; wwiden@law.miami.edu. I am grateful for discussions with Alain Badiou, Susan Haack, Renata Salecl, and Karl Shoemaker about this project.

[1] ALAIN BADIOU, BEING AND EVENT (Oliver Feltham trans., 2005) (1998) [hereinafter BADIOU, BEING AND EVENT]. I include in this group of writings for consideration, ALAIN BADIOU, INFINITE THOUGHT, TRUTH AND THE RETURN TO PHILOSOPHY (Oliver Feltham & Justin Clemens trans. & eds., 2005) [hereinafter BADIOU, INFINITE THOUGHT] and ALAIN BADIOU, THEORETICAL WRITINGS (Ray Brassier & Alberto Toscano trans. & eds., 2004) [hereinafter BADIOU, THEORETICAL WRITINGS].

[2] *See, e.g.*, BADIOU, BEING AND EVENT, *supra* note 1, at 23; BADIOU, THEORETICAL WRITINGS, *supra* note 1, at 41-42.

[3] *See, e.g.*, BADIOU, BEING AND EVENT, *supra* note 1, at 355-58.

[4] *See* PAUL J. COHEN, SET THEORY AND THE CONTINUUM HYPOTHESIS (1966). The continuum hypothesis is described *infra* in note 56.

[5] *See* KURT GODEL, COLLECTED WORKS: VOL. II: WORKS 1938-1974 (Solomon Feferman et al. eds., 1990).

and Cohen establish the independence of the continuum hypothesis from set theory.

The foregoing summary likely seems far from legal considerations. I hope to point out a few ways in which this judgment might be premature. Importantly, *Being and Event* is not a mathematics book. It does not contain proofs or other matters of direct interest to working mathematicians. Rather, it contains a close and detailed study of the structure of modern set theory and some of its concepts, often reflecting extensively on the meaning of individual axioms of Zermelo-Frankel axiomatic set theory with the axiom of choice (ZFC).

As I read Badiou, he calls for increased attention to rigor in philosophical thought by challenging philosophers (particularly continental/French philosophers) to adopt/adapt methods found in logic and mathematics to their scholarship. This stance is not one designed to make friends. Unlike Jacques Lacan's relatively simple use of symbols and mathemes to illustrate points (as today one might make abbreviations on a Powerpoint slide),[6] Badiou actually expects (even demands) close study of the axioms of set theory. I do not want to address here the degree to which Badiou's exposition is successful. Rather, I want to consider the ways in which his project makes sense and to identify some structural challenges for that project.

Professional mathematicians have little interest in Badiou's project because his focus is not on expanding knowledge by traditional methods of proof and inquiry. How dare someone simply reflect on past achievements and formalizations and then consider what they might mean in a broader context? "We don't like it (despite Badiou's insistence that we mathematicians are the true philosophers/ontologists)," or so I expect a typical mathematician would say, even in a kinder, reflective moment.[7]

Continental philosophers (to the extent any such broad grouping makes sense) are, I expect, no more welcoming of the approach. At a surface level, continental philosophy might be contrasted with analytic philosophy as poetry might be contrasted with physics.[8] For those

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[6] I do not mean to imply that Lacan's theories are simple or easy to understand. My point relates to the form of presentation. I have in mind, as an example, the Graph of Desire from SUBVERSION OF THE SUBJECT AND DIALECTIC OF DESIRE IN THE FREUDIAN UNCONSCIOUS (1960), *reprinted in* ECRITS: A SELECTION (Alan Sheridan trans., 1977).

[7] In harsher moments, mathematicians might group Badiou with other alleged pseudo-academics that the hard scientists seem to despise (examples include Derrida and Lacan). Regardless of how one feels about this debate, Badiou's close attention to the actual axioms of ZFC set him apart. My experience is that gems can be found in Badiou's works which might motivate thinking about law and other social structures. By adopting a stance of formal rigor, I take Badiou's procedure as one designed to make gem extraction easier.

[8] I mean no disrespect for either poetry or physics by this remark. Indeed, I later refer to an important observation of Parmenides which appears in a poem. *See infra* note 34. Badiou has spoken about three divisions in modern philosophy: the hermeneutic tradition (strong in

accustomed to presenting their views in a literary style (often relying on analogy, metaphor and similar techniques to make points), studying the axiom of extensionality, for example, generates little enthusiasm. Thus, while Badiou's project might be seen as a vehicle for continental philosophers to enter dialog with analytic philosophers, we find a predictable hostility and skepticism in both camps.

In this essay, I want to defend one instance of using basic results in mathematics and logic as a source for analogies useful for thinking about social phenomenon, including law. In particular, I consider Badiou's focus on the concept of a multiple without a one.[9] (I accept that this may be a conservative and underwhelming use of Badiou's approach—but it is one way I see his work applying most directly to legal thinking.)[10] In the defense, I make three general points and a technical point. First, I provide a few examples of cases in which logicians speculate about the applicability of methods and results in their field to social phenomena such as law and government. This analysis motivates the idea that applying formal logic to law may be a plausible enterprise from the perspective of logicians themselves. Second, I provide an example of how some basic results in meta-mathematics illustrate (or may charitably be taken to illustrate) points made by continental thinkers, particularly Lacan (my prime example being the Lowenheim-Skolem theorem).[11] This example motivates the idea that continental philosophers might usefully explore structures developed in mathematics and philosophy that are of interest to analytic philosophers. Third, I provide an example to illustrate how mathematical analogy might be useful for thinking about aspects of theories of justice. Fourth, I consider limitations on the successful use

Germany), the analytic tradition (originating in Austria but now dominating Anglo-American philosophy), and the post-modern tradition (originating with Derrida and Lyotard, and now active in France, Spain, Italy, and Latin America). *See* BADIOU, INFINITE THOUGHT, *supra* note 1, at 31. I intend to contrast the analytic tradition in philosophy with both the hermeneutic and the post-modern traditions which I consider collectively as continental in style for purposes of this essay (recognizing the risks of oversimplification by using any broad categorizations such as these).

[9] BADIOU, BEING AND EVENT, *supra* note 1, at 38-51; BADIOU, THEORETICAL WRITINGS, *supra* note 1, at 68-82.

[10] I do not address the vast legal literature discussing analogical reasoning in legal argument. For an avenue into these writings, see Scott Brewer, *Exemplary Reasoning: Semantics, Pragmatics, and the Rational Force of Legal Argument by Analogy*, 109 HARV. L. REV. 923 (1996).

[11] English translations of the original theorems of Lowenheim and Skolem may be found in FROM FREGE TO GODEL: A SOURCEBOOK IN MATHEMATICAL LOGIC (Jean van Heijinoort ed., 1977). Skolem proved a generalized version of Lowenheim's theorem, and the results achieved are referred to collectively as the Lowenheim-Skolem theorem. Contemporary proofs of the Lowenheim-Skolem theorem appear in JANE M. BRIDGE, BEGINNING MODEL THEORY 97-100 (1977). The portion of the theorem relevant to this discussion is known as the "downward" theorem and simply states that if S is a countable, satisfiable set of sentences, then S has a countable model.

of analogy, focusing particularly on Badiou's use of the forcing technique in set theory (in which the Lowenheim-Skolem theorem plays a significant role). I provide an alternate example from mathematics for use in an analogy to motivate the idea of a multiple without a one. I conclude with some additional examples of how the general idea of the contrast between a multiplicity without ones and a multiplicity composed of ones might be used to think about social structures.

I. LOGICAL AND MATHEMATICAL THINKING APPLIED TO LAW

I joined the faculty at the University of Miami while John Hart Ely[12] was still alive. We often would speak in his office on a variety of topics—rarely directly related to law. One conversation drifted toward the fact that we had both studied philosophy as undergraduates yet decided to go to law school. The topic came up while discussing one of his last papers—a piece entitled *Ely's Wager*.[13] It turns out we both opted for law rather than philosophy, in part, out of a concern we each had to the effect that we would not have been able to make contributions to logic at the highest level. We each selected law as something at which we might succeed. In that setting, Professor Ely remarked that maybe logicians would find law difficult. I replied that there was some evidence to suggest a difference of opinion between Alan Turing and Kurt Godel on the role that logical reasoning of a formal sort might play in thinking about the nature of law—particularly the U.S. Constitution. This hypothetical dispute particularly interested me because each of Godel and Turing had arrived (by different routes) at essentially equivalent results demonstrating the limits of formal methods—Turing by demonstrating that the "halting problem" for a computing machine was undecidable,[14] and Godel by proving the incompleteness of any first order formal system powerful enough to be a model for simple number theory.[15] Both methods point to the

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12 Professor Ely was one of the leading theorists of the U.S. Constitution during the last century. His most cited work is JOHN HART ELY, DEMOCRACY AND DISTRUST (1980).

13 5 GREEN BAG 393 (2002).

14 Alan Turing, *On Computable Numbers, with an Application to the Entscheidungsproblem*, 2 PROC. LONDON MATHEMATICAL SOC'Y 42 (1937), *available at* http://www.turingarchive.org/browse.php/B/12. The halting problem asks whether, given a specified program and specified input, the program will stop running or continue in an infinite loop when processing the input. The halting problem is undecidable—there is no computable function that can answer this question for all inputs. The term "halting problem" was coined by Martin Davis.

15 KURT GÖDEL, ON FORMALLY UNDECIDABLE PROPOSITIONS OF PRINCIPIA MATHEMATICA AND RELATED SYSTEMS (B. Meltzer trans., 1992) (1962). In the introduction to this translation, R. B. Braithwaite summarizes Godel's 1931 result: "Every system of arithmetic contains arithmetical propositions, by which is meant propositions concerned solely with relations between

undecidability of certain statements in a first order formalization.[16]

As evidence that Kurt Godel believed formal logical methods might be applied directly to law, I cited the often recounted tale of events leading up to Godel's citizenship meeting with a Federal judge. Prior to that meeting, Godel had studied the U.S. Constitution with some care. He told Albert Einstein and Oskar Morgenstern that he had found a logical flaw in the document which would allow the United States to become a dictatorship. He was very agitated by this "discovery." Einstein and Morgenstern calmed him down and told him not to discuss the matter during the meeting with the judge. Unfortunately, during the meeting the judge noted that Godel was from Germany (a misperception, as Godel was Austrian) and commented on how terrible it was that Germany had been taken over by a dictator—a result that could not happen in the United States. At that moment, Godel blurted out his discovery. After some fast talking by Einstein and Morgenstern, the meeting returned to the business at hand, and Godel received his U.S. citizenship.[17]

As evidence that Alan Turing had a different view of how formal logical methods might be applied to law, I cited a less well known remark made by Turing in a famous 1950 article on artificial intelligence. In *Computing Machinery and Intelligence*, Turing wrote:

> The idea of a learning machine may appear paradoxical to some readers. How can the rules of operation of the machine change? They should describe completely how the machine will react whatever its history might be, whatever changes it might undergo. The rules are thus quite time-invariant. This is quite true. The explanation of the paradox is that the rules which get changed in the learning process are of a rather less pretentious kind, claiming only an ephemeral validity. The reader may draw a parallel with the Constitution of the United States.[18]

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whole numbers, which can neither be proved nor be disproved within the system." *Id.* at 1. The best extended, non-technical exposition of Godel's incompleteness result appears in ERNEST NAGEL & JAMES R. NEWMAN, GODEL'S PROOF (1958).

[16] A third method for addressing this problem was formulated by Alonzo Church through his development of the lambda calculus. *See* Alonzo Church, *An Unsolvable Problem of Elementary Number Theory*, 58 AM. J. MATHEMATICS 345 (1936); Alonzo Church, *A Note on the Entscheidungsproblem*, 1 J. SYMBOLIC LOGIC 40 (1936). J. B. Rosser explains the essential equivalence of the results of Godel, Turing, and Church in J. B. Rosser, *An Informal Exposition of Proofs of Gödel's Theorem and Church's Theorem*, 4 J. SYMBOLIC LOGIC 53 (1939). I am not aware of a circumstance in which Church considered the U.S. Constitution. The relevant historical papers on these related problems, including the paper by Rosser, are collected in MARTIN DAVIS, THE UNDECIDABLE: BASIC PAPERS ON UNDECIDABLE PROPOSITIONS, UNSOLVABLE PROBLEMS AND COMPUTABLE FUNCTIONS (1965).

[17] The story is recounted in many places, including recently in REBECCA GOLDSTEIN, INCOMPLETENESS: THE PROOF AND PARADOX OF KURT GODEL 232-34 (2005). The most serious biographical work on Godel has been done by the late logician Hao Wang. *See* HAO WANG, REFLECTIONS ON KURT GODEL (1987).

[18] This is reprinted in MINDS AND MACHINES 29 (Alan Ross Anderson ed., 1964).

The important point of the comparison is not to identify the parameters of the hypothetical debate. Rather, for present purposes significance comes from two logicians of the highest rank turning their gaze to the U. S. Constitution and reasoning about its structure. It is not known what "flaw" Godel uncovered. Professor Ely cautiously sided with Turing (to the extent either of us could ascribe larger views to the citizenship hearing and these cryptic remarks), though he was uncomfortable with the degree to which Turing might have thought the rules in the Constitution had only ephemeral validity.

In thinking about how formal logic might apply to organic law, such as the U.S. Constitution, our discussion ruled out several avenues of inquiry. First, we were not focused solely on parsing logical connectives such as "and" and "or." Though the ability to read language closely plays a role in law and interpreting legal documents, this was not of prime interest because it was only part of the story.[19] Further, we were not interested in making broad (and, to my mind, somewhat extravagant) claims such as have been made in the legal academy from time to time—for example, the claim that Godel showed that law must be essentially incomplete.[20] Rather, we focused on the more traditional debate between Langdell and Holmes. Is law more like a science, as championed by Langdell (and is legal reasoning in principle able to be modeled using formal logic) or is the life of the law not logic, as observed by Holmes?[21] We tentatively paired Godel with Langdell and Turing with Holmes. Our ongoing discussions on this topic were cut short by Professor Ely's untimely death.

I later came across a specific example of how close consideration of mathematical structure might be applied to social phenomena to reach a political or ethical conclusion (as described by the philosopher Stewart Shapiro).[22] Levi-Strauss studied kinship systems of aboriginal tribes in Australia.[23] Andre Weil provided comments in a mathematical

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[19] Similar language parsing concerns arise in application of computer technology and computation methods to legal materials. *See, e.g.*, AUTOMATED ANALYSIS OF LEGAL TEXTS: LOGIC, INFORMATICS, LAW (Antonio A. Martino & Fiorenza Socci Natali eds., 1986). I would certainly endorse using formal logic to help parse clauses in contracts, and for similar exercises. This use of logic was simply not the focus of our discussion.

[20] In order to tap into literature of this sort, see David R. Dow, *Godel and Langdell—A Reply to Brown and Greenberg's Use of Mathematics in Legal Theory*, 44 HASTINGS L. REV. 707 (1993).

[21] For a comprehensive analysis of this debate, see Susan Haack, *On Logic in the Law: "Something, but not All"*, 20 RATIO JURIA 1 (2007). In this essay, Professor Haack explains her view that, despite advances in the methods of formal logic (which do have some applications to law), Holmes the pragmatist remains correct in his basic view that formal logical methods do not capture all aspects of legal reasoning.

[22] STEWART SHAPIRO, PHILOSOPHY OF MATHEMATICS: STRUCTURE AND ONTOLOGY 249 (1997).

[23] CLAUDE LEVI-STRAUSS, THE ELEMENTARY STRUCTURES OF KINSHIP (1969).

appendix to Levi-Strauss' book on the subject.[24] He noted that some aboriginal kinship systems are isomorphic to structures in mathematical groups. Professor Shapiro observes:

> A tribe is divided into four classes, and there is a certain function for determining the class of a child from the classes of his or her parents. There is an "identity class" in the sense that when a member of this class mates with any member of the tribe, the offspring are members of the other's class. If two members of the same class mate, then their offspring are of the identity class. The classes and the function exemplify the Klein group, a well-known finite structure . . . .
>
> This simple example illustrates a central feature of the application of mathematics via structure exemplification (or isomorphism). The properties of a structure apply to any system that exemplifies it. The Klein group has an identity element, and so the Kariera system has an identity class. Because the Klein group is abelian, we see that in one respect, the Kariera system is *egalitarian*.[25]

More recently, the MIT computer science and artificial intelligence laboratory has conducted research on how people learn about kinship groups and causal theories by examining mathematical structures.[26]

I believe many other examples similarly could support the notion that details of mathematical structures often find application to social phenomena. I want to be clear, however, that I am not limiting this claim to the idea that logical and mathematical techniques merely can be tools to help analyze social science data by, for example, running statistical tests. I want to make the broader point that considering logical and mathematical structures illuminates what, in the abstract, is possible for thought. This expansion of our horizon of conceptual possibilities may actually help with theory creation. I find the illustration from Professor Shapiro particularly apt not simply because a mathematical structure is applied to a social phenomenon but because he highlights how a mathematical structure may support a political or ethical claim: the Kariera system is *egalitarian*. I do not consider my observations on this point either novel or provocative.

Badiou identifies Georg Cantor's theory of the transfinite as a pivotal moment.[27] Cantor's work on infinity provides a signature

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24 Interestingly, Andre Weil was a member of the influential French group of mathematicians and logicians known as "Bourbaki" which advanced a logicist program of mechanical deduction and reasoning in mathematics, the limits of which were demonstrated by Kurt Godel. *See generally* AMIR D. ACZEL, THE ARTIST AND THE MATHEMATICIAN (2006).

25 SHAPIRO, *supra* note 22, at 249 (emphasis added).

26 CHARLES KEMP, THOMAS L. GRIFFITHS & JOSHUA B. TENENBAUM, DISCOVERING LATENT CLASSES IN RELATIONAL DATA (2004), *available at* http://web.mit.edu/~ckemp/www/papers/blockTR.pdf. I raise the recent MIT research to highlight the research principle involved because the fieldwork of Levi-Strauss may be criticized; the recent MIT research program suggests that the research principle retains vitality.

27 BADIOU, BEING AND EVENT, *supra* note 1, at 38-43.

example of how mathematical structures expand the horizon of what is possible for thought.[28] Prior to Cantor, notions of levels of infinity did not arise. People tended to think of infinity in terms of potentiality—the potential to add an additional number at the end of a list, and then another, and another (a conception going back at least as far as Aristotle).[29] The idea of the infinite as a type of potentiality contrasts with the idea of the infinite as a completed whole. Cantor's development of a theory of infinite cardinals showed that it was possible to conceive of infinity both as multiples of completed wholes and as having different orders of magnitude—in effect, as a hierarchy of an infinite number of infinities, each one larger than its predecessor in the hierarchy. His theory showed how one might consistently entertain ideas of infinity as a complex structure rather than a simple potentiality.

I do not want to speculate here about how these ideas of infinity might be used to structure a broad or comprehensive theory of law. The point is that Cantor's work provides an additional conceptual apparatus that might find application to a variety of phenomena, including law and other social structures. As an historical matter, Cantor believed his work gave insights into the nature of God—another application of mathematics to the world (at least in Cantor's view).[30] For years, he was taken more seriously by the Catholic Church than by working mathematicians.[31]

Though Badiou's claim that "mathematics is ontology"[32] may have surprised some, today I find the position unremarkable. More recently, Professor Shapiro convincingly has advanced the notion that mathematics should be viewed as the science of structure.[33] Mathematicians develop and expand our notions of what structures can be conceptualized without contradiction (as well as providing insights into the limits of those conceptualizations). As such, mathematicians

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28 *Id.* at 2 (noting that "it will be held that the mathematico-logical revolution of Frege-Cantor sets new orientations for thought"). Another often used example of mathematics expanding the horizon of what is possible for thought is the discovery of non-Euclidian geometries.

29 *See, e.g.*, ARISTOTLE, PHYSICS, Book III 205-08; *accord* JOSEPH WARREN DAUBEN, GEORG CANTOR: HIS MATHEMATICS AND PHILOSOPHY OF THE INFINITE 122-23 (1979) (discussing Aristotle).

30 *See* DAUBEN, *supra* note 29, at 140-48.

31 *Id.*

32 This statement appears in many places. *See, e.g.*, BADIOU, BEING AND EVENT, *supra* note 1, at 4. The notion that mathematics is ontology will remind analytic philosophers of the maxim from Quine that "to be is to be a value of a variable." *See* Willard Van Orman Quine, *On What There Is*, 2 REV. METAPHYSICS (1948), *reprinted in* FROM A LOGICAL POINT OF VIEW 1, 15 (1953).

33 *See supra* note 25 and accompanying text. Even though Badiou equates use of the phrase "philosophy of mathematics" with the "little style" that subordinates the importance of mathematics, *see* BADIOU, THEORECTICAL WRITINGS, *supra* note 1, at 3, I do not find fault with Professor Shapiro's use of the phrase "philosophy of mathematics" from the perspective of *Being and Event* precisely because Shapiro identifies mathematics as the science of structure.

are in the business of identifying what is possible for thought. Though translations of the pre-Socratic "Way of Truth" may differ in the details, Parmenides clearly equated "what exists for thinking" with "what exists for being."[34] In this direct sense, mathematics is ontology. By paying close attention to the structures available for thought, the non-mathematician may expand his or her horizons for theory creation in other disciplines.[35] On reflection, this is a very old idea.

II. USING A META-MATHEMATICAL RESULT TO ILLUSTRATE A LACANIAN CONCEPT

Jacques Lacan admired the work of his friend, the anthropologist Claude Levi-Strauss; in particular he appreciated Levi-Strauss' use of the concept of a mathematical group.[36] A central theme in structural anthropology is that symbolic structures might explain the operation of social groups without the individuals in the social group being aware of the symbolic structure.[37] These structures might even explain the minds of individuals. Lacan hoped to bring a similar type of mathematical rigor to problems in psychoanalysis.[38] As Lacan shifted his emphasis from a study of speech to a study of language, possibilities expanded for Lacan to adopt a mathematical approach because one can view language as a symbolic structure independent of the subject.[39] Lacan did not focus exclusively on the role of language in psychoanalysis but rather

[34] The relevant passage reads: "το γαρ αυτο νοειν εστιν τε και ειναι" Parmenides, Fragment 344, in G. S. KIRK & J. E. RAVEN, THE PRESOCRATIC PHILOSOPHERS 269 (1976). Badiou refers to this aphorism of Parmenides directly in BEING AND EVENT, *supra* note 1, at 38, and in THEORETICAL WRITINGS, *supra* note 1, at 52.

[35] For example, Susan Haack uses the analogy of the crossword puzzle to motivate her thinking about the structure of the development of scientific theories.

[36] *See, e.g.*, Jacques Lacan, *The function and field of speech and language in psychoanalysis*, *reprinted in* ECRITS, A SELECTION, *supra* note 6, at 30, 73 ("Isn't it striking that Levi-Strauss, in suggesting the implication of the structures of language with that part of the social laws that regulate marriage ties and kinship, is already conquering the very terrain in which Freud situates the unconscious."); *see also* ANTHONY ELLIOT, SOCIAL THEORY & PSYCHOANALYSIS IN TRANSITION: SELF AND SOCIETY FROM FREUD TO KRISTEVA 123 (2d ed. 1999) (discussing Lacan's incorporation of ideas from structural anthropology into his theory).

[37] For example, Levi-Strauss stated that structuralism "claims to show, not how men think in myths, but how myths operate in men's minds without their being aware of the fact." CLAUDE LEVI-STRAUSS, THE RAW AND THE COOKED 12 (1969).

[38] *See, e.g.*, GILBERT D. CHAITIN, RHETORIC AND CULTURE IN LACAN 214 (1996) ("During the later stages of his career, Lacan reformulated his conception of the relations we have been describing between the signifier, the subject and repetition in light of set theory.").

[39] Lacan's use of structuralist ideas did not result in his denial of the subject, though his view of the subject as mathematized is non-standard. As Chaitin writes, "The subject of Lacan's theory is not a matter of subjectivity in the usual, psychological, sense, but a quasi-mathematical function which relates a single signifier taken separately, S1, to a set of letters, the collection of other signifiers that mark the subject's history, S2." *Id.*

on the relationship of language to the "real" (in particular his concept of *jouissance* is located in the real)—the real consisting in a core that escapes representation in the symbolic order of language.[40] As I understand the point, there is always some truth that exceeds the attempt to capture it with symbols.

I make no claim to be an expert on Lacan. However, the notion that in any situation something always escapes symbolization is a complex one. It is not the simple idea that a picture replaces a thousand words nor is it the more complex idea that in theories of sufficient power some statements may be undecidable. To try to understand this idea of "escaping symbolization" in formal terms, one might profitably turn to the Lowenheim-Skolem theorems and the so-called "Skolem's Paradox."[41]

The Lowenheim-Skolem theorems are a basic result in model theory. In summary, the theorems state that in any first order theory expressed in a language with a denumerable number of symbols, the theory will be satisfiable by a model with a denumerable universe of discourse.[42] This result gives rise to Skolem's Paradox because set theory is a first order theory expressed in a language with a denumerable number of symbols. There exists a long and complicated expression in set theory that has as its intended interpretation: "This set has an uncountable number of members." The Lowenheim-Skolem theorems show that this expression is satisfiable in a model with a denumerable universe.

At first read, this might seem like a disaster for set theory. Indeed, Skolem himself thought he had shown why set theory could not ever form the foundation for mathematics (a point highlighted by Paul Cohen in a paper shortly before his death).[43] As an historical matter, Skolem's intutition was proved wrong because most working mathematicians accept set theory as the foundation of their subject.[44] Instead, they see Skolem's Paradox as illustrating a gap between the syntax used to express a theory and the semantics or meaning of that theory (i.e. the interpretation given to the symbols by specification of a model).[45] What should we make of the fact that the axioms of ZFC have an uncountable

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40 *Id.* at 215-17 (discussing the relationship between the real, the impossible, and *jouissance*).

41 These theorems are reproduced in JEAN VAN HEIJENOORT, FROM FREGE TO GÖDEL: A SOURCE BOOK IN MATHEMATICAL LOGIC 1879-1931 (1967). An easy explanation of the results appears in JANE BRIDGE, BEGINNING MODEL THEORY: THE COMPLETENESS THEOREM AND SOME CONSEQUENCES 97-100 (1977).

42 MARY TILES, THE PHILOSOPHY OF SET THEORY: AN HISTORICAL INTRODUCTION TO CANTOR'S PARADISE 179-80 (1989).

43 *See* Paul J. Cohen, *Skolem and pessimism about proof in mathematics*, 363 PHIL. TRANS. R. SOC. A 2407 (2005).

44 *See* MICHAEL POTTER, SET THEORY AND ITS PHILOSOPHY: A CRITICAL INTRODUCTION 4 (2004).

45 *See* MOSHÉ MACHOVER, SET THEORY, LOGIC AND THEIR LIMITATIONS 279-82 (1996).

model alongside a countable model? Rather than conclude that ZFC is a failure, we might simply note that it failed as a model only insofar as we intended that the symbolic expression of the model provide a unique specification of its domain. In this explanation, ZFC fails because the Lowenheim-Skolem theorems show that any first order formal specification of a theory with axioms that allow a description of uncountable infinities (such as ZFC) will also be subject to an interpretation (i.e. can be satisfied by a model) in which the theory describes a denumerable universe. If we call the universe of the non-standard model "NS," from the point of view of NS there will be a notion of "uncountable in NS." Not confined to NS, we see that the notion of "uncountable in NS" can be satisfied in a countable model. The syntactic structure contains an ambiguity of sorts. Something of importance escapes symbolization—a truth about the subject matter exceeds our ability to say it, to symbolize it. This is not the same as conceptualizing a truth for which there is no truth procedure, such as Godel demonstrated with his incompleteness theorem.[46]

Though I believe the Lowenheim-Skolem theorems provide a formal example of how a truth may escape symbolization, my example differs from the use of algebraic groups by Levi-Strauss. In the case of the study of kinship, the procedure was to discover an isomorphism between observed behavior and a structure from group theory. This application of mathematics to a social phenomenon is much more direct and powerful than the use of a mathematical theorem to illustrate, by analogy, a concept that appears in a social science theory. Theory development may move from analogy to isomorphism.[47] I want to make the points both that developments in logic and mathematics may be useful as a source of analogy (i.e. to make use of a concept in another field plausible) as well as to learn structural facts about a social practice by discovering an isomorphism at a time when theory and data collection have become more advanced.

I do not know whether an expert on Lacan would find my analogy to the Lowenheim-Skolem theorems useful. From the perspective of analytic philosophy, I am suspicious anytime someone attempts to name the "unnameable." At a surface level, when one describes the "real" as "that which escapes symbolization," I sense a contradiction lurking. Did we not just name or symbolize the "real" by describing it as "that which escapes symbolization?"[48] Consideration of the Lowenheim-

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[46] To be more precise, Godel's formulation was conditional and involved a choice. If number theory is consistent, then there is a statement that we take as true for which no truth procedure exists. That is to say, there is a true sentence that is not a theorem so, in this sense, number theory is incomplete. *See* R. W. Braithwaite, *Introduction* to GODEL, *supra* note 15, at 18 (summarizing the import of Proposition VI in Godel's proof).

[47] *See supra* note 35.

[48] A classic semantic paradox of this sort is Richard's Paradox, simplified in 1906 by G. G.

Skolem theorems, however, provides an example of how a matter of potential importance in set theory—unique specification of a universe by the syntactic specification of axioms for a theory—is not possible. There is a real sense in which a concept escapes the formalism. For me, the analogy allows further consideration of Lacan's concept because a surface contradiction does not end the inquiry. We are not forced at this point to sit in the corner with Wittgenstein and remain silent. Instead, we might speculate about the implications of a "that" which escapes symbolization in a world in which knowledge may be conceived as increasingly sentential and autonomous, as proposed by Professor Hacking.[49]

III. FORCING ANALOGIES

As a general matter, the usefulness of an analogy is limited by familiarity with the point of comparison. Recently we have seen the personality and the results of Kurt Godel enter and expand in the popular consciousness. Beginning with a short volume explaining the essential ideas behind his incompleteness results,[50] moving through the popular *Godel, Escher, Bach*,[51] and recently finding their way into a novel, *A Madman Dreams of Turing Machines*,[52] Godel and his proofs are on display. Logicians have given ever simplified treatments of his incompleteness results.[53] For better or worse, we are at the point when political or ethical discourse might make effective use of analogies to Godel's work in presentations to a broader audience. In my view, the results in set theory that show the independence of the continuum hypothesis do not yet share this dissemination in the popular understanding (or misunderstanding) to be effective as a validation

Berry: the phrase "the least natural number not nameable in fewer than twenty-two syllables" names in twenty one syllables a natural number which by definition cannot be named in fewer than twenty-two syllables. STEPHEN COLE KLEENE, INTRODUCTION TO METAMATHEMATICS 39 (1950).

49 IAN HACKING, WHY DOES LANGUAGE MATTER TO PHILOSOPHY? (1975). Hacking argues that ideas mattered to 17th century philosophers for the same reason that sentences matter to philosophy today: in each case they perform the function of interface between a knowing subject and what is known. On this view, the role of the sentence becomes increasingly important as the subject is discarded and knowledge is conceived as autonomous "discourse." Sentences matter to philosophy because knowledge has become sentential. Detailed consideration of the implications would require taking a position on the relationship between a formal language and a natural language—a project well beyond the scope of this essay.

50 ERNEST NAGEL & JAMES R. NEWMAN, GODEL'S PROOF (1958).

51 DOUGLAS R. HOFTSTADTER, GODEL, ESCHER, BACH: AN ETERNAL GOLDEN BRAID (1979).

52 JANNA LEVIN, A MADMAN DREAMS OF TURING MACHINES (2006).

53 *See, e.g.*, George Boolos, *Godel's Second Incompleteness Theorem Explained in Words of One Syllable*, *in* LOGIC, LOGIC AND LOGIC 411 (Richard Jeffrey ed., 1998).

strategy for a broad audience. Given the complexity of the articulation of these ideas, I wonder whether a simplification will ever materialize. While many informed readers have a vague idea that Godel's results have something to do with the paradox of the liar (i.e. the person who states, "Everything I say is false") currently there is no easy way into Paul Cohen's forcing technique. This more restrictive understanding does not prevent the analogy, but it limits the scope of the analogy's persuasiveness. Limiting scope obviously does not change the historical fact of providing inspiration to Badiou for his theories.

The Lowenheim-Skolem theorems, however, provide an essential background structural idea behind the independence results that may be more accessible. To give an idea of this background structure, assume that the axioms of set theory consist of a set of statements named "ZFC." In addition to this set of statements, we have several rules of inference. These rules of inference allow us to use as inputs one or more axioms of ZFC. The outputs constitute theorems of ZFC. The continuum hypothesis (CH) has various formal statements: in ordinary parlance it states that there is no level of infinity between the cardinal number of the natural numbers and the cardinal number of the real numbers.[54] We now know that CH is not a theorem of ZFC because of the combined work of Godel and Cohen, who jointly demonstrated the independence of CH from ZFC.[55] To show that ZFC is consistent with CH, the strategy is to construct a model by supplementing ZFC with additional statements (call this supplement "G-Supp") in which the statements in the ZFC set are true and in which the negation of CH is not a theorem.[56] To show that ZFC is consistent with the negation of CH, the strategy is to construct a model by supplementing ZFC (call this second supplement "C-Supp") in which the statements of ZFC are true and in which CH is not a theorem. G-Supp represents the supplement provided by Godel to prove the consistency of ZFC with CH, and C-Supp represents the supplement provided by Cohen to prove the consistency of ZFC with the negation of CH. As background for the proof, you need to show that each of ZFC + G-Supp and ZFC + C-Supp can be satisfied in some model. CH is independent from ZFC because one can construct one model that satisfies ZFC + CH and another that satisfies ZFC + ~CH. The statements in G-Supp define what is known as an "inner model" and might be thought to narrow the universe in which ZFC is true. The statements in C-Supp create what is known as a

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54 I adapt the following illustration from GREGORY CHAITIN, META MATH! 103-04 (2005): #{reals} = #{points in line} = #{points in plane} = c, #{positive integers} = #{rational numbers} = #{algebraic real numbers} = aleph0. The continuum problem asks whether there is a set S such that aleph0 < #S < c. The continuum hypothesis is simply that there is no such set S.

55 *See, e.g.*, TILES, *supra* note 42, at 137.

56 The additional statement in G-Supp amounts to a restriction that limits the universe of discourse to constructible sets.

"generic extension" and might be thought to expand the universe in which ZFC is true. The generic extension employed by Cohen forms the basis for Badiou's notion of a multiplicity without a "one."[57] The generic extension C-Supp is constructed without making statements that provide specific information about the universe structured by C-Supp. To understand how this might be done, contrast the statement, "The universe contains element p," with the statement, "If the universe contains p, then X is true."[58]

The general notion that ZFC can be satisfied in universes with domains of different cardinalities comes directly from the Lowenheim-Skolem theorems. We know that ZFC has a standard model in which its axioms are true in a non-denumerable domain. We also know that ZFC has a non-standard model in which its axioms are true in a denumerable domain. The syntactic structure of the axioms does not uniquely determine the size of the model in which the axioms may be satisfied. Given that ZFC itself can be satisfied by models with universes of different sizes, it should be intuitively plausible to search for models with additional statements (e.g. ZFC + Supp) that satisfy either CH or ~CH because the hypothesis and its negation assume universes with different cardinalities.

IV. ANOTHER WAY TO THINK MULTIPLE-WITHOUT-ONE?

If the example of the generic set created by Paul Cohen does not resonate for thinking about a multiplicity without a one, I believe a simple exemplar for this idea is at hand. The technique of forcing requires creation of certain generic sets that might be conceived of as multiplicities without a one. The strategy employed by Badiou is easy to understand (though details of the forcing technique are not easy to understand, and I do not pretend to have mastered them). Identification of the concept of a multiplicity without a one in the rigorous environment of the foundations of mathematics provides validation for use of this concept in other contexts. As discussed above, I believe the difficulties in understanding the forcing technique limit its general persuasive efficacy as a validation strategy, but there are other ways to motivate acceptance for the idea of multiplicities without a one. The basic structure of transfinite numbers may provide a more effective example of a multiple without a one for a wider audience. I do not

57 *See, e.g.*, BADIOU, BEING AND EVENT, *supra* note 1, at 355.

58 The shortest detailed explanation of forcing that I have found appears on an MIT website. *See* Timothy Y. Chow, *Forcing for Dummies*, http://www-math.mit.edu/~tchow/mathstuff/forcingdum (last visited Apr. 16, 2008). The explanation takes over ten printed pages and is for "dummies" only in the MIT universe of discourse.

mean to suggest that use of a complex exemplar (such as forcing in set theory) does not work as a validation strategy. Rather, the complex exemplar compels readers unable or unwilling to address the complexity to accept propositions on the same basis as they might be asked to accept a religion. Potential critics who have not mastered the details of the validation example are placed at a disadvantage (and, Badiou might say, with good reason).[59]

In that regard, the work of Gregory Chaitin provides an accessible way to understand how multiplicities might exist without the concept of a "one."[60] Chaitin notes that the real numbers, while natural geometrically, remain elusive in the extreme:

> Why should I believe in a real number if I can't calculate it, if I can't prove what its bits are, and if I can't even refer to it? And each of these things happens with probability one! The real line from 0 to 1 looks more and more like Swiss cheese, more and more like a stunningly black high-mountain sky studded with pinpricks of light.[61]

One of the requirements one might impose on finding a "one" in any field is to specify individuation criteria and persistence criteria for the object that constitutes the "one." If individuation criteria and persistence criteria are not forthcoming, then one has no "one" in any meaningful sense (or so I think it cogently might be argued). If this is so, then perhaps Chaitin has identified in simple language the signature example of multiplicities without the concept of the one—the real numbers that cannot be named, for they are too large in number all to have names. Thus, the real numbers provide the exemplar of the multiple without one. For this idea, we need not turn to Cohen, but simply to the real numbers themselves. Assuming agreement that there can be a meaningful conception of a multiple without a one, how might that idea be employed in relation to law?

V. LOGICAL AND MATHEMATICAL THINKING APPLIED TO THEORIES OF JUSTICE

To answer this question, let us turn to two questions traditionally central to metaphysics: what is being, as such (or being *qua* being), and what is change (an event)? For Badiou, the study of set theory provides an answer to the first question.[62] The second question might be put

[59] Badiou would recount here the sign above Plato's Academy admonishing those without knowledge of geometry not to enter the school.

[60] *See supra* note 54 and accompanying text.

[61] *See* Chaitin, *supra* note 54, at 115.

[62] *See, e.g.*, BADIOU, THEORETICAL WRITINGS, *supra* note 1, at 16.

simply: what is something new? Though the second question may be more interesting for Badiou, one needs to know "what is" before one can answer "what is new." Setting this stage for the question of the event is what I believe motivates Badiou's interest in set theory.

Recall that set theory is the study of abstract groupings of objects[63] and the possible relationships among these groupings.[64] Seen as the study of abstract groupings and their relationships, set theory provides a framework for thinking about multiplicities. To convey this idea, I am tempted to write (and, in fact, write): MULTIPLICIES/THOUGHT = set theory.[65] Badiou says, in various formulations, that "mathematics is ontology."[66] More precisely, the language of mathematics provides the discourse in which being *qua* being is discussed. Badiou also says that "ontology is mathematics"[67] (the converse order of "mathematics is ontology") because it is the mathematicians who work out the structure of being *qua* being by engaging in the actual practice of mathematics. For Badiou, philosophy (properly understood) does not consider the question of being as such; instead, philosophy is seen as a meta-discipline in which the consequences of this core observation might be considered.[68] Thus, Badiou strips philosophy of one of its self appointed historical missions—the investigation of being as such. Mathematicians are the true ontologists. Accordingly, there is no philosophy of mathematics.

Our core question is why study of the particular abstract discipline of set theory has any bearing on questions of more direct interests to society. Below, I provide an example of how fundamental questions of

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63 Abstract groupings of objects need not be groupings of "existents" but can begin with the null set (a nothing) or might be objects that are not identical with themselves—e.g. the collection of objects that satisfy the formula ~(x=x).

64 For Badiou, the question of being *qua* being is a decidedly different question from investigation into existents, though answers to the former question may inform the later question. *See* BADIOU, BEING AND EVENT, *supra* note 1, at 8 ("The thesis that I support does not in any way declare that being is mathematical, which is to say composed of mathematical objectivities. It is not a thesis about the world but about discourse.").

65 I use this rather odd typographical presentation in an attempt to convey a direct thinking about the subject matter without mediation by language, a subject, etc. This is intended to signal a return to original or primordial thinking much in the mode of the Greeks for which thinking about the basic structure of the world was not mediated by a Cartesian subject. I interpret Badiou as wanting to start from such a subjectless position in developing a basic ontology before providing a theory that (re)constructs a modern theory of the subject. To effect such a (re)construction, Badiou first addresses being, and second addresses change (or the event). These steps logically precede development of his theory of the subject. This hierarchy is found in the progression of *Being and Event*, which first addresses being, then proceeds to a consideration of the event, and finally arrives at a theory of the subject. *Accord* SLAVOJ ZIZEK, THE TICKLISH SUBJECT: THE ABSENT CENTRE OF POLITICAL ONTOLOGY 130 (1999) (noting that "the subject comes *after* the Event").

66 *See, e.g.*, BADIOU, BEING AND EVENT, *supra* note 1, at 4.

67 *See, e.g.*, BADIOU, THEORETICAL WRITINGS, *supra* note 1, at 1.

68 *See, e.g.*, BADIOU, BEING AND EVENT, *supra* note 1, at 10, 13-15.

structure addressed by set theory may be relevant to developing social and political theory at a foundational level. To do so requires a brief detour through ways in which we might conceptualize collectives, groups, multiplicities, and pluralities (to spin out a number of labels).

There is a classic, commonsense way of thinking about multiplicities: at least since Greek times, a multiplicity often was viewed as a "one" that provided the basis for a count.[69] The "one" is conceptually prior to the "multiplicity." This view conceives of a multiplicity as a collection of ones. In this framework, infinity is not a completed whole. Rather it is a potentiality: the ability to keep adding "just one more" to a collective. It is not possible in this framework to think of an infinity either as a completed whole, or without the "one" in the background as foundation for the count.

A typical Greek question might be "What is it to be a man, a horse, brave, etc.?" If you can answer the "What is it to be?" question, you have provided individuation and persistence criteria for an object—a "one"—that might become the basis for a count. In some translations of Greek philosophy, the "What it is to be" is given the elusive translation "essence." [70] Far from deep or mysterious, "essence" in such a translation is merely the answer to the "What is it to be?" question.[71] The answer to this question is a possible explanation for the concept "substance."

The question of "substance" in the sense used here arises in the dialectic of question and answer (in my English language example, by transforming the question, "What is it to be an X?" into the schema for an answer). The generic form of the answer supplies the "what it is to be" in response to the question. In the schema, the phrase "what it is to be" is nothing more than a concatenation of words that provides a technical name for an answer to a particular type of question. Q: "What is it to be an X?" A: "The 'what it is to be' of X is [fill in details]." In Aristotelian scientific thinking, the answer to this question might tell the questioner something about an actual existent or class of existents. In

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69 *See, e.g.*, Aristotle, Metaphysics 1052b; *see generally* JACOB KLEIN, GREEK MATHEMATICAL THOUGHT AND THE ORIGIN OF ALGEBRA 8-9, 100-113 (Eva Brann trans., 1968) (discussing Greek conceptions of number). Badiou suggests that Greek thinking employed a language for speaking about multiplicities without a one (πλαθοσ) as distinct from pluralities made up of ones (i.e. the concept of the many, or πολλα). *See* BADIOU, BEING AND EVENT, *supra* note 1, at 35. I am not making that distinction in my choice to use "multiplicity" rather than "plurality" in my textual discussion. I merely want to convey the general idea of numerosity.

70 THE BASIC WORKS OF ARISTOTLE (Metaphysics Book VII) 784 (Richard McKeon ed., 1941) (describing different possible meanings for the word "substance" or ουσια). After translation into English, this produces seemly deep conclusions such as "the substance is the essence"—phrases for which I find it hard to ascribe a meaning or a use.

71 For Aristotle, many meanings were attached to the general concept of substance (or ουσια). One concept for substance was the "whatness" of a thing or the "το τι ηv ειναι"—the "What it is to be." *See generally* F. E. PETERS, GREEK PHILOSOPHICAL TERMS: A HISTORICAL LEXICON 149-51 (1967) (discussing different meanings assigned to the term "ουσια").

Greek ethical theorizing, the answer to the question might identify the parameters of concepts, such as "good" or "brave"—these concepts were sometimes theorized as objects with a form of being outside the ordinary world of perception (Platonism being the standard-bearer for this notion).[72]

For Badiou, it is important that rigorous thought be capable of conceptualizing a multiple without the underlying concept of a "one" that forms the basis of a count.[73] He sometimes refers to this idea as the "multiple-without-one."[74] There must be another way to think about multiplicities without contradiction and confusion. Badiou identifies the work of Paul J. Cohen, the late American mathematician, as providing this other way for thinking about multiplicities.[75] He identifies the technique of "forcing" in set theory discussed above as an example of a milieu in which formal recognition and use of the concept of multiplicities without a one does not involve contradiction or confusion because forcing uses the idea of the generic extension of a set.

Without a formal example from set theory (or another rigorous discipline), I would expect a vigorous assault against speaking about multiplicities without a one as a brand of nonsense. The criticism would flow from the commonsense understanding of what it is to be a multiple that traces back to the Greeks. Such an attack would have the same quality as the attacks made against Georg Cantor for thinking of infinities as completed wholes rather than as potentialities. Until Cantor's theory of the transfinite had been worked out on a formal basis, Cantor was attacked by fellow mathematicians for incoherent or crazy thinking. The rigorous formulation of a theory of transfinite numbers eventually allowed Cantor's ideas to triumph over his detractors.

Very basic thinking about the nature of collectives, groups, multiples, and pluralities figures prominently in construction of some political and social theories. For example, in thinking broadly about egalitarian social principles, we often find what I would term a "subtractive" method in which various attributes of human subjects are peeled away to create a stylized society of multiple persons—a collective of distilled subjects, if you will. The philosophy of John

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72 *See, e.g.*, STEPHAN KORNER, THE PHILOSOPHY OF MATHEMATICS: AN INTRODUCTORY ESSAY 14-18 (2d ed. 1968) (describing Platonism in mathematical thinking).

73 BADIOU, THEORETICAL WRITINGS, *supra* note 1, 41-43 (explaining how the project of *Being and Event* was to develop a concept of radical multiplicity not subordinated to the concept of the "one" which he termed the "multiple-without-one"). A detailed critique of the use of this concept by Badiou and its efficacy is beyond the scope of this essay, though I provide below an example from the Anglo-American tradition of how such a concept might prove useful.

74 *Id.*

75 *Id.* at 130-31.

Rawls provides a familiar Anglo-American example of this subtractive method when, as a thought experiment, Rawls posits a plurality of subjects in "the original position."[76] In the original position, subjects are required to subscribe to rules organizing allocation of resources in society. From this stance, Rawls investigates the type of society that subjects so situated would rationally choose to construct from a position behind a "veil of ignorance" in which they are unaware of their particular attributes and potentialities.[77]

At the extreme, as a logical matter, the subtractive method results in creating a plurality of individuals with no individuation criteria. (I use the term "individuals" advisedly to distinguish from subjects for a variety of reasons, not the least of which are that a logical individual is not necessarily a subject and it is an open question whether such a distilled subject remains a subject at all). Indeed, when individuation criteria are removed, in what sense are we even thinking about a plurality of individuals rather than a single individual? Use of the subtractive method raises a fundamental logical question: is it coherent to think about a plurality of individuals without individuation criteria?[78] Perhaps a logical inconsistency lurks at the foundation of the structure Rawls posits to think about questions of justice. If our conceptual framework requires that the concept of a multiple depends upon a prior concept of a one which measures a count, then the subtractive method is at risk of incoherence. Any framework using a subtractive method runs the risk of being stillborn before it can be applied to do substantive work within the context of a political or social theory if we adopt the Greek conception of a multiple composed of ones as our limit.

Individuation criteria must perform double duty both by providing the measure of a count and by allowing distinctions that permit the count to be performed under the rubric of the measure. An example

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76 The classic statement of this argument appears in JOHN RAWLS, A THEORY OF JUSTICE (1971). Rawls had been working out his theory as an alternative to utilitarian conceptions of justice during the previous twenty years prior to publication of *A Theory of Justice*. A short predecessor article that uses the concept of persons situated in the original position behind a veil of ignorance is John Rawls, *Distributive Justice* (1967), *reprinted in* COLLECTED PAPERS 130, 132-33 (Samuel Freeman ed., 1999).

77 *Id.*

78 An objection of this sort is discussed in detail in MICHAEL J. SANDEL, LIBERALISM AND THE LIMITS OF JUSTICE 79 (2d ed. 1998), and echoed by David Grey Carlson, *Philosophy in Bankruptcy*, 85 MICH. L. REV. 1341 (1987) (citing an earlier edition of Sandel's book). It appears that Nozick originally may have spotted this type of difficulty in Rawls. *See* ROBERT NOZICK, ANARCHY, STATE, AND UTOPIA 228 (1974). Oddly, when Rawls provided a restatement of his theories shortly before his death, he does not appear to have singled out this potential logical problem for comment or correction. *See* JOHN RAWLS, JUSTICE AS FAIRNESS, A RESTATEMENT (2000). One reason for the absence of a response to this sort of criticism may be Rawls's conviction that his argument was a political argument and not a metaphysical one. *See* John Rawls, *Justice as Fairness: Political not Metaphysical* (1985), *reprinted in* COLLECTED PAPERS, *supra* note 76, at 388.

from set theory illustrates the point. The concept of "set" might provide the measure of a count.[79] We can ask, "How many sets are there in a given presentation?" Though "set" provides the measure of the count, differences must exist among the various sets counted for a count to be possible. In set theory, this difference principle is provided by the criteria of identity for sets: different sets have different elements; it is not possible to have two distinct sets with the same elements.[80]

Absent individuation criteria with this dual nature, a multiple composed of ones may not be constructed. If, however, it is possible to think of pure multiplicities in the absence of a one (i.e. in the absence of individuation criteria) without contradiction or confusion, then a subtractive method remains a conceptual option for theory construction. Modern mathematics (since Cantor) provides a setting in which multiplicities may be conceptualized coherently without the concept of the one. The basic example of multiplicities without a one is transfinite numbers with the power of the continuum (or greater), as discussed above. In short, Rawls may have made use of Badiou's set theory ontology in order to address a particular criticism directed at his subtractive method.[81] Though Rawls strives to formulate his theory as a political theory divorced from metaphysical considerations, his attempt may not succeed in divorcing itself from foundational questions of logic.

To be clear, a great distance exists between an uncountable infinity of real numbers and a society composed of multiple persons. In no sense does reflection on the structure of set theory disprove all

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[79] The choice of the concept "set" for this example raises the technical point that, within formal set theory, the concept of "set" is not the subject of a formal definition. Rather, the notion of set is elucidated by the structure of the axioms and the relationship "is a member of" or "$\in$." Thus, the formal theory does not directly answer the question, "What is it to be a set?"

[80] This idea might be traced back to Leibniz and the principle of the identity of indiscernables. *See* Wilhelm Gottfried Liebniz, *Discourse on Metaphysics*, Section 9, *in* PHILOSOPHICAL PAPERS AND LETTERS 308 (L. Loemker ed. & trans., 1969). In one of Bertrand Russell's less lucid moments, he attempts to cast doubt on this principle in his introduction to the second edition of Wittgenstein's Tractatus. LUDWIG WITTGENSTEIN, TRACTATUS LOGICO-PHILOSOPHICUS 16-17 (2d ed. 1933); the acceptability of the principle remains a matter of philosophical debate. These are difficult logical concepts even though they display a surface simplicity.

[81] I do not mean to equate Rawls's mild, liberal defense of modest egalitarian social policies within a capitalist regime, operated under the rubric of "democracy," with Badiou's political thought. At a minimum, Badiou would strongly reject Rawls's use of a democracy as a starting point—indeed, conceding that the debate must start with a democracy reflects a shifting of political debate that Badiou would find most unfortunate. I merely use Rawls's theories as a familiar example for Anglo-American readers of the subtractive method. The Rawlsian example illustrates how more rigorous thinking at the outset of theory formation might place a theory on a sounder footing by addressing logical criticisms of the sort raised by Sandel and Nozick. Badiou's attention to foundations in *Being and Event* reflects the opposite approach because, unlike Rawls, who attempts to sidestep metaphysics, Badiou addresses foundational issues before attempting to construct a theory of the subject. This is true quite apart from whether one regards Badiou's project as a success.

conceptual objections to Rawls's use of a subtractive method. Nor are reflections on the structure of set theory absolutely required to engage in Rawls's project. However, reflection on different conceptions of multiplicities found in set theory does blunt a particular kind of criticism—the criticism that there is a fundamental incoherence involved in thinking about multiples without a one. The construct of the original position coupled with the veil of ignorance mirrors the structure of a multiple without a one.

CONCLUSION

I want to close with a few cautionary remarks and an avenue for further reflection. I think it makes tremendous sense to equate mathematics with ontology to the extent that we conceive of mathematics as the science of structure. It also makes perfect sense to examine structures in mathematics to inspire development of theory in other areas—which, I take it, is at the core of Badiou's project. I would, however, be cautious about aligning any derived theory too closely with any particular mathematical structure or development (unless I had found an isomorphism). My colleague at the University of Miami, Professor Susan Haack, has long held the view that logic may be revisable in some fundamental sense.[82] I would be reluctant to conclude that ZFC provides a finished structure for ontology. Even to take such a position would involve a decision and not a deduction. This step need not be taken if all one is searching for is analogies to draw upon to motivate theory construction.

I think the contrast between thinking about multiplicities as a collection of individuals and thinking about multiplicities without a one reflect two different ways of thinking about possible models for social structures and their administration. That is to say, the two methods reflect very different paradigms that we might observe generally in describing or designing social systems. I provide a tentative example of each model. As I read Foucault, he theorized that our prison systems—and other mechanisms of discipline—have evolved (or are evolving) from systems that manage individuals into systems that manage populations.[83] This is a case of managing multiples without individuation criteria for persons—without ones.[84] In contrast, our

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[82] *See, e.g.*, A LADY OF DISTINCTIONS, SUSAN HAACK, THE PHILOSOPHER RESPONDS TO HER CRITICS 41, 53 (Cornelis de Waal ed., 2007).

[83] *See* MICHEL FOUCAULT, DISCIPLINE AND PUNISH 195-228 (Alan Sheridan trans., Pantheon Books 1977) (1975) (discussing Panopticism).

[84] As always happens in such a case, a new "one" is theorized, in this case "the population." The result of finding this new one, however, results in the erasure of the individuals that made up

current administration of foster care systems seems unwilling to embrace structures that ignore individuals in this fashion—we attempt to place each child in a suitable family, rather than run orphanages. The preferred social solution, in the foster care setting, might be envisioned as managing a multiplicity composed of individuals.[85] In this setting, we are unwilling to move up a level and manage foster children from the perspective of a population.

To be sure, consideration of set theory is not required to make these observations. However, the detour through concepts in set theory may sharpen our sensitivity to the possibility of the operation of different social structures and provide us with a vocabulary for discussing and evaluating those structures. Having noticed that certain social systems structure themselves as managing multiples without regard for individuals, whereas other social systems structure themselves as managing a multiple composed of individuals, we are positioned to ask why this should be the case and whether the different approaches are a good idea.

Though use of set theory and similar formalisms may stimulate thinking, it also carries a risk. Whenever we engage in projects of this sort, it is important to keep in mind the different roles that may be played by the development of a specialized technical vocabulary. On the one hand, a specialized technical vocabulary may be developed to speak about phenomena for which the current state of language is inadequate. On the other hand, a specialized vocabulary may amount to mere jargon. Susan Haack makes this point directly:

> There is something wrong, however, with jargon the main purpose of which is simply to impress others with your supposedly arcane knowledge. William Gilbert made the essential distinction long ago, when he announced that he would "sometimes employ words new and unheard of, not (as alchemists are wont to do) in order to veil things with a pedantic terminology and make them dark and obscure, but in order that hidden things with no name . . . may be plainly and fully published."[86]

Just as the Greeks struggled to develop a vocabulary within which to discuss philosophical ideas (and, thus, we find odd Greek language constructions such as the concept of a "What it is to be"), I take Badiou's adaptation of concepts from set theory in much the same spirit. The project is to formulate a way of speaking about social phenomenon and justice that allows us to speak "plainly and fully publish" aspects of our experience that currently are "hidden things with

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the former multiple. What remains of interest is the implications of shifting perspectives in such a hierarchy.

[85] I am indebted to Robert Rosen for this example.

[86] SUSAN HAACK, DEFENDING SCIENCE—WITHIN REASON: BETWEEN SCIENTISM AND CYNICISM 133-34 (2003).

no name."[87] This enterprise reflects the general iterative process of a struggle to symbolize some aspect of experience that currently escapes symbolization. If a new vocabulary and symbolization is successful, then we turn our attention to another hidden thing with no name. Though a complete symbolization may be a vain hope, the constant search for a new technical vocabulary to symbolize the un-named well describes the process in which we are engaged.

Returning to the two visions of law suggested by my hypothetical debate between Godel and Turing on the nature of the United States Constitution, I envision two scenarios. Godel perhaps imagined that, in some sense, the law was powerless to prevent the event of change from democracy to dictatorship despite the fact that the law was designed to prevent this very eventuality. In contrast, Turing perhaps suggested a more plastic vision of law that might adapt to changing circumstances—the sort of structure that might prevent an event such as conversion to dictatorship from taking place. One might read Badiou in search of a vocabulary within which these different visions of law might be enunciated and evaluated.

[87] Badiou's concern with developing the right terminology is evident in the interview reproduced under the title *Ontology and Politics* in BADIOU, INFINITE THOUGHT, *supra* note 1, at 169.