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———. *Science and Religion, 400 B.C. to A.D. 1550: From Aristotle to Copernicus.* Greenwood Guides to Science and Religion. Westport, CT: Greenwood, 2004.

Land, Barbara. *The Telescope Makers: From Galileo to the Space Age.* New York: Crowell, 1968.

Lindberg, David C. *The Beginnings of Western Science: The European Scientific Tradition in Philosophical, Religious, and Institutional Context, Prehistory to A.D. 1450.* 2nd ed. Chicago: University of Chicago Press, 2007.

———. "Galileo, the Church, and the Cosmos." In *When Science and Christianity Meet,* edited by David C. Lindberg and Ronald L. Numbers, 33–60. Chicago: University of Chicago Press, 2003.

Lindberg, David C., and Ronald L. Numbers. *When Science and Christianity Meet.* Chicago: University of Chicago Press, 2003.

Mando, Pier Andrea, Luca Mercatelli, Giuseppe Molesini, Maurizio Vannoni and Marco Verità. "The Quality of Galileo's Lenses." In *Galileo's Telescope, the Instrument That Changed the World,* edited by Giorgio Strano, 63–85. Milan: Giunti, 2011.

"Open Culture." http://www.openculture.com/2014/01/galileos-moon-drawings.html.

Pederson, Olaf. "Astronomy." In *Science in the Middle Ages,* edited by David C. Lindberg, 303–37. Chicago: University of Chicago Press, 1978.

Van Helden, Albert, Elizabeth S. Burr, Krist Bender, Adam C. Lasics, Adam J. Thornton, Martha A. Turner, Nell Warnes, Lisa Spiro, and Megan Wilde. "The Galileo Project." http://galileo.rice.edu/about.html.

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Galileo's Contribution to Mechanics

Asim Gangopadhyaya

We generally remember Galileo for the fantastic advances he made in observational astronomy and his brilliant defense of the heliocentric theory of our solar system. He also made enormous contributions to the foundation of mechanics which are often overlooked.

In today's language, Galileo would be called a theoretical as well as an experimental physicist. In addition to carefully performed experiments, Galileo is also known as a master of *Gedanken* experiments (thought experiments). While we will discuss some of his thought experiments in this chapter, the main emphasis will be on the experiments he conducted and his theoretical deductions that profoundly influenced Newton. In a recent translation of *Principia*, the authors claim that Newton gave credit to Galileo for both the first and the second laws of motion that form much of the basis for classical mechanics.[1]

In this chapter I will review his contribution to mechanics including his work on pendulums, the Galilean Theory of Relativity—the precursor to Einstein's Theory of Relativity, investigations of uniformly accelerated objects including freely falling bodies, objects on frictionless, inclined planes, as well as his adventures with the "New Machine." For an extensive description of his wide

1. Cohen and Whitman, *The Principia.*

ranging work in this area, we refer the readers to the work by Roberto Vergara Caffarelli.[2]

Galileo and Mechanics

Experimental investigations of mechanical systems involve a careful study of the time evolutions of positions, i.e., the recording of a sequence of positions parameterized by increasing values of time. All such measurements require collection of accurate data for both position and time. While measurement of positions can be done very accurately and consistently, the same cannot be said about the measurements of temporal intervals. The sundial and its variants have been used since antiquity to measure time, but these are of value only when the sun is visible. Hour glasses that use the flow of a fixed amount of sand and water clocks were great advancements in time measurement due to their portability and because they could be used at night. However, the measurement of time was revolutionized when oscillatory systems came into use.

Measurement of Time

Use of oscillatory mechanics to measure time intervals was in existence in Europe before Galileo's time. However, many such measurements, as far as we know, were made using a trial and error method. During his student days in Pisa (1583–85), Galileo is known to have carried out the first controlled scientific study of a simple pendulum, a device consisting of a ball hanging by a flexible string. One variable that affects the period of a pendulum is its length: the period is proportional to the square root of the length l ($\tau \propto \sqrt{l}$) when all other variables are kept fixed. Some authors have argued that Galileo did not discover this functional dependence till much later in his life since he did not write about it before his *Discorsi*. But that is hard to believe considering the extensive analysis Galileo had done on the isochronicity of pendulums; i.e., the

2. Vergara Caffarelli, *Galileo Galilei and Motion*.

equality of frequencies for all pendulums of the same length.[3] He also found that two simple pendulums of the same length, but with different suspended masses (of the same size), will have exactly the same period, but the one with smaller mass will die out earlier, a phenomenon we can easily explain today using Newton's laws.[4] It would be almost another one hundred years before Christian Huygens would derive the dependence of the time period τ on l and g (acceleration due to gravity $\approx 9.81\text{m/s}^2$): $\tau = \frac{1}{2\pi}\sqrt{\frac{l}{g}}$ and build a clock based on the compound pendulum.

Galilean Relativity

One of the basic tenets of relativity is that the laws of physics appear to be the same for all inertial observers[5]—observers that are moving with constant velocities[6] with respect to each other and with respect to distant stars. For practical purposes, the frames that are moving with constant velocity with respect to the surface of Earth can be considered to be inertial frames. As we all know, we can drink a cup of coffee just as easily on a plane or a train that is coasting with a constant velocity as standing at a station—all three are examples of inertial frames. However, drinking the same coffee in an accelerated system such as an accelerating car or a roller coaster is a very different proposition. In 1632, Galileo introduced this principle of relativity; i.e., the equivalence of all inertial frames, in order to argue that the motion of the Earth around the Sun, or the Earth's spin about its axis, should make no more discernible difference on the free falls of terrestrial bodies than what would be expected on a stationary Earth. This was, of course, to

3. Ibid.

4. Two balls with same size and speed will experience exactly the same resistive force due to air. From Newton's second law, this implies that the less massive ball will go larger deceleration and hence will slow down faster.

5. Also called inertial frames of reference.

6. By velocity, we mean speed and the direction of motion. An object moving with a constant velocity travels along a straight line with constant speed.

support the Copernican heliocentric system. In particular, Galileo introduced a thought experiment in *Dialogues Concerning the Two Chief World Systems* where he surmised that a ball dropped from the top of the mast of a ship will hit the ship at the base of the mast independent of whether the ship was stationary or moving with a constant velocity. A uniform motion of the ship will not cause the ball to fall behind the mast. His claim was not without basis. He had seen the independence of the vertical and horizontal motions of a projectile and explained that the uniform horizontal motion of the ship should have no effect on the falling ball. In other words, an experiment carried out below the deck of a cruising ship would not be able to determine whether the ship was stationary or moving with a constant velocity. This claim was apparently substantiated by the observations of his friend, Giovanni Francesco Sagredo, a mathematician and an avid traveler, and experimentally verified by an empiricist and mathematician, Pierre Gassendi.[7]

No one before Galileo had stated this principle of relativity as clearly. The impact of this principle on the developments of mechanics, electrodynamics, and special relativity cannot be exaggerated.

Freely Falling Bodies

When things move under the force of gravity alone, the resulting motion is called free fall. This is probably one area of mechanics where Galileo's contributions are well known. His observations of free fall formed the basis of Newton's and Einstein's laws of gravity. He supposedly dropped two objects of different mass from the top of the tower of Pisa, and they both landed at the same time.[8] From this, he observed that all masses accelerate at the same rate under gravity. Similar experiments are commonly performed in classrooms today, leading to exactly the same result every time. This observation regarding free fall violated Aristotle's conjecture

7. Gassendi, *De Motu.*

8. It is widely believed that this was just another thought experiment and Galileo never really carried out this experiment.

that the velocities of such falling objects are proportional to their masses,[9] i.e., an object twice as heavy as another should move with twice the velocity and should reach Earth in half the time.

Galileo buttressed his experimental observation with one of the best thought experiments in the history of physics. To understand his arguments let us assume that Aristotle was right, i.e., a lighter object did come down slower than a heavier object and took a longer time to reach the earth than a heavier object starting from the same height. If we tie these two objects together and let them fall, the heavier one will be pulling down on the lighter one and hence speeding it up, and at the same time the lighter one will be pulling the heavier one up and thus slowing it down. The combination should then come down with a common velocity which would be between those of the lighter and the heavier objects had they fallen independently. But then the combination should have a higher weight than both objects, and by Aristotelian principle, it should come down with even greater velocity than either one of them. Thus, we reach a conundrum. The only way to solve it is that each would come down with the same velocity and acceleration, independent of their masses.[10]

According to Newton's second law of motion, the acceleration depends on the ratio of force to the inertia of the body, called the "inertial mass"; i.e., $a = \frac{F}{m_i}$. The gravitational force arises due to interaction between the object and the Earth, and is proportional to m_G, another inherent attribute of the object that we call the "gravitational mass," i.e., $F = m_G g$. The acceleration for an object in free fall is then given $a = \frac{F}{m_i} = (\frac{m_G}{m_i})g$. For this acceleration a to be independent of the mass requires that $m_G = m_i$. This intriguing equality is the basis for Newton's law of gravity and Einstein's principle of equivalence, the starting point for the General Theory of Relativity.

9. Heavier objects do fall faster than lighter objects of the same size due to air resistance.

10. In fact we often see bodies falling at different rates. This is due to air resistance (friction), and it was Galileo's genius to remove that variable.

Accelerated Linear Motion

In the last section, we saw the foundational influence Galileo's observations of free fall had on the work of Newton and Einstein. However, vertical free fall was too fast to measure with the technologies of the sixteenth century. Accurate recording of the time and positions of a falling object was virtually impossible. So, he needed to slow down the motion of the objects, and he did that in two ingenious ways:

1. by letting objects slide on inclined planes;
2. by using a newly invented machine that partially balanced the weight of a falling object using a counter weight.

First we discuss Galileo's use of inclined planes to reduce the accelerations of objects. Inclined planes allowed Galileo to carry out careful measurement of the positions of accelerating objects as functions of time. One of the major observations he made was that the distance traveled in time t along an inclined plane increased as the square of the elapsed time, i.e., $d \propto t^2$. This implied that the speed of the object increased at a constant rate along the plane, and that this rate depended on the inclination. From these experiments, and through theoretical reasoning, he derived the law that governs the motion of objects on inclined planes. We know that if we place a ball on a horizontal plane, it does not move at all. This is because the weight of the ball is supported by an equal but opposite force exerted by the plane. What would happen if the plane were inclined? Galileo answers this through the following argument. On an inclined plane the gravitas of the ball would be *partially* balanced by a force perpendicular to the plane. What force parallel to the plane would we then need to insure that the ball does not roll? He argued that for the ball not to move the moments of the weight (what we call "torque") and the balancing force[11] must cancel each other out. He then showed that this required that the ratio of the balancing force to the gravitas be the same as the height

11. It is important to note that the history of moments of forces goes back to Archimedes.

of the inclined plane to its length. In other words, the balancing force needs to be equal to (*height/length*)×*weight*. The height and length are shown in Figure 5.1. In today's language this would be written as $F = W \times \sin \vartheta$

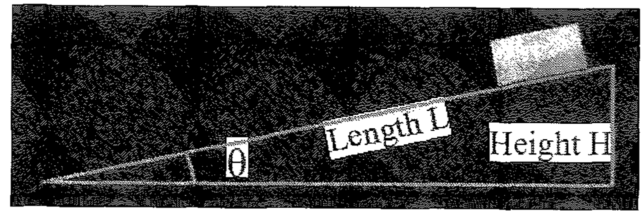

Figure 5.1: A block sliding on an inclined plane of height H and length L.

where the angle ϑ is the angle of inclination of the plane, $\sin \vartheta = (height/length)$. From this he deduced that the distances traveled by an object on planes of varying inclinations within a fixed time interval Δt were proportional to the ratios of their heights to lengths. In fact, he was flirting with Newton's second law.

From Newton's laws we know that the distance traveled on an inclined plane is given by

$$d = \tfrac{1}{2}a(\Delta t)^2 = \tfrac{1}{2}g \sin \vartheta \, (\Delta t)^2$$

Thus, the ratios of distances traveled in time Δt on planes of varying inclinations would be given by

$$\frac{d_1}{d_2} = \frac{\tfrac{1}{2}g \sin \vartheta_1 (\Delta t)^2}{\tfrac{1}{2}g \sin \vartheta_2 (\Delta t)^2} = \frac{\sin \vartheta_1}{\sin \vartheta_2} = \frac{H_1/L_1}{H_2/L_2}$$

This is exactly what we get from the application of Newton's laws.

Two-dimensional Motion of a Projectile

As the following diagram shows (Figure 5.2), Galileo launched projectiles with different initial speeds from a horizontal launcher into a two-dimensional vertical plane. As the starting speed increased, so did the range of the projectile,[12] and they were linearly proportional to each other ($R \propto v$). Galileo was among the first to discover one of the most important aspects of two-dimensional motion: the motion along the horizontal direction was completely independent of the motion along the vertical direction. He also noted that the uniform velocity along the horizontal direction and free-fall along the vertical resulted in a parabolic trajectory.

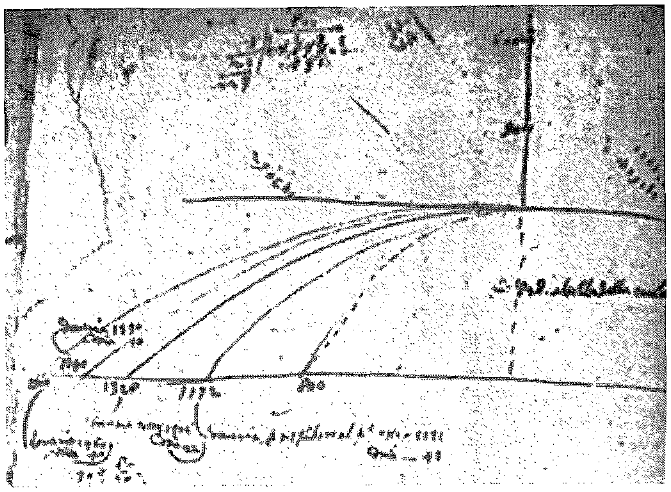

Figure 5.2: Trajectories of projectiles for varying starting velocities (Galilean manuscripts, 1608).

Let us present here another example in which two independent motions are superimposed on each other. In this ingenious experiment, a ball is released off center on an inclined trough. If the trough is kept horizontal the ball executes oscillatory motion with a fixed period determined by the curvature of the trough. When the ball is released on an inclined trough (see Figure 5.3), it comes down along the axis of the trough with an increasing velocity and a transverse oscillation. When this experiment is carried out in a laboratory, one finds that the extreme rightward point of each

12. I.e., how far these projectiles landed from the point of projection along a horizontal axis denoted by numbers such as 800, 1172 and 1320 units of length.

oscillation provides us positions along the length of the trough at equal intervals, and the distances of these points[13] from the starting position increases in ratios of squares of natural numbers, i.e. $d_n = n^2 d_1$. Thus, we see that motion along the axis of the trough obeys the laws of an inclined plane; it is completely independent of transverse oscillations. Even though this experiment might have been performed by Galileo, we do not find any mention of it in the literature.[14]

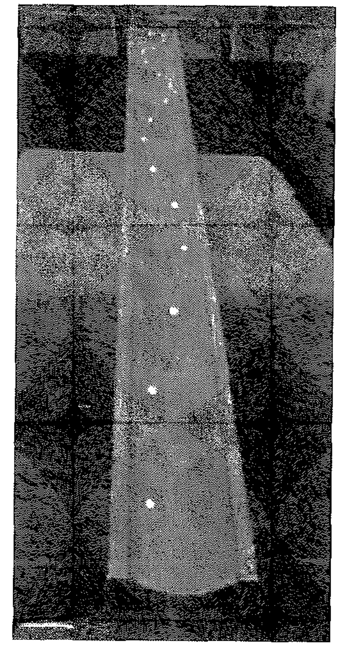

Figure 5.3: Combination of a linear motion on an inclined plane and a transverse oscillatory motion. Courtesy of Harvard Natural Sciences Lecture Demonstrations and Prof. Owen Gingerich. For more details, see http://sciencedemonstrations.fas.harvard.edu/presentations/scantling-and-ball.

13. The distance of the n-th peak on the right from the starting position is denoted by d_n.

14. Communications with Profs. Roberto Vergara Caffarelli and Paolo Palmieri.

Newton's Laws of Motion: Inertia

According to Aristotle, to keep an object moving even at a constant velocity, we need an external force on the object. Galileo put this principle to the test. In Figure 5.4, a ball released from point A rises to point B on a second plank. If the inclination of the plank were lowered, it would rise to C. Each time it goes up to the same height as the starting point A. If we keep on lowering the inclination of the second plank, it will travel farther and farther to reach the same height. What if we make the second plank horizontal? In this case the ball should keep going farther and farther in its futile effort to reach the height of A—this is the law of inertia. That is, we do not need to keep pushing an object to keep it moving with constant velocity on a frictionless horizontal surface. This is the statement of Newton's First law which Newton himself attributed to Galileo.

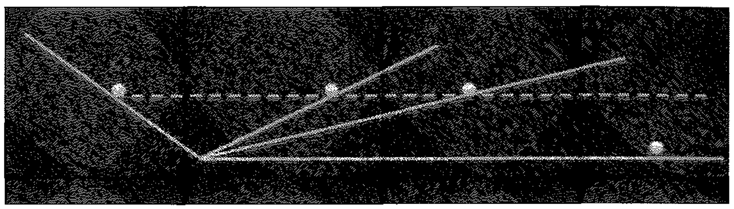

Figure 5.4

Galileo's New Machine

We now describe an experiment that was carried out by Galileo in his later years which brings us very close to the laws of dynamics as we know them today. A similar piece of equipment was built by George Atwood about 150 years later and is now known as the Atwood Machine. We have schematically shown the apparatus in Figure 5.5. Two masses, m_1 and m_2, are supported by pulleys. Caffarelli's reconstruction of the equipment shows one pulley; we have chosen to use two pulleys to ensure that the pulleys could be made

small and have relatively little effect on the vertical motions of the weights.

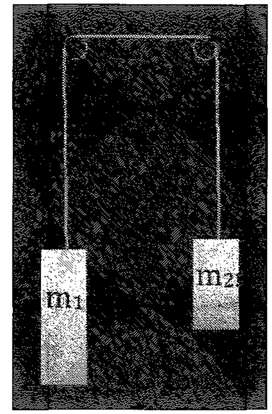

Figure 5.5: Galileo's New Machine

Galileo experimented with different relative values of these masses. Choosing equal masses, he demonstrated that one need not keep pushing these objects for them to have uniform velocities - a statement of the law of inertia, but this time for vertical linear motion. He also carried out quantitative studies of different masses. (This may well be why Newton gave him credit for the second law of motion, $F = ma$.) Very importantly, Galileo carried out an interesting experiment on this apparatus that would yield the principle of momentum conservation. He kept one mass (say m_2) on a support and lifted the other mass m_1 to a certain height h and slackened the rope linking them together. When he the released the mass m_1, it came down with increasing speed. Eventually the rope tightened and the second mass started moving in what he called a "violent motion." Galileo discovered that the velocity v_1 of m_1 right before the rope tightens bears a relationship to the common speed V of the combination just after the rope tightens: $m_1 v_1 = (m_1 + m_2)V$. This is actually the statement of momentum conservation for a perfectly inelastic collision, a corollary of Newton's second and third laws of motion.

Concluding Remarks

We use the phrase "Renaissance person" to refer to an individual who excels in many areas at the same time. Galileo was a figure not only historically from the late European Renaissance, but also was one whose mind and its achievements touched many areas: physics, astronomy, mechanics, religion, and politics. Although often overshadowed by the controversies he was a part of, his contributions to the understanding of motion, inertia, acceleration, and gravity are remarkable. Explaining both his real and his thought experiments in mechanics gives us yet another way to appreciate his lasting contributions to physics and humanity.

Acknowledgment

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Bibliography

Gassendi, Pierre. *De Motu Impresso a Motore Translato*. (1642) New World Encyclopedia. http://www.newworldencyclopedia.org/entry/Pierre_Gassendi.

Newton, Isaac. *The Principia: Mathematical Principles of Natural Philosophy*. Translated by I. Bernard Cohen and Anne Whitman, assisted by Julia Budenz. Berkeley: University of California Press, 1999.

Vergara Caffarelli, Roberto. *Galileo Galilei and Motion: A Reconstruction of 50 Years of Experiments and Discoveries*. Berlin: Springer, 2009.