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Number Theory

Transforming Instruction in Undergraduate Mathematics via Primary Historical Sources (TRIUMPHS)

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Babylonian Numeration

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BABYLONIAN NUMERATION

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You may know that people have developed several different ways to represent numbers throughout history. The simplest is probably to use tally marks, so that the first few numbers might be represented like this:

$$|\qquad ||\qquad |||\qquad ||||\qquad |||||\qquad ||||||$$

and so on.

You may have learned a more efficient way to make tally marks at some point, but the method above is almost certainly the first the humans that ever used. Some ancient artifacts, including one known as the "Ishango Bone", seem to show that people were using symbols like these over 20,000 years ago! [1]

By 2000 BCE, the Sumerian civilization had developed a considerably more sophisticated method of writing numbers – a method which is quite unlike what we use today. Their method was adopted by the Babylonians after they conquered the Sumerians, and today these symbols are usually referred to as *Babylonian Numerals.*

Fortunately, we have a very good record of the way the Sumerians (and Babylonians) wrote their numbers. They didn't use paper, but instead used a small wedge-shaped ("cuneiform") tool to put marks into tablets made of mud. If these tablets were baked briefly, they hardened into a rock-like material which can survive with little damage for thousands of years.

On the next page you will find a recreation of a real tablet, presented close to its actual size. It was discovered in the Sumerian city of Nippur (in modern-day Iraq), and dates to around 1500 BCE. We're not completely sure what this is, but most scholars suspect that it is a homework exercise, preserved for thousands of years. Of course, it's not preserved perfectly, and dealing with the parts that have become damaged is part of the challenge (and part of the fun) of working with old texts like this. If you study the picture closely, you may discover that you can discover a lot about Babylonian numerals.

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[1] See, for example, Rudman, Peter Strom (2007). How Mathematics Happened: The First 50,000 Years. Prometheus Books. p. 64.

Study the image below, and try to use it to answer three questions.

(1) How do Babylonian numerals work?
(2) Describe the mathematics on this tablet.
(3) Write the number 72 in Babylonian numerals.

Cuneiform Texts, ed. Hilprecht, Vol. XX, Part 1. (1906). It's on Plate 16, No. 27.

1. Instructor Notes

The purpose of these notes is to share with instructors considering the use of this project the author's experience in implementing it in classes. Individual instructors, of course, should feel free to modify its use as they see fit.

This "mini-Primary Source Project" is intended to introduce students to Babylonian numeration. It can replace part of a lecture concerning Babylonian numerals by giving students a chance to discover for themselves several details about Babylonian numeration, and to construct an understanding of this foreign-seeming system before seeing it presented by an instructor.

I have used this project in Elementary Education and History of Mathematics courses. I have found that it makes a very good in-class group project. I divide students into groups (or let them form their own) of size about 3, and pass out a copy of the project to each student. Before this, they should have had no introduction to Babylonian numeration in our class – neither in lecture nor in the reading.

I generally introduce the project with a very short discussion, asking students to imagine that they have an Archaeologist friend who has unearthed this somewhat-damaged tablet, which she suspects contains mathematics, and that she brings it to them. Their task is both to explain how Babylonians wrote numerals and to work out the mathematics on this table in 15 minutes.

The students seem to respond almost invariably with disbelief – they do not believe such a task is possible. I encourage them to try, and usually most groups make some early progress. It's not too hard to guess that the first column represents the positive integers, for example, and most groups are able to guess that the "left-pointing" wedge represents "10." If, after 5-10 minutes' struggle, a group can get no further, I suggest they look at lines 4 and 6, which are often enough for students to reason (correctly") that this is a table of squares.

After about 15 minutes, I then ask each group to get together with a second group to compare their progress (about 3–5 minutes), and then lead a classroom discussion in which we reason through the tablet together. I find that student suggestions or questions concerning the tablet are usually sufficient to naturally bring up all the major features of Babylonian numeration (the sexigesimal system, the lack of a 0) that I would have covered in a lecture.

As a final thought, I would add that no other classroom project I use regularly leaves students feeling as proud as they are of their success. I remind them that they have just worked out for themselves a completely foreign system of writing numbers. Even those groups which needed a few hints to make progress have seemed very pleased with themselves by the end of class, and I regularly hear that students have recounted the day's work later to friends or other instructors. This instilling of confidence and pride in students' ability to tackle an initially impossible-seeming project is not the primary goal of this Primary Source Project, but I have often found it to be a fortunate outcome.