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Experimental Evidence for Heterogeneous Expectations in a Simple New Keynesian Framework

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Contents

Chapter 0

Abstract

This paper is a two-dimensional analysis of agent behavior in a standard New Keynesian (NK) Macroeconomic model. On the dimension of pure mathematics, we analyze the parameters of the NK model and of possible prediction rules. On the other dimension we continue a practice of empirical study of heterogeneous expectations with an experiment. The experiment will ask participants to make predictions of future output and inflation. Their responses will create a data-set upon which analysis will be performed to illuminate and corroborate current theories of economic decision making. The literature has shown that most agents' forecasting rules can be modeled by basic linear formulae. We conclude that some subject's predictions are consistent with recursively updating coefficient models, while others still use more inscrutable methods. Despite this, and regardless of which model fit them best, the subjects' errors were all of similar magnitudes.

Chapter 1

Introduction

A feature that distinguishes economics from the natural sciences is that the beliefs of participants in an economic system actually have an effect on what that economic system does. This effect, known collectively as "expectations," is an integral part of all economic theory; however, due to the very large number of participants in a given system, quantitatively assessing every agent's expectations is extremely cumbersome at best and completely impossible at worst. Economists have developed many simplifications and assumptions to deal with expectations, such as "naïve" expectations, where agents predict the future will be the same as the past. The reigning theory is the Rational Expectations Hypothesis, which is well-liked for its elegance and computational simplicity. The following paragraphs describes the history of expectations, concluding with the development of learning, the method that is the focus of this paper.

Macroeconomics is the study of economic systems with interest in large-scale development and patterns across all agents in the economy. This is in contrast to microeconomics where one is interested in profit-maximizing decisions for oneself or one's firm. Many aspects of economics can be exactingly represented mathematically, i.e. an interest or inflation rate. At its core, however, economics must quantify the decisions and actions of real agents (firms and people). Their decisions and actions have direct effect on the outcomes of any given system, and agents take those actions based on what they expect the system do in the future. Actions can take such varied forms that to quantitatively account for each would be meaningless. Therefore economists instead account for agents' expectations of the future as a way to represent the intent of any particular action an agent takes.

In microeconomics, this additional input is relatively trivial. The only "expectation" one cares about is one's own and therefore a single-input based on the firm's/person's goals is all that is needed to fill this gap. In macroeconomics, however, outcomes are affected by every agent in the economy and their aggregate expectations. Given that macro-models are usually scaled to the size of a country, this means at least hundreds of thousands of agents. Collecting data on individual expectations would be exhaustively expensive, if not impossible, not to mention that the data would probably be outdated by the time collection was complete. To solve this problem, macroeconomists have been deriving laws and assumptions of expectations that might be broadly applied to each agent and approximately represent aggregate expectations. One of the earliest methods to tackle this problem was called "naïve" expectations. It can be described in the following manner:

$$x_t^e = x_{t-1}$$

Where x_t with no superscript is the value of the economic variable, x, for the time period t. The superscript e denotes that this is not the actual value of x_t, but the value that will be expected of x for time t. The assumption of naïve expectations, as expressed above, states that the expectation of a variable for the current period, x_t^e, is equal to the observed value of that variable in the previous period, x_{t-1}.

This assumption worked well as a placeholder, but soon proved to be too simple. Agents are more than capable enough to see and extrapolate trends, as well as have even more sophisticated predictions based on other knowledge. The great breakthrough in expectations theory came in the form of the Rational Expectations Hypothesis (REH).

For a completely deterministic model, it can be expressed as follows:

$$x_{t+1}^e = x_{t+1}$$

This theory, introduced in Muth (1961) and extended in Lucas (1967), assumes that all agents are "rational" in the sense that they can compute the future the same way an economist does: with exact knowledge of the framework (often called the "law of motion" of the given system) within which they are an agent and that all other agents in the framework are rational as well. The only thing a "rational" agent cannot predict is any random structural shock an economy exhibits in a given period. It should be noted that this hypothesis requires significant and nontrivial assumptions to be taken at face value. All agents must not only be very sophisticated, but they must all be homogenous in their sophistication. These objections were brought to the REH, and were answered to some degree. Proponents argued that even though agents may not be homogenously sophisticated, there are enough sophisticated agents (banks, finance professionals, etc) to influence the expectations of less sophisticated agents and guide the model towards rational expectations equilibria that can be well approximated by an REH model. The REH is still used in most models today, even though its assumptions are not often fulfilled, partly because it is entrenched, and partly because a better alternative has yet to solidify.

The Rational Expectations Hypothesis works very well in terms of computation and allowed economists to analyze models with higher orders of complexity. However, many economists were (and are) skeptical that the average economic agent does, or even can do, what rational expectations assumes they do. The main assumption that does not seem plausible is that all agents somehow have perfect knowledge of the framework within which they exist, including the true value of each parameter. For perspective, this is knowledge about which even the economic profession has yet to come to a consensus. However, it cannot be argued that agents make uniformly random or unintelligent decisions; therefore, they must have some non-naïve way of processing information. Learning theorists posit that agents do not know the parameters and structure of the law of motion, nor do they delight in substitution and linear algebra, but they *do* have some model (a *perceived* law of motion) that they rely upon to inform their expectations. This model probably begins rudimentary and erroneous, but as an agent is able to update the model with more data, it becomes better over time. This model can become so good that the agent makes choices that a truly rational agent would make, but in other cases this convergence may never happen; it all depends on the framework and what incentives exist within it. The field began with thought experiments such as in Haltiwanger and Waldman (1985) and has since developed into experiments with real human subjects to divine what "perceived laws of motion" real agents form. Learning models have had success in explaining some stylistic qualities found in real world data that the REH cannot explain.*

Numerous studies have been conducted to understand how agents form these perceived laws of motion, how they perform, and what effect they have in feedback into a given economic system. These studies focus on asking real humans to predict some economic variable over a time period. This paper is similar in intention to Pfajfar and Zakelji (2013) and Assenza, Heemeijer, Hommes and Massaro (2013), in that it wishes to understand real agent expectation formation in a macroeconomic framework (as opposed to an asset pricing framework). However, our study will break new ground by asking subjects to predict both the future inflation rate and the future output gap simultaneously.

Prior to the experiment, we also analyze the effects and forces of expectation formation from a mathematical perspective. As will be shown, economists often describe the economy using dynamical systems. While real world data is extremely valuable, and indeed statistical analysis of such data is an indispensible part of the discipline, an economy is a complex system with too many variables to track at once, inlcuding

*For an in depth discussion of Learning's history see Evans and Honkapohja (2001).

many that are inscrutable to the point of seeming random. Dynamical mathematical systems are used in this discipline, like in other disciplines, to represent the motion of a complex real-world phenomenon. Variables and parameters model the levels and rates of economic factors. With a good model, that is, one that represents the real world well, researchers can not only study system outcomes, but processes and relationships. One can see not only what happens under normal conditions, but also under extreme, or unusual conditions; conditions that otherwise require rare "natural experiments" to study.

This paper is structured as follows: First recent literature is discussed, followed by an explanation of the of the New Keynsian model. We continue into a mathematical analysis of the system using an exemplary learning rule to furnish expectations. Finally, we describe the design, execution, and results of the experiment, and conclude.

1.1 Literature Review

Economists often argue that the Rational Expectations Hypothesis (REH) assumes too much. The representative agent cannot be credited with having the knowledge neccesary to find the Rational Expectations Solution every time they have to make an economic choice. Arthur (1992) argues that there exists a "Problem Complexity Barrier" beyond which agents cannot use deductive (rational) methods to find a solution. In these cases they instead employ inductive reasoning: they create a mental model (schema) of the framework using similar past experience, and improve that model with any new information gathered as time goes on. That mental model can be approximated mathematically, and illustrative decomposition of that model can give great insight into how agents interact with the system they work in.

Even if many of its underlying assumptions are unrealistic, the REH reigns supreme because of its elegance and computational simplicity. Furthermore, it is considered "robust" because in many cases, even if agents are not homogenously rational, the existence of *some* rational agents can cause convergence to Rational Expectations Equilibria (REE). However Haltiwanger and Waldman (1985) shows that this convergence only happens under certain circumstances, specifically when expectation feedback is negative, i.e., when agents are rewarded for predicting differently from one another. When a system exhibits positive expectation feedback, i.e., when agents are rewarded for coordinating with each other, non-rational agents can cause volatility leading to oscillation, explosive growth, and even convergence to non-rational equilibria. Understanding what "sign" of expectations feedback is present gives great insight into whether the REH will be sufficient in describing the model.

Even though they note that most frameworks exhibit both "signs" of expectation feedback, as our framework will, Heemeijer, Hommes, Sonnemans and Tuinstra (2009) conducts equivalent experiments on two asset pricing models, one with positive and one with negative expectations feedback, to isolate "sign" as the experimental variable and analyze the results for rationality. Subjects, in groups of 6 or 7, are asked in each case to predict the future price of an asset. The price in the positive model persistently oscillates even though subjects coordinate with each other very quickly, whereas the price in the negative model converges slowly to the fundamental price of the asset. This shows that for any system with positive expectations feedback, the REH may yield false equilibria.

Patterns of non-rationality have always been visible in real world data sets, especially in the data sets of asset markets, such as a stock market, where expectations feedback *is* positive. The REH and Efficient Market Hypothesis (EMH) state that an asset's price will always reflect its fundamental value; however, time and time again stock prices have exhibited bubbles and other stylized non-rational behavior. Brock, Hommes and Wagener (2005) creates an asset pricing model that allows for any number of different trader types, any of which can be non-rational. These trader types use learning models to guide their actions. A simulation of this model is able to quantitatively match the vital statistics of 20 years of observed financial data (from the

S&P 500). Learning models that outperform the REH in terms of descriptive ability continue to legitimize Learning as the superior option for accounting for expectations in economic analysis. Since learning rules are derived from the thought process of real agents, recent experiments by and large consist of asking human subjects to make predictions of some variable's future value in a given economic framework. Our experiment follows in this vein.

The second goal of this experiment, in line with convention in the field, is to analyze the gathered data and attempt to model how subjects form their expectations in the game. Most papers have remarked that a large percentage of their subject's predictions can be well approximated by a linear regression rule (Pfajfar and Zakelji (2013) Heemeijer et al. (2009)). Brock and Hommes (1998) develops a more sophisticated "heuristic switching" model, where agents have the choice of a set of different forecasting rules, and switch between them depending on which has been recently most successful. This model is cited in Assenza et al. (2013) for fitting data in learning-to-forecast experiments even better than linear models, and we will be examining the same set of forecasting rules in relation to our data; however, the differences in our experimental design may lead to new rules that have yet been unobservable.

Pfajfar and Zakelji (2013) performs an experiment using a very similar New Keynesian framework as will be used in this paper. In contrast to our intention to ask subjects to predict both output and inflation, Pfajfar and Zakelji (2013) asked participants only to predict inflation. The results showed that expected inflation was always higher than realized inflation, suggesting negative expectations feedback. Assenza et al. (2013) also performs a very similar experiment, running three treatments to separate the prediction of each variable. The subjects predicted inflation, while output expectations are (in the first treatment) expected to be in equilibrium or (in the second treatment) assumed to be naïve. In the third treatment, subjects were separated into two groups, one predicting output and one predicting inflation.

Our project builds upon these papers' experiments. Our experiment has each subject predicting both the inflation rate and the output gap, as economic agents do in the real world. We expect this difference will allow agents to make better predictions because they will receive feeback from the effects of both variables.

Chapter 2

Framework

2.1 New Keynesian Macroeconomic Model

The New Keynesian (NK) framework that describes the economy's law of motion, as assumed by the experiment, is written below. A full discussion of the model's foundations and justifications can be found in Woodford (2003). The monetary policy rule was derived in Branch and McGough (2009) and previously used for modeling heterogeneous expectations in Assenza et al. (2013).

$$x_t = x^e_{t+1} - \phi(i_t - \pi^e_{t+1}) + g_t$$
$$\pi_t = \beta\pi^e_{t+1} + \lambda x_t + u_t$$
$$i_t = \bar{\pi} + \theta_\pi(\pi_t - \bar{\pi}) + \varepsilon_t$$

The endogenous errors g_t and u_t are autocorrelated in the following manner:

$$g_t = \delta g_{t-1} + \tilde{g}_t$$
$$u_t = \mu u_{t-1} + \tilde{u}_t$$

Furthermore, $\tilde{u}_t$ and $\tilde{g}_t$ are random stochastic error terms with mean 0 and standard deviation 0.2.

This is a Dynamic Stochastic General Equilibirum Model (DSGE). The output gap and inflation rate laws of motion are based on micro-foundations of utility maximizing representative agents and profit-maximizing representative firms. The monetary policy rule (interest rate) is based on the perceived actions of the central bank; this rule in particular is based on the propensities of the U.S. Federal Reserve. The economy is described by three variables (and the expectations thereof):

- The inflation rate π_t (subject to random shock u_t)
- The output gap x_t (subject to random shock g_t)
- The interest rate i_t (subject to random shock ε_t)

The inflation rate measures the percentage change in the price level of the economy. In each period, the inflation rate depends on expectations of itself and output gap, as well as on a random minor price shock u_t. There is a positive relationship between the actual inflation rate and the expectations of both the output gap and the inflation rate. This means, for example, that if all other factors are the same, an increase in the expectations of the inflation rate will cause an increase in the real inflation rate. The minor price shocks have an equal chance of affecting inflation positively or negatively.

The output gap measures the percent difference between Gross Domestic Product(GDP) and the natural GDP. The GDP of a country is the value of all goods produced during a period in the economy. Natural GDP is the value that total production would have been if the economy had acheived full employment. If the output gap is positive (negative), the economy has produced more (less) than the natural GDP. In each period, the output gap depends on both the expecations of itself, expectations of the inflation rate, the interest rate, and on a minor economic shock g_t. There is a positive relationship between the output gap and the expectations of both the interest rate and output gap. There is a negative relationship between the output gap and the interest rate. The minor economic shocks have an equal chance of positively or negatively affecting the output gap.

The interest rate measures the price of borrowing money and is determined by the central bank. There is a positive relationship between the interest rate and the inflation rate.

The subscripts t are indicative of the period of the variable. i.e. π_6 would indicate the inflation rate in period 6. The framework assumes that expectations of the output gap and inflation are made from two periods behind. That is to say that they are expectations of the next period $(t+1)$ made with only the information available in the last period $(t-1)$. As it will become a necessary distinction in the next section, this is clarified as follows:

$$x^e_{t+1} = E_{t-1}[x_{t+1}] \text{ and } \pi^e_{t+1} = E_{t-1}[\pi_{t+1}]$$

We will modify this assumption in the next chapter, so that predictions are only made one period behind. The rest of the framework will remain unchanged for the remainder of this paper's discussion.

2.2 Operating Law of Motion

For the model to be used to simulate an economy, and therefore yield any useful results, we are ourselves faced with the question of how to quantify expectations. In order to perform parameter tests, we allowed expectations to be formed by an established learning rule from Evans and Honkapohja (2001). Expectations are created in the following manner:

$$x^e_t = a_{1_t} + b_{1,1_t} x_{t-1} + b_{1,2_t} \pi_{t-1}$$

$$\pi^e_t = a_{2_t} + b_{2,1_t} x_{t-1} + b_{2,2_t} \pi_{t-1}$$

The coefficeints a_{i_t}, b_{i,j_t} are updated each period with the below process:

$$\begin{bmatrix} a_{1_t} \\ b_{1,1_t} \\ b_{1,2_t} \end{bmatrix} = \psi_t = \psi_{t-1} + \gamma R_t Z_{t-1}(x_t - Z'_{t-1}\psi_{t-1})$$

$$\begin{bmatrix} a_{2_t} \\ b_{2,1_t} \\ b_{2,2_t} \end{bmatrix} = \psi_t = \psi_{t-1} + \gamma R_t Z_{t-1}(\pi_t - Z'_{t-1}\psi_{t-1})$$

Where:

$$Z_{t-1} = \begin{bmatrix} 1 \\ x_{t-1} \\ \pi_{t-1} \end{bmatrix} \text{ and } R_t = R_{t-1} + \gamma(Z_{t-1}Z'_{t-1} - R_{t-1})$$

This updates the coefficients with a Regressive Least Squares process on past information.

For simplicity's sake, this model assumes that expectations are formed from only *one* period behind. It gives the expectations of the output gap and inflation rate in the current period, given knowledge of the period immediately previous:

$$x_t^e = E_{t-1}[x_t] \text{ and } \pi_t^e = E_{t-1}[\pi_t]$$

This diverges from an earlier discussed assumption of the NK framework in Section 2.1, but makes little practical difference, and results found in later parameter analysis are applicable to a framework with a two-period assumption. We will note later that this contemporaneous assumption does seem to make the system more stable.

Chapter 3

Numerical System Analysis

This chapter will discuss the mathematical analysis performed before the experiment to study the behavior of the New Keynesian framework when furnished by the chosen learning rule. Both the system and the learning rule were constructed in MATLAB 2012. The results in this chapter were generated by that construction.*

Table 3.1 below summarizes each parameter in the system as well as γ, the gain parameter from the learning rule. All calibrated values from Gali and Gertler (1999)†.

Table 3.1: Each parameter in the New Keynesian Framework and the description, possible range, and calibrated values of each. Also includes the range of γ, the gain parameter from the learning rule.

Notation	Description	Range	Calibrated Value
λ	The slope of the Phillips curve	$\lambda > 0$	0.3
β	The Global discount factor	$0 < \beta < 1$	0.99
ϕ	The inter-temporal elasticity of substitution	$\phi > 0$	1
θ_π	The interest rate's responsiveness to inflation	$\theta_\pi > 0$	1.5
γ	Learning gain parameter	$0 \leq \gamma \leq 1$	n/a

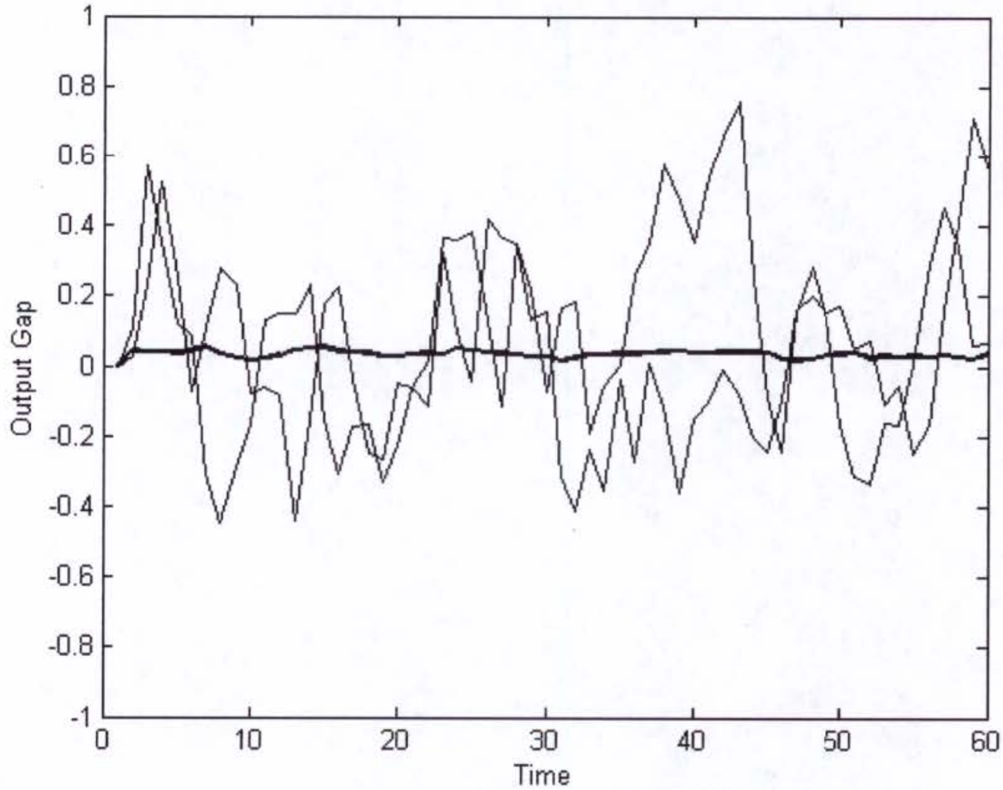
Figure 3.1 below shows a 60-period simulation of the system, as represented by

*The MATLAB code used for all of this chapter's analysis is available in the Appendix.

†Calibration is done through mathematical programming based on fundamental economic theory.

the outcomes of the output gap. The two thin lines show two out of the thousand simulated paths to represent the shape that a single simulation might take and illustrates the stochastic nature of the system. The bold line is the average of all one thousand paths at each period. It is notable that the average path stays very close to zero, which is the value that a model using rational expectations would converge to, but it does not quite stay there.This deviation is likely the result of using a learning rule to generate expectations for the model, and exhibits change throughout the experiments below.

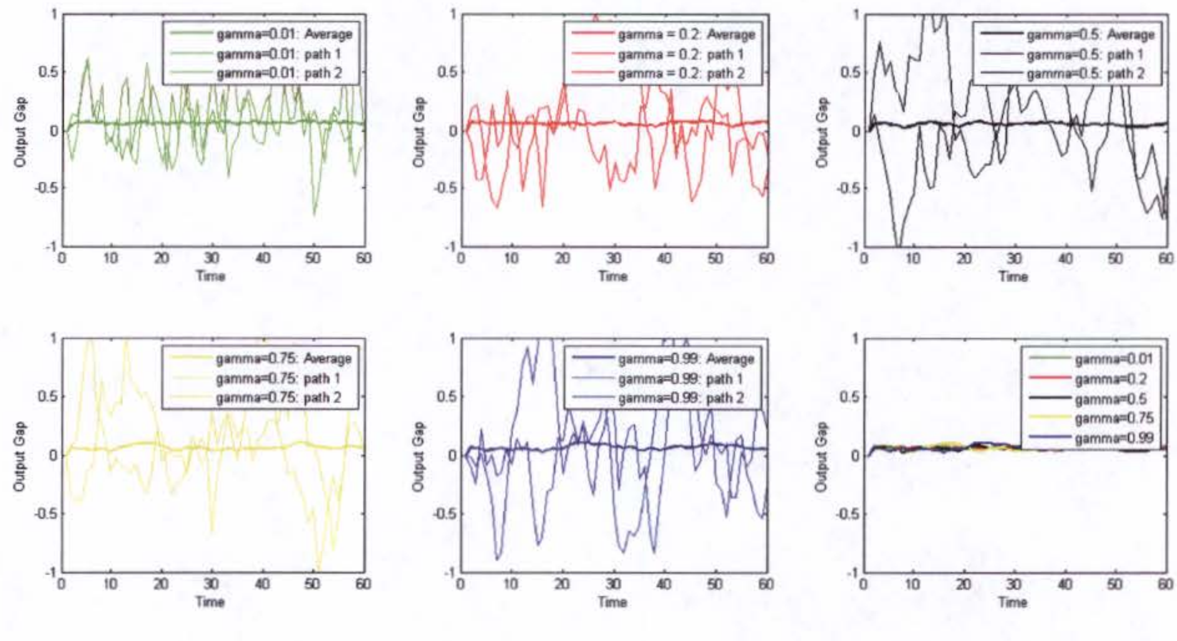
Figure 3.1: The Output Gap over 1000 simulations with the parameters set to their calibrated values, and $\gamma = 0.2$. The bold line is the average path over 1000 simulations. The two thinner lines in each graph are representative of what each of the 1000 individual paths could look like.



3.1 Effects of γ

The parameter γ describes how much an agent weights the most recent information when they form their expectations. Figure 3.2 below shows the effect on the output gap when γ is assigned different values in the interval $(0, 1)$. The bold line shows the average over 1000 simulations, and the thinner lines show representative paths. The last graph plots all of the averages from the five treatments on the same axes.

Figure 3.2: The Output Gap over 1000 simulations for each of 5 different values of γ. As γ gets closer to 1, the more output gap becomes more sensitive to shocks. The average trend, however, stays relatively constant, though it floats above the Rational Expectations outcome of 0.



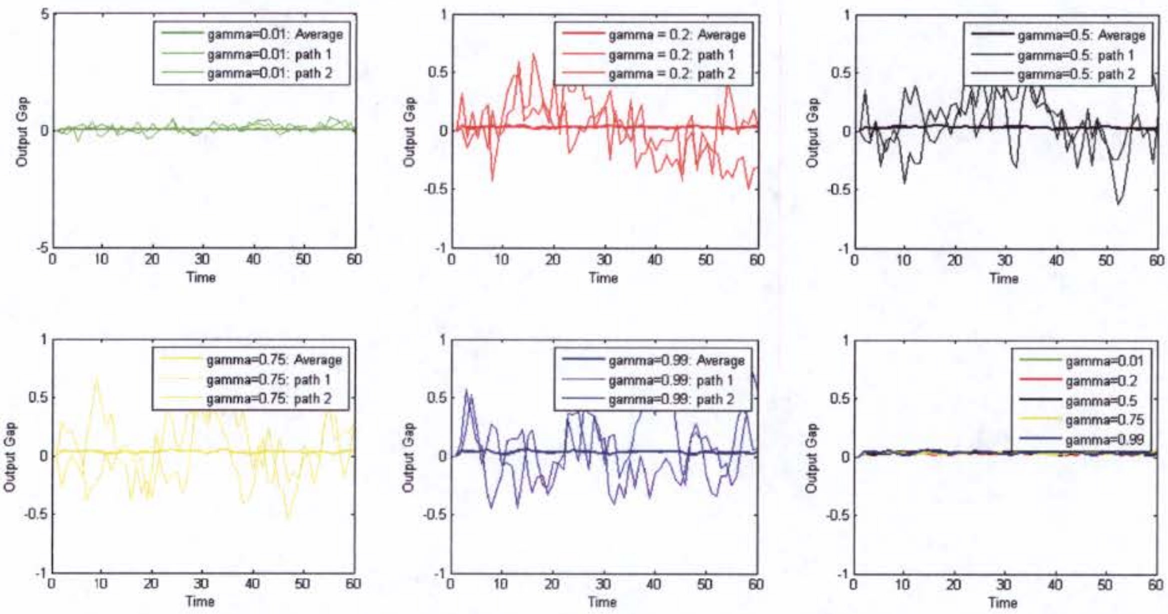
Though particular paths oscillate at varying intensities, the long term trend (shown by the average) is similar no matter the value of γ. This is best illustrasted by the bottom right graph of averages which all seem to trace the same line. Since the trend is not being affected, we will find the standard deviation of the output gap for each treatment. Table 3.2 at the end of Section 3.2 shows the result.

In addition, we were surprised that the system was so stable. It has been obsrvered in the past that learning rules with large gains do not provide stable equilibria. To test this stability we ran the same simulation as in Figure 3.2 for 3000 periods instead of 60, and found that the system was stable for the entire range, and for each value of γ. We beleive this stability comes from the contemporaneous assumption.

3.2 Effects of λ

Since conceivable values of γ change the variability but not the trend of the output gap, we changed the calibrated value of λ to 0.6. λ is the slope of the Phillips curve, which describes the relationship between unemployment and inflation. In other words, it is a parameter that describes the "stickiness" of the economy's prices. It can also been thought of as the "speed of price adjustment." When prices are less "sticky", or update to accomodate new conditions swiftly, the economy can better handle shocks and volatility is reduced. When prices are more "sticky," random shocks have more inertia, which is to say the system takes longer to recover from their influence. Figure 3.3 shows the effects of changing γ in a significantly less "sticky" environment than the calibrated value. Again, The bold line shows the average over 1000 simulations, and the thinner lines show representative paths.

Figure 3.3: Effects of $\lambda = 0.6$ on the Output Gap for 5 different values of γ. The higher λ simulates an economy with less "sticky" prices, therefore stochastic shocks are dealt with more flexibly and the paths stay closer to the average trend. The average trend is dampened vertically and closely approaches the Rational Expectations outcome at 0.



As is expected, the variability in the output gap is visibly decreased when λ is larger. This implies that quickly adjusting prices helps agents forecast better, which makes intuitive sense in addition to agreeing with standard economic theory. Table 3.2 below shows that the standard deviation of the output gap when $\lambda = 0.6$ varies little with respect to a changing γ, though the amount of volatility was positively correlated to γ when λ was equal to its calibrated value of 0.3.

Table 3.2: Standard deviations of the Output Gap given differing values of γ and λ. Variation is significantly lower with higher λ.

Std Dev	$\gamma = 0.01$	$\gamma = 0.2$	$\gamma = 0.5$	$\gamma = 0.75$	$\gamma = 0.99$
$\lambda = 0.3$	0.2448	0.3969	0.3805	0.4172	0.6485
$\lambda = 0.6$	0.1898	0.2580	0.3036	0.2814	0.3255

3.3 Sensitivity Analysis of λ and ϕ

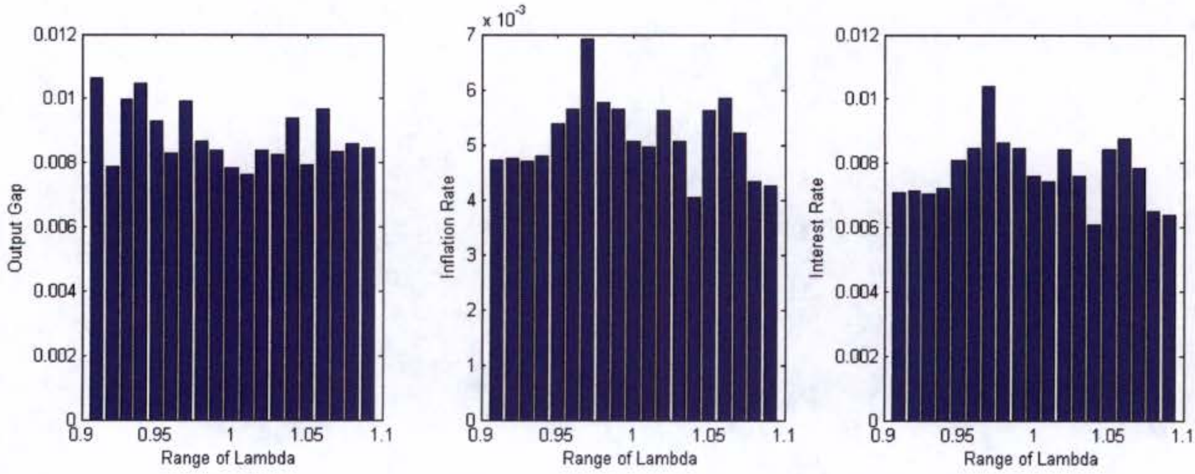
Noting the damping effect of λ, we decided to perform sensitivity analyses to determine whether there exist particularly sensitive ranges of the parameter that lead to particularly intense changes in the outcome variables. If λ is particularly sensitive in some ranges, then economic systems exhibiting λ's within those ranges might be considered vulnerable to a smaller change than a system with a more "stable" value of λ. We performed the same analysis with ϕ, the intertemporal elasticity of substitution. In both cases, results were inconclusive.

To examine the sensitivity of λ, we used the same numerical framework as above in terms of the operating law of motion and calibrated parameter values. To isolate λ's effects away from the stochastically generated noise, we again use the average of one thousand ($N = 1000$) simulations. Let v_t' be the value of one of the three variables of interest (output gap, inflation rate, or interest rate) when λ is 1% less than the value of lambda for the computation of v_t. Then, the below quantity gives us the change in a variable's outcome, given a small change in λ.

$$\Delta v = \frac{1}{T}\sum_{i=1}^{T}\left(\left|\frac{1}{N}\sum_{j=1}^{N} v_t' - \frac{1}{N}\sum_{j=1}^{N} v_t\right|\right)$$

We then compute Δv for all $\lambda = .9\lambda, .91\lambda, .92\lambda, ..., 1.09\lambda, 1.1\lambda$, giving us a spectrum of fluctuations caused by equal changes in λ. Figure 3.4 shows these spectra for all three variables of interest.

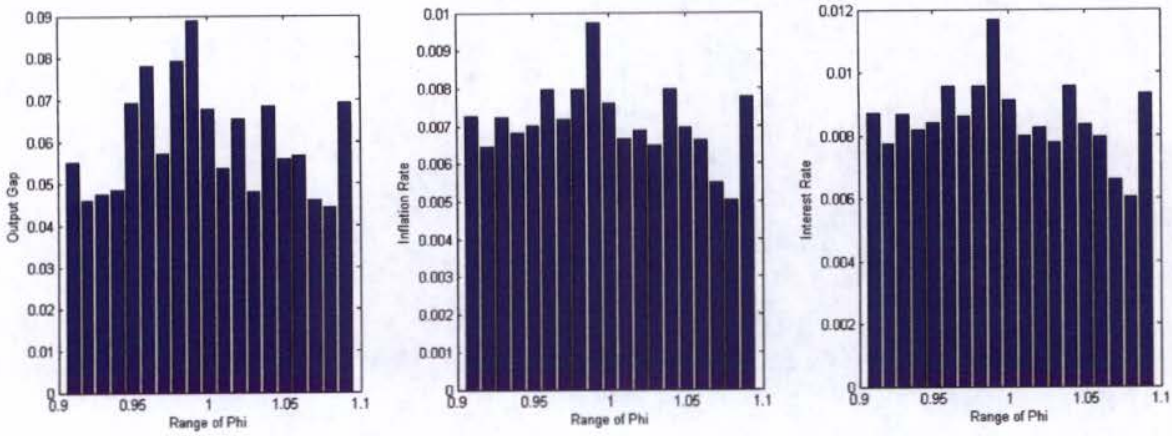
Figure 3.4: Sensitivity of the Output Gap, Inflation Rate and Interest Rate to uniform changes to λ. Uniformity indicates an insensitive interval of λ and intervals where adjacent values exhibit significant height difference migh be considered sensitive. The very small scale of these differences, and the lack of consistency over repeated trials, leads to the conclusion that the variation seen below is likely just muted stochastic noise.



At first glance it seems as if λ does have sensitive points along this spectrum; however, two factors indicate otherwise. First, despite all of the averaging to reduce the effect of the stochastic shocks, repetition of this simulation produced different resultant curves. Second, the scale of the changes is extremely small and the changes probably are themselves only noise. While this is dissapointing, it is not surprising; this economic model, especially when a recursive data-based learning rule furnishes the expectations, is known to be particularly stable. Therefore this inconclusive sensitivity analysis seems only to provide evidence for conventional economic wisdom.

We arrive at the same conclusion in the case of ϕ. Concluding this chapter, Figure 3.5 below looks very similar to the above graph for λ, and was produced using the same Δv equation as above. It succumbs to the same failings as the sensitivity analysis of λ, in that any perceivable variation is miniscule in absolute scale, and that repeated trials yield inconsistent results. It is worth noting that both ϕ and λ are both compound parameters that economists are still uncertain about. Calibrations for both are difficult to ascertain and often unreliable. This impenetrability could be contributing the the inconclusiveness of these results.

Figure 3.5: Sensitivity of the Output gap, Inflation Rate and Interest Rate to uniform changes to ϕ. Uniformity indicates an insensitive interval of ϕ and intervals where adjacent values exhibit significant height difference migh be considered sensitive. The very small scale of these differences, and the lack of consistency over repeated trials leads to the conclusion that the variation seen below is most likely just muted stochastic noise.



Chapter 4

Results

Over three experiments, twenty-seven (27) subjects completed the simulation. We proceed in this section to analyze this data set, and compare the subject's predictive behavior to that of various posited learning rules, including the one used for mathematical analysis in earlier chapters.

4.1 Experimental Design

The following procedure was performed on three separate occasions, on the dates of March 19th, 22nd, and 30th.

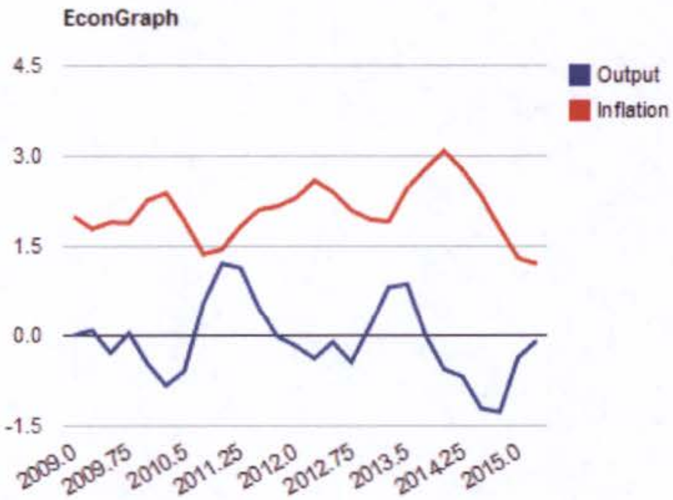
Subjects were recruited via on-campus advertisement and were exclusively volunteers. Prior to agreeing to be a part of the experiment they were given only the information available from a campus-wide email advertisement. Subjects arrived at room Pfhaler 106 at Ursinus College, with instructions to bring their school issued laptops.* Subjects signed a consent form to participate in accordance with prevailing IRB protocols. When they signed the form, they were each given a random username. The subjects were then asked to navigate to a webpage hosted on a local Ursinus server

*Students at Ursinus College are given ubiquitous laptops as part of entry into the undergraduate program.

where they were instructed to log in using their given username.

Subjects were given a basic description of the economy via a written digital information packet.[†] The researcher gave a brief verbal description of the experiment and its most important details, and then subjects were let into the main game page at the same time. Figure 4.1 below is an example of what the subjects saw on that screen:

Figure 4.1: Game Screen as seen by participants



Expected output:
Expected inflation:

Submit

Last quarter comparison		
Field	Your guess	Actual Value
Output	-0.5	-0.091
Inflation	1.467	1.205

As a starting point, five periods of inflation and output data already populated each user's database, and were presented on the graph as when the subject saw it for the

[†]Similar to qualitative descriptions in Section 2.1. An exact copy of the participant information packet is available in appendix.

first time. Subjects were then allowed to proceed at their own pace and make guesses about the next period as quickly as they liked. As soon as a subject inputted estimations for both the output gap and inflation rate, the web application used those values to calculate the next values of output inflation using the exact New Keynsian model described in Chapter 2. Each subject participated in their own personal "economy" and was the singular source of expectations.[‡] This continued for sixty (60) iterations.

Each iteration, the program keeps track of each individual subject's absolute error in guessing both the output gap and the inflation rate. At the end of the game, either the output gap error data or inflation rate error data was chosen at random for each participant. Each participant was compensated based only on their predictive error with regard to only the variable that the game chose for them randomly. This method was meant to preserve the incentive for subjects to continually exert effort in predicting *both* variables over the arduous sixty-period game. Each subject was also given a "base pay" of \$5 for simply participatingl added onto their total earnings. A full compensation schedule and other game materials are available in the appendix.

4.2 Groupings

Table 4.1 is a list of rules that agents might conceivably use to predict inflation. It is reproduced from Pfajfar and Zakelji (2013). Though this table refers only to the forecasting of the inflation rate, the structures can be used to predict other variables, such as the output gap, as well. Our operating law of motion, as outlined in 2.2, is of a "Recursive" form, similar to the models marked as M7 through M10.

[‡]We have been laying the groundwork for a future experiment where a large number of subjects interact in a single virtual economy. See Chapter 5.

Table 4.1: Possible learning rules for agents predicting the inflation rate. Reproduced from Pfajfar and Zakelj (2013)

Model (Eq.)	Specification
AR(1) process (M1)	$\pi^k_{t+1\|t} = \alpha_0 + \alpha_1 \pi^k_{t\|t-1}$
Sticky information type (M2)	$\pi^k_{t+1\|t} = \lambda_1 \eta_0 + \lambda_1 \eta_1 y_{t-1} + (1-\lambda_1)\,\pi^k_{t\|t-1}$
Adaptive expectations CGL (M3)	$\pi^k_{t+1\|t} = \pi^k_{t-1\|t-2} + \vartheta(\pi_{t-1} - \pi^k_{t-1\|t-2})$
Adaptive expectations DGL (M4)	$\pi^k_{t+1\|t} = \pi^k_{t-1\|t-2} + \frac{\iota}{t}(\pi_{t-1} - \pi^k_{t-1\|t-2})$
Trend extrapolation (M5)	$\pi^k_{t+1\|t} = \tau_0 + \pi_{t-1} + \tau_1\,(\pi_{t-1} - \pi_{t-2})\,;\ \tau_1 \geq 0$
General model (M6)	$\pi^k_{t+1\|t} = \alpha_0 + \alpha_1 \pi_{t-1} + \alpha_2 y_{t-1} + \alpha_3 y_{t-1} + \alpha_4 i_{t-1}$
Recursive - lagged inflation (M7)	$\pi^k_{t+1\|t} = \phi_{0,t-1} + \phi_{1,t-1}\pi_{t-1}$
Recursive - lagged output gap (M8)	$\pi^k_{t+1\|t} = \phi_{0,t-1} + \phi_{1,t-1} y_{t-1}$
Recursive - trend extrapolation (M9)	$\pi^k_{t+1\|t} = \phi_{0,t-1} + \pi_{t-1} + \phi_{1,t-1}\,(\pi_{t-1} - \pi_{t-2})$
Recursive - AR(1) process (M10)	$\pi^k_{t+1\|t} = \phi_{0,t-1} + \phi_{1,t-1}\pi^k_{t\|t-1}$
Lagged output gap (M11)	$\pi^k_{t+1\|t} = \phi_0 + \phi_1 y_{t-1}$
Lagged inflation (M12)	$\pi^k_{t+1\|t} = \phi_0 + \phi_1 \pi_{t-1}$

We will attempt to group our subjects into categories based on the similarity of their expectation formation to those that would have been created by these learning rules. Specifically we focus on recursive rules, like our own earlier stated operating law of motion, and the models M7 and M8 above, which rely on only past inflation rate values and only past output gap values respectively. In the case of our experiment, however, our subjects had to predict both the inflation rate and output gap, thus we have more options for what dataset is used to predict which future variable. To be precise, we will try to reconcile our results with three different recursive rules operating on different sets of data:

$$\phi_{OLR}\begin{cases} x^e_t = f(x_{t-1}, \pi_{t-1}) \\ \pi^e_t = f(x_{t-1}, \pi_{t-1}) \end{cases} \qquad \phi_{x,\pi}\begin{cases} x^e_t = f(x_{t-1}) \\ \pi^e_t = f(\pi_{t-1}) \end{cases} \qquad \phi_{\pi,x}\begin{cases} x^e_t = f(\pi_{t-1}) \\ \pi^e_t = f(x_{t-1}) \end{cases}$$

We will compare the results to these rules by visual comparison, as well as by comparing the autocorrelation coefficients of each subject's predictions. As a baseline, Figure ?? shows the paths and autocorrelations for the three simulation learning rules. The following Figures 4.2 and 4.3 show each of the subject's expected output and inflation paths respectively. The 27 subjects have been separated according to their autocorrelation coefficient's similarity to our baseline learning rules. If the subject's autocorrelations were dissimilar entirely to the posited learning rules, they were placed in a fourth category.

4.3 Conclusions

After this experiment, a couple of facts are immediately clear. First, the absolute error of the human subjects predictions is much larger in magnitude than the error of any of the proposed learning rules if $\gamma = 0.01$, as it was in our earlier numerical analysis. This is clear from Table 4.2; to even see the contours of the rules' paths, the scale has to be so reduced that the predictions of the subjects seem like random noise. However, when γ is increased, the rules get closer in volatility to the predictions pf the subjects. For $\phi_{x,\pi}$, letting $\gamma = 0.25$ makes it so volatile that the system soon exhibits nonsensical values of arbitrary size. This is consistent with a phenomenon observed in Marcet and Sargent (1989): when predicitons are based on lagged endogenous information, an extreme stochastic shock can destabilize the system. Second, many of the subjects could loosely be categorized by their autocorrelation coefficients to fit the predicitive pattern of one of the three learning rules. However, many of the subjects had autocorrelation coefficients lower than 0.5 for both predictions of the output gap and the inflation rate.

Table 4.2: Behaviors of ϕ_{OLR}, $\phi_{x,\pi}$ and $\phi_{\pi,x}$ over 60 periods with shocks identical to those faced by experiment participants. The black, blue and red lines are the simulated behavior of the rules with gain parameter γ (as discussed in Section 3.1) inhabiting the values 0.01, 0.1 and 0.25 respectively. The yellow lines are the paths of all 27 subjects. The autocorrelation coefficeint of each path is shown as well, and is indicative of each rule's pattern. The Middle rule does not show the path for $\gamma = 0.25$ (red) because for that value, both output gap and inflation rate predictions became arbitrarily large.

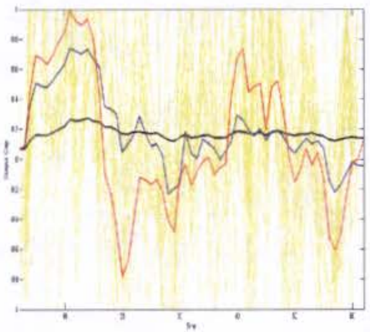
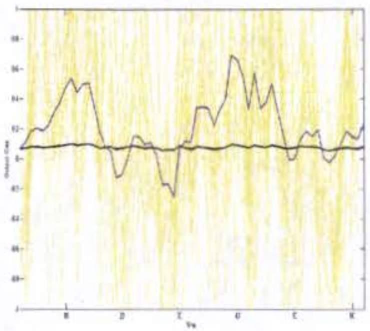
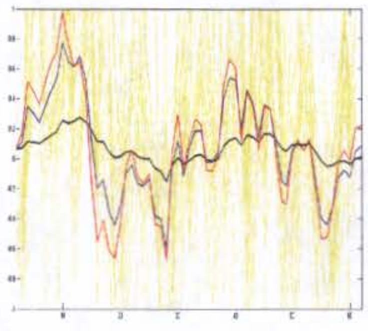
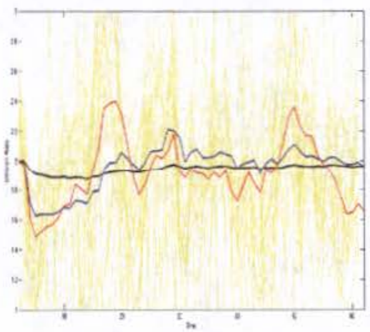
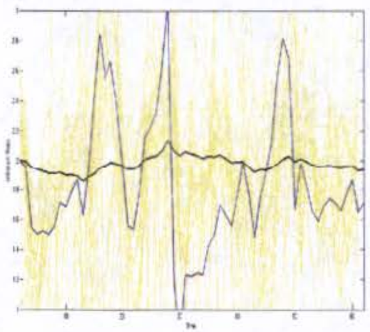
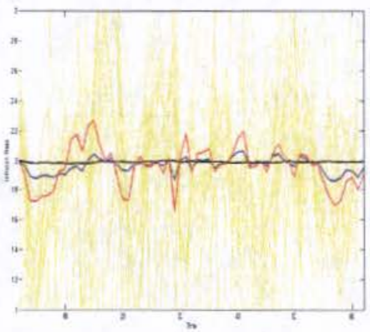
Variable	ϕ_{OLR}	$\phi_{x,\pi}$	$\phi_{\pi,x}$
Output Gap			
Autocorrelation Coefficient	0.9142 ($\gamma = 0.01$) 0.9068 ($\gamma = 0.1$) 0.9051 ($\gamma = 0.25$)	0.6047 ($\gamma = 0.01$) 0.8335 ($\gamma = 0.1$)	0.9023 ($\gamma = 0.01$) 0.8319 ($\gamma = 0.1$) 0.7992 ($\gamma = 0.25$)
Inflation Rate			
Autocorrelation Coefficient	0.9231 ($\gamma = 0.01$) 0.9109 ($\gamma = 0.1$) 0.8680 ($\gamma = 0.25$)	0.8889 ($\gamma = 0.01$) 0.6907 ($\gamma = 0.1$)	0.6326 ($\gamma = 0.01$) 0.5787 ($\gamma = 0.1$) 0.6029 ($\gamma = 0.25$)

Figure 4.2: Subjects' output gap expectations, grouped according to similarities in autocorrelation coefficients to ϕ_{OLR}, $\phi_{x,\pi}$ and $\phi_{\pi,x}$.

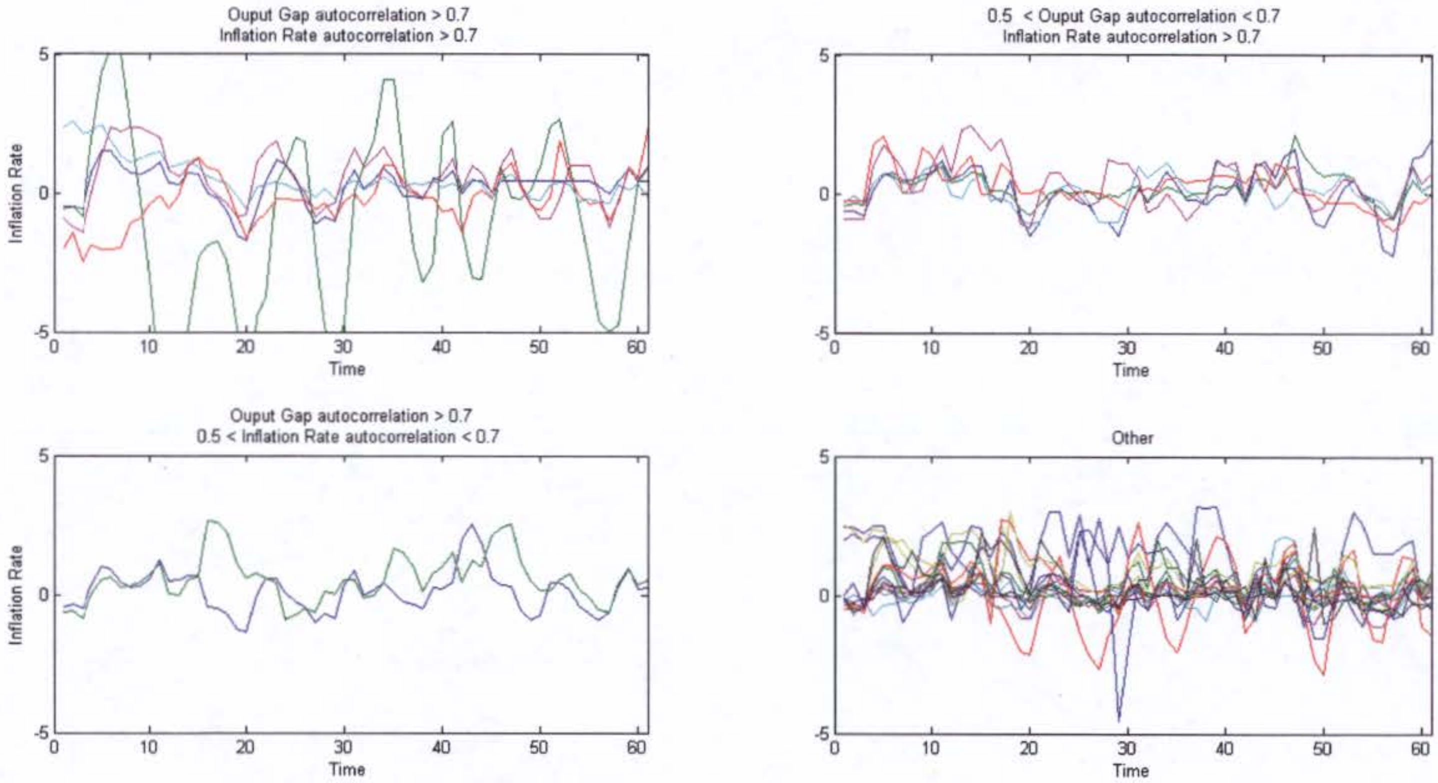


Figure 4.3: Subjects' inflation rate expectations, grouped according to similarities in autocorrelation coefficients to ϕ_{OLR}, $\phi_{x,\pi}$ and $\phi_{\pi,x}$.

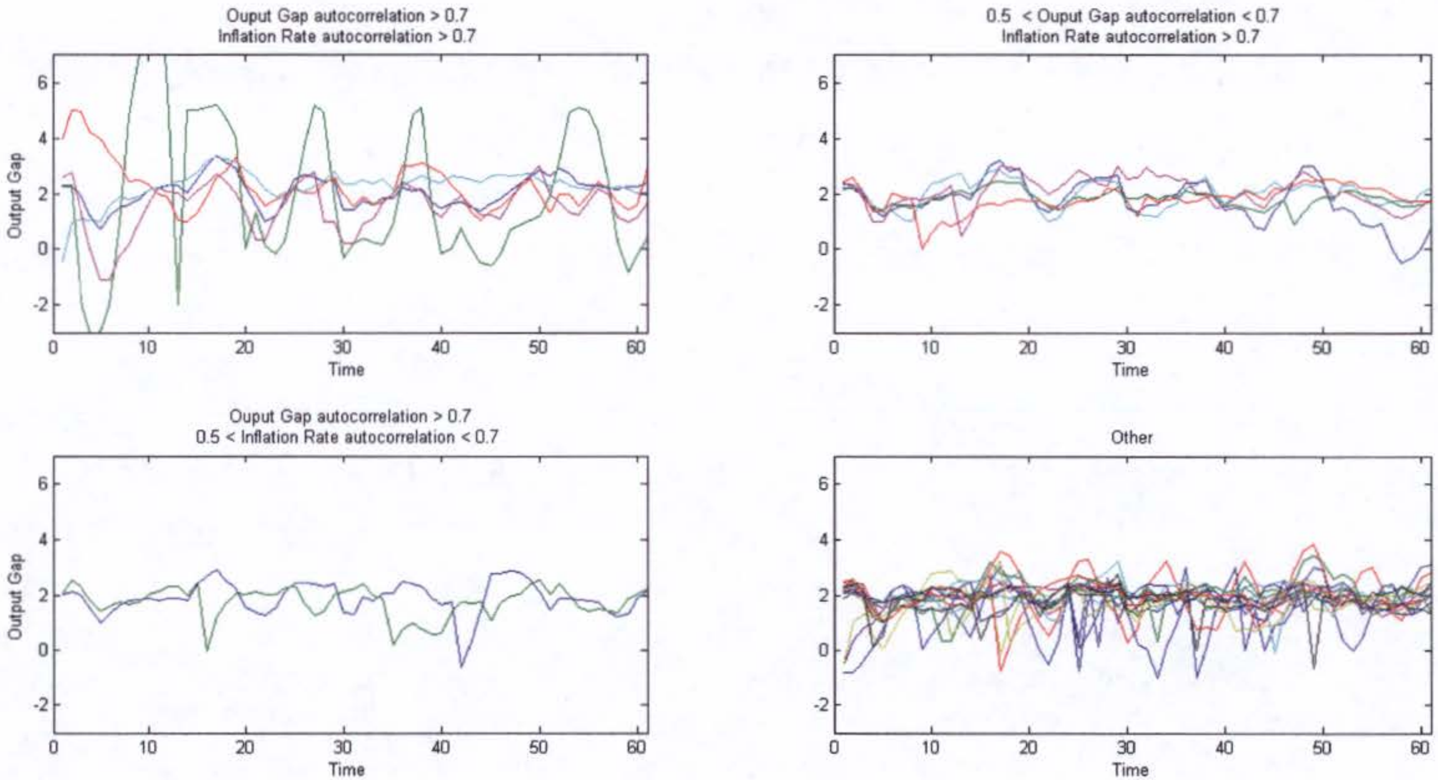
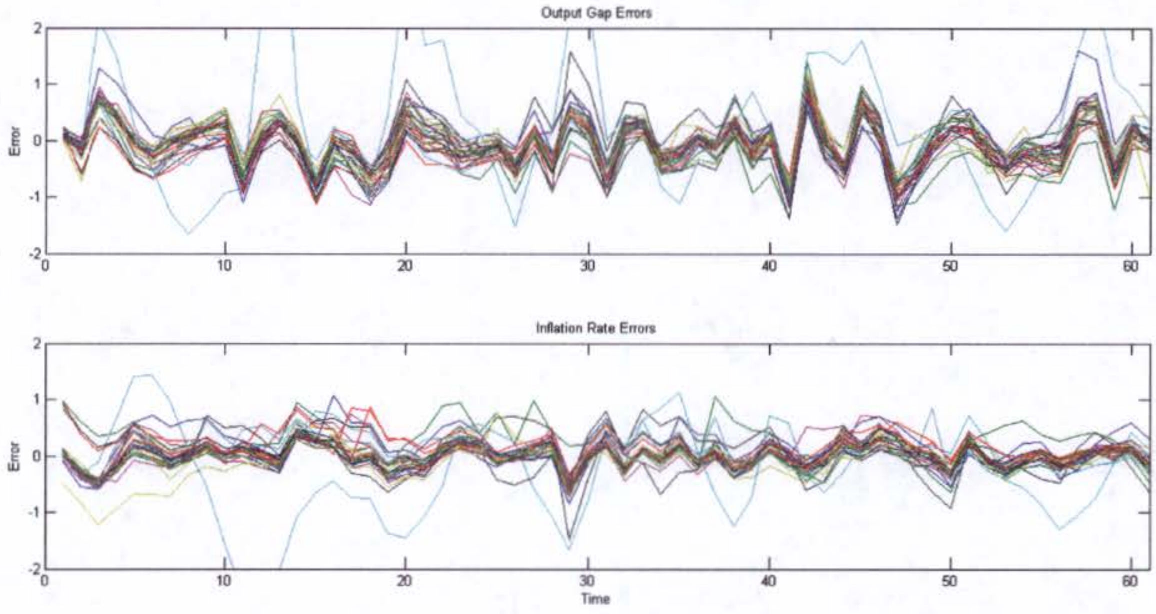


Figure 4.4: The errors of each subject for every period. The top graph shows the errors for the Output Gap and the bottom shows the errors for the Inflation Rate. While there are outliers, most subjects' errors share a tight band.



None of the three rules suggested, which together cover all possible recursive uses of the information available on the two variables of the output gap and inflation rate, produce an autocorrelation coefficient lower than 0.5. All of these rules were quite accurate in their predictions, so it seems a resonable conclusion that using past expectations to inform future ones (which is the cause of autocorrelation) is a "good" strategy for having accurate predictions. The rules come even closer to the subject's predictive behavior when γ is in the range of 0.1 to 0.25. However, many of the subjects' autocorrelation coefficients were below 0.5, even some as low as 0.2. Despite this difference, Figure 4.4 shows that the errors were for the most part tightly clustered, and that subjects performed comparably relative to those subjects whose predictions do fit a recursive rule. This not only implies that there are many predicitve rules that agents can use, but also that those rules can all be similarly effective.

Chapter 5

Future Work

The future of this line of experimentation lies in coordinating the experiment to have every agent be a part of the same simulated economy. This would give each individual subject a more realistic economy, where they are not the only all-powerful predictive agent. It would also allow the researchers to examine what interaction and coordination occurs among subjects, even when they do not conciously know that there are others in the same economy. We are close to having the software infrastructure neccesary to be able to perform this higher complexity experiment. Even beyond this goal, further research could examine the effects of giving the forecasted values of a random participant in a multi-subject economic simulation disproportionate weight. This would give insight into the power that influential institutions or people have when they make economic forecasts.

On the mathematical dimension, more research can be done on the behavior of the model under different learning rules, and with different parameter values. With more data, optimization techniques could be employed to estimate the coefficients of a general learning rule that does not rely on recursively updating coefficients. This rule might help describe the actions of those subjects who did not fit one of our proposed rescursive rules. Unchanging coefficients would be consistent with lower autocorrelation.

Chapter 6

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Chapter 7

Appendix

7.1 Compensation Schedule

This compensation method adapted from Hommes, Sonnemans, Tuinstra and van de Velden (2005)

Subjects will receive a base compensation of $5 for arriving to participate in the experiment. Average earnings per participant should be near $15 per session, including the $5 "show-up" fee. This compensation schedule is similar to the asset pricing model experiments conducted by Cars Hommes, with the difference being that the error term used in the below computation will be chosen randomly for each participant at the end of the experiment. This formula uses a points-based system, where points have a fixed exchange rate for dollars at the end of the game. During preliminary trials we found that subjects were much better at predicting the inflation rate than the output gap, so we adjusted the exchange rates so that average performance would be rewarded equally for whichever variable is chosen. The exchange rates, for the output gap and inflation rate respectively, are:

$$\rho_x = 5200 \text{ points / dollar}$$

$$\rho_\pi = 8350 \text{ points / dollar}$$

The formula for earning points is displayed below. Let v_i be the variable, either the output gap (x) or the inflation rate (π), randomly chosen for subject i. Let v_{it} be exactly equal to that variable in period t, and v_{it}^e be subject i's prediction for that varirable in period t.

$$e_{it} = \max\{2000 - \frac{2000}{0.156}[(v_{it} - v_{it}^e)]^2, 0\}$$

Where e_{it} is the point earnings in period t of subject i. Total earnings for subject i in dollar terms can be calculated by:

$$5 + \frac{\sum_{t=0}^{T}(e_{it})}{\rho_i}$$

Where T is the total number of periods, and ρ_i is the appropriate exchange rate. In the case of this experiment, $T = 60$.

7.2 Participant Packet

Experiment Set-Up

You have volunteered to participate in an experiment of economic decision making. Please read the following pages of instructions before beginning.

- All of the experiment will be conducted on this computer. At the end of the experiment, you will be asked to answer some questions about how it went.
- If you have a calculator you may use it. You do *NOT* need a calculator to participate.
- If you have a question during the experiment, raise your hand. A researcher will come to assist you.

Information about your role

In this experiment, you are a statistical researcher. You are going to make predictions about the **output gap** and **inflation rate** of the fictional in-game economy. You are one of many researchers, and will make predictions about *both* the output gap and inflation rate for each of 60 periods. You need no real statistical knowledge to make these predictions, but you are welcome to use any knowledge you do have. The amount of your compensation, dispensed in the form of Ursinus bookstore credit, will be directly related to how accurate your predictions are.

Information about the Economy

The experiment's economy is described by three variables:

- the inflation rate π_t
- the output gap x_t
- the interest rate i_t

The inflation rate measures the percentage change in the price level of the economy. In each period, the *inflation rate* depends on the inflation rate and output gap *predictions* of all agents in the economy (including those made by yourself and some other statistical researchers) and on random *minor price shocks*. There is a *positive relationship* between the actual inflation rate and *BOTH* the inflation predictions and output predictions of the statistical researchers. This means, for example, that if all other factors are the same, an increase in the *predictions of the inflation rate* will cause an increase in the *real inflation rate*. The *minor price shocks* have an equal chance of affecting inflation positively or negatively.

The output gap measures the percent difference between the *Gross Domestic Product(GDP)* and the *natural GDP*. The *GDP* is the value of all goods produced during a period in the economy. The *natural GDP* is the value the total production would have been if prices in the economy would be fully employed. If the *output gap* is positive (negative), the economy has produced more (less) than the *natural GDP*. In each period, the output gap depends on the *inflation predictions* and *output gap predictions* of the statistical researchers, on the *interest rate* and on *minor economic shocks*. There is a *positive relationship* between the output gap and the inflation and output gap predictions. There is a *negative relationship* between the output gap and the interest rate. The *minor economic shocks* have an equal chance of positively or negatively affecting the output gap.

The interest rate measures the price of borrowing money and is determined by the central bank. There is a *positive relationship* between the interest rate and the inflation rate.

The subscripts t are representative of the period of the variable; i.e., π_6 would indicate the inflation rate in period 6. t will go from 1 to 60 as the experiment proceeds.

Your predictions

In this experiment you will be asked to predict *BOTH* the output gap and inflation rate in each period. When the experiment starts you will be asked to predict inflation rate and output gap, i.e. x_1^e and π_1^e, for the next period based on a short list of the preceding 5 periods (periods -4 to 0) inflation rates and output gaps. (The superscript e denotes that these are predictions.) When you and other participating researchers have entered both predictions, the actual output gap and inflation rate for period 1 will be published and viewable on your screen. You will then make your predictions for the next period, x_2^e and π_2^e, with access to this new information. This process will continue for 60 periods.

The historical range of inflation is between -5% and 15%.

The historical range of the output gap is between -5% and 5%.

About your compensation

You will earn compensation proportional to the accuracy of your predictions. Just for participating you will begin with a "show-up fee" of $5. At the end of the experiment, the computer will randomly choose either the inflation rate or the output gap. You will be compensated for your accuracy in relation to predicting that variable. To be clear, nobody will know which variable determines your compensation until after the conclusion of the experiment, so it is in your best interest to be accurate in your predictions of *both* the output gap and the inflation rate.

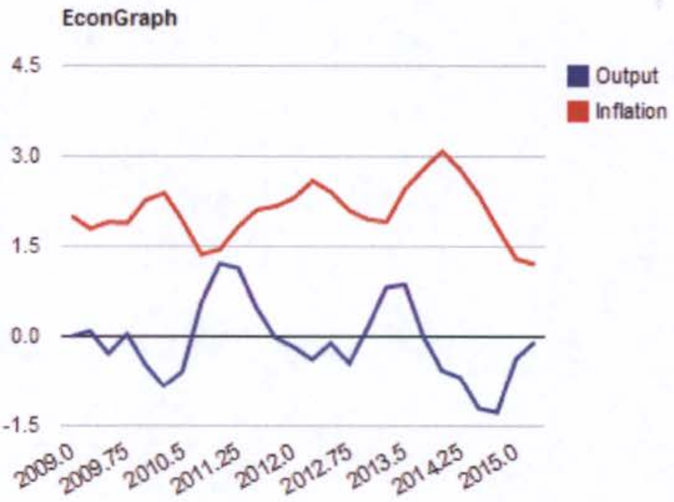
When the computer randomly makes a decision, it will use your predictions from the experiment to find your earnings e_t, for each period t. The sum of all e_t plus your \$5 show up fee is your total compensation. e_t is calculated an error formula, but the below table shows some examples of earning based on accuracy:

Average Error	Total Earnings for Output	Total Earnings for inflation
0.01	\$23.06	\$14.36
0.025	\$22.98	\$14.31
0.05	\$22.71	\$14.14
0.075	\$22.24	\$13.85
0.1	\$21.60	\$13.45
0.125	\$20.77	\$12.93
0.15	\$19.75	\$12.30
0.175	\$18.55	\$11.55
0.2	\$17.16	\$10.69
0.225	\$15.59	\$9.71
0.25	\$13.83	\$8.61
0.275	\$11.89	\$7.40
0.3	\$9.76	\$6.08
0.325	\$7.45	\$4.64
0.35	\$4.96	\$3.09
0.375	\$2.27	\$1.42
0.4	\$0.00	\$0.00

It is important to note that earnings drop off quickly as accuracy decreases. Each period you have oppurtunity for gain, but for any significant profit your prediction must be within a few tenths of a percentage point of the actual outcome. Predicting the inflation rate has been shown to be easier than predicting output, so in fairness predicting the inflation rate is rewarded less. We expect your average earnings to be in the yellow boxed areas. The schedule ensures that the average earnings of each participant, whether selected to be paid on the basis of output or inflation prediction, will be near \$10. **All payment will be given after the fact, in the form of Ursinus bookstore credit**.

About the Computer Program

Reproduced below is a screenshot of the program you are about to use. It is made up of three parts. In the upper left is a time-series graph of the economy's inflation rate and output gap. The vertical axis is marked in percentages, and the horizontal axis shows the time periods. Notice the next period is always on the right-hand side.



Expected output:

Expected inflation:

Submit

Last quarter comparison		
Field	Your guess	Actual Value
Output	-0.5	-0.091
Inflation	1.467	1.205

Below the graph are input boxes where you will type in your predictions for each period. You **MAY** use decimals to express a prediction between two integer percentages, e.g., "1.3." Simply use a period "." as the decimal point. You **MAY** predict negative values. Use a hyphen "-" as the negative sign, e.g., "-5." You **MAY NOT** enter a decimal value out beyond the thousandths' place. If you do, your prediction will be truncated to the nearest thousandth. (If you enter "4.6789" it will be recorded as "4.678".)

Once you have made a prediction, a table will appear below the input boxes showing your last prediction, and the numerical value of the actual outcomes this period. Use this to guide your next guesses.

7.3 Advertisement Email

CALL FOR VOLUNTEERS FOR ECONOMIC STUDY. COMPENSATION PROVIDED BASED ON PERFORMACE.

Come spend an hour of your time to participate in a study into how people form economic expectations about the future. You will be asked to make predictions about the future of a fictional economy over many periods. The more accurate your predictions, the more you'll be paid! All payment will be given in the form of Ursinus book store gift cards. Contact atgraven@ursinus.edu.