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Mathematical Beauty

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Abstract

"With all the brokenness in the world, is the study of the mathematical aspects of Creation worthwhile?"

Posting about the beauty of mathematics from *In All Things* - an online hub committed to the claim that the life, death, and resurrection of Jesus Christ has implications for the entire world.

http://inallthings.org/mathematical-beauty/

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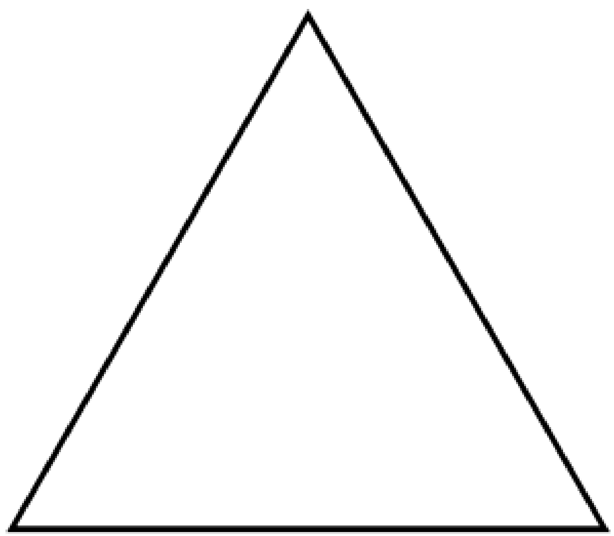
Reformed people can be relentlessly practical. My own discipline, mathematics, is also practically-minded, and is even traditionally divided into two broad areas: applied mathematics (that is, using math to solve problems in other disciplines) and pure mathematics (math for its own sake). In fact, developments in mathematics have been applied in many ways, and have made possible the technologically-inclined, data-driven society in which we live. Still, my heart was stirred to study mathematics not for its practicality or usefulness, but for its *beauty*.

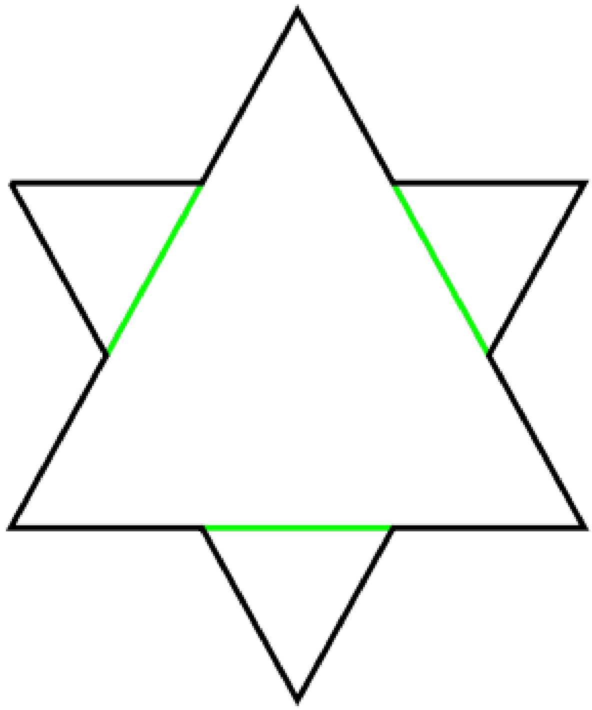
It is my guess that many readers may not have thought about mathematics in these terms. What qualities define beautiful mathematics? There are many which might be familiar from other aesthetically-minded disciplines (e.g., symmetry), but here I want to focus on two in particular: surprise and elegance.

One of the most delightful qualities of pure mathematics is its ability to surprise.

To see what I mean, consider an equilateral triangle with side length 1 and thus a perimeter of 3: [1]

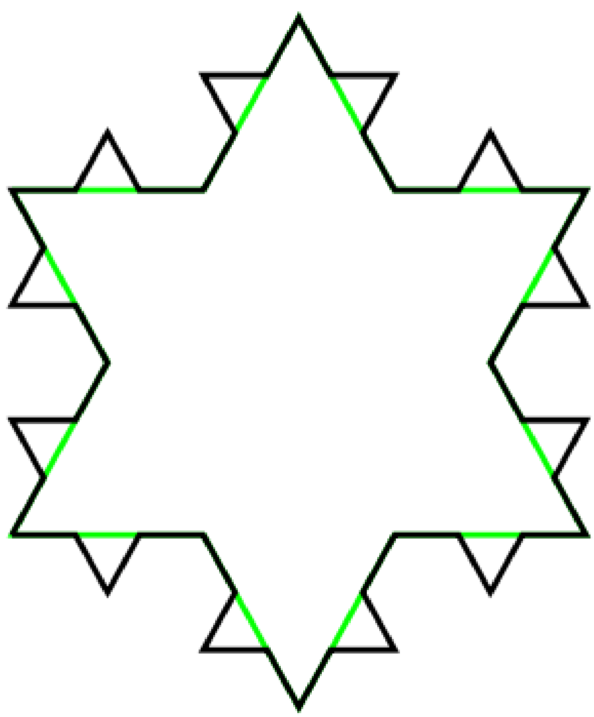


Now cut out the middle one-third of each side and replace each missing portion with two segments of length 1/3 as shown:

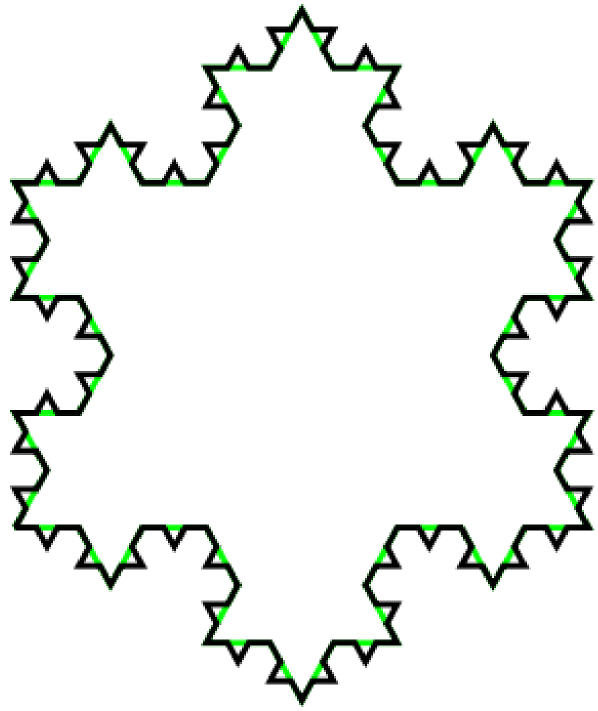


Thus, we have removed one third of each side of the original triangle and replaced the space with segments whose combined length is twice what was removed. We have also increased the perimeter by a factor of 1/3, from 3 to 4.

Do this again:



And again:



And again, and again, *ad infinitum*.

The object we obtain by iterating this process an infinite number of times is called the *Koch snowflake*. What we saw above turns out to be true in general: the perimeter of a given iteration is 4/3 times the perimeter of the previous iteration. Thus, after infinitely many iterations, the Koch snowflake has *infinite* perimeter! However, it encloses a *finite* area! In fact, our Koch snowflake has area $(2\sqrt{3})/5$, or approximately 0.69282. That this is possible should come as a surprise! After all, if I hand you a string of infinite length, you would have a difficult time arranging it so that, when bent appropriately and laid flat, it enclosed an area smaller than the screen on which you are reading this. Yet such things occur in mathematics, and their existence speaks to the deep, transcendent mystery inherent in the mathematical aspects of Creation.

When 20th-century mathematician Paul Erdös saw a particularly beautiful proof, he liked to say that it was from “The Book,” a reference to a book that God kept which contains the most elegant proofs of theorems. An *elegant* proof is one which communicates deep truths with simplicity and brevity. One of the best-known such proofs is Euclid’s proof that there are infinitely many prime integers (recall that a prime is an integer greater than 1 which is only divisible by 1 and itself; examples include 2, 3, 7, 19, and 149). Euclid’s proof runs more or less as follows.

Suppose we have a finite list of primes $p_1, p_2, \ldots, p_n$. Our goal is to find a prime that is not on the list. Write $M = p_1 p_2 \cdots p_n + 1$. Since every integer is either prime or divisible by a prime, and M is an integer, it is either prime or divisible by a prime. If it’s prime, we’ve found a prime not on the list, namely M. If it’s not prime, we know it’s divisible by a prime q, and if q were one of the primes on the list, it would in turn divide 1. Therefore, q is not a prime on the list. In either case, our original list was incomplete, which in turn means that *any* finite list of primes is incomplete. Thus, there are infinitely many primes.

Euclid proved this theorem around 300 BC, and his simple, elegant proof is as true and beautiful today as it was then.

These examples, coming as they do from pure mathematics, should not suggest to the reader that applied mathematics contains no beauty at all. In fact, the “unreasonable effectiveness”—to borrow a term from Eugene

Wigner"—of mathematics in describing the created order suggests a transcendent beauty all its own. Moreover, many ideas which began as pieces of pure mathematics turn out eventually to be useful (see, for example, ideas from group theory, originally concerned with the structure of solutions of polynomial equations, which eventually found use in quantum physics).

Questions I am fond of asking my students are: "With all the brokenness in the world, is the study of the mathematical aspects of Creation worthwhile? Are we right to pursue the beauty of pure mathematics, or should we focus our energy on studying 'practical' mathematical concepts which have immediate, obvious application to solving the world's pressing problems?" My answer, as you may have guessed, is a resounding *yes!* The awe inspired by the transcendent mysteries of mathematics and the delight which an elegant proof engenders in its readers point, in all cases, to the Creator of the universe.

Wolterstorff says it this way: "Aesthetic delight is a component within and a species of that joy which belongs to the *shalom* God has ordained as the goal of human existence and which here already, in this broken and fallen world of ours, is to be sought and experienced."[2] In other words, the surprise and delight felt at the sight of beautiful mathematics is a foretaste of the joy to come. I encourage you to pursue such beauty every day. Even in math class.

Footnotes

1. images adapted from https://commons.wikimedia.org/wiki/File:KochFlake.svg ↩
2. *Art in Action*: *Toward a Christian Aesthetic* (Grand Rapids: Eerdmans, 1980), 169.) ↩