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# Mathematics: Giving Classical, Christian Education Its Voice 

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# Mathematics: Giving Classical, Christian Education Its Voice 


#### Abstract

Classical, Christian education developed in the late twentieth century as the result of the influence of authors and educators such as Dorothy Sayers, Douglas Wilson, and Mortimer Adler. Building upon the educational approach taken in the Middle Ages and earlier, the classical, Christian approach has slowly grown in popularity over the past thirty years. As classical, Christian education has matured, however, some areas of its educational philosophy have developed more slowly than others. In particular, mathematics education within the classical, Christian model has received minimal treatment. This thesis attempts to initiate a more intentional educational philosophy for mathematics in a classical, Christian context. To accomplish this goal, it starts with a review of the history of classical education in the Middle Ages and continues by examining some of the approaches within contemporary classical, Christian education. Then, the thesis surveys the educational philosophy of mathematics from a non-classical, Christian context in order to gain ideas that can be used to begin building a philosophy of education for mathematics in a classical, Christian context. The thesis concludes by proposing some features to be adopted by mathematics education in a classical, Christian educational setting.


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Thesis
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by

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# Mathematics: Giving Classical, Christian Education Its Voice iv 


#### Abstract

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In the late 1980s, author James Nickel produced a provocatively-titled work called "Mathematics: Is God Silent?" Nickel contended that while many in the Christian educational community saw mathematics as something theologically neutral, it was in fact laden with God's character and attributes.

A similar question could be asked today regarding a particular model within the broader Christian schooling framework: the classical, Christian education (CCE) model—Mathematics: is classical, Christian education silent? For example, the ACCS bi-monthly newsletter, The Classis, which provides articles of interest to classical, Christian educators, consistently has articles that describe teaching Latin, English, art, music, and other humanities subjects from a classical and Christian viewpoint. In that time span, only one article discusses mathematics from a classical, Christian perspective, and that article deals not with educational philosophy but a general philosophy of mathematics that is impractical for the classroom. Only since 2010 has any group (Novare, based out of Regents School in Austin, TX) in the CCE community done significant work in the area of mathematics and science education.

## Problem Statement

A significant gap exists in the CCE literature regarding the philosophical basis for mathematics education within the CCE model. Authors such as Douglas Wilson began articulating the CCE framework based upon Dorothy Sayers's short essay, and educators such as Littlejohn and Evans have clarified many aspects of this educational philosophy. No one in the movement questions that mathematics should be taught. The philosophical basis for mathematics instruction, however, remains mostly untouched. CCE needs a philosophy of mathematics education.

To develop this philosophy, we will first examine the history of classical education in the

Middle Ages. Then we will briefly sketch the changes in the educational system that led some to feel the need for a classical education revival. Next, this thesis will present and analyze the modern CCE model in terms of its overall approach as well as its relevance to mathematics. In this context we will briefly assess the CCE model as a vehicle for Christian education. A full discussion of its validity, however, lies beyond the scope of this thesis. Instead, its validity will generally be assumed, although individual aspects of various CCE approaches may be looked at more closely. Since CCE theory is mostly silent on the topic of mathematics instruction, we will turn to examine some of the key ideas in non-classical mathematics education in the Christian schools in order to gain ideas for improving the CCE approach to mathematics. Finally, this thesis will pull together the various concepts and ideas from the classical and non-classical approaches to develop more fully the place of mathematics within the CCE model.

## Research Questions

The major question this thesis seeks to answer is, "What are some of the important components for a philosophy of mathematics education in the CCE model?" Important secondary questions include, "How was mathematics taught in classical times?" and "How is mathematics taught in a Christian manner?"

## Definition of Terms

Trivium - Classically, the trivium consisted of grammar, logic, and rhetoric. As we will see below, in the modern classical, Christian schools, the trivium may be seen as a set of subjects, a guide for the intellectual development of children, and/or a general guide to pedagogy. Quadrivium - Classically, the quadrivium consisted of arithmetic, geometry, music, and astronomy. Some modern classical, Christian theorists use the quadrivium in conjunction with the trivium in their educational philosophies.

Seven Liberal Arts - The trivium and quadrivium grouped together.
Classical Education (CE) - During the Middle Ages, the type of education that served to make free men into better leaders and that prepared them for further study, consisting ultimately of the seven liberal arts

Classical, Christian Education (CCE) - Any of several various approaches to Christian education that builds upon an interpretation of medieval education. The best known method derives from the theories of English novelist Dorothy Sayers and American pastor Douglas Wilson, although others exist.

## Literature Review

## Classical Education in Antiquity and the Middle Ages

In order to understand how mathematics fits into the modern CCE movement, we must first understand how mathematics fit into medieval classical education. Using history as a guide, it may then be possible to develop a coherent educational philosophy for mathematics within a CCE model.

The roots of today's classical, Christian education movement lie in the educational structures of medieval Europe. While space does not permit a full discussion of the history of classical education, a brief overview is warranted. ${ }^{1}$

Education in the Middle Ages was built upon two sets of related subject groupings: the trivium and the quadrivium (Hart, 2006, p. 47). The trivium consisted of three language-focused areas of study: grammar, logic, and rhetoric. The quadrivium, on the other hand, emphasized mathematics. Arithmetic (number theory), geometry, music, and astronomy comprised the

[^0]quadrivium. Medieval scholars grouped these seven areas of study together, calling them the seven liberal arts. These subjects, in one form or another, have survived until this day in education, albeit as just some among many.

The students who received a liberal arts education varied somewhat throughout the Middle Ages. Initially, education served the upper classes (who at the time would be future leaders) and those destined for the priesthood (Littlejohn \& Evans, 2006, p. 29). Eventually, a liberal arts education became a pre-requisite for students wishing to enter university (Hart, 2007, p. 41). As the Renaissance began, and in later centuries as universal education gained in popularity, many students continued to study according to the classical model, although at no time were all students trained classically. As modern approaches gradually gained dominance, they modified and often replaced the classical approach, as will be discussed below (Veith, 1997, p. 50). Thus, a liberal arts education focused primarily on educating society's political and spiritual leaders, which at this time came from the upper class, those people whose children had the leisure time needed for study.

The classical education system of the Middle Ages developed out of the earlier Greek and later Roman approach to education (Marrou, 1956, p. 177). The Ancient Greek schools focused on grammar, rhetoric, and logic (in that order), as well as geometry and music, in educating future leaders (Hart, 2006, p. 15, 16). Arithmetic and astronomy as we now think of them were present to varying degrees, although they were often combined with geometry and called by that name. Likewise, the Romans trained future leaders in the trivium (with rhetoric being the ultimate goal), with aspects of the quadrivium sometimes receiving emphasis.

Classical education, however, taught more than mere facts. It also had a strong moral component (Marrou, 1956, p. 221). From a young age students learned to be upright members of
society. In ancient times, this occurred under the influence of the pedagogue, the slave assigned the task of ensuring the child's education. In modern terms, classical education aimed not only to shape the minds of students; it also aimed to shape their worldview.

After the Fall of Rome, the educational system gradually changed. The trivium and quadrivium eventually gained broad acceptance during the fifth and sixth centuries through work of men such as Capella and Cassiodorus (Hart, 2006, p. 39). Capella, in his $5^{\text {th }}$ century work The Marriage of Philology and Mercury, codified the disciplines now identified as the trivium and quadrivium. His contemporary, Cassiodorus, took these academic areas and developed a "Christian" approach to them, building a curriculum for monks based on these principles.

In the system of Capella and Cassiodorus, education consisted of three language-centered arts-the trivium or three ways-and four mathematics-centered arts-the quadrivium or four ways (Hart, 2006, p. 39). Grammar in medieval times meant study of the Latin and Greek languages, both grammar and content. It was the study of Latin, especially, that freed the student to study on his own, since Latin was the language of scholars in medieval Europe. Logic (or Dialectic, as it was also called) included Aristotelian logic and other forms of reasoning. Rhetoric focused on the eloquent presentation of concepts and ideas. The art of persuasion, in particular, received great emphasis. In most medieval schools, rhetoric served as the ultimate art in the trivium. Sometimes, however, Logic formed the capstone, depending on the needs of the particular culture.

In contrast to the language emphasis of the trivium, the quadrivium focused on the mathematical arts: two theoretical (or pure mathematical) arts and two applied (or mixed) arts (Wagner, 1983, p. 1). While the theoretical/applied distinction is not airtight, it serves to describe the general approach to mathematics taken by each art.

Arithmetic, the first of the theoretical arts, consisted primarily of number theory with secondary discussions of computation. Building upon the tradition of the Pythagoreans, who worshiped the concept of number, medieval scholars wrote numerous books dealing with how numbers related to each other. Often, the discussion also included a detailed section on the mystical (or Scriptural, depending on the worldview of the author) meanings of numbers (Abelson, 1939, p. 96). As the work of later Arabic mathematicians found its way into Europe, the mystical approaches to numbers began to be supplemented with more detailed treatments of number theory and computation, largely thanks to the introduction of Hindu-Arabic numerals along with their associated computational procedures (Abelson, 1939, p. 103).

The second theoretical art, geometry, dealt with shapes and figures. This art could as easily have been called the Elements, for Euclid's work, either in pieces or later in complete form, provided nearly all the content (Abelson, 1939, p. 116). At times, geometry also included practical aspects like surveying and optics (Shelby, 1983, p. 200).

Astronomy, one of two applied mathematical arts, focused on understanding the movement of the stars and planets. This study, obviously, required a solid understanding of mathematics. Of particular importance to medieval scholars was the calculation of the moveable date of Easter, a feat that required a thorough knowledge of arithmetic (especially computation) and the movements of the heavenly bodies (Abelson, 1939, p. 120). Calendar systems and their study also occupied a central place within astronomy (Kren, 1983, p. 231).

The inclusion of music as the fourth mathematical art may surprise modern readers. To the medieval way of thinking, however, music was as much about numbers as it was about producing sounds. According to Cassiodorus, "Music is the discipline which treats of numbers in their relation to those things which are found in sounds" (as cited in Karp, 1983, p. 174). Music
in medieval times focused more on music theory and numerical relationships among different sounds and chords than it did on performance of musical works. As a result, a musician meant not necessarily a skilled performer as much as it meant a student with a thorough knowledge of the numerical properties that underlay music (Abelson, 1939, p. 128).

The selection of these seven areas from among many others might seem arbitrary to a modern educator. Educators in the Middle Ages, however, believed that these seven "arts" had unique characteristics to them. Hugh of St. Victor, an influential scholar and teacher from the twelfth century, wrote:

Out of all the sciences above named, however, the ancients, in their studies, especially selected seven to be mastered by those who were educated. These seven they considered to so excel all the rest in usefulness that anyone who had been thoroughly schooled in them might afterward come to knowledge of the others by his own inquiry and effort rather than by listening to a teacher. For these, one might say, constitute the best instruments, the best rudiments, by which the way is prepared for the mind's complete knowledge of philosophic truth (Hugh, 1120/1961, pp. 86-87).

Two important concepts concerning classical education emerge from Hugh's explanation of the seven liberal arts. First, one goal of classical education was the preparation of students who would go on to investigate other areas of study and discover new concepts and ideas. Contrary to what many believe, classical education in the Middle Ages aimed to encourage students to advance in their knowledge of their specialized content areas. This advancement, while primarily confined to learning what had come before, did lead to improved knowledge in specific content areas. Progress occurred slowly, frequently by individuals supported by wealthy patrons, but those who were classically educated did improve the state of their fields. Admittedly,
investigation was not the primary focus of many within medieval times, but nevertheless the results of classical education included a forward-looking component.

Second, another equally-important goal of a liberal arts education, as Hugh noted, was the preparation of the mind to receive philosophic truth. In modern terms, we might say that these seven liberal arts serve as the best preparation for developing both moral propriety and a correct worldview. The development of morality in liberal arts students had its roots in the ancient Greek education with philosophers such as Aristotle, who argued that character development formed a necessary part of a student's education (Burnet, 1913, p. 3). It continued throughout the Middle Ages as schools sought to prepare leaders who could lead well (Littlejohn \& Evans, 2006, p. 29). Worldview formation, although not explicitly identified as such, also occurred as the result of a combination of the moral training in the schools combined with the influence of the Church, as the Roman Catholic church also ran the schools. The Catholic church, in turn, had "no great difficulty about creating a religious education with the Bible at its centre. The Christians had something of the sort under their very eyes, in the Jewish schools..." (Marrou, 1956, p. 316). Thus, moral/worldview formation naturally grew out of the classical and biblical points of view.

As the Middle Ages progressed, the educational system also evolved. Universities developed from the cathedral schools, initially to teach the trivium and quadrivium to students, but later to teach the quadrivium to students already educated in the trivium (Hart, 2006, p. 41). Throughout the High and Late Middle Ages, the seven liberal arts served as prerequisite general studies prior to specialization at the university in either law, medicine, theology, or philosophy (Hart, 2006, p. 49). Thus, the trivium and quadrivium functioned like primary and secondary education in the modern schools, while the specialized training in some ways paralleled the
modern goal of a university education. This comparison, admittedly, is not exact, given the primary goal of classical education as discussed below. Still, this comparison will aid us as we attempt to formulate a modern classical, Christian approach to mathematics.

Moreover, a liberal education aimed to create model citizens. As Littlejohn and Evans (2006) explained, the word "liberal" in the context of the liberal arts came from the Latin word liber, which meant "free" (p. 29). Liberal arts education, then, focused not on training slaves nor on training craftsman. Rather, a liberal arts education sought to train men "who would be political and cultural leaders in society" (ibid.). Medieval scholars did not consider completing the quadrivium as the pinnacle of achievement. Rather, they desired students to study theology and its cousin philosophy as their ultimate preparation for public service (Hart, 2006, p. 49). Thus, the seven liberal arts as a whole functioned not only to create learners but also to create citizens.

## The Marginalization of Classical Education

Classical learning held its own through the initial philosophical turbulence of the Renaissance. During the $16^{\text {th }}$ century, however, the educational system slowly began to move away from classical education and towards a different system, a change that would occur at an uneven pace and with uneven coverage for several hundred years. Hart (2006) details how the educational system shifted away from its classical roots.

In the early seventeenth century Francis Bacon proposed seminal ideas that became part of the foundation of modern science. In particular, Bacon promoted learning that focused on discovering new knowledge about the natural world by studying it rather than reading what past thinkers had said about it (Hart, 2006, p. 59). As the Enlightenment took hold in Europe a century later, several factors-Bacon's suggestions about education, the Enlightenment's
emphasis on reason, and the enthusiasm caused by successful scientific investigations-slowly led to new scientific discoveries becoming the most important goal of education. Classical education, with its emphasis on the best of the past, found itself increasingly out of step with the educational culture.

By the middle of the nineteenth century, modern research institutions developed, and the liberal arts relinquished their place as the cornerstone of education, becoming instead just additional subjects (Jongsma, 2003, p. 2). The move was not entirely without merit. Classical education, which had increasingly operated in a defensive mode, had become too narrow and backwards-focused. Preservation of the intellectual accomplishments of the past became classical education's rallying cry, a concept that fit well within the humanities but made little sense in the rapidly-expanding fields of mathematics and natural science.

Even as the educational culture drifted from classical ideas, so did educational philosophy. John Locke ( $17^{\text {th }}$ century) and Jean-Jacques Rousseau ( $18^{\text {th }}$ century) took the changes encouraged by Bacon a step farther, urging the creation of an entirely secular moral education (Hart, 2006, p. 61). Both viewed the child as a tabla rasa, denying the Christian doctrine of original sin. In so doing, they contradicted what had become the classical view of the child in the Middle Ages, which was based upon the principle that the child was a sinner in need of moral training through instruction by the Church in the Scriptures. Moreover, Rousseau's approach, which centered on the desires of the child, stood in contrast to a classical approach that emphasized giving the student not necessarily what he wanted but on training the student to love what was best for his development (an Aristotelian approach). These two philosophers of education set the stage for the final $20^{\text {th }}$ century attacks on the remnants of classical education.

In the early twentieth century, the move of modern education away from anything
classical continued with William James and the philosophy of pragmatism (Hart, 2006, pp.6366). Pragmatism emphasized truth as that which worked, compared to classical education's view of truth as that which conformed to idealized forms (the view of Ancient Greece) or to God's revealed will (the medieval viewpoint). As a result of this practical emphasis, usefulness began to be the most important aspect of education. Classical education, with its goal of creating civilized gentlemen (Jongsma, 2003, p. 24) was at odds with pragmatism, which valued skills that could be used directly in employment.

Pragmatism received its authoritative entry into education with the work of John Dewey (Hart, 2006, p. 66). Dewey's progressivism took pragmatism and applied it thoroughly to education, combining it with elements of Rousseau's child-centered education. Progressivist reforms ultimately caused the disappearance of the core curriculum, the last vestige of the liberal arts education that was meant to teach a common body of knowledge to all students. Education instead became about giving students what they wanted.

The disappearance of the core curriculum did not happen overnight, however. Initially, universities instituted general education requirements as a replacement of the core curriculum, starting in the 1920s and 1930s (Jongsma, 1994, p. 3). ${ }^{2}$ These requirements harkened back to the liberal arts without decisively mandating that the student take all of them. Starting at around the same time (1918) vocational training also began to receive greater emphasis at the secondary and tertiary levels (Hart, 2006, p. 69), a trend that gradually continued for the next fifty years. Vocational training became increasingly popular in the 1960s and 1970s. By the 1980s, the constant battle between student choice and a core curriculum (and receiving practical training and between cultivating virtue) led to the creation of "distribution requirements," which required

[^1]students to take a range of courses in various areas (Jongsma, 1994, p. 8). With these changes, students no longer learned a common body of knowledge.

Admittedly, these changes occurred gradually, and some places held on to the liberal arts tradition (at least in some form) far longer than others. Overall, though, by the middle of the twentieth century, only extremely isolated remnants of classical education remained (Hart, 2006, p. 57). American education, which had been a mixture of the classical and modern, as well as the sacred and the secular, finally became entirely modern and entirely secular. ${ }^{3}$

As the twentieth century drew to an end, Douglas Wilson, a pastor in Moscow, Idaho, leveled a strong critique of the current state of the American educational system. He observed that the school system had become too pragmatic and too student-driven, and he believed that it had lost not only the ability to teach students anything about truth and morality, but also the capacity to educate students well at all (Wilson, 1996b, p. 19).

From a religious point of view, Wilson argued, the concepts being taught were so antithetical to a Christian worldview that Christian parents could not accept sending their children to public schools. In addition to advocating (intentionally or not) concepts entirely incompatible with Christianity, such as an anti-Christian bias in textbooks and the dismissal of sexual abstinence as a valid behavior, Wilson also argued that schools failed to teach students the skills necessary for students to learn (Wilson, 1996b, p. 36). Students could neither read nor do mathematics well, and they could not make moral/ethical decisions. To make matters worse, students lacked the ability to gain knowledge on their own. Students had lost the "tools of learning" (Wilson, 1996b, p. 80). With the loss of these tools also came the loss of the ability to evaluate critically the concepts presented to them by the world. In other words, modern secular

[^2]education was turning out graduates who were marginally-employable workers with no heart, no moral code, and no work ethic.

As a result of this two-part failure of education, Wilson believed that Christians needed to found Christian schools and that these schools needed to find an alternative pedagogical and curricular focus to avoid the influence of humanism and pragmatism. To do that, he wanted to avoid creating a school that would be little more than a public school with a Bible class tacked on. What Wilson desired was an education that would support Christian parents as they tried to transmit a biblical worldview to their children, while also giving students the ability to learn on their own. Starting from a biblical view of the student and of knowledge, and influenced by the works of educators from Great Books tradition ${ }^{4}$, Wilson searched for a solution that would allow the student to engage in rigorous study while also gaining an understanding of and an appreciation for Western culture. Eventually, Wilson happened across what he believed to be a solution in an essay by Dorothy Sayers.

## Modern Classical, Christian Education

Sayers, an author and philosopher, proposed her ideas in a 1947 essay, The Lost Tools of Learning, read to a Vacation Class at Oxford University (Hart, 2006, p.71). In the essay, she lamented the state of the English school system, which she believed no longer taught children to think but only created them to be pragmatic workers (Sayers, 1947, p. 3). Reaching back into the past, she drew on her knowledge of medieval history and of her own life to propose a change in the school system. Using her own childhood development as a guide, she observed that children go through three stages: poll-parrot, pert, and poet (Sayers, 1947, p. 9).

[^3]Children in the poll-parrot stage, which she felt ran from ages nine to eleven, ${ }^{5}$ loved to memorize and recite facts, knowing something simply for the pleasure of knowing it. They delighted in chants. Children in the pert stage, from ages twelve to fourteen, wanted to dispute with everyone, taking pride in proving others wrong, especially their elders. Finally, children in the poet stage, between approximately the ages of fourteen to sixteen, desired to express themselves above all else. To Sayers' mind, these stages corresponded with the medieval trivium of grammar, logic, and rhetoric. Grammar, by which Sayers meant the basic facts, fit with the tendency of the poll-parrot stage to memorize. In addition to the traditional subjects taught at the elementary level, Sayers gave two reasons why students at the grammar stage also should learn Latin. First, Latin was one of the important root languages of English. Wilson, in commenting on teaching Latin, clarified Sayers' remarks. "About 80 percent of our English vocabulary comes to us from Latin and Greek" (Wilson, 1996b, p. 87). Studying Latin helps students to understand their own language. Second, Latin had served as the academic lingua franca for over a thousand years. Sayers believed that students needed to learn Latin partially so they could read and understand the great works of the past. Logic, discerning the proper connections between facts, corresponded with the natural tendency of the pert stage. Rhetoric, the capacity to express oneself eloquently, fit well with the poet stage.

At the same time, Sayers also saw the trivium as providing the tools of learning. In order "to be educated in any discipline, you must 1) know its basic facts (grammar); 2) be able to reason clearly about it (logic); and 3) apply it personally in an effective way (rhetoric)" (Veith \& Kern, 1997, p. 12). Therefore, the trivium was not merely a set of subjects, nor did it only provide a general description child development; it also was the best method by which students

[^4]learn anything. In other words, the trivium itself comprised the tools of learning that Sayers felt education had lost. As a result, Sayers argued that after age sixteen, students trained in the trivium would be ready for a university education, which she considered to be akin to the quadrivium of medieval times, although she treated the quadrivium quite loosely, ignoring its mathematical nature.

Douglas Wilson, as he searched for an approach to use in starting a Christian school, remembered Sayers' essay. He applied her ideas to Christian schooling in his 1996 book Recovering the Lost Tools of Learning. Writing to parents as his primary audience, Wilson developed Sayers' ideas further. Wilson desired to create a school that would assist parents in raising their children, since he viewed education as ultimately the responsibility of the parents. "Parents should see the work of the Christian school as a supplement to their own teaching..." (Wilson, 1996b, p. 51). For Wilson, this ideal school had to be thoroughly Christian because education not only sprang from a teacher's worldview but also existed to cultivate the correct worldview in students. The concept of worldview emerged from Wilson's own experiences as a pastor in an independent Reformed church. Wilson's ideal school, therefore, emphasized a Christian worldview as its first and most important defining characteristic. In the classical model, which emphasized moral learning as well as academic, Wilson believed that he had found an approach that would work well in a Christian school.

In addition to being entirely Christian in its approach, Wilson desired a school that would help students understand and engage the culture in which they lived. Without being able to engage their culture, students had little hope of reaching it with the Gospel. The best method for understanding American culture, he argued, lay in studying the classics. "An essential part of the classical mind is awareness of, and gratitude for, the heritage of Western civilization" (Wilson,

1996b, p. 83). Western civilization, while having moved away to some degree from its classical heritage, still clung to its classical roots. Engaging the culture required understanding those roots in order to understand what modern society reacted against. Moreover, engaging American culture meant finding ways to think about the culture differently than modern thinkers. In addition, therefore, to studying the Bible, Wilson argued for studying the classics to allow students to see how other people engaged the problems of society. Through thoughtful reading of the classics, students would learn possible ways to deal with contemporary problems (Wilson, 1996b, p. 84). To study only modern writings meant assuming that writers from any previous era could not teach students anything about how to engage their culture, an assumption that Wilson called "suicidal" (Wilson, 1996b, pp. 84-85).

At the same time, studying ancient authors did not mean blindly accepting what they said. Rather, it meant that students should learn from the great minds of the past in order to better understand the present and be prepared for the future. "Conversation with the past is the heart and soul of classical education. But it is important to guard against a mindless veneration of the past" (Wilson, 1996b, p. 85). Therefore, Wilson's ideal school needed to be both Christian and classical—Christian in order to cultivate a correct worldview and classical in order to understand the culture in which students lived.

For Wilson, the application of the classical model had two different dimensions: a preference for primary sources and the use of the trivium in all of its modes to guide education. Concerning school structure, Wilson stated that at his Logos School, the Grammar stage began in Kindergarten and continued until about the fifth grade (age ten) (Wilson, 1996b, p. 92). Students in the Grammar school engaged in significant amounts of memorization, chants, and other activities meant to help them absorb as much information as possible (Wilson, 1996b, p. 92).

Next, students transitioned to the Logic stage for grades six through eight. During the logic stage, students learned how to reason well and to ask hard questions (Wilson, 1996b, p. 95). Finally, students spent their high school years in the Rhetoric stage, during which students fully learned how to express themselves eloquently, clearly, and with proper supporting evidence (ibid.). The ultimate goal was a senior project (or thesis defense), during which the student must present a meaningful project before his peers (Hart, 2006, p. 81).

For Wilson, these stages concerned emphases, not the totality of learning. He wrote, "[T]his does not mean that young children are not to begin the process of writing or expressing themselves in other ways. It simply means that such early attempts should not be treated as though they were the final product" (Wilson, 1996b, p. 96). Therefore, although grammar students focused on memorizing material, their education was not devoid of logical connections or expression of content. Likewise, students in the rhetoric and logic stages focused on more than expression or reasoning. They engaged in grammar, logic, and rhetoric (as understood by Wilson) in every class. Thus, the distinguishing mark of each stage was its goal, not the presence or exclusion of a portion of the trivium. For example, grammar students focused on knowing facts, dates, and information, not high quality reasoning and expression (ibid.). Students in the logic stage, however, had logical thinking as their primary goal. Even though they learned information, students first and foremost had to develop the ability to reason, something that Wilson saw as the extension of the natural argumentativeness of children that age (ibid.).

As can be seen from this discussion, Wilson (and Sayers) viewed the trivium in three distinct modes ${ }^{6}$. First, they saw the trivium as a guide to child development (developmental mode). In other words, the concepts of grammar, logic, and rhetoric provide an analogy or

[^5]description of the primary emphases for study at different ages. Second, they considered the trivium to be a pedagogical guiding principle (pedagogy mode). Each subject has its own grammar (facts), logic (connectedness), and rhetoric (modes of eloquent expression) (Wilson, 1996b, p. 101). According to this model, teachers at all levels needed to begin by teaching the facts, move to how the facts relate, and then work to get the students to explain the content to the teacher and to each other. Third, Wilson and Sayers saw the trivium as a set of subjects (content mode). Grammar, logic, and rhetoric each have a long, rich tradition as academic disciplines. As a result, Wilson argued, these subjects must be included as part of any classical, Christian school. Wilson's discovery of Sayers' essay launched a minor revolution in Christian education. Although some classical schools came into existence as the result of collegiate experiences with "Great Books" or "Integrated Humanities" programs (Iliff, 2009, personal communication), most resulted from the influence of a group called the Association of Classical and Christian Schools (ACCS), an organization founded by Wilson in the early 1990s to help the nascent CCE schools develop. From a handful of schools initially to 229 members in 2010 (ACCS, 2010), the growth of classical, Christian schools has occurred at a prolific rate. The ACCS publishes a journal, The Classis, holds annual conferences, and assists schools with training and certification. While not all classical, Christian schools affiliate themselves with the ACCS, enough schools do that the ACCS can be considered the defining organization for Wilson's approach to CCE.

Of course, Wilson did not create the CCE movement on his own. He certainly became one of the movement's first well-known proponents, but other thinkers also entered into the picture. Some came from within the ACCS and furthered developed the work of Sayers and Wilson. Others took the concepts of classical education in a different direction.

Developing the ACCS line of thinking occurred primarily through articles published in

The Classis. Wilkins (2004) expounded on Wilson's work by discussing a worldview-related goal of classical, Christian education: the creation of students who can function in a free society while also glorifying God in whatever vocation they find themselves. Seel (2007) added his own worldview-related summary of the purpose of CCE: "Our goal is to equip apprentices of Jesus with a pre-modern mind capable of engaging our postmodern world" (p. 5).

As the ACCS continued to develop its philosophy, other thinkers began to develop variations on the CCE approach. Two educators, Robert Littlejohn and Charles Evans, made adjustments to the approach of Sayers and Wilson that provided a preliminary rationale for mathematics within a CCE model. In the book Wisdom and Eloquence: A Christian Paradigm for Classical Learning, they developed their version of CCE.

Instead of following Sayers' thoughts directly, Littlejohn and Evans used her ideas as a starting point to develop further theories. Thus, Littlejohn and Evans represent a parallel approach to the ACCS. They start from the same sources but sometimes arrive at different conclusions. Their approach could be called the "Seven Liberal Arts" approach to distinguish it from Wilson's "Trivium Only" approach. The goal of education, Littlejohn and Evans argue, was to cultivate in students "a life of faith-filled learning to be Christlike" (Littlejohn \& Evans, 2006, p. 18). A liberal arts education, in which students study the content areas contained in the trivium and quadrivium, they believed, accomplished this objective most effectively (Littlejohn \& Evans, 2006, pp. 22, 185).

Littlejohn and Evans began their examination of CCE by discussing Sayers briefly. After that, however, they departed significantly from her suggestions. They forcefully rejected the idea that the trivium was a way of looking at students' developmental phases (the developmental mode). "A better understanding of the liberal arts and sciences as an educational paradigm,
which long preceded Ms. Sayers, insists that we separate the [liberal] arts from the question of cognitive development altogether." (Littlejohn \& Evans, 2006, p. 39). Based upon their experiences as educators, Littlejohn and Evans argued instead for using "the liberal arts and sciences as the curriculum of choice and giv[ing] careful attention to teaching this curriculum using methods that are sensitive to our students' abilities..." (ibid.).

Littlejohn and Evans also strongly objected to the concept that the trivium in some way provided guidance for pedagogy. "[W]e flatly deny that there is historical precedent or practical necessity for a construct such as the 'grammar of history'.... [W]e could as readily recommend that students be taught 'the astronomy of rhetoric...'" (Littlejohn \& Evans, 2006, p. 39). Instead, they believed that students should be taught with whatever methods were most appropriate for a given group of students at a given time. Thus, they rejected the pedagogy mode of the trivium as well. A classical education, in other words, was about the content, not the pedagogy.

Realizing that they had left educators with only the content and no guidance on how to structure or teach it, Littlejohn and Evans spent the majority of their work attempting to provide some guidance on how to structure a CCE school without resorting to the trivium. First, they argued, teachers must recognize that the tools of learning were not grammar, logic, and rhetoric, but rather "the skills that are learned during one's study of all the liberal arts and sciences" (Littlejohn \& Evans, 2006, p. 39). Second, they described an outcomes-based approach to curriculum development, urging a " $12-\mathrm{K}$ " design approach that considered the desired final outcomes and then worked back from there (Littlejohn \& Evans, 2006, p. 40). Thus, their curriculum structure was not primarily memorization at the elementary level, primarily connectedness in middle school, and primarily expression in high school. Rather, it was "a lifelong study of all the disciplines from day one" that sought to cultivate a Biblical world in the
graduates of the school (Littlejohn \& Evans, 2006, pp. 40, 43). The trivium and quadrivium provided the disciplines which students should learn.

With the reintroduction of the quadrivium to the content areas of study, mathematics once again had a place in the classical model. Expanding on the quadrivium, arithmetic included not only traditional arithmetic and number theory but also algebra, statistics, calculus, and computer science (Littlejohn \& Evans, 2006, p. 87). Geometry, likewise, gained geography and the visual arts as content areas in addition to traditional plane and solid geometry (ibid.). All the natural sciences folded into astronomy, not only as content areas themselves but also as providers of applications for the mathematics learned in the areas covered by arithmetic and geometry. Music lost most of its mathematical basis and became performance-oriented. Thus, Littlejohn and Evans attempted to organize the modern curriculum using the seven liberal arts as their categories. In doing so, they provided a possible justification for mathematics in a CCE curriculum.

A third approach to CCE, partially different from and partially cooperative with the ACCS, developed through the work of the CIRCE Institute, founded by Andrew Kern in 1996 (CIRCE, 2011). Writing with Gene Veith, provost of Patrick Henry College, Kern examined the state of classical education in America, arguing for its use as the preferred method of education. The major distinctive of Veith and Kern's approach to classical education was the presence of the trivium, particularly its pedagogical and content modes. They remained silent on the developmental mode. The pedagogical mode, they believed, provided a critical guide to proper education. "Every type of learning requires knowledge (grammar), understanding (logic), and creativity (rhetoric)" (Veith \& Kern, 1997, p. 12). This trivium-type approach to education worked, they contended, pointing to the success of the (often unwitting) use of the pedagogical
mode in medical schools, law schools, music conservatories, business schools, and religious seminaries (Veith \& Kern, 1997, p. 13).

While Wilson launched the CCE movement, and other authors developed similar models, a discussion of modern CCE would be incomplete without discussing a classical approach that influenced these later developments. This approach came from within the humanities and emphasized the classics in a more general setting. This movement developed primarily in the 1970s and 1980s and, as previously observed, had an impact on Wilson's decision to create a classical school. Advocates of it included professors such as Mortimer Adler of the University of Chicago as well as Dennis Quinn and John Senior from the University of Kansas. Space does not permit a full discussion of this approach, sometimes called the Great Books approach. One distinctive of this approach, however, relates to the study of mathematics: the concept of education instilling a sense of wonder. James Taylor, a humanities teacher and educational philosopher who studied under Quinn and Senior, discusses the importance of wonder in education. Calling an instinctive knowledge of a concept "poetic," he writes, "Poetic knowledge is the wonder of the thing itself" (Taylor, 1998, p. 69). Taylor's mentor, Quinn, in a lecture given in 1977, observed that "wonder both starts education and sustains it" (Quinn, 1977, paragraph 10). Adler, a philosophy professor at the University of Chicago, identified three modes of learning, one of which related to the concept of wonder. This mode, called by Adler the "enlargement of understanding, insight, and aesthetic appeal," "stimulates the imagination and intellect by awakening the creative and inquisitive powers" (Adler, 1982, pp. 23, 29). This mode of knowledge, then, hinged upon students learning to be fascinated and awed by the topics studied, motivating them to further study, ideally for a lifetime. Thus, the concept of wonder within the Great Books program distinguished it from other classical approaches.

Even as each of the above authors argued for a classical approach to Christian education, they knew that others would challenge their support for a classical model. Critics charged that, in adopting the classical approach, Christian educators had uncritically committed the same syncretistic error as the Scholastics of the Middle Ages (Van Dyk, personal communication, 2011). As a result of such concerns, several proponents of CCE offered a defense for choosing the classical model. In so doing, they (often unknowingly) followed in the footsteps of Hugh of St. Victor, who provided an early apologetic for the use of a classical approach to education when he identified the seven liberal arts as being the core of education and extolled their virtues.

In his follow-up work to Recovering the Lost Tools of Learning, Wilson attempted to respond to the worries of uncritical syncretism. In an introductory essay, he explained what he meant by "classical" education. The word "classical," he observed could have three potential meanings. The first meaning referred to "bypass[ing] the last two thousand years of history, and return[ing] to a study of the golden ages of Greece and Rome," an approach Wilson considered contrary to the Christian faith (Wilson, 1996a, pp. 21-22). The second meaning was a syncretistic approach, such as was done by the Scholastics of the Middle Ages, who sought to incorporate uncritically the ideas of Plato and Aristotle, putting those thoughts in Christian words (Wilson 1996a, p. 22). Wilson also considered this approach to be contrary to a Christian approach to education, observing that such an approach "requires a humanistic and autonomous approach to truth that is totally at odds with the biblical revelation of truth in Christ" (Wilson, 1996a, p. 23). Wilson, however, argued for a third meaning of "classical," one that was "thoroughly Christian, and grounded in the great truths of Scripture recovered and articulated at the Reformation" (ibid.). To illustrate this type of classicism, Wilson used the apostle Paul, who knew ancient Greek thought, who quoted pagan poetry and philosophers, and yet who refused to accept a
pagan worldview. After quoting from 2 Corinthians 1 and 2 to illustrate his point, Wilson wrote, "So, we see in Paul a biblical classicist. He does not run from classical culture, nor is he defeated or compromised by it. Rather, he declares the lordship of Jesus Christ over it.... He uses his vast learning in the cause of the Gospel..." (Wilson, 1996a, p. 24). Therefore, Wilson concluded, the type of CCE for which he advocated and desired belonged entirely to the third approach, making it a valid option for Christian schools.

Kertland, a supporter of Wilson's approach and a homeschool educator, argued that the classical model provided the best method for training a student to be a "Daniel," aware of the culture and yet able to reach it for Christ (Kertland, 1997, p. 47). Kertland's support for this contention came from examining the education of the biblical Daniel, who was trained in the language and literature of the Late Babylonian Empire he served. Kertland concluded that Daniel impacted his culture partly because of his understanding of that culture. In the same way, therefore, students who wished to impact American culture needed to understand it. Kertland argued that, because America's cultural roots are Western, a classical structure of education, including studying the great works that influenced Western Civilization, provided students with a solid understanding of the forces underlying American culture. With this base, students could then engage in responsible, Christian examination and critique of the culture.

Concerning their choice of a liberal arts approach, Littlejohn and Evans supplemented their work in an appendix with a detailed argument partially explaining their reasoning. They started by reviewing the contributions of the Hebrews and Greeks to Christian education and then analyzing the similarities and differences between the two approaches. They concluded that both cultures provided insights that Christians could apply to education as long as they started from a biblical worldview in examining the educational structures (Littlejohn \& Evans, 2006, p.
189). As Littlejohn and Evans analyzed the Hebrew and Greek strands, they concluded that the Hebrews provided wisdom (through the Scriptures) and the Greeks provided eloquence (through what later came to be the seven liberal arts, especially rhetoric). To justify this conclusion, they examined in detail Augustine's arguments from On Christian Doctrine, contending that he advocated for this unification of wisdom and eloquence as the goal of education when he stated that Christians should clearly and accurately (eloquence) preach the wisdom of God to "move [their] hearers to faith-filled responsibility" (Littlejohn \& Evans, 2006, p. 199). By the entire line of Augustine's argument, and particularly by this last statement, Littlejohn and Evans concluded that Augustine demonstrated that "the liberal arts constitute a dependable path" toward the goals of Christian education (Littlejohn \&Evans, 2006, p. 201).

Further support for use of a classical model within Christian education came from Veith and Kern. They began by noting the struggles of the current system. "That education in America today is in shambles is obvious and well-documented.... [T]he primary culprit is contemporary education theory" (Veith \& Kern, 1997, p. 1). The contemporary education theory in the late 1990s was a mix of Deweyian pragmatism (truth is what works) and postmodern relativism (truth is socially constructed and relative). To counter this anti-Christian epistemology, Veith and Kern argued for a return to the classical approach, which viewed truth as something revealed (revealed in Christ, as it was frequently understood in the Middle Ages) and as a result knowable. Thus, classical education suited the needs of Christian schools epistemologically, provided that students lived out the knowledge they gained, a common expectation in Christian schools. Moreover, classical education served Christian schooling's academic purposes because it could adapt to the culture, as evidenced by its historical use in various cultures throughout the past two thousand years (Veith \& Kern, 1997, p. ix). This adaptability made it a natural choice for the task
of educating Christians in a postmodern culture without conforming to it. While urging the adoption of a classical model, Veith and Kern also argued against mindlessly duplicating the past. "Obviously, a liberal arts education today would have to be very different from one in ancient Rome or medieval Europe" (Veith \& Kern, 1997, p. 11). They further supported the use of a classical model by providing some details of how this education might look in practice, examining four modern approaches to classical education that occurred within four different ethnic and socioeconomic groups.

While all of these authors said much about classical education in general and some emphasized the humanities in particular, their comments on mathematics varied widely, with few providing much insight into how mathematics education would fit into their overall model. Throughout all of the development of CCE by Sayers, Wilson, and the ACCS, mathematics received scant mention. Sayers only briefly touched on mathematics. Wilson's discussion likewise said little about mathematics, except that it should be taught. Neither offers a good reason for mathematical study, nor do they suggest any methods or materials to use in teaching mathematics.

Sayers wrote novels and poetry, so her emphasis naturally fell upon the humanities as she developed her reasoning. Mathematics, though, did receive a place in Sayers' model. Sayers viewed mathematics as a natural extension of logic, and she argued that it should be treated as such (Sayers, 1947). Sayers, however, provided no details beyond this scant mention.

For Wilson, since the trivium was humanities-focused in content, and since he worked in a humanities-related area (theology), it is not surprising that mathematics, while mentioned, received far less discussion than other areas. Not only was Wilson silent in this area, so also were the other ACCS authors in the Classis. As we noted earlier, only one mathematics-related article
appeared in any of the editions of the newsletter, and it focused on the relationship of mathematics and Christianity. In other words, mathematics was strangely absent from the Sayers/Wilson CCE model, leaving CCE mathematics teachers in a philosophically uncomfortable position.

In contrast to Sayers and Wilson, Littlejohn and Evans provided some additional details about mathematics in a CCE context. They established that mathematics belongs in classical education by reincorporating the quadrivium. Their approach, however, attempted to constrain all of the modern subject areas within the seven medieval categories. The fit was not always a natural one, and as a result, CCE math teachers still did not have a solid philosophical base from which to operate. Moreover, Littlejohn and Evans did not show how to approach the mathematical content in a way that would help inculcate a Christian worldview. They discussed worldview in general, but they did not discuss how to apply it in a mathematical context.

The Great Books educators and the CIRCE Institute did not mention mathematics much, except for Adler, who included mathematics within his approach. Adler considered mathematics and natural sciences to be one of the three key areas of educational content that schools needed to address (Adler, 1982, p. 23). He prescribed teaching simple arithmetic in the lower grades, ending with a semester of calculus as the ultimate goal. Students could make use of calculators and computers as they learned mathematics. These prescriptions, however, formed the extent of his discussion on mathematics. He provided nothing in the way of philosophical support for including mathematics.

Thus, even as modern CCE developed a solid philosophical base overall, its approach to mathematics remained under-developed and unsupported.

## Non-classical, Christian Mathematics Education

While a CCE approach to mathematics has yet to be developed, the wider community of Christian educators has wrestled with the issue of Christian mathematics education far longer and in more depth. Although a broad, evangelical discussion of the topic originated in the 1970s with what is now the Association of Christians in the Mathematical Sciences (ACMS, 2011), some Reformed thinkers and the associated Christian Schools International (CSI) organization had dealt with the issue prior to the founding of the ACMS (Jongsma, 2011, personal communication). Because of the vast amount of work done over the past forty years, an examination of the various threads, themes, and viewpoints within Christian mathematics education would constitute a thesis in itself. Instead of attempting a detailed analysis of these developments, we will examine some foundational concepts that emerge from studying an eclectic but representative sample of the relevant literature, leaving a full treatment of this topic for future research. For further details on these matters, the reader can consult Mathematics in a Postmodern Age: A Christian Perspective, edited by Howell and Bradley (2001) ${ }^{7}$; the many annotated resources listed in the Bibliography of Christianity and Mathematics by Chase and Jongsma (1983); and the proceedings of the various ACMS conferences. The ACMS websitewhich includes a table of contents of the ACMS proceedings as well as a list of current projects-also provides a good starting point for some of this material.

As educators considered Christian mathematics education, they started at the same point as all of Christian education: understanding God and His ways through study of His world. Reformed educators in particular developed an educational philosophy pertaining to knowing God through different content areas. One such educator, William Jellema, a professor at Calvin

[^6]College in the 1940s and 1950s, noted the importance of learning about God in education. "Search till you find [in reality] a revelation of God" (Jellema, 1953/1997, p. 56). For Jellema, this revelation served to confirm a Christian worldview in the student through a study of what God had made, which included mathematics (Jellema, 1953/1997, p. 57). Jellema's focus primarily centered on the humanities, but his remarks easily extended to mathematics.

The idea of tracing God's presence in and intentions for the world through a study of mathematics occurred in the work of numerous authors and organizations over the past forty years, including Van Brummelen, a Canadian mathematics educator and education professor; the Curriculum Development Centre in Toronto (now defunct); and the Kuyers Institute at Calvin College. MacKenzie, et al, of the London-based research group Christian Action Research and Education took the works of these various authors, along with others, and synthesized them into a handbook for Christian teachers. In their section on mathematics, they noted, "A Christian rationale for studying mathematics would probably include the following elements: discovering God's creativity and design, understanding its purpose, and responding to that knowledge by using it in the service of God, humanity, and the rest of creation" (MacKenzie, 1997, p. 137). Christian mathematics education should be about "the nature of God, the unity of creation, the nature of reality, human creativity, and the beauty and wonder of mathematics" (MacKenzie, 1997, p. 141). In particular, in emphasizing the nature of God, MacKenzie affirmed the principle that something of God's character is knowable through mathematics and that students should learn about God through its study.

The importance of knowing about God through mathematical study also received affirmation in the introduction to the work Mathematics in a Postmodern Age: A Christian Perspective. The editors, Russell Howell, a professor at Westmont College, and James Bradley, a
professor at Calvin College, noted that mathematics reveals something about God's nature, "his subtlety, order, beauty, and variety" (Howell \& Bradley, 2001, p. 5). Thus, as students gain a better understanding of mathematics, they grow in their knowledge of God.

Bergman (2001), a mathematics teacher in a CSI school, concurred with the possibility of knowing God through mathematics, noting that mathematics education had two goals, one of them eternal. (We will examine the temporal one later.) This eternal goal focused on studying the things of God as found in the study of mathematics and in the mathematical examination of His creation. Through this study, students would come to a better knowledge of God. Bergman linked mathematics to God in a general way. In other words, the eternal aspects of mathematics existed not because there is one-to-one correspondence between properties of mathematics and God's character but because God created mathematics and thus mathematics reflected "God's perfect wisdom" (Bergman, 2001, p. 31).

Additional support for the idea of knowing God through studying mathematics came from John Byl, a professor at Trinity Western University. He built upon ideas from Reformed philosopher Alvin Plantinga in a paper given to the Thirteenth Conference of the Association of Christians in the Mathematical Sciences. In that paper, Byl noted that learning mathematics meant learning more about the mind and character of God (Byl, 2001, p. 39). ${ }^{8}$

That same year, James Nickel, a mathematics teacher and educational consultant, published the revised and expanded version of his book Mathematics: Is God Silent. In this book, Nickel argued strongly for knowing God through mathematics. In particular, he argued that, to grasp fully the nature of God, one needed to know mathematics. "Since mathematics is a unique, 'alphabetical' description of God's creation, we must expect to find, upon reading it, the invisible things of God" (Nickel, 2001, p. 234). Nickel listed four general objectives for Christian

[^7]mathematics education, including "...reveal[ing] the invisible attributes of God" (Nickel, 2001, p. 235). These attributes manifested themselves in all aspects of mathematics, from orderliness down to counting. This idea originated not with Nickel but from mathematicians and scientists who came before him. The fifth-century theologian Augustine, for example, argued that mathematical ideas originated in the mind and character of God (cited in Kuyers Institute, 2007, p. 4). Likewise, the seventeenth-century scientist Kepler believed that God had "embodied some of his mathematical nature in Creation" (Jongsma, 2001, p. 165). Not all Christian mathematics educators go as far as Kepler or Nickel in directly correlating God's attributes with specific aspects of mathematics, but they all concur that studying mathematics allows students, in some way, to study the attributes of God.

Moreover, as they pondered knowing God through mathematics, some authors saw the connection between God and mathematics as a two-way street. In other words, not only did studying mathematics teach students about God, but knowing about God also helped students better understand the basis for mathematics. Nickel, in commenting on God's attributes, saw the explanatory power of a Christian worldview. Only in a biblical, Christian worldview, he argued, does the existence of mathematics make complete sense. "We can do mathematics only because the triune God exists. Only biblical Christianity can account for the ability to count" (Nickel, 2001, p. 230). For Nickel, therefore, a full study of mathematics required a Christian worldview because only such a worldview could explain all aspects of mathematics. Likewise, Bergman contended for this general idea, although not as specifically as Nickel, arguing, that the teaching of mathematics must be done "in the light of Holy Scripture" (Bergman, 2001, pg. 31, 32). He did not provide any examples, simply stating that mathematics instruction needed to be done from a Christian worldview.

Jongsma, a professor at Dordt College, also analyzed how a Christian worldview affected mathematics in a 2006 lecture that was later published in the Journal of the ACMS. A Christian worldview, he argued, provides the basis for meaning in mathematics (Jongsma, 2006, p. 4). In addition, a Christian worldview also helped to provide focus for mathematical investigation. Mathematics could be seen as "an exploration of various dimensions of the creation God made," with a Christian worldview providing guidelines for what questions to ask, what methods of inquiry to use, and which answers to prefer (Jongsma, 2006, pp. 10, 12). Mathematics for the non-Christian might look similar to mathematics for a Christian, but a Christian worldview influences the study of mathematics whenever the question of meaning or broader context comes into play.

Even as Christian mathematicians considered the possibility of knowing God through mathematics, they realized the importance of a sense of wonder and worship. An early contributor to this idea was C. Ralph Verno, a professor from West Chester State College. He recognized that this wonder should ultimately lead students to deeper worship of God, writing, "The believer should be able to look at mathematics and exclaim to his God, 'How great thou art!' " (Verno, 1979, p. 96). Another advocate for this idea, Vern Poythress, a theologian and mathematician, noted that mathematics included a sense of wonder (Poythress, 1981, p. 37). He went so far as to describe mathematics as poetry, an art form that can induce wonder and joy simply on its own without needing application. MacKenzie, et al, in examining the various thoughts of different authors on this topic, also emphasized the wonder inherent in mathematics when they described "the wonder and beauty of mathematics" as one of the four areas students needed to know (MacKenzie, 1997, p. 141). More recently, authors such as Nickel also discussed the value of wonder and worship. He noted multiple times that Christians should wonder at the
amazing way in which mathematical concepts match reality. This sense of wonder should encourage worship of God (Nickel, 2001, p. 225). Howell and Bradley also saw wonder and worship as appropriate responses to mathematics (Howell and Bradley, 2001, p. 5). Therefore, for some authors, wonder leading to worship formed an important part of the process of teaching mathematics Christianly.

Hand-in-hand with the above concepts of knowing God and exhibiting wonder was the idea that mathematics education should have a Creation (or real world) orientation. Early proponents of such an approach included Van Brummelen (1977), the Curriculum Development Centre's program The Number and Shape of Things (Jongsma and Baker, 1979), and VanderKlok (1981). VanderKlok, a math teacher and educational consultant, noted that a purely algorithmic approach destroyed wonder, interest, and excitement concerning the material being studied (VanderKlok, 1981, p. 8). Instead, he argued for an approach to teaching mathematics that introduced students to the beauty of God's creation and then moved on to study the mathematics discovered therein. This approach, he contended, was inherently Christian because it proclaimed the "wholeness and integrity of God's creation" (VanderKlok, 1981, p. 9).

Later authors also continued to emphasize the value of using the real world in mathematics education. Mathematics instruction based in created reality formed a critical aspect of Nickel's model. In particular, he emphasized the importance of student motivation built upon a study of the world God has made. "It is essential that mathematics appeal to the student at the time he takes the course" (Nickel, 2001, p. 282). He continued this thought, noting "[t]he proper context for true motivation is the context of God's creation" (Nickel, 2001, p. 283). In other words, Nickel argued that teaching mathematics from a Christian viewpoint necessitated the use of relevant examples from Creation, urging that students start from specific examples and work
their way outward to general patterns. Bergman (2001) likewise emphasized the necessity of orienting mathematics instruction toward creation. "The proper pedagogical method for mathematics education in a Reformed, Christian classroom is integration-consistent, deliberate study of mathematics as... part of God's creation" (Bergman, 2001, p. 33). Jongsma, who while at the Curriculum Development Centre was involved in developing its integrated mathematics curriculum for the elementary school (Jongsma and Baker, 1979), also advocated for connecting the mathematics curriculum to real world motivation and applications, observing that mathematics "arises from our experience of certain aspects of creation" (Jongsma, 2006, p. 14). Therefore, for these authors, the use of everyday reality as God's creation, either as a starting point or a referent, formed an important component for the Christian study of mathematics.

A Creation orientation to mathematics education corresponds with recognizing the need for an applied component to mathematics education. Whereas a Creation orientation starts with the world and uses it to aid in learning mathematics and developing mathematical principles, application reverses the process, taking mathematical concepts already learned and putting them in to use in real-world applications. MacKenzie, et al, noted this practical component when they referred to the necessity of students "responding to [mathematical] knowledge by using it in the service of God, humanity, and the rest of creation" (MacKenzie, 1997, p. 137). Bradley, the Calvin College mathematics professor who spearheaded the Kuyers Institute's mathematics project, noted the importance of applying mathematics when considering a Christian approach to the subject (Bradley, 2001, p. 215). Nickel filled his work with various applications of mathematical principles, from "abstract" concepts like the Fibonacci sequence and the Golden Ratio to concrete applications such as compound interest (Nickel, 2001). Similarly, Bergman (2001) emphasized the need for practical mathematics education, calling it the "temporal" goal
of mathematics (p.31). Likewise, Jongsma concurred with these observations when he noted that Christian learning is about allowing students to serve God in their daily lives (Jongsma, 2003, p. 26). Application, therefore, naturally follows from the adoption of a Creation-orientation in Christian mathematics education.

Christian mathematics education does not have to be applied in order to have validity. Room exists for pure and recreational mathematics, as well. Howell and Bradley carefully noted the relationship between pure and applied mathematics in their introduction to Mathematics in a Postmodern Age. Application, they argued, forms an important part of a Christian approach to mathematics, but it does not constitute the whole. For them, mathematics could exist for mathematics' sake because abstract mathematics was an important aspect of humans "cocreating" with God (Howell \& Bradley, 2001, p. 5). Thus, there is room in a Christian approach to mathematics for both the pure and applied branches.

Critical to mathematics in both pure and applied contexts, reasoning skills and their value in Christian education constitute an important theme in the writings of some authors. Students studying mathematics at a Christian school need to learn not only about the God who created mathematics, they argued, but also how to approach mathematics in a way that is consistent with how mathematicians "do mathematics." Van Brummelen observed that mathematics education must include teaching higher-level mathematical reasoning skills (Van Brummelen, 1977, p. 143). Years later, VanderStoep (2001), a professor at Hope College, affirmed the value of teaching mathematical reasoning, noting that students need these skills in order to study and apply mathematics effectively (p. 328). Christian mathematics education, therefore, should include reasoning as an important component.

The components of Christian mathematics education mentioned above-learning about

God, teaching from a Christian worldview, promoting wonder and worship, adopting a realworld orientation, using genuine applications, and teaching mathematical reasoning-form just part of a Christian approach to mathematics education. These ideas, though, provide important guidelines that we can use as we seek to develop a classical, Christian approach.

## Synthesis

In the above pages, we have sketched a brief history of classical education, which came to consist of the trivium and the quadrivium. As the university system developed and scientific thought came to the forefront, the liberal arts gradually receded in importance over the course of several hundred years. In the middle to late twentieth century, different scholars, Christian and non-Christian, attempted to re-capture the good aspects of education from the Middle Ages and adapt it to a post-modern age. For the Christian schools in particular, this movement took the name classical, Christian Education. As CCE has grown, theoreticians have developed curricula and pedagogy for most areas of study. Mathematics, however, has remained mostly untouched. As a result, the philosophical placement of mathematics in the CCE model has not fully been developed. In the broader Christian education community, however, many people over the past forty years have worked to develop a more robust, distinctly-Christian approach to mathematics instruction. Some of their ideas can be adapted to provide CCE instructors with a framework for justifying the place of mathematics within the CCE model.

In developing a rationale for mathematics in CCE, we must take care to incorporate ideas that are consistent both with Biblical Christianity and with classical education. In other words, we should not be satisfied with the refrain of some within the CCE community: "We are Christian because we are classical; we are classical because we are Christian." Each can exist
without the other. As Wilson observed in his three definitions of classical, the classical and the Christian can even be antithetical to each other. Therefore, the goal is what Wilson called "biblical classicism," because it provides a solid base from which to build a CCE philosophy of mathematics. In other words, CCE math should be thoroughly Christian, taking care to ensure that the selected components of a classical approach do not contradict Scriptural principles.

This careful evaluation is necessary because, as the history of education shows, the classical model had its flaws, and these should not be minimized. The classical approach, especially as practiced from the late Middle Ages to the late $19^{\text {th }}$ century, became increasingly backward-focused, and in the opinion of many, had ceased to have much relevance to the modern world. Moreover, the classical approach, especially from the $13^{\text {th }}$ century forward, had fallen into varying degrees of syncretism with pagan thought. Modern education, however, in recognizing the classical model's obsession with the past, overcorrected, leaving behind not only the weaknesses but also the strengths of the classical model. Wilson, Sayers, and other modern CCE theorists have mostly managed to salvage what is good in the medieval classical model and have adapted it to the needs of a postmodern society. Admittedly, their approaches differ, but each contributes important concepts to the CCE model. The aim of this synthesis, therefore, is to take the goals of CCE authors and use the insights put forward by Christian mathematics educators to develop a structure for effective delivery of that education.

To develop this structure, a CCE approach to mathematics must recognize the value in the classical approach without slavish obedience to it. The trivium and quadrivium can guide the CCE model without rigidly prescribing any aspect of it. In other words, this approach does not exclude a modern understanding of the trivium and quadrivium from CCE. Rather, these seven liberal arts and their various understandings provide guidelines that need not be followed
literally. At the same time, they should not be eliminated entirely because of the value noted by various adherents. A modern, Christian use of the classical approach requires analogy as much as prescription. In following this path, we seek a middle ground between the wholesale rejection of the modes of the trivium (Littlejohn and Evans) and a too-slavish obedience to them (Wilson and Sayers). We will take the analogical approach of the former while recognizing the valid structural insights of the latter. Thus, for us the trivium's modes will still influence CCE mathematics curriculum and pedagogy, but we will deviate from the trivium when it is necessary.

Our stipulation of a CCE mathematics program begins by describing some of the components of the classical side of CCE. After discussing some of these aspects, we will attempt to establish how a Christian approach fits together with it.

The starting point for understanding the classical side of CCE mathematics comes, as it did more generally for Sayers in her original essay, from examining medieval classical education. In that approach, the quadrivium contained the four mathematical arts. These four arts constituted the primary core mathematical knowledge in their day. Students schooled in the quadrivium received a comprehensive exposure to the main fields of elementary mathematics at the time. Applying the same principle, therefore, modern CCE mathematics education should aim to teach all developmentally-appropriate mathematical knowledge available to us today. Room still exists for studying the more strictly classical components of mathematics, such as number theory, proofs, area, volume, and a host of other topics contained in works authored by past mathematicians. At the same time, newer fields such as algebra, coordinate graphing, statistics, and other modern content areas should be incorporated into today's "quadrivium." This approach shows how mathematics can be updated within a classical model. A similar approach could be used to expand other content areas outside the humanities, such as the sciences. That
discussion, however, falls outside the bounds of this thesis. The role and scope, then, of modern CCE mathematics derives from that of the quadrivium within classical education and thus includes all aspects of elementary mathematical knowledge.

While the content mode of the trivium (and by extension, the quadrivium) provides a solid base for reasoning by analogy, the other two modes of the trivium stand on varying degrees of shakier ground. As Littlejohn and Evans observed, equating the trivium and quadrivium with developmental stages or with modes of teaching proves at times to be less than instructive, especially when considering the quadrivium (viewing children as being in a "music" stage of development or having teachers include the "astronomy" of a subject as part of their class lessons approaches the level of absurdity). At the same time, these modes (analogies in themselves) may still have some value in deciding what aspects of education to emphasize at various times in students' academic careers, as well as providing some rough ideas about how to teach it. The pedagogical mode, as Veith and Kern noted, has been used successfully for many years in some educational contexts. CCE mathematics theory, therefore, can appropriate insights from these modes.

CCE should especially treat the developmental mode with caution. Sayers' thoughts on the developmental mode, while accurate in a very general way, have substantial limitations. First of all, Sayers' model is incomplete, starting at age nine, several years past the age of entry into school in modern education. Wilson attempts to correct this problem by moving the start of the grammar stage downward. This change, however, occurs without empirical support and seems done more from practical necessity than anything else. Second, the three developmental stages come from Sayers' observations of one child: herself. Thus, her conclusions are necessarily limited and preliminary, something she herself admits. At the same time, the limited sample size
does not automatically invalidate her conclusions. A careful observer will note that younger children tend to memorize information far more easily than their older counterparts, while expression that truly is eloquent occurs more frequently with high school students than with younger students. Further compounding the problem, the developmental mode used by Sayers and Wilson does not seem to conform well with much modern psychological research on child development. The research seems to suggest that the while children do go through phases of development, the development occurs in a more complex pattern than a rigid interpretation of the developmental model suggests. Much of this apparent lack of conformity stems from an erroneous understanding of the developmental mode of the trivium. Both proponents and opponents of CCE tend to attach greater meaning to the developmental mode than they should, treating each stage as if it perfectly described child development. The developmental mode, as already observed, never claims to minutely prescribe a student's psychological development. Rather, as Wilson noted, it merely provides an approximate indication of what the educational emphases should be at a particular age. Therefore, while it may be helpful in some general ways, the developmental mode must not be taken farther than its rough outline can bear. Memorization, interconnectedness, and expression occur at all levels in different forms. The key to applying the developmental mode correctly is to remember that this mode describes what should receive stronger emphasis at a given age rather than what alone must be taught.

While not on as uncertain a base as the developmental mode, the pedagogical mode still requires care in adapting it to mathematics education. Like the developmental mode, the pedagogical mode must not be over-specified. Rather than prescribing a specific method of instruction, the trivium provides a general description of how a lesson should proceed. The pedagogical mode suggests that the three aspects of grammar, logic, and rhetoric must all be
present within a successful lesson; however, it does not forbid the addition of other components, such as motivation or exploration, if a teacher desires to identify them as discrete steps. In describing pedagogy with the trivium, however, other steps in the learning process often fit within or distribute across the trivium's three categories. For instance, motivation and exploration can be understood not necessarily as separate steps but as ways of approaching the grammar, logic, or rhetoric of a given topic.

Moreover, teaching in a grammar-logic-rhetoric sequence does not necessarily mean that the only accepted pedagogy is direct instruction. Inductive approaches also fit within the guidelines, provided the approaches come from a correct epistemological base. For example, a possible deductive approach using the trivium might start with giving students the general principle (grammar), then asking why the principle works (logic), and finally requiring students to apply it (rhetoric). A potential inductive approach might start with having students explore real-world instances of a principle (grammar), then asking them to identify the commonalities in order to derive a general principle (logic), and end with applying the principle in a different context (rhetoric). In each case, the instructional sequence follows the broad guidelines of the pedagogical mode of the trivium. Within this framework, teachers have the freedom to incorporate ideas that might better engage the students (motivation) through a dissonant fact, an intriguing question, or a clever application. Likewise, room also exists for investigating various facts, examining potential connections, and creating innovative applications. The pedagogical mode of the trivium, therefore, provides teachers with a loose guide to lesson structure. Teachers must still fill in the structure in a way appropriate to the content, the developmental level of the students, and the individual teacher's own teaching style.

In addition to the trivium providing some guidance in CCE mathematics education,
particularly regarding content and pedagogy, two other general aspects of CCE education (drawn from the Great Books approach of Adler)—instilling wonder and using primary sources-have their place as well in the CCE mathematics model. As we will see below, wonder provides a natural linking point with non-classical, Christian mathematics education as well as providing a balancing force for the teaching of the abstract concepts that frequently occur in mathematics, while the use of primary sources, when done with discretion, provides a natural integration of history into mathematics education.

Finally, the two-pronged goal of classical education provides the remaining critical piece of the classical side of the CCE approach. One goal of classical education was the creation of citizens for the society. As we will see below, this goal provides an excellent analogy for a Christian approach to cultivating disciples. The second goal of classical education also aids in formulating a CCE model for mathematics education. According to Hugh of St. Victor, education in the seven liberal arts provided students with the foundational knowledge and learning skills necessary for self-learning. Thus, modern CCE must provide students with the necessary mathematical background knowledge and the proper mathematical skills to enable students to learn and apply mathematics on their own once they have completed their formal schooling. ${ }^{9}$

With these key classical components of CCE mathematics education identified, it becomes possible to create a synthesis with Christian mathematics education to create a more robust educational philosophy. The ideas presented below are offered as a beginning. They derive primarily from the common ground observed in classical and non-classical approaches to education.

First, CCE mathematics should have at its core the discipleship of students to be effective

[^8]citizens of the Kingdom of God through their use of mathematics. Classical education in antiquity aimed to create effective citizens for an earthly kingdom, an aspect adapted by modern CCE educators, especially Littlejohn and Evans, into a Christian discipleship emphasis. Other authors from the modern CCE movement also value this discipleship emphasis, noting that one of the goals of classical education is the creation of men and women of God who will engage an increasingly post-modern culture. Non-classical Christian mathematics education also emphatically places discipleship of students at the center of its approach, as supported by the numerous authors discussed above, who observe that a Christian approach to education must be about creating effective citizens for God's Kingdom. Moreover, authors from both CCE and nonclassical, Christian education acknowledge the biblical role of the school in assisting parents in the discipleship process. Thus, any CCE mathematics education should have discipleship as a significant central emphasis.

Second, as part of the process of nurturing students as disciples, CCE mathematics should attempt to cultivate a sense of wonder in the student. Students can respond with amazement and joy to any aspect of mathematics, and, as mentioned above by authors classical and nonclassical, memorization and purely algorithmic study will not produce wonder. On the other hand, investigation of the mathematical features of the world, recognition of some of God's attributes in creation and mathematics, deduction of unexpected conclusions, and development of intuitive ("poetic") knowledge can all contribute to the creation of wonder in students. Therefore, these emphases should also be present in a CCE approach to mathematics. Students schooled in this approach should be able, like many mathematicians throughout history, to praise God as they marvel at the beauty of mathematics. This attitude of wonder should also assist in the discipleship process by enlarging students' ability to worship God. Moreover, when properly
done, wonder provides a motivation for learning additional mathematics, creating the lifelong learner aspired to by many in the education community. Thus, CCE mathematics must include attempts to aid students in seeing the wonder of mathematics as well as encouraging students to worship God as a result of studying mathematics.

Third, as a natural way of developing wonder and worship, CCE mathematics education should include Creation-based examples and explorations where appropriate. These examples and investigations may be very basic in the younger grades, and should increase in complexity as students advance through the grades. In exploring various mathematical aspects of Creation, students should develop a sense of awe at the grandeur of mathematics and (under the influence of the Christian teacher) a deeper worship of God.

Fourth, in addition to providing a natural avenue for developing wonder, the appropriate use of the created world in CCE mathematics education helps maintain a balance between application and theory. As a way to establish this balance, CCE should emphasize the interconnectedness between Creation and mathematics. Part of this interconnection originates in the classical approach to mathematics which included two "arts"-astronomy and music-that focused on how numbers appeared within the context of physical Creation. Modern CCE, unfortunately, mostly remains silent on this point. Perhaps this silence results from a lack of discussion about mathematics. On the other hand, non-classical Christian educators affirm the importance of a Creation-oriented focus, linking a study of creation to motivation as well as to natural opportunities for application of mathematical principles. For many of these authors, Creation forms the starting point for their instruction. In a CCE context, however, focusing on Creation can occur at any of several points in the educational process. Regardless of its location, the presence of relevant aspects of the created world should help provide a balance between
application and theory.
We see the importance of this balance in all of the approaches to mathematics. In medieval classical education, applications of number theory and astronomy to the calculation of the date of Easter and the analysis of music in terms of numerical relationships demonstrate the value of connecting theory with application. The modern CCE approaches likewise recognize the value of putting knowledge to use, particularly in the area of citizenship. Littlejohn and Evans note that to be an effective citizen requires the ability to understand mathematical theory and translate it into action. Similarly, the pedagogical mode of the trivium calls application "rhetoric," which in this context means the eloquent expression of mathematical ideas. This "mathematical rhetoric" comes in abstract and practical forms. In addition, more support for the balance of theory and application comes from the non-classical educators. Most of them consistently discuss the need for a reality-oriented approach to Christian mathematics education, an approach that naturally includes application. Application, of course, is impossible without corresponding theory; thus there must be a balance between the two.

Fifth, CCE aims to use appropriate primary source material while also teaching mathematics relevant to contemporary needs. The inclusion of primary sources comes primarily from the various CCE approaches, especially the Great Books tradition, which advocates heavy use of primary sources. In CCE mathematics education, however, extensive use of primary source material is neither practical nor recommended. Many classical works exist only in Latin. Those that are in English are often outdated or incomplete due to improvements that have occurred since they were written. More recent mathematical works, those from the nineteenth and twentieth centuries, can be highly technical in nature and are often too advanced for K -12 students. The challenge, therefore, of incorporating primary source material in mathematics is
great. Doing so, however, might fit with integrating some history into a CCE mathematics program. To incorporate appropriate historical material successfully, however, teachers and curriculum designers will need not only to know the history of mathematics better but also to exercise caution in selecting works appropriate for the developmental level of the students. Nevertheless, if educators carefully select translated excerpts of the best mathematical works of the past, primary materials can form an important part of the CCE mathematics curriculum. A desire to incorporate historical documents, however, should not prevent CCE math teachers from teaching the content most relevant to the needs of the students as they function as citizens within contemporary society and God's kingdom. Some areas of mathematical instruction that medieval scholars considered essential (use of an abacus, for example, or the calculation of the date of Easter) are now irrelevant, but more recent developments in mathematics (algebra, coordinate geometry, and data analysis, for instance) have become increasingly important. While the classical focus of CCE encourages teachers to give history its due, non-classical, Christian mathematics educators remind us that a mathematics that emphasizes only the "best of the past" will not provide sufficient mathematical training for students preparing to serve the Lord in today's society. Thus, CCE mathematics education should consist of a balanced approach between primary sources and relevant content.

Finally, CCE mathematics education should teach the necessary mathematical "tools of learning." These tools of learning include not only the memorization of basic facts but also highlevel mathematical reasoning skills. The inclusion of memorization counters a recent trend of mathematical instruction in the opposite direction. In the younger grades in particular, the memorization of facts provides an important tool of learning for students in later years. The inclusion of memory in the younger grades draws support partially from the developmental mode
of the trivium, which, despite its flaws, suggests that educators not overlook the value of memorization in learning. The pedagogical mode also provides a reminder of the importance of core concepts ("grammar" in CCE terms) at all levels. In the case of mathematics, this grammar includes the memorization not only of basic facts but also of many foundational properties in various mathematical disciplines. For example, in high school trigonometry, students in a CCE classroom would memorize the basic trigonometric identities in order to work with applications or prove more complex identities. Memorization, therefore, helps students develop an important tool of learning (memory). It also provides students with critical content needed for dealing with later, more advanced mathematical functions. Thus, memorization forms an important part of the CCE approach to mathematics education.

While memorized facts are an important tool, learning mathematics requires more skills than knowledge of mathematical facts. Students also need to develop strong mathematical reasoning skills, excellent mathematical communication skills, and a good level of comfort with using technology. Building upon the pedagogical mode of the trivium (the concept of "logic" in particular), the classical goal of creating students who could be autonomous learners, and the arguments given by several authors for training in mathematical reasoning skills, CCE must include the teaching and development of higher-level mathematical reasoning skills. It is these skills that, when coupled with the memorization of basic information, provide students with two important abilities necessary for success in learning and using mathematics. Teaching mathematical reasoning skills, then, must complement memorization as tools of learning in a CCE mathematical classroom.

In addition, good communication skills comprise another important tool of learning for students. Communication functions as a tool of learning because students learn a concept better
when required to explain it in a way that others understand. Support for this idea derives from considering how the classical concept of rhetoric might look in a mathematical context. Certainly, mathematical rhetoric includes the application of mathematical principles, an aspect already discussed. In a mathematical context, however, rhetoric also means clear communication of mathematical ideas with others. Students should learn how to explain mathematical concepts not only in appropriate technical language but also using non-technical phrasing. For example, students learning geometric proofs should be able to provide proper mathematical justifications in a recognized mathematical format. At the same time, they also should be able to explain clearly in "layman's terms" the various aspects of the proof and its implications.

Finally, technology is a critical tool of learning that should be included in a CCE model, even though the above literature does not address it much. The support that exists for the inclusion of technology comes from the medieval use of mathematical technologies such as the abacus and surveying instruments. Students in the Middle Ages sometimes learned how to use such devices to assist in their study and application of mathematics. While the devices used have changed, the importance of technology in all aspects of life has increased; therefore, students must learn how to use this technology (calculators, graphing calculators, and computers, especially) well. The use of technology in CCE mathematics education should be limited in the lower grades in order to allow students to learn the skill of memorization. In the higher grades, though, students should use technology with increasing regularity, with the older students, who more frequently work with complicated real-world data, using it most often.

As we have seen, CCE mathematics education should contain a number of components, some of them existing in careful balance. First and foremost is the creation and nurturing of disciples. In addition, CCE mathematics education should aim to cultivate wonder of God and
mathematics; appropriately incorporate the created world through examples and investigations; create a balanced approach to application and theory partly by studying the created world; use appropriate primary source material while still teaching content relevant to the circumstances of contemporary society; and train students in the necessary tools of mathematical study, including memorized facts, reasoning and communication skills, and use of technology. While the above facets do not constitute the whole of CCE, they form a significant portion of the philosophical basis for a classical, Christian approach to mathematics education.

## Conclusion

Classical, Christian education, which has its roots in the medieval approach to classical education, has a thoroughly-developed philosophy of education in many content areas. On mathematics, however, it has been mostly quiet. This silence is on the one hand surprising, given that four of the seven core components of medieval education were mathematical in nature. On the other hand, since classical education retreated into the humanities upon the advent of modern education and since mathematics has progressed immensely since then to encompass a vastly expanded and transformed subject area, the lack of development is understandable.

This thesis aims to open a conversation about the philosophical basis for mathematics instruction in classical, Christian education. Using insights from Christian mathematics educators and building upon the role of the quadrivium in medieval times as well as the content mode of the trivium, we developed a description of the mathematical content that could make up CCE mathematics. At the same time, the other modes of the trivium, especially the pedagogical mode, provided some structural guidance to ensure that CCE math education had a methodology for instruction, however general. Finally, these general content and structural components were
combined to provide several foundational principles that should help CCE curriculum designers and educators create materials and methods appropriate for CCE schools.

More components to this area exist than a thesis such as this could cover, and no practical methods have yet been identified for applying the principles mentioned above. Certainly, the CCE community needs to engage in greater thought and dialogue regarding this neglected area of its approach to education. Only by combining the wisdom and talents of many gifted men and women can CCE mathematics instruction rise to the level to which it aspires: cultivating disciples with a Biblical worldview who will go on to make significant contributions to the Kingdom of God via use of their mathematical abilities.

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[^0]:    ${ }^{1}$ For a thorough history of classical education in the Western world, the reader can consult Abelson (1939), Marrou (1956), Wagner (1983), and Hart (2006).

[^1]:    ${ }^{2}$ Columbia University perhaps started this trend in 1919 with an interdisciplinary course on Western Civilization (ibid.)

[^2]:    ${ }^{3}$ This is not to say that classical education was sacred education. Rather, it is simply stating that the two dichotomies existed simultaneously for a period of time, until the modern superseded the classical and the secular replaced the sacred.

[^3]:    ${ }^{4}$ A "Great Books" program is one that emphasizes those works with have stood the test of time. Students read "classical" works going back far into antiquity and continuing up to the late $19^{\text {th }}$ century

[^4]:    ${ }^{5}$ Sayers make no reference to a stage prior to age 9 . This omission indicates how rough her initial thoughts were, implying freedom to deviate from them as necessary.

[^5]:    ${ }^{6}$ The author is indebted to Brian Williams, theologian in residence at First Presbyterian Church in Topeka, KS, and recently of Cair Paravel Latin School, for his help in clarifying this view of the trivium

[^6]:    ${ }^{7}$ As this thesis was being finalized, Howell and Bradley released a new book, Mathematics through the Eyes of Faith. Time did not permit consultation of this work.

[^7]:    $\left.\right|^{8}$ Byl later would expand on these ideas in a 2004 book $_{2}$ Divine Challenge: On Matter, Mind, Math, and Meaning.

[^8]:    ${ }^{9}$ The end of formal schooling in view here is high school, as independent learning at the undergraduate level is highly desirable, albeit increasingly rare.

