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Incorporating Different Number Bases into the Elementary School Classroom.

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Incorporating Different Number Bases into the Elementary School Classroom

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by

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ABSTRACT

Incorporating Different Number Bases into the Elementary School Classroom

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Since becoming an educator and gaining extensive classroom experience, I have concluded that it would be beneficial to elementary school children to learn other number bases and their basic functions and operations. In this thesis, I have developed five units involving lesson plans for incorporating various number bases into the existing curriculum. The units are Decimal (Base 10), Duodecimal (Base 12), Quinary (Base 5), Binary (Base 2), and Octal (Base 8) which are all appropriate for the elementary level.

DEDICATION

I would like to dedicate this work to my family. They have always been there for their "math nerd." This is especially dedicated to my mother, who taught me what it means to be a strong, independent woman in the world. She also never settled for more than the best from me. To my students, who bravely volunteered to be my guinea pigs for each lesson, you are an inspiration and a challenge.

Last, to my Lord and Savior, Jesus Christ, in Him I am a new creation and from Him I draw my strength. "And He said to me, 'My grace is sufficient for you, for My strength is made perfect in weakness.' Therefore most gladly I will rather boast my infirmities, that the power of Christ may rest upon me…For when I am weak, then I am strong." (2 Corinthians 12: 9-10 NKJ).

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1 INTRODUCTION

The following lessons are designed for classroom teachers. Each includes an introduction to the base being discussed, lesson plans, and sample worksheets. According to John McLeish in, The Story of Numbers: How Mathematics Shaped the Universe, "The binary assumption, which in computer science says that switches can be either ON or OFF, suggests that people [students] might also be divided into two kinds: those who are 'turned on' or excited by numbers and those who are depressed by them. The second group is the vast majority"(1). It is with this idea that we approach the most daunting of all forums, the elementary school classroom.

1.1 The Premise

As an elementary school math teacher I have noticed that students, early in their math careers, enjoy studying the subject of mathematics. In my experience, by the time they reach the fifth grade, their "switch" has been flipped, meaning they no longer find joy in mathematics. Sadly, most are permanently stuck on OFF.

As an educator I have wrestled with this problem. How can I make math more enjoyable? How can I flip their switch back to ON? How can I help to make math a favorite subject of my students? I am continually looking for innovative ideas and lessons. With each state saturated with high stakes testing and No Child Left Behind regulations, many educators, like me, are finding the proverbial pond of new ideas dry. Children are becoming percentages. Educators are, increasingly, looking at students as test scores rather than learners with needs (Gross 12). Politics aside, students must be challenged to think. They have to be able to reason beyond "2+2=4." It is with this in mind that I chose my topic and present the following series of units. These units are meant to be a supplement to the current curriculum and provide a new way for students to look at numbers at an early age.

1.2 Selecting the Units

I chose to limit my lessons to the fifth grade, although, they could be used at a lower or higher level. Each base was chosen because it complemented a certain strand of the fifth grade objectives. I narrowed my units to five. The units are: Decimal (Base 10), Duodecimal (Base 12), Hand (Base 5), and Binary (Base 2), and Octal (Base8).

Each unit begins with a brief introduction and history of the selected base. The unit then moves to a description of counting in that base and provides a sample lesson plan, complete with reproducible worksheet and answer key. From there, the unit continues to computation, which I have limited to include only addition. This section also contains a description of addition in that base, sample lesson plan, practice worksheets, and answer key.

The units themselves are designed to be used independently or in sequence. It is recommended that these units be used throughout the school year in conjunction with the current curriculum.

2 ROTE MEMORY

An issue certainly arises when introducing the subject of bases into an elementary curriculum, and that is the place of rote memorization in the classroom. According to an 1850 edition of Noah Webster's dictionary, "rote" is defined as "to fix in memory by means of frequent repetition" (Webster 567). A search at Webster Online (www.webster.com) defines "rote" as:

1. the use of memory usually with little intelligence.
2. routine repetition carried out mechanically or unthinkingly.

Many of my fellow educators today shy away from rote learning. It has become outdated and ineffective to some circles (Hawkins 212). In truth, rote learning has its place in any learning environment. Take the subject of history. As an educator, I have observed many students today cannot recall even the most basic historical facts and dates. Why? They either were not made to memorize the fact because it interrupted the "true learning" of the event, or they chose not to memorize the fact because they found it to be useless. According to a March 2002 article in the New York Times, "A 1999 survey asked college seniors questions like who led our troops at Yorktown; the most common answer was Ulysses S. Grant. In 1986, the government tested 17-year-olds in history; most said the Civil War occurred before 1850" (Rothestein 2). As a result, we have produced generations of Americans who have never memorized much of anything. The same is true in the mathematics classroom. Currently we have thousands, maybe millions, of American school children who cannot provide the answer to eight times seven(Hawkins 213). Progressive educators say that the memorization of facts is outdated and children should be able to learn them in their own way (Slavkin 18). Putting a calculator in front of a child may produce a correct answer, but the child has no way of knowing if what he/she is seeing is correct. In my classroom I have noticed it is harder for a child to go from concrete to abstract mathematics without having basic mathematical facts in memory.

Of course rote memory has its drawbacks. It is a process that begins in your short-term memory. Short-term memory is roughly synonymous with working memory ("Rote Memory" par. 3). As you try to understand the words on this page, your ability to understand these concepts depends on your working memory and your long-term memory.

Short-term memory only lasts about thirty seconds (par. 4). The newly acquired information could be lost if you had not already committed it to long-term memory (par. 4).

Many students study using rote memorization. However, this may not be the best way, especially if the exam is only minutes away. It is not reliable. Studies show that rote memorization is not the most effective way to move information from short-term to long-term memory (Slavkin 19).

Therefore, educators should balance their lessons. Rote memorization is not bad in the classroom, but relying solely on rote memorization is a bad classroom practice. Children must be taught the reasoning behind mathematical facts. Then they should be exposed to many different ways of manipulating that fact. This is true learning. For example, think of a kindergarten child who is just beginning to learn to count. He/she recite the numbers in order from one to twenty. The parents beam. They have the smartest kid on the block. Say that same child gets to kindergarten and the teacher asks him/her to get out two crayons. The child pauses, not sure what to do. Guessing, the child produces four crayons. This is a child that has the number names committed to memory, yet has no concept of what the number two really means. He/she memorized the song with repetition and made his/her parents proud, yet had no real understanding of the numbers. These scenarios are not uncommon in elementary school. As a teacher, I have seen them replayed over and over again, especially in kindergarten and first grade. These problems begin to show themselves as early as kindergarten registration. At registration students are asked to count objects and recite as many numbers in order as they can. Some potential students can recite the numbers and cannot count the objects without difficulty (Gross 50).

Children need a balance of both rote learning and learning via manipulatives included in their curriculum. The purpose of lessons on different number bases is to reinforce place value concepts already in place. It also promotes critical thinking skills, not just rote memorization. A child may tell you a number is in the hundreds place without knowing what the hundreds place means. That same child will not be able to rely on memorization of the places to manipulate numbers in a different base. The rules change. It will then cause the student to think in a different manner, one that will stretch ability to think and hopefully reinforce some basic base 10 principles.

3 NECESSARY TOOLS

3.1 What is a base?

First we must begin with the definition of "base." The base of a system of numerals is the number that determines the place values for all of the numerals in that system ("Base," 1). For example, in Base 10, 234 equals 2 times 100 (10 squared), 3 times 10 (10 to the first power), and 4 times 1 (10 to the zero power). In Base 8, 234 equals 2 times 64 (8 squared), 3 times 8 (8 to the first power), and 4 times 1 (8 to the zero power). Any positive integer greater than one can be used as a base. You can express any number in any base.

3.2 Base Conversion

It is sometimes necessary to convert a number to another base. Most of the time one could do that by using a calculator; however, it is important to know how to do the same operation by hand. There is a trick you can use and with some practice, you will not have to write the steps down.

Let us begin with the number 138 in Base 10 and we are going to convert that number to Base 5.

1. $138 = a * 5 + b$

2. Divide 138 by 5. The quotient is "a" and the remainder is "b."

$138 = 27 * 5 + 3$

3. Now repeat only this time divide 27 by 5.

$27 = 5 * 5 + 2$

Repeat until a = 0

$5 = 1 * 5 + 0$

$1 = 0 * 5 + 1$

4. If you put all of these calculations together, the remainders from bottom to top give the number in Base 5. (The answer is in bold)

138 = 27 * 5 + **3**

27 = 5 * 5 + **2**

5 = 1 * 5 + **0**

1 = 0 * 5 + **1** 138 in Base 5 is 1023!

This is a simple trick that any child on the fifth grade level or higher could master. As an added bonus, it provides basic computation practice as well. This method is, however, a rote form of base conversion.

Keep in mind that this conversion formula is a tool to be used after an understanding of the base has been established. It is important for children to understand the place value aspect of each base before you move into an abstract way to convert. Each lesson contains a counting chart and alternate methods of base conversion, methods that focus more on place value.

3.3 Technology

There are times in the elementary classroom where we want to introduce a skill without the students getting bogged down in the arithmetic. When that happens, it is useful to have a calculator or online applet that will allow the students to find answers or to simply check their answers. Listed below are websites for a base calculator and a base converter. Both will prove useful throughout the lessons.

Base Calculator

http://www.psinvention.com/zoetic/k12addm.htm

Base Converter

http://www.cut-the-knot.org/binary.shtml

4 DECIMAL (BASE 10)

4.1 Introduction

The Decimal (Base 10) system is one of the most commonly used today. It is assumed to have originated because we have ten fingers. People could count on their fingers up to ten, put a mark in the sand, and continue counting on their fingers (“Base Valued Numbers,” 1). However, some cultures do or did use other number bases. The Tzotil use a Base 20 system (using all 20 fingers and toes), the Nigerians use a Base 12 system, and the Babylonians formerly used a Base 60! (“Decimal,” 1)

The Indian culture is credited with developing the decimal system. The Mohenjo Dan culture of the Indus Valley was using a form of decimal numbering some 5,000 years ago (“History of Decimal Writers,” par. 2). Numerous cultural changes in this area developed the decimal system into a rigorous numbering system, including the use of zero by the Hindu mathematicians almost 1,500 years ago (“Numeral Systems,” 2).

The symbol for the digits used around the globe today are called Arabic numerals by Europeans and Indian numerals by Arabs, both referring to the culture from which the system came (“Base 10,” 1). However, the symbols used in different areas of the world are not identical. The forms of “our” numbers differ from those in Arab cultures. The following timeline was adapted from an online media source: Wikipedia, the free encyclopedia.

Decimal Writer Timeline

Decimal writers

- *c.* 3500 - 2500 BC Elamites of Iran possibly use early forms of decimal system. [2] [3]

- *c.* 2900 BC Egyptian hieroglyphs show counting in powers of 10 (1 million + 400,000 goats, *etc.*).
- *c.* 2600 BC Indus Valley Civilization, earliest known physical use of decimal fractions in ancient weight system: 1/20, 1/10, 1/5, 1/2. See Ancient Indus Valley weights and measures.
- *c.* 1400 BC Chinese writers show familiarity with the concept: for example, 547 is written 'Five hundred plus four decades plus seven of days' in some manuscripts.
- *c.* 450 BC Panini uses the null operator in his grammar of Sanskrit
- *c.* 400 BC Pingala develops the binary number system for Sanskrit prosody, with a clear mapping to the base-10 decimal system.
- *c.* 476–550 Aryabhata used an alphabetic cipher system for numbers that used zero.
- *c.* 598–670 Brahmagupta – decimal integers, negative integers, and zero
- *c.* 790–840 Abu Abdullah Muhammad bin Musa al-Khwarizmi – first to expound on algorism outside India
- *c.* 920–980 Abu'l Hasan Ahmad ibn Ibrahim Al-Uqlidisi – earliest known direct mathematical treatment of decimal fractions
- 1548/49–1620 Simon Stevin – author of *De Thiende* ('the tenth')
- 1561–1613 Bartholemaeus Pitiscus– (possibly) decimal point notation
- 1550–1617 John Napier– decimal logarithms ("Decimal Timeline" sec. 3).

4.2 Counting in Base 10

The digits we use are Arabic/Indian. They are: **0, 1, 2, 3, 4, 5, 6, 7, 8, 9.** We use just those ten digits to construct all of our numbers. We can do this because each number occupies a place value. When "9" is reached, the value goes to zero and "1" is added to the next place value.

Example: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, **10**

When we get to 9, we are out of digits. Therefore, we start over and use a zero in what we call the ones place and add 1 to what is the tens place.

Each place value to the left is equal to ten times the place value to the right. Another way to say it is that each value to the right is equal to the place value to the left, divided by ten. A zero is used to represent nothing of a particular place value.

A good way to provide practice and reinforcement in place value in the elementary classroom is to use Base 10 blocks. These blocks can be purchased and used as a hands-on manipulative in the classroom or they can be accessed through numerous online applets. The base 10 block applet used on the following page can be found at the following address: http://www.arctech.org/java/b10blocks.htm.

The figure below (figure 1) is a great example of the technology available. It is recommended that technology such as this be used as a supplement.

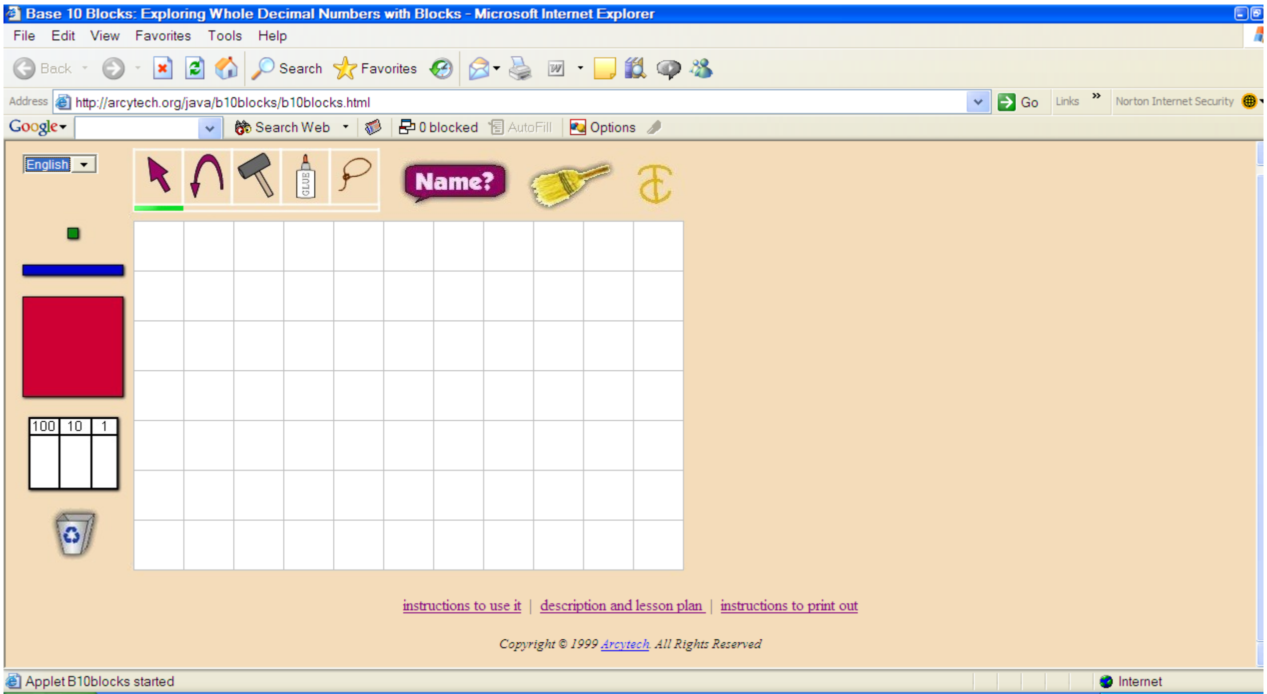

Figure 1. Base 10 Applet

4.3 Use of the Base 10 applet

The program itself is user friendly and students on a fifth grade level should have no trouble navigating through its various uses and commands. As you can see from the previous page, the colored blocks on the left are the manipulative themselves. These blocks can be dragged into the work space and there is no limit on how many you can drag over. Figure 2 shows a short description of the blocks used in this program:

(Note: these descriptions were taken and adapted from the original website)

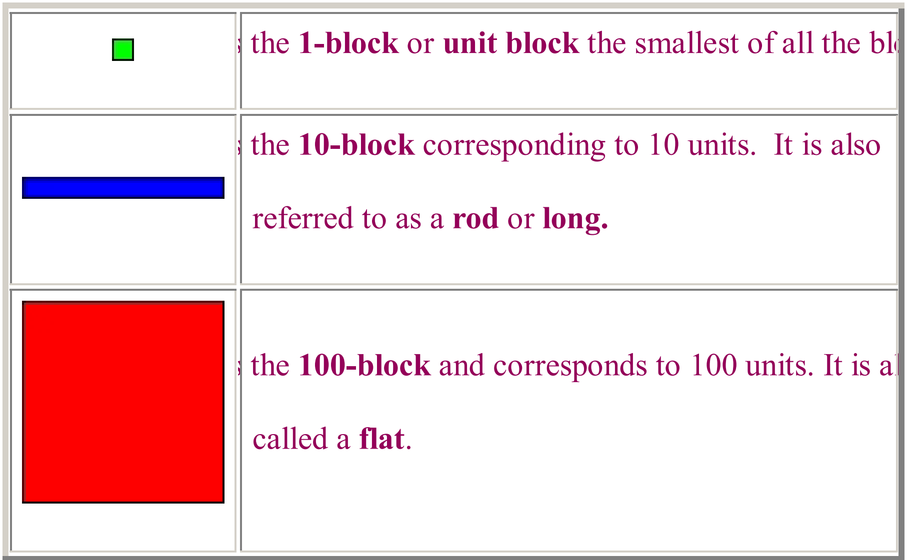

Figure 2. Applet Instructions

It is essential to discuss these blocks and their meaning and uses before beginning a Base 10 block activity. Typically teachers like to give their students time to "play" with the manipulative and develop their own uses for it before telling then how they are actually to be used. This activity is valuable in two ways:

1. It gives the students time to get acquainted with the symbols while satisfying their curiosity.
2. It allows the teacher to assess what the skill level of each student is. If a student begins grouping the blocks and creating numbers, it gives a good indication of understanding.

4.4 Lesson Plan: Place Value: The M&M King

Grade Levels: 2nd—8th

Objective: Through hands-on and role play activities, the student will "discover" and develop the concept of place value in a Base 10 system.

Materials:

- Individual packages of M&M candies (numbers will vary with class sizes)
- Dry erase board or overhead projector

Before you begin:

This activity is designed to allow the students to explore and discover place value. I typically complete this activity BEFORE I continue on with the Base 10 blocks. It is an activity that serves well as an introduction to a lesson. With older students the challenge is to get them NOT to count the way that they are used to. Counting has been so ingrained into them this is not always an easy task. I usually tell them to pretend that they have no knowledge of how to count and are about to learn a useful method.

Procedure:

1. Place students in groups of five.
2. Divide the M&Ms among the groups. Tell them that are not allowed to eat any at this time. (Make sure you are well protected, this can cause a riot!)
3. Read them the story of the Rice King. (attached to lesson plan)
4. Have the students recreate the Rice King's methods of counting with the M&Ms.
5. Have the groups come up before the class and demonstrate the method of counting.
6. Now, allow them to go back to their groups and devise a new method of counting using the M&Ms. This provides a stepping stone for discussion of other bases.

Questions to ask the students:

1. Why do you think that we have only ten digits in our number system? How could this be useful?

2. Why does the first person have no fingers raised? What number does this represent? WHY?

After allowing students to answer the above questions, allow the students to explore various applets on the computer. If possible, use the computer lab within your building so students can work simultaneously. If that is not possible, allow students to work in groups using a classroom computer.

4.5 Attachment: The Rice King (or the M&M King!)

A long, long time ago, before we had a written numerical system, there lived a king. The king ruled a land that was rich and fertile and produced an abundance of rice. The people of the land were required to pay taxes to the king in the form of sacks of rice. Once a month, on the first day of that month, the people would arrive very early to the castle with many sacks of rice. They would stand in line to deliver the sacks into a large pile. The king, being selfish, wanted an accurate count of the number of rice sacks he collected, this way he could always know how much he owned. The priests thought and thought for days but could not come up with an answer. Finally, as they were walking past a village they stumbled upon the answer. A young boy was counting pebbles. After he had used all of his fingers, he would make a mark in the dirt and start all over again. This gave the priest an idea for the king. They rushed back to tell the king their news. On the day that the rice was to be counted, three priests stood by the king. As the farmers started to bring their rice, the first priest began counting on his fingers. When he ran out of fingers, the second priest would hold up one finger. This symbolized the number of times that the first priest had run out of fingers. On a good day, the second priest would run out of fingers and the third would have to begin counting. He could hold up a finger every time the second priest could count no further. And on and on it went for the day. At the end of the day, they would capture finger counts in the form of special symbols, one for each priest and a different symbol for the number of raised fingers. Legend has it that this was the start of the base 10 system.

Note: This tale was adapted from the book, <u>One Grain of Rice: A Mathematical Folktale</u> by Demi. It was shortened for classroom use.

4.6 Practice Worksheet

Write the following in expanded form (Base 10):

1. 123
2. 56
3. 7,843
4. 108, 938

Name the place of the underlined number:

1. 2,3<u>4</u>5
2. <u>8</u>,930
3. 42<u>7</u>
4. <u>8</u>9,003

Write the following in standard form:

1. 100 + 20 + 4

2. 5,000 + 400 + 90 + 1

3. 1,000,000 + 7,000 + 30 + 6

4. 300 + 9

4.7 Answer Key

1. 100 + 20 + 3

2. 50 + 6

3. 7,000 + 800 + 40 + 3

4. 100,000 + 8,000 + 900 + 30 + 8

1. Tens

2. Thousands

3. Ones

4. Ten thousands

1. 124

2. 5,491

3. 1,007,036

4. 309

4.8 Addition in Base 10

Addition in the Base 10 system is a common strand in almost every elementary school grade. The following is a simple explanation and sample lesson plan to promote fluency. Addition is the "putting together" of two groups of objects and finding how many in all. Using a place value chart is a great way of ensuring a digit gets into the correct place. For example, to solve an addition problem, model the addends in the place value chart, regroup the blocks as needed, and then write the answer in standard form. The following example shows 135 + 278. Figure 3, shown below, is an example of an addition chart using Base 10 blocks.

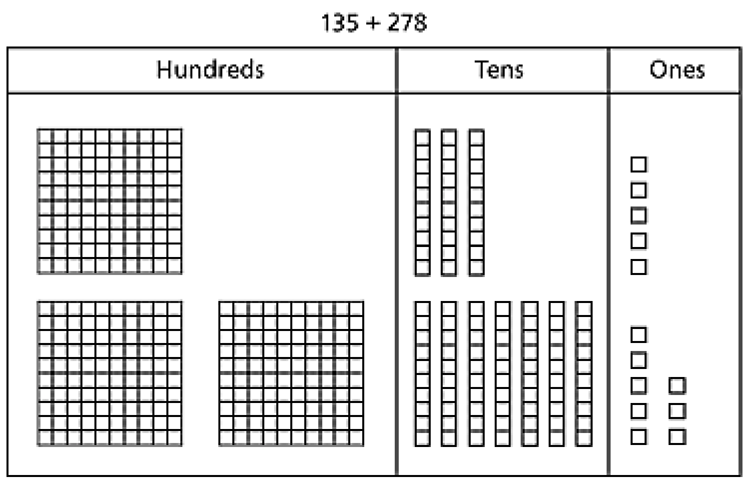

Figure 3 Base 10 Blocks Chart

The sum 413 is now shown. A through mastery of regrouping is essential because it is also used in other operations.

4.9 Lesson Plan: Race to 100

Grade level: 2nd—5th

Materials Needed: die, Base 10 blocks

Procedure:

1. Place the students in pairs.

2. Have each partner roll the die and the one who rolls the highest goes first. If a tie arises, the players must roll the die again.

3. The players will take turns rolling the die, adding each roll onto their previous amount using Base ten blocks and racing to reach 100.

For example:

Player 1 will roll the die. Let's say player 1 rolls a 5, he/she will take 5 single blocks, showing what their score was. Player 2 does the same. Now it is again the turn of Player 1 and let's say he/she rolls a 6. The goal is for students to mentally recognize that they now put back their 5 singles and trade for one tens block and one single. This play continues until one player reaches one hundred.

Extensions and Adaptations:

This game could be played "Race to a Dollar," using pennies, dimes, and dollars. The game could also be played to 1,000 and the students could also graduate from the blocks to paper and pencil addition.

4.10 Addition Worksheet

Solve for the sum:

1. 291 + 165 = _______

2. 189 + 478 = _______

3. 1,580 + 9,875 = _______

4. $5{,}239 + 6{,}346 =$ ______

5. $45{,}234 + 56{,}343 =$ _____

Find the missing value:

1. $567 + 56 + g = 1{,}543$

2. $78{,}432 + a = 85{,}543$

3. $h + h + 762 = 8$

4.11 Answer Key

1. 456
2. 667
3. 11,455
4. 11,585
5. 101,577

1. g = 920
2. a = 7,111
3. h = 68

5 DUODECIMAL (BASE 12)

5.1 Introduction

Twelve is a prolific number. Several examples follow that feature twelve as a central theme.

Time

The clock itself has twelve hours displayed on its face. There are sixty minutes in an hour and sixty seconds in a minute. Sixty is divisible by twelve. There are twenty-four hours in a day, another multiple of twelve. And, of course, there are twelve months in a year.

Degrees

There are three hundred sixty degrees in a circle, a multiple of twelve.

Religion

From a Christian standpoint, twelve is the number of the Apostles of Jesus. Twelve is also the number of signs in the pagan Zodiac.

Practical

Twelve is the amount in a dozen, the most common amount for doughnuts and eggs (to name a few). As a practical matter, people tend to divide groups into thirds and quarters. Both are factors of twelve. A dozen is the smallest that allows one to do this.

5.2 Counting in Base 12

In a system with more than ten numerals, such as the duodecimal, we need to add some other characters in order to count. Therefore, to count in the duodecimal system, the digits look like this:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B

Table 1 shows how some numbers would match up with our familiar Base 10 system:

Table 1. Base 12 Counting Table

Base 10	Base 12	Base 10	Base 12	Base 10	Base 12
0	0	17	15	34	2A
1	1	18	16	35	2B
2	2	19	17	36	30
3	3	20	18	37	31
4	4	21	19	38	32
5	5	22	1A	39	33
6	6	23	1B	40	34
7	7	24	20	41	35
8	8	25	21	42	36
9	9	26	22	43	37
10	A	27	23	44	38
11	B	28	24	45	39
12	10	29	25	46	3A
13	11	30	26	47	3B
14	12	31	27	48	40
15	13	32	28	49	41

5.3 Lesson Plan: Counting in Base 12

Grade Level: 4^{th}—8^{th}

Objective: Use manipulatives and real world examples to model counting in Base 12.

Materials: none

Procedure:

We will begin by teaching the children to count in Base 12 on their fingers. This is a way that they are already comfortable with, why not use that to the advantage?

Have the children curl their fingers of their left hand so they can see the tops of the knuckles. The four tips of the fingers make up one row. The knuckles closest to the tips make a second row. The next set of knuckles make the third row. (You cannot see the knuckles that merge into the back of the hand.) Point out to the children that altogether you see three rows of four, which make twelve. Use the opportunity to point out patterns of halves, thirds, and quarters within a dozen.

Have the children create a counting chart in Base 12. Table 2 is an example chart.

Table 2. Base 12 Counting Table

Base 10	Base 12	Base 10	Base 12	Base 10	Base12
0	0	17	15	34	2A
1	1	18	16	35	2B
2	2	19	17	36	30
3	3	20	18	37	31
4	4	21	19	38	32
5	5	22	1A	39	33
6	6	23	1B	40	34
7	7	24	20	41	35
8	8	25	21	42	36
9	9	26	22	43	37
10	A	27	23	44	38
11	B	28	24	45	39
12	10	29	25	46	3A
13	11	30	26	47	3B
14	12	31	27	48	40
15	13	32	28	49	41

After the children have completed their chart, they can begin practicing conversion techniques. The chart provides a way to convert from Base 10 to Base 12, using Base 10 as a "crutch."

For example, if I wanted to know what the number 40 (Base 10) was in Base 12, I would only need to count to the fortieth number in the Base 12 sequence. Using the chart, it is easy to tell that 40 (Base 10) is 34 (Base 12). Of course, this method may not be the best if a student is asked to convert a large number. We would have to find another way.

Part 3: Use the conversion formula to covert from Base 10 to Base 12.

If this is the first time that the students are using the formula, then go slowly outlining the steps.

Example: Convert 345 (Base 10) to Base 12

345 = a * 12 + b

345= 28 * 12 + 9

28 = 2 * 12 + 4

2 = 0 * 12 + 2

Recall that the list of remainders from bottom to top is the answer. For example, 345 base 10 = 249 base 12.

Other examples to convert to Base 12:

540

875

1,278

5.4 Practice Worksheet

Convert the following from Base 10 to Base 12:

1. 543
2. 691
3. 4,039
4. 2, 002
5. 87,442

Convert the following from Base 12 to Base 10:

1. 45

2. 21

3. 3A

4. 7B

5. 99

5.5 Answer Key

1. 393
2. 497
3. 2,407
4. 1,1AA
5. 42,72A

1. 53
2. 25
3. 46
4. 95
5. 117

5.6 Addition in Base 12

Grade Level: 5^{th}—8^{th}

Objective: Use concrete and abstract examples in order to add Base 12 numbers.

Materials: Graph paper

Procedure:

Begin by showing the children how to construct an addition table in Base 12. It is recommended that graph paper be used for this. The chart itself is easy for children to construct because it is essentially skip counting, a skill that they have been practicing since kindergarten. An example chart appears in Table 3.

Table 3. Base 12 Addition Table

	0	1	2	3	4	5	6	7	8	9	A	B
0	0	1	2	3	4	5	6	7	8	9	A	B
1	1	2	3	4	5	6	7	8	9	A	B	10
2	2	3	4	5	6	7	8	9	A	B	10	11
3	3	4	5	6	7	8	9	A	B	10	11	12
4	4	5	6	7	8	9	A	B	10	11	12	13
5	5	6	7	8	9	A	B	10	11	12	13	14
6	6	7	8	9	A	B	10	11	12	13	14	15
7	7	8	9	A	B	10	11	12	13	14	15	16
8	8	9	A	B	10	11	12	13	14	15	16	17
9	9	A	B	10	11	12	13	14	15	16	17	18
A	A	B	10	11	12	13	14	15	16	17	18	19
B	B	10	11	12	13	14	15	16	17	18	19	1A

Practice is all that it takes to master various bases. The rules are the same as in our standard decimal system. The sum or product of two digits may only produce one or two digit numbers. In the latter case, if necessary, the first digit is carried over to the next column on the left. Use of the table and board examples will help the children understand addition in Base 12. The following are board examples:

135 + 35 =　　　　　　1,257 + 642 =

679 + 409 =　　　　　　8,503+ 3,932 =

Have the children use the addition chart to answer the board examples. Explain that the process is still the same, just different numbers are used.

5.7 Practice Worksheet

1. My age is 23 in Base 12. What is my age in Base 10?
2. What is your age in Base 12?

3. 345 + 322 = __________
4. 567 + 43 = ___________
5. 56 + 76 + 123 = ___________

6. Construct two addition problems and solve them.

5.8 Answer Key

1. 27
2. Answers will vary
3. 477
4. 703
5. 193
6. Answers will vary

6 QUINARY (BASE 5)

6.1 Introduction

The Base 5 numbering system was used primarily before the writing of numbers ("Number Base" par. 2). The use of Base 5 as a grouping value is easily understood as the hand has five fingers. This grouping was used for thousands of years by many cultures around the globe.

The signs or words used were: one hand equal 5, two hands equal 10, one person equals 20 (two hands and two feet). Some cultures would count on fingers with zero being a closed fist ("Number Base" par. 2). Base 5, however, was not formalized as a place value but rather as a grouping value ("Number Base" par. 3).

This is a number base that children take to easily because they are already accustomed to using their fingers to count and perform basic arithmetic. Base 5 can prove to be an easy transition.

6.2 Counting in Base 5

In Base 5 only the first five digits of the Arabic numerals are used.

BASE 5: 0, 1, 2, 3, 4

Therefore, we have no knowledge of the digits 5—9 in a Base 5 world. The students enjoy pretending that they haven't learned those digits, although it proves to be a greater challenge than they anticipate! To the children, this would be equivalent to counting only on one hand. Counting in Base 5 would be as displayed in Table 4.

Table 4. Base 5 Counting Table

Base 10	Base 5	Base 10	Base 5	Base 10	Base 5	
0	0	17	32	34	114	
1	1	18	33	35	120	
2	2	19	34	36	121	
3	3	20	40	37	122	
4	4	21	41	38	123	
5	10	22	42	39	124	
6	11	23	43	40	130	
7	12	24	44	41	131	
8	13	25	100	42	132	
9	14	26	101	43	133	
10	20	27	102	44	134	
11	21	28	103	45	140	
12	22	29	104	46	141	
13	23	30	110	47	142	
14	24	31	111	48	143	
15	30	32	112	49	144	
16	31	33	113	50	200	

In Base 10, there is no single digit that means ten of something. Consequently, in Base 5, there is no single digit that means five of something. Again, this chart is useful in conversion. It is possible to count using the chart until you have achieved the correct Base 5 number.

It is time, however, to introduce another way to convert between bases. The following is a rote form of conversion and uses Base 10 as a guide. It makes it very useful to use with fifth grade students who are still getting a grasp on place value in Base 10.

To explain let us revert to Base 10 and place value. Let us say that you are given the number 547. What does 547 really represent?

It can be broken into a series of values. 547 = seven "ones"+ four "tens" + five "hundreds"

Using this as a model, it is possible to move into Base 5. Instead of ones, tens, and hundreds; we have ones, fives, and twenty-fives.

Example:

Convert 243 (Base 5) into a Base 10 number:

243 (Base 5) = **3** (1) + **4** (5) + **2** (25)

In this example, I have broken 243 into place values. You see that 3x1 + 4x5 + 2x 25 = 73 The result is that 243 (Base 5) is equal to 73 (Base 10). To convert numbers from Base 10 to Base 5, refer to the formula in the third chapter. This formula is good for all base conversions.

Figure 4 is an unusual clock used since 1975 in Berlin, Germany. It is a Base 5 clock. The popular name is “Mengenlehreuhr” or a set theory clock. It displays the hours and minutes in a Base 5 sort of system. The round lamp on top blinks to indicate it is running. The first row shows how many 5-hour blocks have passed since midnight, the second, how many hours in the current 5-hour block. The third row displayed how many 5-minute blocks since the hour, and the bottom row indicates how many minutes in the current 5-minute block. The snapshot was taken at 14:57 (“Set Theory Clock” 1). However, this is the use of Base 5 to get Base 10 answers.

Picture taken from Wikipedia, the free encyclopedia

Figure 4. Base 5 Clock

6.3 Lesson Plan: Base 5

Grade Level: 4^{th}—8^{th}

Objective: Construct a table in Base 5. Count in Base 5

Procedure:

Have students begin by constructing a Base 5 counting table. This table will be helpful throughout the lesson. It is recommended that graph paper be used.

Have the students first practice converting from Base 10 to Base 5 by counting on their chart. For example, if you were to find 10 in Base 5 you would count over to the 10^{th} number on the chart.

Next, have the students convert with place value as covered in the second section of this chapter. With this type of lesson I usually hand out individual dry erase boards and require the students to record their answers and "hide" the answer until I call. After all students have recorded an answer, they turn their boards around simultaneously and I award points to each student with a correct answer. It is a game to accumulate as many points as possible. Also, I have found that students will work longer if they are writing on something other than standard notebook paper. After students have mastered Base 10 to Base 5 conversion, move on to examples in Base 5 to Base 10 conversion.

6.4 Practice Worksheet

Convert from Base 10 to Base 5

1. 14
2. 41

3. 230

4. 133

5. 420

Convert from Base 5 to Base 10

1. 21

2. 110

3. 34

4. 322

5. 212

6. What is your age in Base 5? ________________

7. What are your parents' ages in Base 10? Base 5?

6.5 Answer Key

1. 24

2. 131

3. 1,410

4. 1,013

5. 3,140

1. 11

2. 30

3. 19

4. 87

5. 57

6. Answers will vary

7. Answers will vary

6.6 Addition in Base 5

In addition of numbers in Base 5, you carry "fives" to the left in the same way that you carry "tens" to the left in the Base 10 system. It is important to remember that all of your addition problems can only include digits allowed in the Base 5 system, the digits 0—4.

Example 1:

1,243 Base 5

+_4,243_ Base 5

Begin in the first column. 3+ 3 = 11 (Base 5), so write down 1 and carry 1.

1,243

+ 4,243

1

The next column is then 1 + 4 + 4 = 14 (Base 5): therefore, you write down 4 and carry 1.

That brings us to the next column. Therefore, 1 + 2 + 2 = 10 (Base 5) and write down 0 and carry 1. The last column would then be 1 + 1 + 4 = 11 (Base 5): therefore, write down 11. That brings that answer to 11,041 (Base 5).

6.7 Lesson Plan: Addition in Base 5

Grade: 4th—8th

Objective: Construct an addition chart to add digits in Base 5

Materials: Graph paper

Have the students begin by constructing an addition table for Base 5. Remind the students that these tables are just skip counting. They don't have to be able to add in Base 5 in order to construct the table. Table 5 is an example for addition in Base 5.

Table 5. Base 5 Addition Table

	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	10
2	2	3	4	10	11
3	3	4	10	11	12
4	4	10	11	12	13

6.8 Practice Worksheet

Add. Give answers in Base 10 and Base 5

1. 4 + 3 = __________
2. 23 + 64 = ________

3. 126 + 432 = _______

4. 493 + 794 = _______

5. 56 + 43+ 234 = __________

Construct two addition word problems and solve them in Base 10 and Base 5.

6.9 Answer Key

1. Base 10 = 7; Base 5= 12

2. Base 10= 87; Base 5= 322

3. Base 10= 558; Base 5= 4,213

4. Base 10= 1,287; Base 5= 20,122

5. Base 10= 333; Base 5= 2,313

Word problems and answers will vary. Check individual work.

7 BINARY (BASE 2)

7.1 Introduction

Binary numbers are also known as Base 2. This is because each digit can only be one of two digits, i.e. zero or one. Computers operate on this kind of system.

A transistor, which is the basic unit of a computer, is a switch that can be wired to respond to an input by either turning on or off. "ON" represents the digit 1, and "OFF" represents the digit 0 ("Binary and the Computer" par 2). Wired together the right way, they can add. Wire the adders together correctly and computers can do infinitely more with just these two switches. A programmer decides on a certain plan and the computer just manipulates the bits according to the plan. If programmed correctly, you get the information you want! Here is an example of a binary number: 10011100

As you can see, it is quite simply a bunch of zeros and ones. In the example above, there are eight numerals that make an 8-bit binary number ("Binary and the Computer" par. 3). Bit is short for Binary Digit.

7.2 Counting in Base 2

Remember that the only digits you can use are 0 and 1. Each digit occupies a place value. When 1 is reached, the value goes to either 0 or 1 and is added to the next place value. Each place to the left is equal to two times the place value to the right. In other words, each place value to the right is equal to the place value to the left divided by two. To count, refer to Table 6.

Table 6. Binary Counting Table

Base 10	Base 2	Base 10	Base 2
0 16	100000	17 33	10001111 100001
1	1	18	100011
2	10	19	100111
3	11	20	101111
4	100	21	111111
5	101	22	1000000
6	111	23	1000001
7	1000	24	1000011
8	1001	25	1000111
9	1011	26	1001111
10	1111	27	1011111
11	10000	28	1111111
12	10001	29	10000000
13	10011	30	10000001
14	10111	31	10000011
15	11111	32	10000111

To understand binary numbers, begin by recalling elementary school math. When we first learned about numbers, we were taught that in the decimal system things are organized into columns:

H | T | O

1 | 9 | 3

such that "H" is the hundreds column, "T" is the tens column, and "O" is the ones column. So the number "193" is 1-hundreds plus 9-tens plus 3-ones.

Years later, we learned that the ones column meant 10^0, the tens column meant 10^1, the hundreds column 10^2 and so on, such that:

10^2|10^1|10^0

1 | 9 | 3

193 is really {(1*10^2)+(9*10^1)+(3*10^0)}.

The decimal system uses the digits 0-9 to represent numbers. If we wanted to put a larger number in column 10^n (e.g., 10), we would have to multiply 10*10^n, which would give 10^(n+1) and be carried a column to the left. For example, putting ten in the 10^0 column is impossible, so we put a 1 in the 10^1 column, and a 0 in the 10^0 column, thus using two columns. Twelve would be12*10^0, or 10^0(10+2), or 10^1+2*10^0, which also uses an additional column to the left ("Binary" sec. 1).

The binary system works under the exact same principles as the decimal system, only it operates in base 2 rather than base 10. In other words, instead of columns being

10^2|10^1|10^0

It is:

2^2|2^1|2^0

Instead of using the digits 0-9, we only use 0-1 (again, if we used anything larger it would be like multiplying 2*2^n and getting 2^n+1, which would not fit in the 2^n column. Therefore, it would shift you one column to the left. For example, "3" in binary cannot be put into one column. The first column we fill is the right-most column, which is 2^0, or 1. Since 3>1, we need to use an extra column to the left, and indicate it as "11" in binary (1*2^1) + (1*2^0).

Example: What would the binary number 1011 be in decimal notation?

1011=(1*2^3)+(0*2^2)+(1*2^1)+(1*2^0)

= (1*8) + (0*4) + (1*2) + (1*1)

= 11 (in decimal notation)

7.3 Lesson Plan: Counting in Binary

Objective: Count in Base 2. Construct a counting chart.

Grade Level: 5th—8th

I have found that the easiest way to count in Binary is to set up a table. The numbers can get large quickly! They do, however, begin to follow a pattern that can be seen in table form.

To convert a number from Base 10 to Base 2 you can use any of the previously taught methods from the other bases. For Binary, I prefer the conversion formula first introduced in Chapter 3.

7.4 Practice Worksheet

Change from binary to decimal (Base 10) notation.

1. 10
2. 111
3. 10101
4. 11110

Remember:

2^4	2^3	2^2	2^1	2^0
			1	0
		1	1	1
1	0	1	0	1
1	1	1	1	0

7.5 Answer Key

1. $10=(1*2^1) + (0*2^0) = 2+0 = 2$

2. $111 = (1*2^2) + (1*2^1) + (1*2^0) = 4+2+1=7$

3. $10101= (1*2^4) + (0*2^3) + (1*2^2) + (0*2^1) + (1*2^0)=16+0+4+0+1=21$

4. 11110= (1*2^4) + (1*2^3) + (1*2^2) + (1*2^1) + (0*2^0)=16+8+4+2+0=30

7.6 Addition in Base 2

Consider the addition of decimal numbers: 23 + 48. We begin by adding 3+8=11. Since 11 is greater than 10, a one is put into the 10s column (carried), and a 1 is recorded in the ones column of the sum. Next, add (2+4) +1 (the one is from the carry) =7, which is put in the 10s column of the sum. Thus, the answer is 71.

Binary addition works on the same principle, but the numerals are different. Begin with one-bit binary addition:

```
  0    0    1
 +0   +1   +0
 ___  ___  ___
  0    1    1
```

1+1 carries us into the next column. In decimal form, 1+1=2. In binary, any digit higher than 1 puts us a column to the left (as would 10 in decimal notation). The decimal number "2" is written in binary notation as "10" (1*2^1)+(0*2^0). Record the 0 in the ones column, and carry the 1 to the two's column to get an answer of "10." In our horizontal notation, 1+ 1 = 10.

The process is the same for multiple-bit binary numbers:

```
  1010
 +1111
 _____
```

Step one:

Column 2^0: 0+1=1.

Record the 1.

Temporary Result: 1; Carry: 0

Step two:

Column 2^1: 1+1=10.

Record the 0, carry the 1.

Temporary Result: 01; Carry: 1

Step three:

Column 2^2: 1+0=1 Add 1 from carry: 1+1=10.

Record the 0, carry the 1.

Temporary Result: 001; Carry: 1

Step four:

Column 2^3: 1+1=10. Add 1 from carry: 10+1=11.

Record the 11.

Final result: 11001

Alternately:

```
  11   (carry)
  1010
 +1111
 ------
 11001
```

Always remember

0+0=0

1+0=1

1+1=10

Try a few examples of binary addition:

111	101	111
+110	+111	+111
______	_____	_____

7.7 Lesson Plan: Addition in Binary

Grade Level: 5th—8th

Objective: Use concrete and abstract methods to add digits in Binary.

Materials: Graph paper, Binary addition table

Table 7 is an example of a Binary addition chart.

Table 7. Binary Addition Table

	000	1
0	0	1
1	1	10

7.8 PRACTICE WORKSHEET

Add. Give answers in Base 10 and Base 2. You may use the online base converter.

1. 52 + 3 = Base 10 _______ Base 2 ________

2. 259 + 31 = Base 10 _______ Base 2 ________

3. 48 + 22 = Base 10 ________ Base 2 _______

4. 822 + 913 = Base 10 _______ Base 2 _______

5. 1,001 + 28 = Base 10 _______ Base 2 ______

6. 801 + 290 = Base 10 ______ Base 2 ______

7. What is your age in Binary? _________

8. Mrs. Turner, the principal, is 42 years old. How old is she in Binary? _____

7.9 ANSWER KEY

1. Base 10 = 55; Base 2 = 110111

2. Base 10 = 290; Base 2 = 100100010

3. Base 10 = 70; Base 2 = 1000110

4. Base 10 = 1,735; Base 2 = 11011000111

5. Base 10 = 1,029; Base 2 = 10000000101

6. Base 10 = 1,091; Base 2 = 10001000011

7. Answers will vary

8. 101010

8 OCTAL (BASE 8)

8.1 INRODUCTION TO BASE 8

The **octal** numeral system is the Base-8 number system and uses the digits 0 to 7. According to Wikipedia, the free encyclopedia, it is reported that the Yuki Native Americans of California used an octal system because they counted using the spaces between their fingers rather than the fingers themselves(sec. 1.1). Octal counting may have been used in the past instead of decimal counting, by counting the spaces like the Yuki or by counting the fingers other than the thumbs(sec. 1.1) This may explain why the Latin word *novem* (nine) is so much like the word *novus* (new). It may have meant a new number. Donald Knuth wrote in his book *The Art of Computer Programming* that King Charles XII of Sweden was the inventor of octal in Europe(sec. 1.2). Octal numerals can be made from binary numerals by grouping consecutive digits into groups of three (starting from the right). For example, the binary representation for decimal 74 is 1001010, which groups into 1 001 010 — so the octal representation is 112.

8.2 Counting in Base 8

As stated in the previous lessons, it is best to begin with a counting chart. Children should have less trouble with base 8 because it only differs by 2 digits from our Base 10 system. If you have been completing the lessons in sequence, let the students construct the Octal counting chart on their own. If this is the first lesson, walk the students through the process. Table 8 is a Base 8 counting table.

Table 8. Base 8 Counting Table

Base 10	Base 8	Base 10	Base 8	Base 10	Base 8	
0	0	17	21	34	42	
1	1	18	22	35	43	
2	2	19	23	36	44	
3	3	20	24	37	45	
4	4	21	25	38	46	
5	5	22	26	39	47	
6	6	23	27	40	50	
7	7	24	30	41	51	
8	10	25	31	42	52	
9	11	26	32	43	53	
10	12	27	33	44	54	
11	13	28	34	45	55	
12	14	29	35	46	56	
13	15	30	36	47	57	
14	16	31	37	48	60	
15	17	32	40	49	61	
16	20	33	41	50	62	

8.3 Lesson Plan: Counting in Base 8

Objective: Count in Base 8 and construct a counting table.

Grade Level: 5^{th}—8^{th}

In Base 8, you are only 2 digits short of the regular base 10 system that we are used to. Therefore, counting can be fairly easy. The first thing one need do is construct a Base 8 table as with Table 9.

Table 9. Base 8 Counting Table

Base 10	Base 8	Base 10	Base 8	Base 10	Base 8	
0	0	17	21	34	42	
1	1	18	22	35	43	
2	2	19	23	36	44	
3	3	20	24	37	45	
4	4	21	25	38	46	
5	5	22	26	39	47	
6	6	23	27	40	50	
7	7	24	30	41	51	
8	10	25	31	42	52	
9	11	26	32	43	53	
10	12	27	33	44	54	
11	13	28	34	45	55	
12	14	29	35	46	56	
13	15	30	36	47	57	
14	16	31	37	48	60	
15	17	32	40	49	61	
16	20	33	41	50	62	

Students can know use this table to convert from Base 10 to Base 8 or vice versa.

Let us begin with the number 234 in Base 10. We know that in Base 10 we have four ones (10 to the zero power), three tens (10 to the first power), and two hundreds (10 squared). Similarly, in Base 8, 234 equals 2 times 64 (8 squared), 3 times 8 (8 to the first power), and 4 times 1 (8 to the zero power).

8.4 Practice Worksheet

1. What is you age in Base 8? ____________

2. What is the current year in Base 8? _____________

3. What is your birth year in Base 8? ___________

Convert from Base 10 to Base 8.

1. 34
2. 88
3. 340
4. 748

Convert from Base 8 to Base 10.

1. 77
2. 711
3. 341
4. 566

8.5 Answer Key

1. Answers will vary
2. Answers will vary
3. Answers will vary

1. 42
2. 130
3. 524
4. 1,354

1. 63
2. 457
3. 225
4. 374

8.6 Addition in Base 8

Addition in Base 8 is no different from addition in all of the other bases we have covered. At this point, you can let your students chose the method of addition that best suits them. I have included an addition table (Table 10) that will help in adding in Octal.

Table 10. Base 8 Addition Table

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	7
2	2	3	4	5	6	7	10
3	3	4	5	6	7	10	11
4	4	5	6	7	10	11	12
5	5	6	7	10	11	12	13
6	6	7	10	11	12	13	14
7	7	10	11	12	13	14	14

8.7 Lesson Plan: Addition in Octal

Grade Level: 5th –8th

Objective:

Materials: Graph paper

Have students begin by constructing their own addition table in Base 8. If you have used the previous lessons then let the students work this out on their own. If this is the first base lesson that you are teaching, you will have to guide the students in setting up and constructing their table, of which Table 11 is an example.

Table 11. Base 8 Addition Table

Base 10	Base 8	Base 10	Base 8	Base 10	Base 8	
0	0	17	21	34	42	
1	1	18	22	35	43	
2	2	19	23	36	44	
3	3	20	24	37	45	
4	4	21	25	38	46	
5	5	22	26	39	47	
6	6	23	27	40	50	
7	7	24	30	41	51	
8	10	25	31	42	52	
9	11	26	32	43	53	
10	12	27	33	44	54	
11	13	28	34	45	55	
12	14	29	35	46	56	
13	15	30	36	47	57	
14	16	31	37	48	60	
15	17	32	40	49	61	
16	20	33	41	50	62	

8.8 Practice Worksheet

Add. Give your answer in Base 10 and Base 8 form.

1. 4 + 5 = Base 10 _____ Base 8 _____
2. 40 + 72 = Base 10 _____ Base 8 _____
3. 478 + 938 = Base 10 _____ Base 8 _____
4. 1,002 + 3,902 = Base 10 _____ Base 8 _____

Add. Give your answer only in Octal.

1. 7 + 8 = _____
2. 88 + 21 = _____

3. 90 + 86 = _____

4. 381 + 940 = _____

8.9 Answer Key

1. Base 10 = 9; Base 8 = 11
2. Base 10 = 112; Base 8 = 160
3. Base 10 = 1,416; Base 8 = 2,610
4. Base 10 = 4,904; Base 8 = 11,450

1. 17
2. 155
3. 260
4. 2,451

9 Classroom Experience

In the process of constructing this paper, I consulted eight fifth grade students. These students worked on each unit and provided valuable feedback. This feedback provides insight into what may happen in the everyday classroom.

I found that the students liked to convert between the bases and to count in each individual base. They saw it as a game. The students did not, however, like the addition in another base. Frequently, the students would simply add in Base 10 and convert the sum to the current base of study.

Also, the students struggled in binary. The numbers became too large too quickly. The students also struggled in duodecimal. They had a hard time incorporating letters as symbols for digits. All of these experiences should be taken into consideration. They are certain to reappear in future classroom experiences.

10 CONCLUSION

As a teacher, these lessons were not only for my students, but for me. I found myself stretched beyond my comfort zone in order to present this information to my fifth graders. Teachers can get into educational ruts. Years can pass and you can find the same teacher in the same room teaching the same subject without ever branching out. As professionals, we owe it to ourselves to stretch our own minds as well as the minds of our students. I hope that you, the reader, will take these lessons back to your students as a challenge and as you do, take the challenge yourself to teach them to the best of your ability.

It may be that you find the lessons lacking or the worksheets too easy or too plain. If this is the case, change them! Change the lessons and the practice problems to fit your individual needs. I once had a professor say that all a teacher ever did was borrow good ideas from other teachers. The best, he said, took those borrowed ideas and made them their own. Make these your own.

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