A consumption-based approach to exchange rate predictability

Jair Ojeda-Joya

Banco de la Republica

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Jair N. Ojeda-Joya[*]

We study whether the implications of an international consumption-based asset-pricing model are useful to provide out-sample predictability evidence for the real exchange rate. This model implies a predictability equation that results from the presence of both internal and external consumption habits in the utility function. In this equation, domestic, U.S. and world consumption growth are predictors of the real exchange rate. Our empirical exercises confirm this connection by providing evidence of short-term predictability on the bilateral rates of 15 out of 17 countries vis-à-vis the U.S. over the post Bretton-Woods float. A non-linear GMM estimation of some parameters of the model also brings about evidence of the presence of consumption habits in the utility function.

1. INTRODUCTION

This paper provides a framework to study exchange rate predictability by developing a consumption-based asset-pricing model that includes internal and external habit formation[2]. Within this model, it is possible to show that the presence of habits in consumers' preferences implies that the exchange rate has a predictable component which depends on past consumption growth[3]. That is, if we combine the first order condition for optimal consumption-savings allocation with an international arbitrage condition, it is possible to derive a forecasting equation for the exchange rate due to the presence of habits in the utility function. We estimate this forecasting equation with both linear and non-linear econometric methods using data for 17 industrialized economies over the Post-Bretton-Woods float. We find significant evidence of short-run out-of-sample predictability in 14 countries by computing tests that compare the forecasting power of the model with a random-walk forecast, a regularly used benchmark for evaluating exchange rate models (Rossi, 2013).

We interpret the empirical results in the context of a consumption-based asset-pricing model with N economies, complete markets, imperfect international risk sharing and representative consumers whose preferences include internal and external habit persistence. The economic reason for Real Exchange Rate (RER) predictability in this framework is the habit effect of past consumption growth on current marginal utility and thus on the stochastic discount factors (SDFs) that domestic and foreign investors use to value financial assets. In this framework, the difference between external and domestic SDFs drives RER variations.

As a robustness check for these predictability results, we attempt to measure the degree of habit persistence and its relative importance across countries by estimating the relevant parameters of the utility function using non-linear GMM methods. Results from this estimation show significant and strong habit effects in 15 out of the 17 economies under study.

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[2] In this paper, external habits are very similar to the definition of catching up with the Joneses in Abel (1990) but within an open-economy interpretation. This approach is conceptually similar to that in Campbell and Cochrane (1999) but with a different specification in the utility function.

[3] Since the inputs of the model are real quantities, it is natural to derive all the implications on the real exchange rate instead of the nominal one.

This paper relates to the empirical literature on exchange rate determination models. In particular, the problem has some similarities to the one originally described by Meese and Rogoff (1983) about the poor out-of-sample forecasting power of the monetary approach to exchange rate determination. Several papers have shown that alternative specifications of the monetary model have out-of-sample predictability power for long-run horizons (one year or more); see for example, Mark (1995), Mark and Sul (2001), Groen (2005), Engel et al (2007), and Cerra and Saxena (2010).

Additionally, a few papers show out-of-sample predictability evidence with alternative exchange rate models. Gourinchas and Rey (2007) study an international financial adjustment model in which RER changes are the result of disequilibria of the country's external accounts. Molodtsova and Papell (2009), Byrne et al (2016), Ince et al (2016) among others estimate forecasting equations derived from Taylor-rule specifications for monetary policy in each country. Rogoff and Stavrakeva (2008) perform robustness exercises comparing alternative models and conclude that the out-of-sample predictability evidence is still weak on horizons shorter than one year. One possible reason for this weakness is that the intensity of the relation between exchange rates and alternative fundamentals is time varying. Sarno and Valente (2009) and Fratzscher et al (2015) show the evidence and propose "scapegoat" models based on time-varying elasticities. Rossi (2013) surveys this literature and concludes that the most promising models are those based on Taylor rules or external accounts[4].

The current paper presents a forecasting equation derived from a consumption-based asset-pricing model and shows that it has interesting predictability properties on the one-quarter-ahead horizon. An important difference with previous works on exchange rate predictability is that the model has only implications for the (bilateral) RER. A possible caveat of this approach is that its main predictor is household consumption a quarterly national-accounts variable, which makes it difficult to perform monthly forecast exercises as in most of the literature on the topic.

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[4] Other recent articles study the determinants of professional forecasts (Beckmann and Czudaj, 2017) and the role of sovereign risk expectations (Foroni et al, 2018).

Backus et al (2001) initiated the use of consumption-based asset pricing models to study the necessary conditions to solve the forward premium puzzle. Lustig and Verdelhan (2007) empirically show that low interest rate currencies provide investors with a hedge against consumption-growth risk, which explains the uncovered interest rate parity puzzle. Verdelhan (2010) presents an asset-pricing model with external consumption habits that reproduces the countercyclical risk premium and the observed relation between exchange rates and consumption growth. We partially follow Verdelhan's (2010) approach to derive an exchange rate predictability equation based on consumption data.

The outline of the paper is the following. Section 2 describes the consumption-based asset-pricing framework and its implied forecasting equation for the RER. Section 3 presents the econometrics methods for out-of-sample predictability evaluation. Sections 4 and 5 present results for each country and for alternative forecasting windows, respectively. Section 6 presents the in-sample non-linear estimation of the most relevant parameters of the model. Finally, Section 7 concludes.

2. A CONSUMPTION-BASED ASSET-PRICING MODEL

2.1. Basic Framework

The following consumption-based asset-pricing framework is based on Abel (1990, 2008) but it is extended to include N countries ($i = 1,2,...N$). The representative consumer in each country i maximizes:

$$U_{i,t} = E_t \left[\sum_{j=0}^{\infty} \beta^j \left(\frac{1}{1-\alpha} \right) \left(\frac{C_{i,t+j}}{V_{i,t+j}^{\gamma_i}} \right)^{1-\alpha} \right] \tag{1}$$

In Equation (1), α denotes the risk aversion coefficient, β is the time discount factor, $C_{i,t}$ is the level of household consumption in each country[5] and $V_{i,t}^{\gamma_i}$ is the benchmark level of consumption where the parameter γ_i measures the degree of habit persistence in country i.

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[5] $C_{i,t}$ corresponds to the level of real consumption by households including non-durable goods and services.

Benchmark consumption includes past domestic consumption as well as past world consumption:

$$V_{i,t} = \left[\left(C_{i,t-1} \right)^D \left(C_{w,t-1} \right)^{1-D} \right] \tag{2}$$

In Equation (2), C_w denotes world consumption and D is a weight that measures the importance of domestic consumption relative to world consumption in the composition of the benchmark level of consumption. World consumption is the geometric weighted average of consumption across countries. The weights ω_i in Equation (3) are determined by the relative economic size of country i.

$$C_w = \prod_{i=1}^N C_i^{\omega_i} \tag{3}$$

The utility framework in Equations (1) to (3) nests the standard CRRA case when $\gamma = 0$, since in this case benchmark consumption does not have any influence in utility. When $\gamma > 0$ instead, utility depends on the ratio between domestic and benchmark consumptions. The presence of V_t^γ in the utility function captures both internal and external habit formation. We interpret external habits as the satisfaction from consuming as much as the average world level of consumption or more.

From Equation (1), it is possible to compute the marginal utility of consumption in each country.

$$\frac{\partial U_{i,t}}{\partial C_{i,t}} = \frac{1}{C_{i,t}} E_t \left[\left(\frac{C_{i,t}}{V_{i,t}^{\gamma_i}} \right)^{1-\alpha} - \gamma_i D \beta \left(\frac{C_{i,t+1}}{V_{i,t+1}^{\gamma_i}} \right)^{1-\alpha} \right] \tag{4}$$

Notice that marginal utility in (4) when $\gamma_i = 0$, is exactly equal to the case of a standard CRRA utility function ($C_{i,t}^{-\alpha}$). Therefore, it is possible to partition Equation (4) into three components: standard CRRA, benchmark consumption and habits. We specify these three components in Equation (5).

$$\frac{\partial U_{i,t}}{\partial C_{i,t}} = C_{i,t}^{-\alpha} V_{i,t}^{\gamma_i(\alpha-1)} H_{i,t}. \tag{5}$$

The component $C_{i,t}^{-\alpha}$ is the standard CRRA marginal utility which decreases with current consumption and with the risk-aversion degree (α). The effect of benchmark consumption is measured by $V_{i,t}^{\gamma_i(\alpha-1)}$. Notice that, as long as there is some habit persistence $(\gamma_i > 0)$, the effect of benchmark consumption on marginal utility is positive only if there is enough risk aversion $(\alpha > 1)$. If there is not any habit persistence or in the log-utility case $(\alpha = 1)$, benchmark consumption has not any effect on marginal utility.

Equation (6) defines the component $H_{i,t}$ that measures the effect of internal habits on marginal utility. When there are no internal habits in the utility function, $H_{i,t} = 1$. Otherwise, $H_{i,t}$ is a fraction that considers the fact that a higher consumption today increases the benchmark level of consumption and thus decreases tomorrow's utility. We assume that the parameters of the model are such that $H_{i,t} > 0$, and therefore marginal utility, is strictly positive.

$$H_{i,t} = 1 - \beta D\gamma_i E_t(X_{i,t+1}^{1-\alpha})X_{i,t}^{D\gamma_i(\alpha-1)}X_{w,t}^{(1-D)\gamma_i(\alpha-1)} \tag{6}$$

In Equation (5), $X_{i,t}$ corresponds to the gross rate of consumption. Therefore, we define: $X_{i,t+1} = C_{i,t+1}/C_{i,t}$ and $X_{w,t+1} = C_{w,t+1}/C_{w,t}$.

Equation (5) and the definition of benchmark consumption allow us easily computing the Stochastic Discount Factor (SDF) or pricing kernel, as the product of the time discount factor (β) and marginal utility growth, see Equation (7).

$$M_{i,t+1} = \beta X_{i,t+1}^{-\alpha}X_{i,t}^{D\gamma_i(\alpha-1)}X_{w,t}^{(1-D)\gamma_i(\alpha-1)}\left(\frac{H_{i,t+1}}{H_{i,t}}\right). \tag{7}$$

2.2. Implications for the Real Exchange Rate

We describe the relation between exchange rates and SDFs following the asset-pricing framework of Backus et al (2001). In their model, under free portfolio formation and the law

of one price, there exists a unique SDF in the space of traded assets. Lustig and Verdelhan (2007) derive a similar result and apply it to the cross-section of foreign currency risk.

Let $M_{us,t+1}$ denote the SDF of US investors. Q_t is the real exchange rate (RER) expressed as US goods per foreign good, therefore, if Q_t decreases then the real US dollar appreciates. All investors have access to a foreign-currency return $R_{i,t+1}$. Equations (8) and (9) are the Euler conditions for US and foreign investors, respectively:

$$E_t(M_{i,t+1}R_{i,t+1}) = 1 \tag{8}$$

$$E_t(M_{us,t+1}R_{i,t+1}\,Q_{t+1}/Q_t) = 1 \tag{9}$$

The uniqueness of the SDF in the space of traded assets and Equations (8) and (9) imply the following relationship:

$$M_{i,t+1} = M_{us,t+1}\,Q_{t+1}/Q_t \tag{10}$$

Computing natural logarithms on both sides of Equation (10) we obtain:

$$q_{t+1} - q_t = m_{i,t+1} - m_{us,t+1}. \tag{11}$$

Throughout this paper, lower case letters stand for the natural logarithm of the original variables. In Equation (11), $m_{us,t+1}, m_{i,t+1}$ are the US and country i's log SDFs, respectively. This equation implies that the log variation in the real exchange rate is equal to the difference between the log SDFs across countries. Computing logs on both sides of (7) and inserting this result in (11), we obtain the following expression for the real exchange rate as a function of consumption growth and habit persistence in both countries:

$$\Delta q_{i,t+1} = -\alpha(x_{i,t+1} - x_{us,t+1}) + D\gamma_i(\alpha - 1)x_{i,t} - D\gamma_{us}(\alpha - 1)x_{us,t} +$$
$$(1 - D)(\alpha - 1)(\gamma_i - \gamma_{us})x_{w,t} + \Delta h_{i,t+1} + \Delta h_{us,t+1} \tag{12}$$

In Equation (12), growth rates for the RER and the habit effect are denoted $\Delta q_{i,t+1}$ and $\Delta h_{i,t+1}$, respectively. Notice that we interpret (12) as a forecasting equation in which changes in the real exchange rate are determined by lagged values of domestic, US and world consumption growth. The channel for this effect is the presence of habit persistence and its

implications for asset pricing through its effects on the marginal utility of consumption and thus on SDFs.

There are two necessary conditions for predictability in Equation (12). First, the risk aversion coefficient α should be different from one; otherwise, the RER becomes neutral to the presence of habit persistence. Second, we need $\gamma_i \neq \gamma_{US}$ for the exchange rate to be predictable via the external habit channel.

2.3. Computing a Linear Forecasting Equation

To estimate the expected value of (12) using a linear regression framework, we use a first-order Taylor approximation to $h_{i,t}$ and $h_{us,t}$ since both expressions are nonlinear functions of consumption growth. For this approximation, we define the following:

$$z_{i,t} \equiv D\gamma_i(\alpha - 1)x_{i,t} + (1 - D)\gamma_i(\alpha - 1)x_{w,t} \tag{13}$$

Therefore, inserting (13) in (6) and taking logs, we can write h_t in the following simplified way:

$$h_{i,t} \equiv \log(H_{i,t}) = \log(1 - D\gamma_i\beta E(X_{i,t+1}^{1-\alpha})e^{z_{i,t}}) \tag{14}$$

Once we compute the derivative of (14), it is possible to express the first-order Taylor approximation to $h_{i,t}$ around $E(z_{i,t}) \equiv \bar{z}_i$ in the following way:

$$h_{i,t} \approx \log(1 - D\gamma_i\beta E(X_{i,t+1}^{1-\alpha})e^{\bar{z}_i}) - \frac{D\gamma_i\beta E(X_{i,t+1}^{1-\alpha})e^{\bar{z}_i}}{1 - D\gamma_i\beta E(X_{i,t+1}^{1-\alpha})e^{\bar{z}_i}}(z_{i,t} - \bar{z}_i) \tag{15}$$

From (15), we can compute $\Delta h_{i,t+1}$ which consists of a constant multiplied by $\Delta z_{i,t+1}$. Therefore, using (13) and (15), we can calculate the expected value of $\Delta h_{i,t+1}$, conditional on information through t, in the following way:

$$E_t(\Delta h_{i,t+1}) = -\theta_i\gamma_i(\alpha - 1)g + \theta_i D\gamma_i(\alpha - 1)x_{i,t} + \theta_i(1 - D)\gamma_i(\alpha - 1)x_{w,t} \tag{16}$$

In Equation (16), θ_i is a constant parameter:

$$\theta_i \equiv \frac{D\gamma_i \beta E(X_i^{1-\alpha})e^{\overline{Z_i}}}{1-D\gamma_i \beta E(X_i^{1-\alpha})e^{\overline{Z_i}}} \tag{17}$$

Equation (16) also assumes a log normal distribution for consumption growth in all countries such that in each period t:

$$\log(X_{i,t}) \equiv x_{i,t} \sim N(g, \sigma^2) \tag{18}$$

Using (12), (16) and (18), it is possible to derive a linear forecasting equation for the expected variation of the real exchange rate as a function of past consumption growth in the domestic country, the US and the World:

$$E_t(\Delta q_{i,t+1}) = \psi_{i,0} + \psi_{i,1}\Delta C_{i,t} + \psi_{i,2}\Delta C_{us,t} + \psi_{i,3}\Delta C_{w,t} \tag{19}$$

The coefficients to estimate in Equation (19) are $\psi_{i,0}$, $\psi_{i,1}$, $\psi_{i,2}$ and $\psi_{i,3}$. These are functions of the deep parameters of the model:

$$\psi_{i,0} = (\alpha - 1)g(\theta_{us}\gamma_{us} - \theta_i\gamma_i), \tag{20}$$

$$\psi_{i,1} = (1 + \theta_i)D(\alpha - 1)\gamma_i \tag{21}$$

$$\psi_{i,2} = (1 + \theta_{us})D(\alpha - 1)\gamma_{us} \tag{22}$$

$$\psi_{i,3} = (1 - D)(\alpha - 1)(\gamma_i(1 + \theta_i) - \gamma_{us}(1 + \theta_{us})) \tag{23}$$

Notice that the sign of the coefficients $\psi_{i,0}$ and $\psi_{i,3}$ is determined by the relative size of the parameters γ_i and γ_{us}. Furthermore, $\psi_{i,1}$ and $\psi_{i,2}$ remain different from zero if there is a positive degree of internal habits $(D > 0)$. Additionally, under a sufficiently high-risk aversion coefficient, $\alpha > 1$, we should expect a positive sign for $\psi_{i,1}$ and a negative sign for $\psi_{i,2}$.

3. DATA AND ECONOMETRIC METHODS: OUT-OF-SAMPLE PREDICTABILITY TESTS

3.1 Data Description

Data consists of quarterly real exchange rates (RERs) and real household consumption for 18 advanced economies including the United States (US). We correct for seasonality by computing annual variations of the natural logarithm of these variables. Bilateral RER data are calculated with the consumer price index (CPI) and the average official exchange rate with respect to the US dollar for each country. We retrieve these data from the International Monetary Fund's (IMF) International Financial Statistics (IFS). We use the following formula:

$$q_{i,t} = e_{i,t} - p_{us,t} + p_{i,t} \qquad (24)$$

$q_{i,t}$ is the log RER, $e_{i,t}$ is the log nominal exchange rate and $p_{i,t}$ corresponds to the log CPI for country i. An increase of the RER, according to this definition, corresponds to a real appreciation of the currency vis-à-vis the US dollar. For countries in the European Monetary Union (EMU), we only work with their previous currency before entering the union. In the case of Germany, we only work with data from western Germany before the 1990 unification. For the remaining economies, all data are updated through 2015q2[6].

We construct real consumption with the nominal series on households' consumption of non-durable goods and services for each country. We deflate these series with CPI data, and compute world consumption as described in Equation (3) and using the weights described in Table A1 in the Appendix.

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[6] There is a working paper version available in Ojeda-Joya (2014) in which data is updated until 2007 and EMU real exchange rates include data before and after 2002.

3.2. Three Alternative Tests

Following Rogoff and Stavrakeva (2008), we compute three alternative tests for out-of-sample predictability power: Theil's U (TU), Diebold-Mariano-West (DMW) and Clark-West (CW). When the mean-square forecasting error is significantly smaller than the implied by a random-walk model without drift, we regard it as a good forecast. This criterion has been widely used in the exchange rate predictability literature since Meese and Rogoff (1983), and it is still the toughest benchmark for any exchange rate model (Rossi, 2013).

The first step on the out-of-sample predictability exercise consists of choosing a forecasting window. We initially use a 40-observation window to estimate Equation (19) with quarterly data. Thus, in countries where the total sample spans 1973 q1 through 2015 q3, (173 observations), the forecasting window has approximately 133 observations. The second step consists of using rolling regressions, with 40 observations each, to estimate the parameters in Equation (19). Then we use these estimations to perform forecasts of exchange rates one-quarter ahead. The final step is comparing the resulting 133 forecasts with actual real exchange rate data and using these forecast errors to compute predictability tests.

Assume that $y_t = q_t - q_{t-1}$, is the quarterly variation of the real exchange rate. Let X_t be the matrix that includes the explanatory variables defined in Equation (19) and let ψ be the corresponding vector of constant coefficients. We are interested in comparing the forecasting power of the model in Equation (19) with a random walk without drift. This benchmark model implies: $y_t = e_{1,t}$. We can rewrite the structural model in (19) as: $y_t = X_{t-1}\psi + e_{2,t}$. Innovations terms $e_{1,t}$ and $e_{2,t}$ are assumed to be unobservable.

The estimated forecasts for the random walk and the structural model are $\hat{y}_{,t+1} = 0$, and $\hat{y}_{2,t+1} = X_t\hat{\psi}_t$ respectively, where $\hat{\psi}_t$ is the least-squares estimator of ψ_t. The corresponding forecast errors are $\hat{e}_{1,t}$ and $\hat{e}_{2,t}$, respectively. The Mean Squared Forecast Error (MSFE) for either of the forecasting models is:

$$MSFE = P^{-1} \sum_{t=R+1}^{T} \hat{e}_{t=R+1}^2 \qquad (25)$$

In Equation (25), P is the number of forecasts, T is the sample length and R is the number of observations used to estimate ψ_t on the first forecast. We define the TU test in Equation (26) as the root square of the ratio between the MSFE of the structural model and the random-walk model. Therefore, if TU is significantly lower than 1, the structural model outperforms the random-walk model.

$$TU = \sqrt{MSFE_2/MSFE_1}. \tag{26}$$

The DMW test measures the difference between the MSFE of the random walk model and that of the structural model (Equation 27). Therefore, a significant and positive DMW test implies that the structural model outperforms the random walk.

$$DMW = MSFE_1 - MSFE_2 \tag{27}$$

The literature on forecasting has identified that both statistics, TU and DMW, tend to over-reject the structural model when used to compare projections from nested models like those in the current exercise[7]. In view of this problem, Clark and West (2006, 2007) propose a test statistic (CW) which builds on the DMW test but takes into account that both models are nested by assuming that, under the null hypothesis, the exchange rate follows a random walk. Therefore, the null hypothesis in the CW test is computed under the assumption that the population parameter vector is $\psi = 0$, and that the forecast innovation terms are equal across models: $e_{1,t} = e_{2,t}$.

$$\hat{d} = 2P^{-1} \sum_{t=R+1}^{T} (y_{t+1} X_t \hat{\psi}_t) \tag{28}$$

Clark and West (2006) show that if $\hat{d}$, the quantity defined in (28), is significantly greater than zero, then the structural model outperforms the random walk. Therefore, the CW test is defined in (29) as a significance test for $\hat{d}$ where $\Omega^{\hat{d}}$ is its estimated variance.

$$CW = \frac{P^{0.5}\hat{d}}{\sqrt{\Omega^{\hat{d}}}} \tag{29}$$

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[7] These models are nested because a random-walk model for the real exchange rate holds as a special case of (19) when all the parameters are equal to zero.

We follow Rogoff and Stavrakeva (2008) by computing all three tests (TU, DMW and CW), when performing out-of-sample predictability exercises, and by using bootstrapped critical values in order to correct for the potential size distortion which results from working with nested models.

3.3. Bootstrap Procedure

We use a non-parametric bootstrap procedure to calculate the p-values for the TU and DMW tests, following Mark and Sul (2001). The real exchange rate behaves as a random walk, according to the null hypothesis. For quarterly consumption growth, we fit Equation (30) using least squares, to estimate its autoregressive structure and its correlation with the real exchange rate.

$$\Delta C_t^i = \delta_0 + \sum_{k=1}^{d} \delta_k \, y_{t-k} + \sum_{k=1}^{l} \zeta_k \Delta C_{t-k}^i + \varepsilon_t^i \tag{30}$$

In (30), we select the number of lags, d and l as well as the appropriate trend (constant or linear), by minimizing a Bayesian information criterion. We estimate the residuals from this Equation and resample them 1000 times with replacement. Then, we recursively simulate the real exchange rate and consumption growth. We employ historic averages as initial values for the recursions and discard the first 100 simulated observations to attenuate potential bias related to this choice of starting values. Finally, we estimate the model and calculate again all the test statistics for each resampling. The resulting distribution of test statistics allows computing p-values.

4. OUT-OF-SAMPLE PREDICTABILITY RESULTS

We estimate the forecasting equation (19) country by country using least squares and quarterly data for 17 OECD countries[8]. This set of countries is the same one analyzed by Engel et al (2007) and by Rogoff and Stavrakeva (2008). We compute quarterly bilateral Real

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[8] All implied time series of observable variables are stationary according to unit-root tests. Results are available upon request.

Exchange Rates (RER) with respect to the US for all countries. These quarterly data span the post Bretton-Woods period through 2015Q3[9]. The starting date of the sample is determined by the availability of consumption growth data in each economy, which correspond to nondurable goods and services purchased by households. We retrieve most variables from the International Financial Statistics (IFS).

Table 1 shows the results from the estimation of the out-of-sample predictability tests described in Section 3.1. The null hypothesis in all three tests (TU, DMW and CW) is that both the consumption-based model and a random walk have the same Mean Squared Forecast Error (MSFE); the alternative hypothesis is that the model has lower MSFE than a random walk. We compute all p-values in Table 1 with the bootstrap procedure described in Section 3.2 and using Equation (30) along with 1000 resamplings to construct the consumption series and to re-estimate all the predictability tests. This bootstrap procedure is necessary for the TU and DMW tests to improve their statistical power. The reason for this feature is that the null hypothesis is nested in the alternative one, see Clark and West (2006, 2007).

Results from the TU and DMW tests are similar to each other for most countries in Table 1. There is out-of-sample predictability evidence in 10 out of 17 countries according to these two tests. Countries with no predictability evidence with these tests are Belgium, Denmark, Netherlands, Japan, Switzerland, South Korea and Sweden.

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[9] In the case of Eurozone countries, the sample includes only the flexible regime period, that is, until 1998Q4. The sample for Germany ends in 1990Q4, that is, we only consider the real exchange rate of the mark, before reunification.

TABLE 1

Out-of-Sample Exchange Rate Predictability Tests

Based on One-Quarter Ahead Forecasts

Country	TU	P-value	DMW	P-value	CW	P-value
UK	1.02	0.02	-4.21	0.02	2.44	0.01
Austria	0.95	0.01	14.35	0.01	3.50	0.00
Belgium	1.09	0.72	-17.98	0.57	0.85	0.22
Denmark	1.14	0.99	-30.29	0.93	2.70	0.00
France	0.98	0.01	6.56	0.01	3.01	0.00
Germany	0.90	0.01	43.4	0.01	3.38	0.00
Netherlands	1.18	0.97	-37.4	0.80	1.41	0.09
Canada	1.01	0.01	-0.65	0.01	2.65	0.01
Japan	1.07	0.49	-20.54	0.43	2.20	0.01
Finland	0.86	0.00	47.57	0.00	3.77	0.00
Spain	0.77	0.00	71.54	0.00	5.61	0.00
Australia	1.03	0.04	-7.95	0.04	2.15	0.01
Italy	0.84	0.00	44.83	0.00	3.78	0.00
Switzerland	1.15	0.99	-29.08	0.85	2.81	0.01
South Korea	1.10	0.85	-27.61	0.91	1.66	0.05
Norway	1.02	0.01	-5.36	0.02	2.39	0.00
Sweden	1.06	0.39	-19.02	0.49	2.30	0.02

This table presents country-by-country out-of-sample predictability tests estimated from Equation (19) using 40-observation rolling samples. We describe the tests TU, DMW and CW in equations (25), (26) and (28) respectively. We compute p-values with the bootstrap procedure described in Section 3.
Source: Author's Calculations

This predictability evidence improves when we consider the CW test except in the case of Belgium. If we only accept null-hypothesis rejections with at least 95% confidence degree, the CW test reports predictability evidence in 14 out of 17 countries. In this case, there is no such evidence in Belgium, Netherlands and South Korea.

The reason for the TU and DMW tests to be more stringent than the CW test is that they directly compare the models in terms of mean square forecasting error (MSFE). In contrast, the CW test includes the possibility of using a weighted average between the predictions from the structural and the random-walk models to minimize forecasting errors. In terms of our results, this finding implies that in the case of Japan, Switzerland and Sweden, we should combine the consumption-based with the reference model to improve their real exchange rate forecasts.

In summary, we have found evidence that the consumption-based framework is able to beat a random walk when forecasting real exchange rates variations one quarter ahead, in 14 out of 17 economies, using a 95% confidence degree. We show figures of predicted versus observed real exchange rate variations in the Appendix.

Engel et al (2007) perform similar tests based on panel data regressions, for the same set of 17 countries, using the monetary model of the exchange rate. Although their long-horizon predictability results are positive for most countries, their short-horizon results work well only in 4 countries. The failure of the monetary model in predicting exchange rate variations on short-run horizons relates to its central assumptions. Namely, Purchasing Power Parity (PPP) and Uncovered Interest Parity (UIP) fail to hold in the short run according to the literature on international finance[10]. An alternative explanation for this result is that these fundamentals have a unit root that, with a near-one discount factor, leads to exchange rates behaving almost like a random walk, (Engel and West, 2005).

The consumption-based model presented in Section 2 contains an arbitrage condition for international asset markets and its relation with consumers' stochastic discount factors (Equation 11). Therefore, this approach does not need to assume PPP nor UIP in order to derive the forecasting equation. Additionally, since we use domestic and international consumption growth as fundamentals, we do not deal with I(1) fundamentals. Finally, exchange rate predictability in this framework is an implication of the presence of consumption habits. Namely, it originates on the effects of past consumption growth on current marginal utility and thus on the expected stochastic discount factors that domestic and foreign investors use to price international financial assets.

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[10] See the papers by Rogoff (1996) as well as by Taylor and Taylor (2004) on the failure of the PPP hypothesis, and Fama (1984) on the UIP hypothesis.

5. ALTERNATIVE FORECASTING WINDOWS

Rogoff and Stavrakeva (2008) argue that it is very important to check for robustness to alternative rolling-window sizes to assure that the estimated relationship remains stable.[11] They perform this kind of robustness check to the exchange rate predictability results from the models proposed by Molodtsova and Papell (2009), Engel et al (2007) and Gourinchas and Rey (2007). These exercises show that the out-of-sample predictability evidence weakens when tests use narrower forecast windows or, equivalently, longer samples to compute the parameters of the model. The only exception is the international valuation model (Gourinchas and Rey, 2007) in which the predictability evidence is reasonably stable across rolling-window sizes.

We perform a similar procedure to evaluate robustness to alternative sizes of the forecasting window. We focus on the Clark-West test and compute it for each country and for six alternative sizes that range from 60 to 110 observations, or equivalently, from 80 to 30 observations to compute the regression parameters.

Our results are similar to those in Rogoff and Stavrakeva (2008) for their monetary and Taylor-rule models. Table 2 shows that the good predictability results from the consumption-based model remain true when we use 30 to 50 observations to estimate the parameters of Equation (19). When 60 or 80 observations are employed, this evidence weakens notoriously across countries. However, when we perform the estimations with a sample size of 70, there is again predictability evidence for more than a half of the country sample. These results show the possible time-varying nature of the parameters on the consumption-based model, especially, those related to external and internal consumption habits.

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[11] The size of the forecasting rolling window is the number of out-of-sample forecasts used to compute the predictability tests. We define it as P in Equation (25), i.e. it is the difference between the sample length and the number of observations used to compute the parameters of the regression.

TABLE 2

CW Test for Alternative Rolling Forecasting Windows

	Number of Observations Used for Parameter Estimation					
Country	30	40	50	60	70	80
UK	2.35***	2.44***	1.41*	1.15	1.77**	0.86
Austria	1.96**	3.5***	2.14**	1.52*	2.43***	2.52***
Belgium	-0.90	0.85	1.71**	2.56***	-5.50	NA
Denmark	3.95***	2.70***	1.42*	0.95	-0.96	-0.90
France	3.04***	3.01***	2.11**	2.10**	2.23**	2.53***
Germany	3.54***	3.38***	2.06**	0.53	NA	NA
Netherlands	1.75**	1.41*	1.04	2.48	-0.22	NA
Canada	3.18***	2.65***	0.79	0.21	1.67**	1.48*
Japan	4.28***	2.20**	0.99	-0.64	1.34*	0.71
Finland	4.01***	3.77***	2.90***	3.34***	2.56***	2.68***
Spain	5.16***	5.61***	4.24***	3.39***	2.93***	2.62***
Australia	2.53***	2.15**	0.79	1.64*	1.80**	0.64
Italy	2.17**	3.78***	2.45***	2.52***	2.59***	2.28**
Switzerland	1.60*	2.81***	1.75**	0.69	-1.12	-1.28
Korea	2.08**	1.66**	1.98**	1.84**	1.60*	1.61*
Norway	3.42***	2.39***	1.00	0.95	1.39*	1.28
Sweden	2.73***	2.30**	1.21	1.02	1.88**	1.83**
Overall	16/17	16/17	11/17	8/17	12/17	8/17

* denotes significance at 10% level; ** denotes significance at 5% level; *** denotes significance at 1% level. NA: not available due to short time series.
This table presents the Clark-West (2006) predictability test for alternative forecasting windows. We evaluate the significance of these tests according to the following asymptotic critical values: 1.282 (10%), 1.645 (5%), 2.33 (1%).
Source: Author's calculations

6. IN-SAMPLE ESTIMATION OF THE PARAMETERS OF THE MODEL

The goal of this section is to perform a direct, country-by-country estimation of the structural parameters related to habits, namely: γ_i, γ_{us} in Equations 20 to 23. We perform this estimation with a non-linear GMM approach following Hansen (1982). We assume the remaining parameters to take values according to our data and related literature. Namely, the average annual consumption growth rate across economies in our dataset of 17 economies since 1973 is $g = 2.11\%$, and its average standard deviation is $\sigma = 1.51\%$. We assume an equal weight for each type of consumption habit, therefore $D = 0.5$. Following the equity

premium literature with habits, for instance Abel (1990), standard values for the time discount factor and the risk aversion parameter are $\beta = 0.95$ and $\alpha = 2$, respectively.

The econometric method consists of estimating the sample equivalent of the conditional expectation of Equation (19) by using country-by-country data on real exchange rates and consumption growth. Since Equation (19) includes lagged consumption, it is possible to use contemporaneous consumption-growth measures (domestic, US and world) as instruments for the GMM estimation. By assumption, the errors from the forecasting equation remain orthogonal to contemporaneous consumption innovations. As a result, this set-up gives 4 moment conditions for each country, which allows estimating three parameters.

We apply the continuously updating GMM estimation method in which the initial weighting matrix is proportional to Z, the matrix of instruments. Namely, the initial matrix is: $W_0 = (Z'Z)^{-1}$. In the second step, we apply the optimal weighting matrix, which is the inverse of the spectral density matrix. Then we re-estimate this optimal matrix in the following iterations until an appropriate convergence criterion is reached. Finally, we compute standard errors following Hansen's (1982) GMM asymptotic theory[12].

Our results show that the habit persistence parameter for each country (γ_i) is significantly different from zero in 15 out of 17 countries. The average value of this parameter, across all the significant cases, is 2.8. Table 3 also shows a country-by-country estimation of γ_{us} which is significant in 11 out of 17 countries and its average value across countries is 2.0. Therefore, this estimation helps to understand our predictability results by showing that the presence of habits in the utility function is consistent with the data for most economies. Thus, open-economy models should analyze in more detail the presence of consumption habits.

Table 3 also shows the J-test for over-identifying restrictions. This is a test of the null hypothesis that the estimated parameters are useful to satisfy the moment conditions. The J-

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[12] Cochrane (2005) and Cliff (2003) are references and guides for this GMM estimation.

test detects nine cases, with a 95% confidence degree, for which the selection of instruments may not be the most appropriate. We regard this result as an implication of fitting a non-linear equation with only two free parameters. However, if we exclude the cases in which the J-test detects misspecification with a 95% confidence degree, we still have significant habit parameters for 7 economies with an average (γ_i) of 2.4.

<div align="center">

TABLE 3

In Sample Non-Linear GMM Estimation of Parameters

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Country	A. Gamma i	B. Gamma US	J-Test
UK	5.43*	5.79**	5.07*
Austria	0.35	0.96**	8.89**
Belgium	3.29***	1.26***	2.02
Denmark	3.42***	4.75***	3.47
France	3.26***	1.14***	9.03**
Germany	2.87***	3.99***	7.41**
Netherlands	3.27***	0.82	6.04**
Canada	1.03***	0.96***	1.91
Japan	0.67	-0.13	1.36
Finland	3.55***	1.01***	8.82**
Spain	3.22***	0.76*	9.78***
Australia	1.10***	0.63	3.72
Italy	3.67***	1.01***	6.07**
Switzerland	1.24***	0.45	3.32
Korea	4.68***	0.02	7.08**
Norway	1.03***	0.75*	7.76**
Sweden	1.10***	0.51	5.36*

* denotes significance at 10% level; ** denotes significance at 5% level; *** denotes significance at 1% level.
This table presents country-by-country estimations of habit-related parameters from Equation (19) using the total sample. The method of estimation is non-linear GMM with instrumental variables. The J-test corresponds to the test for over-identifying restrictions.
Source: Author's Calculations

7. CONCLUSIONS

Engel et al (2007), Rogoff and Stavrakeva (2008), Rossi (2013), among others, explain that it is difficult to obtain good out-of-sample predictability evidence for the exchange rate in short-run horizons with the traditional models in the literature. Therefore, the puzzle described by Meese and Rogoff (1983) still seems to hold in such cases. A few new approaches have found positive predictability evidence in short-run horizons. Molodtsova and Papell (2009), Byrne et al (2016) and Ince et al (2016), among others, apply the Taylor-rule approach. Gourinchas and Rey (2007) employ an external-balance model. Finally, Sarno and Valente (2009) as well as Fratzscher et al (2015) study scapegoat models of the exchange rate.

This paper provides an alternative approach to study short-run real exchange rate (RER) predictability using out-of-sample tests. This framework is an open-economy extension of the model studied by Abel (1990, 2008), and can be described as a consumption-based asset-pricing model with N countries and complete markets. In this model, the difference between Stochastic Discount Factors (SDF) across countries determines real exchange rate variations.

We show that when preferences include internal and external habit persistence, SDFs are driven by past consumption growth and therefore RER variations are predictable with consumption data. In other words, habits imply that current consumption growth predict some of the valuation of financial assets through the effects of current marginal utility on future SDFs. Furthermore, the functional form of the utility function allows deriving a linear specification for the RER as function of the following predictors: domestic consumption growth, US consumption growth and world consumption growth.

Predictability tests with data for 17 developed economies, show good out-of-sample evidence in 15 countries. Additionally, we confirm the relevance of this habit-based approach through a direct estimation of the key parameters of the utility function using non-linear GMM methods. The estimated habit-related parameters are statistically significant for most countries.

This consumption-based framework has potential applications for the continuing study of exchange rate determination. This kind of habit-based utility functions can also be incorporated to asset pricing models with disaster risk (i.e. Gourio et al, 2013) or long-run risk (i.e. Colacito and Croce, 2011), in order to develop further results on macro-financial linkages in open-economy environments.

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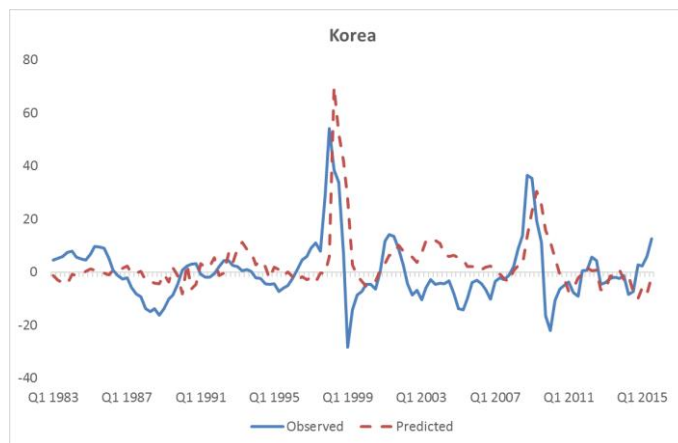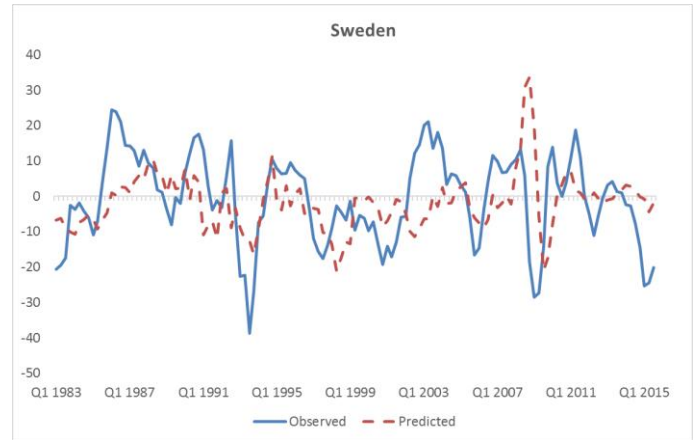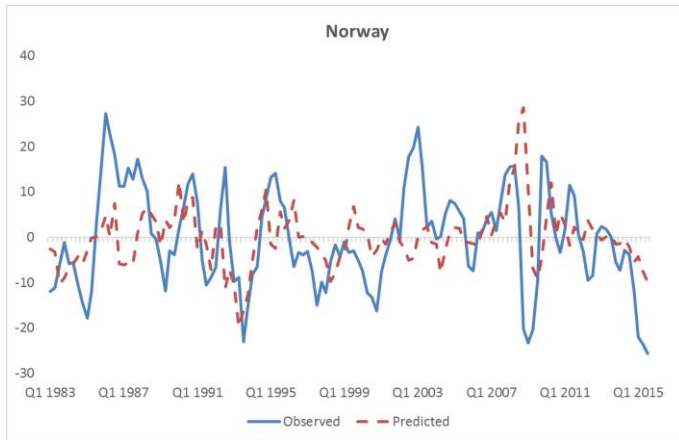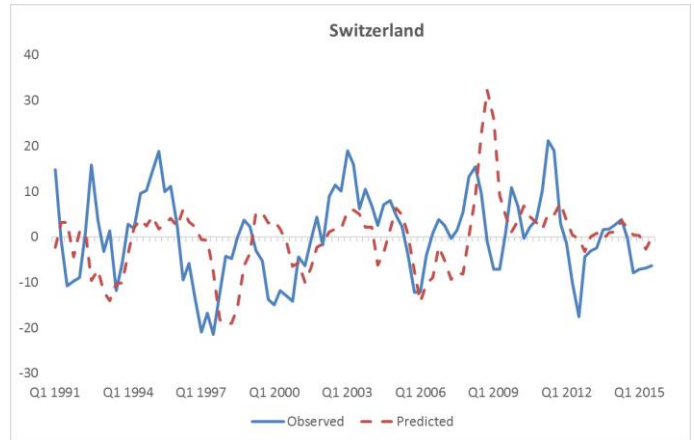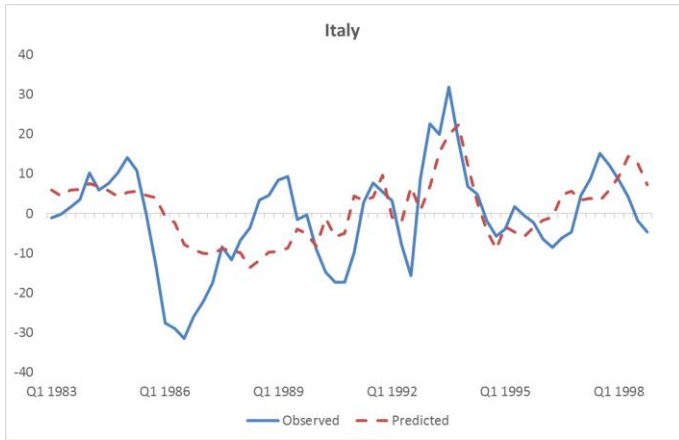Verdelhan, A. (2010). "A Habit-Based Explanation of the Exchange-Rate -risk Premium," *The Journal of Finance*, 65(1): 123-146.

Appendix

Figures for Annual Variations of the Real Exchange Rate: Observed Versus Predicted.

Netherlands

Canada

Japan

Finland

Spain

Australia
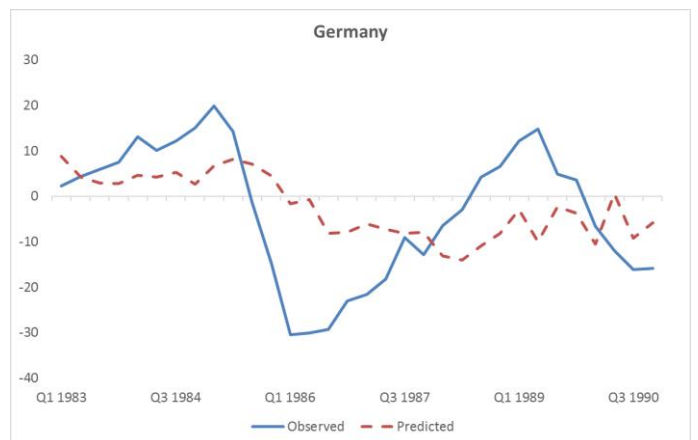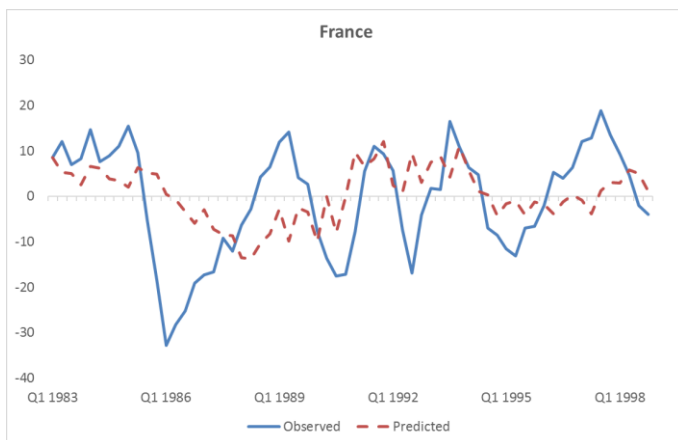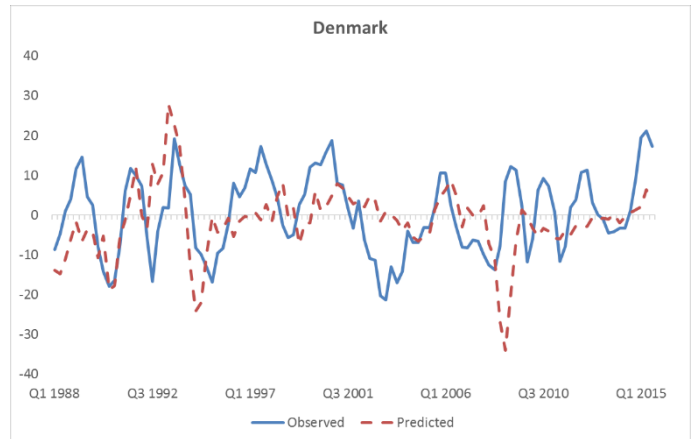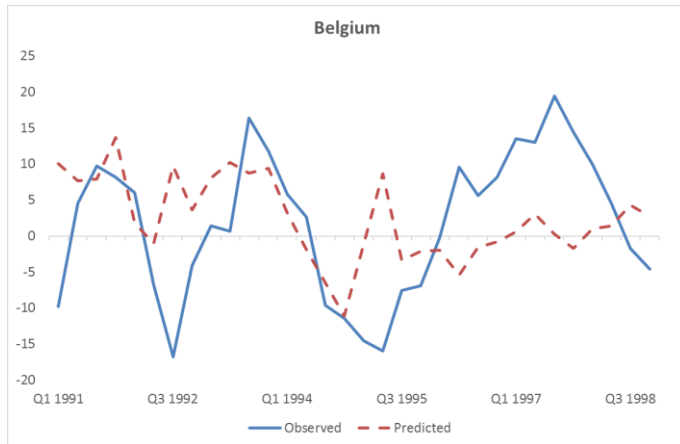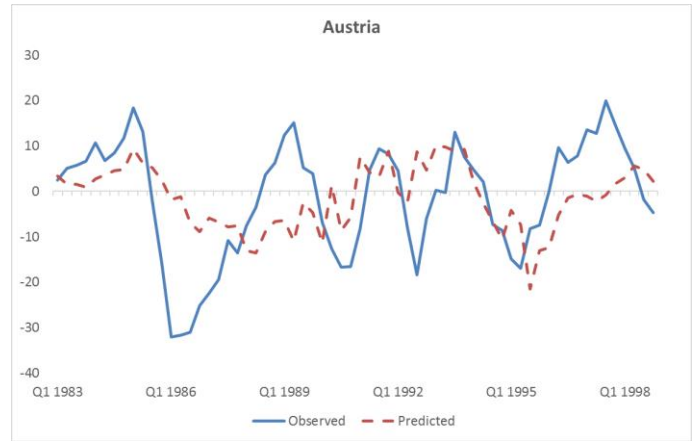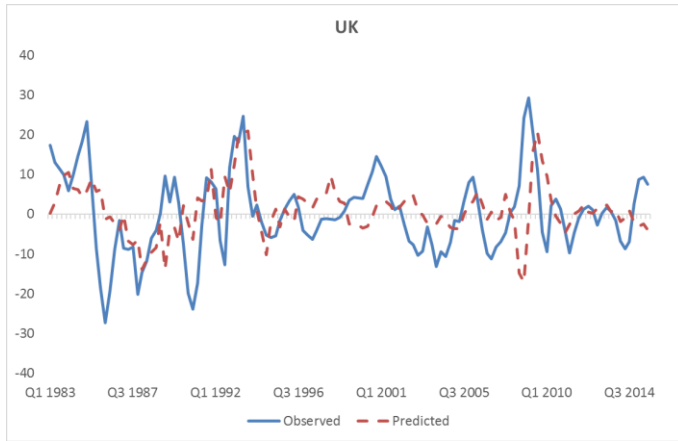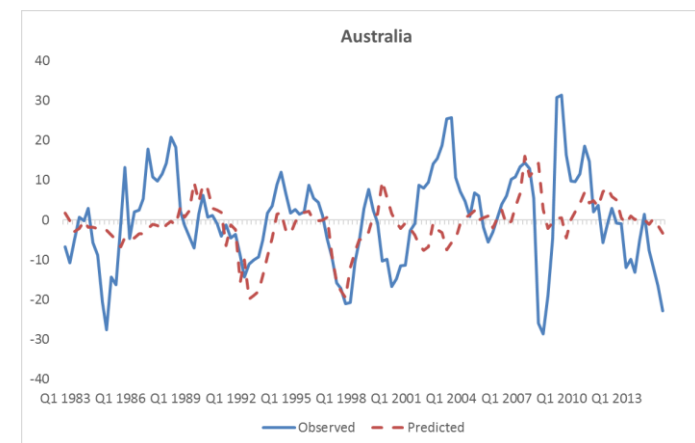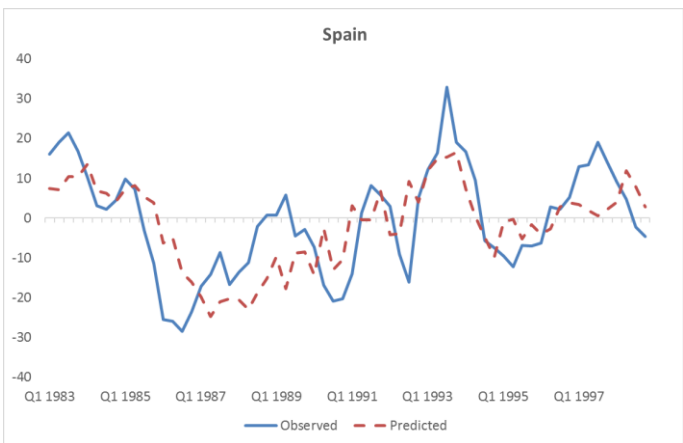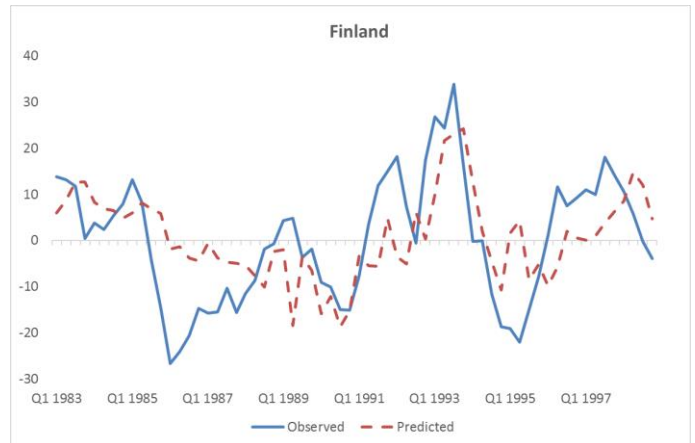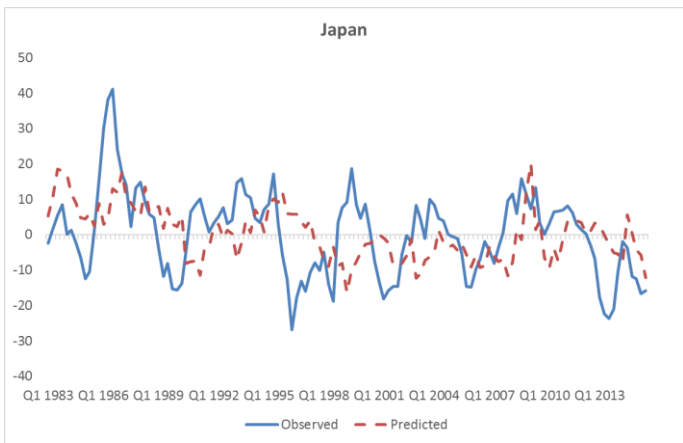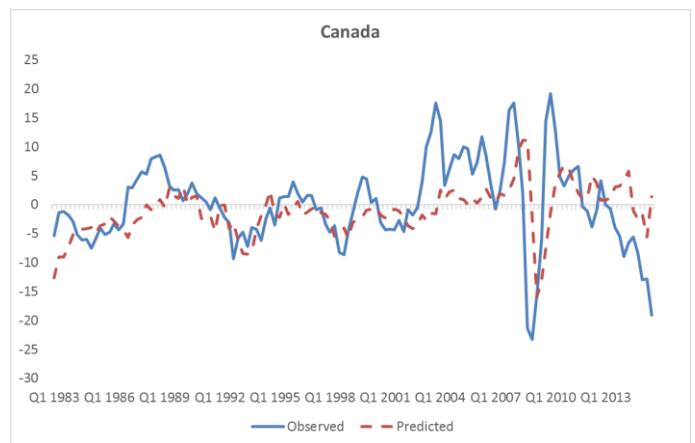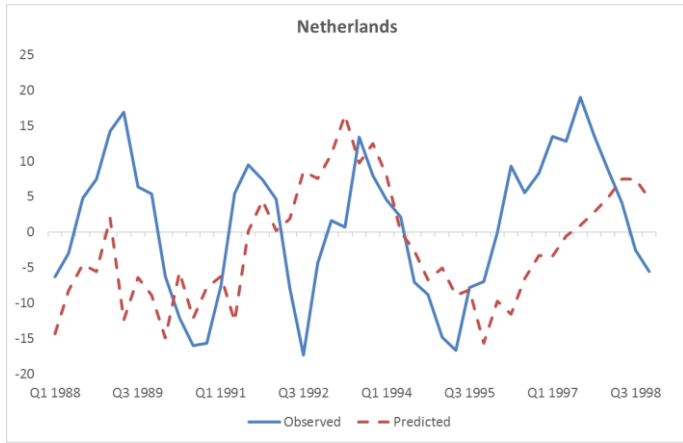
Source: International Monetary Fund and Author's Calculations

TABLE A1

Weights Used for the Computation of World Consumption

Country	GDP 2007 Billions of US Dollars	Weight %
UK	2148	6.6%
Austria	289	0.9%
Belgium	336	1.0%
Denmark	182	0.6%
France	2059	6.3%
Germany	2623	8.0%
Netherlands	567	1.7%
Canada	1127	3.5%
Japan	4229	12.9%
Finland	185	0.6%
Spain	1221	3.7%
Australia	699	2.1%
Italy	1789	5.5%
Switzerland	305	0.9%
Korea	1152	3.5%
Norway	203	0.6%
Sweden	317	1.0%
US	13233	40.5%
Overall	32664	100.0%

This table describes the weights used to compute world consumption in Equation (3). These weights correspond to the relative size of each country's GDP according to the World Development Indicators. The World Bank adjusts these GDP data by purchasing power parity.

Source: World Bank