Information Acquisition with Endogenously Determined Cost in Cournot Markets with Stochastic Demand

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Abstract

This paper presents a model of information acquisition in Cournot Market with stochastic demand where the acquisition cost is endogenously determined. The novelty is to consider the possibility of cost reducing alliances to be formed in the first-stage of a two-stage acquisition game. This paper encompasses the main assumptions found in the current literature on information acquisition regarding the role of information and how it affects firms' profits in a two stage the game. However, we argue that by adding natural assumptions regarding the choices and trade-offs between cost reduction and loss of strategic value we provide a better prediction for the outcome of information acquisition games and welfare implications.

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Key Words: Information acquisition; Cost Sharing Alliances; Information Asymmetry; Strong Nash Equilibrium.

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1. Introduction

Information acquisition in Cournot Markets with stochastic demand has almost exclusively been modelled as a two-stage game.[1] In the first stage of the game firms independently choose to acquire - or not - costly information on the true state of the demand. At the end of the first stage, decisions become common knowledge and firms that chose to acquire information observe the true parameter of the demand function. A Cournot output game takes place in the second stage.

The structure of the two-stage game implicitly assumes that a firm that acquires information bears the full acquisition cost. There are two main reasons that may justify this implicit assumption. First, cooperation to acquire information, even if not forbidden, may raise suspicions of antitrust behavior even if the cooperation is not extended to the output stage. Secondly, the assumption that there is no incentive to cooperation to acquire information may be to cooperation to acquire information may be just an may be based on findings in the exogenously given information sharing literature.

Ponssard (1979), Gal-Or (1985-1986), Ganuza and Jansen (2013), Raith (1996), and Vives (1994) for instance, have shown that firms prefer not to share their information about (linear) market demand as information asymmetry gives a strategic advantage to the informed firms.

Apparently, the lack of incentive for *information sharing* has been associated with a lack of incentive for *sharing the cost of information acquisition* as the later may lead to the former

[1] See, for instance, Chang and Lee, 1992; Hwang (1993); McKelvey and Page (1987); Ponssard, 1979; Vives (1988);

and, consequently a reduction in strategic value. However, what has not been studied so far is the trade-off between cost reduction and loss of strategic value as result of a cost sharing alliance. This paper shows that under certain conditions there may be a beneficial welfare improving trade-off between cost sharing and information sharing that increases the number of informed firms while reducing the cost of information acquisition.

The current literature on information acquisition in Cournot market with stochastic demand has drawn some welfare comparisons. On the one hand, it has shown that there are welfare gains when the level of information regarding the state of the demand increases. There is an increase in consumer surplus and the level of uncertainty in the market is reduced. On the other hand, it has been shown that the two stage information acquisition games also lead to an inefficient allocation of resources because of the duplication of costly research (Hawk 2001).

The objective of this paper is to revisit the issue of information acquisition in Cournot markets with stochastic demand. This paper investigates the conditions under which firms may have incentive to form cost reducing alliances in the first stage of a two-stage information acquisition game. We assume that alliances have the sole purpose to reduce acquisition costs and, when formed, are dissolved at the end of the first stage of the game before firms choose output[2].

In addition, this paper contributes to a literature (Myatt and Wallace, 2015; Palfrey, 1985; Vives, 1988) which considered the welfare impact of information use oligopolies with stochastic demand. The game analyzed has a similar structure found in the literature on information sharing

[2] It's beyond the scope of this paper to examine whether cost-sharing alliances induce cooperation in the output market and/or provide enough synergy to make output cartel stable. See Catilina and Feinberg (2006) for a discussion on this topic.

in oligopolies (Raith, 1996). The informational framework follows the tradition of the literature on the social value of information (Morris and Shin, 2002; Angeletos and Pavan, 2004, 2009; Angeletos, Iovino and La'O, 2011, Amador and Weill, 2012; Llosa and Venkateswaran, 2013; Colombo, Femminis and Pavan, 2014; Colombo, Femminis and Pavan, 2014)

1.2. The Cost of Sharing Cost

The decision to join a cost-sharing alliance is not trivial. Whether or not a firm becomes informed affects its output choice in the second stage of the game and, consequently, its profitability. Firms acquire information when there is a beneficial trade-off between the gains and the cost of information. Under certain circumstance to be *informed* is not enough of an incentive to acquire costly information; a firm may need to be *better informed* than its competitors.

Information has a direct value that is always positive. It refers to the gain from making better (more informed) decisions. In our specific case, it allows firms to choose output using the true parameter of the demand function as opposed of its expected value. Information has also an indirect (or strategic) value - that may be positive or negative. It refers to the gains (or losses) not from being informed (direct gain) but from being *better* informed than the competitors. When firms form an alliance to share the cost of information it may lose strategic value. Even if there is no transaction cost, there is an implicit cost to join a cost reduction alliance that can be measured in term of the loss of strategic value. The cost of sharing cost is the cost of sharing information.

To understand firms' incentive to share acquisition cost this paper introduces modifications that may *enhance* the two-stage game of information acquisition. The main assumption in this paper is that, in the first-stage of the game, firms choose an *information* *acquisition strategy*. After observing the cost of information acquisition, firms face three possible strategies: (*i*) to remain uninformed, (*ii*) to purchase information individually, or (*iii*)to join a cost sharing alliance to purchase information. A firm join a cost sharing alliance if either, (*i*) it doesn't decrease the strategic value of the information purchased, that is, it does not increase the number of informed firms or, (*ii*) the cost sharing scheme attracts otherwise uninformed firms but there is a favorable trade-off between the *endogenous* cost of information acquisition and the value added by the information purchased. Firms would remain uninformed if there is a negative trade-off between cost of information acquisition and its benefits, regardless of the purchasing strategy.

The existing literature on costly information acquisition suggests a socially undesirable and welfare reducing duplication of costly research (see, for instance, Hauks and Hukens. 2001). The alternative approach proposed in this paper shows that for some configuration of the parameters of the model it's optimal for firms to share the cost of information acquisition. It also shows that the cost sharing may increase the total welfare of the economy as it reduces the information asymmetry in the market. In addition, while it may not eliminate duplication of costly research efforts, it shows that it happens to a much lesser degree.

This paper encompasses the main assumptions in the current literature on information acquisition and how it affects firms' profits in a two stage the game. However, we argue that by adding natural assumptions regarding the choices and trade-offs firms face on the first stage of the game we provide a better prediction for the outcome of information acquisition games.

2. The Model Set up

Consider a Cournot oligopoly with n identical firms producing a homogeneous good. The linear demand function is $P = D - \sum_{i=1}^{n} y_i$ where $y_i \geq 0$ is firm i's output choice. $D = d$, $d = H, L$, is stochastic and with equal probability can be high $(D = H)$ or low $(D = L)$ and it's assumed to be common knowledge among all firms. Production is costless. Firms know they can learn the true realization of the demand parameter D at an exogenously given and fixed cost $c > 0$. Demand parameter is assumed to be either perfectly learned or not learned at all.

2.1. Baseline Model 1: The incomplete information game

The baseline model is the classical Cournot Oligopoly with stochastic demand without the possibility of acquiring costly information. In this static game uninformed firms simultaneously choose quantity output, $y_{iu,}$ to maximize their expected profit

$$E(\pi_i|D) = \pi_{iu},$$

where

$$\boldsymbol{\pi_{iu} = E(P)y_{iu}} \qquad (1)$$

Each firm i chooses y_{iu}^* such that

$$y_{iu}^* = argmax\ \pi_{iu}$$

Solving the maximization problem, we find that there is a unique pure strategy Nash Equilibrium to this game in which[3]

[3] Substituting P into (1) we have

$$\boldsymbol{y_{iu}^{*} = \frac{E(D)}{n+1}} \qquad (2)$$

and the expected profit for each uninformed firm at the equilibrium is

$$\boldsymbol{\pi_{iu}^{*} = \left(\frac{E(D)}{n+1}\right)^{2}} \qquad (3)$$

2.2. Baseline Model 2: Two Stage Information Acquisition Game without Cost Sharing

Allow firms to individually acquire costly information. Let

$$\pi_{iI} = E(\pi_i | D = d)$$

be firm i's expected profit when firm knows the realization of the demand parameter D before the output decision is made.

Regardless of the specification of the game and considering that information is costly firm i acquires information if

$$\pi_{iI} \geq \pi_{iu}$$

$$\pi_{iu}(y_{iu}) = \left(E(D) - \sum_{j}^{n} y_{ju}\right) y_{iu}, \qquad j = 1, \cdots, n$$

Solving the maximization problem, we have $y_{iu}^{*}, i = 1, \cdots n$ that satisfies the following set of linear equations

$$\sum_{j=1}^{n} y_{ju} + y_{iu} = E(D), i = 1, \cdots n; j = 1, \cdots, n$$

This set of equations is easily by adding up all equations to obtain $\sum_{j=1}^{n} y_{ju}$ and then substituting to find each value of y_{iu}.

That is, when the expected profit after acquiring information is greater or equal to the expected profit when information is not available.

3. The two-stage information acquisition game with cost sharing.

Now let's consider a two-stage information acquisition game where firms have the possibility to join a cost reducing alliance.

3.1: First-Stage

On the first stage of the game the $n > 0$ firms in the industry observe the fixed cost of information acquisition $c > 0$ and sequentially[4] choose their information acquisition strategy. There are three possible actions to be taken: (*i*) to purchase information by joining cost-sharing alliance[5] (***IS***); (*ii*) to purchase information individually not sharing information (***INS***); or (*iii*) to remain uniformed (***UN***). Formally, the set of actions for each firm i is

$$a_{1i} = \{IS, INS, UN\}\ i = 1, \cdots, n$$

At the end of first stage the uncertainty regarding the state of the demand is resolved for those that decided to become informed.

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[4] We assume sequentially in the first stage to avoid multiple equilibria in the final solution of the game. It doesn't change the nature of quality of the results.
We are mainly interested in the total number of identical firms that, in equilibrium acquire information. Suppose, for instance, that there are $n = 3$ firms and, in equilibrium, only two firms become informed. If we assume simultaneous move there would be three possible equilibria with a cost-reducing alliance of size two. If firms move sequentially, for any sequential protocol, there will be a unique subgame perfect Nash equilibrium in which only the first and second firms to move become informed.
[5] Although a general transaction cost paid by firms for cooperative efforts to share cost is intuitively appealing, we do not include that charge in our model. Instead, we assume that there is a trade association (TA) that shares the information members choose to disclose with all industry (members and non-members). It is as if at the end of the first stage firms, (members and non-members) reveal their informedness to the TA. The TA will be responsible for purchasing the information on behalf of the alliance. Once a firm report its decision, it's bounded by it. As it is in the best interest of firms to inform their informedness, the absence of a report will be understood as a firm being uninformed.

Decisions taken at the first stage are binding (see footnote 5). Define I as the set of informed firms at the end of the first stage and let $0 \leq K \leq n$ be the number of firms in this set. The set I is partitioned into two subsets. The subset $S = \{i|a_{1i} = S\}$ consists of firms participating in a cost sharing alliance, and the subset $N = \{i|a_{1i} = NS\}$ of firms purchasing information individually. Define $0 \leq K^* \leq K$ as the number of firms in S and $(K - K^*)$ the number of firms in N. Let the expected profits of firm $i \in S$ be $\pi_{iS}(c, K^*)$, a function of the cost of information acquisition and the number of firms in the alliance. For a firm $i \in N$, the expected profit is $\pi_{iN}(c)$.

Finally, define $U = \{i|a_{1i} = UN\}$as the set of firms that remain uninformed at the end of the first stage. Then $(n - K)$ is the number of firms in this set. For a firm $i \in U$ the expected profit is given by eq. (3).

3.2 Second-Stage

At the beginning of the second stage K and K^* become common knowledge and firms engage in a Cournot competition to choose output. Let y_{iI} and y_{iU}, be the quantity output to the informed[6] and uninformed firms, respectively. Actions at this stage are in the set

$$a_{2,i\in I} = \{y_{iI}: y_{iI} \in R_+\}$$

$$a_{2,i\in U} = \{y_{iU}: y_{iU} \in R_+\}$$

[6] Notice that the output choice of informed firms is not affected by the purchasing strategies. The information acquisition strategy affects firms' profits.

4. Solution to the two-stage information acquisition game with cost sharing

Let Γ_2 be the two-stage information acquisition game. We use the concept of Subgame Perfect Nash Equilibrium (SPNE) to find a solution to the game. W apply the concept of Strong Nash Equilibrium (SNE) to solve the cost sharing alliance in the first stage of the game.

4.1 Solving the Cournot Subgame

When firms remain uniformed we proceed to find the Bayesian Nash equilibrium to the Cournot subgame with incomplete information. The equilibrium output for each firm i, y_{iU}^*, is given by equation (2) and each firm's profit, π_{iU}^*, is given by equation (3) as described in the Baseline model in section 3.1.

If firms decide to become informed, we proceed to find the Nash equilibrium to the Cournot subgame with complete information. The equilibrium output in this subgame is a function of the total number of informed firms, K, and the realization of the demand parameter D.

The profits for firms in I, the set of informed firms, is a function of the cost of information acquisition c and the information purchasing strategy.

The next Lemma shows the equilibrium to the Cournot subgame with complete information.

Lemma 1: ***In the pure strategy (symmetric) Nash Equilibrium of the Cournot Subgame with complete information***

$$y_{iI}^{*} = \frac{D - (n-K)y_{iU}^{*}}{K+1} \quad \text{when } i \in I \text{ and } K \leq n \leq, \text{or any } K^{*} \leq K$$

In the Bayesian Nash equilibrium of the Cournot Subgame with incomplete information

$$y_{iu}^{*} = \frac{E(D)}{n+1}$$

For an informed firm the optimal output choice depends on the of output choice of the K informed firms and the $(n-K)$ uninformed firms. On the other hand, uniformed firms rely on the expected value of the demand parameter and the total number of firms in the market. The level of information of the competitors is irrelevant to the uninformed firms.

4.2 The Information Acquisition Game (First-Stage) and the Subgame Perfect Nash Equilibrium

On the first stage of the game firms choose whether it's optimal to pay the full cost of information acquisition, to join a cost sharing alliance, or to remain uninformed. The decision will be based on tradeoff between the cost of information (endogenous or exogenous) and its benefits.

We can relate the benefit from information to the optimal equilibrium output of informed firms showed in *Lemma 1*. The information about the true realization of the demand parameter D determines the direct value of information while K determines the indirect or strategic value of information. If a firm decides to purchase information, whether it shares the cost of information, may affect K the total number of informed firms.

In the first stage of the game the cost of information acquisition is endogenously determined and depends on firm's information acquisition strategy. Notice that we assume the cost of information acquisition is evenly shared among the participants in the alliance. Thus, for $K^* \geq 2$ the cost of information acquisition for each firm $i \in S$ is $c_{iS} = \frac{c}{K^*}$ while the cost for a firm $i \in N$ is $c > 0$.[7]

If firm i decides to purchase information and share the cost of information acquisition its profit is given by

$$\pi_{iS}(K, K^*, c) = E\left[\frac{D^2 - (n-K)ED^2 - (n-K)^2(K+1)}{(K+1)^2} - c_i(K^*)\right] \qquad (4)$$

which depends on K, the total number of firms in I, and K^*, the total number of firms in the set S that share the exogenously given cost of information acquisition c. (See Appendix for algebra).

If a firm i decides not to share the cost of information acquisition its profits are

$$\pi_{iN}(K, c) = E\left[\frac{D^2 - (n-K)ED^2 - (n-K)^2(K+1)}{(K+1)^2} - c\right] \qquad (5)$$

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[7] Notice that if only one firm chooses to share the cost of information acquisition the firm will belong to S but will pay the full cost of information acquisition. In other words, the firm would be indifferent between sharing or not sharing the cost of information acquisition if $(n-1)$ firms decide either not share cost or remain uninformed. Firms benefit from a cost sharing scheme if and only if $K^* \geq 2$. In what follows in this paper we will be interested in joint deviations instead of single deviations from not sharing to sharing the cost of information acquisition.

and depends on the total number of firms purchasing information and the exogenously given cost of information acquisition, c.

Thus, in the first stage of the game a firm i shares the cost of information acquisition if,

$$\pi_{iS}^* \geq max\{\pi_{iN}^*, \pi_{iU}^*\} \qquad (6)$$

that is, if firm's profit when purchases information and shares the cost is higher compared to profits when firms either individually purchases information or remain uninformed, whichever is higher.

Firm i becomes informed but does not share the cost of information acquisition if

$$\pi_{iI}^* > \pi_{iU}^* \qquad (7)$$

that is, if individually purchasing information gives higher profit to firm i compared to purchasing information and sharing the cost or remaining uninformed, whichever is higher.

Finally, a firm i does not purchase information and remain uninformed if

$$\boldsymbol{\pi_{iI}^* < \pi_{iU}^*} \qquad (8)$$

where the expected profits of the uninformed firm are given by equation (3) in the baseline model.

In the first stage of the game firms' decisions determine the total number of informed firms, K, and the size of the cost sharing alliance, K^*, if formed. Before proceeding to find the Subgame Nash equilibrium it's important to discuss how firms' profits are affected by these two variables.

Keeping everything else constant, it is straight forward to see that the expected profits for each firm $i \in S$, the set of firms sharing cost of information acquisition, increases as K^* increases.[8]

On the other hand, the expected profit of each firm $i \in I = S \cup N$, that is the set of all informed firms regardless of the purchasing strategy, decreases as K, the number of firms taken informed decisions, increases. As the informational asymmetry in the market decreases, the strategic value of informed firms (the advantage from being better informed than the competitors) decreases that is,

$$\left.\frac{\partial \pi_{iI}}{\partial K}\right|_{y_{iI}^*} < 0 \; for \; all \; K \leq n.$$

The two-stage information acquisition game with cost sharing poses an interesting trade-off. Firms must reconcile the incentive for cost sharing and the disincentive for information sharing. Bearing this in mind we can determine firms' gain from information by calculating the difference between the expected profits of informed and uninformed firms[9].

Let

$$C_i = \begin{cases} c, & i \in N \\ c(K^*) = \dfrac{c}{K^*}, & i \in S \end{cases}$$

then,

$$\underbrace{E[\pi_i|D = d]}_{\text{informed}} - \underbrace{E[\pi_i|D]}_{\text{uninformed}} = \frac{VarD}{(K+1)^2} - C_i \qquad (9)$$

(see appendix for algebra)

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[8] $\left.\frac{\partial \pi_{iS}}{\partial K^*}\right|_{y_{iI}^*} = \left(\left.\frac{\partial \pi_{iS}}{\partial c_i(K^*)}\right|_{y_{iI}^*}\right)\left(\frac{\partial c_i(K^*)}{\partial K^*}\right) > 0 \; for \; all \; K^* \leq K, where \; c_i(K^*) = \frac{c}{K^*} \; and \; \frac{\partial \pi_{iS}}{\partial c_i(K^*)}, \frac{\partial c_i(K^*)}{\partial K^*} < 0 \; for \; all \; K^* \leq K \leq n.$

[9] This result is based on the baseline model II and holds regardless of the specification of firms' actions in the first stage of the game.

If we denote $\frac{VarD}{(K+1)^2} = TV(K)$ the total value of information we can rewrite equation (9) as

$$\underbrace{E[\pi_i|D=d]}_{\text{informed}} - \underbrace{E[\pi_i|D]}_{\text{uninformed}} = TV(K) - C_i$$

Hence, firms become informed if and only if,

$$TV(K) \geq C_i \qquad (10)$$

As discussed before the total value of information (TV) is a decreasing function of the total number of informed firms, K.

$$\frac{\partial TV(K)}{\partial K} = -\frac{2}{(K+1)^3} VarD < 0 \text{ for any } 1 \leq K \leq n \qquad (11)$$

For a firm $i \in S$, the marginal cost of information acquisition is

$$\frac{\partial c(\cdot)}{\partial K^*} = -\frac{c}{(K^*)^2} < 0 \; for \; any \; 2 \leq K^* \leq K \qquad (12)$$

It is straightforward from equation (12), and not surprising, that for any fixed value of c and K, the expression in (12) is minimized when $K^* = K$, that is when all firms purchasing information join the cost sharing alliance. Based on this we state the next *Lemma*.

Lemma 2: ***In the equilibrium of the two-stage information acquisition game in which K is the number of informed firms and $c(K^*)$ is the cost function, when a cost sharing alliance is formed $K^* = K$.***

Proof: See Appendix.

Lemma 2 rules out the possibility of more than one coalition to be formed in equilibrium. Thus, either all firms purchasing information share the cost of information among them or firms purchase information individually. Notice that *Lemma 2* does not say anything about the total number of firms that become informed or how the decision on to share the cost of information is taken. This will be determined by the next *Lemmas* and *Propositions*.

Following *Lemma 2*, we can substitute $K^* = K$ into equation (11) and observe the rate of change in the total value of information (TV) and the rate of change in the marginal cost for any given value of K (equations 11 and 12, respectively).

For any value of $K$and c, a decrease in the marginal cost due to a cost-sharing alliance will be offset by a reduction in the strategic value of information and, consequently, it is not optimal for firms to share the cost of information acquisition if

$$\left|\frac{\partial TV}{\partial K}\right| > \left|\frac{\partial c(\cdot)}{\partial K}\right|$$

that is, if

$$VarD > \frac{c(K+1)^3}{2K^2} \qquad (13)$$

We can now proceed to find the subgame Perfect Nash Equilibrium for the game. According to *Lemma 2* in equilibrium only one cost sharing alliance will be formed and $K = K^*$. It implies that in equilibrium either $S = \emptyset$ or $N = \emptyset$, that is $I = S$ or $I = N$. Let's $\#S$ denote the cardinality of the set S. If $I = S$ then $\#S = K_S$. That is, $K = K_S$ is the total number of informed firms when in equilibrium a cost reducing alliance is formed. Similarly, if $I = N$, we have $\#N =$

K_N, where $K = K_N$ is the number of informed firms that in equilibrium individually purchase information.

To determine the equilibrium of the game we must relate the cost of information acquisition (C_i) with the total value of information (TV). In what follows it is useful to introduce some parameters.

Let

$$\underline{c} = \frac{VarD}{(n+1)^2} = TV(n)$$

be the total value of information when total number of informed firms is $K = n$, and let

$$\overline{c} = \frac{VarD}{(2)^2} = TV(1)$$

be the total value of information when there is only one informed firm, $K = 1$. Recall that the total value of information represents the increment in firms' revenue caused by the acquisition of information and does not directly depend on the purchasing strategy used by firms. It's a function of the total number of informed firms. On the other hand, firms' profit depends on the purchasing strategy.

5.2.1 *Cooperative refinement of Nash equilibrium and the Subgame Perfect Nash Equilibrium*

To find the SPNE, we must determine the equilibrium to the reduced game where firms decide their purchasing strategy. If we use the concept of Nash Equilibrium to solve the first-stage multiple equilibria might arise for different values of c and, as we will discuss later, some of them would be inefficient. We need, therefore, to obtain a sharper prediction for firm's incentive to form a cost-sharing alliance. To this end, we will use an equilibrium concept which allows deviations by a group of firms not just individual deviations from not sharing/not purchasing to sharing the cost.

We will use concept of Strong Nash Equilibrium (SNE)[10]to solve the purchasing strategy subgame. This concept is appropriate because in our model a cost-sharing alliance is formed if and only if at least two firms deviate from purchasing alone or not purchasing information. *Lemma 3* introduces the strong Nash Equilibrium to the first stage of the information acquisition game.

Lemma 3: In the unique Strong Nash Equilibrium of the two-stage game of information acquisition with cost-sharing, in which firms choose their information purchasing strategy:

I. If $c \leq \underline{c}, U = \emptyset, I = S \rightarrow \#S = n$.

II. If $c > \overline{c}, I = \emptyset, U \neq \emptyset \rightarrow \#U = n$

III. If $\underline{c} < c \leq \overline{c}, U \neq \emptyset, I \neq \emptyset, and$

   a. If $VarD \leq \frac{c(K+1)^3}{2K^3}; I = S \neq \emptyset \rightarrow \#S = K_S; U \neq \emptyset \rightarrow \#U = n - K_S$.

   b. If $VarD > \frac{c(K+1)^3}{2K^3}; I = N \neq \emptyset \rightarrow\rightarrow \#N = K_N; U \neq \emptyset \rightarrow \#U = n - K_N$.

[10] The concept of *Strong Nash Equilibrium* was introduced by Aumann (1959). Briefly, a strategy profile is a strong Nash Equilibrium if and only if it's Pareto efficient and immunity to any coalitional or joint deviation. For a definition of SNE see also Myerson (1991).

(See Appendix for complete proof).

In (I) the cost of information acquisition, c, is less than or equal to the increment in firm's revenue (total value of information) when n firms purchase information. In this case, the strategy to share the cost of information acquisition dominates the strategy to purchase individually and to remain uninformed. Notice that in this case firms benefit from a direct gain from information but don't have any strategic advantage from being informed[11].

In (II) if $c > \overline{c}$, to remain uninformed dominates to purchasing information regardless of the purchasing strategy. In this case there is no direct or indirect gain form information.

In (III) if the cost of information acquisition is sufficiently ($\underline{c} < c \leq \underline{c}$), a cost reducing alliance will be form and $K = K_s < n$. If the cost of information acquisition is sufficiently high within the range $\underline{c} < c \leq \underline{c}$, $K = K_N < n$, there is no profitable deviation to form a coalition. Notice that within this cost range there will be an information asymmetry in the market. Informed firms will benefit from the direct and strategic gain from information regardless of the purchasing strategy.

[11]It's interesting to observe the inefficiencies that are avoid by using the concept of Strong Nash Equilibrium as opposed to the Nash Equilibrium to solve the first stage of the game. In *Lemma 3* (I), for $c \leq \underline{c}$ there would be two pure strategy Nash Equilibria. Notice that, keeping everything else constant, if $(n-1)$ firms decide not to share the cost of information acquisition, firm i would be indifferent between joining (forming) or not joining the cost sharing alliance. In this case, firm's profit would be the same regardless of the purchasing strategy. On the other hand, if at least one firm decides to deviate and join a cost sharing alliance, firm i best response is to share the cost of information acquisition. Thus, if $c \leq \underline{c}$ there would be two Nash Equilibria, one with all firms joining the alliance and anther with no firms joining the alliance. Notice however that only one of these profiles would be a strong Nash equilibrium. Only the equilibrium profile in which all firms share the cost of information is not vulnerable to joint deviations. The equilibrium profile with firms purchasing information individually is not a SNE because while a firm individually would not have incentive to deviate it would be a profitable deviation to at least two firms. Hence, an equilibrium in which no firms share information is not stable and cannot be a strong Nash equilibrium.

The next proposition states the Subgame Perfect Nash Equilibrium[12] for the information acquisition game with cost sharing.

Proposition 4: ***The n-tuple***

(a) $(\#S = n;\ y^*_{iS}\ for\ i \in S)\ if\ c \leq \underline{c}$;

(b) $(\#N = n;\ y^*_{iN}\ for\ i\ \in N)\ if\ c > \overline{c}$;

(c) $(\#S = K_S, \#U = n - K_S;\ y^*_{iS}\ for\ i \in S, y^*_{iN}\ for\ i \in U)\ if\ \underline{c} < c \leq \overline{c}\ and\ VarD \leq \frac{c(K+1)^3}{2K^3}$;

(d) $(\#N = K_N, \#U = n - K_N;\ y^*_{iN}\ for\ i \in N, y^*_{iU}\ \ for\ i \in U) if\ \underline{c} < c \leq \overline{c}\ and\ VarD > \frac{c(K+1)^3}{2K^3}$.

Constitute the Subgame Perfect Nash Equilibrium for the information acquisition game with cost sharing.

According to *Proposition 4*, if the cost of information acquisition is sufficiently low ($c < \underline{c}$), all firms in the market purchase information and share the cost of information acquisition. In this case, the total number of informed firms in the model with cost sharing is the same as in the standard models of information acquisition. However, in the former, the total cost of information acquisition is considerably small. If the cost of information acquisition is sufficiently large ($c > \overline{c}$) all firms remain uninformed. This outcome is also like the outcome of

[12] A profile strategy is a subgame perfect Nash equilibrium if it induces a Nash equilibrium in every subgame. Here, in the first subgame, that solves the whole game, we opted for using a refinement of the Nash equilibrium to eliminate inefficient equilibria. However, the fact we used this refinement does not affect the essence of or is inconsistent with the formal definition of a subgame perfect Nash equilibrium.

the standard two stage game of information acquisition. The intuition behind this result goes as follows. If the exogenous cost of information c is too high, it is necessary many firms sharing the cost of information acquisition for the purchase to be affordable. However, as the number of informed firms in the industry increases the strategic value of information for each firm decreases. Consequently, the strategic and informational gain from information would not be enough to offset the cost.

For an intermediate level of information acquisition cost ($\underline{c} < c \leq \overline{c}$), the outcomes of the information acquisition game with cost sharing are substantially different from the standard approach. Within this intermediate range the level of informedness and industry's total profit would be higher compared to the standard approach. That is, the depending the exogenously given cost of information acquisition the costs sharing alliance would allow firms that would otherwise be uniformed to acquire information. On the other hand, firms that would acquire information individually are now able to reduce the cost of information acquisition. At the cost range the trade-off there is profitable trade-off between the increase in the level of unforcedness and the cost decrease in cost per firm due to the cost sharing alliance. The decision on whether to join a cost sharing alliance depends on how the cost sharing affects the strategic value of information and the cost of information acquisition per firm.

5. A Simple Numerical Example

This simple example compares the results of our model to the standard approach to information acquisition.

Consider a Cournot oligopoly model with $n = 3$ firms and inverse demand given by $P = d - Q$, where $d = \{50,100\}$ with equal probability and Q is the aggregate demand. Thus, the expected inverse demand function $EP = 75 - Q$. Information may be acquired at a fixed and exogenously determined cost c.

Using standard calculation to solve a Cournot model, if a firm decides to remain uninformed its expected profit is $\pi_{iU}^{*} = 351.56$.

If $K_N = 3$, that is, all firms decide to individually acquire information, the expected profit for each firm is π_{iN}^{*}=390.625-c. Thus, $K_N = 3$ firms individually purchase information if

$$\pi_{iN}^{*}(K_N = 3) \geq \pi_{iU}^{*}$$

$$c \leq 39.065$$

If $c \leq 39.065$ and all firms are informed, the total value of information. The total value of information (*TV*) is given by the information value of information (*IV*) only. The $K_N = 3$ firms benefit from taken an informed decision. Nevertheless, none of the informed firms have a strategic gain from being better informed than some of all the competitors. In this specific case we have that the total value of information for each firm is

$$TV = IV = \pi_{iN}^{*}(K_N = 3) - \pi_{iU}^{*}$$

$$= 39.065$$

Now consider the possibility of forming a cost sharing alliance. If $c \leq 39.065$ and $K_N = 3$ firms share the cost of information acquisition because for any $c \geq 0$, $\pi_{iS}^{*}(K_S = 3) > \pi_{iN}^{*}(K_N = 3)$.

When only two firms decide to individually purchase information the expected profit for each firm i is

$$\pi_{iN}^{*}(K_N = 2) = 421.09 - c$$

$K_N = 2$ firms would individually information if

$$\pi_{iN}^{*}(K_N = 2) \geq \pi_U^{*}$$

$$39.065 < c \leq 69.53.$$

The two firms that individually acquire information, benefit from the information asymmetry in the market. The strategic value of information (SV) is given by

$$SV = \pi_{iN}^{*}(K_N = 2) - \pi_{iN}^{*}(K_N = 3) = 30.465$$

and the informational value (IV) of information

$$IV = [\pi_{iN}^{*}(K_N = 2) - \pi_{iU}^{*}] - SV = 39.065$$

and the total value of information[13] $(TV)$is

$$TV = IV + SV = 69.53$$

Let's now allow for the possibility of cost sharing. Firms *do not* share the cost of information acquisition if

$$\pi_{iN}^{*}(K_N = 2) \geq \pi_{iS}^{*}(K_S = 3)$$

$$421.09 - c \geq 390.625 - \frac{1}{3}c$$

$$c \leq 45.70$$

[13] Notice that the informational value of information doesn't change as the number of informed firms decreases from 3 to 2. On the other hand, the strategic value of information increases as the number of informed firms decreases.

Thus, if $39.065 < c \leq 45.70$ we have that $\#N = 2,\ S = \emptyset$, and $\#U = 1$. That is, there will be only two informed firms and they will not share the cost of information acquisition. On the other hand, if $45 < c < 69.70$ we that $\#S = 3$, $N = \emptyset$, and $U = \emptyset$. That is, three firms acquire information and share the cost because $\pi_{iN}^*(K_N = 2) < \pi_{iS}^*(K_S^* = 3)$.

If cost sharing is not considered only one firm acquires information, $K_N = 1$, if

$$\pi_{iN}^*(K_N = 1) \geq \pi_{iU}^*$$

$$69.53 < c \leq 156.25$$

The strategic value of information (*SV*) for the informed firm is

$$SV = \pi_{iN}^*(K_N = 1) - \pi_{iN}^*(K_N = 3) = 117.185$$

and the informational value of information (*IV*) is,

$$IV = [\pi_{iN}^*(K_N = 1) - \pi_{iU}^*] - SV = 39.065$$

The total value of information (*TV*) is $TV = IV + SV = 156.25$. Thus, if $69.53 < c \leq 156.25$, only one firm, $K_N = 1$, purchase information if cost sharing is not considered.

Let's now consider the possibility of cost sharing. If $69.53 < c \leq 156.25$ three firms would share the cost if and only if

$$\pi_{iS}^*(K_S = 3) \geq \pi_{iU}^*$$

$$390.625 - \frac{1}{3}c \geq 351.56$$

$$69.53 < c \leq 117.95.$$

Likewise, two firms would share the cost of information acquisition if and only if,

$$\pi_{is}^*(K_S = 2) \geq \pi_{iU}^*$$

$$390.625 - \frac{1}{3}c \geq 351.56$$

$$117.95 < c < 139.06.$$

When $139.06 < c \leq 156.25$, there is no incentive for cost sharing because at this range $\pi_S^*(K_S \geq 2) < \pi_{iU}^*$. As result, only one firm purchase information and $K_N = 1$.

When $c > 156.25$ all firms remain uninformed as $\pi_S^*(K_S \geq 2) < \pi_{iU}^*$ and $\pi_N^*(K_N \geq 1) < \pi_{iU}^*$.

The tables below summarize and compare the results of the two approaches to information acquisition.

Table 1: Standard Two-Stage Game of Information Acquisition

c	K_N	*IV*	*SV*	*TV*	$c(k)$
$c \leq 39.625$	3	39.065	0	39.065	$c(k) \leq 39.625$
$39.53 < c \leq 69.53$	2	39.065	30.465	69.53	$39.625 < c(k) < 69.625$
$69.53 < c \leq 156.25$	1	39.065	117.185	156.25	$69.625 < c \leq 156.25$
$c > 156.25$	0	0	0	0	$c(k) = 0$

Table 2: Two-Stage Game of Information Acquisition Game with endogenous cost

C		K_N	K_S	*IV*	*SV*	*TV*	$c(K^*) = \frac{c}{K^*}$
$c \leq 39.625$			3	39.065	0	39.065	$c(K^*) \leq 13.208$
$39.625 < c \leq 45.70$		2		39.065	30.465	69.53	$39.625 < c(K^*) \leq 45.70$
$45.70 < c \leq 69.53$			3	39.065	0	39.065	$15.625 < c(K^*) \leq 23.10$
$69.53 < c \leq 117.95$			3	39.065	0	39.065	$23.17 < c(K^*) \leq 39.32$
$117.95 < c \leq 139.06$			2	39.065	30.465	69.53	$58.917 < c(K^*) \leq 69.53$
$139.06 < c \leq 156.25$		1		39.065	117.185	156.25	$139.06 < c(K^*) \leq 156.25$
$c > 156.25$		0		0	0	0	0

6. Final Remarks

This paper presents a model of information acquisition in Cournot Market with stochastic demand where the acquisition cost is endogenously determined. The novelty in this paper is the possibility of a cost sharing alliance to be formed in the first stage of the game to reduce the acquisition costs. The paper explores the conditions under which firms may find beneficial to share the acquisition cost even if it implies a decrease in the asymmetry in the market.

The decision on whether to join a cost sharing alliance depends on the trade-off between the endogenously determined cost of acquisition and the benefit of information measured in terms of the direct and strategic value of information. In the information acquisition model with cost sharing firms acquire more information and at lower cost. In addition, profits are generally higher compared to the standard two-stage approach.

When the cost of information acquisition is exogenously given firms trade-off the cost of information acquisition and its benefits (strategic and direct value). However, when the possibility of cost sharing is considered firms are faced with a more complex trade-off. Now the trade-off is between the endogenously determined cost of information acquisition and its effect on the strategic value of information. On the one hand, the trade-off proposed in this paper confirms the lack of incentive to information sharing. On the other hand, it shows that firms may benefit from cost sharing if the benefit of a cost reduction is not off-set by the eventual increase in the total number of informed firms and, consequently, the decrease in the strategic value of each informed firm.

The standard approach to information acquisition reveals the existence of a socially undesirable duplication of costly research. This model shows that although duplication of costly research may occur, it is on a much smaller scale than what has been believed so far. We also show that industries are in general more informed and the total cost of information acquisition is proven to be lower.

In this paper we considered that firms can either perfectly learn the realization of the demand parameter or not learn it at all. A natural extension of this model is to assume that information can be learned at different degrees of precision and the endogenous cost of information is determined based on the precision level chosen by each firm. This treatment has already been applied in the standard two-stage approach to information acquisition. It would be interesting to investigate the characteristics of a cost sharing alliance under this assumption.

7A. Appendix

A.1: Proof of Lemma 1

The informed firms maximize their profits conditional to on *D*, thus

if

$$i \in S, \pi_{iS} = (D - \sum_{j\epsilon I} y_{jI}\,(D) - \sum_{j\epsilon U} y_{jU}(D) y_{iI}\,(D) - c(K^{*})$$

The first order condition is given by

$$\sum_{j\in I} y_{jI}(D) + \sum_{j\in U} y_{jU}(D) + y_{iI}(D) = D$$

for all $i \in S$. For $K = n$ and thus, $\sum_{j\in U} y_{jU} = 0$. ■

A.2 Derivation of Equation 4 and 5

$$E(\pi_{iS}|D = d) = \pi_{iS}(K, K^{*}) = E[y_{iI}^{*}P(Q)] - c(K^{*}) \qquad (*)$$

$$E(\pi_{iN}|D = d) = \pi_{iN}(K, K^{*}) = E[y_{iI}^{*}P(Q)] - c(K^{*}) \qquad (**)$$

where,

$$P(Q) = D - [K(y_{iI}^*) + (n - K)y_{iU}^*].$$

Substituting $P(Q)$ into equation (*) we obtain equation (4). Similarly, if we substitute $P(Q)$ into (**), we obtain equation (5).

A.3 Derivation of equation 9

Following Possard (1979, p.247)'s proof for the difference between expected profits knowing the realization of the demand parameter $D = d$ and the expected profits not knowing d, observe that $E[\alpha Y + \beta]^2 - \left[E[\alpha Y + \beta]\right]^2 = \alpha^2 VarY$, where $\alpha, \beta > 0$. Applying this result to the following expression

$$E[\pi_i | D = d, K, K^*]^2 - (E[\pi_i | D])^2 = E\left[\frac{D}{K+1}\right]^2 - \left(\frac{ED}{K+1}\right)^2 = \frac{VarD}{(K+1)^2}$$

■

A.4 Proof of Lemma 2

Notice that

$$c(K^*) = \frac{c}{K^*}$$

is a decreasing linear function of $2 \leq K^* \leq K$. For any fixed c and K, $c(K^*) \to 0$ as $K^* \to K$.■

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