A closed-form solution to the risk-taking motivation of subordinated debtholders

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Abstract

Black and Cox (1976) claim that the value of junior debt is increasing in asset risk when the firm's value is low. We show, using closed-form solution, that the junior debt's value is hump-shaped. This has interesting implications for the market-discipline role of banks' subdebt.

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1 Introduction

Over the past few decades the size and complexity of financial institutions has increased to an extent that challenges regulators' ability to monitor them. Many proposals suggest greater reliance on junior debt or subordinated debt (hereafter, subdebt) as a tool to discipline banks' risk-taking.[1] It is argued that the negative effect of excessive risk-taking on subdebt price encourages subdebtholders to monitor banks closely in a way that is aligned with the deposit insurer's incentive (Flannery, 2001). Moreover, a change in the subdebt price can signal a change in the bank's risk, allowing the regulator and other market participants to discipline the bank (Evanoff and Wall, 2001; Dewatripont and Tirole, 1993)

The 2007–2009 financial crisis put into question the effectiveness of subdebt as a monitoring tool. Calomiris and Herring (2013) claim that subdebt was ineffective in mitigating excessive risk due to subdebtholders' lack of motivation to monitor banks' asset risk. Intuitively, when a bank is in financial distress, the only way its subdebtholders can be paid is by winning a large and risky bet; hence, the risk preferences of subdebtholders become more like those of shareholders. The risk preferences of the junior debt holders has been analyzed theoretically in Black and Cox (1976) and further developed in Gorton and Santomero (1990), using the contingent claim approach for pricing corporate liabilities (Merton, 1974). They show that the value of a firm's junior debt can be expressed as a "bull spread" position, which is composed of a long call option on the firm's asset with a strike price equal to the face value of the senior debt and a short call option with a strike equal to the total debt of the firm. Specifically, Black and Cox (1976, p. 369) claim that when the firm's asset value is below a certain threshold (the discounted geometric mean of the face value of the deposits and the total debt), then the value of subdebt is an increasing function of asset risk, since subdebt is then effectively the residual claimant (see also a similar claim in Gorton and Santomero, 1990, p. 122).

In this note, we correct and extend the above claim. We use a closed-form solution (as in Black and Cox, 1976) to show that when the firm's value is below the threshold mentioned above, the relationship between the subdebt value and the asset risk is hump-shaped, rather than increasing with asset risk, and we prove that an interior level of risk

[1] The debate, following the 2007–2009 financial crisis, over the ability of subdebt to enforce market discipline, led policymakers to force banks to issue contingent capital (coco), which is a junior debt converted into the issuing bank's common stock in time of financial distress. The effect of coco on bank risk taking is studied in Hilscher and Raviv (2014), Berg and Kaserer (2015), and Martynova and Perotti (2018).

maximizes the value of the subdebt.[2] The revised analysis can have important implications for the expected effect of subdebt on the risk-taking of financial institutions in time of distress. Specifically, the subdebtholders will not be motivated to increase risk as much as possible (which is the stockholders' motivation); rather, they will be motivated to increase risk to some intermediate level. Thus, if the subdebtholders can affect risk-shifting, then they will choose a moderate level of risk and risk-shifting will be limited to the level that maximizes the value of the subdebt.[3] Moreover, we show that the level of asset risk that maximizes the value of the subdebt is an increasing function of the firm's leverage and of the proportion of the senior debt out of the firm's total debt.

Our analysis can explain the empirical results in Ashcraft (2008), which documents that an increase in the amount of subdebt in regulatory capital has a positive effect in helping a bank recover from financial distress. Others also document that including subdebt in a bank's capital reduced risk-shifting during the crisis period (Nguyen, 2013; John, Mehran, and Qian, 2010; Belkhir, 2013; Chen and Hasan, 2011).

2 Analysis

We use the expanded contingent claims valuation model derived by Black and Cox (1976) to encompass the case of multiple debt claims (junior and senior). Our model and notations closely follow Gorton and Santomero (1990).

2.1 Valuation

We begin by describing the firm's liability structure and expressing the values of the different claims.

A firm is funded by stock with market value S, and two types of debt with different priorities as claimants, such that at debt maturity the junior debt is repaid only after the senior debt is repaid in full. Both debt claims are zero-coupon loans maturing at time

[2]We present a closed-form solution to the sensitivity of a bull spread option strategy to asset risk in Peleg Lazar and Raviv (2017) and Peleg Lazar and Raviv (2019). However, the focus of those papers is on the sensitivity of the bank's stock to asset risk when bank assets are a risky debt claim.

[3]The question whether and to what extent subdebtholders can affect the choice of asset risk is a separate question that is beyond the scope of this paper. In Heller, Peleg Lazar, and Raviv (2019) we apply a game-theoretic bargaining analysis to study the equilibrium risk induced by joint control of the bank by shareholders and subdebtholders.

T. The senior debt's face value is F_S and its market value is B_S and the junior debt's face value is F_J and its market value is B_J. The value of the firm's assets, V, follows a geometric Brownian motion under the risk-neutral measure according to the following equation: $dV_t = r \cdot V_t \cdot dt + \sigma \cdot V_t \cdot dW$, where r is the instantaneous risk-free rate of return, σ is the instantaneous volatility of asset value, and dW is a standard Wiener process under the risk-neutral probability measure.

If at debt maturity the value of the firm's assets is greater than the sum of the face values of all its debt, $F_S + F_J$, then both debtholders are repaid in full and stockholders receive the residual. By contrast, if the firm's asset value at maturity is below the sum of the face values of all its debt, but above the face value of the senior debt, $F_S < V < F_S + F_J$, then the senior debt is repaid in full and the junior debtholders are the residual claimants (stockholders do not receive any payoff). Otherwise, if the firm's asset value is below the face value of the senior debt, $V < F_S$, the senior debtholders receive all the asset value and the junior debtholders and stockholders are not repaid at all.

Hence, the senior debtholders' payoff at maturity is the minimum between the firm's asset value and the face value of their debt: $B_{S,T} = \min\{V_T, F_S\}$. This expression can be rearranged and expressed as $B_{S,T} = F_S - \max\{F_S - V_T, 0\}$. As discussed in Merton (1974), this payoff is equivalent to the payoff of a risk-free debt with face value F_S and a short position in a European put option. Therefore, the present value of the senior debt is

$$B_{S,t} = F_S \cdot e^{-r(T-t)} - Put_t(V_t, F_S, \sigma, T-t, r),$$

where $Put_t(V_t, F_S, \sigma, T-t, r)$ is the value of a European put option on the firm's asset value at time t, with a strike price equal to the face value of senior debt F_S, asset risk σ, and time to maturity $T-t$. Under the above described geometric Brownian motion, the value of the option can be found using the Black and Scholes (1973) equation.

The junior debt's payoff at maturity is the minimum between the value of assets left after repaying the senior debt, if any, and the face value of the junior debt $\min\{V_T - F_S, F_J\}$, as long as the payoff is nonnegative, i.e., $B_{J,T} = \max\{\min\{V_T - F_S, F_J\}, 0\}$. This payoff can be rearranged and expressed as $B_{J,T} = \max\{V_T - F_S, 0\} - \max\{V_T - (F_S + F_J), 0\}$, which is equivalent to a long position in a European call option with a strike price equal to the face value of the senior debt, F_S, and a short position in a European call option with a strike price equal to the sum of the face values of all of the firm's debt, $F_S + F_J$. Therefore, the

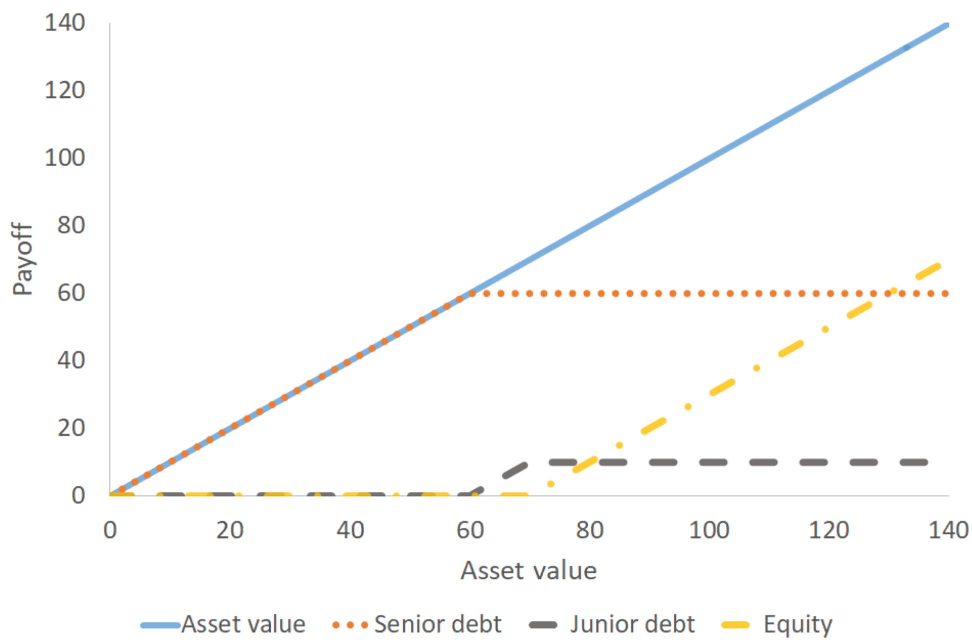


Figure 1: The firm's asset value and the payoff of the senior debt, junior debt, and equity at debt maturity. The face value of the senior debt is $F_S = 60$ and the face value of the junior debt is $F_J = 10$.

value of the junior debt prior to maturity is

$$B_{J,t} = Call_t(V_t, F_S, \sigma, T - t, r) - Call_t(V_t, F_S + F_J, \sigma, T - t, r), \tag{1}$$

where $Call_t(V_t, \cdot, \sigma, T - t, r)$ is the value of a European call option according to the Black and Scholes (1973) equation.

The stock's payoff at debt maturity is $S_T = \max\{V_T - (F_S + F_J), 0\}$. This payoff can be replicated by a European call option on the value of the firm's assets, with a strike price equal to the sum of the face values of all the firm's debt (Galai and Masulis, 1976). Therefore, the value of stock at time t is

$$S_t = Call_t(V_t, F_S + F_J, \sigma, T - t, r).$$

The value of the firm's assets and the payoff to each of its claimholders at debt maturity is presented in Figure 1.

2.2 Junior Debtholder's Asset Risk Preferences

As pointed out by Gorton and Santomero (1990, p. 122), "If the promised payment of the senior debt is close to the value of the firm, then junior debt is effectively the residual claimant and will behave like an equity claim. If, however, the value of the firm is significantly higher than the promised payment on the senior debt, then the junior debt will behave like debt." More precisely, the sensitivity of the value of junior debt to the level of asset risk is divided into two segments defined by the threshold (Black and Cox, 1976, Eq. 10; Gorton and Santomero, 1990, Eq. 7):

$$\hat{V} \equiv e^{-\left(r+\frac{\sigma^2}{2}\right)(T-t)}\sqrt{F_S \cdot (F_S + F_J)}, \tag{2}$$

which is a function of the geometric mean of the face value of senior debt and the sum of the face values of all of the firm's debt. Black and Cox (1976, p. 360) conclude that "Analysis of the function shows J [the bond's value] is an increasing (decreasing) function of σ^2 for V less than (greater than) $\hat{V}$."

We correct this statement by noting that when the value of the borrower's assets is below this threshold, the relationship between the market value of the subdebt claim and asset risk is *hump-shaped*, and the maximum value of the subdebt claim is reached at the level of asset risk defined as follows (the proof is presented in the Appendix):

$$\sigma^{max} \equiv \arg\max_{\sigma} B_{J,t}(\sigma) = \sqrt{\frac{1}{T-t}\ln\left(\frac{F_S \cdot (F_S + F_J)}{V_t^2}\right) - 2r}, \tag{3}$$

if $V \leq V^* \equiv e^{-r\cdot(T-t)}\sqrt{F_S \cdot (F_S + F_J)}$; otherwise there is no internal solution and $\sigma^{max} = 0$. The level of asset risk that maximizes the value of the junior debt, σ^{max}, is an increasing function of the firm's leverage ($F_S + F_J$ to V) and of the ratio of the senior debt to asset value (F_S to V).

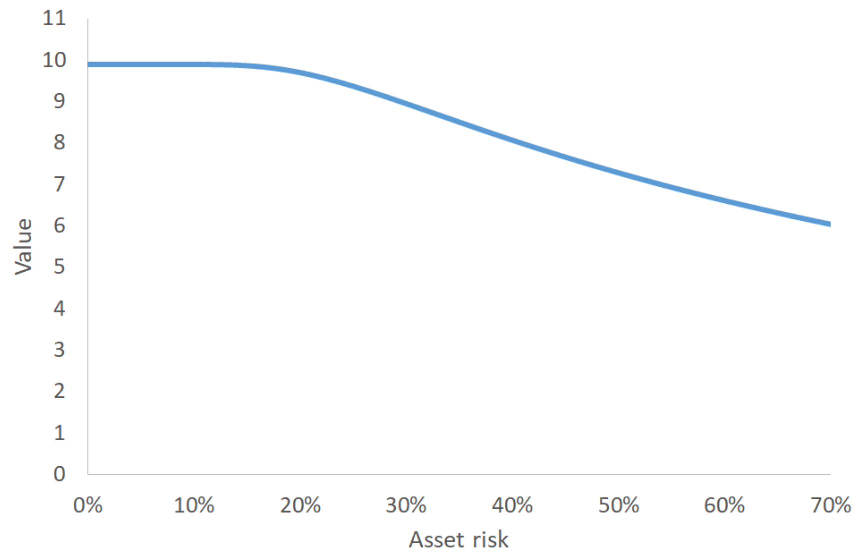
The following result summarizes the above analysis of the value of the bank stock as a function of the asset's risk.[4]

[4]The model can be extended to the case where the firm pays continuous dividends of size q. In this case we find that both the threshold under which risk shifting takes place and the optimal level of risk when risk shifting occurs, are slightly higher and equal $\hat{V} \equiv e^{-\left(r-q+\frac{\sigma^2}{2}\right)(T-t)}\sqrt{F_S \cdot (F_S + F_J)}$ and $\sigma^{max} \equiv \sqrt{\frac{1}{T-t}\ln\left(\frac{F_S\cdot(F_S+F_J)}{V_t^2}\right) - 2r + 2q}$.

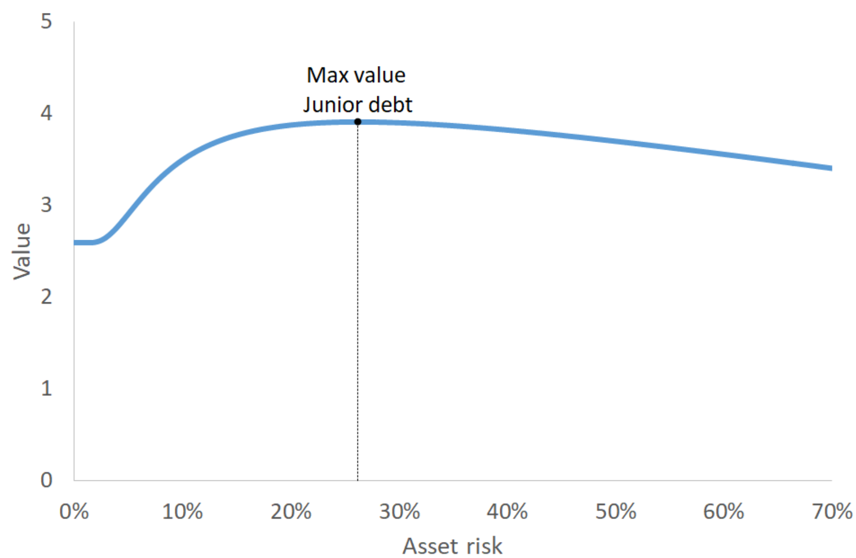
Proposition 1. *The value of the junior debt is (1) decreasing with asset risk if* $V_t > V^*$ *and (2) hump-shaped (unimodal) in asset risk if* $V_t < V^*$*, and, in this case, its maximum is obtained for risk level* σ^{max}*. Moreover, the risk level that maximizes the value of junior debt is higher than the initial risk (i.e.,* $\sigma^{max} > \sigma_0$*) if and only if* $V_t < \hat{V}$*.*

The proposition is demonstrated in Figure 2. In panel (a) the value of assets is above both $\hat{V}$ and V^* and the value of junior debt is decreasing with asset risk. By contrast, in panel (b) the value of junior debt is hump-shaped with respect to asset risk and achieves its maximum value when asset risk equals $\sigma^{max} = 26.2\%$.

The effect of the proportion of junior debt We find that both the range of asset values where risk-shifting takes place and the level of asset risk that maximizes the value of junior debt decrease with the proportion of junior debt. To see this, note that both $\hat{V}$ (Eq. 2) and σ^{max} (Eq. 3) decrease when the face value of junior debt F_J increases while the sum of the face values of the total debt $F_S + F_J$ remains unchanged (where an increase in F_J while keeping $F_S + F_J$ unchanged implies a decrease in F_S). This result is demonstrated in Figure 3, where the sum of the face values of the total debt is constant and equal to $F_S + F_J = 100$ while the face value of the junior debt changes.



(a) Asset value: $V = 100$



(b) Asset value: $V = 62$

Figure 2: The value of the junior debt as a function of the level of asset risk. The face value of the senior debt is $F_S = 60$ and the face value of the junior debt is $F_J = 10$. In addition, the time to maturity is one year and the risk-free rate is $r = 1\%$. Given these values, $\hat{V} = 63.8$.

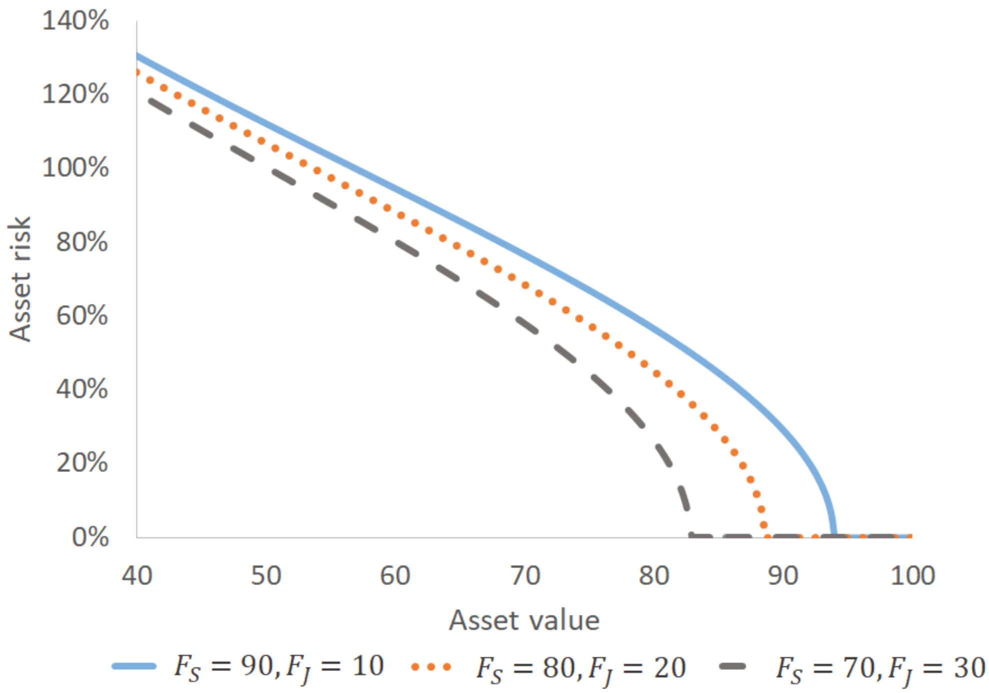


Figure 3: The level of asset risk that maximizes the value of the junior debt for different proportions of the junior debt out of the total debt. The figure depicts the level of asset risk that maximizes the value of the junior debt. The different lines (ranging from 10% to 30%) correspond to different proportions of junior debt out of the total debt. The sum of the face values of both the senior debt and the junior debt is fixed at $F_S + F_J = 100$. The initial level of asset risk is $\sigma_0 = 10\%$. In addition, the time to maturity is one year and the risk-free rate is $r = 1\%$.

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Appendix A: Proofs

The value of junior debt (Eq. 1) is equivalent to a portfolio of two call options on the value of the firm's assets $B_{J,t} = Call_t(V_t, F_S, \sigma, T-t, r) - Call_t(V_t, F_S + F_J, \sigma, T-t, r)$.

To find the level of asset risk that maximizes the value of junior debt, we calculate the derivative of the value of junior debt with respect to asset risk. Since it is well known that

$$\frac{\partial Call_t}{\partial \sigma} = \frac{\sqrt{T}}{\sqrt{2\pi}} \cdot V_t \cdot e^{-\frac{1}{2}\cdot(d_1(F_i))^2},$$

where $d_1(F_i) = \frac{1}{\sigma\sqrt{T-t}} \cdot \left[\ln\left(\frac{V_t}{F_i}\right) + \left(r + \frac{1}{2}\sigma^2\right) \cdot (T-t)\right]$, we get

$$\frac{\partial B_{J,t}}{\partial \sigma} = \frac{\sqrt{T}}{\sqrt{2\pi}} \cdot V_t \cdot e^{-\frac{1}{2}\cdot(d_1(F_S))^2} - \frac{\sqrt{T}}{\sqrt{2\pi}} \cdot V_t \cdot e^{-\frac{1}{2}\cdot(d_1(F_S+F_J))^2}.$$

After rearranging, it can be shown that the derivative equals

$$\frac{\partial B_{J,t}}{\partial \sigma} = \frac{\sqrt{T-t}}{\sqrt{2\pi}} \cdot V_t \cdot e^{-\frac{1}{2\cdot T\cdot\sigma^2}} \left[e^a - e^b\right],$$

where a and b are defined as

$$a = -2 \cdot \ln(V_t) \cdot \ln(F_S) + (\ln(F_S))^2 - 2 \cdot \ln(F_S) \cdot \left(r + \frac{\sigma^2}{2}\right) \cdot (T-t)$$

$$b = -2 \cdot \ln(V_t) \cdot \ln(F_S + F_J) + (\ln(F_S + F_J))^2 - 2 \cdot \ln(F_S + F_J) \cdot \left(r + \frac{\sigma^2}{2}\right) \cdot (T-t).$$

The payoff is maximized with respect to asset risk in cases where the first derivative equals zero. This happens when either $V_t = 0$ or $a = b$. Since the first option is of no interest economically, we focus on the second option. We find that $a = b$ when

$$V_t = e^{-\left(r+\frac{1}{2}\sigma^2\right)\cdot(T-t)} \cdot \sqrt{F_S \cdot (F_S + F_J)}, \tag{A.1}$$

Based on Equation A.1, we define $\hat{V}$ as the borrower's asset value for which the claim's value is maximized given a level of asset risk σ. This threshold is identical to the one found in

Black and Cox (1976, Eq. 10) and Gorton and Santomero (1990, Eq. 7). We also note that the derivative changes its sign from positive below the threshold to negative above the threshold, $\hat{V}$, meaning that the bank's claimholder would like to increase risk below that level and to decrease it above that level of assets.

Next, fixing the level of assets, we find that $a = b$ when

$$\sigma = \sqrt{\frac{1}{T-t} \ln \left(\frac{F_S \cdot (F_S + F_J)}{(V_t)^2} \right) - 2r}. \tag{A.2}$$

However, for both Equations A.1 and A.2 to hold, that is, for an internal solution to exist, the firm's asset value must be below V^* defined as $V^* \equiv e^{-r \cdot (T-t)} \sqrt{F_S \cdot (F_S + F_J)}$.