# Have the log-population processes stationary and independent increments? Empirical evidence for Italy, Spain and the USA along more than a century.

Ramos Arturo

Universidad de Zaragoza

29 April 2019

# Have the log-population processes stationary and independent increments? Empirical evidence for Italy, Spain and the USA along more than a century.

ARTURO RAMOS[a]

April 29, 2019

**Abstract**

We review the classical Gibrat's process for the population of city sizes. In particular, we are interested in whether the log-population process has stationary and independent (Gibrat's Law for cities) increments. We have tested these characteristics for the case of the municipalities of Italy and Spain and the places of USA for a time span of more than one century. The results are clear: stationarity and independence are empirically rejected by standard tests. These results open theoretically the way for the observance of other city size distributions other than the lognormal and the double Pareto lognormal, something that in fact has already happened in the literature.

**Keywords:** Gibrat's process; log-population process; stationary increments; independent increments; Italian cities; Spanish cities; USA cities **JEL:** C46, R11, R12

[a]Department of Economic Analysis, *Universidad de Zaragoza* (SPAIN) `aramos@unizar.com`.

# 1 Introduction

There is an ample amount of work concerning Zipf's Law and Gibrat's Law in the field of Urban Economics. Two of the main references are Gabaix (1999, 2009), where the author finds an explanation for Zipf's Law assuming that the US urban units follow a geometric Brownian motion process with a lower barrier and a Poisson process of city creation. On its side, Eeckhout (2004) proposes the lognormal distribution for describing US city size, and the generation of this distribution is based on the standard multiplicative Gibrat's process, which is another way of considering a geometric Brownian motion. In the firm size distribution literature, Sutton (1997) and Delli Gatti et al. (2005) postulate that if Gibrat's Law holds, the resulting log-size distribution will be normal, and that the log-growth rates are expected to follow a normal distribution as well. However, work by Stanley et al. (1996); Amaral et al. (1997) shows that the log-growth rate distribution of firm sizes is described better by a Laplace distribution. See also Toda (2012) for something similar regarding the income distribution. More recently, Ramos (2017) has shown a new parametric density function for US city log-growth rates that is not empirically rejected by the Kolmogorov–Smirnov (KS), Crámer–von Mises (CM) and Anderson–Darling (AD) tests.

Almost simultaneously, a quite remarkable density function has been proposed for city size (Reed, 2002, 2003; Reed and Jorgensen, 2004), later embraced by Giesen et al. (2010); Giesen and Suedekum (2012, 2014), namely the double Pareto lognormal (dPln). This last distribution can be generated by a variation of the geometric Brownian motion, adding the effects of city age to yield the associated Yule process. Thus, until the year 2015 the dPln offered the best fit for a number of countries in the literature (Giesen et al., 2010; González-Val et al., 2015).

However, the recent papers Puente-Ajovín and Ramos (2015); Luckstead and Devadoss (2017); Luckstead et al. (2017) have proposed new parametric models for which the tails are essentially Pareto, and the body is Singh–Maddala (Singh and Maddala,

1976) or lognormal, the tails and the body delineated by two exact population thresholds. These distributions offer better fits than the lognormal and dPln in the sense that they are often not rejected by standard KS, CM and AD tests for medium-sized and big countries, and that the information criteria Akaike Information Criterion (AIC) and Bayesian or Schwarz Information Criterion (BIC) yield lower (better) values for the former distributions than for the lognormal and dPln.

Even more recently, the paper of Kwong and Nadarajah (2019) proposes a better fit to US and India city size distributions than Luckstead and Devadoss (2017); Luckstead et al. (2017) by considering convex linear combinations of three or five lognormal distributions.

Thus, since the empirically observed distributions (for countries of moderate or high sample size) are not lognormal nor dPln, then the hypothesis of geometric Brownian motion (with a Yule process if needed) may not apply in practice.

The aim of this paper is to review two characteristics of the increments of the log-population process that must hold if the mentioned process qualifies as a *Lévy process*. These kind of processes embrace as particular cases the geometric Brownian motion (with a Yule process, possibly) or geometric Brownian process with drift and/or Poisson process. The two characteristics that will be empirically tested will be the *stationarity* and *independence* of increments of the process. We have at hand census data for Italy, Spain and the USA along all the XX century and the first decade of the XXI one, so the study can be considered as a long-term exercise.

The formal definition of Lévy process is as follows (Lukacs, 1970; Sato, 1999; Kyprianou, 2006):

**Definition 1 (Lévy process)** *A process* $Y = \{Y_t : t \geq 0\}$ *defined on a probability space* $(\Omega, \mathcal{F}, \mathbb{P})$[1] *is said to be a Lévy process if it possesses the following properties:*

[1] $\Omega$ denotes the sample space, i.e. the set of all possible outcomes, $\mathcal{F}$ denotes the $\sigma$-algebra of the set of events, and $\mathbb{P}$ is a function from events to probabilities.

*(i) The paths of $Y$ are $\mathbb{P}$-almost surely right continuous with left limits.*

*(ii)* $\mathbb{P}(Y_0 = 0) = 1$.

*(iii) For $0 \leq s \leq t$, $Y_t - Y_s$ is equal in distribution to $Y_{t-s}$.*

*(iv) For $0 \leq s \leq t$, $Y_t - Y_s$ is independent of $\{Y_u : u \leq s\}$.*

It can be shown, see again Kyprianou (2006) and Sato (1999), that variables that follow Lévy processes can be associated to probability laws that are *infinitely divisible* and reciprocally. The standard geometric Brownian motion with drift (with a Yule process) that can be used to generate the asymmetric double Laplace-normal for the log-population (dPln for the population) (Reed, 2002, 2003; Reed and Jorgensen, 2004) is a Lévy process since the characteristic function of the distribution of the log-population $y = \ln x$ in this case takes the form

$$\phi_y(\theta) = \exp\left(iA_0\theta - \frac{1}{2}B_0^2\theta^2\right)\frac{1}{(1 - i\theta/\alpha)\,(1 + i\theta/\beta)}$$

where $A_0, B_0, \alpha, \beta$ are real constants (and here, $i$ is the imaginary unit). In fact, this characteristic function is the product of the characteristic functions of a normal distribution and of two Gamma distributions, each of them being infinitely divisible. According to Theorem 5.3.2 in Lukacs (1970) the product is infinitely divisible as well and the underlying process of the asymmetric double Laplace-normal distribution for the log-population (dPln distribution for the population) can be associated to a Lévy process. This straightforward result shows the relation between Lévy processes and the dPln distribution.

According to what has been said before, we will explicity test conditions (iii) (stationarity) and (iv) (independence) of Definition 1 which are the ones accessible to our data sets. In short, we will see that they are strongly rejected for the three studied countries, so the log-population process does not qualify as a Lévy one and the appearance of lognormal or dPln distribution are not to be expected to describe well city size distributions *by that reason*, opening the way to other parametric descriptions that begin to appear in the literature as we have mentioned before.

The rest of the paper is organized as follows. Section 2 reviews Gibrat's process. Section 3 describes the databases used. Section 4 studies the stationarity and independence of the log-growth rates for Italy, Spain and the USA along more than a century. Finally, Section 5 offers some conclusions.

## 2 Gibrat's process

Gibrat's process for cities can be understood as follows (we base our development mainly in Sutton (1997) and references therein, Eeckhout (2004) and Delli Gatti et al. (2005)). Let $x_{i,t}$ be the population of city $i$ at time $t$, and $g_{i,t} = \ln x_{i,t} - \ln x_{i,t-1}$ the log-growth rate of city $i$ between times $t-1$ and $t$. From the relation

$$\ln x_{i,t} = \ln x_{i,t-1} + g_{i,t}$$

and assuming that $t$ is a natural number, we can iterate the former and arrive to

$$\begin{aligned}\ln x_{i,t} &= \ln x_{i,t-2} + g_{i,t-1} + g_{i,t} \\ &= \ln x_{i,0} + g_{i,1} + \cdots + g_{i,t-1} + g_{i,t}\end{aligned}$$

Then, if the log-growth rates or increments $g_{i,t}$ are identical and independent variables with mean $\mu$ and variance $\sigma^2$ *for all* $i, t$,[2] by the Central Limit Theorem (see, e.g., Feller (1968)) we have that as $t \to \infty$ the quantity $\ln x_{i,t} - \ln x_{i,0}$ will follow a normal distribution with mean $\mu t$ and variance $\sigma^2 t$.[3]

[2] And therefore the increments are clearly *stationary* and *independent* in the sense of Definiton 1.

[3] Kalecki (1945) modifies this derivation so as to obtain a lognormal distribution for the size with constant variance, by allowing a negative correlation between the log-growth rates and log-size.

In contrast, it has been empirically found (Puente-Ajovín and Ramos, 2015; Luckstead and Devadoss, 2017; Luckstead et al., 2017; Kwong and Nadarajah, 2019) that the normal distribution for log-populations is not observed in practice but other alternative distributions.

Thus the key assumption in obtaining the normal distribution for the log-populations in the previous paragraph, namely that the increments $g_{i,t}$ are stationary and independent, deserves a reconsideration.[4]

Thus it is our main interest in this paper to study empirically, in the most general standard framework, the question of whether the previous log-growth rates $g_{i,t}$ are stationary and independent, based on our relatively ample databases.

## 3 The databases

There is a lively debate about the proper definition of cities from an economic point of view. The usual data sets contain *administratively defined cities*, not always coinciding with the economic sense of a city in terms of, for example, commuting or trade flows, from which metropolitan areas are considered the most descriptive urban agglomerations. See, e.g., Ioannides and Skouras (2013) for the definition of MSAs in the USA. Metropolitan areas have the drawback that they focus essentially on the upper tail of the distribution, or, in other words, they present an implicit cut-off for the considered population values.

On their side, the usual census data sets offer a limitation in the sense that maybe very spatially close different administrative cities are not considered as only one economic entity. This perhaps becomes more relevant for the biggest agglomerations in a country (Giesen and Suedekum, 2012). One way to overcome this problem is to define

[4] One could argue that for US cities the current $t$ is not long enough to give sense to the previous limit. The convergence is known to be of the order $O(t^{-1/2})$ (see, e.g, Feller (1968)) and we will assume that the limiting distribution, if any, should have approximately been reached already.

*clusters* that give economic sense to actual agglomerations, irrespective of their legal borders, as pioneered by Rozenfeld et al. (2008, 2011). The construction of these data sets relies in turn on the availability of previous good data sets of geo-localization of urban settlements. Thus there exist few of these data sets (for the whole city size distribution) currently: those of USA and UK. The availability of data sets imposes therefore a clear constraint on the studies that can be carried out in practice.

Official census have instead a crucial advantage over other types of data for the study that is aimed in this paper, namely the long time span of the data. Indeed, we have collected data from the official census from as early as 1900/1901 and continued until 2010/2011 for three medium sized and big countries as they are Italy, Spain and the USA. Thus, for a long-term study, we are constrained to take data sets available by that time. Moreover official census cover very often almost 100% of the population (a remarkable exception is the census of places in the USA, of use in this paper) without population cut-offs as Eeckhout (2004) advocates.

In this article we use population data, without size restriction, on the one hand, of two European countries: Italy and Spain, and the USA on the other hand.

For the case of Italy and Spain, the administrative urban unit of the data is the municipality. For Italy, the data is obtained from the *Istituto Nazionale di Statistica* (`www.istat.it`), with all the Italian municipalities (*comuni*) for the years from 1901 until 2011. The data for Spain is taken from the *Instituto Nacional de Estadística* (`www.ine.es`). They cover all the municipalities (*municipios*) covering the years from 1900 until 2010.

As we take the data of all cities defined as the lowest administrative subdivision of population settlements for these two European countries, we assume that from an economic point of view they be roughly comparable units of study.

We have used in this article data about US urban centers from the decennial data of the US Census Bureau of "incorporated places" without any size restriction, for the period 1900-1990. These include governmental units classified under state laws as cities, towns, boroughs or villages. Alaska, Hawaii and Puerto Rico have not been considered due to data limitations. The data have been collected from the original documents of the annual census published by the US Census Bureau.[5] These data sets were first introduced in González-Val (2010), see therein for details, and later used in other works like González-Val et al. (2013, 2015). Moreover, despite the slight lack of consistency of the urban units, we have used the data sets of all US urban places in the year 2000 (first used by Eeckhout (2004) and then in many papers, as for example Levy (2009); Eeckhout (2009); Giesen et al. (2010); Ioannides and Skouras (2013); Giesen and Suedekum (2014); González-Val et al. (2015)) and in the year 2010 (used, for example, in Luckstead and Devadoss (2017); Ramos (2017)). Likewise, the datasets of "City Clustering Algorithm" (CCA) (Rozenfeld et al., 2008, 2011) have not been considered because their temporal span is very short (1991-2000) in order to consider a long-term perspective.

We offer in Tables 1, 2 and 3 the descriptive statistics of the population samples obtained from the official censusfor the three countries. Observations with zero population have been removed from the original sources. For Italy the census of 1941 is *not* considered because of WWII but the 1936 one instead. For Spain, the census of 1980 took place on 1981 instead, and then on 1991, 2001 and back on 2010. Observe that for the USA samples the sample size growths considerably with time.

In Tables 4, 5 and 6 we show the descriptive statistics of the log-growth rates samples computed with the previous population samples.

[5] `http://www.census.gov/prod/www/decennial.html` Last accessed: March 1$^{st}$, 2019.

# 4 About the stationarity and independence of the log-growth rates

In this Section we analyze whether the (generally) decennial log-growth rates for the data of Italy, Spain municipalities and the USA places are, respectively, equal in distribution, namely whether requirement (iii) in Definition 1 holds for the process followed by $Y_t = \ln x_t$ applied to each type of data.

If the cited condition (iii) holds, it should happen that

$$\ln x_{i,t} - \ln x_{i,t-1} = g_{i,t} \tag{1}$$

is equal in distribution to $\ln x_{i,1}$ for all $i$. Thus, all $g_{i,t}$ should be equal in distribution for all $t$.

We have available eleven samples of (almost) decennial intervals for each of the three countries under study. We will test whether the corresponding log-growth rates come from the same distribution.

For that, we simply perform the Kolmogorov–Smirnov (KS), Crámer–von Mises (CM) and Anderson–Darling (AD) tests to the empirical log-growth rates of each period compared to all other periods' samples. The null hypothesis in all cases is that the empirical log-growth samples come from the same distribution for a given country.

The results are grouped by test and country, in Tables 7-15. In short, we obtain always a strong rejection of the null hypothesis, with zero $p$-values. Thus we have that the increments of the log-population of Italian and Spanish municipalities and US places are *not* stationary, and requirement (iii) in Definition 1 is not fulfilled.

We should remark however that there are sources for the non-stationarity for the samples of the three countries. For Italy, the incorporation of Trentino, South Tirol, Trieste, Istria, part of Dalmacia and the Friuli by the Treaty of Saint-Germain-en-Laye

(1919) has increased the territories under study so the initial Italian samples are not completely homogeneous with the later ones. Moreover we consider the jumps 1931-1936 and 1936-1951 which are not decennial. For Spain, the jump from 1970 to 1981 takes 11 years instead of 10, and then the jump from 2001 to 2010 takes 9 years. For the USA, the samples' jumps are regularly decennial, but the incorporated places' number increases greatly with time, so the covering of the whole distribution also increases with time. And the last two samples (2000, 2010) contain all places, not only incorporated ones. However, the rejection of the null hypothesis in the previous tests is so strong and so regular that leads us to think that the mentioned sources of non-stationarity have only slight influence, although a faithful assessment of this statement is out of reach given the data.

Also, we will comment about the dependence of log-growth rates on the initial sizes so that requirement (iv) in Definition 1 may not occur. Due to difficulties in preparing the data for other jumps other than one decade (one census), we will limit ourselves to testing whether the log-growth rates (1) are independent of the initial log-sizes $\ln x_{i,t-1}$ for each period $t$.

The test of independence chosen is the standard Hoeffding $\mathcal{D}$ (HD) test (Hoeffding, 1948). The null hypothesis in the test is that the two input vectors are statistically independent, and the alternative hypothesis is that such vectors exhibit some kind of dependence. The HD test allows to detect non-linear dependencies. The results are shown in Table 16.

In short, the HD test yields in all cases a $p$-value of zero, which shows the dependence of the log-growth rates on the initial values of the log-populations for one decade jumps. Occurring this, it is sufficient for saying that requirement (iv) of Definition 1 is not satisfied either. As a byproduct, we obtain rejection of Gibrat's Law in the common conception of it (see, e.g., González-Val et al. (2013, 2014) and references therein).

## 5 Conclusions

We have analyzed whether the Italian and Spanish municipalities and the USA places during more than a century have log-populations that conform processes with stationary and independent increments. The results can be summarized as follows:

a) The log-growth rates or increments are *strongly not* stationary. Thus condition (iii) in Definition 1 is *not* satisfied in the cases under study.

b) The log-growth rates or increments are *not always* independent of initial log-sizes (rejection of Gibrat's Law (González-Val et al., 2013, 2014)). Thus condition (iii) in Definition 1 is *not* satisfied either for these three countries.

Thus the Italian, Spanish and US log-population processes are not Lévy, and as a consequence they are not geometric Brownian processes (eventually, with an added Yule process). This justifies that the lognormal and dPln distributions are not to be expected by the assumption of the last processes taking place. They may appear, however, by other reasons.

In addition, the above results open, theoretically, the way to the existence of newly observed city size distributions other than the lognormal and the dPln as it has happened already in the literature (Puente-Ajovín and Ramos, 2015; Luckstead and Devadoss, 2017; Luckstead et al., 2017; Kwong and Nadarajah, 2019). In fact, until 2015 it seemed that the best fit for the overall city size distribution was obtained by the dPln, but from 2015 onwards, a number of alternative distributions, which fit the data very well and much better than the lognormal and the dPln (for medium-sized and big countries), have appeared. Nowadays the problem is the relative abundance of such parametric models, which are statistically equivalent in many cases one another[6]. And perhaps this abundance is due to the fact that the log-population processes go beyond the framework of Lévy processes to give a much richer structure. However, it is some-

[6]Work under preparation.

what paradoxical this richness with the fact that city size distributions can be extremely well described with relatively simple parametric models (cited above).

**Conflict of Interests** The author declares to have no conflicts of interest regarding the research presented in this article.


## Acknowledgements

This work is supported by the project ECO2017-82246-P of the Spanish Ministry of Economy and Competitiveness and by the Aragon Government, ADETRE Reference Group. I would like to thank Stephen Devadoss, Jeff Luckstead, Rafael González-Val, Luis Lanaspa, Miguel Puente-Ajovín, Fernando Sanz-Gracia and María Vera-Cabello for providing datasets that have been used in this study.


# References


Amaral, L. A. N., Buldyrev, S., Havlin, S., Leschhorn, H., Maas, P., Salinger, M., Stanley, E., and Stanley, M. (1997). Scaling behavior in Economics: I. Empirical results for company growth. *Journal de Physique*, 7:621–633.

Delli Gatti, D., Di Guilmi, C., Gaffeo, E., Giulioni, G., Gallegati, M., and Palestrini, A. (2005). A new approach to business fluctuations: heterogeneous interacting agents, scaling laws and financial fragility. *Journal of Economic Behavior & Organization*, 56:489–512.

Eeckhout, J. (2004). Gibrat's law for (all) cities. *American Economic Review*, 94(5):1429–1451.

Eeckhout, J. (2009). Gibrat's law for (all) cities: Reply. *American Economic Review*, 99:1676–1683.

Feller, W. (1968). *An introduction to probability theory and its applications (third edition). Volume I*. John Wiley & Sons.

Gabaix, X. (1999). Zipf's law for cities: An explanation. *Quarterly Journal of Economics*, 114:739–767.

Gabaix, X. (2009). Power laws in Economics and finance. *Annu. Rev. Econ.*, 2009:255–293.

Giesen, K. and Suedekum, J. (2012). The French overall city size distribution. *Région et Développement*, 36:107–126.

Giesen, K. and Suedekum, J. (2014). City age and city size. *European Economic Review*, 71:193–208.

Giesen, K., Zimmermann, A., and Suedekum, J. (2010). The size distribution across all cities-double Pareto lognormal strikes. *Journal of Urban Economics*, 68(2):129–137.

González-Val, R. (2010). The evolution of US city size distribution from a long term perspective (1900–2000). *Journal of Regional Science*, 50:952–972.

González-Val, R., Lanaspa, L., and Sanz-Gracia, F. (2013). Gibrat's law for cities, growth regressions and sample size. *Economics Letters*, 118:367–369.

González-Val, R., Lanaspa, L., and Sanz-Gracia, F. (2014). New evidence on Gibrat's Law for cities. *Urban Studies*, 51(1):93–115.

González-Val, R., Ramos, A., Sanz-Gracia, F., and Vera-Cabello, M. (2015). Size distribution for all cities: Which one is best? *Papers in Regional Science*, 94(1):177–197.

Hoeffding, W. (1948). A non-parametric test of independence. *The Annals of Mathematical Statistics*, 19(4):546–557.

Ioannides, Y. M. and Skouras, S. (2013). US city size distribution: Robustly Pareto, but only in the tail. *Journal of Urban Economics*, 73:18–29.

Kalecki, M. (1945). On the Gibrat distribution. *Econometrica*, 13(2):161–170.

Kwong, H. S. and Nadarajah, S. (2019). A note on “Pareto tails and lognormal body of US cities size distribution”. *Physica A: Statistical Mechanics and its Applications*, 513:55–62.

Kyprianou, A. E. (2006). *Introductory lectures on fluctuations of Lévy processes with applications*. Springer-Verlag.

Levy, M. (2009). Gibrat’s law for (all) cities: Comment. *American Economic Review*, 99:1672–1675.

Luckstead, J. and Devadoss, S. (2017). Pareto tails and lognormal body of US cities size distribution. *Physica A: Statistical Mechanics and its Applications*, 465:573–578.

Luckstead, J., Devadoss, S., and Danforth, D. (2017). The size distribution of all Indian cities. *Physica A: Statistical Mechanics and its Applications*, 474:237–249.

Lukacs, E. (1970). *Characteristic functions*. Griffin.

Puente-Ajovín, M. and Ramos, A. (2015). On the parametric description of the French, German, Italian and Spanish city size distributions. *The Annals of Regional Science*, 54(2):489–509.

Ramos, A. (2017). Are the log-growth rates of city sizes distributed normally? Empirical evidence for the USA. *Empirical Economics*, 53:1109–1123.

Reed, W. J. (2002). On the rank-size distribution for human settlements. *Journal of Regional Science*, 42:1–17.

Reed, W. J. (2003). The Pareto law of incomes–an explanation and an extension. *Physica A*, 319:469–486.

Reed, W. J. and Jorgensen, M. (2004). The double Pareto-lognormal distribution–a new parametric model for size distributions. *Communications in Statistics-Theory and Methods*, 33(8):1733–1753.

Rozenfeld, H. D., Rybski, D., Andrade, J. S., Batty, M., Stanley, H. E., and Makse, H. A. (2008). Laws of population growth. *Proceedings of the National Academy of Sciences*, 105(48):18702–18707.

Rozenfeld, H. D., Rybski, D., Gabaix, X., and Makse, H. A. (2011). The area and population of cities: new insights from a different perspective on cities. *American Economic Review*, 101:2205–2225.

Sato, K. I. (1999). *Lévy processes and infinitely divisible distributions*. Cambridge University Press.

Singh, S. K. and Maddala, G. S. (1976). A function for size distribution of incomes. *Econometrica*, 44(5):963–970.

Stanley, M. H. R., Amaral, L. A. N., Buldyrev, S. V., Havlin, S., Leschhorn, H., Maass, P., Salinger, M. A., and Stanley, H. E. (1996). Scaling behaviour in the growth of companies. *Nature*, 379:804–806.

Sutton, J. (1997). Gibrat's legacy. *Journal of Economic Literature*, 35:40–59.

Toda, A. A. (2012). The double power law in income distribution: Explanations and evidence. *Journal of Economic Behavior & Organization*, 84:364–381.

Table 1: Descriptive statistics of the Italian population samples.

| | Obs. | Mean | SD | Mean (log scale) | SD (log scale) | Min | Max |
|---|---|---|---|---|---|---|---|
| It 1901 ini | 7711 | 4275 | 14425 | 7.790 | 0.915 | 56 | 621213 |
| It 1911 fin | 7711 | 4648 | 17393 | 7.843 | 0.931 | 58 | 751211 |
| It 1911 ini | 7711 | 4648 | 17393 | 7.843 | 0.931 | 58 | 751211 |
| It 1921 fin | 7711 | 4971 | 20327 | 7.868 | 0.956 | 58 | 859629 |
| It 1921 ini | 8100 | 4864 | 20032 | 7.836 | 0.963 | 58 | 859629 |
| It 1931 fin | 8100 | 5067 | 22560 | 7.839 | 0.991 | 93 | 960660 |
| It 1931 ini | 8100 | 5067 | 22560 | 7.839 | 0.991 | 93 | 960660 |
| It 1936 fin | 8100 | 5234 | 25275 | 7.842 | 1.010 | 116 | 1150338 |
| It 1936 ini | 8100 | 5234 | 25275 | 7.842 | 1.010 | 116 | 1150338 |
| It 1951 fin | 8100 | 5866 | 31138 | 7.895 | 1.049 | 74 | 1651393 |
| It 1951 ini | 8100 | 5866 | 31138 | 7.895 | 1.049 | 74 | 1651393 |
| It 1961 fin | 8100 | 6250 | 39131 | 7.848 | 1.101 | 90 | 2187682 |
| It 1961 ini | 8100 | 6250 | 39131 | 7.848 | 1.101 | 90 | 2187682 |
| It 1971 fin | 8100 | 6684 | 45582 | 7.788 | 1.185 | 51 | 2781385 |
| It 1971 ini | 8100 | 6684 | 45582 | 7.788 | 1.185 | 51 | 2781385 |
| It 1981 fin | 8100 | 6982 | 45329 | 7.793 | 1.246 | 32 | 2839638 |
| It 1981 ini | 8100 | 6982 | 45329 | 7.793 | 1.246 | 32 | 2839638 |
| It 1991 fin | 8100 | 7010 | 42450 | 7.796 | 1.282 | 31 | 2775250 |
| It 1991 ini | 8100 | 7010 | 42450 | 7.796 | 1.282 | 31 | 2775250 |
| It 2001 fin | 8100 | 7021 | 39326 | 7.803 | 1.307 | 33 | 2546804 |
| It 2001 ini | 8081 | 7031 | 39370 | 7.805 | 1.306 | 33 | 2546804 |
| It 2011 fin | 8081 | 7478 | 41531 | 7.848 | 1.339 | 34 | 2761477 |

Table 2: Descriptive statistics of the Spanish population samples.

| | Obs. | Mean | SD | Mean (log scale) | SD (log scale) | Min | Max |
|---|---|---|---|---|---|---|---|
| Sp 1900 ini | 7800 | 2282 | 10178 | 6.966 | 1.063 | 78 | 539835 |
| Sp 1910 fin | 7800 | 2452 | 11221 | 7.013 | 1.079 | 92 | 599807 |
| Sp 1910 ini | 7806 | 2452 | 11217 | 7.013 | 1.079 | 92 | 599807 |
| Sp 1920 fin | 7806 | 2623 | 13506 | 7.025 | 1.108 | 82 | 750896 |
| Sp 1920 ini | 7812 | 2622 | 13501 | 7.025 | 1.107 | 82 | 750896 |
| Sp 1930 fin | 7812 | 2901 | 17575 | 7.060 | 1.142 | 82 | 1005565 |
| Sp 1930 ini | 7875 | 2892 | 17514 | 7.055 | 1.143 | 79 | 1005565 |
| Sp 1940 fin | 7875 | 3184 | 20126 | 7.063 | 1.182 | 11 | 1088647 |
| Sp 1940 ini | 7896 | 3181 | 20100 | 7.063 | 1.182 | 11 | 1088647 |
| Sp 1950 fin | 7896 | 3482 | 26041 | 7.086 | 1.203 | 64 | 1618435 |
| Sp 1950 ini | 7901 | 3489 | 26033 | 7.086 | 1.203 | 64 | 1618435 |
| Sp 1960 fin | 7901 | 3802 | 33671 | 7.033 | 1.272 | 51 | 2259931 |
| Sp 1960 ini | 7910 | 3802 | 33652 | 7.033 | 1.272 | 51 | 2259931 |
| Sp 1970 fin | 7910 | 4258 | 44099 | 6.830 | 1.445 | 10 | 3146071 |
| Sp 1970 ini | 7956 | 4241 | 43972 | 6.829 | 1.442 | 10 | 3146071 |
| Sp 1981 fin | 7956 | 4734 | 46219 | 6.631 | 1.628 | 5 | 3188297 |
| Sp 1981 ini | 8034 | 4701 | 45995 | 6.631 | 1.624 | 5 | 3188297 |
| Sp 1991 fin | 8034 | 4881 | 45336 | 6.529 | 1.715 | 2 | 3084673 |
| Sp 1991 ini | 8077 | 4882 | 45220 | 6.534 | 1.715 | 2 | 3084673 |
| Sp 2001 fin | 8077 | 5039 | 43080 | 6.541 | 1.755 | 7 | 2938723 |
| Sp 2001 ini | 8074 | 5041 | 43087 | 6.542 | 1.755 | 7 | 2938723 |
| Sp 2010 fin | 8074 | 5803 | 47645 | 6.580 | 1.848 | 5 | 3273049 |

Table 3: Descriptive statistics of the USA population samples.

| | Obs. | Mean | SD | Mean (log scale) | SD (log scale) | Min | Max |
|---|---|---|---|---|---|---|---|
| US 1900 ini | 10502 | 3473 | 42606 | 6.695 | 1.263 | 7 | 3437202 |
| US 1910 fin | 10502 | 4574 | 57217 | 6.880 | 1.326 | 4 | 4766883 |
| US 1910 ini | 13543 | 3624 | 50412 | 6.659 | 1.271 | 7 | 4766883 |
| US 1920 fin | 13543 | 4492 | 60692 | 6.772 | 1.336 | 5 | 5620048 |
| US 1920 ini | 15085 | 4093 | 57518 | 6.695 | 1.312 | 3 | 5620048 |
| US 1930 fin | 15085 | 5004 | 70899 | 6.763 | 1.406 | 1 | 6930446 |
| US 1930 ini | 16199 | 4787 | 68431 | 6.739 | 1.405 | 1 | 6930446 |
| US 1940 fin | 16199 | 5119 | 72452 | 6.808 | 1.429 | 1 | 7454995 |
| US 1940 ini | 16416 | 4981 | 71968 | 6.777 | 1.414 | 1 | 7454995 |
| US 1950 fin | 16416 | 5805 | 77656 | 6.865 | 1.505 | 1 | 7891957 |
| US 1950 ini | 16943 | 5668 | 76444 | 6.854 | 1.491 | 3 | 7891957 |
| US 1960 fin | 16943 | 6652 | 77112 | 6.953 | 1.603 | 1 | 7781984 |
| US 1960 ini | 17826 | 6470 | 75195 | 6.943 | 1.594 | 1 | 7781984 |
| US 1970 fin | 17826 | 7248 | 76678 | 7.027 | 1.664 | 3 | 7894862 |
| US 1970 ini | 18321 | 7160 | 75654 | 7.034 | 1.643 | 4 | 7894862 |
| US 1980 fin | 18321 | 7564 | 70278 | 7.143 | 1.658 | 2 | 7071639 |
| US 1980 ini | 18810 | 7450 | 69374 | 7.130 | 1.656 | 2 | 7071639 |
| US 1990 fin | 18810 | 8058 | 72448 | 7.110 | 1.742 | 2 | 7322564 |
| US 1990 ini | 19048 | 8042 | 72015 | 7.115 | 1.739 | 2 | 7322564 |
| US 2000 fin | 19048 | 9023 | 78498 | 7.190 | 1.779 | 1 | 8008278 |
| US 2000 ini | 24684 | 8207 | 69221 | 7.267 | 1.743 | 1 | 8008278 |
| US 2010 fin | 24684 | 8981 | 71316 | 7.302 | 1.805 | 1 | 8175133 |

Table 4: Descriptive statistics of the Italian log-growth rates samples.

| | Obs. | Mean | SD | Min | Max |
|---|---|---|---|---|---|
| It 1901 1911 | 7711 | 0.054 | 0.112 | -0.957 | 1.890 |
| It 1911 1921 | 7711 | 0.024 | 0.095 | -0.802 | 0.683 |
| It 1921 1931 | 8100 | 0.003 | 0.134 | -0.933 | 3.447 |
| It 1931 1936 | 8100 | 0.003 | 0.091 | -0.704 | 2.488 |
| It 1936 1951 | 8100 | 0.053 | 0.126 | -0.531 | 2.324 |
| It 1951 1961 | 8100 | -0.047 | 0.161 | -0.861 | 1.873 |
| It 1961 1971 | 8100 | -0.060 | 0.200 | -1.075 | 2.234 |
| It 1971 1981 | 8100 | 0.004 | 0.145 | -0.900 | 1.108 |
| It 1981 1991 | 8100 | 0.003 | 0.132 | -3.098 | 3.835 |
| It 1991 2001 | 8100 | 0.007 | 0.110 | -1.384 | 1.366 |
| It 2001 2011 | 8081 | 0.043 | 0.117 | -0.580 | 3.303 |

Table 5: Descriptive statistics of the Spanish log-growth rates samples.

| | Obs. | Mean | SD | Min | Max |
|---|---|---|---|---|---|
| Sp 1900 1910 | 7800 | 0.047 | 0.117 | -0.689 | 1.493 |
| Sp 1910 1920 | 7806 | 0.012 | 0.126 | -1.504 | 2.143 |
| Sp 1920 1930 | 7812 | 0.034 | 0.143 | -1.304 | 1.803 |
| Sp 1930 1940 | 7875 | 0.008 | 0.144 | -3.312 | 1.330 |
| Sp 1940 1950 | 7896 | 0.023 | 0.127 | -1.382 | 2.411 |
| Sp 1950 1960 | 7901 | -0.053 | 0.176 | -1.360 | 1.579 |
| Sp 1960 1970 | 7910 | -0.204 | 0.311 | -2.104 | 2.619 |
| Sp 1970 1981 | 7956 | -0.198 | 0.306 | -2.416 | 2.396 |
| Sp 1981 1991 | 8034 | -0.102 | 0.235 | -2.351 | 3.131 |
| Sp 1991 2001 | 8077 | 0.007 | 0.238 | -1.985 | 2.529 |
| Sp 2001 2010 | 8074 | 0.038 | 0.244 | -1.458 | 3.258 |

Table 6: Descriptive statistics of the USA log-growth rates samples.

| | Obs. | Mean | SD | Min | Max |
|---|---|---|---|---|---|
| US 1900 1910 | 10502 | 0.185 | 0.374 | -3.714 | 2.664 |
| US 1910 1920 | 13543 | 0.113 | 0.322 | -3.036 | 3.723 |
| US 1920 1930 | 15085 | 0.068 | 0.346 | -5.053 | 3.393 |
| US 1930 1940 | 16199 | 0.069 | 0.229 | -5.849 | 3.570 |
| US 1940 1950 | 16416 | 0.088 | 0.293 | -5.187 | 5.645 |
| US 1950 1960 | 16943 | 0.099 | 0.347 | -3.235 | 4.810 |
| US 1960 1970 | 17826 | 0.084 | 0.329 | -5.499 | 8.716 |
| US 1970 1980 | 18321 | 0.109 | 0.294 | -2.354 | 4.166 |
| US 1980 1990 | 18810 | -0.020 | 0.269 | -2.735 | 2.770 |
| US 1990 2000 | 19048 | 0.075 | 0.262 | -4.467 | 3.581 |
| US 2000 2010 | 24684 | 0.035 | 0.282 | -5.278 | 6.075 |

Table 7: Results of the Kolmogorov–Smirnov (KS) test for the null that the log-growth rates come from the same distribution. Italian samples. The format is $p$-value (statistic).

| | It 1911 1921 | It 1921 1931 | It 1931 1936 | It 1936 1951 | It 1951 1961 |
|---|---|---|---|---|---|
| It 1901 1911 | 0 (0.142) | 0 (0.180) | 0 (0.291) | 0 (0.037) | 0 (0.370) |
| It 1911 1921 | | 0 (0.120) | 0 (0.152) | 0 (0.158) | 0 (0.331) |
| It 1921 1931 | | | 0 (0.146) | 0 (0.162) | 0 (0.213) |
| It 1931 1936 | | | | 0 (0.298) | 0 (0.326) |
| It 1936 1951 | | | | | 0 (0.341) |

| | It 1961 1971 | It 1971 1981 | It 1981 1991 | It 1991 2001 | It 2001 2011 |
|---|---|---|---|---|---|
| It 1901 1911 | 0 (0.403) | 0 (0.211) | 0 (0.232) | 0 (0.208) | 0 (0.067) |
| It 1911 1921 | 0 (0.377) | 0 (0.156) | 0 (0.122) | 0 (0.101) | 0 (0.088) |
| It 1921 1931 | 0 (0.263) | 0 (0.037) | 0 (0.066) | 0 (0.051) | 0 (0.128) |
| It 1931 1936 | 0 (0.376) | 0 (0.151) | 0 (0.092) | 0 (0.109) | 0 (0.233) |
| It 1936 1951 | 0 (0.369) | 0 (0.189) | 0 (0.222) | 0 (0.197) | 0 (0.075) |
| It 1951 1961 | 0 (0.073) | 0 (0.179) | 0 (0.239) | 0 (0.259) | 0 (0.338) |
| It 1961 1971 | | 0 (0.229) | 0 (0.295) | 0 (0.312) | 0 (0.379) |
| It 1971 1981 | | | 0 (0.071) | 0 (0.085) | 0 (0.163) |
| It 1981 1991 | | | | 0.002 (0.030) | 0 (0.174) |
| It 1991 2001 | | | | | 0 (0.147) |

Table 8: Results of the Kolmogorov–Smirnov (KS) test for the null that the log-growth rates come from the same distribution. Spanish samples. The format is $p$-value (statistic).

| | Sp 1910 1920 | Sp 1920 1930 | Sp 1930 1940 | Sp 1940 1950 | Sp 1950 1960 |
|---|---|---|---|---|---|
| Sp 1900 1910 | 0 (0.156) | 0 (0.087) | 0 (0.146) | 0 (0.101) | 0 (0.415) |
| Sp 1910 1920 | | 0 (0.103) | 0 (0.034) | 0 (0.067) | 0 (0.271) |
| Sp 1920 1930 | | | 0 (0.076) | 0 (0.075) | 0 (0.331) |
| Sp 1930 1940 | | | | 0 (0.063) | 0 (0.272) |
| Sp 1940 1950 | | | | | 0 (0.332) |

| | Sp 1960 1970 | Sp 1970 1981 | Sp 1981 1991 | Sp 1991 2001 | Sp 2001 2010 |
|---|---|---|---|---|---|
| Sp 1900 1910 | 0 (0.604) | 0 (0.585) | 0 (0.462) | 0 (0.266) | 0 (0.225) |
| Sp 1910 1920 | 0 (0.539) | 0 (0.514) | 0 (0.354) | 0 (0.141) | 0 (0.143) |
| Sp 1920 1930 | 0 (0.558) | 0 (0.534) | 0 (0.395) | 0 (0.185) | 0 (0.155) |
| Sp 1930 1940 | 0 (0.526) | 0 (0.500) | 0 (0.344) | 0 (0.130) | 0 (0.146) |
| Sp 1940 1950 | 0 (0.571) | 0 (0.547) | 0 (0.405) | 0 (0.191) | 0 (0.166) |
| Sp 1950 1960 | 0 (0.379) | 0 (0.349) | 0 (0.147) | 0 (0.155) | 0 (0.216) |
| Sp 1960 1970 | | 0 (0.034) | 0 (0.234) | 0 (0.420) | 0 (0.429) |
| Sp 1970 1981 | | | 0 (0.205) | 0 (0.388) | 0 (0.400) |
| Sp 1981 1991 | | | | 0 (0.216) | 0 (0.244) |
| Sp 1991 2001 | | | | | 0 (0.098) |

Table 9: Results of the Kolmogorov–Smirnov (KS) test for the null that the log-growth rates come from the same distribution. USA samples. The format is $p$-value (statistic).

| | US 1910 1920 | US 1920 1930 | US 1930 1940 | US 1940 1950 | US 1950 1960 |
|---|---|---|---|---|---|
| US 1900 1910 | 0 (0.096) | 0 (0.156) | 0 (0.210) | 0 (0.146) | 0 (0.143) |
| US 1910 1920 | | 0 (0.091) | 0 (0.118) | 0 (0.054) | 0 (0.061) |
| US 1920 1930 | | | 0 (0.105) | 0 (0.067) | 0 (0.032) |
| US 1930 1940 | | | | 0 (0.066) | 0 (0.078) |
| US 1940 1950 | | | | | 0 (0.038) |

| | US 1960 1970 | US 1970 1980 | US 1980 1990 | US 1990 2000 | US 2000 2010 |
|---|---|---|---|---|---|
| US 1900 1910 | 0 (0.165) | 0 (0.146) | 0 (0.331) | 0 (0.196) | 0 (0.274) |
| US 1910 1920 | 0 (0.086) | 0 (0.058) | 0 (0.282) | 0 (0.119) | 0 (0.209) |
| US 1920 1930 | 0 (0.043) | 0 (0.081) | 0 (0.196) | 0 (0.087) | 0 (0.122) |
| US 1930 1940 | 0 (0.074) | 0 (0.066) | 0 (0.276) | 0 (0.062) | 0 (0.164) |
| US 1940 1950 | 0 (0.052) | 0 (0.043) | 0 (0.259) | 0 (0.078) | 0 (0.175) |
| US 1950 1960 | 0 (0.027) | 0 (0.057) | 0 (0.224) | 0 (0.063) | 0 (0.149) |
| US 1960 1970 | | 0 (0.041) | 0 (0.209) | 0 (0.044) | 0 (0.127) |
| US 1970 1980 | | | 0 (0.246) | 0 (0.062) | 0 (0.160) |
| US 1980 1990 | | | | 0 (0.223) | 0 (0.142) |
| US 1990 2000 | | | | | 0 (0.110) |

Table 10: Results of the Crámer–von Mises (CM) test for the null that the log-growth rates come from the same distribution. Italian samples. The format is $p$-value (statistic).

| | It 1911 1921 | It 1921 1931 | It 1931 1936 | It 1936 1951 | It 1951 1961 |
|---|---|---|---|---|---|
| It 1901 1911 | 0 (37.072) | 0 (69.877) | 0 (137.82) | 0 (2.039) | 0 (253.842) |
| It 1911 1921 | | 0 (20.110) | 0 (33.638) | 0 (38.413) | 0 (179.175) |
| It 1921 1931 | | | 0 (33.279) | 0 (60.542) | 0 (82.251) |
| It 1931 1936 | | | | 0 (132.203) | 0 (157.719) |
| It 1936 1951 | | | | | 0 (225.312) |

| | It 1961 1971 | It 1971 1981 | It 1981 1991 | It 1991 2001 | It 2001 2011 |
|---|---|---|---|---|---|
| It 1901 1911 | 0 (274.075) | 0 (84.084) | 0 (102.525) | 0 (82.378) | 0 (7.436) |
| It 1911 1921 | 0 (214.083) | 0 (35.641) | 0 (27.994) | 0 (17.189) | 0 (12.233) |
| It 1921 1931 | 0 (114.742) | 0 (2.494) | 0 (5.481) | 0 (3.906) | 0 (35.991) |
| It 1931 1936 | 0 (201.290) | 0 (44.332) | 0 (15.716) | 0 (15.656) | 0 (80.306) |
| It 1936 1951 | 0 (246.156) | 0 (70.366) | 0 (91.269) | 0 (73.992) | 0 (7.383) |
| It 1951 1961 | 0 (8.330) | 0 (59.196) | 0 (92.651) | 0 (107.859) | 0 (207.572) |
| It 1961 1971 | | 0 (89.316) | 0 (132.245) | 0 (147.075) | 0 (236.551) |
| It 1971 1981 | | | 0 (8.721) | 0 (9.940) | 0 (49.805) |
| It 1981 1991 | | | | 0 (1.381) | 0 (56.566) |
| It 1991 2001 | | | | | 0 (41.415) |

Table 11: Results of the Crámer–von Mises (CM) test for the null that the log-growth rates come from the same distribution. Spanish samples. The format is $p$-value (statistic).

| | Sp 1910 1920 | Sp 1920 1930 | Sp 1930 1940 | Sp 1940 1950 | Sp 1950 1960 |
|---|---|---|---|---|---|
| Sp 1900 1910 | 0 (49.052) | 0 (8.607) | 0 (37.899) | 0 (22.536) | 0 (288.817) |
| Sp 1910 1920 | | 0 (17.333) | 0.001 (1.226) | 0 (7.295) | 0 (135.221) |
| Sp 1920 1930 | | | 0 (11.351) | 0 (6.581) | 0 (201.374) |
| Sp 1930 1940 | | | | 0 (4.983) | 0 (134.799) |
| Sp 1940 1950 | | | | | 0 (195.689) |

| | Sp 1960 1970 | Sp 1970 1981 | Sp 1981 1991 | Sp 1991 2001 | Sp 2001 2010 |
|---|---|---|---|---|---|
| Sp 1900 1910 | 0 (566.829) | 0 (539.245) | 0 (368.216) | 0 (107.155) | 0 (68.422) |
| Sp 1910 1920 | 0 (458.948) | 0 (425.319) | 0 (222.746) | 0 (30.694) | 0 (35.867) |
| Sp 1920 1930 | 0 (499.088) | 0 (468.876) | 0 (284.501) | 0 (58.333) | 0 (37.536) |
| Sp 1930 1940 | 0 (440.501) | 0 (408.036) | 0 (216.015) | 0 (29.556) | 0 (31.409) |
| Sp 1940 1950 | 0 (507.843) | 0 (476.278) | 0 (282.054) | 0 (57.555) | 0 (50.383) |
| Sp 1950 1960 | 0 (228.532) | 0 (194.185) | 0 (31.984) | 0 (41.553) | 0 (81.760) |
| Sp 1960 1970 | | 0 (1.562) | 0 (95.208) | 0 (306.334) | 0 (346.244) |
| Sp 1970 1981 | | | 0 (72.798) | 0 (273.646) | 0 (315.096) |
| Sp 1981 1991 | | | | 0 (98.852) | 0 (143.311) |
| Sp 1991 2001 | | | | | 0 (10.901) |

Table 12: Results of the Crámer–von Mises (CM) test for the null that the log-growth rates come from the same distribution. USA samples. The format is $p$-value (statistic).

| | US 1910 1920 | US 1920 1930 | US 1930 1940 | US 1940 1950 | US 1950 1960 |
|---|---|---|---|---|---|
| US 1900 1910 | 0 (21.092) | 0 (74.363) | 0 (98.073) | 0 (50.963) | 0 (57.266) |
| US 1910 1920 | | 0 (26.803) | 0 (35.487) | 0 (7.138) | 0 (13.457) |
| US 1920 1930 | | | 0 (32.300) | 0 (13.355) | 0 (3.563) |
| US 1930 1940 | | | | 0 (15.252) | 0 (23.381) |
| US 1940 1950 | | | | | 0 (3.966) |

| | US 1960 1970 | US 1970 1980 | US 1980 1990 | US 1990 2000 | US 2000 2010 |
|---|---|---|---|---|---|
| US 1900 1910 | 0 (77.211) | 0 (55.485) | 0 (345.072) | 0 (104.995) | 0 (225.134) |
| US 1910 1920 | 0 (23.768) | 0 (10.749) | 0 (280.73) | 0 (41.655) | 0 (147.47) |
| US 1920 1930 | 0 (3.786) | 0 (18.537) | 0 (135.527) | 0 (17.208) | 0 (49.600) |
| US 1930 1940 | 0 (17.693) | 0 (11.589) | 0 (288.229) | 0 (10.534) | 0 (107.024) |
| US 1940 1950 | 0 (7.878) | 0 (2.720) | 0 (253.36) | 0 (19.126) | 0 (112.75) |
| US 1950 1960 | 0 (2.500) | 0 (7.261) | 0 (196.896) | 0 (15.800) | 0 (79.313) |
| US 1960 1970 | | 0 (7.933) | 0 (189.152) | 0 (6.525) | 0 (59.347) |
| US 1970 1980 | | | 0 (282.527) | 0 (12.896) | 0 (109.946) |
| US 1980 1990 | | | | 0 (224.498) | 0 (94.099) |
| US 1990 2000 | | | | | 0 (55.566) |

Table 13: Results of the Anderson–Darling (AD) test for the null that the log-growth rates come from the same distribution. Italian samples. The format is $p$-value (statistic).

| | It 1911 1921 | It 1921 1931 | It 1931 1936 | It 1936 1951 | It 1951 1961 |
|---|---|---|---|---|---|
| It 1901 1911 | 0 (197.116) | 0 (366.799) | 0 (733.396) | 0 (16.872) | 0 (1259.56) |
| It 1911 1921 | | 0 (125.714) | 0 (197.695) | 0 (218.835) | 0 (916.746) |
| It 1921 1931 | | | 0 (239.361) | 0 (316.021) | 0 (406.754) |
| It 1931 1936 | | | | 0 (722.293) | 0 (850.608) |
| It 1936 1951 | | | | | 0 (1118.23) |

| | It 1961 1971 | It 1971 1981 | It 1981 1991 | It 1991 2001 | It 2001 2011 |
|---|---|---|---|---|---|
| It 1901 1911 | 0 (1368.08) | 0 (434.516) | 0 (500.442) | 0 (407.393) | 0 (34.993) |
| It 1911 1921 | 0 (1108.51) | 0 (219.461) | 0 (142.470) | 0 (87.486) | 0 (76.958) |
| It 1921 1931 | 0 (590.312) | 0 (17.867) | 0 (32.155) | 0 (27.106) | 0 (204.367) |
| It 1931 1936 | 0 (1090.48) | 0 (316.188) | 0 (117.575) | 0 (119.939) | 0 (459.932) |
| It 1936 1951 | 0 (1233.46) | 0 (352.771) | 0 (444.941) | 0 (369.843) | 0 (39.049) |
| It 1951 1961 | 0 (56.264) | 0 (298.272) | 0 (483.658) | 0 (560.831) | 0 (1054.75) |
| It 1961 1971 | | 0 (468.167) | 0 (703.992) | 0 (780.230) | 0 (1209.41) |
| It 1971 1981 | | | 0 (59.944) | 0 (70.977) | 0 (273.77) |
| It 1981 1991 | | | | 0 (7.467) | 0 (284.311) |
| It 1991 2001 | | | | | 0 (212.905) |

Table 14: Results of the Anderson–Darling (AD) test for the null that the log-growth rates come from the same distribution. Spanish samples. The format is $p$-value (statistic).

| | Sp 1910 1920 | Sp 1920 1930 | Sp 1930 1940 | Sp 1940 1950 | Sp 1950 1960 |
|---|---|---|---|---|---|
| Sp 1900 1910 | 0 (241.656) | 0 (48.780) | 0 (201.061) | 0 (114.003) | 0 (1392.29) |
| Sp 1910 1920 | | 0 (84.373) | 0 (8.589) | 0 (37.121) | 0 (674.426) |
| Sp 1920 1930 | | | 0 (62.335) | 0 (38.733) | 0 (971.564) |
| Sp 1930 1940 | | | | 0 (32.763) | 0 (644.293) |
| Sp 1940 1950 | | | | | 0 (960.047) |

| | Sp 1960 1970 | Sp 1970 1981 | Sp 1981 1991 | Sp 1991 2001 | Sp 2001 2010 |
|---|---|---|---|---|---|
| Sp 1900 1910 | 0 (2693.83) | 0 (2571.83) | 0 (1801.36) | 0 (578.149) | 0 (433.32) |
| Sp 1910 1920 | 0 (2227.5) | 0 (2075.66) | 0 (1140.83) | 0 (211.693) | 0 (264.526) |
| Sp 1920 1930 | 0 (2392) | 0 (2257.65) | 0 (1411.17) | 0 (322.789) | 0 (252.549) |
| Sp 1930 1940 | 0 (2123.62) | 0 (1975.53) | 0 (1076.63) | 0 (187.177) | 0 (230.686) |
| Sp 1940 1950 | 0 (2439.17) | 0 (2297.04) | 0 (1409.57) | 0 (350.403) | 0 (352.744) |
| Sp 1950 1960 | 0 (1169.68) | 0 (1009.78) | 0 (196.971) | 0 (206.679) | 0 (415.639) |
| Sp 1960 1970 | | 0 (7.415) | 0 (478.733) | 0 (1512.56) | 0 (1726.26) |
| Sp 1970 1981 | | | 0 (372.823) | 0 (1365.48) | 0 (1590.13) |
| Sp 1981 1991 | | | | 0 (513.264) | 0 (752.987) |
| Sp 1991 2001 | | | | | 0 (55.342) |

Table 15: Results of the Anderson–Darling (AD) test for the null that the log-growth rates come from the same distribution. USA samples. The format is $p$-value (statistic).

| | US 1910 1920 | US 1920 1930 | US 1930 1940 | US 1940 1950 | US 1950 1960 |
|---|---|---|---|---|---|
| US 1900 1910 | 0 (122.765) | 0 (385.517) | 0 (597.921) | 0 (291.986) | 0 (293.001) |
| US 1910 1920 | | 0 (128.70) | 0 (229.755) | 0 (39.763) | 0 (64.483) |
| US 1920 1930 | | | 0 (223.299) | 0 (67.338) | 0 (21.580) |
| US 1930 1940 | | | | 0 (110.401) | 0 (175.753) |
| US 1940 1950 | | | | | 0 (23.500) |

| | US 1960 1970 | US 1970 1980 | US 1980 1990 | US 1990 2000 | US 2000 2010 |
|---|---|---|---|---|---|
| US 1900 1910 | 0 (402.27) | 0 (303.727) | 0 (1742.59) | 0 (568.023) | 0 (1172.63) |
| US 1910 1920 | 0 (111.927) | 0 (64.679) | 0 (1348.97) | 0 (216.006) | 0 (716.762) |
| US 1920 1930 | 0 (27.143) | 0 (132.007) | 0 (658.598) | 0 (128.410) | 0 (273.625) |
| US 1930 1940 | 0 (122.035) | 0 (88.263) | 0 (1359.66) | 0 (55.942) | 0 (485.901) |
| US 1940 1950 | 0 (34.689) | 0 (29.886) | 0 (1187.36) | 0 (100.489) | 0 (525.194) |
| US 1950 1960 | 0 (15.691) | 0 (61.723) | 0 (952.803) | 0 (108.324) | 0 (401.064) |
| US 1960 1970 | | 0 (52.193) | 0 (911.969) | 0 (46.946) | 0 (289.817) |
| US 1970 1980 | | | 0 (1401.95) | 0 (66.781) | 0 (532.715) |
| US 1980 1990 | | | | 0 (1103.77) | 0 (476.954) |
| US 1990 2000 | | | | | 0 (263.913) |

Table 16: Results of the Hoeffding $\mathcal{D}$ (HD) test for the null that the log-growth rates are independent of the initial log-populations. Samples from the three countries. The format is $p$-value (statistic).

| | | | |
|---|---|---|---|
| It 1901 1911 | 0 (0.0036) | Sp 1900 1910 | 0 (0.0026) |
| It 1911 1921 | 0 (0.0174) | Sp 1910 1920 | 0 (0.0142) |
| It 1921 1931 | 0 (0.0148) | Sp 1920 1930 | 0 (0.0147) |
| It 1931 1936 | 0 (0.0156) | Sp 1930 1940 | 0 (0.0155) |
| It 1936 1951 | 0 (0.0203) | Sp 1940 1950 | 0 (0.0106) |
| It 1951 1961 | 0 (0.0159) | Sp 1950 1960 | 0 (0.0417) |
| It 1961 1971 | 0 (0.0429) | Sp 1960 1970 | 0 (0.0774) |
| It 1971 1981 | 0 (0.0673) | Sp 1970 1981 | 0 (0.1262) |
| It 1981 1991 | 0 (0.0372) | Sp 1981 1991 | 0 (0.0768) |
| It 1991 2001 | 0 (0.0175) | Sp 1991 2001 | 0 (0.0217) |
| It 2001 2011 | 0 (0.0301) | Sp 2001 2010 | 0 (0.0623) |
| | | | |
| US 1900 1910 | 0 (0.0059) | | |
| US 1910 1920 | 0 (0.0071) | | |
| US 1920 1930 | 0 (0.0142) | | |
| US 1930 1940 | 0 (0.0058) | | |
| US 1940 1950 | 0 (0.0343) | | |
| US 1950 1960 | 0 (0.0348) | | |
| US 1960 1970 | 0 (0.0143) | | |
| US 1970 1980 | 0 (0.0020) | | |
| US 1980 1990 | 0 (0.0381) | | |
| US 1990 2000 | 0 (0.0097) | | |
| US 2000 2010 | 0 (0.0236) | | |