A Baseline Medium-Scale NK DSGE Model for Policy Analysis

Wenli Xu

CIMERS, Anhui University

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Anhui University and CIMERS
xuweny87@hotmail.com

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I. Introduction

During the ten years before the Globe Financial Crisis, the most common framework employed in Macroeconomics had incorporated price/wage rigidity into the DSGE models. We developed a basic medium-scale NK model following Sims,E.(2017, Course notes,A New Keyesian Model with price stickiness). Currently, the extensions of these models are employed extensively by governmental sectors to conduct policy analysis. The pioneering medium scale DSGE models are Christiano, Eichenbaum and Evans(2005,JPE) and Smets and Wouters(2007,AER).

Traditional New Keyesian model is mainly used for analyzing the impact of monetary policy. However, in our model, we focus on not only Monetary Policy the Taylor Rule), but also fiscal policy, including procyclical government expenditure, value-added tax, capital gain tax, and labor income tax policies.

The main characters of our model are:

1. Physical capital accumulation
2. Price stickiness
3. Wage stickiness
4. Backward indexation of non-updated prices and wages
5. Habit formation in consumption
6. Investment adjustment costs
7. Variable capital utilization
8. A fixed cost of production
9. Monetary policy conducted according to a Taylor rule
10. The following fiscal policies:

    a Governmental consumption spending
    b Added value tax
    c Progressive labor income tax
    d Progressive capital rent tax

11. The following shocks:

    a Productive
    b Marginal efficiency of investment
    c Governmental expenditure
    d Added value tax
    e labor income tax
    f capital rent tax
    g Monetary policy
    h Intertemporal preferences
    i Intratemporal preferences(labor supply)

II. Labor Union

Erceg, Henderson and Levin(2000) used Calvo pricing assumptions. The other studies using the wage and price stickiness properties include: Ravenna(2000), Sbordone(2002), Christiano, Eichenbaum and Evans(2005), etc. In labor market, we use $l \in [0, 1]$ to represent the different labor services. Labor union receive labor services. Then, we aggregated the various labor supply.

Aggregate labor supply function:

$$N_{d,t} = \left(\int_0^1 N_t(l)^{\frac{\epsilon_w - 1}{\epsilon_w}} dl \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}, \epsilon_w > 1 \tag{1}$$

where, $N_{d,t}$ is the homogeneous labor input available for production. $N_t(l)$ is the differentiated labor supply by household l in period t.

The parameter ϵ_w shows the elasticity of substitution among different labor service. We assume that the elasticity is large than 1 to ensure that there exists substitutability among different labor services. Based on this, labor union maximizes its profit:

$$\underbrace{max}_{N_t(l)} W_t \left(\int_0^1 N_t(l)^{\frac{\epsilon_w - 1}{\epsilon_w}} dl \right)^{\frac{\epsilon_w}{\epsilon_w - 1}} - \int_0^1 W_t(l) N_t(l) dl \tag{2}$$

In the equation, W_t is the aggregated nominal wage, $W_t(l)$ denotes the nominal wage of labor l. the Labor demand of household is represented by the FOC.

$$\begin{aligned} & W_t \frac{\epsilon_w}{\epsilon_w - 1} \left(\int_0^1 N_t(l)^{\frac{\epsilon_w - 1}{\epsilon_w}} dl \right)^{\frac{\epsilon_w}{\epsilon_w - 1} - 1} \frac{\epsilon_w - 1}{\epsilon_w} N_t(l)^{\frac{\epsilon_w - 1}{\epsilon_w} - 1} - W_t(l) = 0 \\ & \Rightarrow N_t(l) = \left(\frac{W_t(l)}{W_t} \right)^{-\epsilon_w} N_{d,t} \end{aligned} \tag{3}$$

so we can derive an aggregate wage:

$$\begin{aligned} & W_t N_{d,t} = \int_0^1 W_t(l) N_t(l) dl = \int_0^1 W_t(l) \left(\frac{W_t(l)}{W_t} \right)^{-\epsilon_w} N_{d,t} dl \\ & \Rightarrow W_t^{1-\epsilon_w} = \int_0^1 W_t(l)^{1-\epsilon_w} dl \\ & \Rightarrow W_t^{1-\epsilon_w} = \left(\int_0^1 W_t(l) dl \right)^{1-\epsilon_w} \end{aligned} \tag{4}$$

Total labor supplied by household:

$$\begin{aligned} & N_t = \int_0^1 N_t(l) dl \\ & \Rightarrow N_t = \int_0^1 \left(\frac{W_t(l)}{W_t} \right)^{-\epsilon_w} N_{d,t} dl = \int_0^1 \left(\frac{W_t(l)}{W_t} \right)^{-\epsilon_w} dl N_{d,t} \end{aligned} \tag{5}$$

Defining sticky wage diffusion index

$$v_t^w = \int_0^1 \left(\frac{W_t(l)}{W_t} \right)^{-\epsilon_w} dl \tag{6}$$

so we can derive the equilibrium condition of labor market

$$N_t = N_{d,t} v_t^w \tag{7}$$

If $N_t > 1$, then aggregate labor demand in firms will be less than the aggregate labor supply in households, which means the wage stickiness can distort the labor market.

III. Household

We assume that the utility of household is composed of consumption and leisure (also known as the negative utility of labor), household has consumption habit and capital. Household income is composed of labor income, capital income, dividend from firms, and interest income from bonds. Household decides the consumption level, the bond-holding level, and the investment label. They also decide the degree of capital utilization and the supply of capital service. Because the labor services can be differentiated, they have some bargaining power in the labor market. The households' problem can expressed as:

$$\max_{C_t, I_t, W_t(l), u_t, K_{t+1}, B_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t v_t \{\ln(C_t - bC_{t-1}) - \psi_t \frac{N_t(l)^{1+\chi}}{1+\chi}\} \tag{8}$$

where, C_t is the consumption level. u_t is the intension of capital utilization. K_t is the capital stock. B_t is bond. β is the intertemporal discount rate. v_t is intertemporal preference shock. b captures the persistence of habit formation. ψ_t is intratemporal preference (labor supply) shock.

Budget constraint for household:

$$(1+\tau_t^c)P_t(C_t+I_t)+B_{t+1}+\tilde{B}_{t+1} = (1-\tau_t^n)W_t(l)N_t(l)+(1-\tau_t^k)P_tR_tK_tu_t+\Pi_t-P_tT_t+(1+i_{t-1})B_t \tag{9}$$

, where $\tau_t^c, \tau_t^n, \tau_t^k$ are changeable VAT rate, labor income tax, and capital gain tax, separately; I_t is the investment; P_t is the aggregate price level; T_t is the lump-sum tax; Π_t is the dividend from corporation; $R_t and r_t$ are the real interest rate for capital and bonds.

Define the capital service supplied by households:

$$\hat{K}_t = K_t u_t \tag{10}$$

Household accumulation of capital:

$$K_{t+1} = Z_t \left[1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_t + (1 - \delta(u_t))K_t \tag{11}$$

Z_t is the investment shock. κ is the parameter to adjust the investment cost. δ is the deprecation rate. It is a function of the capital utilization rate:

$$\delta(u_t) = \delta_0 + \delta_1(u_t - 1) + \frac{\delta_2}{2}(u_t - 1)^2 \tag{12}$$

As mentioned above, household chooses the optimal consumption, investment, capital utilization rate, capital service, bonds and wage level. We divide the household decision into two parts: first, non-wage decision, which includes consumption, investment, capital utilization rate, capital service, and bonds; second, wage decision.

Step 1: Non-wage decision:
Lagrangian condition:

$$
\begin{aligned}
\mathcal{L} =&E_0 \sum_{t=0}^{\infty} \beta^t (v_t \ln(C_t - bC_{t-1}) + \cdots \\
&+ \overline{\lambda}_t [\cdots + (1-\tau_t^k) P_t R_t K_t u_t + \Pi_t - P_t T_t + (1+i_{t-1}) B_t \\
&- (1+\tau_t^c) P_t (C_t + I_t) - B_{t+1}] + \overline{\mu}_t [Z_t [1 - \frac{\kappa}{2} (\frac{I_t}{I_{t-1}} - 1)^2] I_t \\
&+ (1 - \delta(u_t)) K_t - K_{t+1}])
\end{aligned}
$$

where, $\overline{\lambda}_t$ is the Lagrangian multiplier for budget constraint. It is defined by $\lambda_t = P_t \overline{\lambda}_t$; $\overline{\mu}_t$ is the Lagrangian multiplier for capital accumulation. It is defined as $\mu_t = P_t \overline{\mu}_t$

First order condition for consumption:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_t} =& \frac{v_t}{C_t - bC_{T-1}} - \beta b E_t \frac{v_{t+1}}{C_{t+1} - bC_t} - (1+\tau_t^c) P_t \overline{\lambda}_t = 0 \\
\Rightarrow \lambda_t =& \frac{1}{(1+\tau_t^c)} \left(\frac{v_t}{C_t - bC_{T-1}} - \beta b E_t \frac{v_{t+1}}{C_{t+1} - bC_t} \right)
\end{aligned}
\tag{13}
$$

First order condition for Bond holding:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial B_{t+1}} =& - \overline{\lambda}_t + \beta E_t \overline{\lambda}_{t+1} (1+i_t) = 0 \\
\Rightarrow \lambda_t =& \beta E_t \lambda_{t+1} (1+\pi_{t+1})^{-1} (1+i_t)
\end{aligned}
\tag{14}
$$

where, $1 + \pi_{t+1} = \frac{P_{t+1}}{P_t}$.

First order condition for Capital Utility:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial u_t} =& \lambda_t (1-\tau_t^k) R_t K_t - \mu_t \delta'(u_t) K_t = 0 \\
\Rightarrow& \lambda_t (1-\tau_t^k) R_t K_t = \mu_t \delta'(u_t) K_t
\end{aligned}
\tag{15}
$$

First order condition for Investment:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial I_t} =& - \lambda_t (1+\tau_t^c) + \mu_t Z_t (1 - \frac{\kappa}{2} (\frac{I_t}{I_{t-1}} - 1)^2 - \kappa (\frac{I_t}{I_t - 1} - 1) \frac{I_t}{I_{t-1}}) + \beta E_t \mu_{t+1} Z_{t+1} \kappa (\frac{I_{t+1}}{I_t} - 1)(\frac{I_{t+1}}{I_t})^2 = 0 \\
\Rightarrow& -\lambda_t (1+\tau_t^c) = \mu_t Z_t (1 - \frac{\kappa}{2} (\frac{I_t}{I_{t-1}} - 1)^2 - \kappa (\frac{I_t}{I_t - 1} - 1) \frac{I_t}{I_{t-1}}) + \beta E_t \mu_{t+1} Z_{t+1} \kappa (\frac{I_{t+1}}{I_t} - 1)(\frac{I_{t+1}}{I_t})^2
\end{aligned}
\tag{16}
$$

First order condition for Capital accumulation:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial K_{t+1}} =& - \mu_t + \beta E_t (\lambda_{t+1} (1-\tau_{t+a}^k)) R_{t+1} u_{t+1} + \mu_{t+1} (1 - \delta(u_{t+1}))) = 0 \\
\Rightarrow& \mu_t = \beta E_t (\lambda_{t+1} (1-\tau_{t+1}^k)) R_{t+1} u_{t+1} + \mu_{t+1} (1 - \delta(u_{t+1})))
\end{aligned}
\tag{17}
$$

Step 2: Optimal decision on household wages setting. Supposing that household determines the wage level based on Calvo pricing assumption. In each period, each household has a probability of $1 - \phi_w$ to adjust the wage and a probability of ϕ_w to not adjusting the wage. This means that overall, $1 - \phi_w$ shares of household in the society can adjust their wage while ϕ_w

percentage of household cannot adjust the wage. It is worth noted that for the households who could not adjust their wage, their wage would still be upgraded by the inflation index from the last period. In other words, the nominal wage for the households who could not adjust their wage could be determined by $(1+\pi_{t-1})^{\zeta_w} W_{t-1}(l)$, where ζ_w is the lagged inflation rate, $W_{t-1}(l)$ stands for the nominal wage in the last period. Hence, the nominal wage of a household l in period t is:

$$W_t(l) = \begin{cases} W_t^*(l), & ifW_t(l)beoptimallychosen \\ (1+\pi_{t-1})^{\zeta_w} W_{t-1}(l), & otherwise \end{cases} \tag{18}$$

where, $W_t^*(l)$ is the optimal wage. After households have already chosen the optimal wage level, which is $W_t(l) = W_t^*(l)$, households would not adjust the wage in the following period. Their wage would only be upgraded by the inflation index. This means,

$$\begin{aligned} &W_{t+1}(l) = (1+\pi_t)^{\zeta_w} W_t^*(l) \\ &W_{t+2}(l) = (1+\pi_{t+1})^{\zeta_w} W_{t+1}(l) = ((1+\pi_{t+1})(1+\pi_t))^{\zeta_w} W_t^*(l) \\ &\cdots \\ &W_{t+s}(l) = (1+\pi_{t+s-1})^{\zeta_w} W_{t+s-1}(l) = \left(\prod_{j=0}^{s-1}(1+\pi_{t+j})\right)^{\zeta_w} W_t^*(l) \end{aligned}$$

where

$$\left(\prod_{j=0}^{s-1}(1+\pi_{t+j})\right)^{\zeta_w} = \frac{P_t}{P_{t-1}}\frac{P_{t+1}}{P_t}\cdots\frac{P_{t+s-1}}{P_{t+s-2}} = \frac{P_{t+s-1}}{P_{t-1}}$$

the wage that could not be adjusted at t+s time could be expressed as:

$$W_{t+s}(l) = \left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\zeta_w} W_t^*(l) \tag{19}$$

The Lagrangian for household wage setting:

$$\begin{aligned} \mathcal{L} &= E_t \sum_{s=0}^{\infty} (\beta\phi_w)^s (-\upsilon_{t+s}\psi_{t+s}\frac{N_{t+s}(l)^{1+\chi}}{1+\chi} + \overline{\lambda}_{t+s}((1-\tau_{t+s}^n)W_{t+s}(l)N_{t+s}(l) + ...)) \\ &= E_t \sum_{s=0}^{\infty} (\beta\phi_w)^s (-\upsilon_{t+s}\psi_{t+s}\frac{\left(\left(\frac{\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\zeta_w} W_t^*(l)}{W_{t+s}}\right)^{-\epsilon_w} N_{d,t+s}\right)^{1+\chi}}{1+\chi} \\ &+ \overline{\lambda}_{t+s}((1-\tau_{t+s}^n)\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\zeta_w} W_t^*(l) \left(\frac{\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\zeta_w} W_t^*(l)}{W_{t+s}}\right)^{-\epsilon_w} N_{d,t+s} + ...)) \end{aligned}$$

The FOC is :

$$\frac{\partial \mathcal{L}}{\partial W_t^*(l)} = E_t\sum_{s=0}^{\infty}(\beta\phi_w)^s\Big(- v_{t+s}\psi_{t+s}\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{-\zeta_w\epsilon_w(1+\chi)} W_t^{\epsilon_w(1+\chi)} N_{d,t+s}^{1+\chi}(-epsilon_w)W_{t+s}^*(l)^{-epsilon_w(1+\chi)}$$

$$+ \overline{\lambda}_{t+s}(1-\tau_{t+s}^n)\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\zeta_w(1-\epsilon_w)} W_{t+s}^{\epsilon_w} N_{d,t+s}(1-\epsilon_w)W_t^*(l)^{-\epsilon_w}\Big) = 0$$

$$\Rightarrow \epsilon_w W_t^*(l)^{-\epsilon_w(1+\chi)} E_t\sum_{s=0}^{\infty}(\beta\phi_w)^s\Big(v_{t+s}\psi_{t+s}\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{-\zeta_w\epsilon_w(1+\chi)} W_{t+s}^{\epsilon_w(1+\chi)} N_{d,t+s}^{1+\chi}$$

$$= (\epsilon_w - 1)W_t^*(l)^{-\epsilon_w} E_t\sum_{s=0}^{\infty}(\beta\phi_w)^s\overline{\lambda}_{t+s}(1-\tau_{t+s}^n)\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\zeta_w(1-\epsilon_w)} W_{t+s}^{\epsilon_w} N_{d,t+s}$$

Simplifying further, we have:

$$W_{t+s}^{\star}(l)^{1+\zeta_w\epsilon_w} = \frac{\epsilon_w}{\epsilon_w - 1}\frac{h_{1,t+s}}{h_{2,t+s}} \tag{20}$$

where :

$$h_{1,t} = E_t\sum_{s=0}^{\infty}(\beta\phi_w)^s(v_{t+s})\psi_{t+s}\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{-\zeta_w\epsilon_w(1+\chi)} W_{t+s}^{\epsilon_w(1+\chi)} N_{d,t+s}^{1+\chi} \tag{21}$$

$$h_{2,t} = E_t\sum_{s=0}^{\infty}(\beta\phi_w)^s\overline{\lambda}_{t+s}(1-\tau_{t+s}^n)\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\zeta_w(1-\epsilon_w)} W_{t+s}^{\epsilon_w} N_{d,t+s} \tag{22}$$

defining real wage by $w_t = \frac{W_t}{P_t}$. Then h1 and h2 can transfer to:

$$h_{1,t} = E_t\sum_{s=0}^{\infty}(\beta\phi_w)^s(v_{t+s})\psi_{t+s}\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{-\zeta_w\epsilon_w(1+\chi)} w_{t+s}^{\epsilon_w(1+\chi)} P_{t+s}^{\epsilon_w(1+\chi)} N_{d,t+s}^{1+\chi} \tag{23}$$

$$h_{2,t} = E_t\sum_{s=0}^{\infty}(\beta\phi_w)^s\overline{\lambda}_{t+s}(1-\tau_{t+s}^n)\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\zeta_w(1-\epsilon_w)} w_{t+s}^{\epsilon_w} P_{t+s}^{\epsilon_w} N_{d,t+s} \tag{24}$$

Using real Lagrangian multiplier to exchange for the nominal h2:

$$h_{2,t} = E_t\sum_{s=0}^{\infty}(\beta\phi_w)^s\lambda_{t+s}(1-\tau_{t+s}^n)\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\zeta_w(1-\epsilon_w)} w_{t+s}^{\epsilon_w} P_{t+s}^{\epsilon_w-1} N_{d,t+s} \tag{25}$$

Now we write $h_{1,t}, h_{2,t}$ recursively:

$$h_{1,t} = v_t\psi_t w_s^{\epsilon_w(1+\chi)} P_t^{\epsilon_w(1+\chi)} N_{d,t+s}^{1+\chi} + \beta\phi_w E_t\left(\frac{P_t}{P_{t-1}}\right)^{\zeta_w\epsilon_w(1+\chi)} h_{1,t+1} \tag{26}$$

$$h_{2,t} = \lambda_t(1-\tau_t^n)w_t^{\epsilon_w} P_t^{\epsilon_w-1} N_{d,t} + \beta\phi_w E_t\left(\frac{P_t}{P_{t-1}}\right)^{\zeta_w(1-\epsilon_w)} h_{2,t+1} \tag{27}$$

Define $\hat{h}_{1,t} = \frac{h_{1,t}}{P_t^{\epsilon_w(1+\chi)}}, \hat{h}_{2,t} = \frac{h_{2,t}}{P_t^{\epsilon_w-1}}$. In term of inflation rates, we get

$$\hat{h}_{1,t} = v_t\psi_t w_t^{\epsilon_w(1+\chi)} N_{d,t+s}^{1+\chi} + \beta\phi_w(1+\pi_t)^{\zeta_w\epsilon_w(1+\chi)} E_t(1+\pi_{t+1})^{\epsilon_w(1+\chi)}\hat{h}_{1,t+1} \tag{28}$$

$$\hat{h}_{2,t} = \lambda_t(1-\tau_t^n)w_t^{\epsilon_w} N_{d,t} + \beta\phi_w(1+\pi_t)^{\zeta_w(1-\epsilon_w)} E_t(1+\pi_{t+1})^{\epsilon_w-1}\hat{h}_{2,t+1} \tag{29}$$

So, define $w_t^\star = \frac{W_t^\star}{P_t}$Result of first order condition of wage setting:

$$(w_t^\star)^{1+\epsilon_w\chi} = \frac{\epsilon_w}{\epsilon_w - 1}\frac{\hat{h}_{1,t}}{\hat{h}_{2,t}} \tag{30}$$

Define $\overline{h}_{1,t} = \frac{\hat{h}_{1,t}}{(w_t^\star)^{\epsilon_w(1+\chi)}}$. So we can get

$$\begin{aligned}\overline{h}_{1,t} &= v_t\psi_t\left(fracw_t w_t^\star\right)^{\epsilon_w(1+\chi)} N_{d,t+s}^{1+\chi} + \beta\phi_w(1+\pi_t)^{\zeta_w\epsilon_w(1+\chi)}E_t(1+\pi_{t+1})^{\epsilon_w(1+\chi)}\frac{\hat{h}_{1,t+1}}{(w_t^\star)^{\epsilon_w(1+\chi)}}\\ &= v_t\psi_t\left(fracw_t w_t^\star\right)^{\epsilon_w(1+\chi)} N_{d,t+s}^{1+\chi} + \beta\phi_w(1+\pi_t)^{\zeta_w\epsilon_w(1+\chi)}E_t(1+\pi_{t+1})^{\epsilon_w(1+\chi)}\frac{\hat{h}_{1,t+1}}{(w_{t+1}^\star(^{\epsilon_w(1+\chi)}}\frac{(w_{t+1}^\star)^{\epsilon_w(1+\chi)}}{(w_t^\star)^{\epsilon_w(1+\chi)}}\\ &= v_t\psi_t\left(fracw_t w_t^\star\right)^{\epsilon_w(1+\chi)} N_{d,t+s}^{1+\chi} + \beta\phi_w(1+\pi_t)^{\zeta_w\epsilon_w(1+\chi)}E_t(1+\pi_{t+1})^{\epsilon_w(1+\chi)}\overline{h}_{1,t+1}\frac{(w_{t+1}^\star)^{\epsilon_w(1+\chi)}}{(w_t^\star)^{\epsilon_w(1+\chi)}}\end{aligned} \tag{31}$$

Define $\overline{h}_{2,t} = \frac{\hat{h}_{2,t}}{(w_t^\star)^{\epsilon_w}}$. So we can get

$$\begin{aligned}\overline{h}_{2,t} &= \lambda_t(1-\tau_t^n)\left(\frac{w_t}{w_t^\star}\right)^{\epsilon_w} N_{d,t} + \beta\phi_w(1+\pi_t)^{\zeta_w(1-\epsilon_w)}E_t(1+\pi_{t+1})^{\epsilon_w-1}\frac{\hat{h}_{2,t+1}}{(w_t^\star)^{\epsilon_w}}\\ &= \lambda_t(1-\tau_t^n)\left(\frac{w_t}{w_t^\star}\right)^{\epsilon_w} N_{d,t} + \beta\phi_w(1+\pi_t)^{\zeta_w(1-\epsilon_w)}E_t(1+\pi_{t+1})^{\epsilon_w-1}\frac{\hat{h}_{2,t+1}}{(w_t^\star)^{\epsilon_w}}\frac{(w_{t+1}^\star)^{\epsilon_w}}{(w_t^\star)^{\epsilon_w}}\\ &= \lambda_t(1-\tau_t^n)\left(\frac{w_t}{w_t^\star}\right)^{\epsilon_w} N_{d,t} + \beta\phi_w(1+\pi_t)^{\zeta_w(1-\epsilon_w)}E_t(1+\pi_{t+1})^{\epsilon_w-1}\overline{h}_{2,t+1}\frac{(w_{t+1}^\star)^{\epsilon_w}}{(w_t^\star)^{\epsilon_w}}\end{aligned} \tag{32}$$

Then we get

$$w_t^\star = \frac{\epsilon_w}{\epsilon_w - 1}\frac{\overline{h}_{1,t}}{\overline{h}_{2,t}} \tag{33}$$

IV. Firm

The production of goods could be split into two types: final goods production and intermediate goods production. Intermediate firms (or wholesale goods products) produce different types of goods which are imperfect substitutes (at the origin of the monopolistic competition). Final firms (or retailers) produce an homogeneous good by combining intermediate goods in a CES technology. Because of imperfect substitutes of the intermediates in final production process, the intermediate firms have the pricing power. The intermediate goods firms produce output using labor and capital service and subject to an aggregate productive shock. They follow the Calvo's pricing rule as well.

A. Final Goods Firms

The final products are a CES form of a continuum of intermediate goods:

$$Y_t = (\int_0^1 Y_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p}} dj)^{\frac{\epsilon_p}{\epsilon_p - 1}}, \epsilon_p > 1 \tag{34}$$

where, Y_t is the output of the final goods firm; $Y_t(j)$ is the output of the intermediate j; ϵ_p is the institute elastic between various intermediate products.

The profit maximization of the final products firm reads as:

$$\underbrace{max}_{Y_t(j)} P_t Y_t - \int_0^1 P_t(j) Y_t(j) dj$$

$$\underbrace{max}_{Y_t(j)} P_t (\int_0^1 Y_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p}} dj)^{\frac{\epsilon_p}{\epsilon_p - 1}} - \int_0^1 P_t(j) Y_t(j) dj$$

$$\Rightarrow F.O.C. : P_t \frac{\epsilon_p}{\epsilon_p - 1} (\int_0^1 Y_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p}} dj)^{(\frac{\epsilon_p}{\epsilon_p - 1} - 1)} \frac{\epsilon_p - 1}{\epsilon_p} Y_t(j)^{(\frac{\epsilon_p - 1}{\epsilon_p} - 1)} - \int_0^1 P_t(j) dj = 0$$

$$\Rightarrow P_t (\frac{Y_t}{Y_t(j)})^{(\frac{1}{\epsilon})} - P_t(j) = 0$$

Then, we can get the demand of intermediary goods:

$$Y_t(j) = (\frac{P_t(j)}{P_t})^{-\epsilon_p} Y_t \tag{35}$$

where, P_t is the aggregate price index; $P_t(j)$ is the price of the intermediate good j.

The aggregate price index:

$$P_t^{1-\epsilon_p} = \int_0^1 P_t(j)^{1-\epsilon_p} dj \tag{36}$$

B. *Intermediary Goods Firms*

Intermediate producers have the following production technology:

$$Y_t(j) = A_t \hat{K}_t(j)^\alpha N_{d,t}(j)^{1-\alpha} \tag{37}$$

here A_t is the productivity of the intermediates that faced a common TFP shock. $\hat{K}_t(j)$ is the demand of capital service in the intermediate firm j. α is the share of capital service in the output.

As the intermediary goods are differentiated, the firms have some level of market power. So, the optimal strategy for a intermediary goods firm j can be separated into two steps:

First step: Given the level of productivity, cost can be minimized by changing the investment and labor:

$$\underbrace{min}_{\hat{K}_t(j), N_{d,t}(j)} \quad R_t \hat{K}_t(j) + w_t N_{d,t}(j) - (A_t \hat{K}_t(j)^\alpha N_{d,t}(j)^{1-\alpha} - Y_t(j))$$

$$\Rightarrow F.O.C. :$$

$$R_t = \alpha A_t \hat{K}_t(j)^{\alpha-1} N_{d,t}(j)^{1-\alpha}$$

$$w_t = (1-\alpha) A_t \hat{K}_t(j)^\alpha N_{d,t}(j)^{-\alpha}$$

Combining the FOC conditions, we can get:

$$\frac{w_t}{R_t} = \frac{1-\alpha}{\alpha} (\frac{\hat{K}_t}{N_{d,t}}) \tag{38}$$

Note that this equation no longer relies on the intermediary goods firms j, this is because the optimal input elements of all the intermediary goods firms are the same.

The marginal cost:

$$mc_t = \frac{w_t}{(1-\alpha)A_t(\frac{\hat{K}_t}{N_{d,t}})^\alpha} \tag{39}$$

Second Step: Intermediary goods firms can maximize the profit by adjusting price. We assume that the intermediary goods firms uses Calvo price-setting, which means that a $1-\phi_p$ percentage of firms can set the price at its optimal level $P_t^\star(j)$, the rest ϕ_p percent of firms cannot set the price at its optimal level but can upgrade the price at a velocity of ζ_p according to last period inflation.

$$P_t(j) = \begin{cases} P_t^\star(j), & if P_t(j) be optimally chosen \\ (1+\pi_{t-1})^{\zeta_p} P_{t-1}(j), & otherwise \end{cases} \tag{40}$$

Similar as the way that household adjust the wage, intermediary goods firms adjust their price by:

$$\begin{aligned}
&P_{t+1}(j) = (1+\pi_t)^{\zeta_p} P_t^*(j) \\
&P_{t+2}(j) = (1+\pi_{t+1})^{\zeta_p} P_{t+1}(j) = ((1+\pi_{t+1})(1+\pi_t))^{\zeta_p} P_t^*(j) \\
&\ldots \\
&P_{t+s}(j) = (1+\pi_{t+s-1})^{\zeta_p} P_{t+s-1}(j) = \left(\prod_{j=0}^{s-1}(1+\pi_{t+j})\right)^{\zeta_p} P_t^*(j)
\end{aligned}$$

, where

$$\left(\prod_{j=0}^{s-1}(1+\pi_{t+j})\right)^{\zeta_w} = \frac{P_t}{P_{t-1}}\frac{P_{t+1}}{P_t}\cdots\frac{P_{t+s-1}}{P_{t+s-2}} = \frac{P_{t+s-1}}{P_{t-1}}$$

Meanwhile, at time $t+s$, the price of the intermediary goods firms that cannot adjust the price can be upgraded by:

$$P_{t+s}(j) = \left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\zeta_p} P_t^*(j) \tag{41}$$

Based on this, we consider the profit maximization for the intermediary firms:

$$\underbrace{max}_{P_t(j)^\star} E_t \sum_{s=0}^{\infty} (\beta\phi_p)^s \frac{\hat{\lambda}_{t+s}}{\hat{\lambda}_t} [P_{t+s}(j) Y_{t+s}(j) - P_{t+s} mc_{t+s} Y_{t+s}(j)]$$

$$\Rightarrow \underbrace{max}_{P_t(j)^\star} E_t \sum_{s=0}^{\infty} (\beta\phi_p)^s \frac{\hat{\lambda}_{t+s}}{\hat{\lambda}_t} [\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\zeta_p} P_t^*(j) \left(\frac{\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\zeta_p} P_t^*(j)}{P_{t+s}}\right)^{-\epsilon_p} Y_{t+s}$$

$$- P_{t+s} mc_{t+s} \left(\frac{\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\zeta_p} P_t^*(j)}{P_{t+s}}\right)^{-\epsilon_p} Y_{t+s}]$$

$$\Rightarrow \underbrace{max}_{P_t(j)^\star} E_t \sum_{s=0}^{\infty} (\beta\phi_p)^s \frac{\hat{\lambda}_{t+s}}{\hat{\lambda}_t} [\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\zeta_p} \left(\frac{\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\zeta_p}}{P_{t+s}}\right)^{-\epsilon_p} Y_{t+s} P_t^*(j)^{1-\epsilon_p}$$

$$- P_{t+s} mc_{t+s} \left(\frac{\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\zeta_p}}{P_{t+s}}\right)^{-\epsilon_p} Y_{t+s} P_t^*(j)^{-\epsilon_p}]$$

The FOC is:

$$\Rightarrow (\epsilon_p - 1) P_t^*(j)^{-\epsilon_p} E_t \sum_{s=0}^{\infty} (\beta\phi_p)^s \frac{\hat{\lambda}_{t+s}}{\hat{\lambda}_t} \left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\zeta_p} \left(\frac{\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\zeta_p}}{P_{t+s}}\right)^{-\epsilon_p} Y_{t+s}$$

$$= \epsilon_p P_t^*(j)^{-\epsilon_p - 1} E_t \sum_{s=0}^{\infty} (\beta\phi_p)^s \frac{\hat{\lambda}_{t+s}}{\hat{\lambda}_t} P_{t+s} mc_{t+s} \left(\frac{\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\zeta_p}}{P_{t+s}}\right)^{-\epsilon_p} Y_{t+s}$$

$$\Rightarrow P_t^*(j) = \frac{\epsilon_p}{\epsilon_p - 1} \frac{E_t \sum_{s=0}^{\infty} (\beta\phi_p)^s \frac{\hat{\lambda}_{t+s}}{\hat{\lambda}_t} P_{t+s} mc_{t+s} \left(\frac{\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\zeta_p}}{P_{t+s}}\right)^{-\epsilon_p} Y_{t+s}}{E_t \sum_{s=0}^{\infty} (\beta\phi_p)^s \frac{\hat{\lambda}_{t+s}}{\hat{\lambda}_t} \left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\zeta_p} \left(\frac{\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\zeta_p}}{P_{t+s}}\right)^{-\epsilon_p} Y_{t+s}}$$

Optimal pricing strategy through Lagrangian:

$$P_t^\star = \frac{\epsilon_p}{\epsilon_p - 1} \frac{X_{1,t}}{X_{2,t}} \tag{42}$$

where,

$$X_{1,t} = E_t \sum_{s=0}^{\infty} (\beta\phi_p)^s \hat{\lambda}_{t+s} P_{t+s} mc_{t+s} \left(\frac{\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\zeta_p}}{P_{t+s}}\right)^{-\epsilon_p} Y_{t+s}$$

$$X_{2,t} = E_t \sum_{s=0}^{\infty} (\beta\phi_p)^s \hat{\lambda}_{t+s} \left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\zeta_p} \left(\frac{\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\zeta_p}}{P_{t+s}}\right)^{-\epsilon_p} Y_{t+s}$$

Because of $\lambda_t = P_t\hat{\lambda}_t, 1+\pi_t = \frac{P_t}{P_{t-1}}$, define $x_{1,t} = \frac{X_{1,t}}{P_t^{\epsilon_p}}, x_{2,t} = \frac{X_{2,t}}{P_t^{\epsilon_p - 1}}$, so we write the conditions recursively as:

$$x_{1,t} = \lambda_t mc_t Y_t + \beta\phi_p(1+\pi_t)^{-\zeta_p\epsilon_p} E_t(1+\pi_{t+1})^{\epsilon_p} x_{1,t} \tag{43}$$

$$x_{2,t} = \lambda_t Y_t + \beta\phi_p(1+\pi_t)^{-\zeta_p(\epsilon_p-1)} E_t(1+\pi_{t+1})^{\epsilon_p-1} x_{2,t} \tag{44}$$

Define $1+\pi_t^\star = \frac{P_t^\star}{P_{t-1}}$, then

$$1+\pi_t^\star = \frac{\epsilon_p}{\epsilon_p - 1}\frac{x_{1,t}}{x_{2,t}}(1+\pi_t) \tag{45}$$

V. Central Bank

Taylor's rule of Monetary Policy:

$$i_t = (1-\rho_i)i + \rho_t i_{t-1} + (1-\rho)[\phi_\pi(\pi_t - \pi) + \phi_y(\ln Y_t - \ln Y_{t-1})] + s_i \tag{46}$$

VI. Department of Finance

(1)Government spending policy:

$$\ln G_t = (1-\rho_g)\ln(\omega Y_t) + \rho_g \ln G_{t-1} + \epsilon_g \tag{47}$$

where, G_t is governmental real expenditure. ω is the parameter of a sensitivity to the output. This means that the government expenditure could be positively and endogenously effected by economic growth. This is supported by the fact that the majority of government expenditure is procyclical.

Budget constraint for Department of Finance:

$$P_t G_t + (1+i_{t-1})B_t = P_t T_t + B_{t+1} + \tau_t^n \int_0^1 W_t(l)N_t(l)dl + \tau_t^k P_t R_t K_t u_t + \tau_t^c P_t(C_t + I_t) \tag{48}$$

(2)VAT policy:

$$\tau_t^c = (1-\rho_c)\tau^c + \rho_c\tau_{t-1}^c + \epsilon_c \tag{49}$$

(3)Labor income tax policy:

$$\tau_t^n = (1-\rho_n)\tau^n + (1-\rho_n)\ln\frac{w_t N_{d,t}}{wN_d} + \rho_n\tau_{t-1}^n + \epsilon_n \tag{50}$$

(4)Capital rent tax policy:

$$\tau_t^k = (1-\rho_k)\tau^k + (1-\rho_k)\ln\frac{R_t K_t}{RK} + \rho_k\tau_{t-1}^k + \epsilon_k \tag{51}$$

In most country, labor income and capital income tax are both using progressive tax policy. Hence, we assume that tax rate is determined progressively. This means that if the income is higher than steady state level, the tax rate will increase, vise versa.

VII. Equilibrium and Aggregation

After all markets clear, we get a total resource constraint:

$$Y_t = C_t + I_t + G_t \tag{52}$$

We derive the total output function below. We start by integrating the both sides of the intermediary goods function:

$$\int_0^1 Y_t(j)dj = \int_0^1 A_t \hat{K}_t(j)^\alpha N_{d,t}(j)^{1-\alpha} dj$$

Put the intermediary demand function into the last equation, we get:

$$\int_0^1 (\frac{P_t(j)}{P_t})^{-\epsilon_p} Y_t dj = A_t \left(\frac{\hat{K}_t(j)}{N_{d,t}(j)}\right)^\alpha \int_0^1 N_{d,t}(j)dj$$

Define $v_t^p = \int_0^1 (\frac{P_t(j)}{P_t})^{-\epsilon_p} dj$ as price diffusion index. Then

$$v_t^p Y_t = \hat{K}_t^\alpha N_{d,t}^{1-\alpha} \tag{53}$$

What is v_t^p? Recall that we use Calvo pricing strategy, we split the price diffusion function into two parts:

$$\begin{aligned} v_t^p &= \int_0^1 (\frac{P_t(j)}{P_t})^{-\epsilon_p} dj \\ &= \int_0^{1-\phi_p} (\frac{P_t^\star}{P_t})^{-\epsilon_p} dj + \int_{1-\phi_p}^1 (1-\pi_{t-1})^{-\zeta_p \epsilon_p} \left(\frac{P_{t-1}(j)}{P_t}\right)^{-epsilon_p} dj \\ &= (1-\phi_p)\left(\frac{1+\pi_t^\star}{1+\pi_t}\right)^{-\epsilon} + \phi_p(1+\pi_{t-1})^{-\zeta_p\epsilon_p}(1+\pi_t)^{\epsilon_p} v_{t-1}^p \end{aligned} \tag{54}$$

Total price function can also be split to:

$$\begin{aligned} P_t^{1-\epsilon_p} &= \int_0^{1-\phi_p} (P_t^\star)^{1-\epsilon_p} dj + \int_{1-\phi_p}^1 (1+\pi_{t-1})^{\zeta_p(1-\epsilon_p)} P_{t-1}(j)^{1-\epsilon_p} dj \\ &= (1-\phi_p)(P_t^\star)^{1-\epsilon_p} + \phi_p(1+\pi_{t-1})^{\zeta_p(1-\epsilon_p)} P_{t-1}^{1-\epsilon_p} \end{aligned} \tag{55}$$

We add inflate rate into the equation:

$$(1+\pi_t)^{1-\epsilon_p} = (1-\phi_p)(1+\pi_t^\star)^{1-\epsilon_p} + \phi_p(1+\pi_{t-1})^{\zeta_p(1-\epsilon_p)} \tag{56}$$

following the Calvo pricing strategy, we split the function of wage change into two parts:

$$\begin{aligned} W_t^{1-\epsilon_w} &= \int_0^1 W_t(l)^{1-\epsilon_w} dl \\ &= \int_0^{1-\phi_w} (W_t^\star)^{1-\epsilon_w} dl + \int_{1-\phi_w}^1 (1-\pi_{t-1})^{\zeta_w(1-\epsilon_w)} W_{t-1}(l)^{1-\epsilon_w} dl \\ &= (1-\phi_w)(W_t^\star)^{1-\epsilon_w} + \phi_w(1-\pi_{t-1})^{\zeta_w(1-\epsilon_w)} W_{t-1}^{1-\epsilon_w} \end{aligned} \tag{57}$$

The real wage:

$$w_t^{1-\epsilon_w} = (1-\phi_w)(w_t^\star)^{1-\epsilon_w} + \phi_w(1-\pi_{t-1})^{\zeta_w(1-\epsilon_w)}(1+\pi_t)^{\epsilon_w-1} w_{t-1}^{1-\epsilon_w} \tag{58}$$

VIII. Other Shocks

the productivity shock:

$$\ln A_t = \rho_A \ln A_{t-1} + \epsilon_{A,t} \tag{59}$$

the shock marginal efficiency of investment:

$$\ln Z_t = \rho_z \ln Z_{t-1} + \epsilon_{Z,t} \tag{60}$$

the intertemporal preference shock:

$$\ln v_t = \rho_v \ln v_{t-1} + \epsilon_{v,t} \tag{61}$$

the intratemporal preference(or labor supply) shock:

$$\ln \psi_t = (1-\rho_\psi)\ln \psi + \rho_\psi \ln \psi_{t-1} + \epsilon_\psi \tag{62}$$

IX. Full set of equilibrium conditions

$$\lambda_t = \frac{1}{(1+\tau_t^c)}\left(\frac{v_t}{C_t - bC_{T-1}} - \beta b E_t \frac{v_{t+1}}{C_{t+1} - bC_t}\right) \tag{63}$$

$$\lambda_t = \beta E_t \lambda_{t+1}(1+\pi_{t+1})^{-1}(1+i_t) \tag{64}$$

$$\lambda_t(1-\tau_t^k)R_t K_t = \mu_t \delta'(u_t) K_t \tag{65}$$

$$\lambda_t(1+\tau_t^c) = \mu_t Z_t(1 - \frac{\kappa}{2}(\frac{I_t}{I_{t-1}} - 1)^2 - \kappa(\frac{I_t}{I_t - 1} - 1)\frac{I_t}{I_{t-1}}) + \beta E_t \mu_{t+1} Z_{t+1} \kappa(\frac{I_{t+1}}{I_t} - 1)(\frac{I_{t+1}}{I_t})^2 \tag{66}$$

$$\mu_t = \beta E_t(\lambda_{t+1}(1-\tau_{t+1}^k))R_{t+1}u_{t+1} + \mu_{t+1}(1-\delta(u_{t+1}))) \tag{67}$$

$$\overline{h}_{1,t} = v_t \psi_t \left(fracw_t w_t^\star\right)^{\epsilon_w(1+\chi)} N_{d,t+s}^{1+\chi} + \beta\phi_w(1+\pi_t)^{\zeta_w \epsilon_w(1+\chi)} E_t(1+\pi_{t+1})^{\epsilon_w(1+\chi)} \overline{h}_{1,t+1} \frac{(w_{t+1}^\star)^{\epsilon_w(1+\chi)}}{(w_t^\star)^{\epsilon_w(1+\chi)}} \tag{68}$$

$$\overline{h}_{2,t} = \lambda_t(1-\tau_t^n)\left(\frac{w_t}{w_t^\star}\right)^{\epsilon_w} N_{d,t} + \beta\phi_w(1+\pi_t)^{\zeta_w(1-\epsilon_w)} E_t(1+\pi_{t+1})^{\epsilon_w-1}\overline{h}_{2,t+1}\frac{(w_{t+1}^\star)^{\epsilon_w}}{(w_t^\star)^{\epsilon_w}} \tag{69}$$

$$w_t^\star = \frac{\epsilon_w}{\epsilon_w - 1}\frac{\hat{h}_{1,t}}{\hat{h}_{2,t}} \tag{70}$$

$$\frac{w_t}{R_t} = \frac{1-\alpha}{\alpha}(\frac{\hat{K}_t}{N_{d,t}}) \tag{71}$$

$$mc_t = \frac{w_t}{(1-\alpha)A_t(\frac{\hat{K}_t}{N_{d,t}})^\alpha} \tag{72}$$

$$x_{1,t} = \lambda_t mc_t Y_t + \beta\phi_p(1+\pi_t)^{-\zeta_p\epsilon_p}E_t(1+\pi_{t+1})^{\epsilon_p}x_{1,t} \tag{73}$$

$$x_{2,t} = \lambda_t Y_t + \beta\phi_p(1+\pi_t)^{-\zeta_p(\epsilon_p-1)}E_t(1+\pi_{t+1})^{\epsilon_p-1}x_{2,t} \tag{74}$$

$$1+\pi_t^\star = \frac{\epsilon_p}{\epsilon_p-1}\frac{x_{1,t}}{x_{2,t}}(1+\pi_t) \tag{75}$$

$$Y_t = C_t + I_t + G_t \tag{76}$$

$$v_t^p Y_t = \hat{K}_t^\alpha N_{d,t}^{1-\alpha} \tag{77}$$

$$v_t^p = (1-\phi_p)\left(\frac{1+\pi_t^\star}{1+\pi_t}\right)^{-\epsilon} + \phi_p(1+\pi_{t-1})^{-\zeta_p\epsilon_p}(1+\pi_t)^{\epsilon_p}v_{t-1}^p \tag{78}$$

$$(1+\pi_t)^{1-\epsilon_p} = (1-\phi_p)(1+\pi_t^\star)^{1-\epsilon_p} + \phi_p(1+\pi_{t-1})^{\zeta_p(1-\epsilon_p)} \tag{79}$$

$$w_t^{1-\epsilon_w} = (1-\phi_w)(w_t^\star)^{1-\epsilon_w} + \phi_w(1-\pi_{t-1})^{\zeta_w(1-\epsilon_w)}(1+\pi_t)^{\epsilon_w-1}w_{t-1}^{1-\epsilon_w} \tag{80}$$

$$\hat{K}_t = K_t u_t \tag{81}$$

$$K_{t+1} = Z_t\left[1-\frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2\right]I_t + (1-\delta(u_t))K_t \tag{82}$$

$$i_t = (1-\rho_i)i + \rho_t i_{t-1} + (1-\rho)[\phi_\pi(\pi_t-\pi) + \phi_y(\ln Y_t - \ln Y_{t-1})] + s_i \tag{83}$$

$$\ln G_t = (1-\rho_g)\ln(\omega Y_t) + \rho_g \ln G_{t-1} + \epsilon_g \tag{84}$$

$$\tau_t^c = (1-\rho_c)\tau^c + \rho_c\tau_{t-1}^c + \epsilon_c \tag{85}$$

$$\tau_t^n = (1-\rho_n)\tau^n + (1-\rho_n)\ln\frac{w_t N_{d,t}}{wN_d} + \rho_n\tau_{t-1}^n + \epsilon_n \tag{86}$$

$$\tau_t^k = (1-\rho_k)\tau^k + (1-\rho_k)\ln\frac{R_t K_t}{RK} + \rho_k \tau_{t-1}^k + \epsilon_k \tag{87}$$

$$\ln A_t = \rho_A \ln A_{t-1} + \epsilon_{A,t} \tag{88}$$

$$\ln Z_t = \rho_z \ln Z_{t-1} + \epsilon_{Z,t} \tag{89}$$

$$\ln v_t = \rho_v \ln v_{t-1} + \epsilon_{v,t} \tag{90}$$

$$\ln \psi_t = (1-\rho_\psi)\ln \psi_s s + \rho_\psi \ln \psi_{t-1} + \epsilon_{\psi,t} \tag{91}$$

There are 29 equations with 29 endogenous variables:$\lambda_t, \mu_t, C_t, i_t, \pi_t, R_t, u_t, Z_t, I_t, v_t, \psi_t, w_t, w_t^\star$, $\overline{h}_{1,t}, \overline{h}_{2,t}, N_{d,t}, \hat{K}_t, K_t, mc_t, \pi_t^\star, \hat{x}_{1,t}, \hat{x}_{2,t}, Y_t, G_t, A_t, v_t^p, \tau_t^c, \tau_t^n, \tau_t^k$.

And the parameters:$\beta, b, \delta_0, \delta_1, \delta_2, \kappa, \epsilon_w, \chi, \phi_w, \zeta_w, \alpha, \phi_p, \zeta_p, \epsilon_p, \rho_i, \phi_\pi, \phi_y, \rho_g, \rho_a, \rho_z, \rho_v, \psi_s s, \rho_\psi, \omega$, $\tau_c, \tau_n, \tau_k, \rho_c, \rho_n, \rho_k, \phi_n, \phi_k, \pi_{target}$.

X. The steady state

Basically, we have to remove all the time indices (t,t1,t+1,) on the endogenous and exogenous variables in all the equations, and express the endogenous variables as functions of the exogenous variables and parameters. This cannot be done analytically in all models, and often we will have to resort on a numerical solver to obtain the steady state (for given values of the parameters and exogenous variables).

To get the steady state for the equilibrium, we first remove the subscribts t for all the endogenous variables. In above models, it is possible to obtain an analytical expressions for the steady state of the most endogenous variables.

1. From monetary policy (83), we can get steady state of inflation rate $\pi = \pi_{target}$.
2. From the shock function (85) - (91), we can get $A = 1, Z = 1, v = 1, \psi = \psi_s s, \tau^c = \tau_c, \tau^n = \tau_n, \tau^k = \tau_k$
3. From (64), we get $i = \frac{1+\pi}{\beta} - 1$.
4. $\delta(u_t) = \delta_0 + \delta_1(u_t - 1) + \frac{\delta_2}{2}(u_t-1)^2$, $\delta'(u_t) = \delta_1 + \delta_2(u_t - 1)$. In steady state, we assume taht the capital utility of household is equal to 1.
   $u = 1$ Then, we use the free parameter to calibrate the steady value of capital utilization rate.
   From (66), we get $\lambda(1+\tau^c) = \mu$
   From (65), we get

$$\begin{aligned} &\lambda(1-\tau^k)R = \mu[\delta_1 + \delta_2(u-1)] \\ &\overbrace{\Rightarrow}^{u=1} R = \frac{1+\tau^c}{1-\tau^k}\delta_1 \end{aligned} \tag{92}$$

   From (67),

$$\begin{aligned} &\mu = \beta(\lambda(1-\tau^k)R + \mu(1-\delta_0)) \\ &\Rightarrow R = \frac{1+\tau^c}{1-\tau^k}(\frac{1}{\beta} - (1-\delta_0)) \end{aligned} \tag{93}$$

From (92) and (93), we know that in order to get steady value of capital utilization rate $u = 1$, then

$$\delta_1 = \frac{1}{\beta} - (1 - \delta_0)$$

which means, δ_1 depends on deep parameters β, δ_0.

5. From (79), we get

$$\begin{aligned}(1+\pi)^{1-\epsilon_p} &= (1-\phi_p)(1+\pi^\star)^{1-\epsilon_p} + (1+\pi)^{\zeta_p(1-\epsilon_p)}\phi_p \\ \Rightarrow \pi^\star &= \left[\frac{(1+\pi)^{1-\epsilon_p} - \phi_p(1+\pi)^{\zeta_p(1-\epsilon_p)}}{1-\phi_p}\right]^{\frac{1}{1-\epsilon_p}} - 1\end{aligned} \tag{94}$$

6. From (78)we get

$$v^p = \frac{(1-\phi_p)(\frac{1+\pi^\star}{1+\pi})^{-\epsilon_p}}{1-\phi_p(1+\pi)^{\epsilon_p(1-\zeta_p)}} \tag{95}$$

7. From (73) and (74), we get

$$(1 - \phi_p\beta(1+\pi)^{\epsilon_p(1-\zeta_p)})x_1 = \lambda mcY$$

$$(1 - \phi_p\beta(1+\pi)^{(1-\epsilon_p)(\zeta_p-1)})x_2 = \lambda Y$$

Combining both equations above, we get

$$mc = \frac{1-\phi_p\beta(1+\pi)^{\epsilon_p(1-\zeta_p)}}{1-\phi_p\beta(1+\pi)^{(1-\epsilon_p)(\zeta_p-1)}}\frac{x_1}{x_2} \tag{96}$$

Substituting (75) to (96), we get

$$mc = \frac{\epsilon_p - 1}{\epsilon_p}\frac{1+\pi^\star}{(1+\pi)}\frac{1-\phi_p\beta(1+\pi)^{\epsilon_p(1-\zeta_p)}}{1-\phi_p\beta(1+\pi)^{(1-\epsilon_p)(\zeta_p-1)}} \tag{97}$$

8. Combining (71) with (72), we can get

$$\begin{aligned}\frac{mc}{R} &= \frac{1}{\alpha}\left(\frac{\hat{K}}{N_d}\right)^{1-\alpha} \\ \Rightarrow \frac{\hat{K}}{N_d} &= \left(\frac{\alpha mc}{R}\right)^{\frac{1}{1-\alpha}}\end{aligned} \tag{98}$$

9. Then, we can get the real wage

$$w = \frac{1-\alpha}{\alpha}\left(\frac{\hat{K}}{N_d}\right)R \tag{99}$$

XI. Appendix List of Dynare Files

This appendix lists the dynare files used in this paper. All files and the data used are contained in the file Mscale_NK2019.zip. In case simulated data was used in the estimation, one first needs to run the relevant dynare code to simulate the data, and save it.

- mscale_dsge_baseline.mod - solves the above NK model and simulates data;
- mscale_dsge_baseline_steadystate.m - solves the steady state values of all endogenous variables;
- mscale_dsge_baseline_running.m - runs the mscale_dsge_baseline.mod and saves the simulated data;
- mscale_dsge_baseline_est.mod - estimates the baseline NK model.