Interactions between Social and Topping Up Insurance under ex-post Moral Hazard

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Abstract

As health expenditure and need for corresponding funding rises, resorting to topping up insurance can seem natural. Complementary and supplementary insurances are both topping up contracts and, as such, are treated as one in the theoretical literature on optimal insurance. We argue that distinguishing them is crucial, and should be considered carefully when defining policies impacting the structure of the health insurance system, as these two kinds of insurance can have opposite effects on social insurance coverage.

In this model, the optimal social insurance rate is defined endogenously and varies according to redistribution and the ex-post moral hazard characteristics of the insurance. This game has three stages and is solved through backward induction. The optimal social insurance rate is chosen first, by maximising social welfare. Second, individuals choose their private complementary and supplementary contracts. In the third stage they decide on their level of labour and consumption of health and other goods.

Results indicate that whereas the presence of complementary insurance decreases the optimal size of social insurance, the offset effects of supplementary insurance can improve welfare.

Keywords. Social insurance; health insurance; ex-post moral hazard; topping up; redistribution.

JEL Classification. D82; I13; I18.

1 Introduction

As health expenditure rises faster than social resources, co-pays can be used to mitigate public spending in health. When these co-pays become important, allowing private insurance to cover them can be an appealing way of increasing access to care. This kind of private topping up insurance, known as complementary insurance, is extensively relied on in France, Belgium and Luxembourg. It is purchased by segments of the population in other OECD countries, including the United states (cf Appendix 1 for a classification of mixed systems in the OECD). In the literature, the effects of complementary insurance are assimilated to the effects of other topping up systems. We argue that complementary insurance generates specific inefficiencies.

Mixed systems of health insurance have been emerging all around the world in an attempt to combine the benefits of social and private insurance systems. They are often seen as an opportunity to

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guarantee coverage to all, and avoid increasing public spending. Paradoxically, their efficiency and the consequences of the interactions between private and social insurance have seldom been studied.

There are two types of topping up contracts. Complementary insurance provides insurance for health goods that are part of the social bundle and insures the share which is not covered by public insurance. Supplementary insurance, on the other hand, offers insurance for health goods that are not covered by social insurance. It is common for countries to have a health system characterised by the presence of both complementary and supplementary insurances.

To see whether the nature of mixed systems has a systematic effect on health outcomes, cost and consumption, we turn to the latest OECD analysis of health systems (Paris et al., 2010). Descriptive statistics on life expectancy, spending, out-of-pocket expenses and consumption were computed by type of mixed system (see Appendix 1 for the statistics and a more detailed analysis). These descriptive statistics suggests that complementary insurance may have an ambiguous effect on social welfare - reducing out-of-pocket expenses but potentially generating moral hazard. In this context, a theoretical study of health insurance can make sense of these complex phenomena.

This article is related to two well established strands of the literature on exclusive system of insurance. The first is on the effect of moral hazard on optimal insurance (for seminal papers see the debate between Arrow (1963, 1968) and Pauly (1968); the results from the RAND experiment (Manning et al, 1987); and the contributions of Nyman (1999)). The second strand is related to the rational for social insurance, and the role of redistribution in particular (see Cremer and Pestieau, 1996; Henriet and Rochet, 2006, Pestieau, 2003; Rochet, 1991).
Findings in the literature on optimal coverage show that social insurance is desirable when it generates redistribution and that moral hazard has a negative effect on coverage. In an exclusive system, the first best scenario, characterised by no moral hazard, is full insurance. Introducing ex-post moral hazard acknowledges that insured individuals often disregard the effect their consumption will have on their premiums and consequently consume more than in the first best situation. It is often considered that in the presence of moral hazard, full coverage is no longer optimal. However, according to McGuire (2011), even with some price elasticity, full insurance can be optimal if "the marginal utility when sick is high enough".
When private insurance for the *same* health good (i.e. complementary insurance) is introduced, the moral hazard effect increases and over-consumption rises. When social insurance plays first and anticipates private insurance, this leads to a reduction of social coverage; when the insurers play simultaneously, this leads to over-insurance (Barigozzi, 2006). In these models the only rationale for social coverage is redistribution. Therefore when individuals are homogeneous in productivity or when wage rates are heterogeneous and non-correlated to risk, the optimal social coverage is zero (Barigozzi, 2006). Similarly, Boadway et al. (2006) find that when the social planner observes ability and risk, and can transfer directly through taxation, then, there is no rationale for social insurance coverage. Consistently, Petretto (1999) finds that the optimal social insurance rate is negatively related to moral hazard and the correlation between risk and ability.

The study of mixed systems of insurance focuses on the effect of the introduction of social insurance on the scope of private insurance (designated as the second margin by Barros et al. (2011)) and on the nature of the interactions between the insurances. These interactions can be inefficient, as in the case of increased moral hazard, or efficient as in the case of offset effects. By re-insuring co-payments, complementary contracts deprive social insurance from a tool to limit ex-post moral hazard. The simplest form of interaction is the effect insuring a *second* health good has on an initial insurance contract. When goods are supplementary (and cross elasticities negative) the insurance of the second good will provoke a decrease in the consumption of the first good, known as an offset effect, which is welfare improving. The opposite is true when goods are complementary (and cross elasticities are positive). As articles on optimal insurance often consider a unique composit health good, the interactions between different health goods are generally outside their scope. On the other hand, considerations for interactions between health goods can be found in the literature on the optimal coverage of drugs and formulary contracts.

Apart for the OECD, the World Health Organization and rare empirical articles that make the distinction between complementary and supplementary insurances (notable examples are Paris et al, 2010 and 2016; Mossialos and Thomson, 2004; Keane and Stavrunova, 2016), these terms are still widely used indifferently. In theoretical articles, complementary insurance is systematically designated under the term 'supplementary insurance' (Petretto (1999); Barigozzi (2006)) and even in Boone (2015 and 2018) who points out the possible use of the term "complementary insurance" in this context. In empirical article, the term "supplemental" remains widely chosen to characterise complementary insurance (for example Buchmueller et al., 2004; Jones et al. 2006; Manning and Marquis, 1996; Marquis and Phelps, 1985 and 1987). This choice of terminology makes distinguishing the two topping up contracts impossible and is an indication that the distinction between these types of contracts remains often unclear. By introducing a second health good, this article is able to differentiate these contracts and illustrates why, and when, making the distinction is necessary.

This paper studies the effect of topping up insurance on the optimal rate of social insurance, in the presence of ex-post moral hazard. This article is in the line of normative literature that studies the characteristics of an optimal health insurance system. The aim of this model is to analyse the impact of supplementary and complementary insurance on the optimal social insurance rate. The study of supplementary insurance is made possible with the introduction of a second health good. In the manner of Petretto (1999) we develop a three-stage model of backwards induction. The social planner moves first by choosing the rate of coverage of the social bundle. Second, individuals choose a private insurance contract which includes complementary and supplementary components. Third, households choose their level of labour supply, of expenditure on health and other goods.
Results show that interactions between social and private insurance play a central role in the definition of optimal insurance rates. We find that the optimal social insurance rate depends on the redistributive nature of the insured health good and we argue that when studying the interactions between social and private topping up insurances, complementary and supplementary contracts should be distinguished. Social and complementary insurance have a crowding out effect on each other and increase inefficiencies linked to moral hazard. Conversely, when the scope of social insurance is well defined, supplementary insurance can generate efficient offset effects and avoid reverse redistribution.

2 The model

2.1 Households

In this model, there are n individuals, with $i = 1, ..., n$, who face two states $j = s, h$: being sick (s), with a probability ϕ_i, and being healthy (h), with a probability $1 - \phi_i$. The odds of being sick is an exogenous variable and can be observed by all. The productivity (w_i) is heterogeneous, exogenous and private information. The utility function $u(.)$ is function of labour supply (l_i^j), the quantity of health goods consumed (x_i and z_i, both equal to zero in the event of health) and the level of consumption of a non-medical composite good (c_i^j). The expected utility function can be written: $U(.) = \phi_i u(c_i^s, x_i, z_i, l_i^s) + (1 - \phi_i) u(c_i^h, 0, 0, l_i^h)$. The non-medical good c faces a price normalized to one. The social planner observes income $w_i l_i^j$.

Assumption 1: u is increasing and concave in c_i^j (with $j = h, s$), x_i and z_i and decreasing in l_i^j. Labour supply is lower in the sick state than in the healthy state.

In this model, there are two different types of medical goods. The first health good (x) is insured by social insurance, it has a unit price p (with p positive). The second health good (z) is not insured by social insurance, but by private supplementary insurances only, and has a unit price r (with r positive). The criteria used to define which good should be socially insured are important and outside the scope of this article.

There are three sources to finance the costs of x and z: social insurance, private insurance, and households' out-of-pockets. Social insurance offers the same rate of coverage to each individual and is financed through taxes. To cover the risk that the remaining expenditure represents, individuals can chose ex-ante (before they learn in which state they will be in) to purchase a private insurance contract. Complementary insurance offers coverage on remaining health expenses of good x and supplementary insurance offers coverage on the cost of good z. The residual out-of-pocket rate for the health good x is referred to as q_i^{cpl}, respectively q_i^{spl} for the health good z. In other words, individuals face the prices $q_i^{cpl} p$ and $q_i^{spl} r$ when they choose to buy a unit of respectively x or z. Provision of insurance decreases these prices.

Assumption 2: x and z are normal goods, the demand for health goods is negatively related to prices. In other words, $e_{x_i, q_i^{cpl}} < 0$ and $e_{z_i, q_i^{spl}} < 0$, with $e_{x_i, q_i^{cpl}}$ and $e_{z_i, q_i^{spl}}$, the price elasticity of health good x, and the price elasticity of health good z. pq^{cpl} and rq^{spl} are the prices that individuals face for respectively the health goods x and z, with p and r exogenous constant variables. The direct and crossed price elasticities are: $e_{x_i, q_i^{cpl}} = \frac{\partial x_i}{\partial q_i^{cpl}} \frac{q_i^{cpl}}{x_i}$; $e_{z_i, q_i^{spl}} = \frac{\partial z_i}{\partial q_i^{spl}} \frac{q_i^{spl}}{z_i}$; $e_{x_i, q_i^{spl}} = \frac{\partial x_i}{\partial q_i^{spl}} \frac{q_i^{spl}}{x_i}$; $e_{z_i, q_i^{spl}} = \frac{\partial z_i}{\partial q_i^{cpl}} \frac{q_i^{cpl}}{z_i}$. There is no assumption on the sign of $e_{x_i, q_i^{spl}}$ and $e_{z_i, q_i^{cpl}}$. The two health goods will be complementary if $e_{x_i, q_i^{spl}} < 0$ and substitute if $e_{x_i, q_i^{spl}} > 0$.

When social coverage varies, individuals can adjust their private insurance contracts. The sensitivity of this type of adjustment is measured by the elasticities $\sigma_{\alpha\beta}$ and $\sigma_{\alpha\gamma}$, with α, β and γ the respective rates of social, complementary and supplementary insurance.

$\sigma_{\alpha\beta} = \frac{\partial x_i / \partial \beta_i}{\partial x_i / \partial \alpha} \frac{\alpha}{\beta_i}$ and $\sigma_{\alpha\gamma} = \frac{\partial z_i / \partial \gamma_i}{\partial z_i / \partial \alpha} \frac{\alpha}{\gamma_i}$.

Assumption 3: For calculative reasons, we consider that we are in a simplified Feldstein (1972) framework. This partial equilibrium analysis implies that labour is not influenced by health insurance prices and there is no income effect on health care. In other terms, $\frac{d\bar{l_i}}{d\alpha} = 0$ and $\frac{dx_i}{dt} = \frac{dz_i}{dt} = \frac{dx_i}{dM} = \frac{dz_i}{dM} = 0$.

2.2 The insurance sector

2.2.1 Social insurance

Following the classification from Boadway et al. (2006), the rationale for public intervention in the field of insurance is threefold: lower transactions costs (including administrative costs), market failures (from asymmetric information) and redistribution. In our setting, social insurance can be a means of redistribution as, by assumption, the social planner cannot differentiate income taxes according to risk. No assumption was made on the correlation between risk and income. It is negative (respectively positive), if low income households have higher (lower) expenditure levels for the socially insured health goods than individuals with high income, and social insurance will be redistributive (regressive).

Social insurance is financed through a classical linear payroll tax. Individuals will pay $T^j = tw_i l_i^j - M$, according to their state of health j. The tax rate on labour supply (t) is identical for all individuals. The lump sum transfers (M) can be positive or negative in order to represent the possible non-proportional nature of the tax. The social planner provides social insurance for the health good x at a rate α , (with $0 \leq \alpha \leq 1$) identical for all individuals. If an individual i is sick, the total expenses covered by social insurance for this consumer is $A_i = \alpha p x_i$. There are n individuals in the society, therefore the total cost of social insurance is $\sum_{i=1}^{n} \phi_i A_i$.

The social planner has one objective: maximising the welfare function that is defined as the unweighted sum of each individual utility in the society.

$$max_{\alpha,t,M}\ W = \sum_{i=1}^{n} U_i$$

st $\alpha \sum_{i=1}^{n} \phi_i p x_i = t \sum_{i=1}^{n} w_i l_i - nM$, with μ the Lagrange multiplier of the corresponding maximisation program.

The government's budget constraint is such that the total expenditure in social insurance is equal to the receipt from the linear payroll tax:

$$\sum_{i=1}^{n} \phi_i A_i = t \sum_{i=1}^{n} \phi_i w_i l_i^s + (1-\phi_i) w_i l_i^h - nM$$

$$\Rightarrow \alpha \sum_{i=1}^{n} \phi_i p x_i = t \sum_{i=1}^{n} w_i \bar{l_i} - nM \Rightarrow \alpha p \overline{x} = t\overline{wl} - M$$

Denoting averages with a bar, for example, $\bar{l_i} = \phi_i l_i^s + (1-\phi_i) l_i^h$;
$\bar{x_i} = \phi_i x_i^s + (1-\phi_i) x_i^h = \phi_i x_i + (1-\phi_i)0$; $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} \bar{x_i}$.

2.2.2 Private insurance

In this setting, complementary and supplementary insurance apply to the same risk of being sick (with a probability ϕ_i). Complementary insurance offers coverage on remaining health expenses $(1-\alpha)px_i$ at a premium π_i^{cpl} and supplementary insurance offers coverage on rz_i at a premium π_i^{spl}. These contracts are characterized by their rate of insurance (β_i and γ_i , with $0 \leq \beta_i \leq 1$ and $0 \leq \gamma_i \leq 1$) and their premiums (π_i^{cpl} and π_i^{spl}). Premiums vary according to individuals' risks; as private insurances evolve on a competitive market the premiums are considered fair, i.e. PHIs make zero profit.

Premiums of complementary and supplementary insurance are actuarial, with β_i and γ_i the insurance rates of health goods x and z: $\pi_i^{cpl} = \phi_i(1-\alpha)\beta_i p$ and $\pi_i^{spl} = \phi_i\gamma_i r$. $B_i^{cpl} = \beta_i(px_i - A_i) = \beta_i(1-\alpha)px_i$ and $B_i^{spl} = \gamma_i r z_i$, with B_i^{cpl} and B_i^{spl} the total expenses covered by private insurance.

Households finance social and private insurance through taxes and premiums in both health states. When they are sick, they face the remaining health expenditure not covered by insurances, R_i. $R_i = D_i - A_i - B_i = (1-\alpha)(1-\beta_i)px_i + (1-\gamma_i)rz_i = q_i^{cpl}px_i + q_i^{spl}rz_i$, with q_i^{cpl} and q_i^{spl} the out-of-pocket coefficient rates. Once premiums have been paid their cost is sunk and insurance contracts reduce the prices individual face for the health goods x_i and z_i to $q_i^{cpl}p$ and $q_i^{spl}r$.

2.3 The timing of the model

The game has three sequential steps and is resolved through backward induction. The outcomes of the previous steps are anticipated making the equilibrium sub-game perfect. First, the social planner defines the optimal rate of social insurance (α^*) by maximising the social welfare function. In the second stage, individuals anticipate social coverage and choose a private contract with a supplementary and a complementary component. At this stage, individuals vary according to their productivity (w_i) and their risk of becoming sick (ϕ_i). The information on productivity is private, and the level of risk is known by all. In the last step it is revealed in which state of nature each individual is and they define their optimal individual level of labour supply, consumption of health and other goods.

The sections follow the anti-chronological order of the resolution. Section 3.1 studies the maximisation program of the households once the state is known, Section 3.2 analyses the choice of private insurance contracts, Section 4 presents the social planner's choices and Section 5 concludes.

3 The households' choices

3.1 The households' choice of labour and consumption

In the third stage, it is assumed that premiums are sunk and taxes are given. Individuals maximise their utility with respect to the endogenous variables l_i , c_i (plus x_i and z_i when sick). Premiums being exogenous, the choice of health consumption levels will depend exclusively on prices pq_i^{cpl} and rq_i^{spl}. Individuals maximise their utility under their budget constraints. When they are healthy, the consumption of the non-health good is equal to their income plus the lump-sum transfer, net from taxes and of the cost of the private insurance premium. $c_i^h = w_i(1-t)l_i^h + M - \pi_i^{cpl}x_i - \pi_i^{spl}z_i$. When they are ill, this income is used to buy health goods and other goods.

$$c_i^s + q_i^{cpl}px_i + q_i^{spl}rz_i = w_i(1-t)l_i^s + M - \pi_i^{cpl}x_i - \pi_i^{spl}z_i.$$

According to Assumption 1, labour supply is higher in the healthy state than in the sick state and marginal utility decreases with consumption. Consequently, $c_i^s < c_i^h$ and the associated marginal utility of income will be higher when sick than when healthy.

The optimal choice of the consumer results from the maximisation of the expected utility. By rearranging these First Order Conditions (FOC) we obtain the following optimality conditions:

$$\frac{\partial u}{\partial c_i^h} w_i(1-t) + \frac{\partial u}{\partial l_i^h} = 0 \tag{1}$$

$$\frac{\partial u}{\partial c_i^s} w_i(1-t) + \frac{\partial u}{\partial l_i^s} = 0 \tag{2}$$

$$\frac{\partial u}{\partial x_i} - p q_i^{cpl} \frac{\partial u}{\partial c_i^s} = 0 \tag{3}$$

$$\frac{\partial u}{\partial z_i} - r q_i^{spl} \frac{\partial u}{\partial c_i^s} = 0 \tag{4}$$

The optimal quantities of labour supply and consumption of health goods and non-health goods satisfy the conditions given by equations (1) to (4).

From (1) to (4) we obtain several results of comparative static, presented in Lemma 1. Denoting from now on derivatives with a subscript $\left(\frac{\partial x_i}{\partial \alpha} = x_\alpha\right)$ and $\rho^j = \frac{\partial u}{\partial c^j}$ the marginal utility of the non-health good.

Lemma 1.

(i) $\frac{\partial x_i}{\partial \alpha} = -\frac{\frac{\partial^2 u^s}{\partial x_i \partial \alpha} + p(1-\beta_i)\rho^s}{\frac{\partial^2 u^s}{\partial x_i{}^2}} > 0$

(ii) $\frac{\partial x_i}{\partial \beta_i} = -\frac{\frac{\partial^2 u^s}{\partial x_i \partial \beta_i} + p(1-\alpha)\rho^s}{\frac{\partial^2 u^s}{\partial x_i{}^2}} > 0$

(iii) $\frac{\partial z_i}{\partial \gamma_i} = -\frac{\frac{\partial u^s}{\partial z_i \partial \gamma_i} + r\rho^s}{\frac{\partial^2 u^s}{\partial z_i{}^2}} > 0$

(iv) The elasticity between social and complementary insurance rates is equal to:
$\sigma_{\alpha\beta} = \frac{\frac{\partial x}{\partial \alpha} / \frac{\partial x}{\partial \beta_i}}{\alpha/\beta_i} = \frac{\beta_i}{\alpha} \frac{\frac{\partial^2 u^s}{\partial x_i \partial \alpha} + p(1-\beta_i)\rho^s}{\frac{\partial^2 u^s}{\partial x_i \partial \beta_i} + p(1-\alpha)\rho^s}.$

Proof. See Appendix 2. □

Lemma 1 indicates that consumption of health goods increases with their insurance coverage.

3.2 The household's choice of private insurance

In the second stage, individuals choose their private coinsurance (β_i and γ_i) among a choice of adapted competitive contracts. During the previous step, the optimal levels of consumption according to parameters β_i, γ_i , α ,t and M were found and described by the optimality conditions (1) to (4). By introducing these variables in the utility function we obtain the expected indirect utility function:

$v_i = max\ \phi_i u(c_i^s, x_i, z_i, l_i^s) + (1-\phi_i) u(c_i^h, 0, 0, l_i^h)$. The optimal private rates of private insurance are the results of the maximisation of their expected indirect utility function, with α, t and M exogenous variables at this stage.

The private insurance problem is solved thanks to the following maximisation program:
$Max_{\beta_i,\gamma_i}\ v(\alpha, \beta_i, \gamma_i, t, M) = \phi_i u(c_i^s, x_i, z_i, l_i^s) + (1-\phi_i) u(c_i^h, 0, 0, l_i^h)$.

β_i^* and γ_i^* are solutions of $\frac{\partial v}{\partial \beta_i} = 0$ and $\frac{\partial v}{\partial \gamma_i} = 0$ with:

$$\frac{\partial v}{\partial \beta_i} = -[\rho^s - \bar{\rho}] \frac{\partial q_i^{cpl}}{\partial \beta_i} p\bar{x}_i - \bar{\rho}(\pi_i^{cpl} x_\beta + \pi_i^{spl} z_\beta) \tag{5}$$

$$\frac{\partial v}{\partial \gamma_i} = -[\rho^s - \bar{\rho}] \frac{\partial q_i^{cpl}}{\partial \gamma_i} r\bar{z}_i - \bar{\rho}(\pi_i^{cpl} x_\gamma + \pi_i^{spl} z_\gamma) \tag{6}$$

Lemma 2 presents some results when derivating by the private insurance rates and is proven in Appendix 2.

Lemma 2. *x_i and z_i are normal goods, their consumption increases when their price goes down and the sign of the cross derivatives vary according to if goods are complementary or supplementary.*

(i) $\frac{\partial x_i}{\partial \beta_i} = -(1-\alpha)\, x_{q^{cpl}}$
(ii) $\frac{\partial x_i}{\partial \gamma_i} = -x_{q^{spl}}$
(iii) $\frac{\partial z_i}{\partial \beta_i} = -(1-\alpha)\, z_{q^{cpl}}$
(iv) $\frac{\partial z_i}{\partial \gamma_i} = -z_{q^{spl}}$

Equalizing equations (5) and (6) to zero and rearranging the terms we obtain the optimality conditions presented by equations (7) and (8).

$$Cov(\rho, px_i) = \bar{\rho}\phi_i\beta_i p x_\beta + \bar{\rho}\phi_i \frac{\gamma_i}{(1-\alpha)} r z_\beta \tag{7}$$

$$Cov(\rho, rz_i) = \bar{\rho}\phi_i(1-\alpha)\beta_i p x_\gamma + \bar{\rho}\phi_i\gamma_i r z_\gamma \tag{8}$$

With $Cov(\rho, px_i) = \phi_i p x_i(\rho^s - \bar{\rho})$ and $Cov(\rho, rz_i) = \phi_i r z_i(\rho^s - \bar{\rho})$ the risk-sharing gains of insurance.

Lemma 3. *Optimal positive interior conditions for private insurance rates β^* and γ^* are given by equations (7) and (8). The optimal rates of complementary and supplementary insurance are such that the gain of private insurance is equal to the expected loss from the marginal cost of moral hazard. Private coverage is positively related to the utility of reducing income dispersion and negatively linked to moral hazard.*

Out-of-pocket payments in the sick state create income dispersion between the two health states which is costly to risk averse individuals. The first terms of equations (5) and (6) and the term on the Left Hand Side (LHS from now on) of equations (7) and (8) express the gain caused by the reduction of the risk, in terms of income dispersion. $Cov(\rho, px_i)$ and $Cov(\rho, rz_i)$ are positive as the utility of income is increasing and concave and the marginal utility of income is higher when sick than when healthy (Assumption 1). Note, private insurance, by decreasing the difference in income between the sick and healthy state generates risk-sharing gains, without redistributing between individuals who face a low risk (ϕ_i low) and those who face a high risk of sickness (ϕ_i high).

The second terms of equations (5) and (6) and the Right Hand Side (RHS from now on) of equations (7) and (8) express the cost of moral hazard. The first best solution (i.e. the optimum in a setting with no moral hazard) is achieved when the effects of consumption on premiums are taken into account by consumers. In this setting, by increasing private insurance rates, the perceived cost of health goods x_i and z_i have gone down and their consumption consequently increased, generating inefficiently high premiums. This increase in premiums results in a lower coverage; β^* and γ^* are negatively related to price elasticities and moral hazard are high than when they are low. When goods are inelastic to price, the first best solution is achieved and optimal insurance is reached. In this case, full insurance is optimal.

After solving the system given by equations (7) and (8), we find the optimal values:

$$\beta_i^* = \frac{Cov(\rho, px_i)z_\gamma}{\bar{\rho}\phi_i p(x_\beta z_\gamma - x_\gamma z_\beta)} - \frac{Cov(\rho, rz_i)z_\beta}{\bar{\rho}\phi_i p(1-\alpha)(x_\beta z_\gamma - x_\gamma z_\beta)}$$

$$\gamma_i^* = \frac{Cov(\rho, rz_i)x_\beta}{\bar{\rho}\phi_i p(x_\beta z_\gamma - x_\gamma z_\beta)} - \frac{Cov(\rho, px_i)z_\gamma}{\bar{\rho}\phi_i p(x_\beta z_\gamma - x_\gamma z_\beta)}$$

In the likely case that demand for a health good is more sensitive to its own insurance than the insurance of the other health good (i.e.: $|x_\beta| > |x_\gamma|$ and $|z_\gamma| > |z_\beta|$), then $x_\beta z_\gamma > x_\gamma z_\beta$ and β_i^* and γ_i^* are defined.

Proposition 1. *When x and z are substitutes, some private coverage will always be optimal, with $\beta^* > 0$ and $\gamma^* > 0$.*

Proof. Private insurance is always positive when x and z are substitutes: $\frac{Cov(\rho,px_i)z_\gamma}{\bar{\rho}\phi_i p(x_\beta z_\gamma - x_\gamma z_\beta)} - \frac{Cov(\rho,rz_i)z_\beta}{\bar{\rho}\phi_i p(1-\alpha)(x_\beta z_\gamma - x_\gamma z_\beta)} > 0 \Leftrightarrow \beta_i > 0$ and $\frac{Cov(\rho,rz_i)x_\beta}{\bar{\rho}\phi_i p(x_\beta z_\gamma - x_\gamma z_\beta)} - \frac{Cov(\rho,px_i)z_\gamma}{\bar{\rho}\phi_i p(x_\beta z_\gamma - x_\gamma z_\beta)} > 0 \Leftrightarrow \gamma_i > 0$. As $Cov(\rho, px_i)$, $\bar{\rho}\phi_i px_\beta$, $Cov(\rho, rz_i)$, $\bar{\rho}\phi_i\gamma_i rz_\gamma$ are all positive, the first terms, $\frac{Cov(\rho,px_i)z_\gamma}{\bar{\rho}\phi_i p(x_\beta z_\gamma - x_\gamma z_\beta)}$ and $\frac{Cov(\rho,rz_i)x_\beta}{\bar{\rho}\phi_i p(x_\beta z_\gamma - x_\gamma z_\beta)}$ are positive. Similarly, $\frac{Cov(\rho,rz_i)}{\bar{\rho}\phi_i p(1-\alpha)(x_\beta z_\gamma - x_\gamma z_\beta)} > 0$ and $\frac{Cov(\rho,px_i)z_\gamma}{\bar{\rho}\phi_i p(x_\beta z_\gamma - x_\gamma z_\beta)} > 0$. It follows the sign of the second term is function of z_β and x_γ . When $z_\beta < 0$ and $x_\gamma < 0$ (i.e. when health goods are substitutes), then the second terms are positive.

In this case, risk averse individuals will acquire some coverage ($\beta^* > 0$ and $\gamma^* > 0$) – even if the price-elasticity is positive and insurance generates moral hazard. □

Full-insurance, $\beta^* = 1$ and $\gamma^* = 1$, can be optimal, even in the presence of moral hazard, if the difference between the valuation of marginal utility when sick and healthy is large enough (when $Cov(\rho, px_i) > \bar{\rho}\phi_i{\beta}_i px_\beta + \bar{\rho}\phi_i \frac{\gamma_i}{(1-\alpha)} rz_\beta$). These findings are in line with the literature on the trade-offs between risk aversion and moral hazard.

Conditions (7) and (8) implicitly define the optimal private coverage: $\beta_i(\alpha, t, M)$ and $\gamma_i(\alpha, t, M)$. As in Petretto (1999), we assume that the partial derivative $\frac{\partial \beta_i}{\partial \alpha}$ is negative, making complementary and social insurance strategic substitutes. At the second margin, social insurance crowds out complementary insurance.

Interactions between private insurances are measured by the second term of the RHS. As McGuire (2011, p.351) underlined, when studying the optimal insurance rate, the effect of other insured, and therefore "overused", goods must be accounted for. The contribution of the model is to identify and distinguish the effect of interactions between complementary and supplementary contracts. The effect varies according to the relations between goods. When x_i and z_i are complementary, a rise in one private insurance rate will generate moral hazard and raise both premiums. When x_i and z_i are substitutes, the second private coverage decreases premiums. A rise in one health good insurance rate would provoke efficient offset effects: the consumption of the other good would decrease leading to lower out-of-pockets and premium levels for both health goods. Consequently, private coverage will be higher when goods are substitutes, and lower when they are complementary.

Proposition 2. *When x_i and z_i are substitutes, complementary insurance has an efficient offset effect on supplementary contracts and supplementary insurance has an efficient offset effect on complementary contracts.*

Proof. When x_i and z_i are substitutes, x_γ and z_β are negative, and the second RHS term effects $\bar{\rho}\pi_i^{spl} r z_\beta$ and $\bar{\rho}\pi_i^{cpl} p x_\gamma$ are negative. □

In presence of ex-post moral hazard, households do not anticipate the impact their extra consumption will have on premiums. Similarly, individuals do not anticipate that their coverage of one health good will offset the consumption of the other health good and consequently decrease their premium.

4 The Social planner's choices

At this stage, parameters α, t and M are yet to be determined. The social planner anticipates the results of the previous stages. We assume a simple, unweighted, utilitarist function $W = \sum v_i$ maximised under the social budget constraint.

Section 4.1 presents the optimal rate of social insurance α^* that maximises social welfare and Section 4.2 the optimal rate t^* and transfer M^* of the tax system.

4.1 The optimal social insurance rate

Before giving the optimality condition, Lemma 4 presents the derivation results according to α of: premiums, out-of-pocket levels and health consumption; the proof is in the Appendix 2.

Lemma 4.

(i) $\frac{\partial q_i^{cpl}}{\partial \alpha} = -1 + \beta_i(1 + \frac{1-\alpha}{\alpha}\sigma_{\alpha\beta})$
(ii) $\frac{\partial \pi_i^{cpl}}{\partial \alpha} = -\phi_i\beta_i(1 + \frac{1-\alpha}{\alpha}\sigma_{\alpha\beta})p$
(iii) $\frac{\partial x_i}{\partial \alpha} = [\beta_i(1 + \frac{1-\alpha}{\alpha}\sigma_{\alpha\beta}) - 1]x_{q^{cpl}}$
(iv) $\pi_\alpha^{cpl} = -\phi_i(q_\alpha^{cpl} + 1)$
(v) $\frac{\partial q_i^{spl}}{\partial \alpha} = -\gamma_\alpha\frac{\gamma_i}{\alpha}\sigma_{\alpha\gamma}$
(vi) $\frac{\partial \pi_i^{spl}}{\partial \alpha} = -\phi_i\frac{\gamma_i}{\alpha}\sigma_{\alpha\gamma}r$
(vii) $\frac{\partial z_i}{\partial \alpha} = \frac{\gamma_i}{\alpha}\sigma_{\alpha\gamma}z_{q^{spl}}$
(viii) $\pi_\alpha^{spl} = -\phi_i q_\alpha^{spl}$

By maximising the objective function with respect to α we obtain:

$$\sum_{i=1}^{n} v_\alpha = \mu\sum_{i=1}^{n}\phi_i p x_i + \mu\alpha\sum_{i=1}^{n}\phi_i p x_\alpha \tag{9}$$

As,

$$v_\alpha = \bar{\rho}p\bar{x}_i + (\bar{\rho} - \rho^s)q_\alpha^{cpl}p\bar{x}_i + (\bar{\rho} - \rho^s)q_\alpha^{spl}r\bar{z}_i - \bar{\rho}\pi_i^{cpl}px_\alpha - \bar{\rho}\pi_i^{spl}rz_\alpha \tag{10}$$

equation (9) can be rearranged:

$$\sum_{i=1}^{n} Cov(\bar{b}; q_\alpha^{cpl}p\bar{x}_i) + \sum_{i=1}^{n} Cov(\bar{b}; q_\alpha^{spl}r\bar{z}_i) + n\left(\bar{b} + \bar{b}Cov\left(\frac{b_i}{\bar{b}}, \frac{x_i}{\bar{x}}\right) - 1\right)p\bar{x}$$

$$+n\left(\bar{b} + \bar{b}Cov\left(\frac{b_i}{\bar{b}}, \frac{z_i}{\bar{z}}\right)\right)r\bar{z} = \sum_{i=1}^{n}\alpha\phi_i p x_\alpha + \sum_{i=1}^{n}\bar{b}\pi_i^{cpl}px_\alpha + \sum_{i=1}^{n}\bar{b}\pi_i^{spl}rz_\alpha \tag{11}$$

With $b_i^j = \frac{\rho_i^j}{\mu}$, and $\bar{b}_i = \frac{1}{n}\sum_{i=1}^{n}\phi_i b_i^s + (1-\phi_i)b_i^h$ the private marginal utility of the other good's consumption; $Cov(\bar{b}, q_\alpha^{cpl}p\bar{x}_i) = \phi_i q_\alpha^{cpl}p\bar{x}_i(b^s - \bar{b})$ and $Cov(\bar{b}, q_\alpha^{cpl}p\bar{z}_i) = \phi_i q_\alpha^{cpl}p\bar{z}_i(b^s - \bar{b})$ the social risk-sharing gains of social insurance; $Cov\left(\frac{b_i}{b}, \frac{x_i}{\bar{x}}\right) = \frac{1}{n}\sum_{i=1}^{n}\frac{b_i}{b}\frac{x_i}{\bar{x}} - 1$ and $Cov\left(\frac{b_i}{b}, \frac{z_i}{\bar{z}}\right) = \frac{1}{n}\sum_{i=1}^{n}\frac{b_i}{b}\frac{z_i}{\bar{z}} - 1$ distributive factors of health expenditure, as defined in the literature of indirect taxation.

Lemma 5. *The positive interior condition for the optimal rate of social insurance α^* verifies equation (11). α is optimal when the social gain from risk sharing and the social redistribution gain are equal to the social cost of moral hazard.*
Social coverage is positively related to the gain of risk-sharing, redistribution, and negatively related to moral hazard.

The first effect of increasing social insurance is to reduce the variation between the states of natures, of the remaining cost of the health good x. The decrease in income variation that social insurance provides is the social risk sharing gain. It is captured by the first two terms $\sum_{i=1}^{n} Cov(\bar{b}; q_\alpha^{cpl}p\bar{x}_i)$ and $\sum_{i=1}^{n} Cov(\bar{b}; q_\alpha^{spl}r\bar{z}_i)$, the sum of differences between the marginal utility of income of the expected out-of-pocket expense of good x and z, and the marginal utility of income of the out-of-pocket cost in case of sickness.

The right hand side is equal to the social marginal cost of moral hazard. There are two types of moral hazard effects. $\alpha \sum_{i=1}^{n} \phi_i p x_\alpha$ expresses the negative effect of an increase in social insurance on the government's budget constraint. The second form of moral hazard ($\sum_{i=1}^{n} \bar{b}\pi_i^{cpl} p x_\alpha$ and $\sum_{i=1}^{n} \bar{b}\pi_i^{spl} r z_\alpha$) is the sum of the impact of the increase of coverage on the individual's premiums.

The third component of the LHS expresses the social redistribution gain from social insurance. It has the form of a normalised covariance between social MUI and health expenditure. The covariance between the net social marginal utility of income and the share of consumer i in the total consumption of health goods, $Cov\left(\frac{b_i}{\bar{b}}, \frac{x_i}{\bar{x}}\right)$, is greater for goods that are heavily consumed by individuals with a high net social marginal utility of income. As analysed by Salanié (2011), because of the concave shape of the utility function, "these agents, who are privileged by the government in its objective function, are likely also the poorest". α varies according to the relation between the distribution of income and health expenditure $p\bar{x}$. α will be highest when they are strongly positively related, i.e. when the poorest individuals who consume few other goods are the ones who spend on average the most on the good x. Even in the presence of some moral hazard, full social insurance can be optimal if the redistributive gains are high enough. Note, because individuals with low incomes have a higher value of marginal utility of consumption, the social planner favours low income individuals, even when its objective function is an unweighted sum. Barigozzi (2006) studies the case with homogenous individuals. In this case $Cov\left(\frac{b_i}{\bar{b}}, \frac{x_i}{\bar{x}}\right) = 0$ and there is no gain from redistribution.

Proposition 3. *When the consumption of the health good x is negatively correlated to income, social insurance is redistributive and $\alpha^* > 0$. When reverse distribution is strong, the high-income individuals buy more x than the low-income group and $\alpha^* = 0$.*

Proof. In the absence of supplementary insurance, equation (11) can be rewritten: $\sum_{i=1}^{n} Cov(\bar{b}; q_\alpha^{cpl} p\bar{x_i}) + n\left(\bar{b} + \bar{b}Cov\left(\frac{b_i}{\bar{b}}, \frac{x_i}{\bar{x}}\right) - 1\right) p\bar{x} = \sum_{i=1}^{n} \alpha\phi_i p x_\alpha + \sum_{i=1}^{n} \bar{b}\pi_i^{cpl} p x_\alpha$.
When consumption of the health good x and income are negatively correlated, $Cov\left(\frac{b_i}{\bar{b}}, \frac{x_i}{\bar{x}}\right) > 0$, if $\sum_{i=1}^{n} Cov(\bar{b}; q_\alpha^{cpl} p\bar{x_i}) > \sum_{i=1}^{n} \bar{b}\pi_i^{cpl} p x_\alpha$, then $\frac{\sum_{i=1}^{n} Cov(\bar{b}; q_\alpha^{cpl} p\bar{x_i}) + n\left(\bar{b} + \bar{b}Cov\left(\frac{b_i}{\bar{b}}, \frac{x_i}{\bar{x}}\right) - 1\right) p\bar{x} - \sum_{i=1}^{n} \bar{b}\pi_i^{cpl} p x_\alpha}{\sum_{i=1}^{n} \phi_i p x_\alpha} > 0 \Rightarrow \alpha^* > 0$.
Conversely, when social insurance is regressive, $Cov\left(\frac{b_i}{\bar{b}}, \frac{x_i}{\bar{x}}\right) < 0$, the optimal coverage is null. When $\frac{\sum_{i=1}^{n} Cov(\bar{b}; q_\alpha^{cpl} p\bar{x_i}) + n\left(\bar{b} + \bar{b}Cov\left(\frac{b_i}{\bar{b}}, \frac{x_i}{\bar{x}}\right) - 1\right) p\bar{x} - \sum_{i=1}^{n} \bar{b}\pi_i^{cpl} p x_\alpha}{\sum_{i=1}^{n} \phi_i p x_\alpha} < 0 \Rightarrow \alpha^* = 0$.
Finally, optimal full social coverage can exist, even in the presence of moral hazard if the gains of redistribution and risk sharing are high enough.
When $\frac{\sum_{i=1}^{n} Cov(\bar{b}; q_\alpha^{cpl} p\bar{x_i}) + n\left(\bar{b} + \bar{b}Cov\left(\frac{b_i}{\bar{b}}, \frac{x_i}{\bar{x}}\right) - 1\right) p\bar{x} - \sum_{i=1}^{n} \bar{b}\pi_i^{cpl} p x_\alpha}{\sum_{i=1}^{n} \phi_i p x_\alpha} \geqslant 1 \Rightarrow \alpha^* = 1$. □

The correlation between income and consumption of the socially insured health good is central. There will be no social coverage when the health good is regressive.

As in Barigozzi (2006), optimal social coverage is null if individuals are homogenous.

Proposition 4. *The optimal social insurance rate α^* is higher in an exclusive system without complementary insurance.*

Proof. Complementary insurance decreases the effect of the social risk sharing gain and increases moral hazard, thus reducing α^*. In the previous section, the risk sharing gain of private insurance was independent from social insurance, $Cov(\rho; px_i)$. Because of the timing of the model, this result is not symmetrical and we find here that, even after simplification, the gain of risk sharing of social insurance $Cov(\bar{b}; q_{\alpha}^{cpl} p\bar{x}_i)$ is proportional to the size of the out-of-pocket rate and is therefore decreasing in β_i. The impact of α on individuals' out-of-pocket level will be strongest when there is no complementary insurance or, when the elasticity between private and public insurance is weak. □

Following the classification from Boadway et al. (2006), the rationale for public intervention in the field of insurance, are: transactions costs (administrative costs), market failures (from asymmetric information) and redistribution. Proposition 3 underlines the central role of redistribution in the optimality of social insurance.

The relation between the health goods x and z is what distinguishes complementary and supplementary insurance.

Proposition 5. *Supplementary insurance has an offset effect resulting in an increase of α^* when x and z are substitutes; it increases moral hazard and thus reduces α^* when x and z are complementary.*

Proof. When health goods x and z are substitutes, supplementary insurance will generate efficient offset effects, reducing consumption of x while increasing α^*. Conversely, when health goods are complementary, private coverage will increase moral hazard. Consequently, the optimal α^* will be higher when the goods are substitutes than if they are complementary.

□

When health goods are substitutes, supplementary and complementary have opposite effects on optimal social insurance. Confusing complementary and supplementary insurances can be detrimental in the design of optimal insurance.

In a previous version of this model, we considered that PHIs had higher administrative costs than SHI. Results of the previous model indicated that, when administrative costs are considered, the optimal rate of private insurance is such that the gain of private insurance is equal to the expected loss from the marginal cost of moral hazard and the cost of administrative costs. In the first best scenario, when administrative costs exceed a certain threshold, full insurance is no longer optimal. Conversely, in this setting, social insurance coverage (measured by α) is positively related to private administrative costs. These results are in line with findings from Boadway et al. (2006), administrative costs increase the rationale for SHI coverage. Because this specification did not add any novel results, we chose for simplicity not to include administrative costs.

4.2 The optimal tax structure

According to the maximisation program, the optimal tax rate t^* and transfer M^* are the solutions of respectively $\frac{d\Psi}{dt} = 0$ and $\frac{d\Psi}{dM} = 0$, which gives us:

$$\bar{a} - \frac{1}{n} Cov(b,\, q_{\beta}^{cpl} p\bar{x}_i \beta_M + q_{\gamma}^{cpl} r\bar{z}_i \gamma_M) = 1 \tag{12}$$

$$\frac{t}{1-t} = -\frac{\sum_{i=1}^{n} Cov(b,\, q_{\beta}^{cpl} p\bar{x}_i \beta_t + q_{\gamma}^{spl} r\bar{z}_i \gamma_t) + n\overline{wl}(\bar{a} Cov\left(\frac{a_i}{\bar{a}}, \frac{w_i l_i}{wl}\right) + \bar{a} - 1)}{\sum_{i=1}^{n} \overline{w_i l_i} \epsilon_{ll}} \tag{13}$$

With $a_i = \frac{du}{dy_i}\frac{1}{\mu} + \sum_{i=1}^{n} t_i w_i l_M$ the net social marginal utility of income (MUI), it is equal to the social MUI net from the extra tax caused by the extra income. When individuals are healthy, the marginal utility of consumption is equal to the MUI ($y_i^h = c_i^h$). This identity does not hold when ill ($y_i^s = c_i^s + q^{cpl} p x_i + q^{spl} r z_i$). With $\bar{a} = \frac{1}{n}\sum_{i=1}^{n} a_i$.
With $\epsilon_{\bar{l}l} = \frac{\partial \bar{l}}{\partial w_i(1-t)} \frac{w_i(1-t)}{l} = S\frac{w_i(1-t)}{l}$ the elasticity of the expected labour supply; $Cov\left(\frac{a_i}{\bar{a}}, \frac{w_i l_i}{wl}\right) = \frac{1}{n}\sum_{i=1}^{n} \frac{a_i}{\bar{a}} \frac{w_i l_i}{wl} - 1$ the covariance between the expected social MUI and the expected gross labour income across individuals - a distributional characteristic of income distribution as presented by Feldstein (1972).

Lemma 6. *Positive interior conditions for the tax rate t^* and transfer M^* are given by equation (12) and (13).*
The lump sum transfer M^* is optimal when the net social valuation of the transfer of $1 is equal to the cost of that transfer plus the gain of risk-sharing for the extra out-of-pocket expenditure.
The optimal tax rate t^* follows the Ramsey tax rule, that is, t^* is inversely proportional to the compensated elasticity of labour. It is function of the distribution of income (of the variation of the valuation of w) and of the risk sharing loss caused by the reduction of out-of-pocket expenditure caused by t.

Equations (13) and (14) follow the usual interpretation of optimal linear income taxation. We cannot exclude a corner solution.

Proof. See Appendix 2.

□

5 Conclusion

In 2013 the French Council of Economic Analysis (CAE) proposed to forbid re-insuring co-payments of social insurance, thereby offering to restrict topping up activities to supplementary insurance only (Askenasy et al., 2013).
In this article, we set out to explore whether complementary and supplementary insurances had the same impact on social coverage, and to see if there was a case for banning complementary insurance. In order to do this, we adapted a health insurance model, in presence of moral hazard by introducing a second health good. Thereby making it possible to study the interactions between complementary, supplementary and social insurance.

We find that, when the correlation between income and consumption of the health good is positive, social insurance would be regressive and the optimal social coverage is null, resulting in an exclusive private system. Conversely, when the correlation is positive, optimal social coverage is positive and can be equal to one when price elasticity is low. Consequently, optimal mixed systems with complementary insurance will arise solely when social insurance is redistributive and health demand is elastic.

The main contribution of this article is to give evidence that complementary and supplementary insurances can have opposing interactions with social insurance and should therefore be distinguished from one another. Compared to an exclusive system, complementary insurance increases moral hazard and decreases the beneficial effects of social insurance, resulting in lower coverage of social insurance and consequently lower redistribution. On the other hand, when health goods are substitutes, supplementary insurance will have a positive offset effect resulting in an increase in social coverage. When health goods are complementary, supplementary insurance acts in a similar way to complementary insurance, generating ex-post moral hazard and decreasing optimal social coverage.

The efficiency of the offset effects of private insurance on social insurance are determined by the sign of the interactions. We offer to illustrate these effects using examples from the market of eyewear. In our example, we consider that social insurance participation is restricted to covering part of the cost of eyeglass lenses; complementary insurance covers eyeglass lenses and supplementary insurance finances frames and contact lenses. Frames are complementary to eyeglass lenses whereas contact lenses are substitutes. As the demand for lenses is elastic to price, complementary insurance generates inefficient moral hazard, resulting in a lower optimal social insurance rate. Similarly, supplementary insurance of frames generates inefficient offset effects resulting in inefficient extra-consumption of eyeglass lenses. However, supplementary insurance of contact lenses causes positive offset effects on eyeglass lenses, resulting in an increase of the optimal social insurance rate. In other words, the social insurance rate for eyeglass lenses will be lower if complementary insurance is allowed and when supplementary insurance covers frames; conversely, the social insurance rate for eyeglass lenses will increase when supplementary insurance covers lenses.

Our findings are consistent with past articles. As in Barigozzi (2006), when insuring the health good yields no redistribution gain, social insurance coverage should be null. As in Petretto (1999), the optimal rate of social insurance is positively related to the gain of risk sharing and redistribution and is also negatively related to moral hazard. Consistent with Boadway et al. (2006), the rationale for social insurance are redistribution and relative administrative efficiency between social and private insurance.

These results give ground to the choice made by many countries to ban private insurance from re-insuring social co-pays. In countries where complementary insurance is widespread, banning complementary insurance seems difficult. In this case, reaching towards full social insurance on some health goods and suppressing SHI contribution for other health expenditure can be considered. When considering whether to ban complementary insurance or not, the composition of the social health bundle is crucial. One of the limits of this model is to include only two health goods (one covered by social insurance and one that is not). Selection considerations are outside the scope of our analysis. Our results suggest that the goods that should be put in priority inside the social bundle should be foremost valued by lower income households and have weak price sensitivity (necessary treatments rather than comfort health goods). These findings complement

References

[1] Askenazy P, Dormont B, Geoffard PY, Paris V. Towards a More Efficient Health System. Les notes du conseil d'analyse économique 2013;No 8, July.

[2] Arrow K. Uncertainty and the Welfare Economics of Medical Care. The American Economic Review 1963;53 (5): 941–73. https://doi.org/10.2307/1812044.

[3] Arrow K. The Economics of Moral Hazard: Further Comment. The American Economic Review 1968;58 (3): 537–39. https://doi.org/10.2307/1813786.

[4] Barigozzi F. Supplementary Insurance with Ex-Post Moral Hazard: Efficiency and Redistribution. Annales d'Économie et de Statistique 2006; No. 83/84 (July): 295–325. https://doi.org/10.2307/20079172.

[5] Barros P, Siciliani L. 2011. Public and Private Sector Interface. In: Handbook of Health Economics. Elsevier. Volume 2, 2011, Pages 927-1001 https://doi.org/10.1016/B978-0-444-53592-4.00015-3

[6] Boadway R, Leite-Monteiro M, Marchand M, Pestieau P. Social Insurance and Redistribution with Moral Hazard and Adverse Selection. The Scandinavian Journal of Economics 2006;108 (2): 279–98. https://doi.org/10.1111/j.1467-9442.2006.00446.x

[7] Boone J. Basic versus Supplementary Health Insurance: Moral Hazard and Adverse Selection. Journal of Public Economics 2015;128 (August): 50–58. https://doi.org/10.1016/j.jpubeco.2015.05.009.

[8] Boone J. Basic versus Supplementary Health Insurance: access to care and the role of cost effectiveness. Journal of Health Economics 2018;60: 53–74. https://doi.org/10.1016/j.jhealeco.2018.05.002.

[9] Buchmueller TC, Couffinhal A, Grignon M, Perronnin M. Access to Physician Services: Does Supplemental Insurance Matter? Evidence from France. Health Economics 2004;13 (7): 669–87. https://doi.org/10.1002/hec.879.

[10] Cremer H, Pestieau P. Redistributive Taxation and Social Insurance. International Tax and Public Finance 1996;3 (3): 281–95. https://doi.org/10.1007/BF00418945.

[11] Feldstein M. Tax Incidence in a Growing Economy with Variable Factor Supply. Harvard Institute of Economic Research Discussion Paper no. 263, 1972.

[12] Henriet D, Rochet JC. Is Public Health Insurance an Appropriate Instrument for Redistribution? Annales d'Économie et de Statistique 2006;No 83/84: 61-88. https://doi.org/10.2307/20079163.

[13] Jones AM, Koolman X, van Doorslaer E. The Impact of Having Supplementary Private Health Insurance on the Use of Specialists. Annales d'Économie et de Statistique 2006;No 83/84: 251-75. https://doi.org/10.2307/20079170.

[14] Keane M, Stavrunova O. Adverse selection, moral hazard and the demand for Medigap insurance. Journal of Econometrics 2016;190 (1): 62-78. https://doi.org/10.1016/j.jeconom.2015.08.002.

[15] Manning WG, Newhouse JP, Duan N, Keeler EB, Leibowitz A. Health Insurance and the Demand for Medical Care: Evidence from a Randomized Experiment. The American Economic Review 1987;77 (3): 251-77.

[16] Manning WG, Marquis MS. Health Insurance: The Tradeoff between Risk Pooling and Moral Hazard. Journal of Health Economics 1996;15 (5): 609–39. https://doi.org/10.1016/S0167-6296(96)00497-3.

[17] Marquis MS, Phelps CE. Demand for Supplementary Health Insurance. Santa Monica, Calif.: RAND Corporation, R-3285-HHS, 1985. As of December 12, 2018: https://www.rand.org/pubs/reports/R3285.html

[18] Marquis MS, Phelps CE. Price Elasticity and Adverse Selection in the Demand for Supplementary Health Insurance. Economic Inquiry 1987;25 (2): 299–313. https://doi.org/10.1111/j.1465-7295.1987.tb00741.x.

[19] McGuire TG. Chapter Five - Demand for Health Insurance, 2011. In: Pauly M, McGuire TG, Barros PP, Handbook of Health Economics. Vol. 2 of Handbook of Health Economics. Elsevier, pp. 317-396.

[20] Mossialos E, Thomson SMS. Voluntary health insurance in the European Union. World Health Organization. Regional Office for Europe European Observatory on Health Systems and Policies. Copenhagen : WHO Regional Office for Europe 2004. http://www.who.int/iris/handle/10665/107601

[21] Nyman JA. The economics of moral hazard revisited. Journal of Health Economics 1999;18 (6): 811-24. https://doi.org/10.1016/S0167-6296(99)00015-6.

[22] Paris V, Devaux M, Wei L. Health Systems Institutional Characteristics: A Survey of 29 OECD Countries. OECD, Directorate for Employment, Labour and Social Affairs, OECD Health Working Papers 2010;50. https://doi.org/10.1787/5kmfxfq9qbnr-en.

[23] Paris V, Hewlett E, Auraaen A, Alexa J, Simon L. Health Care Coverage in OECD Countries in 2012. OECD Health Working Papers 2016;88. https://doi.org/10.1787/5jlz3kbf7pzv-en.

[24] Pauly, MV. The Economics of Moral Hazard: Comment. The American Economic Review 1968;58 (3): 531-37. https://doi.org/10.2307/1813785.

[25] Pestieau P. 2003. Social Insurance and Redistribution. In: Alternatives for Welfare Policy, edited by Torben M. Andersen and Per Molander, 238–54. Cambridge: Cambridge University Press, 2003. https://doi.org/10.1017/CBO9780511493508.011.

[26] Petretto A. Optimal social health insurance with supplementary private insurance. Journal of Health Economics 1999;18 (6): 727-45. https://doi.org/10.1016/S0167-6296(99)00017-X.

[27] Rochet JC. Incentives, Redistribution and Social Insurance.The Geneva Papers on Risk and Insurance Theory 1991;16 (2): 143-65. https://doi.org/10.1007/BF02386304.

[28] Salanié B.The Economics of Taxation. The MIT Press, Cambridge, MA, second edition, November 2011. ISBN 978-0-262-01634-6.

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Type of insurance system	OECD Countries
Supplementary insurance only	Australia, Canada, the Netherlands, Sweden and Switzerland
Supplementary and complementary insurance	Austria, Belgium, Denmark, Finland, France, Germany, Korea, Luxembourg and Portugal
Duplicative systems	Australia, Greece, Ireland, Italy, Mexico, New Zealand, Poland, Portugal, Spain and the United Kingdom
No private health insurance	Czech Republic, Hungary, Iceland, Norway, Slovak Republic and Turkey

Table 1: A classification of OECD countries according to their private insurance systems.

Appendix

Appendix 1: Analysis of health expenditure according to the insurance systems

A classification of countries according to the insurance systems

In addition to complementary and supplementary systems, mixed systems can take a duplicative form, also known as opting out. In this setting, individuals can decide to renounce socially covered insurance and consume a private alternative. Private insurance covers the costs of alternative goods that are outside of the social contract, but individuals must pay for social insurance regardless.

As presented in Table 1, according to the most recent data from the OECD (Paris et al., 2016), there are nine countries with supplementary and complementary insurance; five countries with supplementary insurance only; ten countries with duplicative private insurances; six countries with no private health insurance. There are no countries where complementary insurance exists without the presence of supplementary insurance.

Among the fourteen countries where supplementary insurance exists, covering cost-sharing expenses through a complementary contract is allowed in nine countries only.

Two example of complex topping up systems are France and the United States. In France, most health goods, including psychotropic medications, are covered in part by social insurance. Apart from a two euro co-pay per prescription that cannot be re-insured by private insurance companies, the remaining costs may be covered by a complementary insurance contract. On the other hand, other goods such as psychotherapy sessions or ostheopathy are not covered by social insurance and can be privately insured with a supplementary contract. Contracts often combine both types of insurance and enrolees are not aware of this distinction. In France 95% of the populations have a complementary contract.

Complementary systems are present and developing in other countries but, as in the United States, only affect part of the population. For instance, Medicare patients can be covered by complementary and supplementary contracts. Since 1996, the Medicare plan offers social insurance to Americans over 65 who have contributed to its funding. A variety of contracts known as Medigap, offer complementary insurance to cover part of the co-payments left by Medicare. Until 2006, Medicare did not cover any costs related to drugs. Pensioners who wished to insure against this risk could buy specific Medigap contracts which offered this supplementary component.

Health expenditure and outcomes according to the insurance systems

Table 2 presents statistics of health expenditure and health outcomes in the OECD according to private insurance systems. These descriptive statistics do not account for other factors that can influence these outcomes (GDP, environmental factors, how health providers are contracted or regulated...). These statistics reveal a lack of significant difference in life expectancy of the total population across mixed systems of health insurance and an unclear effect on total cost. Overall, public and total health expenditure are higher in countries with supplementary insurance. Indicators of health consumption and life expectancy are also higher. Intuitively, complementary insurance should reduce out-of-pocket levels but increase moral hazard. The evidence shows that indeed, out-of-pocket levels are lower in countries with complementary insurance than in topping up countries with supplementary insurance only (respectively \$46 and \$612 a year and per capita). There also seems to be evidence of extra-consumption: the number of Magnetic Resonance Imaging and Computed Tomography Scans, and the average length of stay in hospital are consistently higher. More surprisingly, the overall level of private funding in countries with both supplementary and complementary insurance is lower than in countries with supplementary insurance only. The level of social insurance is higher in countries with complementary insurance. The annual growth rate of public spending is lower in countries with complementary insurance, suggesting a greater possibility of transferring social spending onto complementary insurance.

Types of private insurance	Suppl & compl		Suppl only		Duplicative		No private ins	
	N	Mean (SD)	N	Mean (SD)	N	Mean (SD)	N	Mean (SD)
Spending indicators								
Total health expenditure (in % of GDP)	9	9.9% (1.6%)	5	10.5% (1.1%)	10	8.7% (1.2)	6	7.8% (1.3)
Public expenditure on health (in % of THE*)	9	75.8% (9.7)	5	72.0% (9.6)	10	70.5% (11.1)	6	76.6% (7.8)
Out-of-pocket payments (in % of THE*)	9	16.9% (8.9)	5	17.1% (8.8)	9	21.4% (12.2)	6	19.2% (5.3)
Private insurance expenditure on health (in % of THE*)	9	5.1% (3.9)	5	7.1% (4.6)	9	4.0% (2.8)	6	0.2% (0.4)
Total expenditure on health per capita (in US$ ppp*)	9	3,999 (1002)	5	4,776(850)	10	2,799 (950)	6	2,774 (1843)
Public expenditure on health per capita (in US$ ppp*)	9	3,102 (1,008)	5	3,405 (516)	10	2,042 (837)	6	2,202 (1,621)
Out-of-pocket payments on health per capita (in US$ ppp*)	9	612 (201)	5	846 (591)	9	520 (132)	6	493 (258)
Private insurance EH per capita (in US$ ppp*)	9	213 (183)	5	354 (235)	9	129 (116)	6	4 (8)
Annual growth rate of THE (in real terms)	9	3.1% (2.2)	5	3.8% (0.7)	10	3.6% (1.6)	6	4.1% (2.1)
Annual growth rate of public EH (in real terms)	9	3.1% (2.7)	5	4.5% (1.5)	10	3.7% (1.7)	6	3.8% (2.06)
Consumption indicators								
Diagnostic exams, Magnetic Resonance Imaging (per 1 000)	9	59.9 (26.3)	3	43.23 (15.0)	8	33.1 (22.7)	5	62.3 (33.9)
Diagnostic exams, Computed Tomography (per 1 000)	9	136.3 (49.6)	3	101.4 (29.3)	8	89.7 (49.9)	5	119.3 (36.6)
Average length of stay, all causes (in days)	9	8.4 (3.5)	5	6.6 (1.3)	10	6.4 (1.2)	6	6.7 (2.4)
Average length of stay single spontaneous delivery (in days)	9	3.3 (0.7)	5	2.4 (0.7)	10	2.5 (0.9)	6	3.4 (1.6)
Life expectancy, Total population at birth (in years)	9	80.9 (0.6)	5	81.8 (0.6)	10	80.2 (2.6)	6	78.1 (3.4)

*ppp: purchasing power parity; THE: total health expenditure

Table 2: Descriptive statistics of health expenditure in the OECD according to private insurance systems (Source: Paris et al., 2010).

Appendix 2: Proofs

First Order Conditions

The budget constraints can be written consumption equal to resources:

$c_i^h = w_i(1-t)l_i^h + M - \pi_i^{cpl} x_i - \pi_i^{spl} z_i$ when individuals are healthy and

$c_i^s + q_i^{cpl} p x_i + q_i^{spl} r z_i = w_i(1-t)l_i^s + M - \pi_i^{cpl} x_i - \pi_i^{spl} z_i$ when they are sick.

The optimal level of labour, consumption of health and non-health goods are defined thanks to a classical maximisation program. We propose to resolve that program using a Lagrangian; the maximisation programs can be presented as follows:

In the healthy state:

$$Max\, u(c_i^h, 0, 0, l_i^h)$$
$$\text{st } c_i^h = w_i(1-t)l_i^h + M - \pi_i^{cpl} x_i - \pi_i^{spl} z_i$$
$$L = u(c_i^h, 0, 0, l_i^h) + \mu(c_i^h - w_i(1-t)l_i^h - M + \pi_i^{cpl} x_i + \pi_i^{spl} z_i)$$

In the sick state:

$$Max\, u(c_i^s, x_i, z_i, l_i^s)$$
$$st\ c_i^s + q_i^{cpl} p x_i + q_i^{spl} r z_i = w_i(1-t)l_i^s + M - \pi_i^{cpl} x_i - \pi_i^{spl} z_i$$
$$L = u(c_i^s, x_i, z_i, l_i^s) + \lambda(c_i^s + q_i^{cpl} p x_i + q_i^{spl} r z_i - w_i(1-t)l_i^s - M + \pi_i^{cpl} x_i + \pi_i^{spl} z_i)$$

First Order Conditions in case of health:

FOC 1: $\frac{\partial L}{\partial c_i^h} = 0 \quad \Leftrightarrow \quad \frac{\partial u}{\partial c_i^h} + \mu = 0 \quad \Leftrightarrow \quad \mu = -\frac{\partial u}{\partial c_i^h}$

FOC 2: $\frac{\partial L}{\partial l_i^h} = 0 \quad \Leftrightarrow \quad \frac{\partial u}{\partial l_i^h} - \mu w_i(1-t) = 0$

We deduce relation (1) by substituting FOC 1 into FOC 2.

First Order Conditions in case of sickness:

FOC 1: $\frac{\partial L}{\partial c_i^s} = 0 \quad \Leftrightarrow \quad \frac{\partial u}{\partial c_i^s} + \lambda = 0 \quad \Leftrightarrow \quad \lambda = -\frac{\partial u}{\partial c_i^s}$

FOC 2: $\frac{\partial L}{\partial l_i^s} = 0 \quad \Leftrightarrow \quad \frac{\partial u}{\partial l_i^s} - \lambda w_i(1-t) = 0$

At this stage social health insurance structure is known and private insurance has already been purchased: $\frac{\partial \pi_i^{cpl}}{\partial x_i} = \frac{\partial \pi_i^{spl}}{\partial z_i} = 0$.

FOC 3: $\frac{\partial L}{\partial x_i} = 0 \quad \Leftrightarrow \quad \frac{\partial u}{\partial x_i} + \lambda p q_i^{cpl} = 0$

FOC 4: $\frac{\partial L}{\partial z_i} = 0 \quad \Leftrightarrow \quad \frac{\partial u}{\partial z_i} + \lambda r q_i^{spl} = 0$

Thanks to the first two order conditions, we deduce relation (1) and (2). Thanks to FOC 1 and 3, we deduce relation (3). Thanks to FOC 1 and 4, we deduce relation (4).

Lemma 1

According to the implicit function theorem, if $F(x(\alpha), \alpha) = 0$ then, $\frac{\partial x}{\partial \alpha} = -\frac{\partial F(x(\alpha),\alpha)/\partial \alpha}{\partial F(x(\alpha),\alpha)/\partial x}$.

The optimality conditions (equations (1) to (4)) give us five relations equal to zero and several results of static comparative are derived.

$F^1(x_i(\alpha), \alpha) = \frac{\partial u}{\partial x_i} - p(1-\alpha)(1-\beta_i)\frac{\partial u}{\partial c_i^s}$ therefore, $\frac{\partial x_i}{\partial \alpha} = -\frac{\partial F^1(x_i(\alpha),\alpha)/\partial \alpha}{\partial F^1(x_i(\alpha),\alpha)/\partial x_i} = -\frac{\frac{\partial^2 u^s}{\partial x_i \partial \alpha} + p(1-\beta_i)\rho^s}{\frac{\partial^2 u}{\partial {x_i}^2}} > 0.$

$F^1(x_i(\beta_i), \beta_i) = \frac{\partial u}{\partial x_i} - p(1-\alpha)(1-\beta_i)\frac{\partial u}{\partial c_i^s}$ therefore,

$\frac{\partial x_i}{\partial \beta_i} = -\frac{\partial F^1(x_i(\beta_i),\beta_i)/d\beta_i}{\partial F^1(x_i(\beta_i),\beta_i)/dx_i} = -\frac{\frac{\partial^2 u}{\partial x_i d\beta_i} + p(1-\alpha)\rho^s}{\frac{\partial^2 u}{\partial {x_i}^2}} > 0.$

Consequently, $\frac{\partial x_i/\partial\alpha}{\partial x_i/\partial\beta_i} = \frac{\frac{\partial^2 u}{\partial x_i \partial\alpha}+p(1-\beta_i)\rho^s}{\frac{\partial^2 u}{\partial x_i \partial\beta_i}+p(1-\alpha)\rho^s}$, which gives us:

$\sigma_{\alpha\beta} = \frac{\frac{\partial x}{\partial\alpha}/\frac{\partial x}{\partial\beta_i}}{\alpha/\beta_i} = \frac{\beta_i}{\alpha}\frac{\frac{\partial^2 u}{\partial x_i \partial\alpha}+p(1-\beta_i)\rho^s}{\frac{\partial^2 u}{\partial x_i \partial\beta_i}+p(1-\alpha)\rho^s}$ the elasticities between the social insurance rate and private complementary insurance rate.

$F^2(z_i(\gamma_i),\gamma_i) = \frac{\partial u}{\partial z_i} - r(1-\gamma_i)\frac{\partial u}{\partial c_i^s}$ therefore,

$$\frac{\partial z_i}{\partial\gamma_i} = -\frac{\partial F^2(z_i(\gamma_i),\gamma_i)/\partial\gamma_i}{\partial F^2(z_i(\gamma_i),\gamma_i)/\partial z_i} = -\frac{\frac{\partial^2 u}{\partial z_i \partial\gamma_i}+r\rho^s}{\frac{\partial^2 u}{\partial {z_i}^2}} > 0.$$

$F^3(c_i^s(t),t) = \frac{\partial u}{\partial c_i^s}w_i(1-t)+\frac{\partial u}{\partial l_i^s}$ therefore,

$$\frac{\partial c_i^s}{\partial t} = -\frac{\partial F^3(c_i^s(t),t)/\partial t}{\partial F^3(c_i^s(t),t)/\partial c_i^s} = \frac{w_i\rho^s - \frac{\partial^2 u}{\partial l_i^s \partial t}}{\frac{\partial^2 u}{\partial {c_i^s}^2}} < 0.$$

Lemma 2

Lemma 2 presents the impact of private insurance rates on out-of-pocket expenses, premiums and health consumption.

Relations (i) to (iv) are derived using simple derivation of $q_i^{cpl}(\beta,\gamma)$, $q_i^{spl}(\beta,\gamma)$, $\pi_i^{cpl}(\beta,\gamma)$, $\pi_i^{spl}(\beta,\gamma)$, $x_i(\beta, q_i^{cpl}(\beta,\gamma), q_i^{spl}(\beta,\gamma))$ and $z_i(\beta, q_i^{cpl}(\beta,\gamma), q_i^{spl}(\beta,\gamma))$ by β_i and γ_i; with: $\frac{\partial x_i}{\partial\beta_i} = \frac{\partial x_i}{\partial q_i^{cpl}}\frac{\partial q_i^{cpl}}{\partial\beta_i} + \frac{\partial x_i}{\partial q_i^{spl}}\frac{\partial q_i^{spl}}{\partial\beta_i}$ and $\frac{\partial x_i}{\partial\gamma_i} = \frac{\partial x_i}{\partial q_i^{cpl}}\frac{\partial q_i^{cpl}}{\partial\gamma_i} + \frac{\partial x_i}{\partial q_i^{spl}}\frac{\partial q_i^{spl}}{\partial\gamma_i}$.

Lemma 3

The optimality condition demands

$$\frac{\partial v}{\partial\beta_i} = \phi_i\left[\frac{\partial u}{\partial c_i^s}\frac{\partial c_i^s}{\partial\beta_i} + \frac{\partial u}{\partial l_i^s}\frac{\partial l_i^s}{\partial\beta_i} + \frac{\partial u}{\partial x_i}\frac{\partial x_i}{\partial\beta_i} + \frac{\partial u}{\partial z_i}\frac{\partial z_i}{\partial\beta_i}\right] + (1-\phi_i)\left[\frac{\partial u}{\partial c_i^h}\frac{\partial c_i^h}{\partial\beta_i} + \frac{\partial u}{\partial l_i^h}\frac{\partial l_i^h}{\partial\beta_i}\right] = 0$$

$$\frac{\partial v}{\partial\beta_i} = \phi_i\frac{\partial u}{\partial c_i^s}\left[\frac{\partial c_i^s}{\partial l_i^s}\frac{\partial l_i^s}{\partial\beta_i} + \frac{\partial c_i^s}{\partial\pi_i^{cpl}}\frac{\partial\pi_i^{cpl}}{\partial\beta_i} + \frac{\partial c_i^s}{\partial\pi_i^{spl}}\frac{\partial\pi_i^{spl}}{\partial\beta_i} + \frac{\partial c_i^s}{\partial x_i}\frac{\partial x_i}{\partial\beta_i} + \frac{\partial c_i^s}{\partial z_i}\frac{\partial z_i}{\partial\beta_i} + \frac{\partial c_i^s}{\partial q_i^{cpl}}\frac{\partial q_i^{cpl}}{\partial\beta_i} + \frac{\partial c_i^s}{\partial q_i^{spl}}\frac{\partial q_i^{spl}}{\partial\beta_i}\right] + \phi_i\frac{\partial u}{\partial l_i^s}\frac{\partial l_i^s}{\partial\beta_i} + \phi_i\frac{\partial u}{\partial x_i}\frac{\partial x_i}{\partial\beta_i} + \phi_i\frac{\partial u}{\partial z_i}\frac{\partial z_i}{\partial\beta_i} + (1-\phi_i)\frac{\partial u}{\partial c_i^h}\left[\frac{\partial c_i^h}{\partial l_i^h}\frac{\partial l_i^h}{\partial\beta_i} + \frac{\partial c_i^h}{\partial\pi_i^{cpl}}\frac{\partial\pi_i^{cpl}}{\partial\beta_i} + \frac{\partial c_i^h}{\partial\pi_i^{spl}}\frac{\partial\pi_i^{spl}}{\partial\beta_i} + \frac{\partial c_i^h}{\partial x_i}\frac{\partial x_i}{\partial\beta_i} + \frac{\partial c_i^h}{\partial z_i}\frac{\partial z_i}{\partial\beta_i}\right] + (1-\phi_i)\frac{\partial u}{\partial l_i^h}\frac{\partial l_i^h}{\partial\beta_i}$$

After simplifying using the FOCs presented in the equations (1) to (4), it follows:

$$\frac{\partial v}{\partial\beta_i} = -\phi_i\rho^s\phi_i(1-\alpha)px_i + \phi_i\rho^s(1-\alpha)px_i - \phi_i\rho^s\pi_i^{cpl}px_\beta - \phi_i\rho^s\pi_i^{spl}rz_\beta - (1-\phi_i)\rho^h\phi_i(1-\alpha)px_i - (1-\phi_i)\rho^h\pi_i^{cpl}px_i - (1-\phi_i)\rho^h\pi_i^{spl}rz_\beta$$

According to the optimality condition,

$$[\rho^s - \bar{\rho}](1-\alpha)p\bar{x}_i - \bar{\rho}\phi_i\gamma_i rz_\beta = 0$$

$$[\rho^s - \bar{\rho}]\phi_i px_i(1-\alpha) = \bar{\rho}\phi_i(1-\alpha)\beta_i px_\beta + \bar{\rho}\phi_i\gamma_i rz_\beta]$$

By definition, $Cov(\rho, px_i) = \rho^s\phi_i px_i - \bar{\rho}p\bar{x}_i = \rho^s\phi_i px_i - \bar{\rho}\phi_i px_i = (\rho^s - \bar{\rho})\phi_i px_i$ therefore:

$$Cov(\rho, px_i)(1-\alpha) = \bar{\rho}\phi_i(1-\alpha)\beta_i px_\beta + \bar{\rho}\phi_i\gamma_i rz_\beta$$

$$Cov(\rho, px_i) = \frac{1}{1-\alpha}\bar{\rho}\phi_i(1-\alpha)\beta_i px_\beta + \frac{1}{(1-\alpha)}\bar{\rho}\phi_i\gamma_i rz_\beta$$

$$Cov(\rho, px_i) = \bar{\rho}\phi_i\beta_i px_\beta + \bar{\rho}\phi_i\frac{\gamma_i}{(1-\alpha)}rz_\beta \quad (7)$$

Similarly, we find:

$$\frac{\partial v}{\partial\gamma_i} = -\phi_i\rho^s\left(\frac{\partial\pi_i^{spl}}{\partial\gamma_i} + \frac{\partial q_i^{spl}}{\partial\gamma_i}\right)rz_i - \phi_i\rho^s\pi_i^{spl}pz_\gamma - \phi_i\rho^s\pi_i^{cpl}px_\gamma$$

$-(1-\phi_i)\rho^h\left(\frac{\partial\pi_i^{spl}}{\partial\gamma_i}rz_i + \pi_i^{spl}rz_\gamma + \pi_i^{cpl}px_\gamma\right)$ and $Cov(\rho, rz_i) = \bar{\rho}\phi_i(1-\alpha)\beta_i px_\gamma + \bar{\rho}\phi_i\gamma_i rz_\gamma$ (8).

Lemma 4

Lemma 4 results from simple derivation.

(i) $\frac{\partial q_i^{cpl}}{\partial \alpha} = -1 + \beta_i - (1-\alpha)\beta_\alpha = -1 + \beta_i(1 + \frac{1-\alpha}{\alpha}\sigma_{\alpha\beta})$

(ii) $\frac{\partial \pi_i^{cpl}}{\partial \alpha} = \phi_i((1-\alpha)\beta_\alpha - \beta_i)p = -\phi_i\beta_i(1 + \frac{1-\alpha}{\alpha}\sigma_{\alpha\beta})p$

(iii) $\frac{\partial x_i}{\partial \alpha} = \frac{\partial x_i}{\partial q_i^{cpl}}\frac{\partial q_i^{cpl}}{\partial \alpha} = [\beta_i(1 + \frac{1-\alpha}{\alpha}\sigma_{\alpha\beta}) - 1]x_{q^{cpl}} = [\beta_i(1 + \frac{1-\alpha}{\alpha}\sigma_{\alpha\beta}) - 1]x_{q^{cpl}}$

(iv) $\pi_\alpha^{cpl} = -\phi_i(q_\alpha^{cpl} + 1)$

(v) $\frac{\partial q_i^{spl}}{\partial \alpha} = -\gamma_\alpha \frac{\gamma_i}{\alpha}\sigma_{\alpha\gamma}$

(vi) $\frac{\partial \pi_i^{spl}}{\partial \alpha} = \phi_i\gamma_\alpha r = -\phi_i\frac{\gamma_i}{\alpha}\sigma_{\alpha\gamma}r$

(vii) $\frac{\partial z_i}{\partial \alpha} = \frac{\partial z_i}{\partial q_i^{spl}}\frac{\partial q_i^{spl}}{\partial \alpha_i} = -\gamma_\alpha z_{q^{spl}} = \frac{\gamma_i}{\alpha}\sigma_{\alpha\gamma}z_{q^{spl}}$

(viii) $\pi_\alpha^{spl} = -\phi_i q_\alpha^{spl}$

Lemma 5

Maximising the objective function

$\Psi = \sum_{i=1}^n v_i - \mu[\alpha\sum_{i=1}^n \phi_i p x_i - t\sum_{i=1}^n w_i\bar{l_i} - nM]$ according to α gives us: $\frac{\partial \Psi}{\partial \alpha} = 0$.

$\sum_{i=1}^n v_\alpha = \mu\sum_{i=1}^n \phi_i p x_i + \mu\alpha\sum_{i=1}^n \phi_i p x_\alpha$ (9)

With, $\frac{\partial v}{\partial \alpha} = \frac{\partial v}{\partial \alpha} + \frac{\partial v}{\partial \beta}\frac{\partial \beta}{\partial \alpha} + \frac{\partial v}{\partial \gamma}\frac{\partial \gamma}{\partial \alpha} = \phi_i\frac{\partial u}{\partial \alpha} + (1-\phi_i)\frac{\partial u}{\partial \alpha}$
(at the optimum $\frac{\partial v}{\partial \beta_i} = \frac{\partial v}{\partial \gamma_i} = 0$).

$$\frac{\partial v}{\partial \alpha} = \phi_i\frac{\partial u}{\partial c_i^s}\left[\frac{\partial c_i^s}{\partial l_i^s}\frac{\partial l_i^s}{\partial \alpha} + \frac{\partial c_i^s}{\partial \pi_i^{cpl}}\frac{\partial \pi_i^{cpl}}{\partial \alpha} + \frac{\partial c_i^s}{\partial \pi_i^{spl}}\frac{\partial \pi_i^{spl}}{\partial \alpha} + \frac{\partial c_i^s}{\partial x_i}\frac{\partial x_i}{\partial \alpha} + \frac{\partial c_i^s}{\partial z_i}\frac{\partial z_i}{\partial \alpha} + \frac{\partial c_i^s}{\partial q_i^{cpl}}\frac{\partial q_i^{cpl}}{\partial \alpha} + \frac{\partial c_i^s}{\partial q_i^{spl}}\frac{\partial q_i^{spl}}{\partial \alpha}\right] + \phi_i\frac{\partial u}{\partial l_i^s}\frac{\partial l_i^s}{\partial \alpha}$$
$$+\phi_i\frac{\partial u}{\partial x_i}\frac{\partial x_i}{\partial \alpha} + \phi_i\frac{\partial u}{\partial z_i}\frac{\partial z_i}{\partial \alpha} + (1-\phi_i)\frac{\partial u}{\partial c_i^h}\left[\frac{\partial c_i^h}{\partial l_i^h}\frac{dl_i^h}{d\alpha} + \frac{\partial c_i^h}{\partial \pi_i^{cpl}}\frac{\partial \pi_i^{cpl}}{\partial \alpha} + \frac{\partial c_i^h}{\partial \pi_i^{spl}}\frac{\partial \pi_i^{spl}}{\partial \alpha} + \frac{\partial c_i^h}{\partial x_i}\frac{\partial x_i}{\partial \alpha} + \frac{\partial c_i^h}{\partial z_i}\frac{\partial z_i}{\partial \alpha}\right] + (1-\phi_i)\frac{\partial u}{\partial l_i^h}\frac{\partial l_i^h}{\partial \alpha}$$

Using the results obtained in Lemma 2:

$$\frac{\partial v}{\partial \alpha} = -\phi_i\rho^s\left[px_i\frac{\partial \pi_i^{cpl}}{\partial \alpha} + rz_i\frac{\partial \pi_i^{spl}}{\partial \alpha} + \pi_i^{cpl}p\frac{\partial x_i}{\partial \alpha} + \pi_i^{spl}r\frac{\partial z_i}{\partial \alpha} + px_i\frac{\partial q_i^{cpl}}{\partial \alpha} + rz_i\frac{\partial q_i^{spl}}{\partial \alpha}\right]$$
$$-(1-\phi_i)\rho^h\left[\frac{\partial \pi_i^{cpl}}{\partial \alpha}px_i + \frac{\partial \pi_i^{spl}}{\partial \alpha}rz_i + p\frac{\partial x_i}{\partial \alpha}\pi_i^{cpl} + r\frac{\partial z_i}{\partial \alpha}\pi_i^{spl}\right]$$
$$\frac{\partial v}{\partial \alpha} = -\bar{\rho}\left(px_i\frac{\partial \pi_i^{cpl}}{\partial \alpha} + \frac{\partial \pi_i^{spl}}{\partial \alpha}rz_i\right) - \phi_i\rho^s p\left(x_i\frac{\partial q_i^{cpl}}{\partial \alpha} + rz_i\frac{\partial q_i^{spl}}{\partial \alpha}\right) - \bar{\rho}\pi_i^{cpl}px_\alpha - \bar{\rho}\pi_i^{spl}rz_\alpha$$

By factorizing thanks to Lemma 4 (viii) we get:

$$\frac{\partial v}{\partial \alpha} = \bar{\rho}px_i\phi_i(1 + q_\alpha^{cpl}) + \bar{\rho}\phi_i q_\alpha^{spl}rz_i - \phi_i\rho^s(px_i\frac{\partial q_i^{cpl}}{\partial \alpha} + rz_i\frac{\partial q_i^{spl}}{\partial \alpha}) - \bar{\rho}\pi_i^{cpl}px_\alpha - \bar{\rho}\pi_i^{spl}rz_\alpha$$
$$\frac{\partial v}{\partial \alpha} = \bar{\rho}p\bar{x}_i + (\bar{\rho} - \rho^s)q_\alpha^{cpl}p\bar{x}_i + (\bar{\rho} - \rho^s)q_\alpha^{spl}r\bar{z}_i - \bar{\rho}\pi_i^{cpl}px_\alpha - \bar{\rho}\pi_i^{spl}rz_\alpha \quad (10)$$

By introducing equation (10) in equation (9) and rearranging the terms we find:

$$-\sum_{i=1}^n \bar{\rho}\pi_i^{cpl}px_\alpha - \sum_{i=1}^n \bar{\rho}\pi_i^{spl}rz_\alpha - \sum_{i=1}^n \bar{\rho}\pi_\alpha^{cpl}px_i - \sum_{i=1}^n \phi_i\rho^s q_\alpha^{cpl}px_i = \mu p\sum_{i=1}^n \phi_i x_i + \mu\alpha\sum_{i=1}^n \phi_i p x_\alpha$$

By dividing by μ and adopting the following notation $b^s = \frac{\rho^s}{\mu}$; $b^h = \frac{\rho^h}{\mu}$ and
$\bar{b} = \phi b^s + (1-\phi)b^h = \frac{\phi\rho^s + (1-\phi)\rho^h}{\mu} = \frac{\bar{\rho}}{\mu}$ we get:

$$-\sum_{i=1}^n \bar{b}\pi_\alpha^{cpl}px_i - \sum_{i=1}^n \phi_i b^s q_\alpha^{cpl}px_i - np\bar{x} = \sum_{i=1}^n \alpha\phi_i p x_{i\alpha} + \bar{b}\pi_i^{cpl}px_{i\alpha} + \sum_{i=1}^n \bar{b}\pi_i^{spl}rz_\alpha$$

Which can also be written: $\sum_{i=1}^n Cov(\bar{b}; q_\alpha^{cpl}p\bar{x}_i) + \sum_{i=1}^n Cov(\bar{b}; q_\alpha^{spl}r\bar{z}_i) - n\left(1 - \bar{b} - \bar{b}Cov\left(\frac{b_i}{\bar{b}}, \frac{x_i}{\bar{x}}\right)\right)p\bar{x} +$
$n\left(\bar{b} + \bar{b}Cov\left(\frac{b_i}{\bar{b}}, \frac{z_i}{\bar{z}}\right)\right)r\bar{z} = \sum_{i=1}^n \alpha\phi_i p x_\alpha + \sum_{i=1}^n \bar{b}\pi_i^{cpl}px_\alpha + \sum_{i=1}^n \bar{b}\pi_i^{spl}rz_\alpha$

Lemma 6

The necessary condition for optimal rates t and M presented in Lemma 6 are solutions to the equations: $\frac{d\Psi}{dt} = 0$ and $\frac{d\Psi}{dM} = 0$.

v_i is the individuals indirect function solution of maximisation program

$$v_i(\alpha,\, \beta_i^*(\alpha,\, t,\, M),\, \gamma_i^*(\alpha,\, t,\, M),\, t,\, M) = max\, u(c_i, x_i, z_i, l_i)$$

According to the simplified Feldstein framework there is no income effect on health care demand

$$x_M = z_M = 0.$$

The individual's budget constraint in both states is $y_i^j = w_i(1-t)l_i^j + M - \pi_i^{cpl}x_i - \pi_i^{spl}z_i$.

Therefore, the expenditure function can be written:

$$e_i(u_i, t, M) = w_i(1-t)l_i^{j*} + M - \pi_i^{cpl}x_i^* - \pi_i^{spl}z_i^*.$$

$$\frac{\partial v}{\partial M} = v_M + v_\beta \beta_M + v_\gamma \gamma_M$$

$$\frac{\partial v}{\partial M} = v_M - (\rho^s - \bar{\rho})q_\beta^{cpl}p\bar{x_i}\beta_M - \bar{\rho}(\pi_i^{cpl}x_\beta + \pi_i^{spl}z_\beta)\beta_M - (\rho^s - \bar{\rho})q_\gamma^{spl}r\bar{z_i}\gamma_M - \bar{\rho}(\pi_i^{cpl}x_\gamma + \pi_i^{spl}z_\gamma)\gamma_M$$

$$\frac{\partial v}{\partial M} = v_M - Cov(\rho,\, q_\beta^{cpl}p\bar{x_i}\beta_M + q_\gamma^{cpl}r\bar{z_i}\gamma_M)$$

According to the envelope theorem:

$$\frac{\partial \Psi}{\partial M} = \frac{\partial}{\partial M}\sum_{i=1}^n v_i - \mu\frac{\partial}{\partial M}[\alpha\sum_{i=1}^n \phi_i p x_i - t\sum_{i=1}^n w_i l_i + nM] = 0$$

$$\sum_{i=1}^n \frac{\partial v}{\partial M} = \mu\sum_{i=1}^n \frac{\partial}{\partial M}[\alpha\phi_i p x_i - t w_i l_i + M]$$

$$\sum_{i=1}^n \frac{v_M}{\mu} - \frac{Cov(\rho,\, q_\beta^{cpl}p\bar{x_i}\beta_M + q_\gamma^{cpl}r\bar{z_i}\gamma_M)}{\mu} = \sum_{i=1}^n[-tw_i l_M + 1]$$

$$\sum_{i=1}^n[\frac{v_M}{\mu} + tw_i l_M] - Cov(b,\, q_\beta^{cpl}p\bar{x_i}\beta_M + q_\gamma^{cpl}r\bar{z_i}\gamma_M) = n$$

According to Roy's identity, $v_M = -\frac{\partial u_i}{\partial y_i}\frac{\partial e_i}{\partial M} = -\frac{\partial u_i}{\partial y_i}$.

By definition, the net social marginal utility of income is equal to the social MUI net from the extra tax caused by the extra income. It can be written $a_i = \frac{\partial u}{\partial y_i}\frac{1}{\mu} + tw_i l_M$ with, $\bar{a} = \frac{1}{n}\sum_{i=1}^n a_i$. This gives us:

$$\frac{1}{n}\sum_{i=1}^n[-\frac{\partial u}{\partial y_i}\frac{1}{\mu} + tw_i l_M] - \frac{1}{n}Cov(b,\, q_\beta^{cpl}p\bar{x_i}\beta_M + q_\gamma^{spl}r\bar{z_i}\gamma_M) = 1$$

$$\bar{a} - \frac{1}{n}Cov(b,\, q_\beta^{cpl}p\bar{x_i}\beta_M + q_\gamma^{cpl}r\bar{z_i}\gamma_M) = 1 \quad (12)$$

$$\frac{\partial v}{\partial t} = v_t + v_\beta\beta_t + v_\gamma\gamma_t$$

According to the simplified Feldstein framework there is no income effect on health care demand

$$x_t = z_t = 0.$$

$$\frac{\partial v}{\partial t} = v_t - (\rho^s - \bar{\rho})q_\beta^{cpl}p\bar{x_i}\beta_t - \bar{\rho}(\pi_i^{cpl}x_\beta + \pi_i^{spl}z_\beta)\beta_t - (\rho^s - \bar{\rho})q_\gamma^{cpl}r\bar{z_i}\gamma_t - \bar{\rho}(\pi_i^{cpl}x_\gamma + \pi_i^{spl}z_\gamma)\gamma_t$$

$$\frac{\partial v}{\partial t} = v_t - (\rho^s - \bar{\rho})[q_\beta^{cpl}p\bar{x_i}\beta_t + q_\gamma^{spl}r\bar{z_i}\gamma_t]$$

$$\frac{\partial v}{\partial t} = v_t - Cov(\rho_i,\, q_\beta^{cpl}p\bar{x_i}\beta_t + q_\gamma^{spl}r\bar{z_i}\gamma_t)$$

The Slutsky equation allows to distinguish the effect of the change in net wage rate ω_i into two components. The first component is the compensated term which shows the substitution of the quantity of labour when the net wage rate changes. It is also known as the Slutsky term, noted S . The second term is the income effect. According to the Slutsky equation: $\frac{\partial l_i}{\partial \omega_i} = (\frac{\partial l_i}{\partial \omega_i})_{\bar{u}} + l_i\frac{\partial l_i}{\partial I_i} = S + l_i\frac{\partial l_i}{\partial I_i}$.

Therefore the effect of taxation on labour supply is $\frac{\partial l_i}{\partial t_i} = (S + l_i\frac{\partial l_i}{\partial I_i})(-w_i) + \frac{\partial l_i}{\partial I_i}\frac{\partial I_i}{\partial t_i} = -w_i(S + l_i l_M)$.

We obtain $S = \epsilon_{ll}\frac{l_i}{w_i(1-t)}$ by rearranging the elasticity of labour supply:

$$\epsilon_{ll} = \frac{\partial l_i}{\partial w_i(1-t)}\frac{w_i(1-t)}{l_i} = S\frac{w_i(1-t)}{l_i}.$$

By definition, the covariance between the expected social MUI and the expected gross labour income across individuals is equal to:

$\theta = Cov\left(\frac{a_i}{\bar{a}}, \frac{w_i l_i}{\bar{wl}}\right) = E\left(\frac{a_i}{\bar{a}}\frac{w_i l_i}{\bar{wl}}\right) - E\left(\frac{a_i}{\bar{a}}\right)E\left(\frac{w_i l_i}{\bar{wl}}\right) = \frac{1}{n}\sum_{i=1}^{n}\frac{a_i}{\bar{a}}\frac{w_i l_i}{\bar{wl}} - 1.$

Therefore, $n\bar{a}\bar{wl}\theta + n\bar{a}\bar{wl} = \sum_{i=1}^{n} a_i w_i l_i$

$$\frac{\partial \Psi}{\partial t} = \frac{\partial}{\partial t}\sum_{i=1}^{n} v_i + \mu\sum_{i=1}^{n}\frac{\partial}{\partial t}[\alpha\phi_i p x_i - t w_i \bar{l_i} + M] = 0$$

$$\sum_{i=1}^{n}\frac{\partial v}{\partial t} = -\mu\sum_{i=1}^{n}[\alpha\phi_i p x_t + w_i l_i + t w_i \frac{dl}{dt}]$$
$$\sum_{i=1}^{n} v_t - Cov(\rho_i,\, q_\beta^{cpl} p\bar{x}_i\beta_t + q_\gamma^{spl} r\bar{z}_i\gamma_t) = -\mu\sum_{i=1}^{n}[\alpha\phi_i p0 + w_i l_i + t w_i l_t]$$
$$\sum_{i=1}^{n}\frac{v_t}{\mu} - Cov(b,\, q_\beta^{cpl} p\bar{x}_i\beta_t + q_\gamma^{cpl} r\bar{z}_i\gamma_t) = -\sum_{i=1}^{n}\overline{w_i l_i}[1 - t w_i(\frac{S}{l_i} + l_M)]$$

According to Roy's identity, $v_t = -\frac{\partial u_i}{\partial y_i}\frac{\partial e_i}{\partial t} = -w_i l_i \frac{\partial u_i}{\partial y_i}$.

$$-\sum_{i=1}^{n}[\frac{\partial u}{\partial y}\frac{1}{\mu}\overline{w_i l_i} + t w_i l_M \overline{w_i l_i}] - Cov(b,\, q_\beta^{cpl} p\bar{x}_i\beta_t + q_\gamma^{cpl} r\bar{z}_i\gamma_t) = -n\overline{wl} + \sum_{i=1}^{n}\overline{w_i l_i} t w_i \frac{1}{l_i}\epsilon_{ll}\frac{l_i}{w_i(1-t)}]$$
$$-\sum_{i=1}^{n} a_i\overline{w_i l_i} - Cov(b,\, q_\beta^{cpl} p\bar{x}_i\beta_t + q_\gamma^{cpl} r\bar{z}_i\gamma_t) = -n\overline{wl} + \frac{t}{1-t}\sum_{i=1}^{n}\overline{w_i l_i}\epsilon_{ll}$$
$$-n\bar{a}\bar{wl}Cov\left(\frac{a_i}{\bar{a}}, \frac{w_i l_i}{\bar{wl}}\right) - n\bar{a}\bar{wl} + n\overline{wl} - Cov(b,\, q_\beta^{cpl} p\bar{x}_i\beta_t + q_\gamma^{cpl} r\bar{z}_i\gamma_t) = \frac{t}{1-t}\sum_{i=1}^{n}\overline{w_i l_i}\epsilon_{ll}$$
$$\frac{t}{1-t} = -\frac{\sum_{i=1}^{n} Cov(b,\, q_\beta^{cpl} p\bar{x}_i\beta_t + q_\gamma^{spl} r\bar{z}_i\gamma_t) + n\overline{wl}(\bar{a}Cov\left(\frac{a_i}{\bar{a}}, \frac{w_i l_i}{\bar{wl}}\right) + \bar{a} - 1)}{\sum_{i=1}^{n}\overline{w_i l_i}\epsilon_{ll}} \quad (13)$$