Revisiting the AA-DD model in Zero Lower Bound

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Abstract

This paper proposes a simple model of a mechanism through which exchange rate can affect the link between output and government spending in zero lower bound (ZLB) periods. In our proposed model, the expected near-future interest rate is added as an endogenous variable. Unlike existing AA-DD models in ZLB, the nominal exchange rate is no longer constant. Our model predicts that the output effect of an increase in government spending in a ZLB period is deflected by an appreciation of the current exchange rate. The AA-DD model is taught in almost all economic departments. The model is also generally used by many central banks and governments. The existing AA-DD model can be misleading. Our new AA-DD model may help to update the existing model in ZLB periods. Our AA-DD model is also consistent with recent dynamic stochastic general equilibrium models in open economies in ZLB periods.

Keywords: Zero lower bound; The exchange rate; Government Spending.

JEL classification: E4, E62, F41.

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1 Introduction

The AA-DD model of Krugman, Obstfeld, & Melitz (2012, page 453) summarises three markets: the foreign exchange market, the money market, and the market of goods and services. The main aim of the AA-DD model is to analyse governments' and central banks' policies using two simple curves. The first curve (named the AA curve), which represents the asset market equilibria, summarises the money market and the foreign exchange markets. The second curve (the DD curve) represents the goods market equilibria. At the intersection of the two curves, each of the three markets is in equilibrium (Krugman, Obstfeld, & Melitz, 2012, page 453).

In existing AA-DD models, the ZLB interest rate implies a fixed exchange rate (Krugman et al., 2012, page 453).[1] The fixed exchange rate leads to a larger output effect of an increase in government spending compared to that in normal periods. Thus, existing AA-DD models are not consistent with predictions provided by recent dynamic stochastic general equilibrium (DSGE) models in open economies. For example, using a DSGE model in open economies, Mao Takongmo (2017) shows that increases in government spending during the ZLB period increase aggregate demand, which leads to an appreciation of the real exchange rate greater than that in normal periods. The appreciation of the real exchange rate then deflates the effect of government spending on total production. The AA-DD model is widely taught in fourth years in economic departments of almost all universities, in part because the intuitions that drive the results are simple and understandable. The AA-DD model is also used by many central banks and governments. It is therefore important that the model provide correct predictions. During the 2007 financial crisis and the recession that followed, the nominal interest rate reached a lower bound and remained at a very low level for a long period of time (called the zero lower bound [ZLB] period). During the ZLB period, central banks lost their conventional monetary policy, which consisted of lowering the nominal interest

[1] The interest parity condition that summarizes the relationship between the output and the nominal exchange rate, when both the money market and the foreign exchange market are in equilibrium, implies a fixed exchange rate during ZLB periods.

rate to increase output. As a result, governments of many countries, including the United States, started to increase government spending to boost output.[2] Using US data, Boskin (2012) provides empirical evidence that the government spending policy did not work in the United States. Boskin observes that the increase in debt exceeded the improvement in total production during the ZLB period. It is important to note that, before observing data, many policymakers usually base their decisions on the widely taught AA-DD model. Existing AA-DD model predictions can be misleading and may not be consistent with the data. Moreover, the AA-DD model predictions are not consistent with the recent DSGE literature in ZLB in open economies. It is therefore very important to revisit the model in ZLB periods.

In ZLB periods, agents usually take into account the expected interest rate in the near future when making their decisions. In this paper, we proposed a theoretical model to allow the expected interest rate in the near future to be endogenous. Our proposed model no longer implies a fixed exchange rate and predicts that the output effect of an increase in government spending is lower than that provided by the existing AA-DD model because of an appreciation in the exchange rate. This prediction is consistent with the recent DSGE literature in open economies in ZLB periods (see Mao Takongmo, 2017).

The remainder of the paper is organized as follows. Section 2 presents the model and the resulting government spending multiplier [GSM] in ZLB periods. The classical model, the classical GSM, and an analytical comparison between our GSM and the classical GSM in ZLB are presented in section 3. Section 4 focuses on a graphical comparison between the two results. The final section concludes the paper.

[2] See, for example, the American Recovery and Reinvestment Act (ARRA), which was designed to increase US government spending by $831 billion between 2009 and 2019. Another example is the European Economic Recovery Plan (EERP), meant to increase European government spending by €200 billion between 2008 and 2010.

2 The new model in ZLB

2.1 *The new foreign exchange market*

Interaction between buyers and sellers of foreign currency bank deposits is assumed to determine the exchange rate in the foreign exchange market. There are two countries: our country, which uses the U.S. dollar (\$), and the rest of the world, which uses the euro (€). There are two periods: the current ZLB period, and the medium period. The interest rate in the current period, $\overline{r}_{ZLB}$, is assumed to be exogenous and fixed. The interest rate in the medium period , r^e_M, is endogenous. The nominal interest rates in the current period and in the medium period abroad, denoted by $\overline{r}^*_{ZLB}$ and r^{e*} , respectively, are all exogenous. The nominal exchange rate, $E_{\$/€}$, is defined as the price of one euro in term of U.S. dollars. The exchange rate in the current period and in the next period are denoted respectively by E_{ZLB}, and E^e_M. The expected exchange rate is assumed to be exogenous.

Definition. 1 The foreign exchange market is said to be in equilibrium when deposits in U.S. dollar and deposits in euro, at the beginning of the first period, offer the same expected value at the end of the second period. The condition for equilibrium in the foreign exchange market is called the new interest parity condition in ZLB periods.

Proposition 1. *The new interest rate parity condition is*

$$r^e_M = \frac{(1+\overline{r}^*_{ZLB})(1+r^{e*})E^e_{M(\$/€)}}{(1+\overline{r}_{ZLB})E_{ZLB(\$/€)}} - 1 \tag{1}$$

Proof. The expected value for 1\$ deposit, in U.S. dollar, is $1 \times (1+\overline{r}_{ZLB})(1+r^e_M)$ \$. The expected U.S. dollar value of 1\$ deposit in euro is $\frac{E^e_{M(\$/€)}}{E_{ZLB(\$/€)}}(1+\overline{r}^*_{ZLB})(1+r^{e*})$. The foreign exchange market is in equilibrium if $\frac{E^e_{M(\$/€)}}{E_{ZLB(\$/€)}}(1+\overline{r}^*_{ZLB})(1+r^{e*}) = (1+\overline{r}_{ZLB})(1+r^e_M)$. Thus the new interest parity condition is $r^e_M = \frac{(1+\overline{r}^*_{ZLB})(1+r^{e*})E^e_{M(\$/€)}}{(1+\overline{r}_{ZLB})E_{ZLB(\$/€)}} - 1$. □

2.2 *The new equilibrium in the money market*

Other things being equal, people prefer assets that offer higher expected returns. Since the current interest rate is fixed at its ZLB value, the expected return increases when the expected interest rate increases. Since the increases in the expected interest rate represent a rise in the expected rate of return of less liquid assets relative to the rate of return on money, agents will want to hold more of their wealth in non-money assets that pay the expected interest rate and less of their wealth in the form of money, if the expected interest rate rises. Thus, a rise in the expected interest rate, r^e_M, causes the demand for money, L, to fall.

We also assume that, agents hold money to avoid cost of barter trade (Hicks, 1937; Mundell, 1963; Baumol, 1952; Rogoff, 1985, for details). The demand for money, L, is then assumed to be an increasing function of output, Y. Let M^s represent the aggregate real money supply, and P the price level. Equilibrium in the money market is achieved when the aggregate real money supply, $\frac{M^s}{P}$, is equal to the aggregate real money demand. That is

$$\frac{M^s}{P} = L(\overline{r}_{ZLB}, r^e_M, Y), \tag{2}$$

with the assumptions that $\frac{\partial L}{\partial r} < 0$, $\frac{\partial L}{\partial r^e_M} < 0$ and $\frac{\partial L}{\partial Y} > 0$.

2.3 *Equilibrium in the good market*

The aggregate demand is a sum of consumption demand (C), investment demand (I), government spending demand (G), and net export demand (NX). The consumption demand is assumed to be an increasing function of disposable income, $Y^d = Y - T$. That is, $\frac{\partial C}{\partial Y^d} > 0$. We assume that the net export is a function of the real exchange rate.[3] We assume that a depreciation of the domestic currency will lead to an increases of the net export ($\frac{\partial NX}{\partial \mathcal{E}} > 0$).[4]

[3]In fact, the net export can also be a function of many other variables, such as the national and foreign disposable income; since our focus in this paper is on the role played by the exchange rate, for simplicity, we assume that the impact of other factors on the net export is negligible.

[4]Note that $\frac{\partial NX}{\partial \mathcal{E}} > 0$ is equivalent to $\frac{\partial NX}{\partial E} > 0$ because in the short run P is fixed by definition, and P^* is exogenous.

Government spending (G), and taxes (T), are assumed exogenous. For simplicity, investment (I), is assumed to be a function of the current ZLB interest rate, and is therefore fixed. By definition, equilibrium is attained when the aggregated output is equal to the aggregate demand for goods and services. That is

$$Y = C(Y - T) + I(\overline{r}_{ZLB}) + G + NX\left(\frac{P^*}{P}E\right) \tag{3}$$

where P^* represent the price index abroad. Assumption A summarises the assumptions presented in this section

Assumptions A

1. $\frac{\partial L}{\partial r} < 0$; $\frac{\partial L}{\partial r^e_M} < 0$ and $\frac{\partial L}{\partial Y} > 0$.
2. $\frac{\partial NX}{\partial E} > 0$ $\frac{\partial C}{\partial Y^d} > 0$

Definition. 2 The new government spending multiplier in ZLB (New GSM_{ZLB}), is defined as the changes in the aggregated output, Y, generated by a change in one unit of government spending, when the new interest rate parity condition holds, and the market of money, as well as the market of goods and services, are both in equilibrium.

Proposition 2. *The new government spending multiplier in zero lower bound is equal to*

$$\textit{New } GSM_{ZLB} = \frac{1}{1 - \frac{dC}{dY} - \frac{dNX}{dE_{ZLB}}\left(\frac{\left(\frac{\partial L}{\partial Y}\right)(1+\overline{r}_{ZLB})E^2_{ZLB}}{\left(\frac{\partial L}{\partial r^e_M}\right)(1+\overline{r}^*_{ZLB})(1+r^{e*})E^e_{M(\$/€)}}\right)}$$

Proof. At the equilibrium of all markets, we have, $\frac{M^s}{P} = L(\overline{r}_{ZLB}, r^e_M, Y)$, $r^e_M = \frac{(1+\overline{r}^*_{ZLB})(1+r^{e*})E^e_{M(\$/€)}}{(1+\overline{r}_{ZLB})E_{ZLB(\$/€)}} - 1$ and $Y = C(Y-T) + I(\overline{r}_{ZLB}) + G + NX\left(\frac{P^*}{P}E\right)$.

The first two equations lead to

$$\frac{M^s}{P} = L(\overline{r}_{ZLB}, \left[\frac{(1+\overline{r}^*_{ZLB})(1+r^{e*})E^e_{M(\$/€)}}{(1+\overline{r}_{ZLB})E_{ZLB(\$/€)}} - 1\right], Y). \tag{4}$$

Applying the total derivative in both side of the equation 4 lead to $\frac{dM^s}{P} = dL = \frac{\partial L}{\partial r}d\overline{r}_{ZLB} + \frac{\partial L}{\partial r^e_M}dr^e_M + \frac{\partial L}{\partial Y}dY$. Since $dM^s = 0$, and $d\overline{r}_{ZLB} = 0$, we have

$$0 = \frac{\partial L}{\partial r^e_M}dr^e_M + \frac{\partial L}{\partial Y}dY. \tag{5}$$

$dr^e_M = -\frac{(1+\overline{r}^*_{ZLB})(1+r^{e*})E^e_{M(\$/€)}}{(1+\overline{r}_{ZLB})E^2_{ZLB}}dE_{ZLB}$. By replacing dr^e_M in equation 5 we have

$$\frac{dE_{ZLB}}{dY} = \frac{\frac{\partial L}{\partial Y}}{\left(\frac{\partial L}{\partial r^e_M}\right)\frac{(1+\overline{r}^*_{ZLB})(1+r^{e*})E^e_{M(\$/€)}}{(1+\overline{r}_{ZLB})E^2_{ZLB}}}. \tag{6}$$

We also have

$$Y = C(Y-T) + I(\overline{r}_{ZLB}) + G + NX\left(\frac{P^*}{P}E\right). \tag{7}$$

Applying the total derivative in both side of the equation 7 lead to $dY = dC + dI + dG + dNX$. Since $dI = 0$,

$$dY = dC + dG + dNX. \tag{8}$$

Note that $dNX = \frac{dNX}{dE_{ZLB}}dE_{ZLB}$. Thus,

$$\begin{aligned} dY &= dC + dG + \frac{dNX}{dE_{ZLB}}dE_{ZLB} \\ 1 &= \frac{dC}{dY} + \frac{dG}{dY} + \frac{dNX}{dE_{ZLB}}\frac{dE_{ZLB}}{dY} \quad \text{(after dividing both side by dY)} \end{aligned} \tag{9}$$

Replacing $\frac{dE_{ZLB}}{dY}$ in 9 by its expression in 6 lead to

$$1 = \frac{dC}{dY} + \frac{dG}{dY} + \frac{dNX}{dE_{ZLB}} \left(\frac{\frac{\partial L}{\partial Y}}{\left(\frac{\partial L}{\partial r_M^e}\right) \frac{(1+\overline{r}_{ZLB}^*)(1+r^{e*})E_{M(\$/€)}^e}{(1+\overline{r}_{ZLB})E_{ZLB}^2}} \right).$$

Thus,

$$\frac{dG}{dY} = 1 - \frac{dC}{dY} - \frac{dNX}{dE_{ZLB}} \left(\frac{\frac{\partial L}{\partial Y}}{\left(\frac{\partial L}{\partial r_M^e}\right) \frac{(1+\overline{r}_{ZLB}^*)(1+r^{e*})E_{M(\$/€)}^e}{(1+\overline{r}_{ZLB})E_{ZLB}^2}} \right)$$

$$\frac{1}{\left(\frac{dY}{dG}\right)} = 1 - \frac{dC}{dY} - \frac{dNX}{dE_{ZLB}} \left(\frac{\frac{\partial L}{\partial Y}}{\left(\frac{\partial L}{\partial r_M^e}\right) \frac{(1+\overline{r}_{ZLB}^*)(1+r^{e*})E_{M(\$/€)}^e}{(1+\overline{r}_{ZLB})E_{ZLB}^2}} \right)$$

$$\begin{aligned} \frac{dY}{dG} &= \frac{1}{1 - \frac{dC}{dY} - \frac{dNX}{dE_{ZLB}} \left(\frac{\frac{\partial L}{\partial Y}}{\left(\frac{\partial L}{\partial r_M^e}\right) \frac{(1+\overline{r}_{ZLB}^*)(1+r^{e*})E_{M(\$/€)}^e}{(1+\overline{r}_{ZLB})E_{ZLB}^2}} \right)} \\ &= \frac{1}{1 - \frac{dC}{dY} - \frac{dNX}{dE_{ZLB}} \left(\frac{\left(\frac{\partial L}{\partial Y}\right)(1+\overline{r}_{ZLB})E_{ZLB}^2}{\left(\frac{\partial L}{\partial r_M^e}\right)(1+\overline{r}_{ZLB}^*)(1+r^{e*})E_{M(\$/€)}^e} \right)}. \end{aligned}$$

This concludes the proof. □

3 The classical government spending multiplier in ZLB

3.1 *The classical equilibrium in the money market in ZLB*

The classical equilibrium in the money market can be written without the expected interest rate as

$$\frac{M^s}{P} = L(\bar{r}_{ZLB}, Y). \tag{10}$$

3.2 *The classical interest rate parity in ZLB*

The classical interest rate parity is

$$\bar{r}_{ZLB} = (1 + \bar{r}^*_{ZLB}) \left(\frac{E^e_{M(\$/€)}}{E_{ZLB(\$/€)}} \right)^{1/2} - 1. \tag{11}$$

In fact, the return for a 1\$ deposit, in our country is $1 \times (1 + \bar{r}_{ZLB})(1 + \bar{r}_{ZLB})\,\$$. The return for a 1\$ deposit abroad is $\frac{E^e_{M(\$/€)}}{E_{ZLB(\$/€)}}(1 + \bar{r}^*_{ZLB})(1 + \bar{r}^*_{ZLB})$ in U.S. dollar unit. The classical interest rate parity is then

$$\frac{E^e_{M(\$/€)}}{E_{ZLB(\$/€)}}(1 + \bar{r}^*_{ZLB})^2 = (1 + \bar{r}_{ZLB})^2$$

or

$$\bar{r}_{ZLB} = (1 + \bar{r}^*_{ZLB}) \left(\frac{E^e_{M(\$/€)}}{E_{ZLB(\$/€)}} \right)^{1/2} - 1. \tag{12}$$

The interest rate parity in the classical case implies that the nominal exchange rate is fixed in ZLB periods, since the ZLB nominal interest rate is fixed.

3.3 *The classical equilibrium in the good market*

In the good market, equilibrium is the same as in the previous section and is

$$Y = C(Y - T) + I(\bar{r}_{ZLB}) + G + NX\left(\frac{P^*}{P}E\right) \tag{13}$$

Definition. 3 The classical government spending multiplier in ZLB (Classical GSM$_{ZLB}$), is defined as the changes in the aggregated output, Y, generated by a change in one unit of government spending, when the classical interest rate parity condition holds, the classical market of money and the classical market of goods and services are both in equilibrium.

Proposition 3. *The classical government spending multiplier is equal to*

$$\textit{Classical } GSM_{ZLB} = \frac{1}{1 - \frac{dC}{dY}}$$

Proof. The classical interest parity is

$$\overline{r}_{ZLB} = (1 + \overline{r}^*_{ZLB}) \left(\frac{E^e_{M(\$/€)}}{E_{ZLB(\$/€)}} \right)^{1/2} - 1$$

In the zero lower-bound period, since the interest rate is fixed, the exchange rate should be fixed. This means that equilibrium in the money market

$$\frac{M^s}{P} = L(\overline{r}_{ZLB}, Y)$$

will just guarantee the fixed interest rate. By taking the total differential in both sides of the equation 13, we have

$$\begin{aligned}
dY &= dC + dI(\overline{r}_{ZLB}) + dG + dNX\left(\frac{P^*}{P}E\right) = dC + dG \qquad \text{(Since exchange rate is fixed), thus} \\
1 &= \frac{dC}{dY} + \frac{dG}{dY} \qquad \text{(By dividing by dY)} \\
\frac{dG}{dY} &= 1 - \frac{dC}{dY}, \\
\frac{dY}{dG} &= \frac{1}{1 - \frac{dC}{dY}}.
\end{aligned}$$

This concludes the proof □

Proposition 4. *If assumption A holds, the new government spending multiplier ($NewGSM_{ZLB}$), will be lower than the classical government spending multiplier (*Classical *GSM_{ZLB}) in the zero lower bound period, with,*

$$New\ GSM_{ZLB}= \frac{1}{1-\frac{dC}{dY}-\frac{dNX}{dE_{ZLB}}\left(\frac{\left(\frac{\partial L}{\partial Y}\right)(1+\bar{r}_{ZLB})E^2_{ZLB}}{\left(\frac{\partial L}{\partial r^e_M}\right)(1+\bar{r}^*_{ZLB})(1+r^{e*})E^e_{M(\$/€)}}\right)}$$

and

$$Classical\ GSM_{ZLB}= \frac{1}{1-\frac{dC}{dY}}$$

Proof. By assumption A, $\frac{\partial L}{\partial r^e_M}<0$; $\frac{\partial L}{\partial Y}>0$; and $\frac{\partial NX}{\partial E}>0$.

Thus

$$\frac{dNX}{dE_{ZLB}}\left(\frac{\left(\frac{\partial L}{\partial Y}\right)(1+\bar{r}_{ZLB})E^2_{ZLB}}{\left(\frac{\partial L}{\partial r^e_M}\right)(1+\bar{r}^*_{ZLB})(1+r^{e*})E^e_{M(\$/€)}}\right)<0$$

and

$$1-\frac{dC}{dY}-\frac{dNX}{dE_{ZLB}}\left(\frac{\left(\frac{\partial L}{\partial Y}\right)(1+\bar{r}_{ZLB})E^2_{ZLB}}{\left(\frac{\partial L}{\partial r^e_M}\right)(1+\bar{r}^*_{ZLB})(1+r^{e*})E^e_{M(\$/€)}}\right)>1-\frac{dC}{dY}$$

and

$$\text{New GSM}_{ZLB}= \frac{1}{1-\frac{dC}{dY}-\frac{dNX}{dE_{ZLB}}\left(\frac{\left(\frac{\partial L}{\partial Y}\right)(1+\bar{r}_{ZLB})E^2_{ZLB}}{\left(\frac{\partial L}{\partial r^e_M}\right)(1+\bar{r}^*_{ZLB})(1+r^{e*})E^e_{M(\$/€)}}\right)}<\frac{1}{1-\frac{dC}{dY}}=\text{Classical GSM}_{ZLB}.$$

This concludes the proof. □

4 Graphical illustration: AA and DD schedule in zero lower bound

4.1 *The market of goods and services: the DD schedule*

The DD schedule is the relationship between exchange rates and output at which the output market is in equilibrium. In this paper the DD schedule is similar to the one proposed by Krugman, Obstfeld, & Melitz (2012, page 429). The equation representing the DD schedule is

$$Y = C(Y - T) + I(\overline{r}_{ZLB}) + G + NX\left(\frac{P^*}{P}E\right). \tag{14}$$

An increase of E is associated with and increases of NX and therefore an increases of Y: the DD curve is upward sloping.

4.2 *The asset market : the new AA schedule*

The AA schedule is defined as the relationship between exchange rates, E_{ZLB}, and output, Y, at which the market of money and the foreign exchange market are both in equilibrium.

The new equilibrium in the market of money is represented by equation (15), and the equilibrium in the foreign exchange market is represented by equation (16).

$$\frac{M^s}{P} = L(\overline{r}_{ZLB}, r^e_M, Y) \tag{15}$$

$$r^e_M = \frac{(1+\overline{r}^*_{ZLB})(1+r^{e*})E^e_M}{(1+\overline{r}_{ZLB})E_{ZLB}} - 1 \tag{16}$$

Proposition 5. *The new AA curve, in ZLB is downward sloping. The derivative of* E_{ZLB}

respect to Y can be written as

$$\frac{dE_{ZLB}}{dY} = \frac{\frac{\partial L}{\partial Y}}{\left(\frac{\partial L}{\partial r_M^e}\right)\frac{(1+\overline{r}^*_{ZLB})(1+r^{e*})E^e_M}{(1+\overline{r}_{ZLB})E^2_{ZLB}}}.$$

Proof. Replacing r_M^e from equation (16) in the equation (15) leads to

$$\frac{M^s}{P} = L(\overline{r}_{ZLB}, \left[\frac{(1+\overline{r}^*_{ZLB})(1+r^{e*})E^e_{M(\$/€)}}{(1+\overline{r}_{ZLB})E_{ZLB(\$/€)}} - 1\right], Y). \tag{17}$$

$$\frac{dM^s}{P} = dL = \frac{\partial L}{\partial r}d\overline{r}_{ZLB} + \frac{\partial L}{\partial r_M^e}dr_M^e + \frac{\partial L}{\partial Y}dY.$$

Money supply is fixed, thus, $dM^s = 0$. Interest rate is fixed, thus, $d\overline{r}_{ZLB} = 0$, and

$$0 = \frac{\partial L}{\partial r_M^e}dr_M^e + \frac{\partial L}{\partial Y}dY. \tag{18}$$

Applying the derivative of r_M^e respect to E_{ZLB} using equation (16) lead to

$$\frac{dr_M^e}{dE_{ZLB}} = -\frac{(1+\overline{r}^*_{ZLB})(1+r^{e*})E^e_{M(\$/€)}}{(1+\overline{r}_{ZLB})E^2_{ZLB}}$$

or

$$dr_M^e = -\frac{(1+\overline{r}^*_{ZLB})(1+r^{e*})E^e_{M(\$/€)}}{(1+\overline{r}_{ZLB})E^2_{ZLB}}dE_{ZLB}.$$

Equation 18 becomes $0 = -\frac{\partial L}{\partial r_M^e}\frac{(1+\overline{r}^*_{ZLB})(1+r^{e*})E^e_{M(\$/€)}}{(1+\overline{r}_{ZLB})E^2_{ZLB}}dE_{ZLB} + \frac{\partial L}{\partial Y}dY$

$$\frac{dE_{ZLB}}{dY} = \frac{\frac{\partial L}{\partial Y}}{\left(\frac{\partial L}{\partial r_M^e}\right)\frac{(1+\overline{r}^*_{ZLB})(1+r^{e*})E^e_M}{(1+\overline{r}_{ZLB})E^2_{ZLB}}}. \tag{19}$$

$\frac{\partial L}{\partial Y} > 0$ and $\frac{\partial L}{\partial r_M^e} < 0$ by assumption A. Additional to that, all other variables are positive. Equation (19) shows that $\frac{dE_{ZLB}}{dY} < 0$. Thus the new AA curve, in ZLB, is downward sloping.

□

4.3 *The classical asset market : the old AA schedule*

In the previous section we derived equation (20) and (21) that represent respectively the classical equilibrium in the money market and in the foreign exchange market. The old AA schedule is defined as the relationship between the exchange rate, E_{ZLB}, and output in which equations (20) and (21) both hold. Recall that in equation (20) money supply will just be adjusted in other to maintain the fixed interest rate. In other words, the central bank loosens its monetary policy, in the classical framework.

$$\frac{M^s}{P} = L(\overline{r}_{ZLB}, Y). \tag{20}$$

$$\overline{r}_{ZLB} = (1 + \overline{r}^*_{ZLB}) \left(\frac{E^e_{M(\$/€)}}{E_{ZLB(\$/€)}} \right)^{1/2} - 1 \tag{21}$$

The interest rate parity in the classical case (equation 21) implied that the nominal exchange rate is fixed in zero lower-bound periods. This is due to the fact that in the zero lower-bound period, the nominal interest rate is fixed. Recall that by assumption, the expected exchange rate is exogenous.

4.4 *Graphical illustration: the old vs the new government spending multiplier*

The classical model is represented in panel (a) and the new model is displayed in panel (b) of figure 1.[5] Each equilibrium is observed when the AA and the DD curves cross each other. The classical AA curve in ZLB is a horizontal line, while the new AA curve is downward-sloping. An increase in government spending shifts the DD schedule to the right by $\frac{dG}{1-MPC}$

[5]Note that increases in exchange rate is a depreciation of the national currency.

($MPC = \frac{dC}{dY}$). In the classical analysis, (panel a), the DD curve shifts from DD_1 to DD_2 and the equilibrium output moves from Y_1 to Y_2. $dY = Y_2 - Y_1 = \frac{dG}{1-MPC}$.

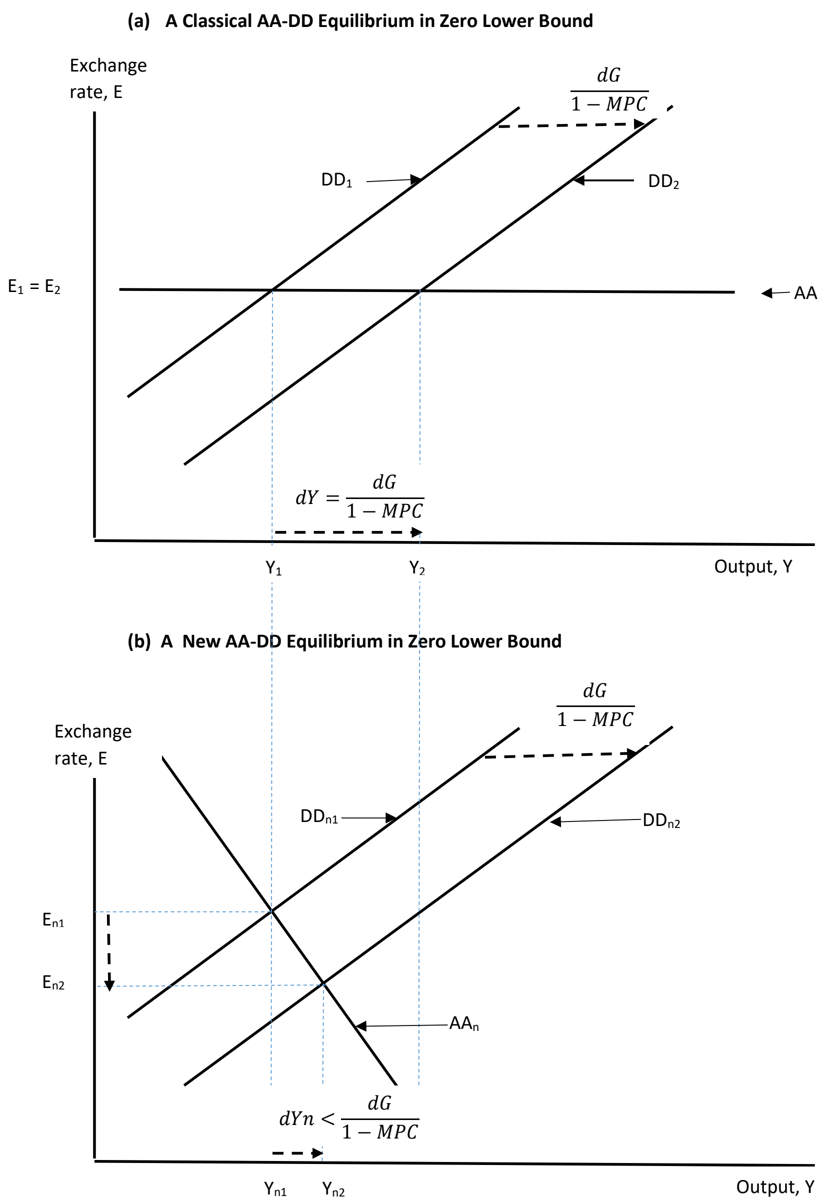
In the new model (panel b), the DD curve shifts from DD_{n1} to DD_{n2} and output moves from Y_{n1} to Y_{n2}. The increases in output are less than those observed in the classical analysis.

$$dY_n = Y_{n2} - Y_{n1} = \frac{dG}{1-\frac{dC}{dY}-\frac{dNX}{dE_{ZLB}}\left(\frac{\left(\frac{\partial L}{\partial Y}\right)(1+\overline{r}_{ZLB})E^2_{ZLB}}{\left(\frac{\partial L}{\partial r^e_M}\right)(1+\overline{r}^*_{ZLB})(1+r^{e*})E^e_{M(\$/€)}}\right)}$$

5 Conclusion

In ZLB periods, the existing AA-DD model proposed by Krugman, Obstfeld, & Melitz (2012) predicts a very large output effect of an increase in government spending compared to that in a normal period. We propose a simple model in which the expected near-future interest rate is endogenous. In our new model, the output effect of an increase in government spending in the ZLB period is deflected by an appreciation in the current exchange rate. The predictions of our new AA-DD model are consistent with recent DSGE literature in open economies in ZLB periods. The AA-DD model is widely taught in many universities. The AA-DD model is also used by many policymakers. Our new AA-DD model will help to update the existing AA-DD model in ZLB periods. Our new model will also help central bankers and governments when building their policies, especially when they do not have access to data.

Figure 1: Effect of Government spending on output in zero lower-bound: comparing the new effect with the classical effect



Note: The classical model is represented in panel (a) and the new model is displayed in panel (b). In panel (a), an increase in Government spending shifts the DD schedule to the right from DD_1 to DD_2 and equilibrium output moves from Y_1 to Y_2. In panel (b), the DD curve shifts from DD_{n1} to DD_{n2} and output moves from Y_{n1} to Y_{n2}, with $dY_n < dY$.

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