Effects of Patents on the Industrial Revolution

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Abstract

This study provides a growth-theoretic analysis on the effects of intellectual property rights on the endogenous takeoff of an economy. We incorporate patent protection into a Schumpeterian growth model in which takeoff occurs when the population size crosses an endogenous threshold. We find that strengthening patent protection has contrasting effects on economic growth at different stages of the economy. Specifically, it leads to an earlier takeoff (i.e., an earlier industrial revolution) but also reduces economic growth in the long run.

JEL classification: O31, O34
Keywords: intellectual property rights, endogenous takeoff, innovation, economic growth

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"England [...] by 1700 [...] had developed an efficient set of property rights embedded in the common law [and...] begun to protect private property in knowledge with its patent law. The stage was now set for the industrial revolution." North and Thomas (1973, p. 156)

1 Introduction

Figure 1 plots the real GDP per capita in the UK.[1] It shows that in the 18th century, income in the UK grew very slowly. Specifically, the average annual growth rate of income in the UK from 1701 to 1800 was 0.4%. Then, the average growth rate from 1801 to 1900 increased to 1.0%. From the 20th century onwards, the average growth rate stabilized at about 1.7%.

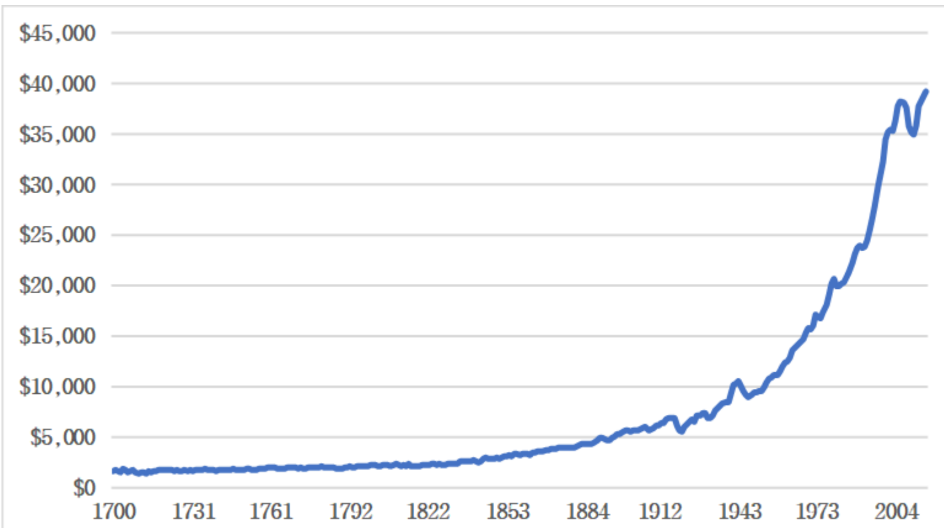

Figure 1: Real GDP per capita in the UK from 1700 to 2016

This study explores the effects of intellectual property rights (IPR) on the endogenous takeoff of an economy. We incorporate patent protection into the Schumpeterian growth model of endogenous takeoff in Peretto (2015). In this model, the economy first experiences stagnation with zero growth in output per capita when the market size (determined by the population size) is small. As the market size becomes sufficiently large due to population growth, innovation takes place and the economy gradually experiences growth. In the long run, the economy converges to a balanced growth path (BGP) with steady-state growth. Within this growth-theoretic framework that is consistent with the growth pattern in Figure 1, we obtain the following results.

Strengthening patent protection leads to an earlier takeoff. Incentives for innovation to take place depend on the market value of inventions, which in turn depends on the level of patent protection and the market size. Therefore, when stronger patent protection increases the market value of patents, it also reduces the market size required for innovation to take

[1] Data source: Maddison Project Database.

place. As a result, the economy starts to experience innovation and growth at an earlier time (i.e., an earlier industrial revolution). Our finding that stronger IPR protection leads to an earlier (but not necessarily immediate) takeoff is consistent with historical evidence on the effects of IPR on the industrial revolution.[2] However, stronger patent protection eventually reduces innovation and growth as recent studies tend to find.[3] Intuitively, although stronger patent protection increases the number of products in the economy, this larger number of products reduces the market size of each product and the incentives for quality-improving innovation, which determine long-run growth.[4]

This study relates to the literature on innovation and economic growth. Romer (1990) develops the seminal variety-expanding growth model in which innovation is driven by new products, whereas Aghion and Howitt (1992) develop the Schumpeterian quality-ladder growth model in which innovation is driven by higher-quality products. Peretto (1998, 1999) and Smulders and van de Klundert (1995) combine the two dimensions of innovation and develop a Schumpeterian growth model with endogenous market structure. This study explores the effects of IPR in this vintage of the Schumpeterian growth model.

In the literature on IPR and innovation, other studies also explore the effects of IPR in the innovation-driven growth model.[5] These studies mostly focus on either variety expansion or quality improvement. Only a few studies, such as Chu, Cozzi and Galli (2012) and Chu, Furukawa and Ji (2016), explore the effects of IPR in the Schumpeterian growth model with both dimensions of innovation. However, none of these studies consider how IPR affects the endogenous takeoff of an economy. The novel contributions of this study are to explore the effects of IPR in a Schumpeterian growth model of endogenous takeoff and to highlight the contrasting effects of IPR on economic growth at different stages of the economy.

This study also relates to the literature on endogenous takeoff and economic growth. Early studies include Galor and Weil (2000), Jones (2001) and Hansen and Prescott (2002). Peretto (2015) develops a Schumpeterian growth model of endogenous takeoff. The Peretto model features both quality improvement and variety expansion, under which endogenous growth in the number of products provides a dilution effect that removes the scale effect of population size on long-run growth. Therefore, although the population size affects the timing of the takeoff, it does not affect the steady-state growth rate. We incorporate patent protection into the Peretto model to explore its effects on endogenous takeoff.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 explores the effects of patent policy at different stages of the economy. Section 4 concludes.

2 The model

The theoretical framework is based on the Schumpeterian growth model with both variety-expanding innovation and quality-improving innovation in Peretto (2015). In this model,

[2] See e.g., North and Thomas (1973), North (1981), Dutton (1984) and Khan (2005).

[3] See Jaffe and Lerner (2004), Bessen and Meurer (2008) and Boldrin and Levine (2008) for evidence.

[4] See Peretto and Connolly (2007) for a theoretical explanation on quality-improving innovation being the only plausible engine of long-run growth.

[5] See e.g., Li (2001), Goh and Olivier (2002), O'Donoghue and Zweimuller (2004), Furukawa (2007), Chu (2009), Iwaisako and Futagami (2013), Cozzi and Galli (2014), Huang *et al.* (2017) and Yang (2018).

labor is used as a factor input for the production of final good. Final good is used for consumption and as a factor input for entry, in-house R&D, the production and operation of intermediate goods. We incorporate a patent policy parameter into the model and analyze its effects on the takeoff, transitional dynamics and the BGP of the economy.

2.1 Household

The representative household has a utility function given by

$$U = \int_0^\infty e^{-(\rho-\lambda)} \ln c_t dt, \tag{1}$$

where $c_t \equiv C_t/L_t$ denotes per capita consumption of final good (numeraire) at time t, and $\rho > 0$ is the subjective discount rate. Population grows at an exogenous rate $\lambda \in (0, \rho)$ with initial population normalized to unity (i.e., $L_t = e^{\lambda t}$). The household maximizes (1) subject to

$$\dot{a}_t = (r_t - \lambda)\, a_t + w_t - c_t, \tag{2}$$

where $a_t \equiv A_t/L_t$ is the real value of assets owned by each member of the household, and r_t is the real interest rate. Each member supplies one unit of labor to earn w_t. Standard dynamic optimization yields

$$\frac{\dot{c}_t}{c_t} = r_t - \rho. \tag{3}$$

2.2 Final good

Final output Y_t is produced by competitive firms using the following production function:

$$Y_t = \int_0^{N_t} X_t^\theta (i) \left[Z_t^\alpha (i)\, Z_t^{1-\alpha} L_t / N_t^{1-\sigma}\right]^{1-\theta} di, \tag{4}$$

where $\{\theta, \alpha, \sigma\} \in (0, 1)$. $X_t(i)$ is the quantity of non-durable intermediate goods $i \in [0, N_t]$. The productivity of $X_t(i)$ depends on its quality $Z_t(i)$ and the average quality of all intermediate goods $Z_t \equiv \int_0^{N_t} Z_t(j)\, dj/N_t$ capturing technology spillovers. The private return to quality is determined by α, and the degree of technology spillovers is determined by $1 - \alpha$. The parameter $1 - \sigma$ captures a congestion effect of variety, and hence, the social return to variety is measured by σ.

Profit maximization yields the following conditional demand functions for L_t and $X_t(i)$:

$$L_t = (1 - \theta)\, Y_t / w_t, \tag{5}$$

$$X_t(i) = \left(\frac{\theta}{p_t(i)}\right)^{1/(1-\theta)} Z_t^\alpha (i)\, Z_t^{1-\alpha} L_t / N_t^{1-\sigma}, \tag{6}$$

where $p_t(i)$ is the price of $X_t(i)$. Perfect competition implies that firms pay $\theta Y_t = \int_0^{N_t} p_t(i)\, X_t(i)\, di$ for intermediate goods.

2.3 Intermediate goods and in-house R&D

Monopolistic firms produce differentiated intermediate goods with a linear technology that requires $X_t(i)$ units of final good to produce $X_t(i)$ units of intermediate good $i \in [0, N_t]$. The firm in industry i incurs $\phi Z_t^{\alpha}(i) Z_t^{1-\alpha}$ units of final good as a fixed operating cost. To improve the quality of its products, the firm devotes $I_t(i)$ units of final good to in-house R&D. The innovation process is

$$\dot{Z}_t(i) = I_t(i), \tag{7}$$

and the firm's (before-R&D) profit flow at time t is

$$\Pi_t(i) = [p_t(i) - 1] X_t(i) - \phi Z_t^{\alpha}(i) Z_t^{1-\alpha}. \tag{8}$$

The value of the monopolistic firm in industry i is

$$V_t(i) = \int_t^{\infty} \exp\left(-\int_t^s r_u du\right) [\Pi_s(i) - I_s(i)] ds. \tag{9}$$

The monopolistic firm maximizes (9) subject to (7) and (8). We solve this dynamic optimization problem in the proof of Lemma 1 and find that the unconstrained profit-maximizing markup ratio is $1/\theta$. To analyze the effects of patent breadth, we introduce a policy parameter $\mu > 1$, which determines the unit cost for imitative firms to produce $X_t(i)$. A larger patent breadth μ allows the monopolistic producer of $X_t(i)$, who owns the patent, to charge a higher markup without losing her market share to potential imitators;[6] see also Li (2001), Goh and Olivier (2002) and Iwaisako and Futagami (2013). The equilibrium price becomes

$$p_t(i) = \min\{\mu, 1/\theta\}. \tag{10}$$

We assume that $\mu < 1/\theta$. In this case, increasing patent breadth raises the markup.

We follow previous studies to consider a symmetric equilibrium in which $Z_t(i) = Z_t$ for $i \in [0, N_t]$ and the size of each intermediate-good firm is identical across all industries $X_t(i) = X_t$.[7] From (6) and $p_t(i) = \mu$, the quality-adjusted firm size is

$$\frac{X_t}{Z_t} = \left(\frac{\theta}{\mu}\right)^{1/(1-\theta)} \frac{L_t}{N_t^{1-\sigma}}. \tag{11}$$

We define the following transformed variable:

$$x_t \equiv \mu^{1/(1-\theta)} \frac{X_t}{Z_t} = \theta^{1/(1-\theta)} \frac{L_t}{N_t^{1-\sigma}}, \tag{12}$$

which is a state variable and not directly affected by μ (but indirectly via N_t). In Lemma 1, we derive the rate of return on quality-improving R&D, which is increasing in x_t and μ.

[6] This setup is consistent with Gilbert and Shapiro's (1990) insight on patent "breadth as the ability of the patentee to raise price" and originates from the patent-design literature; e.g., Gallini (1992) also assumes that a larger patent breadth increases the imitation cost of imitators.

[7] Symmetry also implies $\Pi_t(i) = \Pi_t$, $I_t(i) = I_t$ and $V_t(i) = V_t$.

Lemma 1 *The rate of return on quality-improving in-house R&D is*[8]

$$r_t^q = \alpha \frac{\Pi_t}{Z_t} = \alpha \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} x_t - \phi \right]. \tag{13}$$

Proof. See the Appendix. ■

2.4 Entrants

Following previous studies, we assume that entrants have access to aggregate technology Z_t to ensure symmetric equilibrium at any time t. A new firm pays βX_t units of final good to enter the market with a new variety of intermediate goods and set up its operation. $\beta > 0$ is an entry-cost parameter. The asset-pricing equation implies that the return on assets is

$$r_t = \frac{\Pi_t - I_t}{V_t} + \frac{\dot{V}_t}{V_t}. \tag{14}$$

When entry is positive, free entry implies

$$V_t = \beta X_t. \tag{15}$$

Substituting (7), (8), (12), (15) and $p_t = \mu$ into (14) yields the return on entry as

$$r_t^e = \frac{\mu^{1/(1-\theta)}}{\beta} \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} - \frac{\phi + z_t}{x_t} \right] + \frac{\dot{x}_t}{x_t} + z_t, \tag{16}$$

where $z_t \equiv \dot{Z}_t / Z_t$ is the growth rate of aggregate quality.

2.5 Equilibrium

The equilibrium is a time path of allocations $\{A_t, Y_t, C_t, X_t, I_t\}$ and prices $\{r_t, w_t, p_t, V_t\}$ such that

- the household maximizes utility taking $\{r_t, w_t\}$ as given;
- competitive firms produce Y_t and maximize profits taking $\{w_t, p_t\}$ as given;
- incumbents for intermediate goods choose $\{p_t, I_t\}$ to maximize V_t taking r_t as given;
- entrants make entry decisions taking V_t as given;
- the value of all existing monopolistic firms adds up to the value of the household's assets such that $A_t = N_t V_t$; and
- the following market-clearing condition of final good holds:

$$Y_t = C_t + N_t \left(X_t + \phi Z_t + I_t \right) + \dot{N}_t \beta X_t. \tag{17}$$

[8] Note that $(\mu - 1)/\mu^{1/(1-\theta)}$ is increasing in μ for $\mu < 1/\theta$.

2.6 Aggregation

Substituting (6) and $p_t = \mu$ into (4) and imposing symmetry yield aggregate output as

$$Y_t = (\theta/\mu)^{\theta/(1-\theta)} N_t^{\sigma} Z_t L_t. \tag{18}$$

The growth rate of output per capita is

$$g_t \equiv \frac{\dot{y}_t}{y_t} = \sigma n_t + z_t, \tag{19}$$

where $y_t \equiv Y_t/L_t$ denotes output per capita. Its growth rate g_t is determined by both the variety growth rate $n_t \equiv \dot{N}_t/N_t$ and the quality growth rate z_t.

3 Dynamics of the economy

The dynamics of the economy is determined by the dynamics of $x_t = \theta^{1/(1-\theta)} L_t/N_t^{1-\sigma}$. Its initial value is $x_0 = \theta^{1/(1-\theta)}/N_0^{1-\sigma}$. In the first stage of the economy, there is neither variety expansion nor quality improvement. At this stage, x_t increases solely due to population growth. When x_t becomes sufficiently large, innovation begins to happen. The following inequality ensures the realistic case in which the creation of products (i.e., variety-expanding innovation) happens before the improvement of products (i.e., quality-improving innovation).

$$\alpha < \frac{\mu - 1 - (\rho - \lambda)\beta}{(\rho - \lambda)\beta\phi}\left[\rho - \lambda + \frac{(\theta/\mu)(\mu - 1 - (\rho - \lambda)\beta)}{1 - (\theta/\mu)(\mu - (\rho - \lambda)\beta)}\lambda\right]. \tag{20}$$

Variety-expanding innovation happens when x_t crosses the first threshold x_N defined as

$$x_N \equiv \frac{\mu^{1/(1-\theta)}\phi}{\mu - 1 - (\rho - \lambda)\beta}. \tag{21}$$

Then, quality-improving innovation also happens when x_t crosses the second threshold x_Z defined as

$$x_Z \equiv \underset{x}{\arg\,\text{solve}}\left\{\left[\frac{\mu - 1}{\mu^{1/(1-\theta)}}x - \phi\right]\left[\alpha - \frac{\mu^{1/(1-\theta)}\sigma}{\beta x}\right] = \rho - \sigma(\rho - \lambda)\right\}. \tag{22}$$

The inequality in (20) implies $x_N < x_Z$. In the long run, x_t converges to its steady-state value x^*. The following inequalities ensure that when the economy is on the BGP, the variables $\{x^*, z^*, g^*\}$ are positive:

$$\beta\phi > \frac{1}{\alpha}\left[\mu - 1 - \beta\left(\rho + \frac{\sigma}{1 - \sigma}\lambda\right)\right] > \mu - 1. \tag{23}$$

The following proposition adapted from Peretto (2015) summarizes the dynamics of x_t.

Proposition 1 *When the initial condition of the economy satisfies*[9]

$$\mu^{1/(1-\theta)}\phi/\left(\mu-1\right) < x_0 < x_N, \tag{24}$$

the dynamics of x_t is given by[10]

$$\dot{x}_t = \begin{cases} \lambda x_t > 0 & x_0 \leq x_t \leq x_N \\ \bar{v}\left(\bar{x}^* - x_t\right) > 0 & x_N < x_t \leq x_Z \\ v\left(x^* - x_t\right) \geq 0 & x_t > x_Z \end{cases}, \tag{25}$$

where

$$\bar{v} \equiv \frac{1-\sigma}{\beta}\left[\mu - 1 - \beta\left(\rho + \frac{\lambda\sigma}{1-\sigma}\right)\right],$$

$$\bar{x}^* \equiv \frac{\mu^{1/(1-\theta)}\phi}{\mu - 1 - \beta\left[\rho + \lambda\sigma/\left(1-\sigma\right)\right]},$$

$$v \equiv \frac{1-\sigma}{\beta}\left[\left(1-\alpha\right)\left(\mu-1\right) - \beta\left(\rho + \frac{\lambda\sigma}{1-\sigma}\right)\right],$$

$$x^* \equiv \mu^{1/(1-\theta)}\frac{\left(1-\alpha\right)\phi - \left[\rho + \lambda\sigma/\left(1-\sigma\right)\right]}{\left(1-\alpha\right)\left(\mu-1\right) - \beta\left[\rho + \lambda\sigma/\left(1-\sigma\right)\right]}.$$

Proof. See the Appendix. ■

3.1 Stage 1: Stagnation

When the market size is not large enough (i.e., $x_t \leq x_N$), there are insufficient incentives for firms to develop new products nor improve the quality of existing products. In this case, output per capita is

$$y_t = \left(\theta/\mu\right)^{\theta/(1-\theta)} N_0^{\sigma} Z_0, \tag{26}$$

and the growth rate of y_t is $g_t = 0$. In this regime, strengthening patent protection μ decreases y_t due to monopolistic distortion that reduces intermediate production X_t. However, stronger patent protection also leads to an earlier (but not necessarily immediate) takeoff by decreasing x_N in (21). Intuitively, stronger patent protection increases the profitability of firms and provides more incentives for firms to develop new products. As a result, the economy starts to experience innovation at an earlier time.

Proposition 2 *When $x_t \leq x_N$, stronger patent protection reduces the level of output per capita but leads to an earlier takeoff.*

Proof. Use (21) and (26) to show that x_N and y_t are decreasing in μ. Given that x_t increases at the exogenous rate λ when $x_t \leq x_N$, a smaller x_N implies an earlier takeoff. ■

[9] The inequality $x_0 > \mu^{1/(1-\theta)}\phi/\left(\mu-1\right)$ implies that $\Pi_0 > 0$.

[10] Note that (23) ensures $x_Z < \bar{x}^*$.

3.2 Stage 2: Variety expansion

When the market size is sufficiently large (i.e., $x_t > x_N$), firms have incentives to develop new products. In this case, output per capita is

$$y_t = (\theta/\mu)^{\theta/(1-\theta)} N_t^{\sigma} Z_0, \tag{27}$$

and the growth rate of y_t is $g_t = \sigma n_t$. In the Appendix, we show that whenever $n_t > 0$, c_t/y_t always jumps to a steady state. Therefore, we can substitute r_t^e in (16) into the Euler equation $r_t = \rho + g_t = \rho + \sigma n_t$ in (3) and also use (12) to derive the variety growth rate as[11]

$$n_t = \frac{\mu^{1/(1-\theta)}}{\beta}\left[\frac{\mu-1}{\mu^{1/(1-\theta)}} - \frac{\phi}{x_t}\right] - \rho + \lambda. \tag{28}$$

For a given level of x_t, the equilibrium growth rate $g_t = \sigma n_t$ is increasing in patent breadth μ as in previous studies, such as Li (2001) and O'Donoghue and Zweimuller (2004).

Proposition 3 *For a given $x_t \in (x_N, x_Z)$, stronger patent protection increases the equilibrium growth rate.*

Proof. Use (28) to show that $g_t = \sigma n_t$ is increasing in μ for a given x_t. ■

3.3 Stage 3: Quality improvement and variety expansion

When the market size becomes even larger (i.e., $x_t > x_Z$), firms have incentives to improve the quality of products in addition to inventing new products. Then, output per capita is

$$y_t = (\theta/\mu)^{\theta/(1-\theta)} N_t^{\sigma} Z_t, \tag{29}$$

and the growth rate of y_t is $g_t = \sigma n_t + z_t$. We can then substitute r_t^q in (13) into the Euler equation $r_t = \rho + g_t = \rho + \sigma n_t + z_t$ in (3) to derive the quality growth rate as[12]

$$z_t = \alpha\left[\frac{\mu-1}{\mu^{1/(1-\theta)}} x_t - \phi\right] - \rho - \sigma n_t. \tag{30}$$

For a given level of x_t, the equilibrium growth rate $g_t = \sigma n_t + z_t$ continues to be increasing in patent breadth μ that raises the return to quality-improving innovation.

Proposition 4 *For a given $x_t \in (x_Z, x^*)$, stronger patent protection increases the equilibrium growth rate.*

Proof. Use (30) to show that $g_t = \sigma n_t + z_t$ is increasing in μ for a given x_t. ■

[11] Note from (21) and (28) that $n_t > 0$ if and only if $x_t > x_N$.

[12] It can be shown that $z_t > 0$ if and only if $x_t > x_Z$.

3.4 Stage 4: Balanced growth path

In the long run, x_t converges to x^*. Then, the steady-state quality growth rate is

$$z^* = \alpha \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} x^* - \phi \right] - \rho - \sigma n^*, \tag{31}$$

where $n^* = \lambda/(1-\sigma) > 0$ and

$$x^* = \mu^{1/(1-\theta)} \frac{(1-\alpha)\phi - [\rho + \lambda\sigma/(1-\sigma)]}{(1-\alpha)(\mu-1) - \beta[\rho + \lambda\sigma/(1-\sigma)]}, \tag{32}$$

which is decreasing in μ. Intuitively, stronger patent protection increases the number of products, which leads to a smaller market size for each product. This smaller firm size x^* in turn reduces the incentives for quality-improving innovation and the steady-state equilibrium growth rate $g^* = \sigma n^* + z^*$. This result generalizes the one in Chu *et al.* (2016), who assume zero social return to variety (i.e., $\sigma = 0$).

Proposition 5 *On the BGP (i.e., $x_t = x^*$), stronger patent protection decreases the steady-state equilibrium growth rate.*

Proof. Use (31) and (32) to show that $g^* = \sigma n^* + z^*$ is decreasing in μ. ■

Finally, we conclude this section with Figure 2.[13] It summarizes the entire transition path of the equilibrium growth rate g_t and shows that strengthening patent protection leads to an earlier takeoff but lower long-run growth.

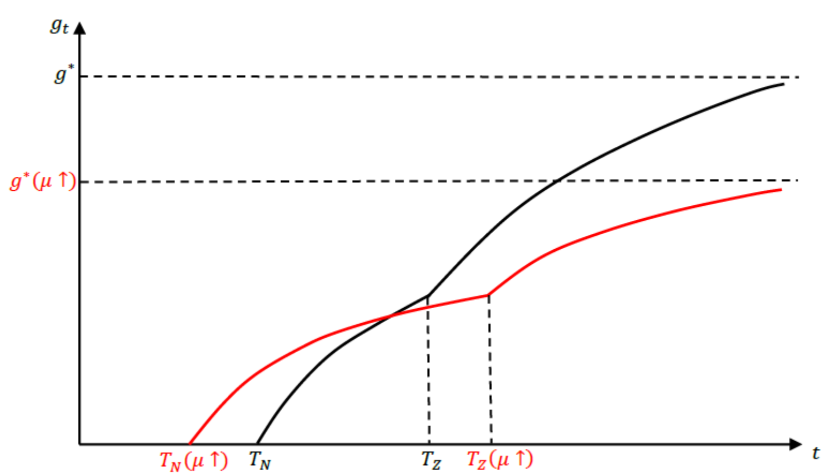

Figure 2: Transition path of the growth rate

[13] T_N (T_Z) is the time when variety-expanding (quality-improving) innovation is activated.

4 Conclusion

In this study, we analyze the effects of IPR in a Schumpeterian growth model with endogenous takeoff and find that strengthening patent protection causes an earlier takeoff by increasing the profitability of firms and providing more incentives for firms to innovate. However, stronger patent protection eventually slows down economic growth by reducing the market size of each product and the incentives for quality-improving innovation. These contrasting effects of IPR at different stages of the economy are consistent with historical evidence on the industrial revolution and recent evidence on the effects of the patent system.

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Appendix

Proof of Lemma 1. The current-value Hamiltonian for monopolistic firm i is

$$H_t(i) = \Pi_t(i) - I_t(i) + \eta_t(i)\dot{Z}_t(i) + \omega_t(i)[\mu - p_t(i)], \tag{A1}$$

where $\omega_t(i)$ is the multiplier on $p_t(i) \leq \mu$. Substituting (6)-(8) into (A1), we can derive

$$\frac{\partial H_t(i)}{\partial p_t(i)} = 0 \Rightarrow \frac{\partial \Pi_t(i)}{\partial p_t(i)} = \omega_t(i), \tag{A2}$$

$$\frac{\partial H_t(i)}{\partial I_t(i)} = 0 \Rightarrow \eta_t(i) = 1, \tag{A3}$$

$$\frac{H_t(i)}{\partial Z_t(i)} = \alpha \left\{ [p_t(i) - 1] \left[\frac{\theta}{p_t(i)} \right]^{1/(1-\theta)} \frac{L_t}{N_t^{1-\sigma}} - \phi \right\} Z_t^{\alpha-1}(i) Z_t^{1-\alpha} = r_t \eta_t(i) - \dot{\eta}_t(i). \tag{A4}$$

If $p_t(i) < \mu$, then $\omega_t(i) = 0$. In this case, $\partial\Pi_t(i)/\partial p_t(i) = 0$ yields $p_t(i) = 1/\theta$. If the constraint on $p_t(i)$ is binding, then $\omega_t(i) > 0$. In this case, we have $p_t(i) = \mu$. Therefore,

$$p_t(i) = \min\{\mu, 1/\theta\}. \tag{A5}$$

Given that we assume $\mu < 1/\theta$, the monopolistic firm sets its price at $p_t(i) = \mu$. Substituting (A3), (12) and $p_t(i) = \mu$ into (A4) and imposing symmetry yield

$$r_t^q = \alpha \frac{\Pi_t}{Z_t} = \alpha \left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} x_t - \phi \right], \tag{A6}$$

which is the rate of return on quality-improving in-house R&D. ■

Before we prove Proposition 1, we first derive the dynamics of the consumption-output ratio C_t/Y_t when $n_t > 0$.

Lemma 2 *When $n_t > 0$, the consumption-output ratio always jumps to*

$$C_t/Y_t = \beta(\theta/\mu)(\rho - \lambda) + 1 - \theta. \tag{A7}$$

Proof. The total value of assets owned by the household is

$$A_t = N_t V_t. \tag{A8}$$

When $n_t > 0$, the no-arbitrage condition for entry in (15) holds. Then, substituting (15) and $\mu X_t N_t = \theta Y_t$ into (A8) yields

$$A_t = N_t \beta X_t = (\theta/\mu)\beta Y_t, \tag{A9}$$

which implies that the asset-output ratio A_t/Y_t is constant. Substituting (A9), (2), (3) and (5) into $\dot{A}_t/A_t = \dot{a}_t/a_t + \lambda$ yields

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{A}_t}{A_t} = r_t + \frac{w_t L_t}{A_t} - \frac{C_t}{A_t} = \rho + \frac{\dot{C}_t}{C_t} - \lambda + \frac{(1-\theta)\,\mu}{\beta\theta} - \frac{\mu}{\beta\theta}\frac{C_t}{Y_t}, \tag{A10}$$

which can be rearranged as

$$\frac{\dot{C}_t}{C_t} - \frac{\dot{Y}_t}{Y_t} = \frac{\mu}{\beta\theta}\frac{C_t}{Y_t} - \frac{(1-\theta)\,\mu}{\beta\theta} - (\rho - \lambda)\,. \tag{A11}$$

Therefore, the dynamics of C_t/Y_t is characterized by saddle-point stability, such that C_t/Y_t jumps to its steady-state value in (A7). ■

Proof of Proposition 1. Using (12), we can derive the growth rate of x_t as

$$\frac{\dot{x}_t}{x_t} = \lambda - (1-\sigma)\, n_t. \tag{A12}$$

When $x_0 \leq x_t \leq x_N$, we have $n_t = 0$ and $z_t = 0$. In this case, the dynamics of x_t is given by

$$\dot{x}_t = \lambda x_t. \tag{A13}$$

When $x_N < x_t \leq x_Z$, we have $n_t > 0$ and $z_t = 0$. In this case, Lemma 2 implies that C_t/Y_t is constant and $\dot{c}_t/c_t = \dot{y}_t/y_t$. Therefore, we can substitute r_t^e in (16) and (A12) into $r_t = \rho + \sigma n_t$ in (3) to obtain (28). Substituting (28) into (A12) yields the dynamics of x_t as

$$\dot{x}_t = \frac{1-\sigma}{\beta}\left\{\phi\mu^{1/(1-\theta)} - \left[\mu - 1 - \beta\left(\rho + \frac{\sigma}{1-\sigma}\lambda\right)\right] x_t\right\}. \tag{A14}$$

Defining $\overline{v} \equiv \frac{1-\sigma}{\beta}\left[\mu - 1 - \beta\left(\rho + \frac{\sigma}{1-\sigma}\lambda\right)\right]$ and $\overline{x}^* \equiv \frac{\phi\mu^{1/(1-\theta)}}{\mu - 1 - \beta\left(\rho + \frac{\sigma}{1-\sigma}\lambda\right)}$, we can express (A14) as

$$\dot{x}_t = \overline{v}(\overline{x}^* - x_t). \tag{A15}$$

When $x_t > x_Z$, we have $n_t > 0$ and $z_t > 0$. In this case, C_t/Y_t is also constant, and $\dot{c}_t/c_t = \dot{y}_t/y_t$. Then, substituting r_t^e in (16) and (A12) into $r_t = \rho + \sigma n_t + z_t$ in (3) yields

$$n_t = \frac{\mu^{1/(1-\theta)}}{\beta}\left[\frac{\mu - 1}{\mu^{1/(1-\theta)}} - \frac{\phi + z_t}{x_t}\right] - \rho + \lambda. \tag{A16}$$

We substitute (30) into (A16) to derive

$$n_t = \frac{\left[(1-\alpha)(\mu-1) - (\rho-\lambda)\beta\right]\left[x_t/\mu^{1/(1-\theta)}\right] - (1-\alpha)\,\phi + \rho}{(\beta x_t)/\mu^{1/(1-\theta)} - \sigma}. \tag{A17}$$

Substituting (A17) into (A12) yields the dynamics of x_t as

$$\dot{x}_t = \frac{1-\sigma}{\beta - \sigma\mu^{1/(1-\theta)}/x_t}\left\{\left[(1-\alpha)\,\phi - \left(\rho + \frac{\lambda\sigma}{1-\sigma}\right)\right]\mu^{1/(1-\theta)} - \left[(1-\alpha)(\mu-1) - \beta\left(\rho + \frac{\lambda\sigma}{1-\sigma}\right)\right] x_t\right\}. \tag{A18}$$

Using $v \equiv \frac{1-\sigma}{\beta - \sigma\mu^{1/(1-\theta)}/x_t}\left[(1-\alpha)(\mu-1) - \beta\left(\rho + \frac{\lambda\sigma}{1-\sigma}\right)\right]$ and x^* in (32), we express (A18) as

$$\dot{x}_t = v\,(x^* - x_t)\,, \tag{A19}$$

where we approximate $\sigma\mu^{1/(1-\theta)}/x_t \cong 0$ for $x_t > x_z$, so $v \cong \frac{1-\sigma}{\beta}\left[(1-\alpha)(\mu-1) - \beta\left(\rho + \frac{\lambda\sigma}{1-\sigma}\right)\right]$ becomes a constant. ■