# Completing incomplete preferences

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# Indecisiveness: A model and a measurement*

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**Abstract**

We propose a model for individuals who have incomplete preferences and attempt to complete them. Based on the model, an incentive-compatible mechanism to measure indecisiveness in preferences is developed. An experimental test provides supporting evidence to our model. Our model can potentially explain the choice probability in the stochastic choice models and the matching law in operant conditioning.

Keywords: incomplete preferences, , indecisiveness, incentive-compatible, measurement mechanism

JEL Classification B40, C91, D81

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# 1 Introduction

That individuals' preferences might be incomplete is an old idea (Von Neumann and Morgenstern, 1944; Aumann, 1962). Indeed, there are many important decisions in our life that we find difficult to make, e.g., which job to accept, which house to buy, which girl or boy to marry. Incomplete preferences arise when there are conflicting motives or beliefs toward alternatives (Ok et al., 2012), and individuals are indecisive when they have incomplete preferences. Much advance has been made on incomplete preferences. Among others, see e.g., Dubra et al. (2004); Ok et al. (2012); Galaabaatar and Karni (2013) for recent developments.

Despite the difficulty in making decisions, most of us do choose a career, buy a house, and marry someone. What would people with incomplete preferences do when they are forced to make a decision? Can we measure indecisiveness? What are the behavioral consequences in the process of completing incomplete preferences? In this paper we go beyond incomplete preferences per se and provide an answer to the above questions. We propose a model for individuals who have incomplete preferences and attempt to complete them. Based on the model, we develop an incentive-compatible mechanism to measure indecisiveness in preferences. Finally, we implement a first experimental test of the model.

Concretely, we focus on incomplete preferences resulting from conflicting objectives by assuming that individuals have not a single but a set of utility functions. This idea is consistent with the characterization of incomplete preferences in, e.g., Dubra et al. (2004). Given any specific utility function in this set, individuals have complete preferences and perform standard expected utility calculations. Individuals' ultimate decision utility is obtained by aggregating across different utility functions. The aggregation process is done by taking a *subjective* expectation of *the concavely transformed* standard expected utilities with respect to the set of utility functions.

Based on the model, we develop an incentive-compatible mechanism to measure indecisiveness in preferences. In the mechanism we propose individuals face a series of tasks. In each of these tasks, individuals face two alternatives: an alternative $x$ over which we are interested in knowing indecisiveness in preferences, and a sure payment $y$, over which individuals are assumed to have complete preferences. Instead of choosing one option out of the two, as in typical pairwise choice tasks, individuals are allowed to choose a $\lambda \in [0, 1]$ and build a simple lottery $(\lambda x, (1 - \lambda)y)$. When individuals' preferences are incomplete and their behavior is in line with our model, we show that (1) there exists a unique optimal $\lambda^*$ that maximizes the individual's decision utility; (2) the value of the optimal $\lambda^*$ can be used to calculate indecisiveness in preferences over alternative $x$. In particular, the value of $\lambda^*$ can be interpreted as the choice probability in the stochastic choice models (e.g., Machina, 1985; Harless and Camerer, 1994) and has a striking similarity to the matching law in operant conditioning (Herrnstein, 1961; McDowell, 2005).[1]

Finally, we ran an experiment to test our model. Option $x$ was a payment of 20 euro in one month's time, while Option $y$ consisted of an immediate payment ranging from 11 euro to 20 euro with an increment of 1 euro. Our experimental results suggest that a vast majority of subjects randomizes over Option $x$ and Option $y$ for at least two values of Option $y$, and the randomization probability of choosing Option $x$ decreases with the value of Option $y$. We show that the deliberate randomization pattern in our experimental results is consistent with our model, but stands in sharp contrast with most models that assume complete preferences, e.g., the expected utility theory and cumulative prospect theory.

Among others Dwenger et al. (2014), Cettolin and Riedl (2015), and Agranov and Ortoleva (2017) experimentally examined individuals' preferences for deliberate randomization. Compared to their methods, our setting has three advantages. First, we need only one choice to reveal preferences for deliberate randomization, whereas Agranov and Ortoleva (2017) need a number of repeated choices to do so. Second, our randomization emerges endogenously and randomization probability varies with choice pairs, whereas in Dwenger et al. (2014) and Cettolin and Riedl (2015) the randomization is exogenously fixed by a

[1] The matching law in operant conditioning states that the probability of an alternative being chosen is based on the relative attractiveness of options.

random device. Third, our measure is continuous, and, with three choices, we can measure the degree of indecisiveness.

The paper proceeds as follows. Section 2 presents a model of valuation under incomplete preferences, and, based on the model, it provides an incentive-compatible measure of indecisiveness in preferences. Section 3 provides supporting evidence of our model from an experiment. Section 4 concludes.

# 2 Completing incomplete preferences: a model and a measurement

## 2.1 A model

There is a close parallel between our model and ambiguity models of multiple priors. Indeed, in ambiguity models of multiple priors, an individual faces an ambiguous scenario and she has a number of probability measures, i.e., multiple priors. In our model, an individual is uncertain which utility functions she should use to evaluate an alternative. Note, however, that the two lines of models are conceptually different. In models of multiple priors, the individual has a unique utility function, and the focus is on the aggregation across different probability measures. In the current model, the individual faces a simple lottery, and the difficulty arises in the aggregation across different utility functions. In the development of assumptions and obtaining the representation theorem, we borrow the modeling technique of multiple priors models, the smooth ambiguity model of Klibanoff et al. (2005) in particular, and apply it to incomplete preferences.

Concretely, let $C$ be a compact metric space, and $c \in C$ be a non-monetary outcome. Note that $C$ includes the empty outcome $\diamondsuit$. Let $X = C \times [w, b]$ be the set of outcomes and $x$ an element of $X$, where $[w, b]$ is the set of monetary outcomes that is large enough and $0 \in [w, b]$. The set of outcomes $X$ that we consider is quite general. It includes all possible aspects of a decision that affect the decision maker's well-being. Thus, besides monetary outcomes, the outcome space also includes, for example, a one-week trip to Paris, an increased safety of a car, or an improvement in air quality. A risky lottery $l \in L$ is then a Borel probability measure over $X$, where $L \equiv \triangle X$. The model is mainly interested in $\succeq$, an individual's preference over $L$. In a standard expected utility framework, the individual's preference is captured by a single utility function, $u : X \to R$, such that for any risky lotteries $l_1$ and $l_2 \in L$, $l_1 \succeq l_2 \iff \int_X u(x)\, dl_1 \geq \int_X u(x)\, dl_2$. When $\succeq$ is potentially incomplete, difficulties arise, and it is unclear what decision an individual makes. Let $\tau \in \mathit{\Gamma}$ be one potential self, $\succeq_\tau$ denote an individual's preference given a self, and $\mathit{\Gamma}$ is a metric space that denotes the set of selfs. Thus, the set of selfs is defined as the state space in the sense of Anscombe and Aumann (1963), with each self corresponding to a potential state. Let $\Pi$ denote the set of Borel probability measures over $\Gamma$. Our main result is summarized in the representation theorem below. A more detailed development of the representation theorem can be found in Appendix 1.

**Theorem 1.** *There exists a countably additive probability measure $\pi \in \Pi$ and a continuous and strictly increasing $\phi : \mathbb{R} \to \mathbb{R}$ subjecting to positive affine transformation such that $\succeq$ is represented by the preference functional $V : L \to \mathbb{R}$ given by*

$$V(l) = \int_{\mathit{\Gamma}} \phi\left[EU_\tau(l)\right] d\pi. \tag{1}$$

In Equation 1 $\pi$ measures indecisiveness in preferences and the curvature of $\phi(\cdot)$ captures an individual's attitudes toward indecisiveness. In principle, the function $\phi(\cdot)$ can be concave, linear, or convex. A concave $\phi(\cdot)$ implies an aversion to indecisiveness as it punishes the disagreement - deviations from the mean expected utility - among different selfs. A linear $\phi(\cdot)$ implies a neutral attitude toward indecisiveness, and a convex $\phi(\cdot)$ implies indecisiveness seeking. Below we assume that individuals are averse to indecisiveness and, hence, $\phi(\cdot)$ is concave. Note that to arrive at a single value when one has many selfs is, in essence, similar to situations where a group of people with different opinions tries to reach a consensus. The more strongly members disagree with each other, the more difficult it is for the group to make compromises and agree on a single opinion. Aversion to indecisiveness in preferences can then be interpreted as the cost of forcing different selfs to agree on a single value. The assumption of aversion to indecisiveness is also consistent with Levitt (2016) where he shows that indecisive subjects - those who having difficulty to make a decision - are often excessively cautious when facing important choices.

Our paper is related to a paper by Cerreia-Vioglio et al. (2015). They start with standard axioms - including the completeness axiom - and replace only the independence axiom with a weaker axiom which they call Negative Certainty Independence. They characterize utility representations for all preferences that satisfy Negative Certainty Independence and other basic rationality postulates. In their representation individuals behave as if they have a set of utility functions and evaluate alternatives according to the utility function giving the lowest certainty equivalent; other utility functions or indecisiveness in preferences play no role. They then show beautifully that the representation can be used to complete an incomplete preference relation. Our model starts with incomplete preferences and aims to complete them. Our set of states is the set of utility functions instead of the set of uncertain outcomes that lie outside of the decision maker. Our model is built on a different set of assumptions, and the representation is characterized by smoothness. In the completion process our individuals consider all utility functions in the set, and indecisiveness in preferences plays a central role. Due to the smoothness of the representation an incentive-compatible measure of indecisiveness in preferences can be developed.

### 2.2 An incentive-compatible measurement of indecisiveness in preferences

As the discussion in the introduction highlights, there have been a number of papers on the existence and importance of incomplete preferences. A natural question to ask, then, given an alternative, is how we measure indecisiveness in preferences. According to Equation 1, an individual's indecisiveness in preferences is captured by the subjective distribution $\pi$ over $u_\tau$. Consistent with the literature in decision making under risk, a natural candidate for the measurement of indecisiveness in preferences is the standard deviation of the subjective distribution $\pi$ over $u_\tau$ ($\sigma_\pi^2$). Hence, if we could somehow measure $\sigma_\pi^2$, we would obtain a proxy for indecisiveness in preferences. Below we propose such a measurement mechanism. As it will be seen shortly, it is incentive-compatible and easy to implement.

More specifically, the mechanism works as follows. An individual faces two options. Denote these two options by $x$ and $y$. Option $y$ is a yardstick, and we are interested in measuring the individual's indecisiveness in preferences over Option $x$. In most preceding studies, an individual would be asked to choose between two options, Option $x$ or Option $y$ (see e.g., Holt, 1986). In the so-called outcome matching method, an individual is asked to compare option $x$ with a list of increasing sure payoffs $y$.[2] However, straightforward choices yield only limited dichotomous information. In particular, there is no room for individuals to express their indecisiveness in preferences.

In the current mechanism, we proceed differently: we ask the individual, instead of choosing one option out of the two, to choose a $\lambda \in [0,1]$ and build a simple lottery: $(\lambda x, (1-\lambda)y)$. The meaning of the lottery $(\lambda x, (1-\lambda)y)$ is easier to understand for decisions with a finite set of outcomes. Let $x$ be $(x_1, p_1; ...; x_n, p_n)$ and $y$ be $(y_1, q1; ...; y_n, q_n)$, then $(\lambda x, (1-\lambda)y)$=$(x_1, \lambda p_1; ...; x_n, \lambda p_n; y, (1-\lambda)q_1; ...; y_n, (1-\lambda)q_n)$. Below we show that, when the individual behaves according to our model, there exist some values of $y$ inducing the individual to strictly prefer the lottery $(\lambda x, (1-\lambda)y)$, with $0 < \lambda < 1$, over Option $x$ *and* Option $y$.

For any particular self $\tau$, the individual's preference over the lottery $(\lambda x, (1-\lambda)y)$ satisfies the expected utility theory.[3] Explicitly, given a self $\tau$, we have $EU_\tau[\lambda x + (1-\lambda)y] = \lambda EU_\tau(x) + (1-\lambda)EU_\tau(y)$. The individual's decision is then to maximize her utility by choosing $0 \leq \lambda \leq 1$ properly:

$$Max_\lambda\ V[\lambda x + (1-\lambda)y] = \int_\Gamma \phi[\lambda EU_\tau(x) + (1-\lambda)EU_\tau(y)]\, d\pi.$$

Taking first order derivative of the above equation gives[4]

[2]In discussion below Option $y$ is often an alternative offering a sure amount of payment $y$. When no confusion is possible we abuse the use of notations slightly and identify $y$ with Option $y$.

[3]See Assumption 1 in Appendix.

[4]The second-order derivative is

$$\frac{d^2V[\lambda x + (1-\lambda y)]}{d\lambda^2} = \int_\Gamma \phi''[\lambda EU_\tau(x) + (1-\lambda)EU_\tau(y)] \times [EU_\tau(x) - EU_\tau(y)]^2\, d\pi.$$

$$\frac{dV\left[\lambda x+(1-\lambda)y\right]}{d\lambda}=\int_{\Gamma}\phi'\left[\lambda EU_{\tau}(x)+(1-\lambda)EU_{\tau}(y)\right]\times\left[EU_{\tau}(x)-EU_{\tau}(y)\right]d\pi=0.$$

In some cases, preferences of $x$ over $y$ can be straightforward, e.g., when options can be ordered by some dominance rules. For example, when options $x$ and $y$ are risky lotteries and option $x$ first degree stochastically dominates option $y$, it seems natural that individuals have $EU_{\tau}(x)>EU_{\tau}(y)$, for $\forall\tau\in\Gamma$. Since $\phi'\left[\lambda EU_{\tau}(x)+(1-\lambda)EU_{\tau}(y)\right]>0$, this leads to a positive first order condition and, hence, $\lambda=1$. Unfortunately, two options cannot in general be ordered via simple dominance rules. In such situations the choice of $\lambda$ would give insights on indecisiveness in preferences over option $x$.

Note that $EU_{\tau}(x)$ and $EU_{\tau}(y)$ are random variables governed by the probability distribution $\pi$. Let $X=EU_{\tau}(x)$ and $Y=EU_{\tau}(y)$, and $\Delta_{\tau}=X-Y$. With these notations, we have

$$\phi'\left[\lambda EU_{\tau}(x)+(1-\lambda)EU_{\tau}(y)\right]=\phi'\left[Y+\lambda\Delta_{\tau}\right].$$

We are mostly interested in the scenario where the individual finds choosing out of Option $x$ and Option $y$ difficult, i.e., when the two options are close. Specifically, we are interested in those situations where $\Delta_{\tau}$ is small relative to $X$ and $Y$. When this is the case, we can use the Taylor expansion and obtain

$$\phi'\left[Y+\lambda\Delta_{\tau}\right]=\phi'(Y)+\phi''(Y)\lambda\Delta_{\tau}+O\left(\lambda\Delta_{\tau}\right)\approx\phi'(Y)+\phi''(Y)\lambda\Delta_{\tau},$$

where $O\left(\lambda\Delta_{\tau}\right)$ is the sum of the terms that have $\lambda\Delta_{\tau}$ with a power of two or higher. The above first order condition can then be written as

Since $\phi(\cdot)$ is concave, $\phi''(\cdot)$ is negative. We are interested in situations where options $x$ and $y$ are not the same, i.e., $EU_{\tau}(x)\neq EU_{\tau}(y)$ for some $\tau\in\Gamma$. Together we have $\phi''\left[\lambda EU_{\tau}(x)+(1-\lambda)EU_{\tau}(y)\right]\times\left[EU_{\tau}(x)-EU_{\tau}(y)\right]^{2}\leq 0$, and the inequality is strict for some $\tau\in\Gamma$. Consequently, $\frac{d^{2}V[\lambda x+(1-\lambda y)]}{d\lambda^{2}}=\int_{\Gamma}\phi''\left[\lambda EU_{\tau}(x)+(1-\lambda)EU_{\tau}(y)\right]\times\left[EU_{\tau}(x)-EU_{\tau}(y)\right]^{2}d\pi<0$. This ensures we are indeed seeking for the maximum.

$$
\begin{aligned}
\frac{dV[\lambda x+(1-\lambda)y]}{d\lambda} &= \int_{\Gamma}\phi'\left[Y+\lambda\Delta_{\tau}\right]\Delta_{\tau}d\pi, \\
&\approx \int_{\Gamma}\left[\phi'(Y)+\phi''(Y)\lambda\Delta_{\tau}\right]\Delta_{\tau}d\pi \\
&= E_{\tau}\left[\phi'(Y)\Delta_{\tau}\right]+\lambda E_{\tau}\left[\phi''(Y)\Delta_{\tau}^{2}\right]=0,
\end{aligned}
$$

where $E_{\tau}(\cdot)$ is the expectation operator with respect to the distribution $\pi$ . Solving for $\lambda$, and we have

$$
\lambda^{*}=\begin{cases} 0 & -\frac{E_{\tau}[\phi'(Y)\Delta_{\tau}]}{E_{\tau}[\phi''(Y)\Delta_{\tau}^{2}]}\leq 0, \\ -\frac{E_{\tau}[\phi'(Y)\Delta_{\tau}]}{E_{\tau}[\phi''(Y)\Delta_{\tau}^{2}]} & 0<-\frac{E_{\tau}[\phi'(Y)\Delta_{\tau}]}{E_{\tau}[\phi''(Y)\Delta_{\tau}^{2}]}<1, \\ 1 & -\frac{E_{\tau}[\phi'(Y)\Delta_{\tau}]}{E_{\tau}[\phi''(Y)\Delta_{\tau}^{2}]}\geq 1. \end{cases}
$$

It seems reasonable to assume that the individual's preference over a sure payment today is complete. Such an assumption is similar to but weaker than the requirement that preferences are complete over constant acts (see e.g., Ok et al., 2012). Specifically, when option $y$ is a sure payment, $Y$ would be a constant. It follows then that $E_{\tau}[\phi'(Y)\Delta_{\tau}]=\phi'(Y)E_{\tau}[\Delta_{\tau}]$, and $E_{\tau}[\phi''(Y)\Delta_{\tau}^{2}]=\phi''(Y)E_{\tau}[\Delta_{\tau}^{2}]$. Similar to decision making under risk, let $-\frac{\phi''(Y)}{\phi'(Y)}$ denote a metric of attitudes toward indecisiveness in preferences, and the optimal value of $\lambda$ becomes:

$$
\lambda^{*}=\begin{cases} 0 & \lambda^{*}\leq 0, \\ \frac{1}{-\frac{\phi''(Y)}{\phi'(Y)}}\times\frac{E_{\tau}[\Delta_{\tau}]}{E_{\tau}[\Delta_{\tau}^{2}]} & 0<\lambda^{*}<1, \\ 1 & \lambda^{*}\geq 1. \end{cases} \tag{2}
$$

Recall $\Delta_{\tau}=X-Y$. We then have $E_{\tau}\left[\Delta_{\tau}^{2}\right]=E_{\tau}\left[(X-Y)^{2}\right]=\sigma_{x}^{2}+\left[E_{\tau}(X)-Y\right]^{2}$ and $E_{\tau}\left[\Delta_{\tau}\right]=E_{\tau}(X)-Y$. Thus, when $0<\lambda^{*}<1$, the optimal value of $\lambda^{*}$ decreases with indecisiveness in preferences over option $x$ and the individual's attitudes toward indecisiveness in preferences. When $\sigma_{x}^{2}>\left[E_{\tau}(X)-Y\right]^{2}$, i.e., indecisiveness in preferences is sufficiently large, $\lambda^{*}$ increases with $\Delta_{\tau}$.

Equation 2 can be used to estimate $\delta_{x}^{2}$. $Y$ is the utility over sure outcomes and is relatively easy to estimate. The metric for indecisiveness attitudes $-\frac{\phi''(Y)}{\phi'(Y)}$ should not change substantially with small variation of $Y$. With $Y$ at hand, only three variables in Equation 2 remain unknown: a measure of indecisiveness in preferences that we want to estimate $\sigma_x^2$, the metric for indecisiveness attitudes $-\frac{\phi''(Y)}{\phi'(Y)}$, and $E_\tau(X)$. Note that there is an optimal $\lambda^*$ for each $Y$. With three $Y$s and accordingly three $\lambda^*$s, one can easily calculate $\sigma_x^2$.

As a concrete illustration, consider the following numerical example: suppose the individual has two selfs $\tau = 1, 2$, and $Prob(u_1) = 0.5$ and $Prob(u_2) = 0.5$. There are two options, Option $x$ and Option $y$. Option $x$ is a lottery, and $EU_1(x) = 1$ and $EU_2(x) = 0$. Option $y$ is a sure payment, and the individual's preference over $y$ is complete, i.e., $EU_1(y) = EU_2(y)$. For the ease of illustration, assume further that $EU_1(y) = EU_2(y) = y$. The function $\phi(\cdot)$ takes the form of $\phi(EU_\tau) = 1 - e^{-EU_\tau}$. The decision utility of choosing option $x$ is then $V(x) = 0.5(1-e^{-1})+0.5\times 0 = 0.316$, and the decision utility of choosing option $y$ is $V(y) = 1-e^{-y}$. It can be easily shown that when $\frac{e}{1+e}<y < \frac{1}{2}$, the individual is better off by opting for the lottery$(\lambda x; (1-\lambda)y)$ instead of choosing option $x$ or option $y$. The decision utility of such a lottery is: $V(\lambda x+(1-\lambda)y) = 0.5\left[1 - e^{-[\lambda\times 1+(1-\lambda)\times y]}\right]+0.5\times\left[1 - e^{-[\lambda\times 0+(1-\lambda)\times y]}\right]$. Simple calculation shows that the optimal $\lambda$:

$$\lambda^* = \begin{cases} 0 & y \geq \frac{1}{2}, \\ -ln(\frac{y}{1-y}) & \frac{e}{1+e}<y < \frac{1}{2}, \\ 1 & y \leq \frac{e}{1+e}. \end{cases}$$

Figure 1 provides the optimal $\lambda$ for the value of $y$. As one can see, when Option $y$ becomes more attractive, the value of $\lambda^*$ decreases. Moreover, $\lambda^*$ approaches to 0.5 when the two options become similar in terms of their decision utilities ($V(y) = 0.314$ versus $V(x) = 0.316$).

Note that in the current setup, $\lambda^*$ is constructed as the ratio according to which the individual is paid with Option $x$. As a first interpretation of $\lambda^*$, note that the value of $\lambda^*$ increases with the utility difference $\Delta_\tau$ of Option $x$ over Option $y$. This feature of $\lambda^*$ is closely related to the error term in stochastic choice models, where the probability of choosing the more attractive option is increasing with the utility difference of the more attractive over the less attractive option. In this sense, the value of $\lambda^*$ can be interpreted

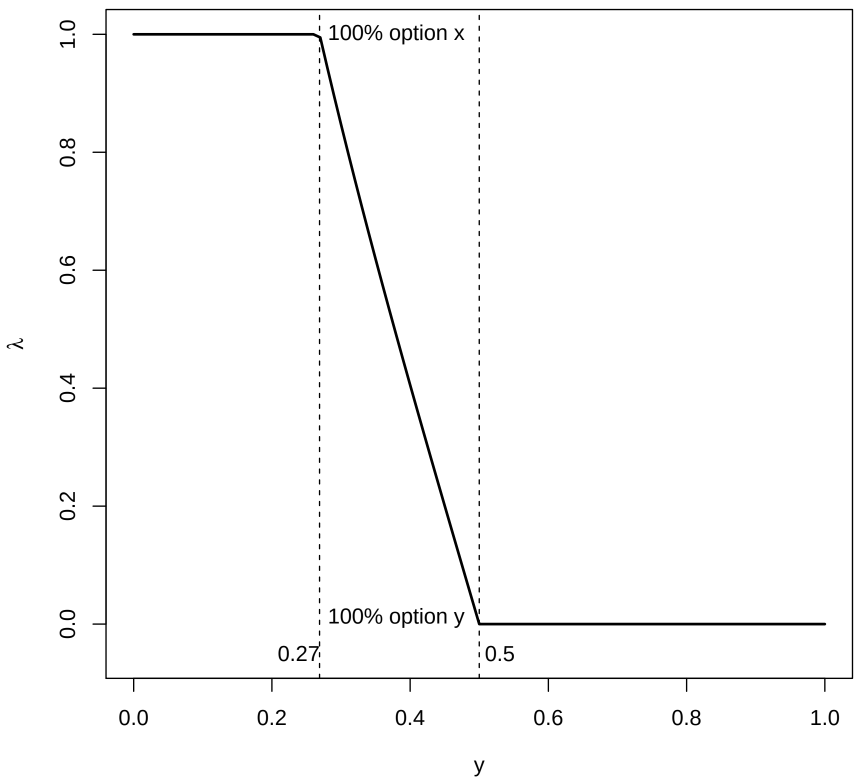


Figure 1: The optimal $\lambda^*$ depending on the value of $y$. Figure is produced by assuming $\phi(EU_\tau) = 1 - e^{-EU_\tau}$, $Prob(u_1) = 0.5$, $Prob(u_2) = 0.5$, $EU_1(x) = 1$,$EU_2(x) = 0$, and $EU_1(y) = EU_2(y) = y$.

as the choice probability in stochastic choice models. When preferences are incomplete, the choice probability is moderated by indecisiveness in preferences. An increase in $\sigma_x^2$, i.e., when the individual's preference over Option $x$ versus Option $y$ becomes less complete, decreases its probability of being chosen. As a second interpretation, the mechanism through which $\lambda^*$ is obtained and the interpretation of $\lambda^*$ have a striking similarity to an old idea in psychology: the matching law in operant conditioning (Herrnstein, 1961; McDowell, 2005). This states that the probability of an alternative being chosen is based on the relative attractiveness of options. It is observed in both animals and human agents and is considered as a clear violation of rational choices. However, Loewenstein et al. (2009) show that the matching law can be made consistent with rational choices if we regard the choices of a single subject as being made up of a sequence of multiple selves - one for each instant of time. Although their result is obtained in an entirely different context, the fundamental idea is surprisingly similar. Last, due to indecisiveness in preferences, the individual orders Option $x$ better than Option $y$ with some utility functions and shows the reverse preference ordering with others, and she considers all utility functions relevant. By choosing a randomization probability $\lambda$ rather than either of the two options, she exhibits a preference for convexity. Cerreia-Vioglio (2009) interprets the preference for convexity as a preference for hedging and links it to uncertainty about future tastes.

## 3 An experiment

To test our model and check the performance of our measure of indecisiveness in preferences, we ran an experiment in which subjects faced choice pairs of Option $x$ versus Option $y$. In each choice pair Option $x$ was a payment of 20 euro in one month's time, while Option $y$ consisted of an immediate payment ranging from 11 euro to 20 euro with an increment of 1 euro.

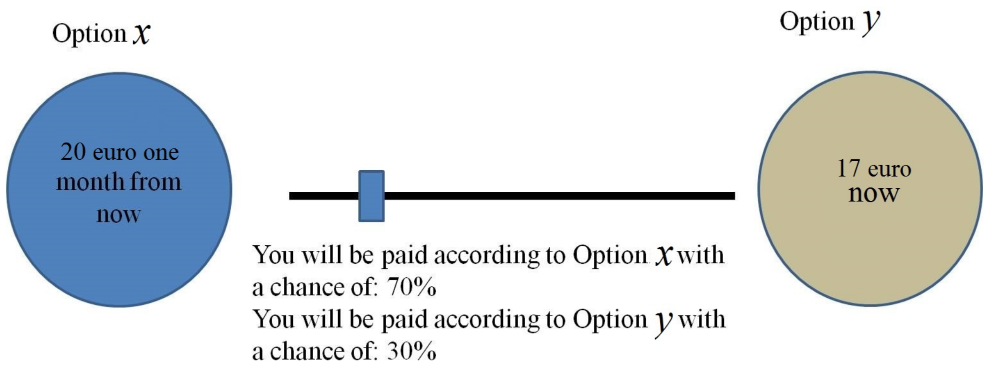


Figure 2: The experimental implementation of the elicitation method for indecisiveness in preferences. Subjects move the slider to decide their optimal $\lambda^*$. When subjects moved the bar, the probabilities below changed to reflect the decision.

### 3.1 Stimuli

Specifically, the experiment consisted of two tasks: the randomization task and the confidence task. In the randomization task, subjects faced 10 decisions. In each decision subjects faced the Option $x$ and an Option $y$, with $11 \leq y \leq 20$. The sequence of Option $y$ was randomized on each individual subject level. As explained in subsection 2.2 subjects, instead of choosing either Option $x$ or $y$, chose a randomization probability $\lambda$. Note that both Option $x$ and Option $y$ are sure payments. This allows us to interpret the randomization probability $\lambda$ as the probability according to which subjects are paid with Option $x$. For example, a value of $\lambda = 0.75$ means subjects choose to be paid accordingly to option $x$ with a probability of 0.75, while to be paid according to option $y$ with a probability of 0.25. The increment of this randomization probability was specified at 0.01. Figure 3.1 illustrates the decision screen. When subjects moved the bar, the probabilities below changed to reflect the decision.

The confidence task is similar to the standard pairwise choice task. Subjects faced a table of ten choice pairs of Option $x$ versus Option $y$, in which Option $y$ increased from 11 euro to 20 euro down the rows. In each choice pair subjects chose one option out of the two. In addition subjects stated - without monetary incentive - how confident they felt about their choice. They could state their confidence in five steps: surely Option $x$, probably Option $x$, unsure, probably Option $y$, and surely Option $y$. This idea is built on a literature closely related to incomplete preferences, the so-called imprecise preferences (see e.g., Butler and Loomes, 2007). As indecisiveness is closely related to the confidence in making decisions, we believe this measure could provide valuable complementary information to our measure.

Our focus is on the randomization task, because decisions in this task allow the distinction of our model from previous models. In the experiment, however, we started with the confidence task. We believe the confidence task is simpler and easier to understand for subjects. Subjects face the more complicated randomization task after having finished the confidence task.

### 3.2 Sample and procedure

***Subjects***

The experiment was conducted online with a random sample of subjects of the DISCON lab in Radboud University. In total, 92 students participated in the experiment, with roughly half being male and half being female.

***Procedure***

It was programmed with OTree (Chen et al., 2016).[5] The whole experiment lasted on average 15 minutes. Invitations were sent in batches via OSEE (Greiner, 2015). Subjects first saw the experimental instruction which explains the decision and the payment procedure. They then completed the confidence task and the randomization task as described above. Finally, they filled in a questionnaire with their fields of study, gender, and email addresses. In the questionnaire they also indicated their preferred way of receiving the payment: either via a bank transfer (they then provided their bank account) or picking up the money at our secretary.

***Motivating subjects***

[5] The experiment contained several other tasks which will be reported in separate papers.

Each student received 2.5 euro participation fee. Additionally, 20% of students were chosen for real payment. One of the two tasks was randomly chosen. If the confidence task was chosen for payment, one row would be randomly chosen and the subjects' preferred choice determined their payment. If the randomization task was chosen for payment, one decision was randomly chosen. Computer then determined the payment option according to the randomization probability that subjects specified in the randomly chosen task. The payment was 20 euro plus 2.50 euro among the chosen students. All subjects chose to be paid via bank transfers. The payment was made either in one month (should Option $x$ was chosen for payment), or in the same day (should Option $y$ was chosen for payment). The participation fee of 2.50 euro was done in the same day should subjects chosen bank transfers.

### 3.3 Experimental results

In the report of our experimental results we mainly focus on the randomization task. Before presenting the experimental results, we offer the predictions for the randomization task under some popular theories of decision making under risk and uncertainty. Cerreia-Vioglio et al.'s (2015) model is an important contribution in completing incomplete preferences and we also derive a prediction under their model.

**Proposition.** *Under EUT, some popular non-EU theories, such as (cumulative) prospect theory, rank dependent utility theory (Quiggin, 1982), and Guls (1991) disappointment aversion theory, as well as Cerreia-Vioglio et al.'s (2015) model: subjects randomize at most once, and* $\lambda^* \in (0,1)$ *occurs only when subjects are indifferent between Option* $x$ *and Option* $y$*.*

Proofs can be found in Appendix.

Our main experimental results are summarized below.

**Result 1.** *Consistent with our model, a large majority of subjects randomized over Option* $x$ *and Option* $y$ *for at least two values of Option* $y$*, and the randomization probability of choosing Option* $x$ *decreases with the value of Option* $y$*.*

| The number of subjects who chose randomization probabilities $\lambda^* \in (0,1)$ for | | | |
|---|---|---|---|
| 0 time | 1 time | more than 1 time | more than 2 times |
| 18 subjects | 6 subjects | 68 subjects | 53 subjects |

Table 1: The distribution of subjects that chose randomization probabilities within 0 and 1 for zero time, for one time, for more than one time, and for more than two times.

Support: First, as we can see from Table 1 68 out of 92 subjects assigned a randomization probability strictly within 0 and 1.0 to Option $y$ more than once. Only 18 out of 92 students could be identified as individuals having complete preferences: they first assigned $\lambda^* = 1$ when Option $y$ was small, and then assigned $\lambda^* = 0$ when Option $y$ was sufficiently attractive (for 14 out of 18 students this occurred when Option $y$ was 20 euro). Six subjects chose a randomization probability within 0 and 1 exactly once. Those subjects could either be indecisive or have a complete preference and be indifferent when choosing $0 < \lambda < 1$ .

Second, consistent with Equation 2 and Figure 1 among the 68 subjects who randomized more than once, 54 subjects' $\lambda^*$ decreased with the value of Option $y$. The median randomization probabilities $\lambda^*$ assigned to Option $x$ were, respectively, 1.00 when Option $y$ was 15 euro or lower, 0.955 when Option $y$ was 16 euro, 0.87 when Option $y$ was 17 euro, 0.65 when Option $y$ was 18 euro, 0.45 when Option $y$ was 19 euro, 0.00 when Option $y$ was 20 euro. Taking together, such a preference for randomization is consistent with our model but stands in sharp contrast with most models of complete preferences as well as Cerreia-Vioglio et al.'s (2015) model.

We now discuss the connection between the binary choices in the confidence task and the randomization probabilities in the randomized task. Subjects chose between Option $x$ and Option $y$ in the confidence task, and they typically started with Option $x$ and switched to Option $y$ when Option $y$ became sufficiently high. We thus find for each subject the switching row at which subjects still preferred Option $x$ but switched to Option $y$ in the following row. We then check the randomization probabilities one row above the switching row, at the switching row, and one row below the switching row.

**Result 2.** *Indecisive subjects were more likely to choose Option y over which they have complete preferences when they were forced to choose between two options than when they*

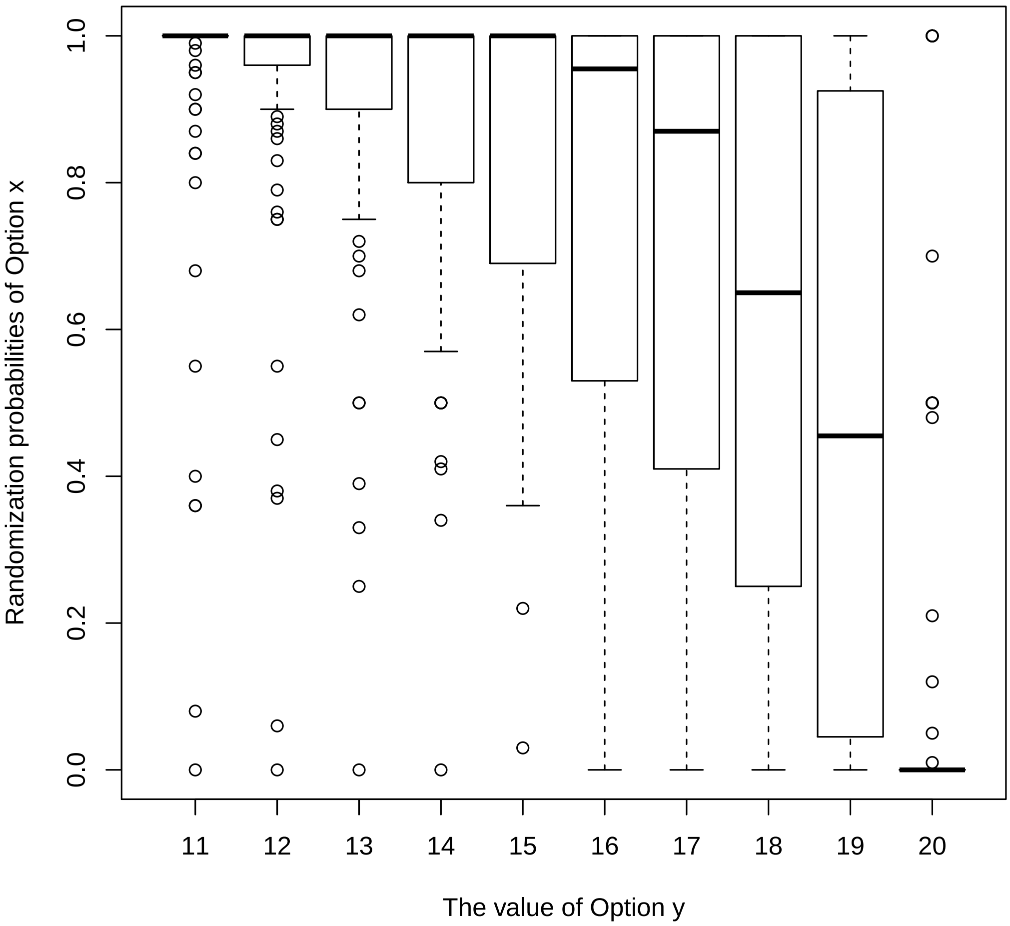


Figure 3: A boxplot of the randomization probabilities that subjects assign to Option $x$, as a function of the value of Option $y$. The thick lines are medians, the upper and lower bars are 1st and 3rd quantiles, respectively.

*were allowed to randomize.*

Support: We find that the median randomization probability is 0.255 for Option $y$ at the switching row. The median randomization probability is 0.70 for Option $y$ one row above the switching row, and 0.10 for Option $y$ one row below the switching. There is thus a sense of caution in the binary choices of choosing between Option $x$ and Option $y$: subjects chose the safer option - Option $y$ with sure payment today - when they were in doubt. When allowed to randomize, however, subjects assigned a larger randomization probability for Option $x$ than for Option $y$ (a median probability of 0.745 versus 0.255).

The measures of confidence statements in, e.g., Butler and Loomes (2007) provide valuable insights beyond dichotomous choices. Below we link the confidence statements to our quantitative measure.

**Result 3.** *There is an asymmetry of the confidence statements and the randomization probabilities between Option $x$ and Option $y$; and given a confidence statement significant heterogeneity in the randomization probability exists among subjects.*

Support: We find, first, that choosing “surely $x$” approximately corresponds to assigning Option $x$ to a median randomization probability of 1.00; choosing “probably $x$” approximately corresponds to assigning Option $x$ to a median randomization probability of 0.90; choosing “unsure” roughly corresponds to assigning Option $x$ to a median randomization probability of 0.70; choosing ”probably $y$” approximately corresponds to assigning Option $x$ to a median randomization probability of 0.56; choosing “surely $y$” corresponds to assigning Option $x$ to a median randomization probability of 0.35. The above result reveals a clear asymmetry: given the same qualitative statements, i.e., ”surely” or ”probably”, subjects assigned a much higher randomization probability to Option $x$ than to Option $y$. A possible interpretation of the above asymmetry is that subjects made qualitative statements such as “surely” or “probably” based on the $E_\tau[\Delta_\tau]$, the expected utility difference between Option $x$ and Option $y$, while the optimal randomization probability $\lambda^*$ depended additionally on indecisiveness of the preference over Option $x$ , as revealed by Equation 2.

Second, confidence statements are self-reports and they could mean differently for different

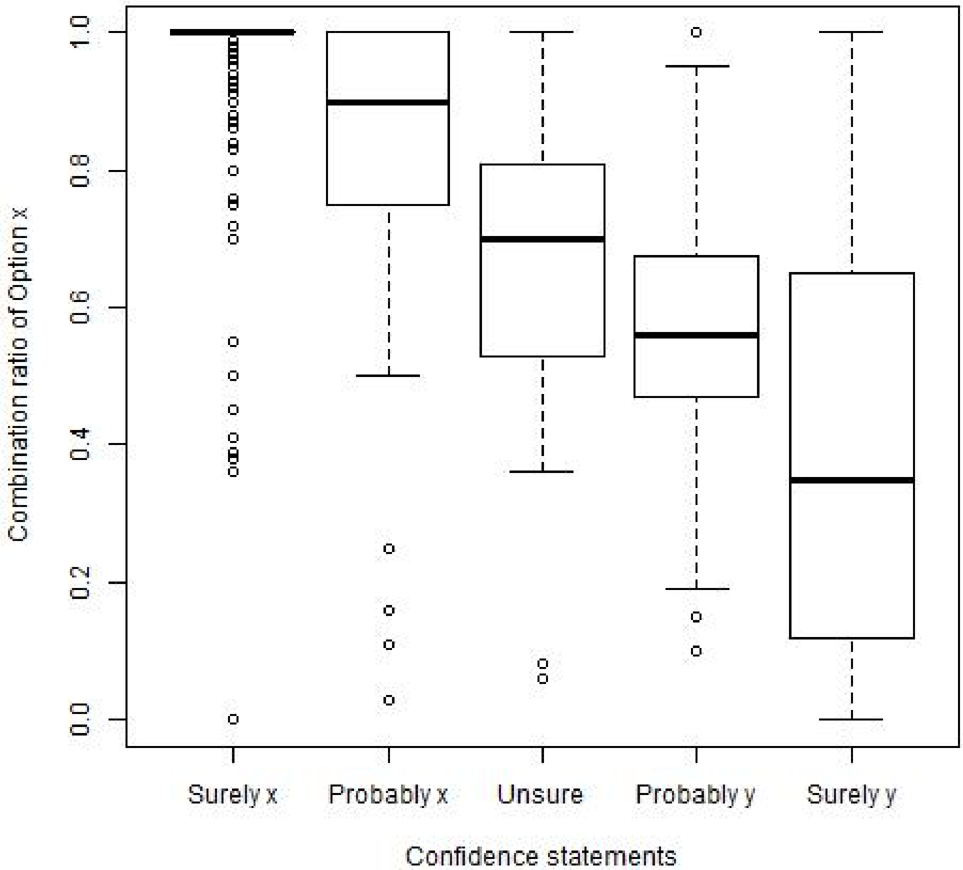


Figure 4: A boxplot of the randomization probability of Option $x$, given each confidence statements. The thick line is median, the upper and lower bars are 2nd and 3rd quantiles, respectively.

people. As one can observe from Figure 4, there is significant heterogeneity of randomization probabilities across subjects in each confidence statement, in particular for "Probably $x$", "Unsure", "Probably $y$ ", and "Surely $y$ ".

# 4 Conclusion

We have developed a model of completing incomplete preferences. Incomplete preferences are captured by individuals having not a single but a set of utility functions. Individuals perform standard expected utility calculations, given any specific utility function, and have an subjective expectation of the *transformed* standard expected utilities with respect to a set of utility functions.

Based on this model, we propose an incentive-compatible mechanism to measure the degree of indecisiveness in preferences. In the mechanism, individuals face a sure payment and an alternative that we are interested in finding out indecisiveness. Instead of choosing either the sure payment or the alternative, as in typical pairwise choice tasks, individuals are allowed to allocate the probabilities according to which they are paid with the alternative or the sure payment. When individuals' preferences are incomplete and their behavior is in line with our model, we show that the value assigned to the alternative provides a proxy for the degree of indecisiveness in preferences over the alternative. The obtained measure can be interpreted as the choice probability in stochastic choice models and has a striking similarity to the matching law in operant conditioning. An experiment provides results consistent with our model but challenges alternative theories.

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The deciding individual

| Self 1 | Self 2 | ... | Self $n$ |
|---|---|---|---|
| $l$ | $l$ | ... | $l$ |

The deciding individual

| | | | |
|---|---|---|---|
| $ce_1(l)$ | $ce_2(l)$ | ... | $ce_n(l)$ |

Table 2: The left panel depicts the decision problem $D(l)$ when the individual faces $l$ and has $n$ selfs. The right panel depicts the decision problem $D(ce_l)$, where $ce_\tau(l)$ is the certainty equivalent of $l$ given $u_\tau$.

## Appendix 1: A detailed development of Theorem 1:

Our representation can be seen a mirror of the smooth model of ambiguity when the states are over the set of utility functions. Below we provide a more detailed development of Theorem 1.

**Assumption 1.** *Expected Utility Over Risky Lotteries Given a Self. Given a self, $\tau$, there exists a unique utility function, $u_\tau$, continuous, strictly increasing, and normalized so that $u_\tau(w) = 0$ and $u_\tau(b) = 1$ such that for all $l_1$ and $l_2 \in L$, $l_1 \succeq_\tau l_2$ if and only if $\int_X u_\tau(x)\, dl_1 \geq \int_X u_\tau(x)\, dl_2$.*

An individual's preference of self $\tau$ can then be captured by utility function $u_\tau(\cdot)$, $\forall \tau \in \mathit{\Gamma}$. Dubra et al. (2004) suggest the following representation when an individual's preference over $L$ is potentially incomplete. There exists a set $\{u_\tau\}_{\tau \in \mathit{\Gamma}}$ of real functions on $L$ such that, for all lotteries $l_1$ and $l_2$ ,

$$l_1 \succeq l_2 \Longleftrightarrow \int_C u_\tau(c)\, dl_1 \geq \int_C u_\tau(c)\, dl_2 \; \forall \tau \in \mathit{\Gamma}.$$

When the set $\{u_\tau\}_{\tau \in \mathit{\Gamma}}$ is of a singleton, we are back to the standard expected utility theory with complete preferences. Given the above setup, the individual essentially faces the situation depicted in the left panel of Table 2. Let $D(l)$ denote the decision problem when the individual faces $l$.

We require that $\forall p \in \triangle C$, $\exists a \in [w, b]$ such that $p \sim_\tau \delta_a$, where $\triangle C$ is the set of Borel probability measures over $C$ and $\delta_a$ is the degenerated lottery obtaining $a$ with probability one. This condition makes sure the existence of $ce_\tau(l) \in [w, b]$, defined as the certainty equivalent of risky lottery $l$ given $u_\tau$. The existence of $ce_\tau(l)$ allows us to transform the decision problem $D(l)$ into the decision problem $D(ce_l)$ that is depicted in the right panel of Table 2.

The difficulty remains: how does an individual aggregate across selfs? Aggregation of similar forms has been discussed extensively in the social welfare literature (see e.g., Hicks, 1939). It is generally agreed that aggregating across different individuals is extremely difficult or even meaningless. Cerreia-Vioglio et al. (2015) do not attempt to aggregate across selfs. Instead, by invoking the negative certainty independence axiom, they obtain a representation similar to the Rawlsian scheme: the alternative is evaluated by the self who gives the lowest certainty equivalent. We believe more can be done. Recall in Assumption 1 that the utility function, $u_\tau$, is normalized such that $u_\tau(w) = 0$ and $u_\tau(b) = 1$. Thus, utility functions, although different across selfs, are based on the same metric. Comparison among selfs is thus not comparing "apples" and "oranges." After all, the aggregation across selfs is performed for the same individual. We believe, the idea that for the same individual there exists a common scale of utility across selfs is a plausible one. For example, Binmore (1998, chapter 4, p.259) concludes that intra-personal comparisons of oneself in different roles in a society are completely acceptable. This observation motivates the next assumption. But before stating it, a new object needs to be defined.

**Definition 1.** An act, $f \in F$, is a function $f : \mathit{\Gamma} \to [w, b]$ that aggregates selfs to a monetary outcome.

The act $f$ is defined over the set of selfs, thus it is different from the act defined in, e.g., Gilboa and Schmeidler (1989); Klibanoff et al. (2005). Yet, it is consistent with the Savage act where the states of the world is the set of selfs. For this reason we still call $f$ an act. Note that $f(\tau) \in [w, b]$ is a monetary outcome given a self $\tau$. Let $\succeq^f$ denote the individual's preference over the acts, and let $\Pi$ denote the set of Borel probability measures over $\Gamma$. We can now state the second assumption.

**Assumption 2.** *Aggregation in the Form of Subjective Expected Utility (SEU) over Acts. There exists a countably additive probability measure $\pi \in \Pi$ and a continuous and strictly increasing function $v : [w, b] \to \mathbb{R}$ such that for all $f_1$, $f_2 \in F$,*

$$f_1 \succeq^f f_2 \Longleftrightarrow \int_{\Gamma} v[f_1(\tau)]d\pi \geq \int_{\Gamma} v[f_2(\tau)]d\pi.$$

The subjective probability distribution $\pi$ captures the individual's subjective assessment of the relevance of the selfs in evaluating $l$. Given a self, $\tau$, by Assumption 1, lottery $l$ is evaluated with the utility function $u_\tau$ according to the expected utility theory. An individual should then be indifferent between facing lottery $l$ as in $D(l)$ or facing $f$ that produces $ce_\tau(l)$ for each self $\tau$ as in $D(ce_l)$. This property motivates the following definition:

**Definition 2.** Given $l \in L$, $f^l \in F$ denotes an act reduced from $l$. The reduced act of $l$, $f^l$, is defined as

$$f^l(\tau) = ce_\tau(l) \text{ for } \forall \tau \in \Gamma.$$

The final assumption relates the preference ordering of lotteries $l \in L$ to the preference ordering of their reduced acts $f^l \in F$:

**Assumption 3.** *Consistency with Preferences over Reduced Acts. Given $l_1, l_2 \in L$ and their reduced acts, $f_1^l, f_2^l \in F$,*

$$l_1 \succeq l_2 \Longleftrightarrow f_1^l \succeq^f f_2^l.$$

Assumption 3 essentially states that the individual regards $D(ce_l)$ and $D(l)$ equivalent. Assumption 3 suggests how preferences over acts $\succeq^f$ are related to preferences over lotteries $\succeq$. Based on assumption 1, 2, and 3, it can then be shown that

**Theorem.** *Given Assumption 1, 2, and 3, there exists a countably additive probability measure $\pi \in \Pi$ and a continuous and strictly increasing $\phi : \mathbb{R} \to \mathbb{R}$ subjecting to positive affine transformation such that $\succeq$ is represented by the preference functional $V : L \to \mathbb{R}$ given by*

$$V(l) = \int_{\Gamma} \phi\left[EU_\tau(l)\right] d\pi. \tag{3}$$

The proof follows largely Klibanoff et al. (2005) and is omitted here.

## Appendix: Proofs

**Proof of prediction under some popular non-EU theories:** The simple lottery $(\lambda x,\ (1-\lambda)y)$ is a binary prospect. In the evaluation of binary prospects, (cumulative) prospect theory, rank dependent utility theory **(Quiggin, 1982)**, and Guls (1991) disappointment aversion theory gives qualitatively the same evaluation (Observation 7.11.1 in Wakker, 2010, p. 231). Below we illustrate the proof under (cumulative) prospect theory. With a slight abuse of notations, let $v(\cdot)$ denote the value function, $V(\cdot)$ denote the prospect value of a lottery, $w(\cdot)$ denote the probability weighting function. Suppose $x \succ_{CPT} y$, where $\succ_{CPT}$ denotes strict preference relation implied by CPT. Under CPT we have$V(x) > V(y)$. Consider now $\lambda x + (1-\lambda)y$. By CPT we have $V[\lambda x + (1-\lambda)y] = w(\lambda)v(x) + [1-w(\lambda)]v(y) = v(y) + w(\lambda)[v(x) - v(y)]$. Since $w(\lambda)$ increases with $\lambda$, we have $\lambda = 1$ when $x \succ_{CPT} y$. The other case $y \succ_{CPT} x$ can be shown similarly.

**Proof of prediction under Cerreia-Vioglio et al.'s (2015) model:** Let $y$ denote a degenerated lottery with a sure payment. It can be shown that (1) $y \prec x \Longrightarrow \lambda x + (1-\lambda)y \prec x$; (2) $y \succ x \Longrightarrow y \succ \lambda x + (1-\lambda)y$. This again implies that $\lambda \in (0,1)$ occurs only when subjects are indifferent between Option $y$ and Option $x$. This result is due to the special role of sure payment in Cerreia-Vioglio et al.'s (2015) model. The first result is simply an implication of the negative certainty independence axiom. To obtain the second result, observe that if $y \succ x$, then $V(y) > V(x)$ by the representation theorem of Cerreia-Vioglio et al. (2015). In particular, there exists $\tau \in \Gamma$ such that $y = V(y) > ce(x,\tau)$, where $ce(x,\tau)$ denotes the certainty equivalence of Option $x$ with respect to the utility function $u_\tau$. It is then immediate to see that for any $\lambda \in (0,1)$ we have $y = ce(y,\tau) > ce(\lambda x + (1-\lambda)y, \tau)$. This implies that for any $\lambda \in (0,1)$ $V(y) = y = ce(y,\tau) > ce(\lambda x + (1-\lambda)y,\tau) \geq inf_{\tau \in \Gamma}\, ce(\lambda x + (1-\lambda)y,\tau) = V(\lambda x + (1-\lambda)y, \tau)$.[6]

[6] The proof of the second result is gratefully provided by Simone Cerreia-Vioglio via a private correspondence.