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Xana Álvarez and María Gómez-Rúa and Juan Vidal-Puga

Universidade de Vigo, Universidade de Vigo, Universidade de Vigo

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# Risk prevention of land flood: A cooperative game theory approach\*

Álvarez, Xana <sup>†</sup>      Gómez-Rúa, María <sup>‡</sup>      Vidal-Puga, Juan <sup>§</sup>

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## Abstract

Protection against flood risks becomes increasingly difficult for economic and hydrological reasons. Therefore, it is necessary to improve water retention throughout catchment with a more comprehensive approach. Strategies in the land use and measures that are designed to prevent flood risks involve land owners. So, justice issues appear. This paper studies the application of game theory through a cooperative game in order to contribute the resolution of possible agreements among owners and to establish cost / benefit criteria. It is a methodological contribution where land use management for flood retention is analyzed. Specifically, we concentrate on enhancing upstream water retention focusing on the role that forests have as natural water retention measures. This study shows a framework for allocating the compensations among participants based on cooperative game theory and taking into account a principle of stability. We show that it is possible to establish distribution rules that encourage stable payments among land owners. This contribution shows the suitability of this method as a flood risk management tool and as a guide to help decision-making. Compensations and benefits could be established to raise awareness and encourage land owners to cooperate.

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<sup>†</sup>Escola de Enxeñaría Forestal. Universidade de Vigo. Campus A Xunqueira. 36005 Pontevedra. Spain. E-mail: xaalvarez@uvigo.es

<sup>‡</sup>Corresponding author. Facultade de Ciencias Económicas e Empresariais. Universidade de Vigo. Campus Lagoas-Marcosende, 36310 Vigo. Spain. Phone: +34986813506. E-mail: mariarua@uvigo.es.

<sup>§</sup>Facultade de Ciencias Sociais e da Comunicación. Universidade de Vigo. Campus A Xunqueira. 36005 Pontevedra. Spain. E-mail: vidalpuga@uvigo.es

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# 1 Introduction

The economic and social development, in the absence of adequate territory and natural resources planning and management, has triggered in different environmental problems. One of the most significant hazards are flood events (Doocy et al., 2013). Floods endanger lives and cause human tragedy as well as heavy economic losses. A study conducted by the European Environment Agency (2011) stated that Europe suffered over 213 major damaging floods between 1998 and 2009, having caused 1,126 deaths, the eviction of about half a million people and at least EUR 52 billion in insured economic losses. According to the latest studies, everything seems to indicate that a higher flood risk and greater economic damage in Europe will happen in the near future (Jongman et al., 2015; Cook, 2017).

There are different factors that lead to damaging floods. Mainly, damages have been attributed to increasing exposure due to high population growth and economic development in areas prone to floods (Bouwer, 2011; Neumayer and Barthel, 2011; Field et al., 2012; Visser et al., 2014). On the one hand, there is the occupation of the territory by the population attending only to criteria of availability and access to resources. In this case, population growth is increasing the likelihood of the overuse of land in flood-prone areas (Larsen, 2009). In addition to this, it must also be considered that most cities are located on these zones. All of these indicate a mistaken territory planning. Risk to human life and property increases considerably in these potentially flooded areas. On the other hand, these are suitable for agriculture (Cobourn and Lewis, 2011) with fertile lands and close to fluvial channels for irrigation. There are also artificial infrastructures that alter natural dynamics of a river and, therefore, the fluvial system (Nilsson and Berggren, 2000; Lehner et al., 2011). Some examples are fish farms, reservoirs, canals, etc. For that reason, those infrastructures that imply water storage must be managed according to flood risk criteria (Plate, 2002). Another factor that affects flood risks is the potential of floodplains and the adjacent land to the rivers for a land use change. Consequently, the land use change affects the hydrology that determines flood hazard (Wheater and Evans, 2009). And finally, climate change also arises as one of the most recent factors that has worsened the incidence of floods (Milly et al., 2002; Brouwer et al., 2007; Wilby and Keenan, 2012; Hirabayashi et al., 2013; van der Pol et al., 2017). A

study conducted by Alfieri et al. (2015) concluded that the socio-economic impact of river floods in Europe would increase by an average 220% due to climate change by the end of the 21st century. Floods are natural phenomena but climate change can influence rainfall patterns and intensities (Kundzewicz et al., 2014) and, consequently, this could influence the flood hazard (Milly et al., 2002; Rojas et al., 2013; Dankers et al., 2014).

In the last decades, floods are being (1) evaluated with the aim to identify the main reasons why they occur, (2) mitigated in order to minimize their frequency and risk, and (3) reduced and limit their impacts with the right measures. The last group corresponds with the protective measures. Traditionally, these measures are based on the so-called *grey infrastructure*, such as dikes, dams, and other concrete structures (Rasid and Paul, 1987; Roth and Winnubst, 2014; Balica et al., 2015). However, due to increase of land use by human populations, this grey infrastructure may be not sufficient by itself to cope with dynamic flood risk (Tempels and Hartmann, 2014; Nquot and Kulatunga, 2014; Mustafa et al., 2018). A promising alternative is the use of nature-based solutions such as the so-called *Natural Water Retention Measures* (NWRM) as a complement to grey infrastructure (Zeleňáková et al., 2017; Brody et al., 2017; Bhattacharjee and Behera, 2017, 2018). The challenge is to consider multifunctional land uses. They have the potential to enable temporary flood retention and storage, stimulating the provision of other ecosystem services.

Since the NWRM are usually and primarily implemented on private land, a compromise between flood risk management and land exploration is needed (Scherer, 1990; Hartmann, 2016; Thaler et al., 2016). Flood management through an integrated approach combining structural and land use planning measures (Rezende, 2010; Barbedo et al., 2014) is an efficient method of reducing flooding (Miguez et al., 2012). According to different experts (e.g., Directorate-General for Environment (European Commission) (2016); Machac et al. (2018)), politics such as Directive 2007/60/EC and the “Blueprint to Safeguard Europe’s Water Resources” (European Commission, 2007), as well as The Working Group F on Floods (2012), there are two main options for flood protection: to control and retain floods upstream and try to adapt land uses downstream. The last alternative has been widely analysed (e.g., Temmerman et al. (2013); Aerts et al. (2014)), mainly because of the urgency of protecting the safety of people. There are large-scale modelling of flood hazard (Milly et al., 2002; Pappenberger et al., 2012; Dankers et al., 2014) and smaller scale too (te Linde et al., 2011; de Moel et al., 2015). However, in the downstream area we can find the largest water volumes, and its topography, normally flat, does not help the drain. For these reasons, one of the most effective flood protec-

tion measures is to provide more capacity for water retention and flows regulation in the headwaters of river basins. This is the focus that will be analysed in this study.

Specifically, we explore floods reduction through actions carried out upstream. The question that arises is the following: what can make the land owners voluntarily decide to change the uses of their lands in order to reduce the flooding risk? The main challenge is to reach the best agreements upstream-downstream (Machac et al., 2018). With this aim, game theory is selected as a negotiation tool in this contribution. In particular, we need to take into consideration multiple aspects such as economic issues (for example, how to compensate for or incentive flood retention services), property rights (e.g., how to allow temporary flood storage on private land), public participation (e.g., how to ensure the involvement of private landowner), and issues of public subsidies (e.g., how to integrate flood retention in agricultural subsidies).

Among these different aspects, in this paper we focus on a key question: How can land owners be encouraged (or compensated) to adapt their land use and its management strategies in a way that allows for an increase in their water retention capacity? In order to do so, we apply cooperative game theory. This mathematical tool, first developed by a seminal book by von Neumann and Morgenstern (1944), allows to analyze and solve allocation situations where two or more agents (or players) have different interests. As opposed to decision theory, where these interests are unique or coincide, and zero-sum games, where these interests are incompatible, cooperative game theory focuses on situations where a mutually beneficial compromise is possible. Moreover, it differs from non-cooperative game theory in that the allocation can be done from a centralized point of view, instead of a non-cooperative bargaining among the players.

Cooperative game theory has been applied to many areas, such as economics (Shapley and Shubik, 1974), social sciences (Myerson, 1992), political science (Baron and Ferejohn, 1989), optimization (Curiel, 2011), health (González and Herrero, 2004) and environmental management (Lin and Li, 2016). In particular, cooperative game theory has been used in water resources management. Parrachino et al. (2006b); Zara et al. (2006); Parrachino et al. (2006a) provide the basics as well as a review of some applications of cooperative game theory to issues of water resources.

Other applications include acid rain pollution (Kaitala and Pohjola, 1998), water resource system models (Lund and Palmer, 1997), water allocation (Wang et al., 2003), groundwater conflicts (Raquel et al., 2016), transboundary river basins (Gengenbac et al., 2010; Alcalde-Unzu et al., 2015; Shi et al., 2016; Li et al., 2018), and negotiation of marine spatial allocation agreements (Kyriazi et al., 2017). Non-cooperative game theory has also

been applied to water management problems (Bogardi and Szidarovszky, 1976; Carraro et al., 2007; Madani, 2010; Lee, 2012) and water right conflicts (Bergantiños and Lorenzo, 2004; Zanjani et al., 2018).

Other research applies game theory to natural disaster management (see Seaberg et al. (2017) for a recent survey), but very few specifically devoted to flood risk, and always using a non-cooperative approach. In particular, Lai et al. (2015) evaluate flood risk in the Dongjiang river basin (China), and Brown and Bhat (2018) focuses on South Florida’s precipitation trends. In a more general setting, Machac et al. (2017) study flood risk management for two-player non-cooperative games.

In this article we have selected a sharing rule function to help the planner to distribute the total benefit among the owners having into account a principles of stability. As far as we know, our paper presents the first cooperative game theory model applied to flood risk management.

## 2 The model

### 2.1 Cooperative games

A *cooperative game* is a pair  $(N, v)$  where  $N$  is a finite set of *agents* (or *players*) and  $v : 2^N \rightarrow \mathbb{R}$  is the *characteristic function* of the game, where  $v(S)$  represents the *worth* of *coalition*  $S \subseteq N$ . The interpretation is that the worth of  $S$  is the benefit that agents in  $S$  can generate by themselves, without the help of the other agents. As usual, we assume  $v(\emptyset) = 0$ .

A cooperative game  $(N, v)$  is *superadditive* if  $v(S \cup T) \geq v(S) + v(T)$  for all  $S, T \subset N$  with  $S \cap T = \emptyset$ . The interpretation is that two different coalitions can obtain at least as much benefit working together than by themselves. A cooperative game  $(N, v)$  is *monotonic* if  $v(S) \leq v(T)$  for all  $S \subseteq T \subseteq N$ . The interpretation is that no coalition can obtain less by adding new members. A cooperative game  $(N, v)$  is *additive* if  $v(S) = \sum_{i \in S} v(\{i\})$  for all  $S \subseteq N$ . The interpretation is that there exists no benefit of cooperation, since no coalition can improve what their members can achieve by themselves alone.

A main objective of cooperative game theory is to select for every cooperative game an allocation, or a set of allocations, admissible for the players. At this point, two main approaches are possible. One of them is based on stability, where the aim to find stable allocations, in the sense that no coalition of players can improve by themselves. The second one is based on fairness, and it aims to find fair allocations based on some idea of justice.

Let  $(N, v)$  be a cooperative game. An *imputation* of  $(N, v)$  is an allocation  $x \in \mathbb{R}^N$  satisfying  $\sum_{i \in N} x_i = v(N)$  (i.e. the worth of the whole coalition is fully allocated among its members), and  $x_i \geq v(\{i\})$  for all  $i \in N$  (i.e. no agent gets less than she would get by herself). We denote as  $I(N, v)$  the set of imputations of  $(N, v)$ . The *core* of  $(N, v)$  is the set of stable imputations, and it is defined as:

$$Core(N, v) = \left\{ x \in I(N, v) : \sum_{i \in N} x_i \geq v(S) \text{ for all } S \subset N \right\}. \quad (1)$$

The interpretation of the core is intuitive: We look for payoff allocations that no coalition of agents can improve by themselves. The main problem with the core is that it may be empty, as we check in Example 2.3 below. Another (minor) problem is that the core may be huge, which makes it necessary to find some criteria to pick up a core allocation. However, if  $(N, v)$  is an additive game, we avoid both problems, since the core is a singleton given by  $Core(N, v) = \{x\}$  where  $x_i = v(\{i\})$  for all  $i \in N$ .

A *sharing rule* is a function that assigns to each cooperative game  $(N, v)$  in some class of games, a vector  $\phi(N, v) \in \mathbb{R}^N$  such that  $\sum_{i \in N} \phi_i(N, v) = v(N)$ . The most known sharing rule in cooperative game theory is the Shapley value (Shapley, 1953). In order to define it formally, we introduce the following notation: Given a finite set  $N$ , let  $\Pi_N$  denote the set of all orders in  $N$ . Given  $\pi \in \Pi_N$ , let  $Pre(i, \pi)$  denote the set of elements of  $N$  which come before  $i$  in the order given by  $\pi$ , i.e.,

$$Pre(i, \pi) = \{j \in N \mid \pi(j) < \pi(i)\}.$$

The Shapley value of the cooperative game  $(N, v)$  is defined as:

$$Sh_i(N, v) = \frac{1}{n!} \sum_{\pi \in \Pi_N} [v(Pre(i, \pi) \cup \{i\}) - v(Pre(i, \pi))]$$

for all  $i \in N$ .

## 2.2 Landflood games

Assume we have a finite number of land owners on a river basin. These will be our agents, i.e.  $N$  is the set of land owners. We denote these agents as  $N = \{1, \dots, n\}$ . On the other hand, we assume that agents are only affected by actions taken by upstream owners, considering the natural direction of run-off. It refers to the amount of water coming from rainfall running over the land surface or through the soil to groundwater and streamflow.

Hence, upstream/downstream is defined by a directed graph  $\mathcal{G}$  with no cycles, whose nodes are the agents. In particular,  $(i, j) \in \mathcal{G}$  is interpreted as that agent  $i$  is upstream agent  $j$ , so that water fallen on region  $i$  eventually ends up on agent  $j$ 's land.

Forest decreases risk of flooding in downstream lands, and they themselves are also less affected by flooding (Laurance, 2007; Van Dijk et al., 2009) because water retention potential tends to increase along with the extent of forest cover in a water basin (Tyszka, 2009; European Environment Agency, 2015) and they themselves are also less affected by flooding due to its retention capacity and water regulation (Licata et al., 2008; Chang, 2012). In general, land owners can use their land for either forests or for other uses. Woodland covers 165 million hectares in the European Union in 2015, representing 38% of the territory (Eurostat, 2017). For example, the Rhine Atlas has six different land uses (te Linde et al., 2011) and in Spain there are thirteen different land uses (SIOSE, 2011), including different types of crops, pastures, scrub, land without vegetation, forest areas, etc. Taking into account the objective of this study, and in order to keep the model simple, we regroup the uses in two types: *Forests* that retain floods and other uses that *accelerate* them. For this particular study, we define “forest” as a large tract of land covered with trees and underbrush (woodland). While term “other uses” includes the rest of coverages, natural or artificial, that are characterized by the absence of vegetation and trees.

The profit of having a forest is given by a vector  $f \in \mathbb{R}_+^N$ , and the profit given by other uses that accelerate floods is given by a vector  $a \in \mathbb{R}_+^N$ , i.e. when agent  $i \in N$  has a forest in her land, she obtains  $f_i \in \mathbb{R}$ , and otherwise she obtains  $a_i \in \mathbb{R}$ .

Moreover, the positive externality for land  $j \in N$  due to the presence of a forest in land  $i \in N$  is given by a matrix  $B = (b_{ij})_{i,j \in N}$  such that  $b_{ij} > 0$  when  $(i, j) \in \mathcal{G}$  and  $b_{ij} = 0$  otherwise (“*water goes downstream*”).

A *landflood problem* is a tuple  $(N, \mathcal{G}, f, a, B)$  with the properties given above.

Finally, the expected damage gradually increases downstream (Petts and Amoros, 1996; Graf, 1998; Begum et al., 2007; Llobet et al., 2018). This has been demonstrated by studies in specific river basins such as te Linde et al. (2011) in the Rhine basin and Papathanasiou et al. (2013) in the basin of the Ardas river. These areas are the floodplains, characterised by not having slopes and for being the final evacuation of all the water that infiltrates in the hydrographic basin, concentrating the highest water flows. In addition, the larger the forest cover, the more water is retained (Petts and Amoros, 1996; Tyszka, 2009; European Environment Agency, 2015). This again lowers the amount of water flowing as surface run-off and at the outlets of the catchments. Therefore and for both reasons, we assume the further the forest, the larger its beneficial effect.

The simplest way to include this assumption in the model is the following:

**Assumption 1**  $(i, j), (j, k) \in \mathcal{G}$  implies  $(i, k) \in \mathcal{G}$  and  $b_{ik} \geq b_{jk}$ .



A *landflood game* is a cooperative game  $(N, v)$  generated by a landflood problem, where the worth of a coalition  $S$  is given by the maximization problem:

$$v(S) = \max_{F \subseteq S} \Psi(F, S, f, a, B)$$

where<sup>1</sup>

$$\Psi(F, S, f, a, B) = \left\{ \sum_{i \in F} f_i + \sum_{j \in S \setminus F} a_j + \sum_{i \in F, j \in S \setminus F} b_{ij} \right\}.$$

In particular, we say that any set in  $\arg \max_{F \subseteq N} \Psi(F, S, f, a, B)$  is an *optimal configuration* for  $S$ .

Notice that  $\sum_{i \in F} f_i$  is the profit for having the forests,  $\sum_{j \in S \setminus F} a_j$  is the profit for having other uses, and  $\sum_{i \in F, j \in S \setminus F} b_{ij}$  is the profit due to externalities.

**Example 2.1** Let  $N = \{1, 2, 3\}$ ,  $f = (1, 0.99, 2)$  and  $a = (2, 1, 1)$ . Moreover, the graph is given by  $\mathcal{G} = \{(1, 2), (2, 3), (1, 3)\}$ , i.e. player 1 is upstream, player 3 is downstream, and player 2 is between both of them (see Figure 1). We assume that the benefit of lands 2 and 3 increase by 2 each when land 1 is a forest, and the benefit of land 3 increases by 1 if land 2 is a forest. Hence,  $b_{12} = b_{13} = 2$ ,  $b_{23} = 1$ , and  $b_{ij} = 0$  otherwise.

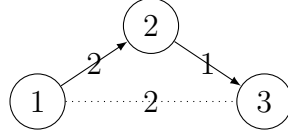


Figure 1: Example of river basin.

The worth of each coalition, as well as the optimal configuration that produces it, is given in the following table:

$S$	$v(S)$	forest	other uses
$\{1\}$	2	-	$\{1\}$
$\{2\}$	1	-	$\{2\}$
$\{3\}$	2	$\{3\}$	-
$\{1, 2\}$	4	$\{1\}$	$\{2\}$
$\{1, 3\}$	4	$\{1\}$	$\{3\}$
$\{2, 3\}$	2.99	$\{2\}$	$\{3\}$
$N$	7	$\{1\}$	$\{2, 3\}$

<sup>1</sup>We use the convention of using  $i$  for a generic element  $F$  (i.e. agents with forests) and  $j$  for a generic element of  $N \setminus F$  (i.e. agents with other uses). When it is undefined whether an agent is a forest or not, we use either term indistinguishably.

Notice that the landflood game given in Example 2.1 satisfies Assumption 1, because  $b_{13} > b_{23}$ , i.e. for agent 3 it is more favorable to have agent 1 as forest alone than agent 2 as forest alone. In this example, the core is nonempty, as for instance the Shapley value  $Sh(N, v) = (2.83, 1.83, 2.33) \in Core(N, v)$ . This allocation is achieved by a two-step procedure: Firstly, optimal configuration  $F = \{1\}$  (i.e. only agent 1 is a forest) is implemented, so that the direct benefit is  $(1, 3, 3)$ . Secondly, in compensation for agent 1 being a forest, agent 2 transfers 1.17 units of utility and agent 3 transfers 0.67 of utility to agent 1.

In order to emphasize the advantage of considering a centralized model as the one we are proposing in this paper, we briefly compare it with the situation in which the agents act in a non cooperative way, i.e., without compensations among themselves.

We represent the problem given in Example 2.1 as follows. Assume that we have two  $2 \times 2$  matrices so that agent 1 chooses the row, agent 2 chooses the column, and agent 3 chooses the matrix.

3 is a forest			3 other uses		
	2 forest	2 other uses		2 forest	2 other uses
1 forest	(1, 0.99, 2)	(1, 3, 2)	1 forest	(1, 0.99, 4)	(1, 3, 3)
1 other uses	(2, 0.99, 2)	(2, 1, 2)	1 other uses	(2, 0.99, 2)	(2, 1, 1)

In order to compute the final payoff allocation in this example, we describe the final payoff allocation as follows: The first component of each vector is the payment to agent 1, the second component is the payment to agent 2, and the third component is the payment to agent 3. Each agent has two possible strategies: or devote her land to have a forest or devote it to other uses.

In this situation, we have the so called *iterated elimination of strictly dominated strategies* (Aumann, 1976), which allows us to predict what would be the final outcome assuming some mild rationality in the agents.

Firstly, agent 1 should choose the second file, since it would give her a larger final payoff (2) than choosing the first file (1), independently of whatever the other agents do. Knowing that, agent 2 should choose the second column, since it would give her a larger final payoff (1) than choosing the first column (0.99), independently of whatever agent 3 does. Knowing that, agent 3 should choose the first matrix, since it would give her a larger final payoff (2) than choosing the second matrix (1).

Then, the only rational choices in this game are that agents 1 and 2 devote their land to other uses and agent 3 will be a forest, resulting in a final payoff allocation of  $(2, 1, 2)$ ,

which implies that each agent is worse off than with the Shapley value  $(2.83, 1.83, 2.33)$ .

Not always the Shapley value belongs to the core, as next example shows:

**Example 2.2** Let  $N = \{1, 2, 3, 4, 5\}$ ,  $f = (0, 0, 0, 0, 1)$ ,  $a = (1, 0, 0, 0, 0)$ . Moreover, the graph (see Figure 2) is given by  $\mathcal{G} = \langle \{(1, 2), (1, 3), (1, 4), (1, 5), (2, 5), (3, 5)\} \rangle$ , and  $b_{ij} = 1$  for all  $(i, j) \in \mathcal{G}$ .

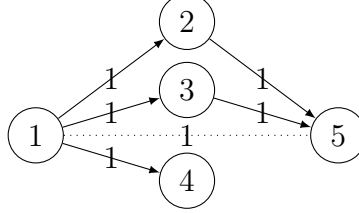


Figure 2: Example of landflood game with the Shapley value out of the core.

There exist multiple optimal configurations, as for example  $F = \{1, 5\}$  and also  $F' = \{1, 2, 3\}$ . Let  $(N, v)$  be the cooperative game generated by this landflood game. In this case,  $Sh(N, v) = (1.5, 0.5, 0.5, 0.42, 1.08)$ . However,  $v(\{1, 2, 3, 4\}) = 3 > 2.92 = \sum_{i=1}^4 Sh_i(N, v)$ . Hence,  $Sh(N, v) \notin Core(N, v)$ . Nonetheless,  $Core(N, v)$  is nonempty, as for example  $(1, 0, 1, 1, 1) \in Core(N, v)$ .

In next Sections, we propose a method to find core allocations.

Given the private ownership of the land use, any allocation that does not belong to the core can be blocked by a group of agents, leading to potential loss of efficiency in the location of the forests.

In general, the assumption “the further the forest, the larger its beneficial effect” is key for the emptiness of the core, as next example (which does not satisfy Assumption 1) shows:

**Example 2.3** Let  $N = \{1, 2, 3, 4, 5\}$ ,  $f = (0, 0, 0, 0, 1)$ ,  $a = (1, 0, 0, 0, 0)$ . Moreover, the graph (see Figure 3) is given by  $\mathcal{G} = \langle \{(1, 2), (1, 3), (2, 5), (3, 4), (4, 5)\} \rangle$ ,  $b_{12} = b_{13} = b_{25} = b_{34} = b_{45} = 1$  and  $b_{ij} = 0$  otherwise.

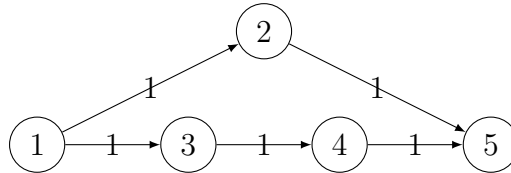


Figure 3: Example of graph that induces a cooperative game with empty core.

The worth of some coalitions, as well as their respective optimal configurations<sup>2</sup>, is given in the following table:

$S$	$v(S)$	forest	other uses
$\{1, 2, 3, 4\}$	2	$\{3\}$	$\{1, 2, 4\}$
$\{1, 2, 3, 5\}$	3	$\{1, 5\}$	$\{2, 3\}$
$\{1, 2, 4, 5\}$	3	$\{2, 4\}$	$\{1, 5\}$
$\{1, 3, 4, 5\}$	3	$\{3, 5\}$	$\{1, 4\}$
$\{2, 3, 4, 5\}$	2	$\{3, 5\}$	$\{2, 4\}$
$N$	3	$\{1, 5\}$	$\{2, 3\}$ .

Notice that a core element  $x$  in the landflood game given in example 2.3 should satisfy  $x_1 + x_2 + x_3 + x_4 \geq 2$ ,  $x_2 + x_3 + x_4 + x_5 \geq 2$ , and  $x_i + x_j + x_k + x_l \geq 3$  otherwise, which implies that  $x_1 + x_2 + x_3 + x_4 + x_5 \geq 3.25$ . This is not possible because  $v(N) = 3$ , and hence the core is empty for this game.

### 3 Saturated landflood games

In order to analyze the nonemptiness of the core in general landflood games, we use the concept of *saturated landflood games*, defined as follows:

**Definition 3.1** *We say that a landflood game is saturated if the two following conditions hold:*

- *For each pair of adjacent lands, there exists an optimal configuration in which both of them are forests.*
- *For each pair of adjacent lands, there exists an optimal configuration in which none of them are forests.*

It is not difficult to check that the landflood problem given in Example 2.3 is saturated. As opposed, the landflood problem  $(N, v)$  presented in Example 2.1 is not saturated, since the only optimal configuration is  $F = \{1\}$ . However, there exists a saturated landflood game  $(N, w)$  satisfying  $v(S) \leq w(S)$  for all  $S \subset N$  and  $v(N) = w(N)$ . In view of (1), this implies that  $Core(N, w) \subseteq Core(N, v)$ . Hence, the nonemptiness of  $Core(N, w)$  implies the nonemptiness of  $Core(N, v)$ , and any core allocation in  $(N, w)$  is also a core allocation in  $(N, v)$ . Next, we show how to generate a possible  $(N, w)$  from  $(N, v)$  in Example 2.1. We follow the next steps:

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<sup>2</sup>For  $S = N$ , it is irrelevant whether agent 4 is a forest or not.

1. Reduce  $b_{12}$  from 2 to 0.99 and increase  $f_1$  from 1 to 2.01. With these changes,  $v(\{1\})$  and  $v(\{1, 3\})$  increase, whereas the rest of  $v(S)$  (including  $v(N)$ ) remain unchanged. Furthermore,  $F = \{1, 2\}$  becomes a new optimal configuration.
2. Reduce  $b_{23}$  from 1 to 0, removing arc  $(3, 4)$ ; and increase  $f_2$  from 0.99 to 1.99. With these changes,  $v(\{2\})$  and  $v(\{2, 3\})$  increase, whereas the rest remain unchanged. Moreover, agent 1 and 3 become adjacent.
3. Reduce  $b_{13}$  from 2 to 1, and increase  $f_1$  from 2.01 to 3.01. With these changes,  $v(\{1\})$  and  $v(\{1, 2\})$  increase, whereas the rest remain unchanged. Furthermore,  $N$  and  $\{1, 3\}$  become two new optimal configurations.
4. Reduce  $b_{12}$  from 0.99 to 0, removing arc  $(1, 2)$ ; and increase  $a_2$  from 1 to 1.99. With these changes, each  $v(S)$  remains the same.
5. Reduce  $b_{13}$  from 1 to 0, removing arc  $(1, 3)$ , and increase  $a_3$  from 1 to 2. With these changes, the landflood problem becomes trivially saturated (because there are no adjacent nodes).

Let  $(N, w)$  be resulting landflood game. Then,  $(N, w)$  is both saturated and additive (since there are no externalities). In particular,  $Core(N, w) = \{(3.01, 1.99, 2)\}$ . We then deduce that  $(3.01, 1.99, 2) \in Core(N, v)$ .

In general, we can replicate this procedure in order to generate a saturated landflood game from each non-saturated one, as next proposition shows:

**Proposition 3.1** *For each landflood game  $(N, v)$ , there exists a saturated landflood game  $(N, w)$  with at most as many arcs and such that*

- $v(S) \leq w(S)$  for all  $S \subset N$ .
- $v(N) = w(N)$ .

*Moreover, if  $(N, v)$  satisfies Assumption 1, it is possible to find such a  $(N, w)$  satisfying Assumption 1 also.*

**Proof.** Let  $(N, v)$  let be a landflood game defined by  $f$ ,  $a$ , and  $B$ . We proceed by double induction on the cardinality of  $\mathcal{G}$ ,  $|\mathcal{G}|$ , and the cardinality of

$$\Omega = \left\{ (i, j) \in \mathcal{G} : i, j \text{ adjacent and } \max_{F \subseteq N: |F \cap \{i, j\}| \neq 1} \Psi(F, N, f, a, B) < v(N) \right\},$$

the set of adjacent nodes for which no optimal configuration has either both or none of them as forests. If  $\mathcal{G} = \emptyset$ , then there are no externalities and  $(N, v)$  is saturated, so we

take  $w = v$ . Assume then the result holds when the cardinality of the graph is  $|\mathcal{G}| - 1$  or lower. If  $\Omega = \emptyset$ , then  $(N, v)$  is saturated, and we take  $w = v$ . Assume now  $\Omega \neq \emptyset$ . We can assume w.l.o.g. that  $(1, 2) \in \Omega$ , so that 1 is before 2 in the graph. Assume that no optimal configuration has both 1 and 2 as forests (the case in which none of them are forest is analogous). We define  $(N, v')$  as follows. Let

$$F' \in \arg \max_{F \subseteq N: \{1,2\} \subseteq F} \Psi(F, N, f, a, B)$$

be the a configuration with maximum value among those in which both 1 and 2 are forests. By assumption, this configuration is not optimal, which implies that there exists some other optimal configuration  $F'' \subset N$  with  $\alpha = \Psi(F'', N, f, a, B) - \Psi(F', N, f, a, B) > 0$ . Since 1 and 2 are adjacent with 1 before 2 in the graph, we deduce  $b_{12} > 0$ . Hence,  $\alpha' = \min \{\alpha, b_{12}\} > 0$ . Now, define  $(N, v')$  by taking  $f'_1 = f_1 + \alpha'$ ,  $b'_{12} = b_{12} - \alpha'$ , and  $b'_i = f_i$ ,  $a'_j = a_j$ , and  $b'_{ij} = b_{ij}$  otherwise. Since, 1 and 2 are adjacent, we deduce that  $(N, v')$  satisfies Assumption 1 when  $(N, v)$  does. It is straightforward to check that  $F''$  is still optimal in  $(N, v')$ , and so  $v'(N) = v(N)$ . Moreover, for each  $S \subseteq N$ , we also have  $v(S) \leq v'(S)$ , with strict inequality when  $1, 2 \in S$  and there exists an optimal configuration in  $S$  in which both 1 and 2 are forests. We have two cases:

1. When  $\alpha' = b_{ij}$ , agents 1 and 2 are not adjacent in  $(N, v')$ .
2. When  $\alpha' = \alpha$ ,  $F$  is optimal in  $(N, v')$ .

In the first case, we apply the induction hypothesis on  $|\mathcal{G}|$ . In the second case, we apply the induction hypothesis on  $|\Omega|$ . In either case, by the induction hypothesis we know that there exists a saturated landflood game  $(N, w)$  satisfying Assumption 1 if  $(N, v)$  does, and such that  $v'(S) \leq w(S)$  for all  $S \subset N$  and  $v'(N) = w(N)$ . Since  $v(S) \leq v'(S)$  for all  $S \subset N$  and  $v'(N) = v(N)$ , we deduce our result. ■

The relevance of Proposition 3.1 is that  $Core(N, w) \subseteq Core(N, v)$ , and hence it is enough to study the nonemptiness of the core for saturated landflood games. Obviously, Assumption 1 plays a role in this study, since by Example 2.3 we know that there exist saturated landflood games with empty core when Assumption 1 does not hold.

In Example 2.1 the resulting saturated game has no externalities and hence it is trivially additive, which allows us to identify a core element. In general, this is not the case as next example shows:

**Example 3.1** *Let  $N = \{1, 2, 3\}$ ,  $f = (0, 0, 1)$  and  $a = (1, 0, 0)$ . Moreover, the graph is given by  $\mathcal{G} = \{(1, 2), (2, 3), (1, 3)\}$ , as in Example 2.1 (see Figure 1). Let  $b_{12} = b_{13} = b_{23} = 1$ .*

The worth of each coalition and the optimal configurations that generate them are given in the following table:

$S$	$v(S)$	optimal configurations
$\{1\}$	1	$\emptyset$
$\{2\}$	0	$\emptyset, \{2\}$
$\{3\}$	1	$\{3\}$
$\{1, 2\}$	1	$\emptyset, \{1\}, \{2\}$
$\{1, 3\}$	2	$\{3\}$
$\{2, 3\}$	1	$\{2\}, \{3\}, \{2, 3\}$
$N$	2	$\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}$

It is clear from the table that this landflood problem is saturated.

The landflood problem presented in Example 3.1 is saturated. Moreover, it has externalities that cannot be reduced by increasing  $a$  or  $f$ , because it would imply an increase in  $v(N)$ . However, it is still possible to remove some adjacent arc (in this case, either  $(1, 2)$  or  $(2, 3)$ ) without changing the associated landflood game. We claim that this is true in general:

**Claim 3.1** *Under Assumption 1, each saturated landflood problem  $(N, \mathcal{G}, f, a, B)$  satisfies one of the following conditions:*

1.  $\mathcal{G} = \emptyset$ , or
2. *there exists some  $(i, j) \in \mathcal{G}$  such that  $v(S) = v^{-ij}(S)$  for all  $S \subseteq N$ , where  $(N, v)$  is the landflood game generated by  $(N, \mathcal{G}, f, a, B)$  and  $(N, v^{-ij})$  is the landflood game generated by  $(N, \mathcal{G} \setminus \{(i, j)\}, f, a, B')$  with  $b'_{kl} = b_{kl}$  for all  $(k, l) \in \mathcal{G} \setminus \{(i, j)\}$ .*

Even though we do not have a formal proof, we have checked it true in more than 640,000 randomly generated landflood games taking natural restrictions. The algorithm used is described in Appendix.

Claim 3.1 allows us to find core allocations.

**Proposition 3.2** *Under Claim 3.1, the core is nonempty in any landflood game satisfying Assumption 1.*

**Proof.** Under Proposition 3.1, for any landflood game  $(N, v)$  satisfying Assumption 1, we can find a saturated landflood game  $(N, w)$  with  $v(S) \leq w(S)$  for all  $S \subset N$  and

$v(N) = w(N)$  and satisfying also Assumption 1. Since  $v(S) \leq w(S)$  for all  $S \subset N$  and  $v(N) = w(N)$ , we have  $Core(N, w) \subseteq Core(N, v)$  and it is enough to prove  $Core(N, w) \neq \emptyset$ . In case  $(N, w)$  has no externalities, it is additive and hence  $Core(N, w) = \{x\}$  where  $x_i = w(\{i\})$  for all  $i \in N$ . Hence,  $x \in Core(N, v) \neq \emptyset$ . In case  $(N, w)$  has externalities, under Claim 3.1.2, we can reduce the cardinality of the graph and repeat the process until the game becomes additive. ■

## 4 Stable sharing rules

Given that we can find core allocations, we look for a way to choose a reasonable one, i.e. we look for a sharing rule in the class of landflood games. A natural candidate can be the Shapley value. However, the Shapley value may lay outside the core (Example 2.2), even when the core is nonempty.

In this section we present three alternative core sharing rules. The first one (Algorithm 1) applies the procedure used in the proofs of Proposition 3.1 and Proposition 3.2 in the most favorable way for agents located upstream. The second one (Algorithm 2) applies the algorithm in the most favorable way for agents located downstream. Finally, we propose an intermediate sharing rule that balances both approaches.

In Algorithm 1, lines 2-10 decrease/remove arcs  $b_{ij}$  by increasing  $f_i$ . Once these transfers are exhausted, lines 11-18 decrease/remove arcs  $b_{ij}$  by increasing  $a_j$ . In case of more than one adjacent arc, the ones that are upstream should be taken first, so that upstream agents are more favoured. Once these transfers are also exhausted, the problem is saturated. Line 19 removes arcs unused by any coalition. Under Claim 3.1, we can always find such arc. In case of more than one, the ones that are downstream should be taken first, so that upstream agents have the chance to be more favored in the loop. Once  $\mathcal{G} = \emptyset$ , the game is additive and line 20 picks up the stand-alone solution.

In Algorithm 2, preference is given to increase  $a_j$  before increasing  $f_i$ . In case of more than one adjacent arc, the ones that are downstream should be taken first, so that downstream agents are more favored. Analogously, line 19 removes arcs unused by any coalition. In case of more than one, the ones that are upstream should be taken first, so that downstream agents have the chance to be more favored in the loop.

Finally, a compromise value among solutions  $x$  and  $y$ , provided respectively by Algorithm 1 and Algorithm 2, is the following:

$$z_i = \frac{x_i + y_i}{2}$$

for all  $i \in N$ . Convexity of the core assures that  $z \in Core(N, v)$ .



---

**Algorithm 1** Optimal sharing rule for upstream agents

---

Input: Landflood problem  $(N, \mathcal{G}, f, a, B)$ Output:  $x \in \mathbb{R}^N$ 

```
1: while  $\mathcal{G} \neq \emptyset$  do
2:   for each  $(i, j)$  adjacent in  $\mathcal{G}$  do
3:      $\alpha \leftarrow v(N) - \max_{F \subseteq N: \{i, j\} \subseteq F} \Psi(F, N, f, a, B)$ 
4:     if  $\alpha > 0$  then
5:       if  $\alpha \geq b_{ij}$  then
6:          $f_i \leftarrow f_i + b_{ij}$ 
7:         remove arc  $(i, j)$  from  $\mathcal{G}$ 
8:       else
9:          $f_i \leftarrow f_i + \alpha$ 
10:         $b_{ij} \leftarrow b_{ij} - \alpha$ 
11:      $\beta \leftarrow v(N) - \max_{F \subseteq N: F \cap \{i, j\} = \emptyset} \Psi(F, N, f, a, B)$ 
12:     if  $\beta > 0$  then
13:       if  $\beta \geq b_{ij}$  then
14:          $a_j \leftarrow a_j + b_{ij}$ 
15:         remove arc  $(i, j)$  from  $\mathcal{G}$ 
16:       else
17:          $a_j \leftarrow a_j + \beta$ 
18:          $b_{ij} \leftarrow b_{ij} - \beta$ 
19:   remove arc in  $\mathcal{G}$  unused by any coalition  $S \subseteq N$ 
20: for each  $i \in N$  do  $x_i \leftarrow \max\{f_i, a_i\}$ 
21: present  $x$  as solution.
```

---

---

**Algorithm 2** Optimal sharing rule for downstream agents

---

Input: Landflood problem  $(N, \mathcal{G}, f, a, B)$ Output:  $y \in \mathbb{R}^N$ 

```
1: while  $\mathcal{G} \neq \emptyset$  do
2:   for each  $(i, j)$  adjacent in  $\mathcal{G}$  do
3:      $\beta \leftarrow v(N) - \max_{F \subseteq N: F \cap \{i, j\} = \emptyset} \Psi(F, N, f, a, B)$ 
4:     if  $\beta > 0$  then
5:       if  $\beta \geq b_{ij}$  then
6:          $a_j \leftarrow a_j + b_{ij}$ 
7:         remove arc  $(i, j)$  from  $\mathcal{G}$ 
8:       else
9:          $a_j \leftarrow a_j + \beta$ 
10:         $b_{ij} \leftarrow b_{ij} - \beta$ 
11:       $\alpha \leftarrow v(N) - \max_{F \subseteq N: \{i, j\} \subseteq F} \Psi(F, N, f, a, B)$ 
12:      if  $\alpha > 0$  then
13:        if  $\alpha \geq b_{ij}$  then
14:           $f_i \leftarrow f_i + b_{ij}$ 
15:          remove arc  $(i, j)$  from  $\mathcal{G}$ 
16:        else
17:           $f_i \leftarrow f_i + \alpha$ 
18:           $b_{ij} \leftarrow b_{ij} - \alpha$ 
19:      remove arc in  $\mathcal{G}$  unused by any coalition  $S \subseteq N$ 
20: for each  $i \in N$  do  $y_i \leftarrow \max\{f_i, a_i\}$ 
21: present  $y$  as solution.
```

---

**Example 4.1** *Following the same procedure as the one previously used to obtain a saturated game in Example 2.1, we compute the three rules for all the examples of the paper. The obtained results are the following:*

	$x$	$y$	$z$
<i>Example 2.1</i>	(3.01, 1.99, 2)	(2, 2, 3)	(2.505, 1.995, 2.5)
<i>Example 2.2</i>	(1, 1, 1, 0, 1)	(1, 1, 1, 0, 1)	(1, 1, 1, 0, 1)
<i>Example 3.1</i>	(1, 0, 1)	(1, 0, 1)	(1, 0, 1)

## Discussion

Our study shows the possibility of continuing to progress in the reduction of flood risk through new alternatives such as incentives to owners to change land use upstream of the river basin, agreements between management and owners, and allocation of benefits in case of uses that enhance water retention. This means an advance at a scientific, technical, political, social and environmental level in this field.

Firstly, the purpose of European Floods Directive (European Commission, 2007) was to establish a framework for the assessment and management of flood risks, aiming at the reduction of adverse consequences for human health, environment, cultural heritage and economic activity associated with floods. With this objective, the Directive requires Member States to carry out flood risk management plans. Accordingly, public bodies carry out specific plans to detect the main risks of flooding of the catchments they manage, as well as to locate the areas with the highest risk of flooding. The objective of flood risk management plans is to improve the territory planning and the flood zones management. In this sense, many studies have tried to model different situations depending on hydrological and weather data. Some examples are those carried out by Kourgialas and Karatzas (2011) and Levy (2005), as well as in the review of (Sanyal and Lu, 2004). Others try to predict future conditions, giving the opportunity to know how will be the natural response if we modify something. For example, Purvis et al. (2008) offer a methodology to estimate the probability of future coastal flooding given uncertainty over possible sea level rise.

On the other hand, the EU Water Framework Directive (European Parliament and Council, 2000) and the Blueprint to Safeguard Europe's Water Resources (European Commission, 2012) also recognise the potential for rural land use change for supporting water management objectives. In addition, the Directorate-General for Environment from the European Commission highlights the role of natural approaches for protecting

water resources and managing flood risks. It emphasizes that NWRM are multi-functional measures that aim to protect and manage water resources and have the potential to provide multiple benefits, for example: flood risk reduction, water quality improvement, groundwater recharge, or habitat improvement (European Commission, 2014).

It seems that there are a wide range of different techniques, analyses and studies that allow us to model situations, make estimates and known risk areas. In addition, there are different recommendations on techniques of land use management to safeguard and enhance the water storage potential of landscape, soil, and aquifers. Therefore, it is recommended to continue advancing on how to avoid these natural risks. It is necessary to identify synergies to reduce flood risk. Sharing this knowledge in different areas and sectors is an exercise that should be carried out more frequently than is currently done. For it, there are other types of tools that are currently not used in the flood risk field but have great potential. An example is the cooperative game theory that we apply in this work.

Taking into account all these advances, it seems that a new perspective is possible by replacing “flood control” with “risk management”. This is possible through influencing some aspects that can reduce flood risk instead of trying to make a total control. In addition, this implies social and economic aspects that require the collaboration of all stakeholders and especially that of the landowners. One of these strategies is to improve water retention in the territory, controlling, as far as possible, the generation of runoff that sometimes results in catastrophic floods. It is important to know and simulate the floods in the cities, because they imply human, economic and environment damages. In order to do that, it is necessary to consider all the factors that influence the flow of water that arrive to these cities. These factors are present in all basin, no only downstream. Therefore, changes, alterations or situations that take place upstream influence what may happen downstream.

As a result, in this work we evaluate the potential of game theory through cooperative games as a useful tool in flood risk preventios, as we evaluate what benefits/costs would cause changes in land use in the upper areas of the catchment.

## Conclusions

With the methodology, alternatives and assumptions that are consider in this study, a more comprehensive and basin-wide approach has been analysed in order to improve and highlight the importance of retaining water in the catchment and upstream. This

has been achieved through the application of cooperative game theory. Moreover, we explore a barely used tool to solve allocation in flood risk negotiations. Our results are positive in the sense that they show that stable incentives are possible in order to encourage landowners to contribute to flood risk reduction. Moreover, our proofs are constructive. We present two algorithms which actually implement stable compensation allocations. We can use one or another depending on which kind of land owners (upstream or downstream) are to be more favoured. An average of both can also be used in order to implement a more balanced allocation.

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## Appendix

Algorithm 3 was used to randomly simulate landflood problems. We consider four different variables whose values depend on the type of basin that is being analyzed. In this study, representative values have been considered according to expert criteria in order to include the widest range of values that can be found in real cases. The main objective is to carry out a first approach to demonstrate that it is possible to apply a cooperative game in this kind of conflicts. Other combinations of values are possible.

A) Number of nodes ( $n = 10$  or  $n = 60$ ). In the first case ( $n = 10$ ), the objective is to represent and analyse different cases, among which they may be: small basins, sub-basins, a small scale of management with special interest (for example in case it is necessary to make decisions on this scale) and homogeneous basins with the same land use. A greater number of nodes ( $n = 60$ ) has been selected to include large basins, more heterogeneous basins in land use and larger units of land management where a larger scale of analysis is necessary.

B) Average number of incoming arcs in each node ( $in = 1$ ,  $in = 2$ , or  $in = 3$ ). This happens when each node has, at least, one direct upstream neighbour. This may be the case of large extensions of land. On the other hand, we consider a maximum of three

incoming arcs, to represent the other real case of small land owners.

C) Average number of outgoing arcs in each node ( $out = 1$ ,  $out = 2$ , or  $out = 3$ ). This is analogous to the incoming arcs case.

D) Ratio between the average benefit/cost of changing from forest to other uses (or vice-versa) and the average benefit of externalities ( $r = 0.5$  or  $r = 1$ ). We establish that the cost / benefit is twice as much upstream than in the node itself ( $r = 0.5$ ) or, at least, equivalent ( $r = 1$ ).

On Algorithm 3, lines 10-12 build  $H$  levels in such a way that by randomly creating  $out$  arcs from each node in a level to a respective number of nodes in the next level, the average number of incoming arcs is  $in$ . Lines 13-18 build the network. Lines 19-20 generate the externalities in such a way that Assumption 1 is satisfied. Lines 21-23 generate vectors  $f$  and  $a$  in such a way that the expected absolute difference  $|f_i - a_i|$  divided by  $\frac{\sum_{(i,j) \in \mathcal{G}} b_{ij}}{|N|}$  is approximately  $r$ . Function  $rand()$  returns a random number between 0 and 1.

Algorithm 3 together with Algorithm 1 were implemented using C++ and run on a 64-bit Intel Core i7-4790K CPU 4.00 GHz with 7,7 GiB. In all the instances Claim 3.1 was satisfied. Sample sizes are summarized in the last Table.

---

**Algorithm 3** Random landflood problem

---

Input:  $n, in, out, r$ Output: random landflood problem  $(N, \mathcal{G}, f, a, B)$ 

```
1: if  $in = out$  then
2:    $H \leftarrow$  random integer between 2 and  $\frac{n}{in}$ .
3:   for  $h = 1, \dots, H$  do  $s_h \leftarrow \frac{n}{H}$ 
4: else if  $in < out$  then
5:    $H \leftarrow$  random integer between 2 and  $\log_{\frac{out}{in}} \left(1 + n \cdot \frac{out-in}{in^2}\right)$ 
6:    $s_1 \leftarrow n \cdot \frac{\frac{out}{in} - 1}{\left(\frac{out}{in}\right)^H - 1}$ 
7:   for  $h = 2, \dots, H$  do  $s_h \leftarrow \frac{out}{in} \cdot s_{h-1}$ 
8: else
9:    $H \leftarrow$  random integer between 2 and  $\log_{\frac{in}{out}} \left(1 + n \cdot \frac{in-out}{out^2}\right)$ 
10:   $s_H \leftarrow n \cdot \frac{\frac{in}{out} - 1}{\left(\frac{in}{out}\right)^H - 1}$ 
11:  for  $h = H - 1, \dots, 1$  do  $s_h \leftarrow \frac{in}{out} \cdot s_{h+1}$ 
12: adjust  $s$  so that  $s_h \in \mathbb{N}$  for all  $h$  and  $\sum_{h=1}^H s_h = n$ 
13: define  $\{S_1, \dots, S_H\}$  partition of  $N$  so that  $|S_h| = s_h$  for all  $h$ 
14: for  $h = 1$  to  $H - 1$  do
15:   for all  $i \in S_h$  do
16:     randomly choose  $in$  nodes  $j_1, \dots, j_{in}$  in  $S_{h+1}$ 
17:     for  $l = 1$  to  $in$  do
18:        $\mathcal{G} \leftarrow \mathcal{G} \cup \{(i, j_l)\} \cup \{(j, j_l) : (j, i) \in \mathcal{G}\}$ 
19: for all  $(i, j) \in \mathcal{G}$  with  $i \in S_l, j \in S_m$  do
20:    $b_{ij} \leftarrow$  random number between  $m - l - 1$  and  $m - l$ 
21: for all  $i \in N$  do
22:    $f_i \leftarrow \frac{3 \cdot r}{|N|} \cdot \sum_{(j,k) \in \mathcal{G}} b_{jk} \cdot rand()$ 
23:    $a_i \leftarrow \frac{3 \cdot r}{|N|} \cdot \sum_{(j,k) \in \mathcal{G}} b_{jk} \cdot rand()$ 
```

---

Nodes	incoming arcs	outcoming arcs	ratio	sample size	Av. comp. time (sec.)	
10	1	1	0.5	20,000	0.00018	
			1	20,000	0.00018	
		2	0.5	20,000	0.00006	
			1	20,000	0.00006	
	2	1	0.5	20,000	0.00006	
			1	20,000	0.00006	
		2	0.5	20,000	0.00063	
			1	20,000	0.00044	
		3	0.5	20,000	0.00025	
			1	20,000	0.00024	
	3	2	0.5	20,000	0.00023	
			1	20,000	0.00023	
		3	0.5	20,000	0.00028	
			1	20,000	0.00029	
	60	1	1	0.5	20,000	1.11
				1	20,000	2.91
2			0.5	20,000	0.04	
			1	20,000	0.03	
3			0.5	20,000	0.06	
			1	20,000	0.05	
2			1	0.5	20,000	0.02
				1	20,000	0.01
		2	0.5	20,000	14.51	
			1	20,000	7.36	
		3	0.5	20,000	2.79	
			1	>20,000	0.64	
3		1	0.5	20,000	0.02	
			1	20,000	0.01	
		2	0.5	>20,000	0.04	
			1	20,000	0.10	
		3	0.5	20,000	15.09	
			1	20,000	9.42	
TOTAL				>640,000		



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