The (intertemporal) equity-efficiency trade-off of fiscal consolidation

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Abstract

We build a dynamic general equilibrium model with heterogeneous households and capital-skill complementarity in the production function to study aggregate and distributional effects of fiscal consolidation policies when government uses a rich set of productivity-enhancing spending instruments along with utility-enhancing spending and tax fiscal instruments. Fiscal policy is conducted through simple fiscal rules. We study both ad-hoc and optimized fiscal rules. Our main results indicate that ad-hoc fiscal consolidation policies, either through spending cuts or tax increases, are recessionary and entail an equity-efficiency trade-off in the short- and medium-run. That is spending-based consolidation policies are less recessionary but come at a higher distributional cost; whereas tax-based consolidation policies result in sharper output losses but have smoother distributional effects. In addition, fiscal consolidation policies through optimized fiscal rules can be expansionary and social welfare enhancing while at the same time balance the equity-efficiency trade-off.

Keywords: Debt consolidation, distributional effects, fiscal policy, optimized fiscal rules
JEL: E62, H52, H54, H63

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1 Introduction

The 2008 world financial crisis found most European countries in a vulnerable fiscal position with high deficits and debts above the 3% and 60% of the Stability and Growth Pact limits.[1] As a result several European governments have been forced to take restrictive fiscal actions the so called fiscal consolidation.

By now most studies find that fiscal consolidation entails an intertemporal trade-off for the main macroeconomic aggregates. That is the early phase of fiscal pain with public spending cuts or/and tax increases to achieve the lower public debt target and the longer-run phase of fiscal gain when debt reduction has been achieved and the resulting fiscal space can be utilized to increase spending or reduce taxes. However, fiscal consolidation seems to also have important distributional implications. For example Furceri et al. (2015) find that during fiscal consolidation periods the lowest income and wealth quantiles of the population became worse off in terms of net income. In addition, consolidation efforts usually come at the expense of growth-friendly spending items such as spending on public investment and education which further harms the prospects of long term growth.[2]

This paper seeks to answer questions like what are the aggregate and distributional implications of fiscal consolidation policies? Does fiscal consolidation generate an equity-efficiency trade-off? Do tax- and spending-based fiscal consolidation policies differ in their distributional implications? We examine whether answers to these questions depend on the fiscal policy mix chosen, on the type of heterogeneity incorporated in the model and the measure of inequality under consideration.

To this end, we develop a dynamic general equilibrium model with heterogeneous households calibrated for Euro Area over the period 2001-2015. The model includes two types of households, capital-skill complementarity in the production function in the spirit of Krusell et al. (2000) and endogenous human-capital accumulation. Households differ both in the type of labour they supply and their access to capital and financial markets.[3] Households that can save in the form of government bonds, own physical capital and firms and supply skilled labour services are referred to as *Rich*. While households that do not have access to financial and capital markets, i.e. they live hand-to-mouth, and supply unskilled labour services are referred to as *Poor*. In a Ricardian

[1] There is by now a tendency of declining public deficits in the Euro Area. This is reflected not only in statistical indicators but also in the number of countries that are still under the Excessive Deficit Procedure (EDP). The structural deficit in the EU was reduced markedly from 4.3 % to 1.0 % in the Euro-Area. At the country level, while only five Member States recorded deficits below the 3% of GDP reference threshold in 2010, 22 did so in 2014. Currently only Spain is over the 3% threshold and subject to the EDP. See European Commission (2017)

[2] See European Central Bank (2017) for a discussion on the trade-offs between fiscal consolidation and reforms.

[3] There are different ways to introduce heterogeneous agents in DSGE models. For instance, Gali et al. (2007), Coenen et al. (2008), Forni et al. (2009), Schwarzmüller and Wolters (2017) use models in which a share of households does not have access to financial or/and capital markets (Ricardian vs Non-Ricardian). Households can also exhibit heterogeneity in terms of their impatience (patient vs impatient) as in Bilbiie et al. (2012), their labour market status (public vs private sector workers) as in Ardagna (2007) and Economides et al. (2015) or with respect to their education and skills as in Angelopoulos et al. (2014), Dolado et al. (2018) and Gomes (2018).

world representative agents can smooth out exogenous fiscal changes like fiscal consolidation through lending or borrowing. In this paper we depart from this world by adding skill and asset heterogeneity; this aggravates the macroeconomic implications of fiscal policy changes and allow for distributional analysis which poses additional challenges in the analysis of fiscal consolidation policies.

Regarding policy, government has a rather rich set of spending and tax instruments at its disposal. Particularly, government levies consumption, labour and capital taxes to finance productivity-enhancing spending like public investment and spending on education, utility-enhancing expenditures like government consumption and public transfers to *Rich* and *Poor* households. Following most of the literature on debt consolidation we follow a rule-based approach to policy. This means that all the fiscal instruments can respond to the gap between public debt and the target of public debt as shares of output. Fiscal policy coefficients in the associated fiscal rules are either set exogenously (referred to as ad-hoc policy) or to maximize a welfare criterion following Schmitt-Grohé and Uribe (2007) (referred to as optimized policy).

Our main policy experiment is fiscal consolidation. That is the economy starts from a steady-state with high debt-to-GDP ratio, say 85% as in EA-18 data average over 2001-2015 and travels towards a new reformed steady-state with lower debt-to-GDP ratio, say 60%. We experiment with various reformed economies varying the fiscal instrument which reaps the benefit of fiscal space after debt consolidation. Government can achieve the transition from the status-quo to the new reformed steady-state by implementing alternative fiscal consolidation policy mixes. We study both ad-hoc and optimized policy. For each scenario, we compute the aggregate effects using variables like output and social welfare and distributional effects using variables like net income, skill premium and household-specific welfare.

Our main results are as follows. First, ad-hoc debt consolidation policies either through tax hikes or spending cuts are recessionary in the short- and medium-run; however, spending cuts induce crowding in effects in private consumption and investment of *Rich* households and as result mitigate output contraction. On the other hand, distortionary tax increases result in negative supply-side effects that cause a sharper and more prolonged recession. Second, in terms of inequality spending-based consolidation comes at a distributional cost as it is more harmful for income, wage and welfare distribution. Reduction in spending instruments like utility-enhancing government consumption and public transfers affect disproportionally *Poor* households while gains from debt consolidation like increases in private investment and capital stock benefits relatively more *Rich* households. The latter is more pronounced in our model due to capital-skill complementarity effect. Third, tax-based fiscal consolidation results in a higher output loss but has a relatively smoother distributional impact. Income tax increases are more harmful for *Rich* households' income and welfare since *Rich* households earn returns from capital and receive higher wages. Fourth, optimized policy can perform better than ad-hoc spending- or tax-based fiscal consolidation policies over all time horizons and for all households. In particular, fiscal consolidation policy through optimized fiscal rules can be expansionary and social welfare enhancing as well as can mitigate (but not eliminate) the equity-efficiency trade-off in the short- and medium-run.

Literature related to our work includes debt consolidation in dynamic general equilibrium models in heterogeneous household setups as in Coenen et al. (2008), Schwarzmüller and Wolters (2017) and Roubanis (2019) who study ad-hoc debt consolidation policies when agents are heterogeneous with respect to their access to financial markets. Economides et al. (2012) focus on the implications of debt consolidation when agents differ with respect to their labor market status, e.g. public sector versus private sector workers. Our work is also related to debt consolidation studies e.g. Forni et al. (2010), Almeida et al. (2013), Bi et al. (2013), Cogan et al. (2013), Erceg and Lindé (2013), Pappa et al. (2015) and Economides et al. (2017) who focus on the aggregate macroeconomic implications of ad-hoc debt consolidation policies in various models. Cantore et al. (2012), Philippopoulos et al. (2015), (2017a) and (2017b) study debt consolidation policies in closed and open economies setups under optimized rules. Our work is also related to papers that study fiscal policy reforms in dynamic general equilibrium models with heterogeneous agents like Garcia-Mila et al. (2010), Angelopoulos, Asimakopoulos and Malley (2017), Angelopoulos, Jiang and Malley (2017), Gomes (2018), Michaud and Rothert (2018), however they do not focus on the distributional effects of debt consolidation.

In our work, by contrast, we consider a joint heterogeneity setup including asset and skill heterogeneity to assess debt consolidation.[4] Moreover, we compare ad-hoc (spending- and tax-based) with optimized policies while we compute optimized fiscal policy rules for a rich set of spending (utility- and productivity-enhancing) and tax instruments.[5] Finally, we provide a systematic framework to assess the aggregate as well as the distributional effects of alternative fiscal consolidation policies.

The structure of the paper proceeds as follows. Section 2 presents the model and solves for the Decentralized Competitive Equilibrium. Section 3 explains our calibration strategy and solves for the status quo steady-state solution. Next, in Section 4 we describe the main policy experiments while in Section 5 we present our results. Section 6 discusses robustness analysis. Section 7 concludes the paper.

2 The model

2.1 Informal description of the model

We develop a closed-economy dynamic general equilibrium model which consists of households, firms and a government. The key feature of the model is household heterogeneity. Households differ in two aspects. First, in the type of labour they supply and second in their access to

[4] See e.g. He and Liu (2008), Angelopoulos et al. (2014), Dolado et al. (2018) and the references therein on the interaction of various fiscal and monetary policies with inequality.

[5] Regarding the productivity-enhancing instruments, apart from public investment which contributes to the accumulation of public capital stock which is growth-enhancing we also examine the role of public spending on education which can work also as a social pillar for the less wealthy households.

financial and capital markets. Thus we incorporate *ex ante* skill and wealth heterogeneity. In particular households that have access to capital and financial markets, supply skilled labour services and own private firms are referred to as *Rich*. Households that do not participate in capital and financial markets and supply unskilled labour services; thus, only consume their after-tax labour income referred to as *Poor*.[6] In addition, both household types can accumulate human capital using a human capital production function à la Lucas while they yield utility from public consumption.

On the production side, firms use physical and public capital, skilled and unskilled labour in order to produce an homogeneous good. In the production sector we incorporate a nested CES-Cobb Douglas production function similar to Krusell et al. (2000) which exhibits capital-skill complementarity. As it is known this feature gives rise to the so called skill premium.

Government has a rather rich set of fiscal policy instruments at its disposal. In particular, it issues public debt and levies consumption, labour and capital income taxes to finance its stream of public expenditures, namely spending on public education and investment, government consumption and transfer payments which are allowed to be allocated unevenly between households.

2.2 Population composition

The population size, N is exogenous and constant. It is comprised by two types of households, i.e. *Rich* households indexed by the subscript $R = 1, 2, ..., N_R$ and *Poor* households indexed by the subscript $P = 1, 2, ..., N_P$ where $N_R > N_P$ and $N_P + N_R = N$ is the total size. No mobility or occupational change is possible between the two types. There are also $f = 1, 2, ..., N^f$ firms. For notational convenience, we assume also that $N^f = N_R$.

2.3 *Rich* households

Each *Rich* household, R, maximizes its expected discounted lifetime utility given by:

$$V_{R,0} \equiv E_0 \sum_{t=0}^{\infty} \beta^t U_{R,t} \left(c_{R,t}, z_{R,t}, \bar{g}_t^c \right) \tag{1}$$

where $c_{R,t}$ and $z_{R,t}$ are consumption and leisure of each household, $\bar{g}_t^c$ is per capita utility-enhancing government consumption[7] and $0 < \beta < 1$ is the discount rate. The period utility function $U_{R,t}\left(.\right)$ is increasing and strictly concave in all its arguments. For convenience we use the following functional forms:

$$U_{R,t} \left(c_{R,t}, z_{R,t}, g_t^c \right) = \mu_1 log \left(c_{R,t} + \xi \bar{g}_t^c \right) + \mu_2 log \left(1 - e_{R,t} - l_{R,t} \right) \tag{2}$$

[6] We follow Michaud and Rothert (2018) by referring to the two types of households as *Rich* and *Poor*.

[7] Notice that $\bar{g}_t^c \equiv \frac{g_t^c}{N}$ where g_t^c is total utility enhancing government consumption.

where μ_1, μ_2 are preference parameters, ξ measures the degree of substitutability between private and public consumption, e.g. if $\xi > 0$ (< 0) private and public consumption are substitutes (complements). Households are endowed with a normalised time unit:

$$z_{R,t} + e_{R,t} + l_{R,t} = 1 \tag{3}$$

where $l_{R,t}$ and $e_{R,t}$ is time devoted to labour and education respectively. The within period budget constraint of each *Rich* household, R, is:

$$(1 + \tau_t^c)\, c_{R,t} + i_{R,t} + d_{R,t} = (1 - \tau_t^k)\,(r_t k_{R,t} + \pi_{R,t}) + \left(1 - \tau_t^l\right) w_{R,t} l_{R,t} h_{R,t} + \overline{tr}_{R,t} + r_t^b b_{R,t} \tag{4}$$

where $i_{R,t}$, is private investment, $k_{R,t}$, physical capital, $b_{R,t}$, government bonds whose gross returns are r_t and r_t^b respectively, $h_{R,t}$, is human capital, $d_{R,t}$, is savings in the form of government bonds, $\pi_{R,t}$ is dividends received from private firms, $w_{R,t}$ is the wage rate earned by *Rich* households, $\overline{tr}_{R,t} \equiv \frac{tr_{R,t}}{N_R}$ is public transfers per *Rich* household and $0 < \tau_t^k, \tau_t^l, \tau_t^c < 1$, are tax rates on capital income, labour income and consumption respectively. Because *Rich* households supply skilled labour services while *Poor* households supply unskilled labour services to firms, *Rich* households receive a relatively higher wage $w_{R,t} > w_{P,t}$ (for more details see section 2.5).[8] To allow for productive education expenditures we use a human capital production function as in Lucas (1988) and Glomm and Ravikumar (1997).[9] Therefore, individual human capital is augmented by time spent in education, $e_{R,t}$, and by public spending on education, $g_{R,t}^e \equiv \omega \bar{g}_t^e$, which is a fixed share ω of per capita public spending on education, $\bar{g}_t^e$.[10] The law motion of human capital of *Rich* household, R, is:

$$h_{R,t+1} = (1 - \delta^h) h_{R,t} + B_R \left[(e_{R,t})^{\theta} \left(g_{R,t}^e \right)^{1-\theta} \right]^{x_R} \tag{5}$$

where $B_R > 0$, $\theta \in (0,1)$, $x_R < 1$ are productivity parameters, and δ^h is the depreciation rate of human capital. Following He and Liu (2008), $x_R < 1$, captures decreasing returns to scale in the production of new human capital. The law of motions of physical capital and government bonds for each R are:

$$k_{R,t+1} = (1 - \delta^k) k_{R,t} + i_{R,t} \tag{6}$$

$$b_{R,t+1} = b_{R,t} + d_{R,t} \tag{7}$$

[8] Throughout the rest of the paper we call labour provided by *Rich* households as skilled labour and labour provided by *Poor* households as unskilled labour. As we explain in Section 3.1 there exists adequate empirical evidence associating wealth and skills.

[9] Trabandt and Uhlig (2011) use a learning by doing specification by including hours of work and education as inputs. On the contrary, in our model, as in Daniel and Gao (2015), we allow for a production function that combines a time input and a good input so as to assess the effects of public education spending as an additional productivity enhancing fiscal instrument.

[10] $\bar{g}_t^e \equiv \frac{g_t^e}{N}$ where g_t^e is total public spending on education.

Each *Rich* household in any given period t, chooses $c_{R,t},\ k_{R,t+1},\ h_{R,t+1},\ b_{R,t+1},\ \ e_{R,t},\ \ l_{R,t}$ to maximize its lifetime utility subject to the constraints (4) (in which we incorporate (6) and (7)) and (5) taking factor prices and policy as given. Defining as $\lambda_{R,t}$ and $\psi_{R,t}$ the Lagrange multipliers associated with (4) and (5) respectively, the first-order conditions are given by:

$$\lambda_{R,t}=\frac{\mu_1}{(1+\tau_t^c)\left(c_{R,t+1}+\xi\bar{g}_{t+1}^c\right)} \tag{8}$$

$$\frac{1}{(1+\tau_t^c)\left(c_{R,t}+\xi\bar{g}_t^c\right)}=\frac{\beta\left[1-\delta^k+\left(1-\tau_{t+1}^k\right)r_{t+1}\right]}{\left(1+\tau_{t+1}^c\right)\left(c_{R,t+1}+\xi\bar{g}_{t+1}^c\right)} \tag{9}$$

$$\psi_{R,t}=\beta\lambda_{R,t+1}\left(1-\tau_t^l\right)w_{R,t+1}l_{R,t+1}+\beta\psi_{R,t+1}(1-\delta^h) \tag{10}$$

$$\frac{1}{(1+\tau_t^c)\left(c_{R,t}+\xi\bar{g}_t^c\right)}=\frac{\beta\left(1+r_{t+1}^b\right)}{\left(1+\tau_{t+1}^c\right)\left(c_{R,t+1}+\xi\bar{g}_{t+1}^c\right)} \tag{11}$$

$$\frac{\mu_2}{1-e_{R,t}-l_{R,t}}=\psi_{R,t}x_RB_R\theta\left(e_{R,t}\right)^{\theta-1}\left[\left(e_{R,t}\right)^{\theta}\left(g_{R,t}^e\right)^{1-\theta}\right]^{x_R-1} \tag{12}$$

$$\frac{\mu_2}{1-e_{R,t}-l_{R,t}}=\lambda_{R,t}\left(1-\tau_t^l\right)w_{R,t}h_{R,t} \tag{13}$$

2.4 *Poor* households

Each *Poor* household, P, maximizes its expected discounted lifetime utility given by:

$$V_{P,0}=E_0\sum_{t=0}^{\infty}\beta^tU_{P,t}\left(c_{P,t},z_{P,t},\bar{g}_t^c\right) \tag{14}$$

The period utility function $U_{P,t}\left(.\right)$ is increasing and strictly concave in all its arguments where we use the same functional form for preferences and the same time constraint as before.[11] The within period budget constraint of each *Poor* household is given by:

$$(1+\tau_t^c)\,c_{P,t}=\left(1-\tau_t^l\right)w_{P,t}l_{P,t}h_{P,t}+\overline{tr}_{P,t} \tag{15}$$

where $w_{P,t}$ is the wage rate received by *Poor* households which supply unskilled labour services, $\overline{tr}_{P,t}\equiv\frac{tr_{P,t}}{N_P}$ is public transfers per *Poor* household. As before, the law motion of human capital of each household of type, P, is:

$$h_{P,t+1}=(1-\delta^h)h_{P,t}+B_P\left[\left(e_{P,t}\right)^{\theta}\left(g_{P,t}^e\right)^{1-\theta}\right]^{x_P} \tag{16}$$

[11] Notation and functional forms are analogous to *Rich* households.

where $g^e_{P,t} \equiv (1-\omega)\,\bar{g}^e_t$ is the amount of total public spending on education services enjoyed by each P.[12] Each *Poor* household, P, maximizes its lifetime utility in any given period t by choosing $c_{P,t}, h_{P,t+1}, e_{P,t},\ l_{P,t}$ subject to the constraints (15) and (16) taking factor prices and policy as given. Defining as $\lambda_{P,t}$ and $\psi_{P,t}$ the Lagrange multipliers associated with (15) and (16) respectively, the first-order conditions are given by:

$$\lambda_{P,t} = \frac{\mu_1}{(1+\tau^c_t)\left(c_{P,t+1} + \xi \bar{g}^c_{t+1}\right)} \tag{17}$$

$$\psi_{P,t} = \beta \lambda_{P,t+1}\left(1-\tau^l_t\right) w_{P,t+1} l_{P,t+1} + \beta \psi_{P,t+1}(1-\delta^h) \tag{18}$$

$$\frac{\mu_2}{1-e_{P,t}-l_{P,t}} = \psi_{P,t} x_P B_P \theta \left(e_{P,t}\right)^{\theta-1}\left[\left(e_{P,t}\right)^{\theta}\left(g^e_{P,t}\right)^{1-\theta}\right]^{x_P-1} \tag{19}$$

$$\frac{\mu_2}{1-e_{P,t}-l_{P,t}} = \lambda_{P,t}\left(1-\tau^l_t\right) w_{P,t} h_{P,t} \tag{20}$$

2.5 Firms

There are $f = 1, 2..., N^f$ identical firms owned by the *Rich* households. Each firm, f, acts competitively taking prices as given. Firm's objective is to maximize profits:

$$\pi^f_t \equiv y^f_t - r_t k^f_t - w_{R,t} l^f_{R,t} - w_{P,t} l^f_{P,t} \tag{21}$$

where y^f_t is firm f's output. Firms utilize four factors inputs to produce an homogeneous good, i.e. physical capital, k^f_t, skilled labour services rented from *Rich* households, $l^f_{R,t}$, unskilled labour services rented from *Poor* households, $l^f_{P,t}$, and aggregate public capital, k^g_t. Production is given by the following constant returns to scale (CRS) and constant elasticity of substitution (CES) function similar to Krusell et al. (2000):

$$y^f_t = A\left[m\left(l^f_{P,t}\right)^{\sigma} + (1-m)\left(\rho\left(k^f_t\right)^{v} + (1-\rho)\left(l^f_{R,t}\right)^{v}\right)^{\frac{\sigma}{v}}\right]^{\frac{\alpha}{\sigma}} \left(k^g_t\right)^{1-\alpha} \tag{22}$$

where $A > 0$ is scale parameter, $0 < \alpha, \rho, m < 1$ are factor inputs share parameters and $\sigma, v \leq 1$ are parameters governing factor elasticities (see below for more details). Each firm f maximizes its profits (21) subject to its production function (22) by choosing k^f_t, $l^f_{R,t}$, $l^f_{P,t}$. First order

[12] This is meant to be not only formal education (i.e. secondary or tertiary education spending), but could resemble other types of educational programmes such as vocational training, on-the-job learning, continuing professional development programmes among others. This type of investment is of special importance for the less skilled or less wealthy members in the society since it increases their productivity and labour earnings.

conditions are given by:

$$r_t = A\alpha(1-m)\left(\rho\left(k_t^f\right)^v + (1-\rho)\left(l_{R,t}^f\right)^v\right)^{\frac{\sigma}{v}-1}\rho\left(k_t^f\right)^{v-1} \times\left[m\left(l_{P,t}^f\right)^\sigma + (1-m)\left(\rho\left(k_t^f\right)^v + (1-\rho)\left(l_{R,t}^f\right)^v\right)^{\frac{\sigma}{v}}\right]^{\frac{\alpha}{\sigma}-1}\left(k_t^g\right)^{1-\alpha} \tag{23}$$

$$w_{R,t} = A\alpha(1-m)\left(\rho\left(k_t^f\right)^v + (1-\rho)\left(l_{R,t}^f\right)^v\right)^{\frac{\sigma}{v}-1}(1-\rho)\left(l_{R,t}^f\right)^{v-1} \times\left[m\left(l_{P,t}^f\right)^\sigma + (1-m)\left(\rho\left(k_t^f\right)^v + (1-\rho)\left(l_{R,t}^f\right)^v\right)^{\frac{\sigma}{v}}\right]^{\frac{\alpha}{\sigma}-1}\left(k_t^g\right)^{1-\alpha} \tag{24}$$

$$w_{P,t} = A\alpha m\left(l_{P,t}^f\right)^{\sigma-1}\left[m\left(l_{P,t}^f\right)^\sigma + (1-m)\left(\rho\left(k_t^f\right)^v + (1-\rho)\left(l_{R,t}^f\right)^v\right)^{\frac{\sigma}{v}}\right]^{\frac{\alpha}{\sigma}-1}\left(k_t^g\right)^{1-\alpha} \tag{25}$$

Notice that each firm, f, makes extraordinary profits given by $\pi_t^f = (1-\alpha)\,y_t^f$ as in Guo and Lansing (1997). Combining equations (24) and (25), the skill premium is given by:

$$\frac{w_{R,t}}{w_{P,t}} = (1-\rho)\frac{1-m}{m}\left(\frac{l_{P,t}^f}{l_{R,t}^f}\right)^{1-\sigma}\left[\rho\left(\frac{k_t^f}{l_{R,t}^f}\right)^\nu + (1-\rho)\right]^{\frac{\sigma}{\nu}-1} \tag{26}$$

The different roles in the production function for skilled (*Rich*) and unskilled (*Poor*) labour give rise to the so called skill-premium, meaning that $\frac{w_R}{w_P} > 1$. In Section 3.1 we calibrate the associated parameters in the production function so that the implied factor input elasticities and the resulting skill premium are in line with empirical studies.[13] The elasticities of substitution between physical capital and skilled labour and between skilled and unskilled labour is $\frac{1}{1-\sigma}$ whereas the elasticity between capital and skilled labour is $\frac{1}{1-v}$. This formulation implies that as long as $\sigma > v$ the production function exhibits capital-skill complementarity. Moreover, this specification implies that the skill premium will be *ceteris paribus* increasing in physical capital, $k_{R,t}^f$ (known as the capital-skill complementarity effect) and decreasing in the skilled to unskilled labour ratio, $\frac{l_R^f}{l_P^f}$ (known as the relative skill supply effect).[14]

2.6 Government

The within-period government budget constraint government is given (in per capita terms):

$$\overline{g}_t^c + \overline{g}_t^i + \overline{g}_t^e + \overline{tr}_{R,t} + \overline{tr}_{P,t} + (1+r_t^b)b_t = b_{t+1} + \tau_t \tag{27}$$

[13] See Krueger et al. (2010) for an empirical investigation on the level and the evolution, over time, of several dimensions of economics inequality.

[14] It is straightforward to show that $\frac{\partial\left(\frac{w_R}{w_P}\right)}{\partial k_R^f} > 0$ and $\frac{\partial\left(\frac{w_R}{w_P}\right)}{\partial\left(\frac{l_R^f}{l_P^f}\right)} < 0$.

where $b_t \equiv n_R b_{R,t}$ denotes the end-of-period stock of government bonds, $\overline{g}_t^i \equiv \frac{g^i}{N}$ is per capita public investment and τ_t denotes total tax revenues in per capita terms defined as:

$$\tau_t \equiv \tau_t^c \left(n_R c_{R,t} + n_P c_{P,t}\right) + \tau_t^k n_R \left(r_t k_{R,t} + \pi_{R,t}\right) + \tau_t^l \left(n_P w_{P,t} l_{P,t} h_{P,t} + n_R w_{R,t} l_{R,t} h_{R,t}\right) \quad (28)$$

Each period t government sets eight fiscal instruments, i.e. five public spending instruments, namely utility-enhancing spending, public education and investment and transfers to *Rich* and *Poor* households and three tax instruments, namely capital, labour and consumption taxes. In our simulations below the residual policy instrument is always public debt. The law motion of public capital is given by:

$$k_{t+1}^g = \left(1 - \delta^g\right) k_t^g + \overline{g}_t^i \quad (29)$$

For notational convenience, concerning public spending policy instruments, we define them in terms of their GDP shares $s_t^{g^i} \equiv \frac{\overline{g}_t^i}{n^f y_t^f}$, $s_t^{g^e} \equiv \frac{\overline{g}_t^e}{n^f y_t^f}$, $s_t^{g^c} \equiv \frac{\overline{g}_t^c}{n^f y_t^f}$, $s_t^{tr_P} \equiv \frac{n_P \overline{tr}_{P,t}}{n^f y_t^f}$, $s_t^{tr_R} \equiv \frac{n_R \overline{tr}_{R,t}}{n^f y_t^f}$, where we also express the number of *Rich* , *Poor* and firms in terms of shares $n_R \equiv \frac{N_R}{N}$, $n_P \equiv \frac{N_P}{N} = 1 - n_R$, $n^f \equiv \frac{N^f}{N} = n_R$.

2.7 Fiscal policy rules

Fiscal policy sets its spending-tax instruments following simple fiscal policy rules, meaning that it reacts to the public debt-to-GDP ratio deviation from a target. In particular we allow all the main policy instruments $\phi_t = \{s_t^{g^c}, s_t^{g^i}, s_t^{g^e}, s_t^{tr_P}, s_t^{tr_R}, \tau_t^c, \tau_t^l, \tau_t^k\}$ to react to the public debt-to-GDP ratio, $q_{t-1} \equiv \frac{n_R b_t}{n^f y_{t-1}}$, as deviation from a target according to a simple linear rule:

$$\phi_t - \phi^* = \gamma_q \left(q_{t-1} - q^*\right) \quad (30)$$

where ϕ^*, q^* denote fiscal policy targets and γ_q are feedback policy coefficients. $\gamma_q \leqslant 0$ if ϕ_t is a spending instrument and $\gamma_q \geqslant 0$ if ϕ_t is a tax instrument (see equations (57)-(64) in Appendix).

2.8 Decentralized Competitive Equilibrium (DCE)

DCE is defined as a sequence of allocations, prices and policies such that: (i) all household types maximize welfare, (ii) firms maximize profits, (iii) goods, capital, labor and bond markets clear, (iv) dividends markets clear, (v) policymakers follow the feedback rules assumed, (vi) all constraints are satisfied.

We thus end up with a first order non-linear dynamic equilibrium system summarized by 29 equations in 29 unknowns $\{y_t^f, c_{R,t}, c_{P,t}, k_{R,t+1}, h_{R,t+1}, h_{P,t+1}, b_{R,t+1}, e_{R,t}, e_{P,t}, l_{R,t}, l_{P,t}, r_t^b, \lambda_{R,t}, \lambda_{P,t}, \psi_{R,t}, \psi_{P,t}, k_t^g, r_t, w_{R,t}, w_{P,t}, q_t\}$ and $\{s_t^{g^i}, s_t^{g_t^c}, s_t^{g^e}, s_t^{tr_P}, s_t^{tr_R}, \tau_t^c, \tau_t^l, \tau_t^k\}$. This is given initial conditions for the state variables and the values of the feedback fiscal policy coefficients in the associated fiscal policy rules. We present the full equilibrium system in the Appendix.

2.9 Plan of the rest of the paper

In the rest of the paper we work as follows: First, using commonly employed structural parameter values and fiscal policy data from the EA-18 over the period 2001-2015 we solve for the steady state solution of the model in Section 3. We explain our calibration strategy in Section 3.1. The long-run solution is computed in Section 3.2. Throughout the paper we refer to this solution as the "status-quo" economy. In our policy experiments we use this solution as point of departure in order to evaluate alternative debt consolidation policies.

Second, we compute various steady state reformed economies in which public debt-to-GDP ratio is lower; details are given in Section 4. Notice that, thanks to public debt reduction one fiscal instrument can adjust in the reformed steady state to reap the benefit of the fiscal adjustment (fiscal gain). We study various reformed economies depending on which fiscal instrument adjusts in the new steady state. Aggregate and distributional long-run effects are computed in section 5.1.

Third, we compute the transition dynamics from the status quo economy with high debt-to-GDP ratio to the various reformed economies with lower public debt-to-GDP ratio. During the transition, fiscal policy should decrease spending or/and increase tax instruments to bring public debt-to-GDP ratio down to its new lower target (fiscal pain). In what follows, we study ad-hoc and optimized consolidation policies. In particular, we study two ad-hoc policy scenarios, tax- and spending-based policies, and we compare these scenarios with an optimized policy scenario. Details on policy scenarios are discussed in Section 4 while results on the transition are presented in Section 5.2 and 5.3.

3 Calibration and status-quo long-run equilibrium

In the following section we discuss how we choose the value of the model parameters and present the long-run solution of the model.

3.1 Parameter values and fiscal policy data

In Table 1a we report the values of the structural parameters. In Table 1b we report the fiscal policy instruments values using fiscal data averages for the EA-18 over the period 2001-2015. Both parameter values and fiscal policy instruments are chosen so that the model's long-run solution mimics various key macroeconomic ratios of the EA-18 economy. We use data from the AMECO database of the European Commission, and Eurostat's databases, COFOG (Classification of Functions of Government), LFS (Labour Force Survey), EU-SILC (Social Income and Living Conditions), Household Finance and Consumption Survey (HFCS). Below we analyse in detail our calibration strategy.

Population shares As said above households differ in two dimensions, access to financial markets and skills. We set $n_P = 0.3$ and $n_R = 0.7$ so that 30% of total population do not participate in capital and financial markets which is in the range reported by Coenen et al.

(2008). This is in line with data on household savings in HFCS which reports that the assymetric savings distribution is also reflected in income distribution. For instance in the Euro Area the richest 20% income group holds over 60% of total savings. Turning to the skills distribution as those are defined by educational attainment (ISCED), data from Eurostat indicate that in the EA-18 around 30% to 35% has at least attained lower secondary education (ISCED 0-2; 10 years of education) while the rest has attained at least upper-secondary, post-secondary non-tertiary and tertiary education (ISCED 3-8) which roughly matches our parameter choices for n_R and n_P. Finally, data from EU-SILC reveal that high income groups as well as high savings groups in the population show relatively higher educational attainment rates. Thus, we believe there exists enough evidence to associate savings and income with skills and education.

Preferences and parameters common to all agents The time discount factor β is set to give an annual real interest rate of about 2.25% which is consistent with data on EA-18 (see AMECO database). The preference parameters $\{\mu_1, \mu_2, \xi\}$ are calibrated so that the weighted average of skilled and unskilled hours worked is around 0.25. It also implies that in steady-state *Rich* households devote more time to labour relative to *Poor* (see Table 2 in the next subsection).[15] We set the depreciation rates of physical and public capital $\left\{\delta^k, \delta^g\right\}$ equal to 6%, as in Coenen et al. (2008). Given that there is not a clear consensus on the magnitude of the depreciation of human capital we assume $\delta^h = \delta^k = \delta^g = 6\%$.

Production We normalize the scale parameter A to 1. We use the estimates of Krusell et al. (2000) for the elasticities of substitution between capital and skilled labour, $v = -0.495$,$1/(1-v) = 0.668$, and between capital/skilled labour and unskilled labour, $\sigma = 0.401$,$1/(1-\sigma) = 0.1666$. We then choose the remaining parameters of the production function, $\{\rho, m\}$, so that the model's status quo solution is consistent with data on factor inputs shares such as labour income share, capital income share and inequality variables like skill premium (for the latter see Krueger et al. (2010)). The choice of the parameter α along with the depreciation rate of physical and public capital imply a physical capital to GDP ratio around 2.5 and a public capital to GDP ratio around 0.15.

Human capital Next, we set the parameters governing the production of new human capital of each household type. The sets of parameters $\{B_R, B_P\}$ and $\{x_R, x_P\}$ both relate to technology and ability in the creation of new human capital and skills. For this reason, similar to He and Liu (2008) and Angelopoulos et. al (2017) we set $B_R = B_P$ and let $x_R = 0.450 > x_P = 0.400$ to capture differences in ability between the two household types. This choice reflects the idea that *Rich* households, due to their higher education status, obtain higher returns. The literature has not reached a consensus for the value of the elasticity parameter of education time with respect to new human capital θ. We set a value of 0.8 so that $(1-\theta) = 0.2$ as in Blankenau et al. (2004). This implies that households devote around 9% on average of their time endowment to skill enhancing activities. Note that in the model both time spent on education and public

[15] This is accordance with the Eurostat's Labour Force Survey series which reports that workers in skilled occupations (e.g. managers, professionals, engineers) record higher weekly hours of work than less skilled occupations (e.g. clerical stuff, technicians etc).

education spending are meant to be post-schooling. This implies that both agents have already acquired a minimum of 10 years of education.

Policy We set fiscal policy variables in steady state equal to the EA-18 fiscal data averages over the period 2001-2015. In particular, we use effective average tax rates following Mendoza et al. (1994). Namely, effective tax rate on consumption is 19.6%, effective tax rate on labour is 46.6% and effective tax rate on capital is 36.7%.[16] Regarding the public spending instruments we set the share of total government expenditure as a share of GDP to be around 49% and transfers as a share of GDP around 15%; this gives a public debt to GDP ratio around 85% which is consistent with data from AMECO database.[17] As said in the previous sections we assume that transfers are unevenly distributed between the two household groups favouring *Poor* households. Given the difficulty to pin-down the exact share allocated to each household type we assume that *Poor* households receive double the amount of transfers relative to *Rich* households as a share of GDP. The rest of the public expenditure sub-components are extracted from Eurostat's COFOG database which breaks down public spending per functional use. This helps us to disentangle total public spending into its main components. For instance public spending on education s^{g^e}, is set at 1%, which is close to the post schooling public spending on education. For simplicity we assume that this share, is equally allocated between the two household types, i.e. we set $\omega = 0.5$. Spending on public investment as a share of GDP s^{g^i}, is set at 3%, based on data reported in the Economic Affairs function of the COFOG database. Finally utility-enhancing government consumption $s^{g_t^c}$ is set equal to 30%.[18]

[16] Effective tax rates are taken from Kostarakos and Varthalitis (2018).

[17] Particularly we use the time series "General government consolidated gross debt - Excessive deficit procedure based on ESA 2010".

[18] According to the COFOG dataset we can define this share to include a broad range of government functions such as general public services, public order and defence, recreation and culture, environmental protection, household and community amenities, health which is close to our chosen value as a share of GDP.

Table 1a: Parameter values

Parameter	Definition	Value
	Households	
$0 \leq \beta \leq 1$	time discount factor	0.978
$0 < \xi < 1$	public consumption weight in composite consumption	0.100
$\mu_1 > 0$	preference weight in the utility	0.400
$\mu_2 > 0$	preference weight in the utility	0.600
$0 \leq \delta^k \leq 1$	depreciation rate of physical capital	0.060
$0 \leq \delta^h \leq 1$	depreciation rate of human capital	0.060
$0 \leq \delta^g \leq 1$	depreciation rate of public capital	0.060
$0 < n_R < 1$	population share of *Rich*	0.700
$0 < n_P < 1$	population share of *Poor*	0.300
	Production	
$0 < \alpha < 1$	share of composite input	0.980
$\sigma < 1$	capital and skilled labour to unskilled labour substitution	0.401
$v < 1$	capital to skilled labour substitution	-0.495
$0 < m < 1$	labour share of *Rich*	0.300
$0 < \rho < 1$	share of physical capital in the composite input	0.400
$0 \leq \theta \leq 1$	elasticity of education time	0.800
$0 \leq 1 - \theta \leq 1$	elasticity of public education spending	0.200
$A > 0$	scale parameter	1.000
$B_R > 0$	human capital technology parameter of *Rich*	1.000
$B_P > 0$	human capital technology parameter of *Poor*	1.000
$0 < x_R < 1$	returns to scale for new human capital of *Rich*	0.450
$0 < x_P < 1$	returns to scale for new human capital of *Poor*	0.400

Table 1b: Fiscal policy instruments

Instrument	Definition	Value
	Tax rates	
τ^k	capital tax rate	0.367
τ_t^l	labour tax rate	0.466
τ^c	consumption tax rate	0.197
	Public spending	
s^{g^e}	GDP share of public education spending	0.010
$s^{g_t^c}$	GDP share of government consumption	0.300
s^{g^i}	GDP share of public investment	0.030
s^{tr_R}	GDP share of government transfers to *Rich*	0.050
s^{tr_P}	GDP share of government transfers to *Poor*	0.100

3.2 Status quo steady-state solution

The steady-state solution of the model, when we use the parameter values and the policy instruments of Tables 1a-1b, is reported in Table 2. In what follows, we refer to this steady-state solution as the "status-quo" economy and will serve us as the point of departure for the various policy experiments studied in the next sections. The implied numerical solution mimics some key macroeconomic ratios observed for the EA-18 like consumption as a share of output, physical capital as a share of output, debt-to-GDP and skill premium.

Table 2: Status quo solution

Main variables		Model	
c_R	consumption of *Rich*	0.1555	
c_P	consumption of *Poor*	0.1091	
l_R	skilled labour	0.2748	
l_P	unskilled labour	0.1772	
e_R	time in education of *Rich*	0.1399	
e_P	time in education of *Poor*	0.0850	
r	real return to physical capital	0.0562	
r^b	return to bonds	0.0224	
y	output	0.3584	

Key ratios		Model	Data
$\frac{c}{y} = \frac{n_R c_R + n_R c_R}{n^f y}$	consumption as share of GDP	0.5680	0.5460
$\frac{n_R k_R}{n^f y}$	physical capital as share of GDP	2.5297	2.9600
$\frac{n_R b_R}{n^f y}$	debt as a share of GDP	0.8500	0.8700
$\frac{w_R}{w_P}$	skill premium	1.3801	1.7100

4 Fiscal policy experiments

In this section we define in more detail the fiscal policy experiments studied. Our thought experiment is the following. The economy starts from its status-quo steady-state computed in Table 2 and travels towards a new reformed steady-state with lower public debt-to-GDP ratio. Since public debt-to-GDP ratio is lower in the reformed economy, government can exploit the fiscal space by increasing public spending or/and reducing distortionary taxation. In what follows we study various reformed economies adjusting one fiscal instrument at a time. More specifically, in the new reformed economy debt-output ratio reduces from 85% which is the EA-18 data average over the period 2001-2015 to 60%; at the same time this reduction allows one spending (tax) instrument to increase (decrease) taking advantage of the fiscal space created by the debt reduction. The 60% is chosen simply to reflect the criteria set by the Stability and Growth Pact.

Government can achieve the transition from status-quo to the new reformed steady-state by implementing different fiscal policy mixes. In the transition fiscal policy sets its fiscal instruments following fiscal feedback rules given by equations (57)-(64) in the Appendix. We study both ad-hoc and optimized policy scenarios. In the ad-hoc policy scenarios the associated feedback policy coefficients are set ad-hoc (see below) to reduce debt-output ratio either via tax hikes (referred to as the tax-based scenario) or, via spending cuts (referred to as the spending-based scenario).

In the tax-based scenario, we set the feedback policy coefficients in the associated fiscal rules so as tax revenues increase by around 2% on impact and over the next 5 years of debt consolidation; while all spending instruments are kept constant to their data averages. Similarly, in the spending based consolidation we set the feedback policy coefficients so as total public spending decrease by around 2% over the same period; while all tax instruments are kept constant at their data averages. For reasons of comparison of the two ad-hoc scenarios, we also impose additional restrictions on the feedback policy coefficients. That is debt-to-GDP ratio should reduce at the same speed under both scenarios while fiscal instruments should fluctuate close to their historical data averages.

On the other hand, in the optimized policy scenario we compute the optimized values of feedback policy coefficients in the associated fiscal rules following Schmitt-Grohé and Uribe (2007) meaning that fiscal policy chooses its feedback policy coefficients to maximize a welfare criterion. The welfare criterion is the weighted conditional welfare of the *Rich* and *Poor* households as defined in (1) and (14) respectively, i.e. $W_0 = \eta_R V_{R,0} + (1 - \eta_R) V_{P,0}$ where η_R denotes the weight assigned to *Rich* households' lifetime welfare.[19] Notice that welfare is computed conditional on the initial conditions which are given by the status quo solution computed in Table 2.[20]

In Section 5.1 we report the implications of debt consolidation in the long-run. In turn, in Section 5.2, we present results in the transition under various ad-hoc and optimized scenarios. Finally in Section 5.3 we conduct welfare analysis.

5 Results

5.1 Aggregate and distributional effects of debt consolidation in the long-run

In the reformed economies, once debt-output ratio has been reduced government can increase spending or/and decrease tax instruments to take advantage of the fiscal space; this is the so called long-run fiscal gain of debt consolidation (see e.g. Coenen et al. 2008 and Philippopoulos et al. 2015). In this section we discuss the long-run aggregate and distributional implications of fiscal consolidation by varying the fiscal instrument that adjusts in the new reformed steady state. In particular, we rank alternative scenarios according to their effects on the aggregate economy as well as to their effects on income and wage distribution. To do this, we compute variables such as aggregate output, net income ratio (income inequality) and skill premium (wage inequality). Net income ratio is defined as the ratio of net income earned by *Rich* households

[19] We study the case of a Benthamite, or utilitarian government in the sense that the weights η_R and η_P in the social welfare function are equal to the population shares,n_R and n_P of *Rich* and *Poor* agents respectively.

[20] In particular, we take a second-order approximation to both the equilibrium conditions and the welfare criterion. First, we compute a second-order approximation of both conditional welfare and the decentralized equilibrium around the reformed steady state as functions of the vector of feedback policy coefficients. Then, we use an optimization routine like *fminsearch.m* to compute the values of the feedback policy coefficients that maximize the conditional welfare criterion. For more details see Philippopoulos et al. 2017a and 2017b. Dynare and Matlab routines are available upon request.

to net income earned by *Poor* households and allow us to evaluate the effect of a fiscal reform on the income distribution of the economy. If the latter increases (decreases) the income gap between *Rich* and *Poor* widens (shrinks).

In Table 3 we vary the residual fiscal instrument that adjusts in the long run and present the associated values of output, net income ratio and skill premium. All values in Table 3 are reported as percentage deviations from their status quo values; notice that a positive (negative) value implies an increase (decrease) vis-à-vis its status quo value. In particular, in the first column of Table 3 we report which fiscal instrument adjusts in the new steady state to take advantage of the post-consolidation fiscal space while in the last column we compute the magnitude of the associated adjustment. The fiscal instruments which adjusts in the new reformed economies are respectively: the output share of government consumption, $s^{g^c_t}$, the output share of government investment, s^{g^i}, the output share of public transfers to *Poor* households, s^{tr_P}, the output share of public spending on education, s^{g^e} consumption, τ^c, labour, τ^l, and capital, τ^k, tax rates. Finally, from second to fourth columns we report the implied percentage deviations of aggregate output, y, net income ratio, $\frac{y_R^{net}}{y_P^{net}}$, and skill premium, $\frac{w_R}{w_P}$ respectively.[21] To understand the mechanisms of each reform we experiment with one spending/tax policy instrument at a time keeping the others constant at their status quo value.

Table 3: Steady state output and distributional effects
in the various reformed economies (as % deviations from status-quo)

Fiscal Instr.	y	$\frac{y_R^{net}}{y_P^{net}}$	$\frac{w_R}{w_P}$	$\Delta Inst$
s^{g^c}	0.0093	-0.0117	-0.0102	0.0047
s^{g^i}	0.0152	-0.0110	-0.0093	0.0047
s^{tr_P}	0.0046	-0.0429	-0.0419	0.0054
s^{g^e}	0.0270	-0.0107	-0.0153	0.0047
τ^c	0.0107	-0.0118	-0.0097	-0.0098
τ^l	0.0181	-0.0067	-0.0034	-0.0101
τ^k	0.0196	-0.0036	-0.0027	-0.0121

A key message that can be derived from Table 3 is that debt consolidation is always output enhancing; notice that output increases vis-a-vis its status quo value in all reformed economies (see column 2 in Table 3). Such policies can also induce positive effects on the income distribution, e.g. net income ratio decreases in almost all reformed economies (see column 3 in Table 3). Thus, in the long-run debt consolidation polices enhance both equity and efficiency (for short-run see discussion in Section 5.2 and 5.3). Below we discuss in more detail results on output, net income ratio and skill premium by reformed economy.

Regarding output, as expected debt consolidation is more productive in the long-run when government increases productive spending (like public spending on education or investment) or

[21] Net income is defined as gross income minus all types of taxes.

reduces distortionary taxation.

Net income ratio decreases in almost all reformed economies which means that debt consolidation policies benefits relatively more *Poor* households. This is mostly driven by the sharp decrease in income from government bonds earned by *Rich* households in all reformed economies.[22] The reduction in net income of *Rich* households is less striking when government reduces income taxes (see the last two rows in Table 3) for two reasons. First, lower income taxes imply higher wealth; recall that *Rich* households earn capital income while receive relatively higher wages than *Poor* households. Second, due to complementarity between physical capital and skilled labour the resulting increase in output requires additional physical capital and as a result more skilled than unskilled labour. These moderate the adverse effects on net income of *Rich* households due to to the decrease in income from bond holdings.

In terms of wage inequality (see column 4 in Table 3), debt consolidation always reduces skill premium.[23]

5.2 Aggregate and distributional effects of debt consolidation in the transition

In this section, we focus our analysis on the transition implications of public debt consolidation. The economy departs from its status quo steady state and moves towards a new reformed economy with lower debt-to-GDP ratio equal to 60%.[24] This requires fiscal policy to use one (or more) fiscal instruments to react to debt deviations from its new target. We experiment with ad-hoc and optimized policies as analyzed in Section 4. Regarding ad-hoc policies, we distinguish between two debated scenarios referred to as the tax-based and the spending-based scenario respectively. Table 4 presents model-based simulations for output, net income ratio and skill premium under ad-hoc policies.[25] We report results over various time horizons, in particular we compute the average percentage deviation of each endogenous variable from its status quo value. For example, under tax (spending)-based consolidation scenario average recession is -1.95% (-0.4%) for the first two years of debt consolidation.

[22] These findings are in line with Schwarzmüller and Wolters (2015). However, they focus on consumption inequality rather than net income and wages.

[23] The net effect on skill premium depends on which of the capital-skill complementarity or the relative skill supply effect dominates. On the one hand, the increase in output requires more physical capital pushing skill premium upwards. On the other hand, relative skill supply increases pushing skill premium in the opposite direction. In our experiments the latter effect is stronger.

[24] To save space, we present results for the transition to the reformed economy in which government consumption is the fiscal instrument that adjusts to reap the benefit of debt reduction. Results from the associated transitional dynamics when the economy travels towards the rest of the reformed economies reported in Table 3 are available upon request. Here we report that our main qualitative results do not change.

[25] Table 4 and 5 present results from model-based simulations generated by the first-order approximation of the equilibrium system.

Table 4: Output, net income ratio and skill premium
over various time horizons with ad-hoc policies (% deviations from SQ)

Panel A: Tax-based consolidation

	2 years	4 years	6 years	10 years
y	-0.0195	-0.0185	-0.0175	-0.0153
$\frac{y_R^{net}}{y_P^{net}}$	-0.0613	-0.0555	-0.0502	-0.0419
$\frac{w_R}{w_P}$	0.0215	0.0177	0.0143	0.0089

Notes: Feedback (tax) coefficients are
$\gamma_q^c = 0.07$, $\gamma_q^l = 0.07$, $\gamma_q^k = 0.07$,
$\gamma_q^{g^c} = \gamma_q^{g^e} = \gamma_q^{g^i} = \gamma_q^{tr_R} = \gamma_q^{tr_P} = 0$.

Panel B: Spending-based consolidation

	2 years	4 years	6 years	10 years
y	-0.0040	-0.0028	-0.0018	0.0000
$\frac{y_R^{net}}{y_P^{net}}$	0.0035	0.0014	0.0000	-0.0035
$\frac{w_R}{w_P}$	0.0357	0.0328	0.0301	0.0251

Notes: Feedback (spending) coefficients are
$\gamma_q^{g^c} = 0.05, \gamma_q^{g^e} = 0.002$,
$\gamma_q^{g^i} = 0.007$, $\gamma_q^{tr_R} = 0.02$, $\gamma_q^{tr_P} = 0.02$,
$\gamma_q^c = \gamma_q^l = \gamma_q^k = 0$

Comparison of y in Panel A and B of Table 4 implies that spending-based fiscal consolidation is less recessionary than tax-based. That is in aggregate (output) terms spending-based consolidation is more productive over all time horizons. However, spending-based consolidation comes at a distributional cost as it seems to be more harmful for income and wage distribution. Tax-based and spending-based consolidation policies have different implications on the equity-efficiency trade-off generated by debt consolidation in the short/medium run. In particular, tax-based consolidation comes at a higher aggregate cost but smoother distributional impact while spending-based consolidation policies cause negative distribution effects but are less harmful on aggregate. The logic of these results is the following.

Regarding output, debt consolidation either through tax hikes or spending cuts results in a contraction; however under spending cuts the latter is mitigated due to crowding in effects of private consumption and investment of *Rich* households. While as it is well known distortionary tax increases have negative supply-side effects that cause a stronger and more prolonged output reduction. In terms of income distribution though, relatively higher income taxes (especially on capital) and the sharper fall of real rates under the tax-based scenario close the income gap over the first 10 years of consolidation. Finally, in terms of wage inequality, skill premium rises relatively more and for a prolonged period under spending based scenario. Both relative labor supply and capital-skill complementarity effect shift upward the skill premium in the short- and medium-run.[26]

In Table 4 we study ad-hoc debt consolidation policies, meaning that fiscal policy sets ad-hoc its feedback policy coefficients in the associated fiscal rules. However, it is well known that the fiscal policy mix (see e.g. Leeper 2010) chosen to bring public debt down have important macroeconomic implications. In our case, the fiscal policy mix is governed by the choice of feedback policy coefficients. For that reason, we also study a more ambitious debt consolidation scenario that does not depend on an ad-hoc choice of these policy coefficients. That is fiscal policy sets feedback policy coefficients optimally to maximize social welfare as analyzed in Section 4. We call this fiscal consolidation scenario optimized policy. Results are reported in Table 5 (which is comparable to Table 4). The main results are as follows.

In terms of policy reaction, the optimized policy mix implies that fiscal policy should cut government consumption sharply and increase consumption tax to consolidate its debt while at the same time keep constant income taxation and productive public spending. The resulting optimized values of feedback policy coefficients reported in the notes of Table 5 suggest that increasing distortionary income taxation or decreasing productive spending to reduce public debt is not recommended.

Table 5 also presents simulations for output, net income ratio and skill premium with optimized fiscal rules. Inspection of Table 5 implies that debt consolidation can be productive over all time horizons in terms of output. Actually, debt consolidation can be expansionary under the optimally chosen fiscal consolidation mix. On the other hand, although optimized fiscal policy rules cannot eliminate the equity-efficiency trade-off of debt consolidation can balance between equity and efficiency better than ad-hoc policies. To sum up, comparison of Table 5 with 4 implies that the transition aggregate cost of debt consolidation consolidation can be avoided while the equity-efficiency trade off can be mitigated if the fiscal policy chooses optimal its fiscal consolidation mix.

Table 5: Output, net income and skill premium over various time horizons with optimized policy

	2 years	4 years	6 years	10 years
y	0.0049	0.0153	0.0173	0.0177
$\frac{y_R^{net}}{y_P^{net}}$	-0.0383	-0.0361	-0.0340	-0.0305
$\frac{w_R}{w_P}$	0.0447	0.0259	0.0191	0.0121

Notes: Optimized policy coefficients are $\gamma_q^{g^c} = 0.80$ $\gamma_q^c = 0.05$, $\gamma_q^l = \gamma_q^k = \gamma_q^{g^e} = \gamma_q^{g^i} = \gamma_q^{tr_R} = \gamma_q^{tr_P} = 0$.

5.3 Welfare effects

So far our analysis focuses on key endogenous variables of the model that captures aggregate and distributional effects. Similar studies usually examine the effects of fiscal consolidation on welfare

[26] Implied response functions from status quo steady state to the new reformed steady state of all endogenous variables are available upon request.

(see e.g. Bi and Kumhof 2011 and Philippopoulos et al. 2017a and 2017b). Policy should also be concerned with welfare since a welfare criterion can summarize the multiple trade-offs faced by policy makers when they plan their policy actions. To this end, in this section we compute social, *Rich* and *Poor* households' welfare over various time horizons for all fiscal consolidation scenarios developed above. Table 6 presents social and household-specific welfare as percentage deviations from a reference regime[27] under ad-hoc fiscal consolidation policies while Table 7 presents similar welfare computations under optimized policy.[28] The main results are, first, ad-hoc debt consolidation policies are harmful for social welfare over all time horizons; this is consistent with output findings in Table 4. Second, *Rich* households are better off while *Poor* households are worse off over the first 10 years of debt consolidation. This implies that ad-hoc policies can increase inequality in the short- and medium-run in terms of welfare as well. In addition, the increase in inequality is more striking under spending-based than tax-based fiscal consolidation. As can be seen in Table 6 welfare of *Poor* (*Rich*) households reduces (increases) more under spending-based consolidation (compare Panel A 4th and 5th row with Panel B 9th and 10th row). Welfare results in Table 7 imply that optimized policy always performs better than ad-hoc policies and this is over all time horizons and across all households. That is as also highlighted in the previous section with optimized policy short-run costs of debt consolidation can be avoided for society as a whole (see second row of Table 7). At the same time, optimized policy can mitigate the short-run negative inequality effect meaning that the reduction in *Poor* households' welfare is lower than the one caused by ad-hoc policies (compare Tables 7 and 6). Finally, both households are better off after the first six years of debt consolidation policies.

[27] We can use multiple reference regimes, for simplicity we compute percentage deviations from a reference regime in which the economy stays at its status-quo steady state forever.

[28] Table 6 and 7 present results from model-based simulations generated by the second-order approximation of the equilibrium system and welfare criterion.

Table 6: Welfare over various time horizons under ad-hoc policies

Panel A: Tax-based consolidation

	2 years	4 years	6 years	10 years
Social Welfare	-0.002	-0.0081	-0.0164	-0.0357
Rich	0.0089	0.0099	0.0058	-0.0108
Poor	-0.0273	-0.0502	-0.0683	-0.0939

Notes: Feedbacks as in Table 4.

Panel B: Spending-based consolidation

	2 years	4 years	6 years	15 years
Social Welfare	-0.0032	-0.0055	-0.0072	-0.0099
Rich	0.0117	0.0206	0.027	0.0338
Poor	-0.0378	-0.0662	-0.0869	-0.1119

Notes: Feedbacks as in Table 4.

Table 7: Welfare over various time horizons with optimized policy

	2 years	4 years	6 years	10 years
Social Welfare	0.0097	0.0280	0.0415	0.0594
Rich	0.0252	0.0418	0.0539	0.0679
Poor	-0.0265	-0.0041	0.0125	0.0394

Notes: Feedbacks as in Table 5.

6 Robustness

This section reports our robustness analysis. In particular, we have experimented with larger and lower values of the complementarity parameter, v, i.e with lower and higher elasticity of substitution between physical capital and skilled labor in the range of $0.33 < \frac{1}{1-v} < 0.90$. We have also experimented with the case in which *Poor* households have access to capital and financial markets. In this way we eliminate heterogeneity in asset holdings. That is *Poor* households can now smooth their consumption and thus capital-skill complementarity affects their capital and income to a smaller extent. We report that our main qualitative results do not depend on this parameter and modelling changes.

7 Conclusions and possible extensions

In this paper using a dynamic general equilibrium model with heterogeneous agents and capital-skill complementarity in the production function we assess the aggregate and distributional implications of public debt consolidation. To conduct our fiscal policy experiments we compare ad-hoc and optimized feedback fiscal policy rules. Since the main results have been summarized in the Introduction, we close with possible extensions.

A possible extension is to depart from the closed economy setup and study similar questions in an open economy setup allowing for international mobility of capital and labour (i.e. migration). Consequently, this leaves room of introducing cross-border effects. Due to the mobility of capital and labour, fiscal consolidation policies could affect aggregate and distributional outcomes through additional channels e.g. changes in the national tax base or household-biased capital or/and migration controls among others. We leave these ideas for future work.

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Appendix

Market clearing conditions

Market clearing conditions in the capital market, the dividends market, the labour (skilled and unskilled) market, the government bonds market are respectively:

$$n^f k_t^f = n_R k_{R,t} \tag{31}$$

$$n^f \pi_t^f = n_R \pi_{R,t} \tag{32}$$

$$n^f l_{R,t}^f = n_R l_{R,t} h_{R,t} \tag{33}$$

$$n^f l_{P,t}^f = n_P l_{P,t} h_{P,t} \tag{34}$$

$$b_t = n_R b_{R,t} \tag{35}$$

The economy's aggregate resource constraint is given by:

$$n_R \left[c_{R,t} + k_{R,t+1} - \left(1 - \delta^k\right) k_{R,t} \right] + n_P c_{P,t} + g_t^i + g_t^c + g_t^e = n^f y_t^f$$

where we express the number of *Rich* and *Poor* households in terms of shares $n_R \equiv \frac{N_R}{N}$, $n_P \equiv \frac{N_P}{N} = 1 - n_R$.

Full equilibrium system

The full equilibrium system is given in detail by the following 29 equations in 29 unknowns which are, $\{y_t^f, c_{R,t}, c_{P,t}, k_{R,t+1}, h_{R,t+1}, h_{P,t+1}, b_{R,t+1}, e_{R,t}, e_{P,t}, l_{R,t}, l_{P,t}, r_t^b, \lambda_{R,t}, \lambda_{P,t}, \psi_{R,t}, \psi_{P,t}, k_t^g, r_t, w_{R,t}, w_{P,t}, q_t\}$ and $\left\{ s_t^{g^i}, s_t^{g_t^c}, s_t^{g^e}, s_t^{tr_P}, s_t^{tr_R}, \tau_t^c, \tau_t^l, \tau_t^k \right\}$, given feedback policy coefficients:

$$\frac{\mu_2}{1 - e_{R,t} - l_{P,t}} = \lambda_{R,t}(1 + \tau_t^c)\left(c_{R,t} + \xi g_t^c\right) \tag{36}$$

$$\lambda_{R,t} = \frac{\mu_1}{(1 + \tau_t^c)\left(c_{R,t+1} + \xi g_{t+1}^c\right)} \tag{37}$$

$$\frac{\mu_2}{1 - e_{R,t} - l_{R,t}} = \psi_{R,t} B_R \theta \left(e_{R,t}\right)^{\theta-1} \left[\left(e_{R,t}\right)^{\theta} \left(g_{R,t}^e\right)^{1-\theta} \right]^{x_R - 1} \tag{38}$$

$$\frac{\mu_2}{1 - e_{R,t} - l_{R,t}} = \frac{\mu_1 \left(1 - \tau_t^l\right) w_{R,t} h_{R,t}}{(1 + \tau_t^c)\left(c_{R,t} + \xi g_t^c\right)} \tag{39}$$

$$\frac{1}{(1 + \tau_t^c)\left(c_{R,t} + \xi \bar{g}_t^c\right)} = \frac{\beta \left[1 - \delta^k + \left(1 - \tau_{t+1}^k\right) r_{t+1}\right]}{\left(1 + \tau_{t+1}^c\right)\left(c_{R,t+1} + \xi g_{t+1}^c\right)} \tag{40}$$

$$\frac{1}{(1 + \tau_t^c)\left(c_{R,t} + \xi g_t^c\right)} = \frac{\beta \left(1 + r_{t+1}^b\right)}{\left(1 + \tau_{t+1}^c\right)\left(c_{R,t+1} + \xi g_{t+1}^c\right)} \tag{41}$$

$$\psi_{R,t}=\beta\lambda_{R,t}\left(1-\tau_t^l\right)w_{R,t+1}l_{R,t+1}+\beta\psi_{R,t+1}\left(1-\delta^h\right) \tag{42}$$

$$h_{R,t+1}=(1-\delta^h)h_{R,t}+B_P\left[(e_{R,t})^{\theta}\left(g_{R,t}^e\right)^{1-\theta}\right]^{x_R-1} \tag{43}$$

$$\begin{aligned}&(1+\tau_t^c)\,c_{R,t}+k_{R,t+1}-(1-\delta^k)k_{R,t}+b_{R,t+1}-b_{R,t}=\\&=(1-\tau_t^k)\,(r_tk_{R,t}+\pi_{R,t})+\left(1-\tau_t^l\right)w_{R,t}l_{R,t}h_{R,t}+\overline{tr}_{R,t}+r_t^bb_{R,t}\end{aligned} \tag{44}$$

$$\lambda_{P,t}=\frac{\mu_1}{(1+\tau_t^c)\left(c_{P,t+1}+\xi g_{t+1}^c\right)} \tag{45}$$

$$\frac{\mu_2}{1-e_{P,t}-l_{P,t}}=\psi_{P,t}B_P\theta\left(e_{P,t}\right)^{\theta-1}\left[(e_{P,t})^{\theta}\left(g_{P,t}^e\right)^{1-\theta}\right]^{x_P-1} \tag{46}$$

$$\frac{\mu_2}{1-e_{P,t}-l_{P,t}}=\lambda_{P,t}(1+\tau_t^c)\left(c_{P,t}+\xi g_t^c\right) \tag{47}$$

$$\psi_{P,t}=\beta\lambda_{P,t}\left(1-\tau_t^l\right)w_{P,t+1}l_{P,t+1}+\beta\psi_{P,t+1}\left(1-\delta^h\right) \tag{48}$$

$$h_{P,t+1}=(1-\delta^h)h_{P,t}+B_P\left[(e_{P,t})^{\theta}\left(g_{P,t}^e\right)^{1-\theta}\right]^{x_P-1} \tag{49}$$

$$n_{R,t}\left[c_{R,t}+k_{R,t+1}-(1-\delta^k)k_{R,t}\right]+n_{P,t}c_{P,t}+g_t^e+g_t^i+g_t^c=n^fy_t^f \tag{50}$$

$$\begin{aligned}&\left(s_t^{g_t^i}+s_t^{g_t^c}+s_t^{g_t^e}+s_t^{tr_R}+s_t^{tr_P}\right)n^fy_t^f+\left(1+r_t^b\right)n_{R,t}b_{R,t}\\=\;&n_{R,t}b_{R,t+1}+\tau_t^c\left(n_{R,t}c_t^s+n_{P,t}c_t^u\right)+\tau_t^kr_tn_{R,t}k_{R,t}+\tau_t^l\left(n_{P,t}w_{P,t}l_{P,t}h_{P,t}+n_{R,t}w_{R,t}l_{R,t}h_{R,t}\right)\end{aligned} \tag{51}$$

$$k_{t+1}^g=(1-\delta^g)\,k_t^g+s_t^{g_t^i}n^fy_t^f \tag{52}$$

$$n^fy_t^f=A\left[m\left(n_{P,t}l_{P,t}h_{P,t}\right)^{\sigma}+(1-m)\left(\rho\left(n_Rk_{R,t}\right)^{v}+(1-\rho)\left(n_{R,t}l_{R,t}h_{R,t}\right)^{v}\right)^{\frac{\sigma}{v}}\right]^{\frac{\alpha}{\sigma}}\left(k_t^g\right)^{1-\alpha} \tag{53}$$

$$\begin{aligned}r_t\;=\;&A\alpha\,(1-m)\left(\rho\left(n_Rk_{R,t}\right)^{v}+(1-\rho)\left(n_Rl_{R,t}\right)^{v}\right)^{\frac{\sigma}{v}-1}\rho\left(k_t^f\right)^{v-1}\\&\times\left[m\left(l_{P,t}^f\right)^{\sigma}+(1-m)\left(\rho\left(n_Rk_{R,t}\right)^{v}+(1-\rho)\left(n_Rl_{R,t}\right)^{v}\right)^{\frac{\sigma}{v}}\right]^{\frac{\alpha}{\sigma}-1}\left(k_t^g\right)^{1-\alpha}\end{aligned} \tag{54}$$

$$\begin{aligned}w_{R,t}\;=\;&A\alpha\,(1-z)\left(\rho\left(n_Rk_{R,t}\right)^{v}+(1-\rho)\left(n_Rl_{R,t}\right)^{v}\right)^{\frac{\sigma}{v}-1}(1-\rho)\left(l_{R,t}^f\right)^{v-1}\\&\times\left[m\left(n_Pl_{P,t}\right)^{\sigma}+(1-m)\left(\rho\left(n_Rk_{R,t}\right)^{v}+(1-\rho)\left(n_Rl_{R,t}\right)^{v}\right)^{\frac{\sigma}{v}}\right]^{\frac{\alpha}{\sigma}-1}\left(k_t^g\right)^{1-\alpha}\end{aligned} \tag{55}$$

$$w_{P,t} = A\alpha m\left(n_P l_{P,t}\right)^{\sigma-1}\left[m\left(n_P l_{P,t}\right)^{\sigma} + (1-m)\left(\rho\left(n_R k_{R,t}\right)^{v} + (1-\rho)\left(n_R l_{R,t}\right)^{v}\right)^{\frac{\sigma}{v}}\right]^{\frac{\alpha}{\sigma}-1}\left(k_t^{g}\right)^{1-\alpha} \tag{56}$$

where we use $s_t^{g_t^i} \equiv \frac{g_t^i}{n^f y_t^f}$, $s_t^{g_t^e} \equiv \frac{g_t^e}{n^f y_t^f}$, $s_t^{g_t^c} \equiv \frac{g_t^c}{n^f y_t^f}$, $s_t^{tr_R} \equiv \frac{tr_{P,t}}{n^f y_t^f} = \frac{n_P \overline{tr}_{P,t}}{n^f y_t^f}$, $s_t^{tr_{R,t}} \equiv \frac{tr_{R,t}}{n^f y_t^f} = \frac{n_R \overline{tr}_{R,t}}{n^f y_t^f}$, $g_{R,t}^e \equiv \omega \bar{g}_t^e$, $g_{P,t}^e \equiv (1-\omega)\bar{g}_t^e$,

Feedback fiscal policy rules are given by:

$$s_t^{g^c} - s^{g^c} = \gamma_q^{g^c}\left(q_{t-1} - q\right) \tag{57}$$

$$s_t^{g^i} - s^{g^i} = \gamma_q^{g^i}\left(q_{t-1} - q\right) \tag{58}$$

$$s_t^{tr_P} - s^{tr_P} = \gamma_q^{tr_P}\left(q_{t-1} - q\right) \tag{59}$$

$$s_t^{tr_R} - s^{tr_R} = \gamma_q^{tr_R}\left(q_{t-1} - q\right) \tag{60}$$

$$s_t^{g^e} - s^{g^e} = \gamma_q^{g^e}\left(q_{t-1} - q\right) \tag{61}$$

$$\tau_t^c - \tau^c = \gamma_q^c\left(q_{t-1} - q\right) \tag{62}$$

$$\tau^l - \tau^l = \gamma_q^l\left(q_{t-1} - q\right) \tag{63}$$

$$\tau_t^k - \tau^k = \gamma_q^k\left(q_{t-1} - q\right) \tag{64}$$

$$q_{t-1} \equiv \frac{n_R b_{R,t}}{n^f y_{t-1}^f} \tag{65}$$