Cost-share induced technological change and Kaldor's stylized facts

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2018

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Abstract

This paper presents a theory of induced technological change in which firms pursue a random, local, and bounded search for productivity-enhancing innovations. Firms implement profitable innovations at fixed prices, which then spread through the economy. After diffusion, all firms adjust prices and wages. The model is consistent with a variety of price-setting behaviors, which determine equilibrium positions characterized by constant cost shares and productivity growth rates. Target-return pricing yields Harrod-neutral technological change with a fixed wage share as a stable equilibrium, consistent with Kaldor's stylized facts, while allowing for deviations from equilibrium, as observed in the longer historical record.

Keywords: classical; post-Keynesian; evolutionary; induced technological change; Harrod-neutral technological change

JEL: E12, E14, O33

Acknowledgements: The author is grateful for the critical comments and suggestions of two anonymous reviewers. Their critiques led to substantial improvements to the manuscript. The work was supported in part by the Swedish Development Cooperation Agency (Sida).

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1 Introduction

In a well-known paper, Kaldor (1961, pp. 178–179) introduced six "stylized facts", of which the first four are: rising labor productivity and output; rising capital per worker; a steady rate of profit; and steady capital-output ratios. The last two imply a steady profit share, and thus a steady wage share. However, shortly after the publication of Kaldor's paper, the wage share rose briefly in many high-income countries and then began to fall, a trend that continues today. By compiling long historical data series, Maddison (1994, p. 10) demonstrated that capital productivity wanders over a wide range, without exhibiting a long-run trend, suggesting that over many decades technological change appears to have a weak Harrod-neutral tendency, but with substantial variations. Observations such as these put Kaldor's stylized facts into question and renewed interest in a major concern of the classical economists – the intersection between growth, distribution and technological change (Acemoglu, 2002; Brugger & Gehrke, 2016; Tavani & Zamparelli, 2017).

The present paper is a contribution to this literature. It presents a theory of cost-share induced technological change, an idea first proposed by Hicks (1932, p. 124 ff.). We show that when combined with a price and wage-setting regime that leaves the profit and wage shares fixed, the theory is consistent with Marx-biased technological change (although other outcomes are possible). A regime of target-return pricing generates a stable dynamic with Harrod-neutral technological change as the equilibrium position, while allowing for substantial variation.

We build on a theory proposed by Duménil and Lévy (1995), who sought, as we do, to motivate a theory of cost-share induced technological change in order to explain the observed behavior of labor productivity and capital-output ratios. Their theory is evolutionary, in the sense of Nelson and Winter (1982), in that innovation is stochastic, locally bounded, and shaped by past decisions. They also follow Nelson and Winter

(1982, pp. 168–169) by separating innovation into a series of distinct processes. With Duménil and Lévy (1995), we view firms as seeking innovations in a bounded region close to their existing technologies. They only accept innovations that increase their profit rate under prevailing prices, which induces a bias in the direction of technological change. Productivity change is random, and constrained by the potential for discovery.

In the theory presented in this paper, innovation is a microeconomic phenomenon, located within the firm. It translates into macroeconomic impacts as innovations are taken up by other firms (Nelson & Winter, 1982, p. 202). Diffusion of innovations weakens the temporary monopoly earned by the innovating firm, and all firms adjust their prices and the wages they pay in response to the new cost structure, resulting in a new functional income distribution. The innovating firm thus faces unintended consequences from its innovation, as proposed by Marx. The functional income distribution which results from their and other firms' innovation, followed by diffusion and price and wage adjustment, induces subsequent rounds of innovation and price setting, thereby setting up a repeating dynamic.

The price- and wage-setting step avoids the implication of Okishio's theorem (Okishio, 1961; Bowles, 1981), which states that with fixed wage rates and the establishment of a new set of Sraffian production prices, the rate of profit must rise. When wages and prices are allowed to adjust dynamically after innovation and diffusion, the profit rate can rise, fall, or stay the same, depending on the price- and wage-setting regime.

While we begin with Duménil and Lévy's (1995) model, we go significantly beyond it. First, we relax their restrictive assumptions about technological discovery. Second, we expand their model from two inputs to production – labor and capital – to an arbitrary number. Third, we follow Kaldor (1961) by tying the rate of technological change to the pace of capital accumulation. We illustrate the theory by applying it to a classically-inspired model along lines proposed by Foley and Michl (1999),[2] showing that the direction of technological change depends on firm pricing strategy: a fixed wage share leads to Marx-biased technological change, whereas target-return pricing leads to Kaldor's stylized fact of trendless capital productivity as an equilibrium position. We generate scenarios with the model that exhibit changes in productivity and cost shares similar to those experienced by the US in the latter half of the 20th century.

2 Theories of induced technological change

The existing body of theory on induced technological change offers competing representations of technological potential and processes of wage and price determination. The classical economists,[3] writing at a time when European countries had large pools of agricultural workers, developed theories with subsistence wages. They saw that labor productivity could be improved both through embodied technological change, which arises from the capabilities of new machines, and disembodied technological change, which arises from learning-by-doing and procedural improvements. Technological progress was reflected in increasing output for a given level of total labor, which was taken to include both the contemporary labor directly used in producing goods and the historical labor embodied in machines. For Ricardo, a rising money wage rate motivated technological innovation, thereby driving down the cost of subsistence. Marx, observing a growing array of machine-built goods, saw more clearly than Ricardo the importance of cumulative technological change and the increasingly intertwined system of industrial production. Famously, Marx concluded that an endogenous process drives a falling rate of profit. Competition for profits drives a substitution of labor for machinery, but the gain is short-lived as the innovation changes the demand for, and cost of, labor. Marx's

[2] The author is applying the theory in a more elaborate post-Keynesian Kaleckian-Harrodian model (Kemp-Benedict, 2017). That model exhibits Kondratieff-type long waves driven in part by cost-induced technological change and in part by wage and price dynamics that operate with lags.
[3] This discussion draws mainly on Kurz's (2010) paper on the views of Smith, Ricardo and Marx on technological progress and income distribution.

general story of rational action at a microeconomic level being frustrated by macroeconomic processes (Kurz, 2016, p. 44) finds an echo in the theory presented in this paper.

The neoclassical theory of technological change was initiated by Hicks (1932), who assumed, on marginalist grounds, that wages and prices are determined endogenously by cost-minimizing firms given a production function. Technical progress was captured in changes to the production function. Hicks' theory was criticized by both Keynesian and neoclassical theorists. Kaldor (1961) argued that neoclassical theory could not account for the stylized facts he had documented, and proposed a "technical progress function" that relates capital accumulation and output growth in a Keynesian theory in which firms have a minimum profit margin and money wages are exogenously specified. Salter (1966 [1960], pp. 43-44) argued that Hicks' entrepreneur, focused on total cost savings, should be indifferent to whether those savings come from capital or labor costs. Kennedy[4] (1964) responded to Salter by arguing that the entrepreneur should seek to minimize unit costs within an "innovation-possibility frontier", which he acknowledged is a "disguised" form of Kaldor's technical progress function. Given rather broad conditions for the shape of the frontier, but arguing that it should not depend on the cost shares themselves, Kennedy showed that his assumption implies technological change biased in the direction of the factor with the highest cost share. Samuelson (1965), while disagreeing with much of Kennedy's argument, nevertheless took it as a starting point for a thorough development of induced technological change with a neoclassical production function, something Kennedy had sought to avoid with the introduction of the innovation-possibility frontier (Kennedy, 1966). This burst of activity faded because of internal

[4] Kennedy credits an unpublished manuscript by Ahmad for inspiration, while Samuelson (1965) credits an unpublished manuscript by Weizäcker with proposing ideas similar to Kennedy's.

problems with the model (Acemoglu, 2002, p. 785; Brugger & Gehrke, 2016; Nordhaus, 1973).

Research within the neoclassical tradition resumed with the emergence of endogenous growth theories (Romer, 1994). This literature follows convention by assuming a production function, but abandons perfect competition. Instead, firms engage in monopolistic competition, gaining monopoly rents for their inventions. Endogenous growth theories thus change the price and wage-setting assumptions of conventional neoclassical theory. Acemoglu (2002) built on this body of work to develop a theory of "directed technological change" in which technological developments are guided by the interaction of a "price effect", under which relative prices drive substitution, and a "market size effect", under which increasing availability of a factor lowers its price and raises demand for goods produced with that factor.

Nelson and Winter (1982) sharply criticized the neoclassical theory of technological change, arguing that it is disconnected from discoveries about the microeconomic processes of innovation. They proposed an evolutionary theory in which firms engage in a search for new techniques, starting from their existing technology, knowledge and practices; technological potential is represented by the possible and probable outcomes of that search. Nelson and Winter offered a rather complex simulation model featuring multiple interacting processes of innovation and imitation. Duménil and Lévy (1995) took their ideas and built a formal model of cost-share induced technological change that can be more easily incorporated into pen-and-paper models of growth and distribution.

The theories of Kennedy (1964) and Duménil and Lévy (1995) have been applied to classical and post-Keynesian models. Julius (2005) argued that to some extent Duménil and Lévy's theory is formally equivalent to that of Kennedy and he applied their common core to different distributional regimes. Among other exercises, he introduced endogenous technological change into a Goodwin (1967) model, as did Shah and Desai

(1981). Both papers found that the Goodwin cycles disappeared when coupled with cost-shared induced technological change. Zamparelli (2015) applied a modified form of Kennedy's theory to a classical model, one in which the innovation-possibility frontier can be altered through R&D expenditure. Julius (2005, p. 110) noted incidentally that Duménil and Lévy's theory places few restrictions on the relationship between the cost share and technological progress; one of the novel contributions of this paper is to discover such restrictions.

Foley (2003, p. 42 ff.) approached biased technological change from a classical perspective. He noted that labor is different from other inputs, and developed a theory based on that proposition. In particular, he made the plausible suggestion that labor-saving innovation is easier to generalize from one process to another, whereas capital-saving innovation is more likely to differ from one process to another. Foley considered, but dismissed, Duménil and Lévy's (1995) theory out of concern that their findings are dictated by the particulars of their model. In this paper we do not address Foley's own theory, which offers a plausible alternative, but propose a model along Duménil and Lévy's lines that addresses his critique.

Post-Keynesian theorists typically endogenize technological change by assuming the Kaldor-Verdoorn law, in which labor productivity growth increases with the pace of capital accumulation. The Kaldor-Verdoorn law has undergone extensive empirical testing, and appears to be robust (Lavoie, 2014, pp. 428–430). However, it applies solely to labor productivity, while, following Kaldor, the capital-output ratio is held fixed. We derive a generalization of the Kaldor-Verdoorn law by drawing a distinction between embodied and disembodied technological change.

The model presented in this paper is more consistent with evolutionary, classical, or post-Keynesian theory than it is with neoclassical theory. Unlike in neoclassical theory, we do not assume a smooth production function, and firms need not know the production possibilities frontier. Along evolutionary lines, firms make a local search relative to their current technology, and implement discoveries that increase their return on capital in the short run. We consider this to be a plausible conception of technological change. The history of technology is one of incremental improvements on existing technologies, rather than a series of leaps to close the gaps opened up by new advances (Basalla, 1988). Consistent with our non-neoclassical orientation, we assume that prices are set by firms engaged in oligopolistic competition (Coutts & Norman, 2013), while wages are influenced by social processes. Thus, both wage and price setting involves choice and conflict, which adds a degree of freedom – and realism – lacking in neoclassical models.

3 The model of Duménil and Lévy

We begin by presenting a version of Duménil and Lévy's (1995) evolutionary model, which has one sector and two factors of production – labor and capital. Firms are continually searching for innovations, which may or may not be biased towards saving on one input or another. A bias will arise in any case because, as argued by Okishio (1961), firms only adopt those innovations that increase their return on capital at fixed prices and wages.

Duménil and Lévy's theory incorporates a version of Marxian unintended consequences. Firms take the individually rational decision to adopt technologies that increase their rate of profit at constant prices and wages. This gives them a temporary monopoly and the excess profits that come with it. In the model, the advantage is erased by the following period, thereby abstracting from the details of diffusion, imitation, and adaptation. Under oligopolistic competition and guided by institutional norms for wage setting, firms then adjust prices and wages to reflect their new cost structure. Thus, pricing and wage-setting behaviors translate the microeconomic behavior proposed by Okishio into macroeconomic outcomes. Duménil and Lévy assumed a fixed mark-up, but as shown by

Julius (2005), their theory is consistent with a variety of pricing and wage-setting strategies.

Firms have a capital productivity κ and labor productivity λ. They have considerable flexibility to set both wages and prices, employing labor at a wage w and setting a price P. The profit rate r is then

$$r = \kappa\pi = \kappa\left(1 - \frac{w}{P\lambda}\right), \qquad (1)$$

where π is the profit share. If a firm makes a discovery that would, if implemented, change productivities by amounts $\Delta\kappa$, $\Delta\lambda$, the firm then asks whether implementing it will raise profitability in the short run while keeping prices fixed. Using a hat to denote a growth rate, $\hat{x} = \Delta x / x$, we have

$$\hat{r} = \hat{\kappa} + \frac{\omega}{\pi}\hat{\lambda}, \qquad (2)$$

where $\omega = 1 - \pi$ is the wage share. Firms adopt an innovation if it raises the profit rate at constant costs, giving the viability condition

$$\pi\hat{\kappa} + \omega\hat{\lambda} > 0. \qquad (3)$$

We illustrate this condition for the average viable technology in . In the figure we suppose, with Duménil and Lévy (1995), that innovation is entirely neutral. Innovations move productivities in a random direction, with a probability distribution that is circularly symmetric and also symmetric around the $\hat{\kappa} = \hat{\lambda}$ line. With these assumptions, the firm is just as likely to make a labor-saving or a capital-saving innovation. However, viable technologies must lie on the positive side of the vector perpendicular to a line that passes through the origin and the point (π, ω); this introduces a bias.

Two such lines are shown, one with $\pi = 0.3$ (so $\omega = 0.7$) and one with $\pi = 0.4$ (so $\omega = 0.6$). The average viable technology lies along a vector perpendicular to the lines, and in a positive direction away from them, shown in the figure by the vectors labeled $(\hat{\kappa}_1, \hat{\lambda}_1)$ and

$(\hat{\kappa}_2, \hat{\lambda}_2)$. As shown, in both cases capital productivity is falling, because both $\hat{\kappa}_1$ and $\hat{\kappa}_2$ are negative. As the profit share rises, the vector rotates clockwise, so when π = 0.4, capital productivity is falling more slowly, and labor productivity rising more slowly, than when π = 0.3.

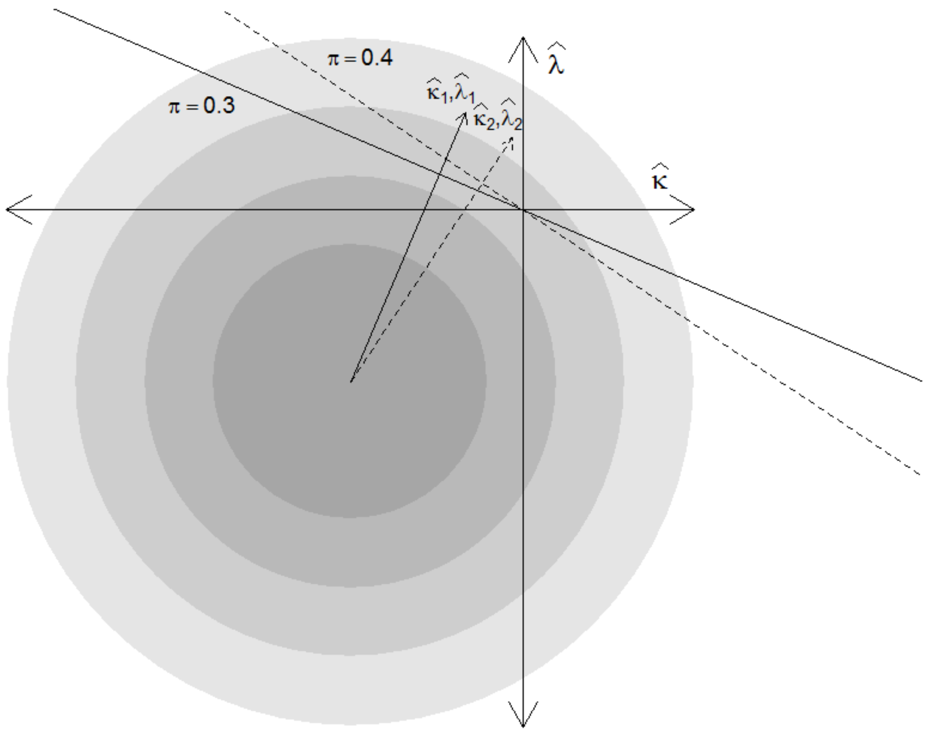

Figure 1: Biased technological change in the model of Duménil and Lévy

As successful discoveries are implemented through innovation and emulation, firms subsequently adjust their prices. They engage in explicit or implicit bargaining over wages and may reduce prices to expand their market or discourage entry by rival firms. Duménil and Lévy (1995, p. 237) argued that labor productivity growth depends on the wage share such that the system approaches an equilibrium with constant labor productivity growth and a fixed wage share. In the example shown in , this drives a steady improvement in labor productivity and a steadily falling rate of profit. However, the result depends on the probability distribution of technological innovation. As the origin of the probability distribution shifts, and as its boundary expands and contracts, it is reasonable to suppose that labor productivity growth continues to be biased in a positive direction, but capital productivity can either rise or fall, as the average rate of capital productivity growth becomes positive or negative. In this way Duménil and Lévy (1995) explained why labor productivity could grow, while the capital-labor ratio wanders across a broad range.

4 A general model of biased technological change

Duménil and Lévy's model is compelling, but it has some limitations. As noted by Foley (2003), their model requires a specific form for the probability distribution of technological discoveries. Furthermore, neither Marx-biased nor Harrod-neutral change is guaranteed, because capital productivity growth is determined by exogenous changes in the probability distribution of technological discovery, and can be positive, negative, or zero. In this section we generalize their model and show how the capital productivity growth rate can emerge endogenously.

First, we allow for an arbitrary number of inputs to production and replace the symmetric probability distribution of technological potential shown in with an arbitrary probability distribution. Next, we separate embodied and disembodied technological change and show how the modified theory implies the Kaldor-Verdoorn law. Finally, we show, in general terms, how cost share induced technological change, when combined with specific price and wage-setting dynamics, sets up equilibrating forces with an equilibrium characterized by constant productivity growth rates and cost shares.

4.1 A multi-factor viability condition

We continue to denote capital productivity by κ and the price level by P, but we now allow for an arbitrary number of inputs, N, indexed by $i = 1,\ldots N$, with costs per unit input q_i. Those inputs, including labor, are used in production with productivity ν_i, so the profit rate is

$$r = \kappa\left(1 - \frac{1}{P}\sum_{i=1}^{N}\frac{q_i}{\nu_i}\right). \tag{4}$$

Suppose that a firm makes a discovery that would, if implemented, change productivities at rates $\hat{\nu}_i$. The change in the profit rate at constant prices is then

$$\hat{r} = \hat{\kappa} + \left(1 - \frac{1}{P}\sum_{i=1}^{N}\frac{q_i}{\nu_i}\right)^{-1}\frac{1}{P}\sum_{i=1}^{N}\frac{q_i}{\nu_i}\hat{\nu}_i. \qquad (5)$$

The expression in parentheses is the profit share,

$$\pi = \left(1 - \frac{1}{P}\sum_{i=1}^{N}\frac{q_i}{\nu_i}\right), \qquad (6)$$

while the cost share for each factor is

$$\sigma_i = \frac{1}{P}\frac{q_i}{\nu_i}. \qquad (7)$$

Using this notation, we can write equation (5) as

$$\hat{r} = \hat{\kappa} + \frac{1}{\pi}\sum_{i=1}^{N}\sigma_i\hat{\nu}_i. \qquad (8)$$

This equation, a generalization of equation (2), is the change in the profit rate that would obtain if a firm were to introduce the innovation. An innovation is viable if it increases the profit rate, and should be rejected if it does not. That is, a viable technology should satisfy $\hat{r} > 0,$ or

$$\pi\hat{\kappa} + \sum_{i=1}^{N}\sigma_i\hat{\nu}_i > 0. \qquad (9)$$

To simplify the notation, we extend the set of factors to include capital, defining a cost share and productivity for index $i = 0$,

$$\sigma_0 \equiv \pi, \quad \nu_0 \equiv \kappa. \qquad (10)$$

We can then write equation (9) compactly as

$$\boldsymbol{\sigma} \cdot \hat{\mathbf{v}} > 0. \qquad (11)$$

This is the viability condition for new discoveries – when it holds, discoveries tend to be accepted, and when it fails, discoveries are certainly rejected.

In reality the decision will be less clear-cut. First, innovation is a messy process; in its transit from the R&D department's bench to the plant floor, a new invention or proposed innovation will undergo numerous changes that ultimately affect productivity. Second, while firms do track costs, it is not always straightforward to assign those costs to the specific product line or process where technological innovation takes place, so firms may not be confident in their ability to estimate the cost implications of a particular innovation. Third, a firm may decide to take a strategic loss in order to gain experience with a new technology or method. These reflections suggest that the crisp cutoff in expression (9) be replaced with a fuzzy boundary.

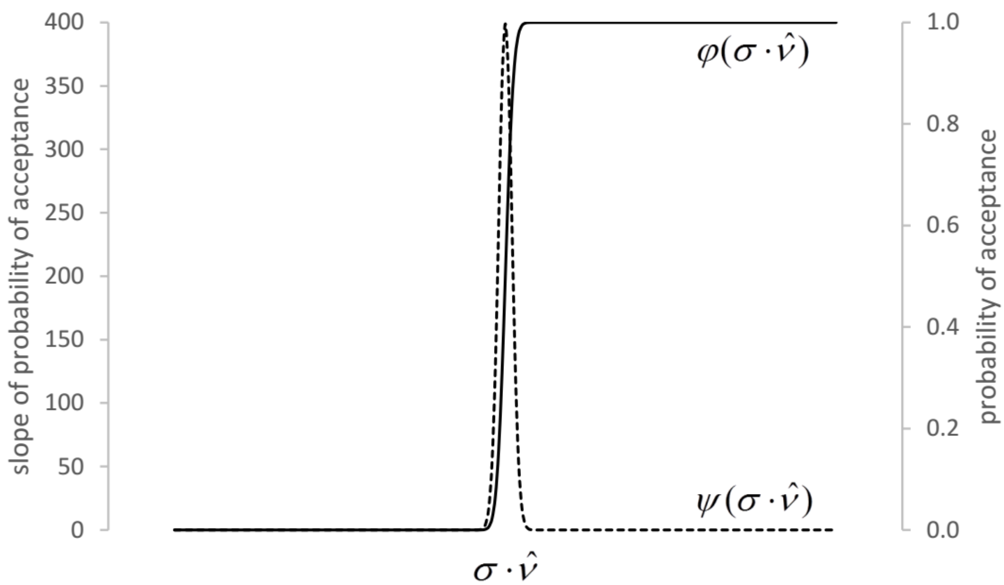

Figure 2: Probability that a firm will accept a candidate innovation

The probability of acceptance $\varphi(\boldsymbol{\sigma}\cdot\hat{\mathbf{v}})$ of a candidate innovation is shown schematically in Figure 2. It drops quickly to zero when the estimated change in profit rate is negative and rises quickly to one when the estimated change is positive. The slope of the curve $\psi(\boldsymbol{\sigma}\cdot\hat{\mathbf{v}})$ is very high near $\boldsymbol{\sigma}\cdot\hat{\mathbf{v}}=0,$ dropping rapidly towards zero on either side. In the ideal case, in which the viability condition in equation (11) holds exactly, $\varphi(\boldsymbol{\sigma}\cdot\hat{\mathbf{v}})$

becomes the Heaviside (or step) function, while $\psi(\boldsymbol{\sigma}\cdot\hat{\mathbf{v}})$ becomes the Dirac delta function.

4.2 Induced bias in a random innovation process

We follow Hicks (1932, p. 125), Nelson and Winter (1982), and Duménil and Lévy (1995) in supposing that innovation is a random process. Specifically, we label innovations by their effect on productivities, and assume that any individual innovation is drawn from a probability distribution $f(\hat{\mathbf{v}})$. The average productivity improvement, $\langle\hat{\mathbf{v}}\rangle$, is then given by integrating over all possible values of $\hat{\mathbf{v}}$ that satisfy the viability condition (11). We enforce the viability condition in a probabilistic form using the probability of acceptance function $\varphi(\boldsymbol{\sigma}\cdot\hat{\mathbf{v}})$ shown in Figure 2, which is equal to one when its argument becomes slightly positive and zero when its argument becomes slightly negative,

$$\langle\hat{\mathbf{v}}\rangle = \int d\hat{\mathbf{v}}\,\varphi(\boldsymbol{\sigma}\cdot\hat{\mathbf{v}})\hat{\mathbf{v}}f(\hat{\mathbf{v}}). \tag{12}$$

The cost share $\boldsymbol{\sigma}$ only appears within the acceptance probability function in equation (12), so we can take the derivative of the *i*th element of $\langle\hat{\mathbf{v}}\rangle$, $\langle\hat{v}_i\rangle$, with respect to σ_j to find

$$\frac{\partial\langle\hat{v}_i\rangle}{\partial\sigma_j} = \int d\hat{\mathbf{v}}\,\psi(\boldsymbol{\sigma}\cdot\hat{\mathbf{v}})\hat{v}_j\hat{v}_i f(\hat{\mathbf{v}}). \tag{13}$$

In this expression we have introduced the slope (or derivative) of the acceptance probability function, $\psi(\boldsymbol{\sigma}\cdot\hat{\mathbf{v}})$. As shown in Figure 2, the acceptance probability has zero slope everywhere except very close to zero, where its slope becomes extremely large.

Equation (13) is an essential result. It holds for any innovation probability distribution $f(\hat{\mathbf{v}})$, so it is a more general and flexible expression than that found by Duménil and Lévy (1995), who assumed a specific form for the probability distribution. By allowing for an arbitrary innovation probability distribution, we address Foley's (2003) objection that Duménil and Lévy's result might depend crucially on their assumption of a symmetric distribution.

We can express the right-hand side of equation (13) as the entries M_{ij} in a matrix $\mathbf{M}$ (the Jacobian matrix),

$$M_{ij} = \frac{\partial \langle \hat{v}_i \rangle}{\partial \sigma_j} = \int d\hat{\mathbf{v}}\, \psi(\boldsymbol{\sigma} \cdot \hat{\mathbf{v}}) \hat{v}_j \hat{v}_i f(\hat{\mathbf{v}}). \tag{14}$$

As shown in the mathematical appendix, $\mathbf{M}$ is, to a good approximation, an $(N+1)\times(N+1)$ dimensional positive semi-definite matrix of rank N that satisfies[6]

$$\boldsymbol{\sigma} \cdot \mathbf{M} = \mathbf{M} \cdot \boldsymbol{\sigma} \approx 0. \tag{15}$$

The properties of the $\mathbf{M}$ matrix characterize acceptable models of cost-share induced technological change consistent with the assumptions that led to equation (12): aggregate productivity change is an average over the randomly-distributed efforts of many firms, with each firm accepting only those innovations that increase its profit rate at prevailing prices and wages. From these properties, we get the immediate result that the own-response of the productivity growth rate to a change in the cost share is positive,

$$\frac{\partial \langle \hat{v}_i \rangle}{\partial \sigma_i} = M_{ii} > 0. \tag{16}$$

By itself, this is not a statement of cost share induced technological change, which obtains when the *total* change in the productivity growth rate depends positively on the change in the cost share,

$$\frac{\Delta \langle \hat{v}_i \rangle}{\Delta \sigma_i} > 0. \tag{17}$$

As explained in the mathematical appendix, this inequality can be violated for complementary inputs – for example, capital and energy.[7] However, the resulting dynamics are unstable, and so unlikely to persist. We expect (17) to hold except for possible transient conditions for complementary factors.

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[6] This relationship holds if the cost shares are treated as independent of each other. In fact, they must sum to one, but if that condition is imposed at this stage, then constraints on the resulting Jacobian are not as easy to state. In applying the theory it is best to impose the condition at a later stage in the analysis.
[7] The author treats this case in a separate paper (Kemp-Benedict, 2018).

It is convenient, when building explicit models, to assume that the approximate equality in equation (15) is exactly true. With two factors, capital and wages, and associated profit and wage shares π and ω, it becomes

$$\begin{pmatrix} M_{00} & M_{01} \\ M_{10} & M_{11} \end{pmatrix}\begin{pmatrix} \pi \\ \omega \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{18}$$

Because **M** is symmetric, $M_{01} = M_{10}$, and we find

$$M_{01} = M_{10} = -\frac{\pi}{\omega}M_{00} = -\frac{\omega}{\pi}M_{11}. \tag{19}$$

All entries can be expressed in terms of a single entry, for example M_{00}, so conditions on **M** restrict the relationship between cost shares and productivity growth rates. This addresses the critique of Julius (2005) that Duménil and Lévy's theory placed few constraints on the functional form of the cost share-productivity relationship.

4.3 Capital accumulation and the Kaldor-Verdoorn law

Up to now, we have treated technological innovation as though it were independent of capital accumulation. For disembodied innovation this is the case: changing operating procedures, retrofitting, and learning-by-doing can lead to higher capital and labor productivity. However, the resource-saving qualities of some innovations are embodied in new capital. In this section we extend the theory to take capital accumulation into account.

We start with a generic input X that is associated with a given capital stock, K. For example, X could be the amount of labor, energy or materials required to operate a given stock of capital. We use a lower-case x to denote the average amount of X per unit of capital stock,

$$x = \frac{X}{K}. \tag{20}$$

New capital has a marginal X-to-capital ratio x_m. If capital of all vintages depreciates at a uniform rate δ (the perpetual inventory model), the time rate of change of X (indicated by a dot) is given by

$$\dot{X} = -x\delta K + x_m I. \tag{21}$$

Dividing both sides by $X = xK$, the growth rate of x is found to be

$$\hat{x} = -\hat{K} - \delta + \frac{x_m}{x} g_K, \tag{22}$$

where $g_K = I/K$ is the gross investment rate. In a perpetual inventory model, the growth rate of the capital stock is

$$\hat{K} = -\delta + g_K. \tag{23}$$

Substituting this expression into equation (22) gives

$$\hat{x} = g_K \left(\frac{x_m}{x} - 1 \right). \tag{24}$$

The quantity in parentheses on the right-hand side of this equation is the relative difference between the marginal and average values of the X-to-capital ratio, which we denote by r_x. The rate of change of the output-to-X ratio – the X-productivity v_x – is then given by

$$\hat{v}_x = \hat{\kappa} - \hat{x} = \hat{\kappa} - g_K r_x. \tag{25}$$

Using the same expressions, but setting X equal to total output, Y, equation (24) gives an expression for capital productivity growth,

$$\hat{\kappa} = g_K \left(\frac{\kappa_m}{\kappa} - 1 \right) = g_K r_Y. \tag{26}$$

Substituting into equation (25) then gives

$$\hat{v}_x = g_K \left(r_Y - r_x \right). \tag{27}$$

This is the expression for X-productivity growth due to embodied technological change.

Regardless of the source of productivity change, whether embodied or disembodied, the choice of technology is governed by the viability condition (9). In the case of embodied technological change, the equations above allow us to write the viability condition as

$$\pi r_Y + \sum_{i=1}^{N} \sigma_i \left(r_Y - r_i \right) > 0. \tag{28}$$

By defining a vector with elements

$$z_0 = r_Y,\, z_i = r_Y - r_i, \tag{29}$$

we find an expression equivalent to equation (11),

$$\boldsymbol{\sigma} \cdot \mathbf{z} > 0. \tag{30}$$

Proceeding by analogy with the derivation for disembodied technical change, we have

$$\langle \hat{\mathbf{v}} \rangle = g_K \int d\mathbf{z}\, \varphi_z(\boldsymbol{\sigma} \cdot \mathbf{z}) \mathbf{z} f_x(\mathbf{z}), \tag{31}$$

where φ_z and f_z are the equivalents, for the **z** variables, to the functions φ and f. This expression is of precisely the same form as equation (12), and it has similar properties.

Firms may discover, in any time step, a number of possible innovations. Some will involve embodied technological change and others disembodied technological change. The first type are governed by equation (31), which we summarize by g_K multiplied by a function $\Psi_i(\boldsymbol{\sigma})$. The second are characterized by equation (12), which we summarize by a function $\Phi_i(\boldsymbol{\sigma})$. Combining the two sources of change, the average rate of productivity change is given by

$$\langle \hat{v}_i \rangle = g_K \Psi_i(\boldsymbol{\sigma}) + \Phi_i(\boldsymbol{\sigma}). \tag{32}$$

This is a generalization of the Kaldor-Verdoorn law. In the Kaldor-Verdoorn law, $\Psi_i(\boldsymbol{\sigma})$ and $\Phi_i(\boldsymbol{\sigma})$ are constants, but here they are functions of the cost shares. The matrices of their partial derivatives,

$$M_{ij} \equiv \frac{\partial \Psi_i}{\partial \sigma_j}, \quad N_{ij} \equiv \frac{\partial \Phi_i}{\partial \sigma_j}, \tag{33}$$

are positive $(N+1)\times(N+1)$ dimensional matrices that are approximately semi-definite. The gross investment rate g_K may also depend on cost shares, so, through equation (32), productivity growth rates can have a complex dependence on the cost share.

Separating embodied and disembodied technological change thus allows for richer dynamics, and the theory developed in this paper offers a generalized form of the Kaldor-Verdoorn law. However, in order to simplify the examples, we do not explore it further in this paper.

4.4 An equilibrating dynamic

The theory of cost share induced technological change present above suggests that aside from a transient condition for some complementary factors, we expect the following to hold:

$$\frac{\Delta\langle\hat{v}_i\rangle}{\Delta\sigma_i} > 0. \tag{34}$$

Thus, a rise (fall) in the cost share of a factor stimulates faster (slower) productivity growth in that factor. We have provided a justification for the assumption using a general probabilistic model of innovation, along with constraints on the relationship between cost shares and productivity growth rates.

After innovation and diffusion, all firms respond to productivity changes by setting prices (following their pricing policies) and wages (through tacit or explicit negotiation with their employees). From the expression for the cost shares in equation (7), the subsequent growth in cost shares is, using the average productivity growth rate,

$$\hat{\boldsymbol{\sigma}} = \left(\hat{\mathbf{q}} - \hat{P}\right) - \langle\hat{\mathbf{v}}\rangle. \tag{35}$$

The term in parentheses is a vector of growth rates of real unit costs. These, in turn, can be expected to respond to changes in productivity. From equation (34), an increase in the cost share associated with input i leads to a rise in its productivity growth rate. Subsequently, from equation (35), the cost share is driven downward, so this is an equilibrating process. The equilibrium is reached when cost shares and productivity growth rates are not changing: $\Delta\hat{\mathbf{v}} = \Delta\boldsymbol{\sigma} = 0$.

An equilibrium, once established, can be disturbed by changes in pricing strategy – e.g., from a fixed mark-up to target-return pricing – or by changes in technological potential as captured in the probability distribution $f(\hat{\mathbf{v}})$. We explore this possibility in the next section.

5 Applying the theory to a classically-inspired growth model

In this section we apply the theory elaborated above to a simple classically-inspired growth model with two inputs to production – labor and capital. In constructing the model we largely follow Foley and Michl (1999).

Different model closures lead to quite different outcomes. We introduce two closures, which we express in terms of the gross profit rate r. Because it is a product, $r = \pi\kappa$, the growth rate of the gross profit rate is the sum of the growth rates of the profit share and capital productivity,

$$\hat{r} = \hat{\pi} + \hat{\kappa}. \tag{36}$$

We first close the model by fixing the profit and wage shares, which corresponds to Foley and Michl's (1999, p. 118) model of Marx-biased technological change and to the standard post-Keynesian assumption of mark-up pricing. In this case,

$$\text{Fixed profit share:} \quad \hat{r} = \hat{\kappa}. \tag{37}$$

A fixed profit (or wage) share is a realistic assumption when labor has a strong bargaining position and is able to defend a constant real wage. When labor is not strong, firms are freer to set prices so as to target a desired rate of return. While target-return pricing was not considered by Foley and Michl, it is a recognized pricing procedure (Lee, 1999) used particularly by large firms with sufficiently sophisticated accounting systems

(Lavoie, 1995, p. 793, 2016, p. 174). Assuming a steady depreciation rate, target-return pricing implies a fixed profit rate. In that case,

$$\text{Fixed profit rate:} \quad \hat{\pi} = -\hat{\kappa}. \tag{38}$$

The conditions expressed by equations (37) and (38) distinguish different pricing regimes that depend on firms' accounting practices and the ability of workers to influence their wages.

5.1 Capital and labor productivity

For purposes of presentation it is helpful to have a specific model for capital and labor productivity. We construct one by setting $M_{10} = M_{01} = -a/\pi$ in equation (19).[9] That determines the other two coefficients, which allows us to write

$$\frac{\partial \hat{\kappa}}{\partial \pi} = M_{00} = a\frac{\omega}{\pi^2}, \quad \frac{\partial \hat{\kappa}}{\partial \omega} = M_{01} = -a\frac{1}{\pi}, \tag{39}$$

$$\frac{\partial \hat{\lambda}}{\partial \omega} = M_{11} = a\frac{1}{\omega}, \quad \frac{\partial \hat{\lambda}}{\partial \pi} = M_{10} = -a\frac{1}{\pi}. \tag{40}$$

These partial derivatives correspond to the following two functions,

$$\hat{\kappa} = k - a\frac{\omega}{\pi}, \tag{41}$$

$$\hat{\lambda} = \ell + a\ln\frac{\omega}{\pi}. \tag{42}$$

The expression for capital productivity growth will prove to be particularly convenient when considering target-return pricing.

5.2 Marx-biased change with a fixed profit share

If the profit share (and therefore the wage share) is fixed at a value $\bar{\pi}$, then capital and labor productivity growth rates are given by

[9] The condition $\boldsymbol{\sigma} \cdot \mathbf{M} \approx 0$ implies that $\hat{\kappa}$ and $\hat{\lambda}$ are (approximately) homogeneous functions of $\boldsymbol{\sigma}$ of order zero. Their first derivatives with respect to the profit share are then homogeneous of order negative one. So, **M** could scale like π^{-1}, ω^{-1}, $(\pi^2 + \omega^2)^{-1/2}$, or any other function of order negative one in the cost shares. This particular choice leads to a convenient expression for the capital productivity growth rate.

$$\hat{\kappa} = k - a\frac{1-\overline{\pi}}{\overline{\pi}}, \tag{43}$$

$$\hat{\lambda} = \ell + a\ln\frac{1-\overline{\pi}}{\overline{\pi}}. \tag{44}$$

These expressions will not change unless the parameters *k*, *ℓ*, or *a* change. Suppose that, initially,

$$\frac{k}{a} = \frac{1-\overline{\pi}}{\overline{\pi}} \quad \Rightarrow \quad \overline{\pi} = \frac{a}{a+k}. \tag{45}$$

With this assumption, capital productivity is constant (by design), while labor productivity is growing at a rate

$$\hat{\lambda} = \ell + a\ln\frac{k}{a}. \tag{46}$$

This situation might have characterized high-income countries from the 1960s through the early 1970s, with a stable wage share and steady profit rates. In the early 1970s that pattern changed.

Gordon (1999, 2012, 2016) has suggested that the pattern changed because the potential for innovation was declining at the end of a wage of technological progress that started in the late 19th century[10]. In the theory presented in this paper, Gordon's suggestion translates into a contraction of the probability distribution $f(\hat{\mathbf{v}})$ of new discoveries. We can implement Gordon's hypothesis by proposing that *k* changes to a new value $k' < k$. After the change, capital productivity growth becomes

$$\hat{\kappa} \to k' - a\frac{1-\overline{\pi}}{\overline{\pi}} = a\left(k'-k\right) < 0. \tag{47}$$

[10] We do not advocate either for or against Gordon's thesis. The validity of the cost-share induced theory of technological change does not depend on it. Oil crises, war, social conflict, the decline of industrial production, and gradually weakening unions could all have plausibly influenced cost shares, wage determination, and pricing strategies. For the purposes of this paper, the virtue of Gordon's mechanism is that it can be implemented in a straightforward manner by changing one or two model parameters.

Thus, under mark-up pricing, after the probability distribution of new discoveries contracts, capital productivity begins to shrink, and continues to shrink indefinitely. Labor productivity continues to grow at the same pace.

The combination of a constant profit share and falling capital productivity corresponds to the conditions in high-income countries that prevailed through the 1970s to the early 1980s. From equation (37), with a fixed profit share, if the capital productivity is falling then the profit rate is also falling. We therefore find Marx-biased technological change as a special – but important – case of a fixed profit share.

5.3 Harrod-neutral technological change under target-return pricing

During the 1980s, the position of labor was considerably weakened, effectively vanishing by the 1990s, a few years into what Goldstein (2012) has called the "shareholder value era". Whether labor's decline was driven by falling profitability or other causes is a moot point, but the increasing strength of firms and their investors vis-à-vis labor allowed firms to set prices so as to achieve desired returns.

Under target-return pricing, firms adjust their profit margins to maintain a fixed value for the gross profit rate, *r*. From equation (38), this implies that the growth rate of the profit share is the negative of the growth rate of capital productivity. The time rate of change of the profit share, indicated by a dot, is then given by

$$\dot{\pi} = -\pi\hat{\kappa}. \tag{48}$$

Substituting from equation (41), we can write this as

$$\dot{\pi} = -\pi\left(k - a\frac{1-\pi}{\pi}\right) = -\left(a+k\right)\pi + a. \tag{49}$$

The solution is a trajectory that asymptotically approaches a new equilibrium,

$$\pi = e^{-(a+k)t}\pi_0 + \frac{a}{a+k}\left(1-e^{-(a+k)t}\right), \tag{50}$$

where π_0 is the value for the profit share at time $t = 0$. Suppose that initially the profit share is $\pi_0 = \bar{\pi}$, where $\bar{\pi}$ satisfies equation (45). Then the exponential terms in equation (50) cancel and we find (as we should) that the profit share is constant,

$$\pi = \frac{a}{a+k} = \bar{\pi}. \tag{51}$$

If k then changes to $k' < k$, as we assumed above, while $\pi_0 = \bar{\pi}$, then we find

$$\pi = \frac{a}{a+k'} - a\frac{k-k'}{(a+k)(a+k')}e^{-(a+k')t}. \tag{52}$$

Over time, the profit share approaches its new steady-state value of $a/(a+k')$. Because k' is less than k, the new profit share is higher than the original. Thus, under target-return pricing we expect a constant profit rate but a rising profit share, until it has reached its new steady-state value. During that time, capital productivity continues to fall. From equation (42), labor productivity growth slows as the profit share increases at the expense of the wage share.

The combination of steady profit rate, falling capital productivity, slowing labor productivity growth, and rising profit share, characterized the 1990s in high-income countries. The 1980s were a transitional period.

5.4 Growth and distribution

We now consider the two cases described above – fixed profit share and fixed profit rate – in a simple classically-inspired growth model, by adding capital accumulation. Following Foley and Michl (1999, p. 98) capitalists' net income is the net profit rate (the gross profit rate, r, less the depreciation rate δ) multiplied by the previous-period capital stock, K_{-1}. That income is then divided between net investment (the difference between current and previous-period capital stocks) and capitalists' consumption C_c,

$$C_c + (K - K_{-1}) = (r - \delta)K_{-1}. \tag{53}$$

If the capitalists' marginal rate of saving out of end-of-period wealth is β, then

$$C_c = (1-\beta)(1+r-\delta)K_{-1}. \quad (54)$$

Substituting into equation (53) gives a growth rate of the capital stock g_K that depends on the rate of profit,

$$g_K \equiv \frac{K}{K_{-1}} - 1 = \beta(1+r-\delta) - 1. \quad (55)$$

Output, Y, is given by the product of capital productivity and the capital stock,

$$Y = \kappa K, \quad (56)$$

while the real wage per worker depends on both the profit share and labor productivity,

$$w = (1-\pi)\lambda. \quad (57)$$

Employment, L, is the ratio of GDP to labor productivity,

$$L = \frac{Y}{\lambda}. \quad (58)$$

In Figure 3 we show indices for κ, r, π, w, Y, and L, as generated by this model. We assume firms to set a target profit rate starting in year 20. The profit rate ramps up to its final value, which is achieved in year 30.

To generate the curves in Figure 3, we set a to 1.75%/year and k to 3.25%/year, corresponding to a profit share of 35%. Labor productivity growth is determined by setting ℓ to 0.40%/year. Capital productivity, κ, starts at 0.3/year. In the first decade, steady capital accumulation and a constant capital productivity lead to steadily rising GDP. The real wage rises at the same as the growth rate of labor productivity, so the profit share remains at 35%. Employment expands modestly because GDP is growing faster than labor productivity.

In year 10, we assume that k falls to 80% of its initial value, causing capital productivity to begin to fall, while ℓ declines to -0.04%/year, causing labor productivity growth to slow. As the wage continues to grow at historical rates, the profit share falls, and as the GDP growth rate falls below the labor productivity growth rate, employment begins to fall as well. Falling capital productivity and declining profit share combine to drive the profit rate down.

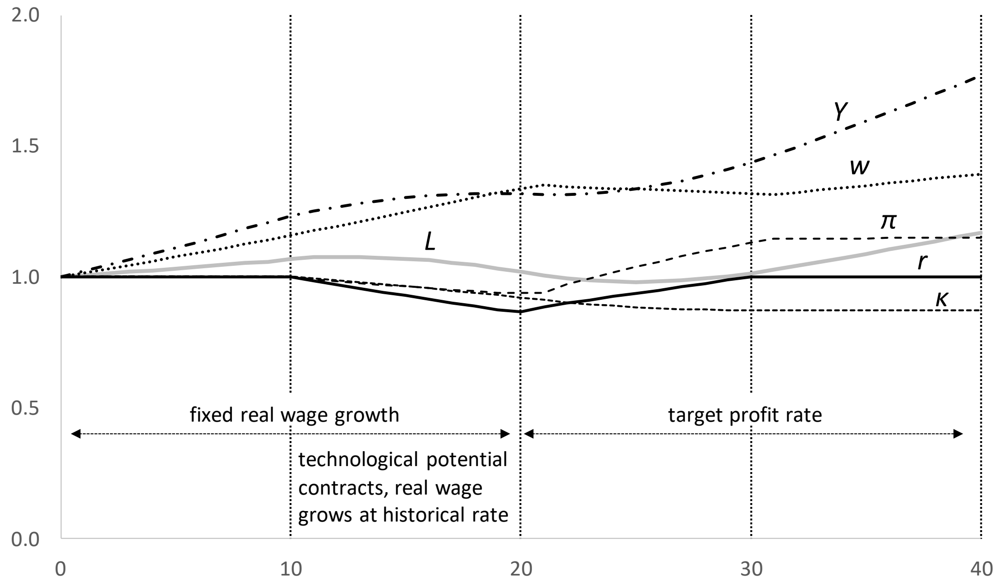

Figure 3: Growth and distribution in a classically-inspired model

In year 20, firms and investors react to falling profitability by targeting returns. The target level rises between years 20 and 30, eventually returning to its initial level. Capital productivity continues to fall, stabilizing around year 30, while the profit share rises. GDP growth outstrips labor productivity and wage growth, leading to rising employment with stagnant wages.

The event that drives the scenario is a contraction in the technological frontier starting in year 10. This corresponds to Gordon's (1999, 2012, 2016) proposed mechanism. The divergent trajectory after that event in Figure 3 follows from changes in price- and wage-setting behaviors. Initially, firm wage- and price-setting strategies are aligned, in which a steady increase in the wage rate is consistent with a fixed mark-up. However, the alignment depends on a steady labor and capital productivity growth rates. Once they begin to slow in year 10, the contradictions become apparent. Firms shift to target-return pricing. The result is rising employment with a sharply rising profit share. This is similar in broad outline to trends in the US from the 1960s through the 1990s.

6 Discussion

The classical preoccupation with the link between growth and distribution has returned to the research agenda. Hicks' (1932) theory of cost share induced technological change has proven both fruitful and frustrating in this effort. Above, we built on a promising but incomplete theory of cost share induced technological change proposed by Duménil and Lévy (1995), who sought to construct an evolutionary theory along the lines laid down by Nelson and Winter (1982). This paper generalized Duménil and Lévy's theory in important ways. We allowed for an arbitrary probability distribution of productivity-enhancing innovations and expanded from two to an arbitrary number of inputs to production. The generalized theory suggests that aside from an edge case involving complementary inputs, cost shares are related positively to the associated productivity growth rate. That is, the theory implies cost share induced technological change. When embodied and disembodied technological change are disentangled, it implies a generalized form of the Kaldor-Verdoorn law.

The expressions that relate cost shares to productivity growth rates are not arbitrary. To a good approximation, the Jacobian matrix **M** is symmetric and positive semi-definite. It is also of order negative one in the cost shares. This finding addresses Julius' (2005) concern that Duménil and Lévy's theory did not constrain the relationships. The constraints are sufficient to ensure cost share induced technological change, with the possible exception of complementary factors. For example, if energy is cheap then energy-intensive capital can readily substitute for labor without regard to the energy costs of operating the capital. However, this exceptional case cannot persist because it generates an unstable dynamic that pushes the cost share of one of the factors upward without limit. At some point the cost will be high enough that we expect it to spur a shift to a different technological regime; for example, toward more energy-efficient capital.

The constraints on the matrix **M** can potentially be tested empirically, but tests using macroeconomic data will be difficult. The reason is that **M** characterizes the change within a firm, before the technology spreads, triggering changes in prices and wages. Macroeconomic data reflect the full process of firm-level innovation followed by diffusion and wage and price adjustment. Tests of the constraints on **M** must be carried out using microeconomic data at the level of the appropriate decision unit.

Alternatively, the underlying assumptions of the model can also be explored using surveys. If firms evaluate potential innovations in a manner consistent with Okishio's (1961) viability criterion, then it increases confidence in Duménil and Lévy's theory and models derived from it, including the one in this paper. Surveys might also ask about R&D processes. We have assumed that the probability of improving productivities with a particular innovation is independent of the cost share, at least in the short run. But if R&D research programs adapt rapidly to a changing cost structure, then that assumption may be violated. We further assumed that innovations made by any firm can be rapidly taken up by other firms. However, diffusion takes time, while, as argued by Foley (2003), labor-saving innovations may diffuse more readily than capital-saving innovations. An extended theory could take such additional biases and time lags into account.

7 Conclusion

Kaldor's stylized facts faced an almost immediate challenge, as the patterns he observed were violated soon after his paper was published. This drove considerable work on technological change, resurrecting the classical research question of the links between growth and distribution. Building on the work of Duménil and Lévy (1995), in this paper we developed a quite general model of cost-share induced technological change. The theory makes specific predictions that can potentially be tested empirically, although the tests are likely to be challenging. When combined with a particular price- and wage-setting regime, the model generates different distributional and growth outcomes. We illustrate this point with a classically-inspired growth model.

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9 Mathematical Appendix

We start with equation (14), which we repeat here for convenience,

$$M_{ij} = \frac{\partial \langle \hat{v}_i \rangle}{\partial \sigma_j} = \int d\hat{\mathbf{v}}\, \psi(\boldsymbol{\sigma} \cdot \hat{\mathbf{v}}) \hat{v}_j \hat{v}_i\, f(\hat{\mathbf{v}}). \tag{A1}$$

Note that if we exchange the *i* and *j* indices under the integral the expression is unchanged, because

$$\hat{v}_i \hat{v}_j = \hat{v}_j \hat{v}_i. \tag{A2}$$

This means that the off-diagonal elements of **M** are symmetric, $M_{ij} = M_{ji}$. The diagonal elements are positive, because after setting $i = j$, all factors under the integral are positive,

$$M_{ii} = \int d\hat{\mathbf{v}}\, \psi(\boldsymbol{\sigma} \cdot \hat{\mathbf{v}}) \hat{v}_i^2\, f(\hat{\mathbf{v}}) > 0. \tag{A3}$$

For a similar reason, for an arbitrary vector **x**, a quadratic form constructed with **M** is non-negative,

$$\mathbf{x}^T \cdot \mathbf{M} \cdot \mathbf{x} = \int d\hat{\mathbf{v}}\, \psi(\boldsymbol{\sigma} \cdot \hat{\mathbf{v}}) \left(\sum_{i=0}^{N} x_i \hat{v}_i \right)^2 f(\hat{\mathbf{v}}) \geq 0. \tag{A4}$$

We can derive a further useful property. Compute

$$\mathbf{M} \cdot \boldsymbol{\sigma} = \int d\hat{\mathbf{v}}\, \boldsymbol{\sigma} \cdot \hat{\mathbf{v}}\, \psi(\boldsymbol{\sigma} \cdot \hat{\mathbf{v}}) \hat{v}_i\, f(\hat{\mathbf{v}}). \tag{A5}$$

Because $\psi(\boldsymbol{\sigma} \cdot \hat{\mathbf{v}})$ is sharply peaked near zero (see Figure 2), and is zero everywhere else, we have

$$\boldsymbol{\sigma} \cdot \hat{\mathbf{v}}\, \psi(\boldsymbol{\sigma} \cdot \hat{\mathbf{v}}) \approx 0, \tag{A6}$$

to a very good approximation. We conclude, from equation , that

$$\mathbf{M} \cdot \boldsymbol{\sigma} = \boldsymbol{\sigma} \cdot \mathbf{M} \approx 0, \tag{A7}$$

again to a very good approximation. The quadratic form in equation is then seen to be (approximately) zero only for vectors **x** that are proportional to **σ**.

From the approximate relationship in equation (A7) we also find that each productivity growth rate $\langle \hat{v}_i \rangle$ is (approximately) homogeneous of order zero in the cost shares,

$$\mathbf{M}\cdot\boldsymbol{\sigma} = \sum_{j=1}^{N} \sigma_j \frac{\partial\langle \hat{v}_i\rangle}{\partial \sigma_j} \approx 0. \tag{A8}$$

If the probability distribution $f(\hat{\mathbf{v}})$ does not change, the total change in $\langle \hat{v}_i\rangle$ is given by

$$\Delta\langle \hat{v}_i\rangle = \sum_{j=0}^{N} \Delta\sigma_j \frac{\partial\langle \hat{v}_i\rangle}{\partial \sigma_j} = \sum_{j=0}^{N} M_{ij}\Delta\sigma_j. \tag{A9}$$

Because $\boldsymbol{\sigma}$ is a vector of shares that must sum to one, the change $\Delta\boldsymbol{\sigma}$ has to sum to zero, so it cannot be proportional to $\boldsymbol{\sigma}$. From this fact, and the positive semi-definiteness of $\mathbf{M}$, we find that

$$\Delta\boldsymbol{\sigma}^T \cdot \Delta\langle \hat{\mathbf{v}}\rangle = \Delta\boldsymbol{\sigma}^T \cdot \mathbf{M}\cdot\Delta\boldsymbol{\sigma} > 0. \tag{A10}$$

When the approximate inequality in (A8) holds exactly, it says that the productivity growth rate for each input is a homogeneous function of the cost shares of order zero. That means that the matrix of derivatives, $\mathbf{M}$, must be of order negative one in the cost shares.

9.1 Complementary factors as a special case

The positivity of the own-response in (16) does not guarantee that the total response of the productivity growth rate of a particular factor to a rise in the cost share will generally be positive. A possible exception can arise when two factors are complementary. We say that two factors *i* and *j* are complements if M_{ij} is positive, and substitutes if negative.

The distinction between complements and substitutes is important when determining the total change in productivity growth rates. Cost shares must sum to one, so any increase in the cost share of one factor must be compensated by a net decrease in the cost shares of other factors. The most problematic case arises when price and wage setting dynamics are such that an increase *z* in the cost share for factor *i* is compensated entirely by a fall in the cost share for a complementary factor *j*,

$$\Delta\sigma_i = z, \quad \Delta\sigma_j = -z, \quad \Delta\sigma_k = 0 \text{ for } k \neq i, j. \tag{A11}$$

Then the total change in $\langle \hat{v}_i\rangle$ is equal to

$$\Delta\langle \hat{v}_i \rangle = z\left(\frac{\partial\langle \hat{v}_i \rangle}{\partial \sigma_i} - \frac{\partial\langle \hat{v}_i \rangle}{\partial \sigma_j} \right) = z\left(M_{ii} - M_{ij} \right). \tag{A12}$$

If factors i and j are substitutes, this is certainly positive, because in that case M_{ij} is negative. However, if they are complements then this expression could, in principle, be negative. To constrain the possibilities, we use a property of positive semi-definite matrices, that the absolute value of the off-diagonal elements is bounded by the average of the corresponding diagonal entries,[11]

$$\left| M_{ij} \right| \le \frac{1}{2}\left(M_{ii} + M_{jj} \right). \tag{A13}$$

If i and j are complements, then $|\mathrm{M}_{ij}| = \mathrm{M}_{ij}$, and we have

$$\Delta\langle \hat{v}_i \rangle \ge \frac{z}{2}\left(M_{ii} - M_{jj} \right) = \frac{z}{2}\left(\frac{\partial\langle \hat{v}_i \rangle}{\partial \sigma_i} - \frac{\partial\langle \hat{v}_j \rangle}{\partial \sigma_j} \right). \tag{A14}$$

As we have assumed z to be positive, the total change in $\langle \hat{v}_i \rangle$ is certainly positive when factor i's own-response exceeds that for factor j. However, using the same argument, a rise in the cost share of the other factor, j, could be negative, which can introduce a positive feedback. Under the assumed conditions, when the jth factor's cost share rises and the ith factor's cost share falls by the same amount, processes that use more of factor i become profitable even if they drive unit inputs of the jth factor upward. If they are adopted, we expect the cost share σ_j to increase, driving a further decline in v_j.

This positive feedback, which drives the cost share of factor j steadily upward and that of factor i downward, clearly cannot continue indefinitely; at some point, factor j figures so heavily in costs that firms can no longer accept increases in its use even if accompanied by declines in the cost of factor i. We therefore argue, on economic grounds, that pairs of factors should normally satisfy

[11] This can be shown by defining $\mathbf{x}^{\pm}$ such that $x_i^{\pm} = 1$, $x_j^{\pm} = \pm 1$, $x_k = 0$ for $k \neq i, j$, and constructing quadratic forms with $\mathbf{M}$.

$$\frac{\partial\langle \hat{v}_j \rangle}{\partial \sigma_i} = \frac{\partial\langle \hat{v}_i \rangle}{\partial \sigma_j} \leq \min\left(\frac{\partial\langle \hat{v}_i \rangle}{\partial \sigma_i}, \frac{\partial\langle \hat{v}_j \rangle}{\partial \sigma_j} \right). \tag{A15}$$

For complementary factors, this inequality may be temporarily violated, accompanied by an unstable dynamic that halts when cost shares have shifted sufficiently that the inequality is re-established.