Temporal disaggregation by dynamic regressions: recent developments in Italian quarterly national accounts

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Abstract

In this paper we discuss the most recent developments of temporal disaggregation techniques carried out at ISTAT. They concern the extension from static to dynamic autoregressive distributed lag ADL regressions and the change to a state-space framework for the statistical treatment of temporal disaggregation. Beyond the development of a unified procedure for both static and dynamic methods from one side and the treatment of the logarithmic transformation from the other, we provide short guidelines for model selection. The inclusion in the regressions of stochastic trends has been also discussed. From the empirical side we evaluate the new dynamic methods by implementing a large scale temporal disaggregation exercise using ISTAT annual value added data jointly with quarterly industrial production by branch of economic activity over the period 1995-2013. The main finding of this application is that ADL models either in levels and logarithms can reduce the errors due to extrapolating disaggregated data in last quarters before the annual benchmarks become available. When the attention moves to the correlations with the high-frequency indicators the ADL disaggregations are also generally in line with those produced by the static Chow-Lin variants, with problematic outcomes limited to few cases.

Keywords*:* temporal disaggregation; state-space form; Kalman filter; ADL models; linear Gaussian approximating model; quarterly national accounts.

1. Introduction

Since mid-eighties, when Italian quarterly national accounts releases became systematic, temporal disaggregation methods gathered larger attention as a decisive tool for their production. At that time ISTAT adopted new information technology instruments borrowing most of the work already developed at the Bank of Italy. Temporal disaggregation methods like both the Chow and Lin (1971) solution – see the development by Barbone et al. (1981) – and the approach by Denton (1971) were adopted for estimating quarterly national accounts. A second renovating phase dates back to mid-nineties when a critical analysis of temporal disaggregation methods by Lupi and Parigi (1996) came out inspiring the development of a

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sophisticated procedure for diagnostic checking. Such procedure still largely supports the operational phases of the quarterly accounts process since it offers a complete diagnostic report of quarterly disaggregations.

Later on, between 2004 and 2005, an ISTAT study commission was set up with the task of formulating new proposals for temporal disaggregation. The final remarks in Di Fonzo (2005) provided evidence of the ISTAT commitment to modernize both conceptual and technical tools used within quarterly accounts, largely implemented in the following years.

Most recently the literature has proposed further methodological developments mainly due to the initiative by Eurostat. See Frale et al. (2010 e 2011), Grassi et al. (2014) e Moauro (2014), among others. At the same time ISTAT favoured a relatively more pragmatic approach. In particular, the effort was aimed at technically implementing temporal disaggregation methods based on autoregressive distributed lag (ADL) models according to Proietti (2005).

Such extension, on one hand significantly broadened the range of the available models to be used for temporal disaggregation, on the other, entailed a number of practical benefits for quarterly accounts analysis, given that it is based on the *Kalman filter* (Kalman, 1960). Namely, the computation of innovations, the development of diagnostics concerning the extrapolations and the resorting to models based on the logarithmic transformation of the data to be disaggregated.

The present work describes the innovative elements of temporal disaggregation recently introduced at ISTAT concerning the enlargement of the range of models to be selected and the development of statistics for diagnostic checking. These tools are largely used in the empirical section of the paper where we present the results of an extensive temporal disaggregation experiment. We compare the performances of the enlarged class of models, providing some guidelines for model selection and highlighting the critical points. Advantages and disadvantages of alternative solutions are discussed by taking into account the main features of the exercise.

This paper is structured as follows: section 2 describes the analytics of the reference ADL(1,1) model and its linkage to the static regression setup; the main features of the state space representation including the case of logarithmic transformation, diagnostic checking and temporal disaggregation evaluation are discussed in section 3; section 4 presents the results of the empirical experiment and section 5 shortly concludes. Finally, appendix A includes a set of tables describing the features of the dataset and the main results of the application, and appendix B the state space form of disaggregation methods under stochastic trends.

2. Dynamic regression methods

2.1 The ADL(1,1) model

Temporal disaggregation methods here discussed are based upon dynamic regression models. They encompass a linear univariate relationship between the dependent variable y_t, its lagged values y_{t-1} and a series of regressors x_t in a given time span t = 1, …, T. The problem is that y_t is available only as a temporal aggregate Y_t over s periods, i.e. only annual values are available over the quarterly time span. In case the annual aggregate value results as the sum of quarters, Y_t is defined as $Y_t = y_t + \cdots + y_{t-s+1}$ with s=4. Alternatively $Y_t = (y_t + \cdots + y_{t-s+1})/s$ reflecting the case of averaged stocks. Hence, Y_t is only observed in periods t=s, 2s, …, $[T/s]$, where $[T/s]$ is the largest integer of the ratio T/s. On the other hand, the k covariates $x_t = (x_{1t}, \dots, x_{kt})$ are observable at each quarter t=1, …, T.

A general representation of the relation between the two sets of variables is given by the dynamic autoregressive distributed lag models ADL(1,1) that is specified at the higher frequency as:

$$\Delta^d y_t = \phi\, \Delta^d y_{t-1} + m + gt + \Delta^d x'_t \beta_0 + \Delta^d x'_{t-1} \beta_1 + \epsilon_t\,,\ \ \epsilon_t \sim NID(0, \sigma^2). \quad (1)$$

In equation (1) Δ is the difference operator such that $\Delta y_t = y_t - y_{t-1}$; d represents the differencing order; ϕ is the autoregressive term such that $-1 < \phi < 1$; m and gt are the deterministic components i.e., respectively, a constant term and a linear trend; β_0 and β_1 are the regression coefficients vectors at lag 0 and 1, respectively; ϵ_t is the vector of stochastic errors for which a normal distribution with zero mean and constant variance equal to σ^2 is assumed.

When in equation (1) x_t only appears at time t (or $\beta_1 = 0$), the ADL(1,1) model (1) switches to the ADL(1,0) form considered in the literature of temporal disaggregation by Santos, Silva and Cardoso (2001) and also treated in the empirical part of the present analysis.

Concerning the differencing order it should be noted that if d=0 the ADL form (1) adequately fits both the cases of stationary and nonstationary cointegrated data, being a simple reparametrization of the error correction model. When instead d=1, the ADL form (1) implies non-stationary and non-cointegrated series.

The deterministic components m and gt in model (1) are only included to let the ADL(1,1) form general enough to fit a wide range of situations where, for example, both a level and a slope require to be accounted for. The case when these components are stochastic will be treated in section 2.4. Indeed in the empirical application both the models with and without the deterministic components will be widely tested, as well as the stochastic variant by a smaller scale exercise.

The ADL(1,1) model was made popular by Hendry e Mizon (1978) who pointed out that the stability of the model would hold even if the regressors determined a spurious relationship in level and were uncorrelated in differences.

Within the domain of temporal disaggregation Proietti (2005) suggested a methodology based both on the parametrization of ADL(1,1) models in the state space form (SSF) and on the use of the Kalman filter for its statistical treatment. In particular the use of the Kalman filter is intended for: log-likelihood computation, model parameters estimation, high-frequency (e.g. quarterly) distribution or temporal disaggregation of data observed as the sum/average in a lower frequency time span (e.g. annually) and the extension to the non-linear temporal disaggregation in case of logged data.

Concerning maximum likelihood estimation of parameters of model (1), the most appropriate solution is the generalised least squares (GLS) method. Indeed, all model regression coefficients, i.e. m, g, β_0, β_1 and the variance term σ^2 of ε_t residuals can be concentrated out of the log-likelihood function, thereby originating a profile likelihood depending only from the autoregressive parameter ϕ. Its estimation can be conveniently set up either as a grid search of ϕ over the interval (-1, 1) or by resorting to a Newton-type optimizing method (like BFGS) which, if available and appropriately set, could require a smaller number of iterations.

An alternative measure to the profile log-likelihood is the 'diffuse' profile log-likelihood shown in Proietti (2006b) (eq.15, p. 264) which is based on data in differences.

Either regression models with AR(1) residuals -including I(1) models- and ARIMA(1,1,0) models are nested in the ADL(1,1) model (1); within the temporal disaggregation domain these specific cases correspond, respectively, to the Chow and Lin (1971), Fernàndez (1981) and Litterman (1983) methods.

2.2 From ADL(1,1) to AR(1) Chow-Lin model

Under suitable hypothesis on initial conditions and under proper linear restrictions on regressor parameters, the model ADL(1,1) nests a linear regression model whose residuals follow an AR(1) process. For example, model (1) in levels without deterministic components can be rewritten in terms of the lag polynomial $\beta_0 + \beta_1 L$ such that L is the lag operator for which $Lx_t = x_{t-1}$:

$$y_t = \phi y_{t-1} + x'_t(\beta_0 + \beta_1 L) + \epsilon_t.$$

Under this form and given the condition $\beta_1 = -\phi \beta_0$, it becomes

$$y_t(1 - \phi L) = x'_t \beta_0 (1 - \phi L) + \epsilon_t$$

and, therefore

$$y_t = x'_t \beta_0 + \alpha_t,\ \alpha_t = \phi \alpha_{t-1} + \epsilon_t \qquad (2)$$

where the residual term α_t follows a first order autoregressive stationary process.

2.3 From ADL(1,1) in differences to the Litterman (1983) and Fernàndez (1981) models

Under suitable initial conditions reflecting the non-stationarity of ADL(1,1) model in differences, both Fernàndez (1981) and Litterman (1983) models can also be derived from model (1). They will result as linear regression models with residuals following a I(1) random walk process in the former case and an ARIMA(1,1,0) process in the latter.

In formulas, when d=1 then:

$$\Delta y_t = \phi\, \Delta y_{t-1} + \Delta x'_t \beta_0 + \Delta x'_{t-1} \beta_1 + \epsilon_t \qquad (3)$$

That, if $\phi = 0$ and $\beta_1 = 0$, corresponds to the Fernàndez model, i.e.:

$$\Delta y_t = \Delta x'_t \beta_0 + \epsilon_t,$$

that is:

$$y_t = x'_t \beta_0 + u_t$$
$$u_t = u_{t-1} + \epsilon_t$$

where u_t follows a random walk process.

Under the condition $\beta_1 = -\phi\ \beta_0$, model (3) nests the Litterman model such that:

$$\Delta y_t = \phi\, \Delta y_{t-1} + \Delta x'_t \beta_0 - \Delta x'_{t-1} \phi\ \beta_0 + \epsilon_t,$$
$$y_t = x'_t \beta_0 + u_t$$
$$\Delta u_t = \phi\, \Delta u_{t-1} + \epsilon_t$$

where u_t follows an ARIMA(1,1,0) process.

2.4 From deterministic to stochastic trend components

An immediate generalization of the ADL(1,1) model (1) is to consider a stochastic form for both the constant and the trend components like

$$y_t = \phi\, y_{t-1} + \mu_t + x'_t\beta_0 + x'_{t-1}\beta_1 + \epsilon_t\,,\ \ \epsilon_t \sim NID(0, \sigma^2), \qquad (4)$$
$$\mu_t = \mu_{t-1} + \upsilon_t + \eta_t\,,\ \ \eta_t \sim NID(0, q_\eta \sigma^2), \qquad (5)$$
$$\upsilon_t = \upsilon_{t-1} + \varsigma_t\,,\ \ \varsigma_t \sim NID(0, q_\varsigma \sigma^2). \qquad (6)$$

The ADL form (4) relates the levels of y_t to those of the regressors x_t and a stochastic level μ_t which takes the place of $m + gt$ of equation (1). The level μ_t is modelled as the 'local linear trend' of equations (5)-(6) where υ_t is a stochastic slope whose form is provided by the simple random walk of equation (6). The terms η_t and ς_t are the white noise disturbances associated respectively to the level μ_t and the slope υ_t. Their variances are scaled with respect to σ^2 (i.e. the variance of ϵ_t) by respectively the couple of 'signal-noise ratio' terms q_η and q_ς, assuming both non-negative values.
When both q_η and q_ς are strictly positive the form (5)-(6) is an I(2) trend for which both the level and the slope components are allowed to change over time, a circumstance which not always matches to data. However the form (5)-(6) is flexible enough to describe also other trend types by restricting the signal-noise ratio terms. A first restriction occurs when μ_t is modelled as a 'smooth trend', corresponding to set $q_\eta = 0$ and $q_\varsigma > 0$. Notice that in this case μ_t is still an I(2) process but it evolves in a smoother way than the unrestricted case. When $q_\eta > 0$ and $q_\varsigma = 0$ the model (5)-(6) becomes a random walk with constant drift υ, which is now an I(1) process particularly appropriate to model an erratic behaviour of a time series around a constant slope. If furthermore $\upsilon = 0$, the random walk is drift-less, equation (5) becomes $\mu_t = \mu_{t-1} + \eta_t$ and equation (6) is dropped. Finally the model (5)-(6) falls into an I(0) deterministic trend when $q_\eta = q_\varsigma = 0$, $\mu_t = m > 0$, $\upsilon_t = g > 0\ \forall t$ and it can be easily shown that $\mu_t = m + gt$.
Like the ADL model (1), also the AR(1) Chow-Lin model (2) can be augmented to include the stochastic trend μ_t of equations (5)-(6), therefore becoming

$$y_t = \mu_t + x'_t \beta_0 + \alpha_t, \quad \alpha_t = \phi\, \alpha_{t-1} + \epsilon_t, \qquad \epsilon_t \sim \text{NID}(0, \sigma^2). \tag{7}$$

The statistical treatment of both the ADL model (4)-(5)-(6) and the Chow-Lin model (7)-(5)-(6) is carried out by setting appropriate state space forms (see Appendix B) and running the Kalman filter and smoother to produce the disaggregated values. The main practical difference from the deterministic models (1) and (2) respectively is that hyper-parameters, i.e. those remaining after concentrating out β_0, β_1 and σ^2 from the log-likelihood are now given by the set $\psi = \{\phi, q_\eta, q_\varsigma\}$ and not only by ϕ. Therefore maximum likelihood estimation of ψ requires the use of a Newton-type optimizing method (like the BFGS routine) which should be make available among the tools for current production of official statistics.

3 Statistical treatment and diagnostic checking

3.1. Essentials of the state space representation

In general, the state space representation within the temporal disaggregation domain is defined by two equations: the former defines the time series structure (measurement equation), the latter how the latent structural components evolve from one state to the following one (transition equation). The SSF representation allows to resort to the Kalman filter methodology which in turn allows to compute the optimal estimator of the state variables vector at time *t* for *t=1, ..., T*, given the information available by the same horizon. The Kalman filter is usually associated to a smoothing algorithm which allows to optimally estimate the state vector conditioned to the whole information set. The application of the SSF approach to temporal disaggregation was formerly introduced by Harvey and Pierce (1984) and then developed by Harvey (1989), Harvey and Chung (2000), Harvey and Koopman (1997) and Moauro and Savio (2002), among others. The peculiarities of the SSF representation applied to the temporal disaggregation techniques have been subsequently treated by Proietti (2005; 2006a; 2006b) whose contributions are the essential basis of this work.

Advantages of the SSF are the following: i) a suitable treatment of the initial conditions in presence of a non-stationary time series; ii) the availability of more effective diagnostics oriented to evaluate the quality of maximum likelihood estimates like the innovations; iii) the chance to easily obtain extrapolations of the series in case of models without covariates. Among disadvantages we find a relatively larger complexity due to the Kalman filter which in some environments implies slower computations.

According to Harvey (1989, sec. 6.3) temporal disaggregation traces back to a "missing observations" problem which is appropriately treated by augmenting the SSF representation of a general model, and therefore the ADL model of equation (1), by a cumulator variable y_t^C observable only at time $t = s, 2s, 3s, \ldots$ For a quarterly-annual disaggregation exercise of flow series $s = 4$ and y_t^C is such that:

$$
\begin{array}{llll}
y_1^C = y_1, & y_2^C = y_1 + y_2, & y_3^C = y_1 + y_2 + y_3, & y_4^C = y_1 + y_2 + y_3 + y_4 \\
y_5^C = y_5, & y_6^C = y_5 + y_6, & y_7^C = y_5 + y_6 + y_7, & y_8^C = y_5 + y_6 + y_7 + y_8 \\
\ldots & & &
\end{array}
$$

or in Markovian terms $y_{\mathrm{t}}^C = \psi_t \, y_{\mathrm{t}-1}^C + y_{\mathrm{t}}$, where ψ_t is such that

$$\psi_t = \begin{cases} 0, & t = 1, 5, \ldots \\ 1, & otherwise \end{cases}$$

As far as the statistical treatment is concerned, the required steps are: the cumulator variable y_t^C is added to the state vector of the SSF of the model defined at the highest frequency of observation; the measurement equation is adjusted so that the Kalman filter could take into account the missing observations of y_t^C; then, the likelihood function of the given model is computed, its maximization with respect to the unknown parameters vector is carried out and both missing observations and disaggregated data are estimated through the smoothing algorithm. For full details see Proietti (2005).

3.2 The case of log-transformed series

The logarithmic transformation of data is a common practice in time series econometrics, and in particular when the series refer to variables defined as the ratio of flow aggregates (Proietti, 2005). Applying the logarithmic transformation to the series implies a number of well-known advantages such as the downsizing of the series volatility, or the larger plausibility of the hypothesis underpinning the regression model (model linearity, homoscedasticity and normality of residuals). Furthermore, contrary to the case of untransformed data, the logarithmic transformation ensures that disaggregated data assume only positive values.

Within the logarithmic context, both the temporal aggregation constraints that must hold for the disaggregated (unknown) series and the constraints represented by the cumulator variable are defined in non-linear terms. For the sake of clarity, we now assume $z_\tau = \log(y_\tau)$ and we consider the following relationship holding between z_τ and the correspondent aggregated series Y_τ:

$$Y_\tau = \sum_{j=0}^{s-1} \exp\left(z_{\tau s-j}\right), \quad \tau = 1, \dots, \left[\frac{T}{s}\right]. \tag{8}$$

where τ refers to the low frequency time span and s denotes the ratio between high and low frequencies involved in the disaggregation, with the cumulator variable becoming $y_{\mathrm{t}}^C = \psi_t \, y_{\mathrm{t}-1}^C + \exp(z_{\mathrm{t}})$.

Disaggregated data $\hat{y}_t$ are computed applying an iterative method converging towards the constrained posterior mode estimate of the unknown solution which satisfies exactly the restrictions of equation (8). Given a trial initial estimate $\tilde{y}_t$ (e.g. a series of ones) of y_{t}, iterations start from and the first order Taylor approximation of $\exp(z_{\mathrm{t}})$ around that trial estimate. This allows to expand the SSF defined for the linear disaggregation case to a linear Gaussian approximating model (LGAM) for log-transformed data. In a second step running the Kalman filter and smoother of the LGAM computed at the first step produces a first disaggregated series $\hat{y}_t$. Then $\tilde{y}_t = \hat{y}_t$ is set and a new LGAM is computed producing a second disaggregation $\hat{y}_t$. This process is iterated until convergence, which usually requires not more than 6-7 rounds. For full detail refer to Proietti (2005) and Proietti and Moauro (2006).

3.3 Test statistics and diagnostic checking of temporal disaggregation

Among the main features of the SSF representation and the Kalman filter there is the estimation of forecasting errors or innovations as a by-product of the application of the Kalman algorithm. In particular, the main diagnostic statistics implemented within the new procedures rely upon standardized innovations $\hat{v}_t$ which, in temporal disaggregation problems, assume real values at time *t=s,2s, ...,[T/s]* but are missing otherwise. Innovations v_t are such that $v_t = y_t^c - E(y_t^c|\mathrm{I}_{t-1})$ where $I_{t-1} = \{\mathrm{y}_{t-1}, x_t\}$ is the information set of the lagged dependent variables y_{t-1} and the exogenous regressor x_t. Therefore standardized innovations are defined as

$$\hat{v}_t = \frac{v_t}{\sqrt{f_t}}$$

where f_t are the estimated variances of v_t for *t = 1, ..., T* also computed by the Kalman filter.

Two remarks: first, both for static and dynamic models the whole set of statistics resulting by the standardized innovations $\hat{v}_t$ are consistent to the standard regression formulas. For an exhaustive and comprehensive discussion on this topic see Harvey (1989, sec.5.4 p. 256 ff.). Second, within temporal disaggregation, the innovations measure the one-step-ahead forecast error over the lowest frequency of observations, as v_t cumulates the errors from the first to the s^{th} sub-period. In other terms the s extrapolations $\hat{y}^C_{t+1\,/t,}, \ldots, \hat{y}^C_{t+s/\;t+s-1}$ for *t=s,2s, ...,[T/s]* are such that

$$\hat{y}^C_{t+1\,/t} = E(y^c_{t+1}|\mathrm{I}_t)$$
$$\hat{y}^C_{t+2\,/t+1} = E(y^c_{t+2}|\mathrm{I}_{t+1}) = E(y^c_{t+2}|\mathrm{I}_t) = \hat{y}^C_{t+2\,/t}$$
$$\ldots$$
$$\hat{y}^C_{t+s\,/t+s-1} = E(y^c_{t+s}|\mathrm{I}_{t+s-1}) = E(y^c_{t+s}|\mathrm{I}_t) = \hat{y}^C_{t+s\,/t}$$

and therefore, apart the regressors x_t that in this context are exogenous, conditional to the last available low-frequency aggregate Y_t.

A first popular statistic is the determination coefficient R^2 and its corrected formula R_c^2 respectively defined as:

$$R^2 = 1 - SSR/SST, \tag{9}$$

$$R_c^2 = 1 - \left[\frac{SSR/_{[T/s]-k}}{SST/_{[T/s]-1}}\right] \tag{10}$$

where *SSR* is the sum of squared residuals that within the state-space framework reads as

$$\mathrm{SSR} = [T/s] \cdot \hat{\sigma}^2$$

with $\hat{\sigma}^2$ resulting from the biased maximum likelihood estimation of σ^2 of equation (1) obtained by the Kalman filter and *SST* is the sum of squared deviations of first differences of Y_t from its mean, as suggested by Harvey (1989, p.268-9) to properly cope with non-stationary data. In equation (10) k is the number of covariates x_t including the constant and the linear trend if present.

Furthermore, models goodness of fit statistics are given by the standard error of regression (SER) and maximum log likelihood value, respectively, defined as follows:

$$\text{SER} = \sqrt{SSR/([T/s] - k)}, \tag{11}$$

$$\hat{L}_c = -0.5 \cdot \{d_{T+1} + [T/s][ln\hat{\sigma}^2 + \ln(2\pi) + 1]\} \tag{12}$$

where d_{T+1} is the sum of innovations variances taken in logarithms at time *t=s, 2s, …,* $[T/s]$. The diffuse profile log likelihood is given by

$$\hat{L}_\infty = -0.5 \cdot \left\{d_{T+1} + \left(\frac{T}{s} - k\right)[ln\hat{\sigma}^2 + \ln(2\pi) + 1] + ln|S_{T+1}|\right\}$$

where $|S_{T+1}|$ is the determinant of the covariance matrix of β computed over the Kalman filter iterations.

A further measure of goodness of fit is the 'finite prediction error variance' (PEV) computed as $PEV = \hat{\sigma}^2 \cdot f_T$.

The information criteria AIC and BIC employed to compare alternative model specifications can also be recovered as functions of $\hat{\sigma}^2$ as shown in Harvey (1989, p.270, eq. 5.5.18) , respectively:

$$\text{AIC} = \frac{2\text{k}}{\left[\frac{T}{s}\right]} + \ln\hat{\sigma}^2, \tag{13}$$

$$\text{BIC} = \text{k} \cdot \frac{\ln[T/s]}{[T/s]} + \ln\hat{\sigma}^2. \tag{14}$$

Notice that, the two information criteria can be alternatively formulated in terms of log likelihood. In particular, using the maximized diffuse profile log likelihood, they can be formulated as $\text{AIC} = -2\hat{L}_\infty + 2k$ and $\text{BIC} = -2\hat{L}_\infty + k2\log\left[\frac{T}{s}\right]$ that allow rigorous comparisons also among linear and non-linear models (i.e. based on log-transformed series).

Durbin-Watson (1950, 1951) test statistic used to detect the presence of first-order autocorrelation in the residuals is defined with respect the standardized innovations $\hat{v}_t$ as follows:

$$\mathrm{DW} = \frac{\sum_{t=k+2}^{[T/s]}(\hat{v}_t - \hat{v}_{t-1})^2}{\sum_{t=k+2}^{[T/s]} \hat{v}_t^{\,2}}.$$

The Jarque-Bera test statistic N for the normality of the residuals is derived from the formulas in Harvey (1989, p.560 eq.5.4.10 and 5.4.11) respectively for the third and the fourth moment of standardized residuals $\hat{v}_t$. In particular:

$$\sqrt{b_1} = \hat{\sigma}_*^{-3} \sum (\hat{v}_t - \bar{v})^3 / T^*$$

$$b_2 = \hat{\sigma}_*^{-4} \sum (\hat{v}_t - \bar{v})^4 / T^*$$

where the relation between $\hat{\sigma}^2$ and $\hat{\sigma}_*^2$ (unbiased residual variance) is

$$\hat{\sigma}_*^2 / \hat{\sigma}^2 = T/(T-k) \text{ and } T^* = T - k.$$

Hence, the N statistic of residual normality results as:

$$\mathrm{N} = \frac{T^*}{6} \cdot b_1 + \frac{T^*}{24} \cdot (b_2 - 3)^2, \tag{15}$$

that under to the null-hypothesis follows a chi-squared distribution with 2 degrees of freedom for large samples.

An option for testing the statistical significance of the first P residual autocorrelations is given by the Ljung-Box Q-test statistic computed as:

$$\mathrm{Q} = T^*(T^* + 2) \sum_{\tau=1}^{P} (T^* - \tau)^{-1} r_v^2(\tau) \tag{16}$$

where $r_v(\tau)$ are the sample autocovariances of standardized innovations. See Harvey (1989) p.259, eq.5.4.7 for details. Within the context of ADL(1,1) models, we compare the related computed statistic to a chi-squared with degrees of freedom equal to $\sqrt{[T/s]} - 1$.

The last test is the *H(h)* statistic for checking the heteroscedasticity of residuals given by:

$$H(h) = \sum_{t=T-h+1}^{T} \hat{v}_t^2 / \sum_{t=k+1}^{k+1+h} \hat{v}_t^2. \tag{17}$$

where *h* is an integer closer to $T^*/3$. In this case the statistic h·H(h) is compared to a chi-squared with h degrees of freedom. See again Harvey (1989) p.259 eq.5.4.9, for a comprehensive treatment.

3.4 Evaluation of a temporal disaggregation

The diagnostic statistics described so far are important tools for evaluating the temporal disaggregation performance but they are not exhaustive to guarantee the quality of an exercise. Further criteria are required, especially when the analyst copes with incomplete dataset and time constraints.

A first remark of the above mentioned diagnostics is that they are based on residuals computed at low-frequency of observation. Therefore they are not able to provide insight about the quality of disaggregations in terms of high-frequency co-movement with the related indicator. In other words - and this point will become clearer in next section - there are situations in which the exercise appears well specified according to the residuals-based statistics but yields disaggregated data either relatively too smooth or too volatile compared to the indicator. It is thus recommended to complement the usual analysis of residual diagnostics with both graphical inspection of disaggregations and a set of statistics of correlation between the indicator and the resulting disaggregation.

A second limit of innovation-based statistics within national accounts concerns their reduced statistical power because the length of time series rarely exceeds 20-30 annual observations.

Therefore, within national accounts a multi-step approach is usually adopted: at first, a preliminary analysis of general goodness of fit of the indicator is undertaken, like comparing the pattern between the annualized indicators and target data adopting both graphical and synthetic statistics tools; then model estimation is carried out, followed by diagnostic checking through residual-based statistics complemented by revisions statistics generated by the disaggregations and correlations between indicators and disaggregated data. In the standard practice these latter statistics are computed in terms of both quarterly and annual growth rates.

4 The empirical application

4.1 Design of the exercise

In this section we present the main results of an extensive exercise of temporal disaggregation based on Italian data and providing evidence of a comparative analysis of the alternative classes of models presented in section 2. A further reduced scale application has been undertaken over the extensions of both ADL and Chow-Lin models with stochastic constant and trend components of section 2.4 (see section 4.6).

Such a large-scale exercise aims at reproducing the current practice of quarterly national accounts both for its extension and for the nature of implied time series. It shows a selection of quarterly disaggregations based on annual national accounts and short term indicators by ISTAT: annual data are relative to the industrial value added split into 17 branches of economic activity (sections B-E of NACE Rev.2) according to the compilation detail of the Italian practice; quarterly indicators are industrial production indexes (IPI) at same detail of activity. The sample period covers the interval 1995-2013. Table 1A of appendix A provides a summary description of the data employed in the exercise, while figures 1-4 present four graphs for both indicators and annual data by branch of activity. In these graphs nominal and volume annual data are presented separately in figures 1 and 3, whereas the two sets of figures 2 and 4 are devoted to the quarterly indicators shown together in both raw and seasonal adjusted forms.

The exercise has been carried out under a double perspective: the former looks at the performance of disaggregations with respect to type of data correction, i.e. taking into account the distinction between seasonal adjusted and unadjusted data; the latter at type of evaluation, i.e. looking at both current price and volume data (chain linked values with reference year 2010). In total 68 cases have been investigated. Each case reviews the full set of methods, from both the static regression approaches by Chow-Lin and Fernandez to the ADL class in the two specifications ADL(1,0) and ADL(1,1).[3] With the exception of the Fernàndez approach, the estimations have concerned the unrestricted form of model (1) and the restricted variants without trend and with neither constant nor trend. ADL models have been estimated in both levels and first differences. Finally, using the approaches by both Proietti (2006a) and Proietti and Moauro (2006), each form has been treated also in the logarithms. In total 2176 temporal disaggregations have been carried out.

Estimation of a so large variety of models and specifications implies the risk that some models could be not significant. An example is when the estimation leads to values for some regression coefficients $\hat{\beta}$ either close to zero or to low values of t-statistics. Indeed, the aim of the exercise is to mimic the current practice of quarterly national accounts when it is rather high the risk of not selecting the best specification, due to lack of either data, or time, or for the presence of organizational constraints. In this respect, we opted for the profile log-likelihood as penalty function to be maximized rather than the 'diffuse' variant since the latter was problematic in terms of convergence, especially with the Fernàndez and the ADL(1,1) models in differences. Dealing with such aspects could be ground of future research.

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[3] A similar exercise concerned also the Litterman model. However, these results, available upon request, were generally very problematic for the uncertain estimate of the autoregressive term ϕ occurring in log-likelihood maximization. For full details in this respect see Proietti (2005), pp.104-106.

The exercise allows to appreciate main advantages of the SSF-based estimate with respect to the standard regression methods, like the possibility to handle both a wider range of models, the logarithmic transformation and the availability of a wider set of uniform diagnostics based on standardized innovations.

Computations of the large-scale exercises of sections 4.2-4.5 have been carried out under Red Hat Linux Modeleasy+ release 5.6 by Econometric Modeling and Computing Corporation (2009a and 2009b), which represents the standard production environment for ISTAT. Computations of the application presented in section 4.6 have been implemented under Ox console version 7.01 (Windows/U) by Doornik (2013), where the BFGS optimization algorithm is available. Comparability of results within the two environments has been accurately checked.

A further check has concerned the efficiency of the grid search versus the BFGS optimization. Over 10 estimated logarithmic models of chain-linked values relative to branch 19, the optimization carried out by grid search, where the grid of ϕ ranges between -0.99 and +0.99 (199 values), requested on average 6.83 seconds, whereas the BFGS only 1.01 seconds. By contrast, the same experiment conducted over the corresponding linear specifications requested on average 0.06 and 0.55 seconds respectively for the grid search and the BFGS methods. Both the experiments have been carried out under Ox. Overall, the gain of the BFGS procedure over the grid search is especially remarkable for large scale applications of non-linear models, whereas it seems useless for linear models. However further work is requested to ensure accuracy in convergence.

Figure 1 – Value added by branch: annual chain-linked values reference year 2010 over the years 1995-2003

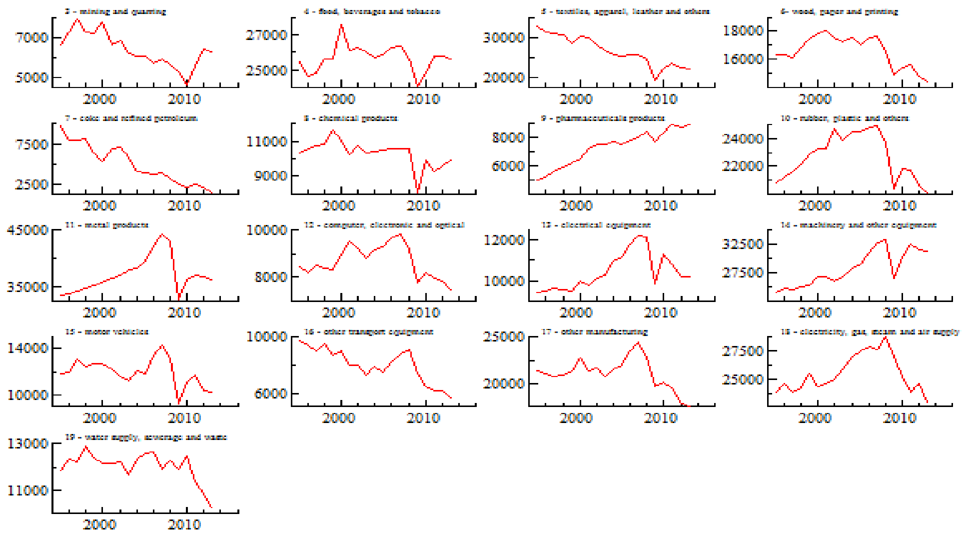


Figure 2 - Industrial production by branch: seasonal adjusted (in blue) and raw data (in red) over the quarters 1995q1-2003q4

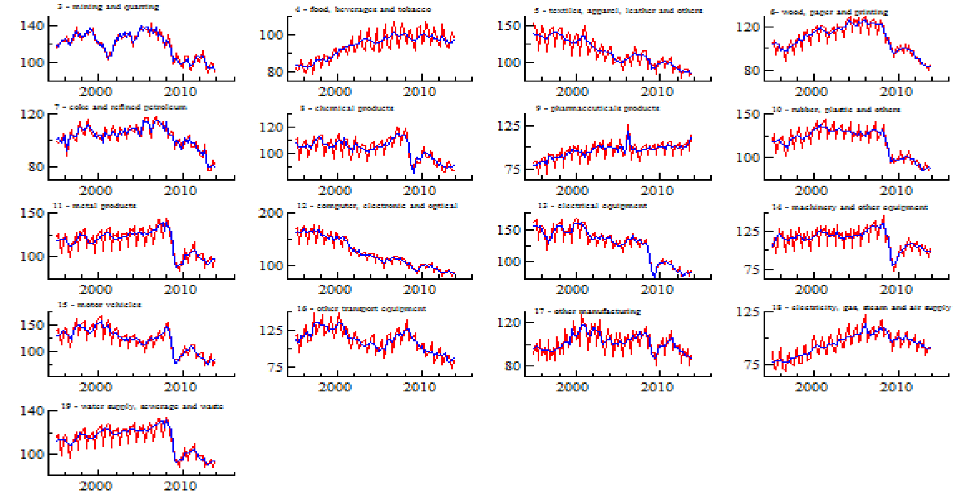


Figure 3 – Value added by branch: annual values at current prices over the years 1995-2003

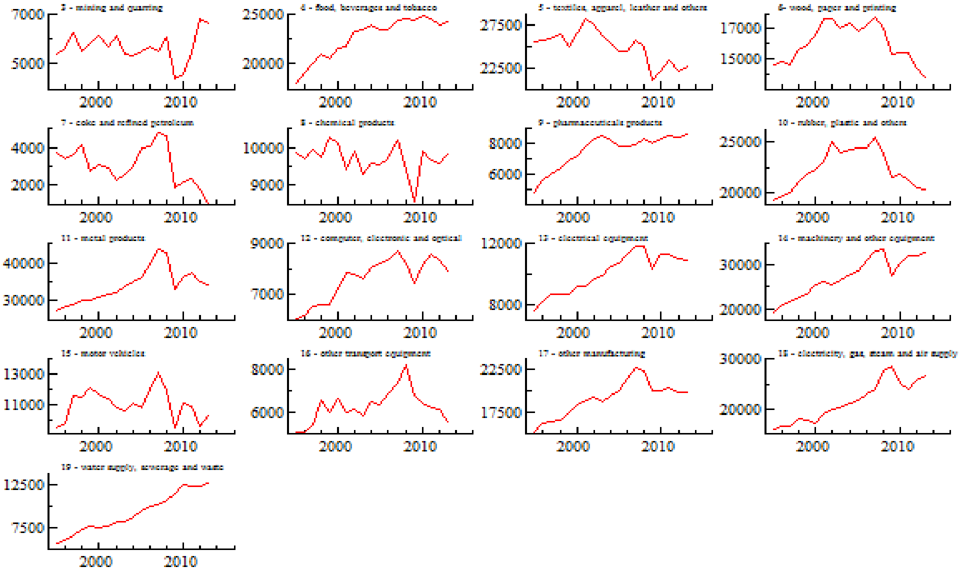
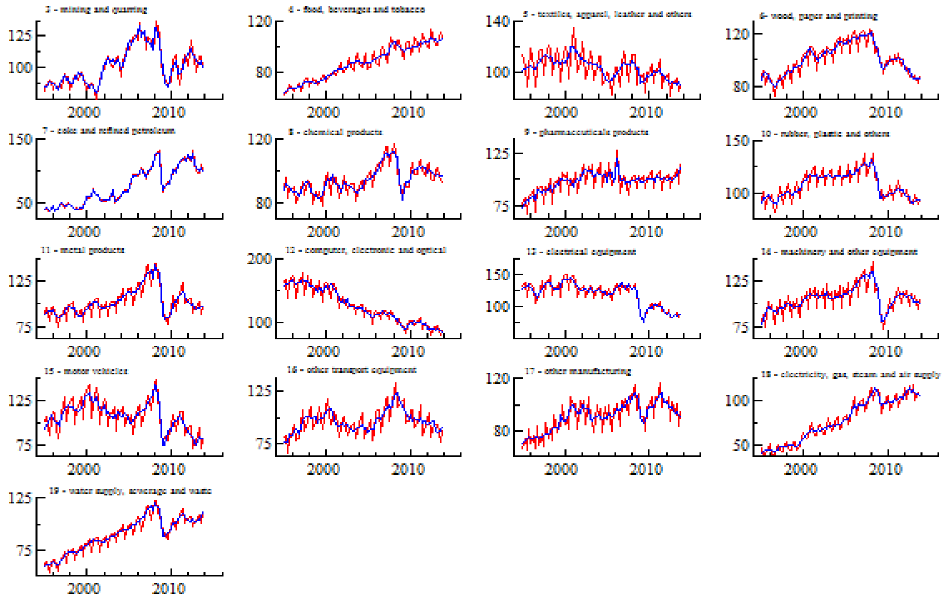


Figure 4 – Inflated industrial production by branch: seasonal adjusted (in blue) and raw data (in red) over the quarters 1995q1-2003q4



4.2 Comparison of temporal disaggregation models

In the comparative analysis of performances between alternative models we have considered two statistics. The former is given by the mean absolute forecast error (MAE) of annual growth rates over the period 2006-2013 (i.e. over the last 8 years of the sample period) measured as the average difference in absolute terms between growth rates of the annual dependent variable and the sum of four extrapolated quarters of present year over the annual totals of previous year:

$$MAE = \frac{1}{8}\sum_{t=2006}^{2013}\left|g_t - g_t^{(q)}\right| \qquad (18)$$

where g_t denotes the yearly growth rates of Y_t on Y_{t-1} and $g_t^{(q)}$ denotes the yearly growth rates based on the extrapolated quarters $\sum_{q=1}^{4} y_{q,t}$ of year t with respect to the annual total Y_{t-1}. The latter statistic is based upon the correlation between the indicator and the disaggregated series in terms of quarterly growth rates over the whole sample period.

The choice of such two comparative statistics rests on the fact that the quality of a temporal disaggregation exercise is assessed from one hand on the basis of the average amplitude of revisions of extrapolated data and from the other on the extent to which the disaggregated pattern follows that of the quarterly indicator. Indeed, MAE statistics provide a measure of goodness of fit for extrapolated quarters, whereas correlations are a synthetic measure of co-movement of disaggregations with the reference indicator. Of course, when the latter well depicts the annual variable the disaggregation exercise ensures both low MAE and high correlations independently by the choice of method.

Tables 2A-3A in appendix A provide, respectively for nominal and volume data, the lowest MAEs by branch (in column) and by class of model (in row) relative to the quarterly disaggregations of value added based on seasonal adjusted indicators; analogously tables 4A-5A show, respectively for nominal and volume data, the highest correlations between quarterly growth rates of the indicator and the disaggregated series by branch (in column) and by class of model (in row).

In both the couples of tables the reported statistics have been selected deleting the specifications with unsatisfactory fit. The selection criteria have been: the statistically significance of the estimated coefficients (over the 0.05 confidence level), a positive autoregressive estimated parameter (larger than -0.2 for ADL models in differences) and positive correlations between the indicator and disaggregations in terms of both quarterly and annual growth rates. For a diagnosis of disaggregations over the whole exercise, in terms of both selection criteria and admissible results, see section 4.5.

Looking at tables 2A-3A, the first evidence is that the values of MAE span over a wide range of values, reflecting therefore both problematic branches -see for instance the MAEs of branch 7 ranging over the interval 14.61-20.35 for current price estimates- and virtuous branches –like MAEs of branch 6 ranging over 1.47-2.20. The good performance of the ADL class of models emerges as, looking at MAE statistics, ADL models outperform static disaggregations in 13 and 5 cases over a total of 17, respectively for nominal and volume data. Hence in 18 times over 34 occurrences (52,3%) dynamic models are relatively more performant than static forms, providing a tool able to reduce revisions of extrapolations.

In appendix A, tables 6A-7A complete the analysis of results. Here the details of the best model specification in terms of MAE relative to each branch are provided: in particular, type of specification, possible log transformation and differentiation, the maximum log-likelihood value and all parameter estimates are presented. Additionally, a small set of statistical diagnostics concerning errors autocorrelation (Durbin-Watson, Ljung-Box Q(4)), heteroscedasticity (H-statistics), and normality (Jarque-Bera) are provided.

From a joint analysis of tables 6A and 7A it emerges that ADL models in differences are more suited to nominal time series (table 6A) than data in volumes (table 7A). This is not in contrast with the evidence that nominal data include the inflationary component which usually features more evolutionary trends or higher order of integration, both elements properly treated by models in differences. Concerning the logarithmic transformation, its effectiveness stands out in several cases for modelling both nominal and volume data: a brief look at the relative diagnostics show that errors are normally distributed and not (positively) autocorrelated at lag 1, whereas they exhibit a positive autocorrelation at lag 4 in just a couple of cases, notably branch 4 for nominal data and branch 19 for volume data. Such branches are particularly problematic since the number of unsatisfactory disaggregations is relatively high. Concerning heteroscedasticity, 4 and 5 cases are problematic respectively for nominal and volume data as the relative H statistics are statistically significant.

Turning to the correlation tables 4A-5A, it emerges that the models where the quality of disaggregations is maximum in terms of fit to the indicator are the static ones. In particular the model by Fernàndez guarantees maximum correlation in 12 branches over 17 and the model by Chow-Lin in the remaining 5 cases. Concerning data in volumes, the exercise also shows a clear prevalence of the Fernàndez approach over the others, notably in 12 out of 17 cases. However also ADL(1,1) models are satisfactory, prevailing in the remaining 5 branches.

Tables 8A-9A provide the details of the estimated model specifications and the associated statistical diagnostics of the best model by branch, according to the correlation criterion whose statistics are shown respectively in tables 4A-5A for nominal and volume data. The selected models are quite satisfactory as their associated test statistics confirm that the estimated errors well behave according to normality and absence of serial correlation criteria apart, once again, branches 4 and 19.

Concerning heteroscedasticity, the number of problematic cases signaled by H statistics is particularly high: 10 and 8 occurrences respectively for nominal and volume data. From a graphical inspection of innovations it emerges that these values heavily depend on the structural break of 2009 analogously to the cases detected in tables 6A-7A. However, the number of heteroscedasticity cases under the correlation criterion, mostly static models, is the double with respect to models selected according to minimum MAE which in most cases takes a dynamic form. Hence, we infer that dynamic forms perform better in presence of outliers than static models which, then, require a larger use of intervention analysis.

Moreover, from these tables we learn that the log-transformation is relatively more performant in the majority of cases, notably 26 out of 34 occurrences (76,5%), compared to the untransformed data. Therefore we conclude that log transformation, by reducing data volatility, guarantees a higher connection between growth rates of disaggregated series and the indicator.

4.3 Model selection

In this section we discuss a specific temporal disaggregation example in order to provide standard elements of model selection, identification and diagnosis among the enlarged class of models presented so far. In particular we focus on the quarterly disaggregation of the Italian annual value added relative to manufacture of machinery and equipment n.e.c (branch 14, NACE Rev.2 A*38 code CK) at current prices, which represents 2.1% of total value added. The quarterly indicator is the industrial production inflated by output prices relative to the same branch of economic activity. The sample period is 1995:q1-2013:q4.

Figure 5 - Quarterly disaggregated value added of machinery and equipment and inflated industrial production (seasonal adjusted data at current prices)

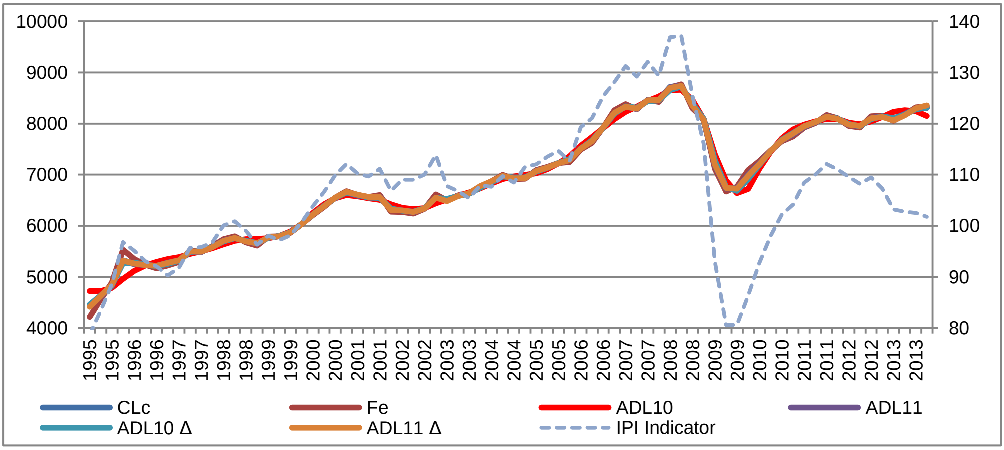
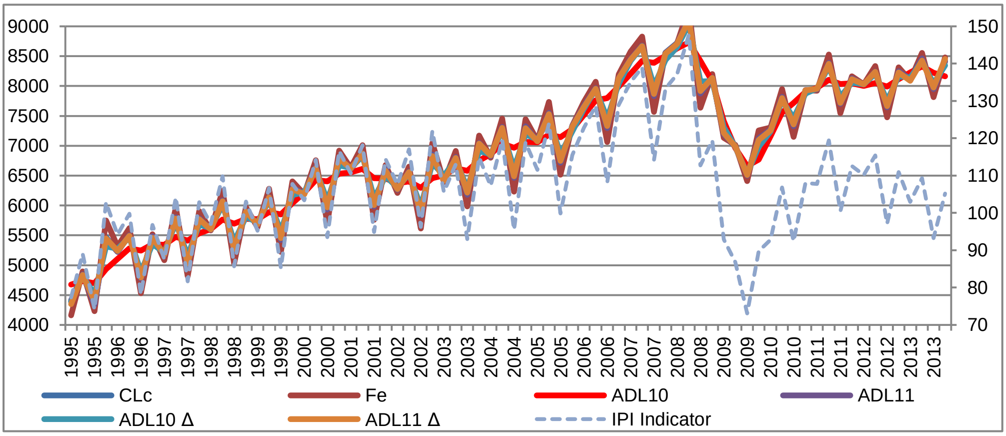


Figure 6 - Quarterly disaggregated value added of machinery and equipment and inflated industrial production (unadjusted data at current prices)



Figures 5 and 6 show, respectively, the seasonal adjusted and unadjusted disaggregated value added of machinery and equipment according to several model specifications: Chow-Lin (including the constant term and therefore denoted as CLc), Fernàndez (denoted as Fe), ADL(1,0) and ADL(1,1) both in levels and first differences (denoted in figures 5-6 with the suffix Δ to be distinguished by the corresponding models in levels). In this application we opted to focus on linear models only.

Both figures 5 and 6 show that the alternative models produce similar disaggregated data: their patterns are so similar that it is very difficult to distinguish one disaggregation from the others by graphical inspection. Nevertheless, the pattern produced by the ADL(1,0) model in levels (red line) is relatively smoother than other disaggregations. In the case of figure 6, devoted to unadjusted data, the seasonal component of disaggregated data almost disappears, whereas in the case of seasonal adjusted data the pattern of the ADL(1,0) model tend to interpolate the alternative disaggregations.

Both tables 1 and 2 provide a synthetic comparative view of statistics for six alternative model specifications, where the former is for seasonal adjusted data and the latter for unadjusted data. Concerning AIC and BIC, we refer to equations (13) and (14) based on innovations, rather than the log-likelihood-based versions, which in this application ensures a proper comparison among models. A further element of comparability is that the alternative disaggregation models share a common state-space form and, hence, the same number of observations. The only limit is that the two information criteria cannot be consistently compared across integrated and stationary models: accordingly, the Chow-Lin formulation will be compared to ADL(1,0) and ADL(1,1), whereas Fernàndez to the two ADL models in differences.

A first evidence is that the models identified by the former set of data are very similar to the latter as it results from a comparative view of tables 1 and 2. Both tables reports that the autoregressive parameters, the estimated regression coefficients, the statistics relative to MAE, correlations, information criteria AIC and BIC and R squared are almost identical.

Moreover, all the models show a strong significance -as shown by F-statistics– and fit quite well the data as the residuals diagnostics confirm they are normally distributed (except in the case of Fernàndez model on seasonally unadjusted data), and not serially correlated.[4] As far as residuals variability is concerned, as already remarked in section 4.2, the presence of heteroscedasticity is always detected (except in the ADL(1,1) in differences) and it depends on the 2009 outlier.

A second result which pops up from both tables 1 and 2 is the sub-optimality of the model ADL(1,0) in levels as emerged by graphical inspection. Although residuals well behave as shown by the tests statistics on autocorrelation (DW, Q(4)),

[4] Durbin-Watson statistics (DW) are compared with the Savin and White critical lower and upper values , with the number of observations equal to 19, either for k=1 or k=2 depending on the number of regressors in the model, at 1% of significance. Concerning the lower bound, we refer to the the Farebrother tables for the models without intercepts, i.e. all the models except Chow-Lin model.

and normality (JB), we observe that the related correlations between disaggregated data and the indicators in terms of quarterly growth rates are the lowest and that the values of MAE are the highest (6.32 and 6.39 for seasonal adjusted and unadjusted data respectively). Accordingly, the PEV is the second highest among the alternatives.

Table 1 - Estimated parameters and other statistics on quarterly disaggregations of value added for machinery and equipment (seasonal adjusted data at current prices)

	$\hat{m}$	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\phi}$	R^2	R_C^2	$\hat{L}_C$	F-test	AIC	BIC	DW	N	Q(4)	H	$\hat{\sigma}^2$	PEV$\cdot 10^{-3}$	Corr var.4	Corr var.1	MAE
CLc	11.31 **	38.23**		0.996	0.996	0.996	-154.93	4184.82**	9.76	9.86	1.53	1.13	4.1	9.46**	13987	625	0.96	0.94	2.91
Fe		51.75**		0.000	0.992	0.992	-158.97	2327.08**	10.29	10.34	1.99	4.58	5.1	1293.1**	26536	1136	0.93	0.92	3.24
ADL(1,0)		8.66**		0.871	0.976	0.976	-166.78	726.02**	11.44	11.49	1.74	3.23	0.9	727.97**	83638	2740	0.76	0.89	6.32
ADL(1,1)		37.27**	-35.92**	0.982	0.996	0.996	-152.91	4579.60**	9.67	9.77	1.43	1.15	3.4	5.90**	12786	575	0.96	0.94	2.70
ADL(1,0)Δ		32.96**		0.156	0.997	0.997	-151.89	6550.82**	9.31	9.41	1.25	0.79	5.6	2.72*	8948	3962	0.88	0.92	2.49
ADL(1,1)Δ		36.60**	-24.22**	0.676	0.999	0.999	-151.06	11575.20**	8.09	8.24	1.74	3.17	3.1	1.16	2388	322	0.95	0.93	1.78

Table 2 - Estimated parameters and other statistics on quarterly disaggregations of value added for machinery and equipment (unadjusted data at current prices)

	$\hat{m}$	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\phi}$	R^2	R_C^2	$\hat{L}_C$	F-test	AIC	BIC	DW	N	Q(4)	H	$\hat{\sigma}^2$	PEV$\cdot 10^{-3}$	Corr. var.4	Corr. var.1	MAE
CLc	11.22**	37.66**		0.996	0.996	0.996	-155.04	3965.66**	9.77	9.87	1.55	1.03	4.0	9.82**	14123	630	0.99	0.93	2.98
Fe		51.48**		0.000	0.992	0.992	-159.2	2172.18**	10.32	10.37	2.08	6.07*	5.3	1753.6**	27191	1207	0.99	0.91	3.19
ADL(1,0)		8.33**		0.876	0.976	0.976	-166.63	716.75**	11.41	11.46	1.75	3.42	1.3	2699.4**	81053	2585	0.62	0.88	6.39
ADL(1,1)		36.72**	-35.44**	0.982	0.996	0.996	-153.11	4308.13**	9.68	9.78	1.43	0.79	3.3	6.90**	13004	572	0.99	0.93	2.82
ADL(1,0)Δ		32.21**		0.162	0.997	0.997	-151.86	6365.61**	9.29	9.39	1.24	0.51	5.8	2.38*	8812	3976	0.97	0.92	2.51
ADL(1,1)Δ		35.68**	-23.54**	0.675	0.999	0.999	-151.06	11044.49**	8.10	8.25	1.65	1.96	3.9	1.34	2395	320	0.99	0.93	1.91

Furthermore, the AIC and BIC in tables 1 and 2 for ADL(1,0) amount to 11.44 and 11.49 for adjusted data and to 11.41 and 11.46 for unadjusted data, respectively, which are higher than any other model specified in levels. Sub-optimality of this model is also confirmed by the R^2 statistics whose values for the ADL(1,0) form are the lowest, as well as by the log-likelihood which also exhibits the minimum value.

Concerning the remaining ADL models their performance are overall in line with the Chow-Lin form. In particular, the ADL(1,1) in differences appears the best solution in terms of overall fit, as indicated by the log-likelihood and the coefficient of determination which are, respectively, the lowest and the highest reported, in each table. Moreover, the AIC and BIC (8.09 and 8.24 for seasonally adjusted series; 8.10 and 8.25 for raw data) are the lowest among the models in differences. Concerning the quality of extrapolations, the value of MAE of the ADL(1,1) in differences (1,78 and 1.91, respectively, for seasonal adjusted and unadjusted data) is lower than the other specifications (ranging between the values 2.49-6.39) coherently with the lowest PEV. Finally, a brief residuals analysis rules out the presence of serial correlations at both lags 1 and 4 as denoted by the DW and Q(4) statistics respectively and it confirms they are homoscedastic and normally distributed.

4.4 Quarterly disaggregation of raw data

Main statistics on the overall fit of temporal disaggregation models as well as parameter estimates are usually invariant with respect to the use of adjusted or unadjusted data. The results reported in tables 1 and 2 are in line with this observation since all the regression coefficients, the autoregressive parameters and the log-likelihood almost coincide in the two cases. Nevertheless, we observe that the ADL(1,0) model in levels estimated over unadjusted data implies an imperfect transfer of the seasonal pattern from the indicator to the disaggregations.

The remarkable smoothness of disaggregated data produced by the ADL(1,0) model in levels emerges from the analysis of both figures 7 and 8 relative to the disaggregated value added of coke and refined petroleum products, respectively, at current prices and chain linked values. Here the pair of disaggregations of the ADL(1,0) form, in red lines, are accompanied by those by Chow-Lin (in blue) and Fernàndez (in gray), which both reproduce the seasonal pattern of the indicator (in dashed red lines).

To complement the graphical inspection of figures 7-8, table 10A in appendix A shows the correlations between the indicator and disaggregated data in terms of both quarterly and annual growth rates by main class of model[1]. In table 10A correlations of ADL(1,0) models are lower than other forms in the 82% of cases when the

[1] It is the average of correlations of the 3 options adopted in model specification, i.e. the unrestricted and the 2 restricted forms without trend and without both constant and trend. Note that quarterly growth rates of raw data, albeit without apparent economic sense, are particularly useful in this context since correlations are more discriminant for the selection of the best specification when the criterium is the fit to the pattern of the indicator.

comparison concerns quarterly growth rates. In this comparison we have considered only the models with correlations higher than 0.2. Concerning correlations in terms of annual growth rates the distance between ADL(1,0) and other models is mitigated, as these statistics provide information more appropriate to measure the model goodness of fit.

Figure 7 – Quarterly disaggregated value added of coke and refined petroleum products and inflated industrial production (unadjusted data at current prices)

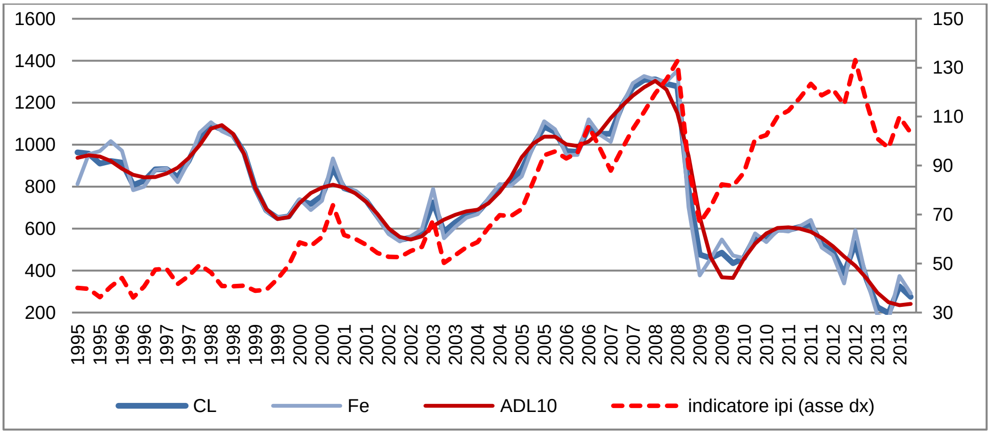
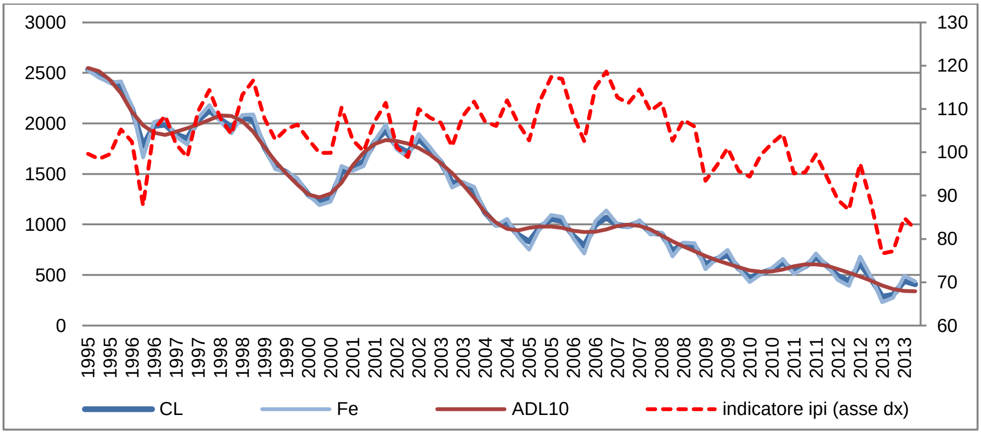


Figure 8 – Quarterly disaggregated value added of coke and refined petroleum products and industrial production (unadjusted chain linked values)



4.5 Diagnosis of disaggregations: selection criteria and admissible results

There is a trade-off between the enlargement to a wider class of temporal disaggregation methods and model selection: on one hand, a larger set of modeling tools can increase the estimation performance, on the other, the risk of obtaining non satisfactory estimations rises. The solution suggested by the literature is adopting a general-to-specific strategy of model selection following given rules which allow to move from a general unrestricted form to restricted and more parsimonious models. Within dynamic regressions refer to Castle at al. (2011).

In the context of ADL regressions, the general-to-specific strategy implies the following steps: getting rid of the deterministic components from the unrestricted form including constant and trend; reducing the order of differentiation applied to the data (from 1 to 0); restricting the lag order of the ADL model. A further dimension is the choice between modelling the data in levels or in logarithms.

By adopting a pragmatic ex-post approach, we started from a wide set of estimated models and then we proceeded by short-listing them according to the following criteria: i) the statistical significance (at least 5%) of estimated model parameters; ii) the positive sign of the estimated autoregressive parameter to avoid volatility of estimated disaggregations (higher than -0.2 for ADL in differences); iii) positive and reasonable high correlations between the indicator and disaggregated data in both quarterly and annual growth rates; iv) general co-movement between disaggregated data and the indicator by graphical inspection.

Table 3 – Percentage shares of admissible disaggregations in total and by class of model

	Seasonal adjustment		Type of evaluation		Type of transformation		Total shares
	Raw data	Seasonal adjusted data	Current prices	Chain linked values	Models in Levels	Models in logarithms	
Total	49.2	49.6	50.9	47.9	47.0	51.8	49.4
CL	71.1	72.1	70.1	73.0	64.2	78.9	71.6
FE	100.0	100.0	100.0	100.0	100.0	100.0	100.0
ADL(1,0)	58.8	63.2	58.8	63.2	61.3	60.8	61.0
ADL(1,1)	52.0	52.5	56.9	47.5	49.0	55.4	52.2
ADL(1,0)Δ	33.3	32.4	36.3	29.4	30.4	35.3	32.8
ADL(1,1)Δ	12.3	10.8	14.7	8.3	10.3	12.7	11.5

Note: The Chow-Lin, ADL(1,0), ADL(1,1), ADL(1,0)Δ and ADL(1,1)Δ classes are estimated in both the unrestricted form of equation (1) and the restricted form (i.e. without either trend or constant and trend).

Table 3 presents the shares of admissible disaggregations with respect to totals, type of seasonal adjustment, type of evaluation and type of transformation. Taking into account the 4 criteria listed above, admissible disaggregations are 49% out of the entire 2176 cases. The share slightly increases to 51% for current price data and decreases to 48% for chain-linked values. Log-transformation provides a higher share of admissible results (51.8%) compared to estimations in levels (only 47%), meaning a larger stability of models in logarithms. A similar evidence (not shown in table 3) is found also looking separately at raw and seasonal adjusted data.

Looking at the class of models, the shares of table 3 are 100% for Fernàndez (of which we have only considered the specification without any deterministic component, differently form the alternative models), 71.6% for Chow-Lin, 61% for the ADL(1,0) form in levels, 52.2% for ADL(1,1). Lower shares have been found for the ADL models in differences for which we have obtained 32.8% and 11.5% of admissible cases respectively for ADL(1,0) and ADL(1,1) forms. Not surprisingly a higher complexity of model specification is accompanied by a lower chance of obtaining significant coefficients from estimation, which turned out to be the most frequent cause for rejecting admissible models.

4.6 The application with stochastic trends

The empirical application of models with stochastic trends (see section 2.4) has been limited to the 17 current price cases related to seasonal adjusted data. Four model specifications have been fitted to the data over the sample 1995q1.2013q4: both an ADL(1,0) and an ADL(1,1) models plus a simple random walk, an ADL(1,1) model plus a local linear trend and a Chow-Lin model plus a random walk. Optimization has concerned the diffuse log-likelihood by the BFGS algorithm in the Ox 7.01 environment by Doornik (2013).

Table 4 – Estimated parameters of disaggregation models with stochastic trend and performance statistics for branch 6

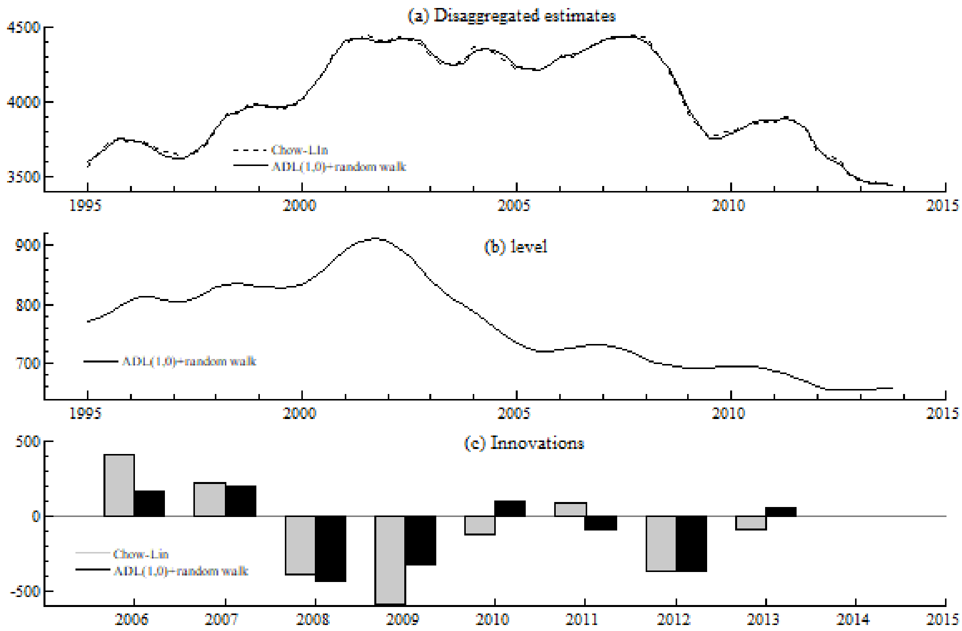
	$\hat{L}_\infty$	$\hat{\phi}$	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\sigma}^2$	$\hat{q}_\eta$	$\hat{q}_\varsigma$	MAE	Correlation
Chow-Lin	-134.55	0.00	20.83	-	12910.29	0.20	-	1.83	0.73
			(5.04)						
ADL(1,0)	-133.44	0.49	12.83	-	4922.33	0.12	-	1.31	0.70
			(5.91)						
ADL(1,1)	-129.20	0.00	0.32	22.44	2549.67	1.42	-	1.61	0.38
			(0.04)	(2.41)					
ADL(1,1)	-126.14	0.00	0.17	22.34	7445.40	0.35	0.00	1.68	0.37
			(0.02)	(2.22)					

A synthetic view of results are shown in tables 4 and 5 respectively for branches 6 (Manufacture of wood and paper products, and printing) and 10 (Manufacture of rubber and plastic products, and other non-metallic mineral products), which represent the most performing cases together with that of branch 8 not shown here for space reasons. A first evidence is that the estimate of the autoregressive coefficient ϕ converged to 0 for all the cases apart the ADL(1,0) model of branch 6. In this respect the optimization of the entire exercise required to restrict ϕ over the interval [0,1) to avoid unreliable disaggregated estimates.

Concerning the results of branch 6, disaggregated estimates of the ADL(1,0)+random walk model are shown in figure 9 panel (a) in comparison with those resulting from the standard Chow-Lin model; panel (b) of the same figure shows the stochastic level of the ADL(1,0)+random walk model and panel (c) the relative innovations over the years 2006-2013 (in black) in comparison with those by Chow-Lin (in grey).

From table 4 it emerges that the ADL(1,0) plus random walk model resulted the most performant for branch 6 in terms of MAE (1.31), also compared to the linear specifications of table 2A (1.47), whereas the correlation statistic (0.70) is in line with the corresponding model of table 4A (0.71) but lower than the standard Fernandez model (0.83). Also the two ADL(1,1) models of table 4 show lower MAEs than the linear models of table 2A, but with a significant decrease in terms of correlations (0.38 and 0.37).

Figure 9 – Disaggregation results of the ADL(1,0)+random walk model for branch 6



Concerning table 5 devoted to branch 10, the most performant model in terms of MAE is the ADL(1,1) (1.90), where both the Chow-Lin and the ADL(1,0) forms also perform better than models of table 2A. Notice also the almost identical results among the Chow-Lin and ADL(1,0) specifications, due to the occurrence that $\hat{\phi} = 0$ in both cases. Finally, the ADL(1,1) model converges to a linear trend.

Table 5 – Estimated parameters of disaggregation models with stochastic trend and performance statistics for branch 10

	$\hat{L}_{\infty}$	$\hat{\phi}$	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\sigma}^2$	$\hat{q}_{\eta}$	$\hat{q}_{\varsigma}$	MAE	Correlation
Chow-Lin	-141.71	0.00	29.07	-	55164.63	0.05	-	1.95	0.70
			(5.95)						
ADL(1,0)	-141.71	0.00	29.06	-	55147.34	0.05	-	1.95	0.70
			(5.94)						
ADL(1,1)	-137.85	0.00	36.43	-7.24	59284.56	0.04	-	1.90	0.74
			(2.05)	-(0.42)					
ADL(1,1)	-133.55	0.00	32.74	-9.61	56440.49	0.00	0.00	2.34	0.71
			(2.04)	-(0.61)					

Overall the exercise has resulted of few utility to the set of data at hand and the full I(2) trend over-identified. Moreover since in some cases $\hat{\sigma}^2$ tends to 0, the optimization should be reset, switching the scale parameter σ^2 from variance of ϵ_t to either variance of η_t or ς_t.

5. Conclusions

The paper has presented the most recent developments carried out at ISTAT within temporal disaggregation. The description of the new methodologies have been followed by a discussion of the results of a large scale experimental exercise based on ISTAT data with the aim of evaluating their performances under multiple criteria. A smaller experiment has also concerned disaggregation models with stochastic trends.

Main contributions concern: the enlargement of disaggregation methods based on regressions from the static to the dynamic class of ADL models; the adoption of the state space approach for model estimation, computation of disaggregated data and diagnosting checking; the introduction of the non-linear disaggregation for the treatment of the log-transformed data; the full integration of the dynamic setup within the standard procedures currently used in the quarterly national accounts process.

Concerning the empirical application, the results highlight a general good performance of dynamic models especially in terms of their predictive capacity. Their appeal is higher when the disaggregation concern nominal values with respect to volumes. By contrast, traditional static disaggregation methods maintain their superiority with respect to the fit of disaggregated results to the indicator overall the sample. Therefore ADL models are preferred when one gives prominence to low revisions of disaggregated estimates in most recent periods, whereas static methods performs better if it prevails the need that the estimates are adherent to the reference indicator. However, static methods turned out to be less performant than dynamic ones in dealing with structural breaks in series, often resulting in heteroscedastic errors and, hence, calling for not rare intervention analysis.

Another finding is that ADL models are difficultly acceptable when seasonal indicators are used, since they smooth too much the disaggregated estimates. In this field static regressions work well.

In general, the non-linear treatment of data in logarithms for both static and dynamic models has been found particularly effective: very often such specifications outperform linear ones. We conclude that temporal disaggregation of data in logarithms is an evolution of remarkable significance.

The limited example over models with stochastic trends shows that their positive impact in terms or error statistics is limited to few cases and in the majority of cases the models appear over-identified.

In conclusion, the integration of the static and dynamic class of models under a uniform analytical setting, the large set of innovations-based statistics and the non-linear treatment of logged data represent undeniable advantages due to the state space approach which then is a recommended tool within the temporal disaggregation practice.

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Appendix A – Tables

Table 1A - Time series used in the exercise listed by branch of economic activity, evaluation and sample period

	Branch	ISIC rev.4 NACE rev.2	NACE Divisions	Evaluation	Period
3	Mining and quarrying	B	05 - 09	Current prices / Chain-linked values	1995-2013
4	Manufacture of food products, beverages and tobacco products	CA	10 - 12	Current prices / Chain-linked values	1995-2013
5	Manufacture of textiles, apparel, leather and related products	CB	13-15	Current prices / Chain-linked values	1995-2013
6	Manufacture of wood and paper products, and printing	CC	16-18	Current prices / Chain-linked values	1995-2013
7	Manufacture of coke, and refined petroleum products	CD	19	Current prices / Chain-linked values	1995-2013
8	Manufacture of chemicals and chemical products	CE	20	Current prices / Chain-linked values	1995-2013
9	Manufacture of pharmaceuticals, medicinal chemical and botanical products	CF	21	Current prices / Chain-linked values	1995-2013
10	Manufacture of rubber and plastics products, and other non-metallic mineral products	CG	22-23	Current prices / Chain-linked values	1995-2013
11	Manufacture of basic metals and fabricated metal products, except machinery and equipment	CH	24-25	Current prices / Chain-linked values	1995-2013
12	Manufacture of computer, electronic and optical products	CI	26	Current prices / Chain-linked values	1995-2013
13	Manufacture of electrical equipment	CJ	27	Current prices / Chain-linked values	1995-2013
14	Manufacture of machinery and equipment n.e.c.	CK	28	Current prices / Chain-linked values	1995-2013
15	Manufacture of motor vehicles, trailers and semi-trailers	CL	29	Current prices / Chain-linked values	1995-2013
16	Manufacture of other transport equipment		30	Current prices / Chain-linked values	1995-2013
17	Other manufacturing, and repair and installation of machinery and equipment	CM	31-33	Current prices / Chain-linked values	1995-2013
18	Electricity, gas, steam and air-conditioning supply	D	35	Current prices / Chain-linked values	1995-2013
19	Water supply, sewerage, waste management and remediation	E	36-39	Current prices / Chain-linked values	1995-2013

Table 2A – Mean absolute errors (MAE) of annualized extrapolations in growth rates over the years 2006-2013 (seasonal adjusted data at current prices) (a) (b) (c)

Model	branch 3	branch 4	branch 5	branch 6	branch 7	branch 8	branch 9	branch 10	branch 11	branch 12	branch 13	branch 14	branch 15	branch 16	branch 17	branch 18	branch 19
CL	6.64	2.44	**2.98**	1.70	**14.61**	3.72	2.48	2.47	1.89	3.43	2.98	2.15	4.12	**2.19**	2.17	4.63	5.27
FE	6.98	3.18	3.27	2.20	20.35	4.46	2.40	4.18	2.71	**3.36**	2.30	3.30	4.84	2.69	4.13	7.45	6.44
ADL(1,0)	**6.53**	2.48	3.02	**1.47**	15.59	5.76	**2.35**	2.45	**1.77**	3.39	**2.29**	**1.49**	2.26	2.89	2.24	3.70	**2.99**
ADL(1,1)	7.52	**2.38**	3.09	1.85	14.62	**3.53**	2.40	**2.29**	2.28	3.53	2.37	1.59	**1.88**	2.53	**1.91**	**3.56**	3.08

(a) In bold the minimum MAE by branch.
(b) For each branch and class of models the value refers to the model specification with minimum MAE.
(c) Each class of models estimated in both levels and logs. The Chow-Lin, ADL(1,0) and ADL(1,1) classes estimated in the unrestricted form of equation (1) and the restricted forms without trend and both constant and trend. The ADL classes include also the models in differences.

Table 3A – Mean absolute errors (MAE) of annualized extrapolations in growth rates over the years 2006-2013 (seasonal adjusted data of chain linked values) (a) (b) (c)

Model	branch 3	branch 4	branch 5	branch 6	branch 7	branch 8	branch 9	branch 10	branch 11	branch 12	branch 13	branch 14	branch 15	branch 16	branch 17	branch 18	branch 19
CL	**7.58**	2.19	**3.90**	**1.85**	**7.24**	5.70	3.20	**1.60**	2.66	4.18	3.63	2.22	**2.90**	2.55	3.41	**3.31**	4.49
FE	8.36	**1.97**	4.65	2.22	10.17	7.67	4.27	1.86	**2.36**	3.46	**3.23**	2.72	3.12	**2.03**	3.98	3.61	7.54
ADL(1,0)	8.67	2.54	4.42	2.00	7.96	**5.57**	**3.09**	2.39	2.82	4.15	3.48	**1.71**	3.66	2.36	3.18	3.54	**3.57**
ADL(1,1)	7.65	2.27	3.99	1.87	10.67	5.63	3.27	2.51	3.13	**3.45**	3.60	1.73	3.13	2.56	**3.13**	3.56	4.23

(a) In bold the minimum MAE by branch.
(b) For each branch and class of models the value refers to the model specification with minimum MAE.
(c) Each class of models estimated in both levels and logs. The Chow-Lin, ADL(1,0) and ADL(1,1) classes estimated in the unrestricted form of equation (1) and the restricted forms without trend and both constant and trend. The ADL classes include also the models in differences.

Table 4A – Correlations of quarterly growth rates between indicator and disaggregated data over the quarters 1996q1-2013q4 (seasonal adjusted data at current prices) (a) (b) (c)

Model	branch 3	branch 4	branch 5	branch 6	branch 7	branch 8	branch 9	branch 10	branch 11	branch 12	branch 13	branch 14	branch 15	branch 16	branch 17	branch 18	branch 19
CL	**0.91**	0.85	0.93	0.80	0.91	0.88	0.97	0.82	**0.97**	0.92	**0.92**	**0.96**	0.96	0.94	**0.94**	0.72	0.63
FE	0.91	**0.85**	**0.94**	**0.83**	**0.92**	**0.88**	**0.97**	**0.84**	0.95	**0.92**	0.91	0.93	**0.96**	**0.94**	0.94	**0.73**	**0.63**
ADL(1,0)	0.81	0.45	0.89	0.71	0.81	0.62	0.91	0.76	0.97	0.67	0.92	0.95	0.90	0.91	0.83	0.18	0.10
ADL(1,1)	0.88	0.26	0.92	0.79	0.91	0.87	0.90	0.75	0.97	0.84	0.92	0.96	0.96	0.92	0.94	0.16	0.31

(a) In bold the maximum correlation by branch.

(b) For each branch and class of models the value refers to the model specification with maximum correlation.

(c) Each class of models estimated in both levels and logs. The Chow-Lin, ADL(1,0) and ADL(1,1) classes estimated in the unrestricted form of equation (1) and the restricted forms without trend and both constant and trend. The ADL classes include also the models in differences.

Table 5A – Correlations of quarterly growth rates between indicator and disaggregated data over the quarters 1996q1-2013q4 (seasonal adjusted data of chain linked values) (a) (b) (c)

Model	branch 3	branch 4	branch 5	branch 6	branch 7	branch 8	branch 9	branch 10	branch 11	branch 12	branch 13	branch 14	branch 15	branch 16	branch 17	branch 18	branch 19
CL	0.87	0.90	0.94	0.87	0.90	0.89	0.96	0.91	0.97	0.91	0.95	0.97	0.99	0.94	0.94	0.89	0.64
FE	**0.87**	**0.90**	**0.94**	**0.89**	0.85	**0.89**	**0.96**	**0.92**	0.97	**0.91**	0.94	0.96	0.98	**0.94**	**0.94**	**0.89**	**0.65**
ADL(1,0)	0.78	0.49	0.90	0.80	0.71	0.85	0.89	0.89	0.94	0.67	0.95	0.97	0.99	0.90	0.56	0.58	0.22
ADL(1,1)	0.77	0.77	0.90	0.88	**0.91**	0.84	0.91	0.90	**0.97**	0.79	**0.95**	**0.97**	**0.99**	0.93	0.93	0.66	0.26

(a) In bold the maximum correlation by branch.

(b) For each branch and class of models the value refers to the model specification with maximum correlation.

(c) Each class of models estimated in both levels and logs. The Chow-Lin, ADL(1,0) and ADL(1,1) classes estimated in the unrestricted form of equation (1) and the restricted forms without trend and both constant and trend. The ADL classes include also the models in differences.

Table 6A – Model specifications and estimated parameters of estimations for which the minimum MAE by branch is obtained (seasonal adjusted data at current prices) (a)

Branch	Model	Specification	Log-likelihood	$\hat{\phi}$	$\hat{m}$	$\hat{g}$	$\hat{\beta}_0$	$\hat{\beta}_1$	AIC	BIC	DW	N	Q(4)	H
3	ADL(1,0)	Δ	-142.73	0.31			8.90		8.0	8.1	2.2	0.4	1.9	1.3
							(5.52)**					(0.83)	(0.59)	(0.25)
4	ADL(1,1)	Δ - log	-148.82	0.23			-0.60	0.85	-8.3	-8.2	3.2	0.3	8.7	0.0
							(-1.33)**	(2.00)**				(0.36)	(0.3*)	(1.00)
5	Chow-Lin	--	-152.76	0.81			60.15		10.2	10.2	1.9	0.7	2.6	4.0
							(71.77)**					(0.69)	(0.47)	(0.00**)
6	ADL(1,0)	Δ - log	-140.88	0.54			0.31		-9.3	-9.2	2.6	0.6	5.6	3.0
							(5.59)**					(0.73)	(0.13)	(.01**)
7	Chow-Lin	--	-150.09	0.98			9.37		9.3	9.3	2.0	0.2	1.0	10.5
							(4.52)**					(0.92)	(0.8)	(0.00**)
8	ADL(1,1)	--	-142.01	0.96			16.00	-14.85	8.6	8.7	2.6	0.6	7.8	0.6
							(4.19)**	(-3.88)**				(0.75)	(.05*)	(0.7)
9	ADL(1,0)	Δ	-131.93	0.51			6.24		6.4	6.5	2.6	1.9	2.8	0.1
							(3.32)**					(0.39)	(0.42)	(1)
10	ADL(1,1)	log	-153.43	0.99			0.38	-0.37	-7.6	-7.5	2.6	1.2	4.4	4.1
							(3.43)**	(-3.26)**				(0.55)	(0.22)	(0.00**)
11	ADL(1,0)	Δ	-156.38	-0.11			58.90		10.0	10.1	1.3	1.1	0.7	0.2
							(15.66)**					(0.59)	(0.88)	(0.98)
12	ADL(1,1)	log	-132.67	0.93	0.05	0.00	0.57	-0.48	-7.1	-6.9	1.6	0.3	1.6	1.2
					-0.34	(4.41)**	(3.56)**	(-3.04)**				(0.88)	(0.66)	(0.3)
13	ADL(1,0)	Δ-log	-132.52	0.03	0.01	0.00	0.39		-7.9	-7.7	2.2	0.7	4.1	0.2
					(3.30)**	(-1.61)	(7.51)**					(0.71)	(0.25)	(0.97)
14	ADL(1,0)	Δ-log	-147.34	0.20	0.00		0.44		-8.8	-8.6	2.3	1.0	6.6	0.7
					(4.04)**		(13.30)**					(0.6)	(0.09)	(0.65)
15	ADL(1,1)	--	-132.46	0.37	333.74	3.59	34.35	-22.76	9.5	9.7	1.1	0.4	7.6	0.0
					(3.26)**	(6.58)**	(9.85)**	(-6.83)**				(0.81)	(0.06)	(1)
16	Chow-Lin	--	-136.01	0.75			16.30		8.6	8.6	2.3	0.5	5.4	0.4
							(51.104)**					(0.79)	(0.15)	(0.9)
17	ADL(1,1)	Δ	-139.15	0.83			24.93	-19.45	6.4	6.5	1.9	1.3	1.5	1.7
							(10.39)**	(-7.57)**				(0.53)	(0.68)	(0.14)
18	ADL(1,1)	--	-152.74	0.53	1113.98		-35.90	54.86	11.1	11.3	1.6	1.2	0.8	7.2
					(12.69)**		(-2.21)**	(3.38)**				(0.56)	(0.86)	(0.00**)
19	ADL(1,0)	--	-130.59	0.81	353.58	5.42	-1.26		8.0	8.2	1.2	0.9	2.4	0.3
					(7.30)**	(9.96)**	(-1.69)*					(0.64)	(0.5)	(0.94)

(a) Probability value in parenthesis: * p-value ≤ 0.05; ** pvalue ≤ 0.01.

Table 7A – Model specifications and estimated parameters of estimations for which the minimum MAE by branch is obtained (seasonal adjusted data of chain linked values) (a)

Branch	Model	Specification	Log-likelihood	$\hat{\phi}$	$\hat{m}$	$\hat{g}$	$\hat{\beta}_0$	$\hat{\beta}_1$	AIC	BIC	DW	N	Q(4)	H
3	Chow-Lin	log	-144.90	0.89	0.54		0.51		-5.6	-5.5	1.9	0.57	0.5	0.0
					(3.99)*		(1.95)*					(4.06)	(0.92)	(0.99)
4	Fernàndez	--	-156.41	0.00			76.91		10.0	10.1	2.9	0.13	9.9	0.1
							(34.46)**					(1.53)	(0.02*)	(0.99)
5	Chow-Lin	--	-158.05	0.80			58.58		10.7	10.8	1.8	0.46	4.4	1.1
							(60.05)**					(1.01)	(0.22)	(0.34)
6	Chow-Lin	log	-137.16	0.87	0.75		0.55		-8.2	-8.1	1.6	0.6	1.8	1.5
					(16.76)**		(7.55)**					(0.24)	(0.61)	(0.19)
7	Chow-Lin	log	-144.12	0.70	0.39	-0.01	1.40		-4.2	-4.1	1.4	0.89	4.4	0.0
					-0.61	(-11.3)**	(3.05)**					(0.34)	(0.22)	(1)
8	ADL(1,0)	--	-145.41	0.25			18.68		10.8	10.8	1.5	0.84	1.7	0.0
							(82.29)**					(4.14)	(0.63)	(1)
9	ADL(1,0)	--	-131.18	0.63	-59.89	3.2	6.53		8.6	8.8	1.9	0.13	1.2	7537.1
					(-.36)	(5.5)**	(3.39)**					(2.39)	(0.75)	(0.00**)
10	Chow-Lin	--	-152.81	0.99			31.67		9.4	9.4	2.9	0.3	5.4	0.1
							(6.09)**					(1.51)	(0.15)	(1)
11	Fernàndez	--	-158.95	0.00			69.94		10.3	10.3	2.4	0.47	4.7	2.9
							(41.13)**					(1.01)	(0.19)	(0.01**)
12	ADL(1,1)	--	-142.49	0.98			6.84	-6.6	8.6	8.7	1.8	0.6	3.6	1.4
							(1.87)*	(-1.8)*				(1.26)	(0.3)	(0.2)
13	Fernàndez	--	-144.17	0.00			14.83		8.7	8.8	1.9	0.53	5.9	0.6
							(24.81)**					(0.74)	(0.11)	(0.72)
14	ADL(1,0)	Δlog	-147.63	0.00	0.00		0.64		-8.4	-8.2	1.9	0.69	3.8	0.0
					(3.09)**		(15.08)**					(0.76)	(0.29)	(1)
15	Chow-Lin	log	-136.92	0.91	0.34	0.00	0.85		-7.6	-7.5	1.5	0.68	7.3	7.8
					(13.64)**	(4.4)**	(15.22)**					(2.88)	(0.06)	(0.00**)
16	Fernàndez	--	-144.02	0.00			21.70		8.7	8.8	2.8	0.24	4.2	0.4
							(25.78)*					(1.76)	(0.25)	(0.89)
17	ADL(1,1)	--	-153.12	0.94			37.55	-34.4	9.9	10.0	1.9	0.41	1.0	6.1
							(5.143)**	(-4.7)**				(2.08)	(0.81)	(0.00**)
18	Chow-Lin	--	-159.57	0.92			66.89		10.5	10.6	2.0	0.35	7.4	0.2
							(28.26)**					(0.09)	(0.06)	(0.98)
19	ADL(1,0)	log	-145.38	0.96			0.07		-7.1	-7.1	2.8	0.96	13.7	0.1
							(132.33)**					(0.96)	(0.00**)	(1)

(a) Probability value in parenthesis: * p-value ≤ 0.05; ** pvalue ≤ 0.01.

Table 8A – Model specifications and estimated parameters of estimations for which the maximum correlation by branch is obtained (seasonal adjusted data at current prices) (a)

Branch	Model	Specification	Log-likelihood	$\hat{\phi}$	$\hat{m}$	$\hat{g}$	$\hat{\beta}_0$	AIC	BIC	DW	N	Q(4)	H
3	Chow-Lin	log	-149.00	0.94			1.57	-5.2	-5.1	1.6	0.6	1.1	1.4
							(62.78)**				(0.83)	0.77)	0.23)
4	Fernàndez	log	-163.86	0.00			2.00	-8.3	-8.2	3.2	0.3	8.7	0.0
							(169.21)**				(0.36)	(0.3*)	(1.00)
5	Fernàndez	log	-166.48	0.00			1.89	-6.4	-6.3	2.3	2.4	1.2	5370.0
							(170.37)**				(0.3)	(0.76)	(0.00**)
6	Fernàndez	--	-151.28	0.00			39.49	9.5	9.5	2.5	1.5	4.4	1.1
							(25.59)**				(0.48)	(0.22)	(0.34)
7	Fernàndez	log	-152.08	0.00			1.85	-3.6	-3.6	1.9	0.4	1.8	4.1
							(34.35)**				(0.81)	(0.61)	(0.00**)
8	Fernàndez	--	-145.73	0.00			26.72	8.9	9.0	2.4	0.4	3.6	24.8
							(23.68)**				(0.81)	(0.31)	(0.00**)
9	Fernàndez	log	-141.93	0.00			1.63	-6.6	-6.5	3.0	0.5	6.2	39.9
							(148.65)**				(0.78)	(0.1)	(0.00**)
10	Fernàndez	--	-160.44	0.00			48.21	10.5	10.5	2.8	3.2	6.4	9.0
							(20.67)**				(0.2)	(0.09)	(0.00**)
11	Chow-Lin	log	-150.37	0.81	1.14	0.00	0.64	-8.0	-7.8	0.9	1.4	4.1	71.3
					(29.63)**	(6.62)**	(14.49)**				(0.5)	(0.26)	(0.00**)
12	Fernàndez	log	-147.42	0.00			1.44	-6.0	-6.0	1.2	0.7	4.9	0.2
							(118.08)**				(0.69)	(0.18)	0.97)
13	Chow-Lin	log	-131.94	0.83	0.93	0.00	0.43	-7.5	-7.4	1.9	0.8	5.6	27.0
					(17.78)**	(12.15)**	(6.97)**				(0.66)	(0.13)	(0.00**)
14	Chow-Lin	log	-144.04	0.68	1.92	0.00	0.56	-7.8	-7.7	2.0	0.3	1.5	3.8
					(31.15)**	(21.21)**	(13.39)**				(0.88)	(0.68)	(0.00**)
15	Fernàndez	log	-163.64	0.00			1.67	-5.1	-5.0	2.7	10.9	3.9	13.2
							(75.57)**				(0.00**)	0.27)	(0.00**)
16	Fernàndez	log	-143.2	0.00			1.64	-6.1	-6.0	2.5	0.1	2.0	0.8
							(119.58)**				(0.95)	(0.57)	(0.55)
17	Chow-Lin	--	-144.18	0.97	69.63		28.23	8.8	8.9	1.3	1.0	3.6	4.0
					(5.22)**		(6.82)**				(0.6)	(0.31)	(0.00**)
18	Fernàndez	log	-181.61	0.00			2.18	-4.5	-4.4	2.3	1.5	3.0	0.0
							(61.05)**				(0.46)	(0.39)	(1)
19	Fernàndez	log	-160.8	0.00			1.76	-5.0	-4.9	2.3	24.1	6.5	2.0
							(68.16)**				(0.00**)	(0.09)	(0.06)

(a) Probability value in parenthesis: * p-value ≤ 0.05; ** pvalue ≤ 0.01.

Table 9A – Model specifications and estimated parameters of estimations for which the maximum correlation by branch is obtained (seasonal adjusted data of chain linked values) (a)

Branch	Model	Specification	Log-likelihood	$\hat{\phi}$	$\hat{m}$	$\hat{g}$	$\hat{\beta}_0$	$\hat{\beta}_1$	AIC	BIC	DW	N	Q(4)	H
3	Fernàndez	log	-150.59	0.00			1.54		-5.3	-5.3	1.9	0.5	1.3	8.1
							(83.63)**					(0.83)	(0.74)	(0.00**)
4	Fernàndez	log	-159.15	0.00			1.98		-8.3	-8.2	3.2	0.3	8.7	0.0
							(260.92)**					(0.36)	(0.3*)	(1.00)
5	Fernàndez	log	-166.86	0.00			1.83		-6.5	-6.4	1.9	0.5	2.0	6.9
							(184.2)**					(0.76)	(0.58)	(0.00**)
6	Fernàndez	--	-145.66	0.00			38.46		8.9	8.9	1.9	0.5	3.8	1.9
							(38.71)**					(0.77)	(0.29)	(0.08)
7	ADL(1,1)	log	-152.33	0.99	0.12		2.35	-2.36	-4.3	-4.2	1.8	0.9	2.5	7.4
					(0.15)		(2.68)**	(-2.49)**				(0.63)	(0.48)	(0.00**)
8	Fernàndez	log	-152.32	0.00			1.68		-6.1	-6.1	2.8	0.4	4.6	4686.0
							(134.43)**					(0.82)	(0.21)	(0.00**)
9	Fernàndez	log	-143.71	0.00			1.63		-6.3	-6.3	2.7	0.6	7.2	1.3
							(133.25)**					(0.75)	(0.07)	(0.24)
10	Fernàndez	--	-152.31	0.00			42.89		9.6	9.6	2.6	0.2	3.0	13.7
							(35.13)**					(0.89)	(0.39)	(0.00**)
11	ADL(1,1)	log	-150.45	0.58	2.29	0.00	0.81	-0.50	-7.4	-7.2	1.4	1.0	2.9	0.3
					(19.97)**	(12.73)**	(8.61)**	(-5.14)**				(0.6)	(0.4)	(0.93)
12	Fernàndez	log	-149.26	0.00			1.50		-6.1	-6.0	1.4	0.9	4.2	0.0
							(129.31)**					(0.65)	(0.24)	(1)
13	ADL(1,1)	Δ log	-140.50	0.57			0.73	-0.49	-8.5	-8.3	2.9	0.3	9.5	0.2
							(7.86)**	(-4.73)**				(0.85)	(0.02*)	(0.98)
14	ADL(1,1)	Δ log	-148.08	0.40	0.00		0.66	-0.28	-9.1	-8.9	2.2	0.9	3.8	0.0
					(2.84)**		(10.50)**	(-3.77)**				(0.64)	(0.29)	(1)
15	ADL(1,1)	--	-136.01	0.89	124.41	0.86	23.48	-21.88	10.0	10.1	2.1	1.0	1.4	0.4
					(2.06)**	(2.69)**	(14.61)**	(-13.38)**				(0.6)	(0.72)	(0.87)
16	Fernàndez	log	-146.54	0.00			1.65		-6.2	-6.2	2.4	3.1	3.7	20.8
							(138.97)**					(0.21)	(0.29)	(0.00**)
17	Fernàndez	--	-156.22	0.00			54.52		11.2	11.3	2.1	1.4	1.6	9.7
							(29.45)**					(0.5)	(0.67)	(0.00**)
18	Fernàndez	log	-168.65	0.00			1.99		-6.2	-6.2	2.3	0.4	3.9	0.1
							(153.7)**					(0.83)	(0.28)	(0.99)
19	Fernàndez	log	-163.90	0.00			1.68		-5.2	-5.2	2.2	17.4	6.5	6.7
							(85.60)**					(0.00**)	(0.09)	(0.00**)

(a) Probability value in parenthesis: * p-value ≤ 0.05; ** pvalue ≤ 0.01.

Table 10A – Correlations between disaggregated series and quarterly indicator in terms of quarterly (Δq) and annual (Δy) growth rates (a)

		Current prices data						Chain-linked data					
br		CLc	Fe	ADL10	ADL11	ADL10 Δ	ADL11Δ	CLc	Fe	ADL10	ADL11	ADL10Δ	ADL11Δ
3	Δq	0.89	0.91	**0.39**	0.88	0.51	0.58	0.76	0.91	**0.23**	0.37	0.76	0.83
	Δy	0.67	0.68	0.58	0.67	0.63	0.38	0.40	0.47	0.27	0.36	0.41	0.37
4	Δq	0.94	0.98	0.40	**0.21**	0.53	-0.82	0.77	0.99	**0.25**	-0.29	-0.74	-0.82
	Δy	0.45	0.58	0.26	0.22	0.36	-0.21	0.37	0.58	0.22	0.12	0.02	-0.43
5	Δq	0.99	0.99	**0.80**	0.99	0.97	-0.25	0.99	0.99	**0.80**	0.98	0.98	0.98
	Δy	0.81	0.82	0.70	0.79	0.78	0.41	0.75	0.75	0.66	0.73	0.76	0.77
6	Δq	0.98	0.98	**0.64**	0.97	0.85	-0.28	0.99	0.99	**0.69**	0.99	0.94	-0.68
	Δy	0.74	0.75	0.71	0.74	0.72	0.70	0.81	0.82	0.78	0.81	0.80	0.61
7	Δq	0.76	0.70	**0.41**	0.76	0.77	0.76	0.48	0.82	-0.01	**0.57**	0.69	0.67
	Δy	0.72	0.73	0.63	0.73	0.72	0.72	0.36	0.64	0.25	0.36	0.55	0.55
8	Δq	0.59	0.97	**0.36**	0.96	0.57	0.92	0.67	0.96	0.86	**0.76**	0.96	-0.14
	Δy	0.54	0.72	0.49	0.70	0.56	0.67	0.57	0.70	0.69	0.59	0.72	0.65
9	Δq	0.89	0.99	**0.70**	0.72	0.97	0.97	0.99	0.99	0.64	**0.61**	0.96	0.95
	Δy	0.68	0.77	0.55	0.46	0.69	0.70	0.55	0.56	0.29	0.31	0.48	0.53
10	Δq	0.48	0.98	0.86	**0.40**	0.84	0.84	0.83	0.99	**0.61**	0.92	0.97	0.90
	Δy	0.57	0.70	0.68	0.55	0.65	0.65	0.76	0.86	0.65	0.73	0.85	0.84
11	Δq	0.99	0.99	**0.88**	0.99	0.99	0.98	0.99	0.99	**0.71**	0.99	0.93	0.94
	Δy	0.95	0.94	0.92	0.95	0.94	0.94	0.94	0.94	0.88	0.94	0.92	0.91
12	Δq	0.93	0.95	**0.20**	0.62	0.80	0.31	0.94	0.96	**0.21**	0.95	0.85	-0.69
	Δy	0.67	0.70	0.56	0.44	0.62	0.55	0.56	0.61	0.46	0.56	0.53	0.23
13	Δq	0.97	0.97	**0.46**	0.97	0.96	0.82	0.98	0.98	**0.45**	0.98	0.98	0.89
	Δy	0.84	0.84	0.73	0.84	0.84	0.83	0.89	0.89	0.79	0.89	0.89	0.89
14	Δq	0.99	0.99	**0.75**	0.99	0.98	0.91	0.99	0.99	**0.29**	0.99	0.99	0.99
	Δy	0.93	0.91	0.90	0.93	0.94	0.93	0.95	0.95	0.65	0.95	0.95	0.95
15	Δq	0.76	0.99	**0.64**	0.97	0.03	0.93	0.99	0.99	**0.78**	0.99	0.98	0.99
	Δy	0.81	0.90	0.64	0.87	0.04	0.85	0.96	0.96	0.88	0.96	0.96	0.96
16	Δq	0.99	0.99	**0.89**	0.99	0.98	0.08	0.99	0.99	**0.80**	0.99	0.96	-0.03
	Δy	0.79	0.79	0.77	0.79	0.79	0.65	0.78	0.79	0.72	0.78	0.77	0.63
17	Δq	0.99	0.99	**0.76**	0.99	0.94	0.99	0.99	0.99	**0.62**	0.99	0.89	0.89
	Δy	0.79	0.78	0.68	0.79	0.73	0.77	0.76	0.77	0.65	0.76	0.58	0.55
18	Δq	0.90	0.96	**0.45**	-0.34	0.59	-0.72	0.98	0.99	**0.58**	0.91	0.97	-0.12
	Δy	0.15	0.24	0.09	0.01	0.08	-0.08	0.56	0.64	0.33	0.45	0.49	0.32
19	Δq	0.15	0.96	0.03	-0.13	-0.27	**0.83**	0.91	0.97	**0.41**	0.60	0.77	-0.65
	Δy	-.06	0.17	-0.06	-0.07	-0.06	0.02	0.25	0.29	0.17	0.18	0.19	0.12

(a) Average correlations over the alternative specifications (standard, with constant, with constant and trend) by each class of model in levels are presented.

Appendix B – State space forms of ADL and Chow-Lin models under stochastic constant and trend components

The ADL(1,1) plus stochastic trend model (4)-(5)-(6) can be cast in the same general state space form of Proietti (2005)

$$\mathrm{y}_t = \mathbf{z}'\boldsymbol{\alpha}_t, \quad t = 1, \dots, T, \tag{B.1}$$

$$\boldsymbol{\alpha}_t = \mathbf{T}\boldsymbol{\alpha}_{t-1} + \mathbf{W}_t\boldsymbol{\beta} + \mathbf{H}\boldsymbol{\varepsilon}_t, \; t = 2, \dots T, \tag{B.2}$$

$$\boldsymbol{\alpha}_1 = \mathbf{W}_1\boldsymbol{\beta} + \mathbf{H}_1\boldsymbol{\varepsilon}_1, \;\; \boldsymbol{\varepsilon}_t \sim \mathrm{NID}(0, \sigma^2\boldsymbol{I}), \tag{B.3}$$

with slight modifications in the state and the disturbance vectors $\boldsymbol{\alpha}_t$ and $\boldsymbol{\varepsilon}_t$, the vector of regression coefficients $\boldsymbol{\beta}$, as well as the system matrices $\mathbf{z}'$, $\mathbf{T}$, **W,** $\boldsymbol{H}$, $\mathbf{W}_1$, $\mathbf{H}_1$. Notably they become,

$$\boldsymbol{\alpha}_t = (\mathrm{y}_t, \mu_t, \upsilon_t)', \qquad \boldsymbol{\varepsilon}_t = (\epsilon_t, \eta_t, \varsigma_t)', \qquad \boldsymbol{\beta} = (\mu_1, \upsilon_1, \beta_0', \beta_1')',$$

$$\mathbf{z}' = (1,0,0), \qquad \mathbf{T} = \begin{pmatrix} \phi & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \mathbf{H} = \begin{pmatrix} 1 & \sqrt{q_\eta} & \sqrt{q_\varsigma} \\ 0 & \sqrt{q_\eta} & \sqrt{q_\varsigma} \\ 0 & 0 & \sqrt{q_\varsigma} \end{pmatrix},$$

$$\mathbf{W}_t = \begin{pmatrix} 0 & 0 & x_t' & x_{t-1}' \\ 0 & 0 & 0' & 0' \\ 0 & 0 & 0' & 0' \end{pmatrix}$$

$$\mathbf{W}_1 = \begin{pmatrix} \frac{1}{1-\phi} & \frac{1}{1-\phi} & \frac{x_1'}{1-\phi} & \frac{x_1'}{1-\phi} \\ 1 & 1 & 0' & 0' \\ 0 & 1 & 0' & 0' \end{pmatrix}, \qquad \mathbf{H}_1 = \begin{pmatrix} \frac{1}{\sqrt{1-\phi^2}} & \frac{\sqrt{q_\eta}}{\sqrt{1-\phi^2}} & \frac{\sqrt{q_\varsigma}}{\sqrt{1-\phi^2}} \\ 0 & \sqrt{q_\eta} & \sqrt{q_\varsigma} \\ 0 & 0 & \sqrt{q_\varsigma} \end{pmatrix}$$

The case of the ADL(1,0) model takes a straightforward development. When μ_t is restricted to a simple random walk $\mu_t = \mu_{t-1} + \eta_t$, $\boldsymbol{\alpha}_t = (\mathrm{y}_t, \mu_t)'$, $\boldsymbol{\varepsilon}_t = (\epsilon_t, \eta_t)'$ and $\boldsymbol{\beta} = (\mu_1, \beta_0', \beta_1')'$, with the system matrices accordingly modified. Under this latter restriction the state space form of the Chow-Lin model (7) is obtained by including the regression effects in the measurement equation such that $\mathrm{y}_t = \mathbf{z}'\boldsymbol{\alpha}_t + x_t'\beta$ and considering the following formulations for state and disturbance vectors and system matrices:

$$\boldsymbol{\alpha}_t = (\alpha_t, \mu_t)', \qquad \boldsymbol{\varepsilon}_t = (\epsilon_t, \eta_t)', \qquad \boldsymbol{\beta} = (\mu_1, \beta')',$$

$$\mathbf{z}' = (1,0), \qquad \mathbf{T} = \begin{pmatrix} \phi & 1 \\ 0 & 1 \end{pmatrix}, \qquad \mathbf{H} = \begin{pmatrix} 1 & \sqrt{q_\eta} \\ 0 & \sqrt{q_\eta} \end{pmatrix}, \qquad \mathbf{W}_t = \begin{pmatrix} 0 & 0' \\ 0 & 0' \end{pmatrix}$$

$$\mathbf{W}_1 = \begin{pmatrix} \frac{1}{1-\phi} & 0' \\ 1 & 0' \end{pmatrix}, \qquad \mathbf{H}_1 = \begin{pmatrix} \frac{1}{\sqrt{1-\phi^2}} & \frac{\sqrt{q_\eta}}{\sqrt{1-\phi^2}} \\ 0 & \sqrt{q_\eta} \end{pmatrix}$$

Notice that these forms should be augmented by the cumulator variable y_t^c to handle the temporal aggregation constraint as described in section 3.1 and with all details in Proietti (2005).