A Dynamic Analysis of Demand and Productivity Growth in a Two-sector Kaleckian Model

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April 2018

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Abstract

This study extends a two-sector Kaleckian model of growth and income distribution by incorporating the dynamics of labour productivity growth. The economy is composed of investment goods and consumption goods producing sectors, with the sectoral demand and productivity growth interaction dynamically formalized. The study analyses the conditions for the cyclical demand and productivity growth phenomena in a two-sector economy. The model reveals that each sector may present a different response in capacity utilization rate to a change in sectoral income distribution. These phenomena are specific to two-sector models, and cannot be observed with a conventional aggregate growth model.

Keywords: Kaleckian model, Two-sector economy, Effective demand, Productivity growth

JEL Classification: E25, E32, O41.

*Professor, Faculty of Economics, Hannan University, 5-4-33, Amami Higashi, Matsubara-shi, Osaka 580-8502, Japan. E-mail: nishi@hannan-u.ac.jp I am grateful to Shinya Fujita, Norihito Shimano, Sonoda Ryunosuke, Engelbert Stockhammer, Tim Gooding, and Christina Wolf for their valuable comments on earlier versions of this paper. This article was written while I was visiting Kingston University, London. The hospitality of Kingston is gratefully acknowledged. Any remaining errors are of course mine.

1 Introduction

This paper builds a two-sector Kaleckian model composed of investment goods and consumption goods producing sectors, with focus on the dynamic interaction of demand, productivity, and income distribution. The model is based on the standard Kaleckian setup and particularly extended to incorporate the effects of labour productivity growth in both sectors. That is, a demand-led Kaleckian model is augmented by supply-side effects in a two-sector framework. In this manner, the current model tries to show the different output responses to changes in income distribution by sector in an economy. It also shows the possibility of cyclical demand and productivity growth interaction through the transaction of different sectors. These results are normal in a two-sector framework, but cannot be observed in the aggregate macro model that many Kaleckian studies employ.

Since Rowthorn (1981), Dutt (1984), Taylor (1985), and Bhaduri and Marglin (1990), the Kaleckian (or Kalecki–Steindlian) model has been extended to a variety of fields. The Kaleckian model can explain economic growth from the principle of effective demand and income distribution. Debates on wage-led demand and growth (WLG) and profit-led demand and growth (PLG) regimes have brought fruitful research outputs in post-Keynesian economics.[1] These models establish a wage-led demand regime if the rise in wage share stimulating aggregate consumption is more than the fall in profit share restraining investment demand (and net exports demand in an open economy). In contrast, a profit-led demand regime is established if a rise in profit share stimulating investment demand (and net exports demand in an open economy) is more than the fall in wage share restraining aggregate consumption.

Numerous Kaleckian studies have explained the stability, instability, and cycles in demand-driven growth models, which are based on the aggregate model. In the aggregate model, differences in the production, expenditure, and distribution specific to particular sectors are not explicitly introduced by structure.[2]

[1]Works in special issues of the *Review of Keynesian Economics* have tried to both take stock of and advance the debate continuing since Bhaduri and Marglin (1990) on the interaction of growth and distribution (Setterfield (2016)).

[2]For example, considering conflicting claim models, Blecker (2011) and Sasaki et al. (2013) investigated the stability conditions of growth and distribution in an open economy. Besides, in a response to Sraffian critics, Lavoie (1995) and Cassetti (2006) present long-run models where the actual capacity utilization rate adjusts to the normal standard rate. They thus show the conditions for (in-)validity of the Kaleckian results, such as the cost and thrift

In contrast to the aggregate models, only a few studies examine growth and distribution using a two-sector framework. Dutt (1990, 1997), Lavoie and Ramirez-Gaston (1997), Park (1997), Franke (2000), Fujita (2015), and Murakami (2017) are works related to the current study. Dutt (1997) and Park (1997) contribute to solve the possible over-determination problems in multi-sector Kaleckian models. In Dutt (1990) chapter 6 and Lavoie and Ramirez-Gaston (1997), a rise in profit share (target return rates) leads to a fall in accumulation rates in both sectors. Specifically, they revealed the WLG regime in both sectors. Franke (2000) introduced the optimal use of input and degree of capital utilization rates to maximize the sectoral profit rate. Fujita (2015)'s model with intermediate goods is also unique in that it reveals different demand regimes in different sectors; this study sheds more light on this aspect. Similar to Murakami (2017), the current study reveals the emergence of cyclical growth, but his Kaldorian business cycle model differs from our model in that it is a Kaleckian model introducing the effects of income distribution and productivity growth.

These studies explain economic growth based on effective demand. On the other hand, some issues still remain to be cleared with the two-sector Kaleckian model. In particular, none of the above studies consider the role of productivity growth in each sector. Consequently, the macroeconomic outcome of the interactions of demand, productivity, and income distribution has not been explained. In addition, apart from Fujita (2015), the uneven impact of changes in income distribution on demand at the sectoral level has not been clarified sufficiently. Also, the existing two-sector models exclusively focus on the mechanism of economic growth, but no study has exclusively considered the mechanism of business cycles that arise from sectoral interactions, with the exception of Murakami (2017).

The two-sector model in this study reveals how the changes in income distribution, demand, and productivity growth in each sector affect both sectoral and macroeconomic performance, and also addresses certain remaining issues. The framework of this paper is similar to the models in Dutt (1990), Lavoie and Ramirez-Gaston (1997), and Fujita (2015), but differs from them in the following points.

First, the current model explores the effects of endogenous productivity growth change in each sector. Recent empirical studies emphasize the role of productivity change in response to

paradox. Moreover, Onaran and Obst (2016) and Stockhammer (2017) empirically provide evidence of profit-led and wage-led demand regimes in different countries and periods. All of these briefly surveyed studies consist of aggregate analyses.

growth and distribution (Taylor (2004); Barbosa-Filho and Taylor (2006); Storm and Naastepad (2012, 2017)), but none of their two-sector models have examined its importance. By endogenizing labour productivity growth, this paper closely explores its interaction with demand and income distribution in a two-sector framework. In this paper, the pattern of the income distribution impact on capacity utilization rate is called a "wage-led or profit-led outcome" (i.e. WLO or PLO) instead of the conventional term of "wage-led or profit-led demand regime" because its impact goes through both the demand and supply sides. Second, it shows different output (capacity utilization rate) responses to a change in income distribution by sector in an economy. For example, from the current model, even if WLO arises in one sector, PLO may be realized in another sector. In this case, the fallacy of composition between industry- and macro-level performances emerges, where the impact of a change in income distribution on the aggregate capacity utilization rate necessarily conflicts with the impact in at least one of the two sectors. Then, the question of which (wage or profit share) or where (sector 1 or 2) to target in order to expand the economic activity level becomes quite puzzling. This is an important difference from the standard aggregate Kaleckian model. Finally, it reveals both economic growth and the emergence of a business cycle. Transitional dynamics to the steady state in the two-sector models of Dutt (1990), Lavoie and Ramirez-Gaston (1997), and Fujita (2015) are monotonic and consequently stable in economic growth. In contrast, the model in this paper illustrates the emergence of business cycles by the interaction of demand and productivity growth in two sectors.

The remainder of the paper is organized as follows. Section 2 sets up a two-sector model. Section 3 analyses the dynamics of the capacity utilization rates and the relative labour productivity level. The conditions for the cyclical phenomena of demand and productivity growth in a two-sector economy are also analysed, which are numerically confirmed in the appendix. Section 4 explores the effects of income distribution change on the capacity utilization rates and output growth rate through a comparative statics analysis. Section 5 concludes the paper.

2 Model

This section presents a closed economy model with two production sectors, one the investment goods production sector (sector 1), and the other the consumption goods production sector (section 2). Both sectors are supposed to be vertically integrated according to what they materially produce. Thus, there is no intermediate input good, and the model exclusively focuses on the transaction of final goods.

The following are the basic notations used for setting up the model. X_i: output in real term, D_i: demand in real term, L_i: labour demand, K_i: capital stock in real term, C_i: consumption demand in real term, I_i: investment demand in real term, a_i: labour productivity level, g_i: capital accumulation rate, u_i: capacity utilization rate, p_i: commodity price, w: nominal wage rate, π_i: profit share, r_i: profit rate, where $i = 1, 2$ refers to the sector number.

Assume that workers supply labour force to firms in a capitalist closed economy having no government sector. The former receives wage and latter receives profit income. Firms in each sector operate under a Leontief-type fixed coefficient production function using capital stock and labour as follows:

$$X_i = \min[(u_i/v_i)K_i, a_iL_i] \tag{1}$$

where $a_i = X_i/L_i$ denotes the labour productivity level. The capacity utilization rate is defined as $u_i = X_i/\overline{X_i}$, where $\overline{X_i}$ denotes the potential output. Coefficient $v_i = K_i/\overline{X_i}$ represents the constant capital stock to potential output ratio, which I assume to be unity. By this assumption, keeping the capacity utilization rate constant, the capital stock and the actual and potential output growth rates are the same.

Furthermore, assume that once installed, capital stock cannot be moved between sectors, but since there are no labour supply constraints, workers can move between the two sectors. Following Lavoie (2014), I introduce three Kaleckian features into each sector, (i) mark-up pricing, (ii) excess capacity, and (iii) an investment function independent of the saving constraint. Apart from the investment function, these parameters are set to differ by sector because different industries have particular production, distribution, and expenditure patterns.

The price system determines the income distribution and pricing, whereas the quantity system determines the expenditure and income generation. The income distribution of the economy can be defined as follows:

$$p_1X_1 = wL_1 + p_1r_1K_1, \tag{2}$$

$$p_2X_2 = wL_2 + p_2r_2K_2, \tag{3}$$

where the nominal wage rates $w_1 = w_2 = w$ are equalized for simplicity. The focus of this paper is not on the wage rate, but on the profit (wage) share. Equations (2) and (3) show that the total nominal income ($p_i X_i$) is distributed as wages (wL_i) to workers and as profits ($p_i r_i K_i$) to capitalists.

In an oligopolistic environment, firms set the mark-up price over the unit labour cost in each sector; this is formalized as the following pricing equations:

$$p_1 = (1+\theta_1)\frac{w}{a_1}, \tag{4}$$

$$p_2 = (1+\theta_2)\frac{w}{a_2}, \tag{5}$$

where θ_i is a positive mark-up rate. The mark-up rate, which is assumed to be exogenous, is supposed to be affected by the degree of monopoly and relative strength of the workers' and firms' bargaining power. Equations (2) through (5) determine the income distribution share in each sector in the following manner:

$$\pi_1 = \frac{p_1 r_1 K_1}{p_1 X_1} = \frac{\theta_1}{1+\theta_1} \quad \Longrightarrow \quad \theta_1 = \frac{\pi_1}{1-\pi_1}, \tag{6}$$

$$\pi_2 = \frac{p_2 r_2 K_2}{p_2 X_2} = \frac{\theta_2}{1+\theta_2} \quad \Longrightarrow \quad \theta_2 = \frac{\pi_2}{1-\pi_2}. \tag{7}$$

Since the mark-up rate is constant, the income distribution is also constant. Thus, the mark-up rate and income distribution have a one-to-one relationship, with a rise in mark-up leading to a rise in profit share and fall in wage share. When the income distribution share is replaced by mark-up pricing, the relative price level becomes as follows:

$$p \equiv \frac{p_1}{p_2} = \frac{(1-\pi_2)wa_2}{(1-\pi_1)wa_1} = \left(\frac{1-\pi_2}{1-\pi_1}\right)z, \tag{8}$$

where the relative productivity growth level z is defined by $z \equiv \frac{a_2}{a_1}$.[3]

The economy's quantity system is presented as follows:

$$p_1 D_1 = p_1(I_1 + I_2), \tag{9}$$

$$p_2 D_2 = p_2(C_1 + C_2). \tag{10}$$

Equation (9) indicates sector 1's demand as the final demand for the investment goods in both sectors, whereas Equation (10) shows sector 2's demand as the final demand of workers for the consumption goods in both sectors.

[3]In the current model, price level is a dependent variable of profit share, nominal wage, and labour productivity. Since the profit share and nominal wage are assumed to be constant over time, a change in labour productivity growth is reflected in the inflation rate. Consequently, the model presents a productivity growth rate differential inflation similar to Baumol (1967).

Using Kaleckian ideas, I introduce behavioural assumptions on consumption and investment activity. First, I assume that the common investment function determines the capital accumulation rate of both sectors, which is formalized as follows:

$$g_1 \equiv \frac{I_1}{K_1} = g, \tag{11}$$

$$g_2 \equiv \frac{I_2}{K_2} = g. \tag{12}$$

Now, by extending Bhaduri and Marglin (1990), investment demand normalized by capital stock can be an increasing function of the profit share as well as capacity utilization rate of both sectors as follows:

$$g = \alpha + \beta_1\pi_1 + \beta_2\pi_2 + \gamma_1 u_1 + \gamma_2 u_2, \tag{13}$$

where α is an autonomous investment demand, β_i captures the profit effect, and γ_i identifies the accelerator effect driven by the change in profit share and capacity utilization rate in each sector i, respectively. The introduction of a uniform capital accumulation rate may seem a strong assumption, but by doing so, I assume that the capital accumulation rate of each sector is affected by the conditions in the other sector. For example, a rise in profit share in sector 1 principally induces its own capital accumulation, but also works as a signal of profit opportunity for sector 2, affecting the capital accumulation rate in sector 2, and vice-versa. Thus, Equation (13) indicates that the sectoral capital accumulation between the two sectors is synchronized.[4]

The total demand in the investment goods production sector normalized by capital stock is

$$\frac{D_1}{K_1} = (1 + k)g, \tag{14}$$

where $k \equiv \frac{K_2}{K_1}$ denotes the sectoral ratio of capital stock, which remains constant because the capital in each sector grows at the same rate.

Now, assume that while workers spend all their wage income on consumption goods, capitalists save all their profit income in both sectors. Then, the consumption demand of each sector

[4]Besides, the uniform capital accumulation rate function helps to reduce the number of state variables in the model. For example, when I define the capital accumulation rate by different functions, I have to consider the dynamics of the relative capital size k as well, because the dynamics of k follows the sectoral difference in capital accumulation rate. In this case, the model involves four state variables, making the analytical argument extremely complicated without contributing to significant insights.

normalized by the capital stock will be as follows:

$$\frac{p_2 D_2}{p_2 K_2} = \frac{wL_1 + wL_2}{p_2 K_2} = \frac{wL_1}{p_1 X_1}\frac{p_1}{p_2}\frac{X_1}{K_1}\frac{K_1}{K_2} + \frac{wL_2}{p_2 X_2}\frac{X_2}{K_2}$$
$$= (1-\pi_1)p\frac{u_1}{k} + (1-\pi_2)u_2. \tag{15}$$

From the relative price level (Equation 8), the total demand in the consumption goods production sector normalized by capital stock is

$$\frac{D_2}{K_2} = (1-\pi_2)\frac{z}{k}u_1 + (1-\pi_2)u_2. \tag{16}$$

I introduce the endogenous determination of labour productivity growth rate in each sector as a supply-side effect on sectoral performance. This idea is based on the theoretical and empirical studies of Taylor (2004), Barbosa-Filho and Taylor (2006), and Storm and Naastepad (2012, 2017), but the impact of income distribution is augmented. In formalizing the productivity growth dynamics, these studies assumed that the labour productivity growth rate depends on the Kaldor–Verdoon effect as well as on the labour-saving technological progress driven by wage increase.[5] In this paper, the former is embodied approximately using the capacity utilization rate, whereas for the latter, I consider the possibilities of both wage and profit shares stimulating labour productivity growth. Their studies regard productivity growth as an increasing function of wage variables and show evidence from advanced economies, which I also introduce as a case. However, a rise in profit share also helps firms increase their productivity growth. Normally, productivity growth is driven by the introduction of new machines and requires a huge amount of money, and therefore firms will need funds to introduce them. The first candidate to finance this innovation is internal funds, as the pecking order hypothesis suggests (Fazzari et al. (1988)), and *ceteris paribus* a rise in profit share increases internal funds. It is plausible that a rise in profit share contributes to labour productivity growth through this channel.

Assume that different sectors experience different labour productivity growth rates over the boom and bust periods. The labour productivity growth rate in each sector is formalized by the

[5]The equation that formalizes the potential labour productivity gains is called the productivity regime equation. In this equation, the Kaldor–Verdoon effect, or the dynamic increasing returns to scale, explains the growth of aggregate demand, or capital accumulation stimulates the labour productivity growth. Also, Taylor (2004), Barbosa-Filho and Taylor (2006), and Storm and Naastepad (2012, 2017) explain labour-saving technological change as a higher wage growth that induces firms to invest in new labour-saving machines. That is, a rise in real wage stimulates labour productivity growth.

following function:

$$\hat{a}_1 = q_1(\pi_1, u_1), \quad q'_{1\pi} \gtrless 0, \quad q'_{1u} > 0, \tag{17}$$

$$\hat{a}_2 = q_2(\pi_2, u_2), \quad q'_{2\pi} \gtrless 0, \quad q'_{2u} > 0, \tag{18}$$

where $q'_{iu} = \partial q_i(\pi_i, u_i)/\partial u_i$ indicates the capacity utilization rate effect on productivity growth, and $q'_{i\pi} = \partial q_i(\pi_i, u_i)/\partial \pi_i$ represents the profit share effects on the capacity utilization rate. The former is positive, whereas the latter can be either positive or negative for the reason mentioned above. The positive sign of $q'_{i\pi}$ is called "profit-led productivity regime," whereas the negative sign is called "wage-led productivity regime." Along with the dynamics of the capacity utilization rates, the determination of productivity growth brings about different demand response patterns to the changes in income distribution.

Finally, by taking the logarithm of z and differentiating it with respect to time, I obtain the change in relative productivity level as follows:

$$\dot{z} = z(\hat{a}_2 - \hat{a}_1), \tag{19}$$

where $\hat{a}_i$ represents the labour productivity growth rate in each sector, as defined in Equations (17) and (18).

3 Analysis

3.1 Dynamic system and steady state

The dynamic system consists of the capacity utilization rate adjustments in both sectors and the change in relative labour productivity level. The former is driven by effective demand, whereas the latter is driven by endogenous labour productivity growth.

Following the Keynesian–Kaleckian modelling, excess demand (supply) leads to a rise (fall) in capacity utilization rate. From Equations (14) and (16), the capacity utilization rates in sectors

1 and 2 are, respectively,

$$\dot{u}_1 = \phi_1 \left(\frac{D_1}{K_1} - u_1 \right)$$
$$= \phi_1[(1+k)(\alpha + \beta_1\pi_1 + \beta_2\pi_2) + (1+k)\gamma_2 u_2 - (1-(1+k)\gamma_1)u_1], \tag{20}$$
$$\dot{u}_2 = \phi_2 \left(\frac{D_2}{K_2} - u_2 \right)$$
$$= \phi_2 \left[\left(\frac{1-\pi_2}{k} \right) z u_1 - \pi_2 u_2 \right], \tag{21}$$

where ϕ_i represents the adjustment speed of change in capacity utilization rate in response to the disequilibrium in each sector.

The third state variable is the change in relative productivity level. Since the labour productivity growth rate in each sector determined by Equations (17) and (18) affects the relative productivity level of the two sectors, the dynamics of the relative productivity level is given by

$$\dot{z} = z[q_2(\pi_2, u_2) - q_1(\pi_1, u_1)]. \tag{22}$$

The dynamic system of a two-sector economy consists of Equations (20), (21), and (22). Since the steady state is defined by $\dot{u}_1 = \dot{u}_2 = \dot{z} = 0$, it can be given by the following conditions:

$$0 = (1+k)(\alpha + \beta_1\pi_1 + \beta_2\pi_2) + (1+k)\gamma_2 u_2^* - (1-(1+k)\gamma_1)u_1^*, \tag{23}$$
$$0 = \left(\frac{1-\pi_2}{k} \right) z^* u_1^* - \pi_2 u_2^*, \tag{24}$$
$$0 = q_2(\pi_2, u_2^*) - q_1(\pi_1, u_1^*), \tag{25}$$

where the asterisk represents the steady-state value of each variable. Equations (23) and (24) indicate no excess demand (supply) in each sector, whereas Equation (25) indicates that the labour productivity growth rates are eventually equalized. Since there are three endogenous variables and three equations, the system is complete. For the moment, assume that there is a unique and positive value of (u_1^*, u_2^*, z^*) that satisfies the steady-state condition, the existence of which I will confirm by a numerical study later.

3.2 Stability, instability, and cycles

In order to investigate the local asymptotic stability of the steady state, I linearize the system of differential equations (20), (21), and (22) around the steady state. The linearized system is given by

$$\begin{pmatrix} \dot{u}_1 \\ \dot{u}_2 \\ \dot{z} \end{pmatrix} = \underbrace{\begin{pmatrix} j_{11} & j_{12} & 0 \\ j_{21} & j_{22} & j_{23} \\ j_{31} & j_{32} & 0 \end{pmatrix}}_{\boldsymbol{J}^*} \begin{pmatrix} u_1 - u_1^* \\ u_2 - u_2^* \\ z - z^* \end{pmatrix}, \tag{26}$$

where $\boldsymbol{J}^*$ is the Jacobian matrix. The non-zero elements of the Jacobian matrix and their signs are as follows:

$$j_{11} \equiv \frac{\partial \dot{u}_1}{\partial u_1} = -\phi_1(1 - (1 + k)\gamma_1), \tag{27}$$

$$j_{12} \equiv \frac{\partial \dot{u}_1}{\partial u_2} = \phi_1(1 + k)\gamma_2 > 0, \tag{28}$$

$$j_{21} \equiv \frac{\partial \dot{u}_2}{\partial u_1} = \phi_2 \left(\frac{1 - \pi_2}{k} \right) z^* > 0, \tag{29}$$

$$j_{22} \equiv \frac{\partial \dot{u}_2}{\partial u_2} = -\phi_2 \pi_2 < 0, \tag{30}$$

$$j_{23} \equiv \frac{\partial \dot{u}_2}{\partial z} = \phi_2 \left(\frac{1 - \pi_2}{k} \right) u_1^* > 0, \tag{31}$$

$$j_{31} \equiv \frac{\partial \dot{z}}{\partial u_1} = -z^* q'_{1u} < 0, \tag{32}$$

$$j_{32} \equiv \frac{\partial \dot{z}}{\partial u_2} = z^* q'_{2u} > 0. \tag{33}$$

where all the elements are evaluated at the steady state. Certainly, there are sectoral capacity utilization rate interactions, as Equations (28) and (29) show, and also feedback to productivity growth rate, as Equations (32) and (33) show. Moreover, a change in relative productivity level induces a variation in the capacity utilization rate of sector 2, as Equation (31) indicates.

I define the characteristic equation associated with the Jacobian matrix $\boldsymbol{J}^*$ as follows:

$$\lambda^3 + b_1\lambda^2 + b_2\lambda + b_3 = 0, \tag{34}$$

where λ denotes a characteristic root. Coefficients b_1, b_2, and b_3 are given as follows:

$$b_1 = -\text{tr}\boldsymbol{J}^* = -(j_{11} + j_{22}), \tag{35}$$

$$b_2 = \begin{vmatrix} j_{11} & j_{12} \\ j_{21} & j_{22} \end{vmatrix} + \begin{vmatrix} j_{22} & j_{23} \\ j_{32} & 0 \end{vmatrix} + \begin{vmatrix} j_{11} & 0 \\ j_{31} & 0 \end{vmatrix} = j_{11}j_{22} - j_{12}j_{21} - j_{23}j_{32}, \tag{36}$$

$$b_3 = -\det \boldsymbol{J}^* = -j_{23}(j_{12}j_{31} - j_{32}j_{11}), \tag{37}$$

where tr $\boldsymbol{J}^*$ denotes the trace of $\boldsymbol{J}^*$, b_2 is the sum of the principal minors' determinants, and b_3 is the determinant of $\boldsymbol{J}^*$. The necessary and sufficient condition for local stability is that all the characteristic roots of the Jacobian matrix have negative real parts, which, from the Routh–Hurwitz condition, is equivalent to

$$b_1 > 0, \quad b_2 > 0, \quad b_3 > 0, \quad b_1 b_2 - b_3 > 0. \tag{38}$$

From the Jacobian matrix elements, b_1, b_2, and b_3 can be expressed as follows:

$$b_1(\phi_1, \phi_2) = \phi_1(1 - (1 + k)\gamma_1) + \phi_2\pi_2, \tag{39}$$

$$b_2(\phi_1, \phi_2) = \frac{\phi_2}{k}(\phi_1\Theta_1 - \Theta_2), \tag{40}$$

$$b_3(\phi_1, \phi_2) = \frac{\phi_1\phi_2}{k}(1 - \pi_2)u_1^* z^* \Theta_3, \tag{41}$$

$$b_1 b_2 - b_3 = \frac{\phi_2}{k}\left[(1 - (1 + k)\gamma_1)\Theta_1\phi_1^2 + (\pi_2\Theta_1\phi_2 - \Theta_4)\phi_1 - \pi_2\Theta_2\phi_2\right], \tag{42}$$

where the coefficients are factorized with respect to ϕ_1 and ϕ_2. In addition, Θ_1 through Θ_4 are defined as follows:

$$\Theta_1 \equiv k(1 - (1 + k)\gamma_1)\pi_2 - (1 + k)(1 - \pi_2)\gamma_2 z^*, \tag{43}$$

$$\Theta_2 \equiv (1 - \pi_2)u_1^* z^* q'_{2u}, \tag{44}$$

$$\Theta_3 \equiv (1 + k)\gamma_2 q'_{1u} - (1 - (1 + k)\gamma_1)q'_{2u}, \tag{45}$$

$$\Theta_4 \equiv (1 - \pi_2)(1 + k)u_1^* z^* \gamma_2 q'_{1u}. \tag{46}$$

The signs of Θ_2 and Θ_4 are obviously positive. As for Θ_i, the following assumptions are imposed, and one can obtain economically meaningful solutions.

Assumption 1. $(1 + k)\gamma_1 < 1$, *and the signs of* Θ_1 *and* Θ_3 *are positive.*

Note the necessities of this assumption. The assumption that $(1 + k)\gamma_1 < 1$ means that the Keynesian stability condition for sector 1 is imposed; this ensures that b_1 is positive. That is, the quantity adjustment in sector 1 is self-stable. $\Theta_1 > 0$ excludes the explosive path due to strong accelerator effects.[6] $\Theta_3 > 0$ excludes the saddle-path dynamics in the current three-dimensional model. Without this assumption, the analysis of comparative statics does not present any economically meaningful interpretation.

[6]To be more precise, this assumption excludes saddle-path dynamics and ensures the local stability of u_1^* and u_2^*, when the current model is reduced to a two-dimensional model without the dynamics of relative productivity level.

Next, I examine how the conditions in Equation (38) hold. First, Assumption 1 ensures that b_1 and b_3 are positive. Second, for b_2 to be positive, the adjustment speed of capacity utilization in sector 1 must satisfy the following condition:

$$\phi_1 > \frac{\Theta_2}{\Theta_1} \equiv \underline{\phi_1}, \tag{47}$$

where $\underline{\phi_1}$ is the lower bound of the adjustment speed of the capacity utilization rate in sector 1. Therefore, for steady-state local stability, the quantity adjustment in sector 1 needs to be fast to a certain extent.

From Equation (42), the last condition depends on both parameters ϕ_1 and ϕ_2. Therefore, local stability analysis is conducted with regard to each parameter. First, given $\phi_2 > 0$, I investigate the last condition with regard to ϕ_1. Since $\phi_2 > 0$, I focus on the brackets in Equation (42) and analyse the last condition in terms of the following quadratic function of ϕ_1:

$$f(\phi_1, \phi_2) \equiv (1 - (1 + k)\gamma_1)\Theta_1\phi_1^2 + (\pi_2\Theta_1\phi_2 - \Theta_4)\phi_1 - \pi_2\Theta_2\phi_2. \tag{48}$$

Assumption 1 ensures that the graph of $f(\phi_1, \phi_2)$ is convex downward in terms of ϕ_1. When $\phi_1 = 0$, I have

$$f(0, \phi_2) = -\pi_2\Theta_2\phi_2 < 0. \tag{49}$$

On the other hand, because

$$\frac{\partial f(\phi_1, \phi_2)}{\partial \phi_1} = \underbrace{2(1 - (1 + k)\gamma_1)\Theta_1}_{+}\phi_1 + (\pi_2\Theta_1\phi_2 - \Theta_4), \tag{50}$$

there exists a positive ϕ_1 that makes $f(\phi_1, \phi_2)$ an increasing function with respect to ϕ_1.[7] Therefore,

$$\lim_{\phi_1 \to \infty} f(\phi_1, \phi_2) = \infty, \tag{51}$$

[7]The ϕ_1 axis of the graph for $f(\phi_1, \phi_2)$ is

$$\tilde{\phi}_1 = \frac{\Theta_4 - \pi_2\Theta_1\phi_2}{2(1 - (1 + k)\gamma_1)\Theta_1}.$$

Therefore, the position of $\tilde{\phi}_1$ depends on the value of ϕ_2. First, given $0 < \phi_2 < \Theta_4/(\Theta_1\pi_2)$, the axis of the graph for $f(\phi_1, \phi_2)$ comes to $\phi_1 > 0$. Then, $f(\phi_1, \phi_2)$ is decreasing in $0 < \phi_1 < \tilde{\phi}_1$ but increasing in $\phi_1 > \tilde{\phi}_1$. Second, given $\Theta_4/(\Theta_1\pi_2) < \phi_2$, the axis of the graph for $f(\phi_1, \phi_2)$ comes to $\phi_1 < 0$, and $f(\phi_1, \phi_2)$ is monotonously increasing in $\phi_1 > 0$.

is established. Hence, there exists at least one positive value of ϕ_1^* such that $f(\phi_1^*, \phi_2) = 0$.

From investigating Equations (47) through (51), I obtain the following proposition with regard to stability, instability, and cycles:

Proposition 1. *Assume a positive fixed value for ϕ_2. Now, there exists at least one positive value ϕ_1^* such that a unique steady state is locally unstable for $0 < \phi_1 < \phi_1^*$ but locally stable for $\phi_1 > \phi_1^*$, and that by a Hopf bifurcation for ϕ_1, a limit cycle occurs sufficiently close to ϕ_1^*.*

Proof. First, Assumption 1 ensures that $b_1 > 0$ and $b_3 > 0$. Second, given a positive value of ϕ_2, the sign of b_2 is positive as long as $\phi_1 > \underline{\phi_1}$. Third, as I have proved above, there exists a positive value of ϕ_1^* such that $f(\phi_1^*, \phi_2) = 0$. If ϕ_1^* is larger than $\underline{\phi_1}$, then there exists ϕ_1^* such that it satisfies $b_1 > 0$, $b_2 > 0$, $b_3 > 0$, and $b_1 b_2 - b_3 = 0$. Thus, a Hopf bifurcation occurs for ϕ_1 sufficiently close to ϕ_1^*.

Then, the existence of Hopf bifurcation can be proved as follows. By substituting $\underline{\phi_1}$ in $f(\phi_1, \phi_2)$ in Equation (48) and arranging, I obtain

$$f(\underline{\phi_1}, \phi_2) = -\frac{\Theta_2}{\Theta_1}(1 - \pi_1) u_1^* z^* \Theta_3. \tag{52}$$

Since Θ_3 is positive, the value of $f(\underline{\phi_1}, \phi_2)$ is obviously negative. When the graph of $f(\phi_1, \phi_2)$ is convex downward in terms of ϕ_1, it means that ϕ_1^* is larger than $\underline{\phi_1}$.

Thus, I obtain the following results: (i) $b_1 > 0$, $b_2 < 0$, $b_3 > 0$, and $b_1 b_2 - b_3 < 0$ within the range $\phi_1 \in (0, \underline{\phi_1})$; (ii) $b_1 > 0$, $b_2 > 0$, $b_3 > 0$, and $b_1 b_2 - b_3 < 0$ within the range $\phi_1 \in (\underline{\phi_1}, \phi_1^*)$; and (iii) $b_1 > 0$, $b_2 > 0$, $b_3 > 0$, and $b_1 b_2 - b_3 > 0$ within the range $\phi_1 > \phi_1^*$. Indeed, at $\phi_1 = \phi_1^*$, I obtain

$$b_1 > 0, \quad b_2 > 0, \quad b_3 > 0, \quad \left.\frac{\partial (b_1 b_2 - b_3)}{\partial \phi_1}\right|_{\phi = \phi_1^*} \neq 0. \tag{53}$$

Consequently, a Hopf bifurcation occurs for ϕ_1 sufficiently close to ϕ_1^*. □

Second, the last condition can be further examined in terms of ϕ_2. I then obtain the following proposition:

Proposition 2. *Suppose the speed of adjustment of the goods market in sector 1 lies within a certain range. Then, there exists at least one positive value ϕ_2^* such that a unique steady state is locally unstable for $0 < \phi_2 < \phi_2^*$, the unique state is locally stable for $\phi_2 > \phi_2^*$, and the limit cycle occurs by a Hopf bifurcation for ϕ_2 sufficiently close to ϕ_2^*.*

Proof. As long as $\phi_1 > \underline{\phi_1}$ is satisfied, $b_1 > 0$, $b_2 > 0$, and $b_3 > 0$ are ensured. I define $g(\phi_1, \phi_2)$ based on Equation (42) as follows:

$$\begin{aligned} b_1 b_2 - b_3 \equiv g(\phi_1, \phi_2) &= \left(\underbrace{\frac{\pi_2}{k}(\Theta_1\phi_1 - \Theta_2)}_{A} \right) \phi_2^2 + \left(\underbrace{(1-(1+k)\gamma_1)\Theta_1\phi_1 - \Theta_4}_{B} \right) \phi_1\phi_2 \\ &= A\phi_2^2 + B\phi_1\phi_2. \end{aligned} \tag{54}$$

When $\phi_1 > \underline{\phi_1}$, the sign of A is positive. Therefore, given $\phi_1 > \underline{\phi_1}$, the graph of $g(\phi_1, \phi_2)$ is convex downward in terms of ϕ_2. In addition, because

$$\frac{\Theta_4}{(1-(1+k)\gamma_1)\Theta_1} > \frac{\Theta_2}{\Theta_1} \equiv \underline{\phi_1}, \tag{55}$$

the sign of B can be positive or negative depending on the value of ϕ_1. Here, I denote $\overline{\phi_1} \equiv \dfrac{\Theta_4}{(1-(1+k)\gamma_1)\Theta_1}$. By a simple calculation, (i) if $\underline{\phi_1} < \phi_1 < \overline{\phi_1}$, then the sign of B is negative. On the other hand, (ii) if $\overline{\phi_1} < \phi_1$, then the sign of B is positive.

By factorizing $g(\phi_1, \phi_2)$ with respect to ϕ_2 and equalizing it to zero, I obtain

$$g(\phi_1, \phi_2) = \phi_2(A\phi_2 + B\phi_1) = 0. \tag{56}$$

Obviously, the solutions that satisfy $g(\phi_1, \phi_2) = 0$ are $\phi_2^* = 0$ and $\phi_2^* = \dfrac{-B\phi_1}{A}$. Since the adjustment speed is positive, $\phi_2^* = 0$ is excluded. In case (i) above, where the sign of B is negative, $\phi_2^* = \dfrac{-B\phi_1}{A}$ is positive. Hence, within the range $\underline{\phi_1} < \phi_1 < \overline{\phi_1}$, there exists a positive value of ϕ_2^* such that $g(\phi_1, \phi_2^*) = 0$.

Suppose the speed of adjustment of the goods market in sector 1 lies within $\underline{\phi_1} < \phi_1 < \overline{\phi_1}$. Then, I obtain the results that $b_1 > 0$, $b_2 > 0$, $b_3 > 0$, and $b_1b_2 - b_3 < 0$ within the range $\phi_2 \in (0, \phi_2^*)$, and $b_1 > 0$, $b_2 > 0$, $b_3 > 0$, and $b_1b_2 - b_3 > 0$ within the range $\phi_2 > \phi_2^*$. Consequently, a Hopf bifurcation occurs at ϕ_2^*. Indeed, at $\phi_2 = \phi_2^*$, I obtain

$$b_1 > 0, \quad b_2 > 0, \quad b_3 > 0, \quad \left.\frac{\partial(b_1b_2 - b_3)}{\partial\phi_2}\right|_{\phi=\phi_2^*} \neq 0. \tag{57}$$

Thus, all the conditions for the existence of the Hopf bifurcation are satisfied. When the speed of adjustment of the goods market in sector 1 lies within $\underline{\phi_1} < \phi_1 < \overline{\phi_1}$, the limit cycle occurs by a Hopf bifurcation for ϕ_2 sufficiently close to ϕ_2^*. □

Although I limited the speed of adjustment of the goods market in sector 1 to within a certain range in order to prove Proposition 2, if the speed goes beyond the range, the parametrical configuration of ϕ_2 that determines stability changes. This result can be presented as a corollary of Proposition 2

Corollary 1 (Corollary of Proposition 2)**.** *Suppose that the speed of adjustment of the goods market in sector 1 is sufficiently large. Then, the steady state is locally stable for any positive value of ϕ_2.*

Proof. By argument (ii) above for the proof of Proposition 2, the sign of B is positive when $\overline{\phi_1} < \phi_1$. For this case, I show that the speed of adjustment of the goods market in sector 1 is sufficiently large. Then, the non-trivial solution for $g(\phi_1, \phi_2) = 0$ is $\phi_2^* = \dfrac{-B\phi_1}{A}$, which is negative. Therefore, within the range $\overline{\phi_1} < \phi_1$, any positive values of ϕ_2 will ensure that $g(\phi_1, \phi_2) > 0$. Consequently, I obtain the results that $b_1 > 0$, $b_2 > 0$, $b_3 > 0$, and $b_1 b_2 - b_3 > 0$ for any positive values of ϕ_2. Thus, the steady state is locally stable. □

I have thus shown the conditions for stability, instability, and the cycle of demand and productivity growth in a two-sector economy, and the cyclical phenomena are confirmed through numerical simulation in the appendix. A numerical study shows that the capacity utilization rate in the consumption goods and investment goods sectors move almost in a synchronized manner. From Proposition 1, Proposition 2, and Corollary 1, a large value for both ϕ_1 and ϕ_2 ensures local stability of the steady state.

However, certain quantitative adjustment speed combinations in an economy can lead to unstable or cyclical dynamics. First, when the quantitative adjustment in sector 1 is comparatively slow, given a positive speed for ϕ_2, the economy suffers from unstable dynamics, as Proposition 1 states. Second, when the quantitative adjustment in sector 1 takes place at a certain speed $\phi_1 \in (\underline{\phi_1}, \overline{\phi_1})$ but the quantitative adjustment in sector 2 is comparatively slow, the economy falls into unstable dynamics, as Proposition 2 states. From Corollary 1, as long as the quantitative adjustment speed in sector 1 is sufficiently fast, the speed in sector 2 does not matter for local stability. Finally, from Propositions 1 and 2, the speed of adjustment in the investment goods sector plays a dominant role in generating business cycles. A necessary and sufficient condition for the emergence of a business cycle in sector 1 is that the speed of quantitative adjustment lie in a certain range. However, in sector 2, it is neither a necessary nor a sufficient condition for the emergence of a business cycle that the speed of quantitative adjustment should lie in a certain range.[8] A cyclical movement in output and productivity growth arises when these two sectors

[8]If the Hopf bifurcation occurs, the value of ϕ_1 must take a certain value, as Propositions 1 and 2 state. On the

produce goods at an intermediate speed, implying that it is necessary to coordinate the quantitative adjustment speed between sectors to prevent potential business cycles. Once cycles begin, the labour productivity growth rates fluctuate sustainably without being equalized. Therefore, in light of the current model, the sectoral labour productivity growth rate differential is evidence of a business cycle.

4 Comparative statics analysis

This section investigates the effects of shifts in income distribution on the capacity utilization rate of each sector at the steady state. I exclusively focus on these impacts of change on the capacity utilization rates and accumulation rate, excluding the impacts on the relative productivity level.[9] The purpose of this study is to reveal which of WLO and PLO is established for the growth regime and under what condition. When a rise in profit share increases (decreases) the capacity utilization rate, the sector is characterized as a PLO (WLO). The mathematical explanations for the impacts are given in the appendix.

Now, note the direct impact of income distribution on the change in capacity utilization rate in each sector. Equation (20) indicates that a rise in profit share in both sectors 1 and 2 positively stimulates the capacity utilization rate in sector 1. Equation (21) indicates that a rise in wage share in sector 2 positively stimulates the capacity utilization rate in sector 2. In other words, sector 1 has a profit-led demand regime, whereas sector 2 has a wage-led demand regime. However, when a change in income distribution spurs labour productivity growth, the outcome cannot be determined by a demand regime only.

Table 1 summarizes the results of comparative statics analysis according to the change in profit share in sector 1 (Part A) and sector 2 (Part B). The outcome that arises will mainly depend on the relative size of the productivity growth ($q'_{i\pi}$) and investment demand (β_i) impacts of change in income distribution, given the other parameters.

contrary, even if ϕ_2 takes the value of ϕ_2^*, the Hopf bifurcation will not exist because of the value of ϕ_1, as Proposition 2 and its corollary state.

[9] The impact of change in these parameters on the relative productivity level z^* can be traced by using Cramer's rule. However, there are several complicated routes for a rise in profit share to lead to both positive and negative changes in z^*. Therefore, it is not worthwhile to investigate all these possibilities in detail.

Table 1: Comparative statics analysis

(A) The impact of change in π_1 on:	Sector 1 (u_1^*)	Sector 2 (u_2^*)	Output growth (g^*)
(A1) $-\frac{q'_{1\pi}}{\beta_1} > \frac{(1+k)q'_{1u}}{(1-(1+k)\gamma_1)}$	PLO	PLO	PLG
(A2) $\frac{q'_{2u}}{\gamma_2} < -\frac{q'_{1\pi}}{\beta_1} < \frac{(1+k)q'_{1u}}{(1-(1+k)\gamma_1)}$	PLO	WLO	PLG
(A3) $-\frac{q'_{1\pi}}{\beta_1} < \frac{q'_{2u}}{\gamma_2}$	WLO	WLO	WLG
(B) The impact of change in π_2 on:	**Sector 1 (u_1^*)**	**Sector 2 (u_2^*)**	**Output growth (g^*)**
(B1) $-\frac{q'_{2\pi}}{\beta_2} > -\frac{q'_{2u}}{\gamma_2}$	WLO	WLO	WLG
(B2) $-\frac{(1+k)q'_{1u}}{1-(1+k)\gamma_1} < -\frac{q'_{2\pi}}{\beta_2} < -\frac{q'_{2u}}{\gamma_2}$	PLO	WLO	PLG
(B3) $-\frac{q'_{2\pi}}{\beta_2} < -\frac{(1+k)q'_{1u}}{1-(1+k)\gamma_1}$	PLO	PLO	PLO

Note: If a sector has profit-led productivity growth regime, the sign of $q'_{i\pi}$ is positive. If a sector has wage-led productivity growth regime, the sign of $q'_{i\pi}$ is negative.

In part (A), in case of a rise in profit share in sector 1 under a profit-led productivity growth regime ($q'_{1\pi} > 0$), the only possible case is A3. It necessarily decreases the capacity utilization rate of both sectors 1 and 2. That is, the WLO is realized in both sectors. However, if sector 1 involves a wage-led productivity growth regime ($q'_{1\pi} < 0$), there could be three sectoral capacity utilization rate outcomes. If the productivity growth effect of the profit share is strong but its effect on the investment is weak (i.e. a large absolute value of $\frac{q'_{1\pi}}{\beta_1}$), then both sectors exhibit PLO (case A1). On the contrary, if the former is weak but the latter is strong (i.e. a small absolute value of $\frac{q'_{1\pi}}{\beta_1}$), then both sectors exhibit WLO (case A3). The interesting case is A2, where the capacity utilization rate of each sector responds differently to a rise in profit share. If the relative impact of income distribution on productivity growth and investment demand is intermediate, then sector 1 exhibits a PLO regime whereas sector 2 presents a WLO regime. Thus, the impact of income distribution on the economy is hybrid.[10]

[10] A case in which sector 1 exhibits WLO and sector 2 presents PLO does not arise, because the condition for this case violates the stability condition examined in Section 3. This is true also for a rise in sector 2's profit share, as examined below.

A lengthy explanation may be needed to answer why there are three outcomes under sector 1's wage-led productivity regime. A rise in sector 1's profit share first stimulates the investment demand in sector 1, and this initially increases sector 1's capacity utilization rate. The magnitude depends on the profit effect on the investment demand (β_1). A rise in capacity utilization rate in sector 1 raises its labour productivity growth rate through the Kaldor–Verdoon effect, decreasing the relative labour productivity level as well as the relative price level. Furthermore, a rise in profit share decelerates sector 1's labour productivity growth in the wage-led productivity regime. The magnitude of this depends on the profit effect on productivity growth ($|q'_{1\pi}|$). An increase in the relative productivity level raises the relative price level. Thus, a rise in sector 1's profit share *ceteris paribus* either decreases or increases the relative price level p depending on the size of β_1 and $|q'_{1\pi}|$. A rise (fall) in relative price means an increase (decrease) in the real income of sector 1's workers measured by the consumption good prices. An increase (a decrease) in this real income directly changes the effective demand for consumption goods, because the marginal propensity to consume is unity.

How a change in sector 1's profit share affects the capacity utilization rate in each sector depends on the change in real income from the variation in the relative labour productivity and price level. First, if the profit effect on the productivity growth is strong (i.e. large $|q'_{1\pi}|$) but its impact on the investment demand is weak (i.e. small β_1), there will be a significant rise in the real income of sector 1's workers. Because the effective demand for sector 2's goods would show a large increase, the capacity utilization rates of both sector 1 and sector 2 would rise. In this case, following the rise in profit share in sector 1, there would be an expansion of the capacity utilization rates in both sectors (Case A1). When the relative profit effect on the productivity growth is weak (i.e. small $|q'_{1\pi}|$) but its impact on the investment demand is strong (i.e. large β_1), there would be a fall in real income for sector 1's workers. Thus, the effective demand of sector 1's workers for consumption goods would decrease. Second, when this effect is modest (i.e. intermediate $|q'_{1\pi}|/\beta_1$), although the capacity utilization rate in sector 1 is still sustained by the initial rise in investment demand, the capacity utilization rate in sector 2 will decrease from the fall in real income. Thus, a different impact of the profit share arises on the sectoral capacity utilization rate in an economy, where sector 1 experiences PLO and sector 2 experiences WLO (Case A2). Third, a substantial fall in the real income of sector 1's workers (i.e. small $|q'_{1\pi}|/\beta_1$) leads to a fall in their demand for consumption goods, and this leads to fall in the capacity utilization rate in sector 1. In this case, the economy will experience a lower capacity utilization rate in both sectors following a rise in sector 1's profit share (Case 3).

In the steady state, because the capacity utilization rates in both sectors are constant, the output growth rates in both sectors will be the same as the capital accumulation rate. The impact of a change in income distribution on output growth can also be examined, and this will be classified as a WLG or PLG regime. By differentiating Equation (13) considering sector 1's profit share and summarizing the related terms, the economy is found to exhibit the PLG regime in the A1 and A2 cases. On the other hand, the economy exhibits the WLG regime in the A3 case.

In a similar manner, the impacts of a rise in profit share in sector 2 are summarized in case (B) of Table 1. When sector 2 establishes a wage-led productivity growth regime ($q'_{2\pi} < 0$), the capacity utilization rate of both sector 1 and sector 2 necessarily decreases. That is, the WLO regime is realized in both sectors (case B1). In contrast, when sector 2 shows a profit-led productivity growth regime ($q'_{2\pi} > 0$), there are three different configurations for the sectoral capacity utilization rate. If the productivity growth effect of the profit share is strong, both sectors exhibit the PLO regime (case B3). On the contrary, if the effect is weak, then both sectors exhibit the WLO regime (case B1). Case B2 is a hybrid economy, where the capacity utilization rate in each sector shows a different response to a rise in sector 2's profit share. If the profit share's productivity growth effect is intermediate, sector 1 will exhibit the PLO regime whereas sector 2 will present the WLO regime. A case in which sector 1 exhibits WLO and sector 2 presents PLO does not arise.[11]

The same earlier exercise identifies which of the WLG and PLG regime is established under what conditions. The economy exhibits the WLG regime in case B1. On the other hand, the economy exhibits the PLG regime in cases B2 and B3.

A comparative statics analysis presents two important implications for the Kaleckian growth and distribution analyses. The first is that the income distribution impact on the capacity utiliza-

[11]The basic mechanisms by which a rise in profit share in sector 2 leads to the three outcomes under sector 2's profit-led productivity regime are related to (i) a fall in sector 2's wage share, (ii) a fall in sector 1 workers' real income from a change in distributional ratio, and (iii) a rise in sector 1 workers' real income from a change in relative productivity level. By almost the same token on the discussion in case A, depending on the relative strength of (i), (ii), and (iii), three outcomes arise in this productivity growth regime. I do not explain them here to avoid a lengthy argument again.

tion and growth rates hinges on the demand side as well as supply side parameters. In a standard Kaleckian macro model, the establishment of a WLG or PLG regime crucially depends on the profit share's relative impact on the investment and consumption demand (Bhaduri and Marglin (1990); Lavoie and Stockhammer (2013)). In contrast, the two-sector model in the current study has a more complicated configuration. A change in capacity utilization rate is the outcome of distributional impacts on the demand and productivity growth in each sector as well as on their interactions. In this vein, the income-led demand as well as productivity determinations should be taken into account to find the income distribution impact on the capacity utilization rate and economic growth.[12]

The second is that the impact of income distribution between sectors on the capacity utilization rate is not always unique. When the ratio of productivity growth and investment demand effect of income distribution lies within a certain range, the impact of income distribution may differ from sector to sector. That is, each sector's capacity utilization rate may move in a direction opposite to the same distributional shock. When each sector responds differently to a change in income distribution in a sector, the impact of a change in income distribution on the aggregate capacity utilization rate necessarily conflicts with at least that in one of the two sectors. That is, there is a fallacy of composition between industry-level and macro-level performance.[13] Thus,

[12] Fujita (2015) reveals a hybrid income distribution impact on the sectoral capacity utilization rates. The mechanism in his paper relies on the existence of intermediate goods causing the relative price effect. In contrast, the current paper reveals the hybrid impact in terms of change in labour productivity growth rate.

[13] In cases A2 and B2, sector 1 involves the PLO regime but sector 2 involves the WLO regime. The aggregate capacity utilization rate u_A, which can be defined as

$$u_A = \frac{X_1 + X_2}{K_1 + K_2} = \frac{1}{1+k}u_1 + \frac{k}{1+k}u_2$$

is affected by either the profit-led or wage-led pattern. Therefore, at least one of the sectoral outcomes is necessarily different from the determined pattern of the aggregate capacity utilization rate. In the current model, the impact of a change in sector 1's profit share on the aggregate capacity utilization rate is

$$\frac{\partial u_A}{\partial \pi_1} = -\frac{1}{\Theta_3}\left[(\beta_1 + k\beta_2)q'_{2u} + \gamma_2(q'_{1\pi} - kq'_{2\pi})\right].$$

Therefore, if $\frac{kq'_{2\pi} - q'_{1\pi}}{\beta_1 + k\beta_2} > \frac{q'_{2u}}{\gamma_2}$, the aggregate capacity utilization rate is determined on profit-led considerations. In contrast, if $\frac{kq'_{2\pi} - q'_{1\pi}}{\beta_1 + k\beta_2} < \frac{q'_{2u}}{\gamma_2}$, the aggregate capacity utilization rate is determined on wage-led considerations.

Similarly, the impact of a change in sector 2's profit share on the aggregate capacity utilization rate is

$$\frac{\partial u_A}{\partial \pi_2} = -\frac{1}{\Theta_3}\left[(1+k)(\beta_1 + k\beta_2)q'_{1u} + (1 - (1+k)\gamma_1)(q'_{1\pi} - kq'_{2\pi})\right].$$

the effectiveness of the income policy becomes more complicated because a certain change in income distribution is not always beneficial for an individual industry. A rise in profit share may increase the capacity utilization rate of one sector and the aggregate rate but may decrease that of the other sector. A two-sector model can elucidate such a sectoral conflict with regard to the impact of income distribution, which cannot be explored by an aggregate growth model.

5 Conclusion

This paper analysed the dynamics of demand and labour productivity growth, and the income distribution impacts on them in a two-sector economy. The model incorporated endogenous productivity growth determination, which enhances supply side analyses; this has not been explored much in Kaleckian demand-led growth models. The model has a feature that generates the emergence of cyclical fluctuation in demand and productivity growth and a variety of distributional impacts on the capacity utilization rates, including the hybrid outcome. These points are summarized as a conclusion.

This paper analysed the stability conditions, mainly considering the speed of quantitative adjustment in each sector. As long as the adjustment is fast, local stability of the steady state can be ensured. However, when an economy involves certain quantitative adjustment speed combinations, there could be instability or a cycle even if the quantitative adjustment is self-stable. The existence of a limit cycle is proved by the Hopf bifurcation. A stability analysis reveals that the adjustment speed of the investment goods sector plays a dominant role in generating business cycles. This may cause a cyclical behaviour regardless of the adjustment speed of the consumption goods sector. Cyclical movement in output and productivity growth also emerges when two sectors produce goods at an intermediate speed. The sectoral coordination of the quantitative adjustment speed is required to prevent potential business cycles.

A comparative statics analysis showed three types of outcomes in an economy. An economy may have a case in which both sectors realize PLO or WLO, or a hybrid case in which sector 1 realizes the PLO regime but sector 2 realizes the WLO regime. A Kaleckian model explained the

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Therefore, if $\frac{kq'_{2\pi} - q'_{1\pi}}{\beta_1 + k\beta_2} < \frac{(1+k)q'_{1u}}{1-(1+k)\gamma_1}$, the aggregate capacity utilization rate is determined on profit-led considerations. In contrast, if $\frac{kq'_{2\pi} - q'_{1\pi}}{\beta_1 + k\beta_2} > \frac{(1+k)q'_{1u}}{1-(1+k)\gamma_1}$, the aggregate capacity utilization rate is determined on wage-led considerations.

impact of the income distribution on the capacity utilization rate and growth rate considering the demand side parameters. In addition, the current model indicates that the impact also depends on supply-side parameters, such as the wage-led or profit-led productivity regime. The sectoral and macroeconomic outcome of a change in income distribution in a sector is a complex effect of both demand- and supply-side effects.

The existence of a hybrid outcome should be especially emphasized, because it means that the economy involves uneven industrial expansion with regard to the impact of income distribution. In such a case, a very puzzling question that one faces is which (wage or profit share) and where (sector 1 or 2) to change in order to expand the economic activity level in terms of economic growth and fairness of functional income distribution. Besides, when both sectors respond differently to a change in income distribution in a sector, each sector's capacity utilization rate moves in the opposite direction, implying that its macroeconomic capacity utilization rate may conflict with that of the other sector with regard to a change in income distribution. Even if the output in a sector is expanding in a wage-led manner, it does not necessarily mean that the aggregate output expansion follows the same manner. Thus, the current two-sector model sheds light on the possibility of the fallacy of composition between industry-level and macro-level performance, which cannot be observed by an aggregate model.

Appendix A: Numerical study

Using numerical simulations, Appendix A shows that the Hopf bifurcation with regard to ϕ_1 and ϕ_2 actually exists. The approach here is qualitative and the purpose is to show how the two-sector Kaleckian model behaves cyclically, which Propositions 1 and 2 state. The basic parameters are as follows:

$$\alpha = 0.01, \quad \beta_1 = 0.01, \quad \beta_2 = 0.01, \quad \gamma_1 = 0.8, \quad \gamma_2 = 0.02, \quad \pi_1 = 0.2, \quad \pi_2 = 0.2,$$
$$\theta_1 = 0.010, \quad \theta_2 = 0.0325, \quad \delta_1 = 0.025, \quad \delta_2 = 0.0001, \quad \eta_1 = 0.0025, \quad \eta_2 = 0.0025, \quad k = 0.2.$$

Using these parameters, I define the function of productivity growth rate $\hat{a}_1 = \theta_1 + \eta_1\pi_1 + \delta_1 u_1$ and $\hat{a}_2 = \theta_2 + \eta_2\pi_2 + \delta_2 u_2$. In this numerical example, the parameters are set to satisfy assumption 1. In addition, they also satisfy the Hopf bifurcation conditions. In solving the differential equation systems, the initial conditions of the capacity utilization rates and the relative productivity level are $u_1(0) = 0.9$, $u_2(0) = 0.8$, and $z(0) = 0.04$, respectively. Using these parameters, the steady-state values of the endogenous variables are $u_1^* = 0.903221$, $u_2^* = 0.805369$, and $z^* = 0.0445831$, respectively.

First, I consider a cyclical phenomenon given by Proposition 1. Then, I set $\phi_2 = 0.01$; these parameters satisfy $b_1 > 0$, $b_2 > 0$, and $b_3 > 0$. I obtain the positive bifurcation parameter $\phi_1^* = 0.599849$; this is larger than $\underline{\phi_1} = 0.00432992$. Using $\phi_1 = 0.6$, which is in the neighbourhood of ϕ_1^*, the dynamic behaviour of the endogenous variables is presented in Figure 1.[14]

Second, I derive a cyclical phenomenon based on Proposition 2. In addition to the above parameters, I set $\phi_1 = 0.1$ as given; these parameters satisfy $b_1 > 0$, $b_2 > 0$, and $b_3 > 0$. I thus obtain the positive bifurcation parameter $\phi_2^* = 0.114871$. Using $\phi_2 = 0.115$, which is in the neighbourhood of ϕ_2^*, the dynamic behaviour of the endogenous variables is presented in Figure 2.

Thus, I numerically confirm that a Hopf bifurcation actually generates a periodic orbit in the two-sector model. Both Figures 1 and 2 present a similar configuration regarding the behaviour of capacity utilization rates and labour productivity level. In both case, I find that the capacity utilization rates in the consumption goods and investment goods sectors change almost in a synchronized manner. The dynamics of the effective demand in the course of a cycle basically consists of two phases, one where there is a cumulative fall in both sectors' capacity utilization rates, and the other where there is a cumulative rise in the rates.

Appendix B: Mathematics for comparative statics analysis

The steady-state values of the capacity utilization rates and relative productivity level satisfy equations (23), (24), and (25). By totally differentiating these variables with respect to profit shares π_1 and π_2, and arranging the result in a vector and matrix form, I obtain

$$\underbrace{\begin{pmatrix} -(1-(1+k)\gamma_1) & (1+k)\gamma_2 & 0 \\ \left(\frac{1-\pi_2}{k}\right) z^* & -\pi_2 & \left(\frac{1-\pi_2}{k}\right) u_1^* \\ -q'_{1u} & q'_{2u} & 0 \end{pmatrix}}_{\boldsymbol{J}_C} \begin{pmatrix} du_1^* \\ du_2^* \\ dz^* \end{pmatrix} = \begin{pmatrix} -(1+k)\beta_1 \\ 0 \\ q'_{1\pi} \end{pmatrix} d\pi_1 + \begin{pmatrix} -(1+k)\beta_2 \\ \frac{z^*}{k} u_1^* + u_2^* \\ -q'_{2\pi} \end{pmatrix} d\pi_2. \qquad (58)$$

[14] The solution path is from $t = 50$ to $t = 3000$ for both Figures 1 and 2. Further calculations over this period show that the path is simply asymptotically close to a closed orbit.

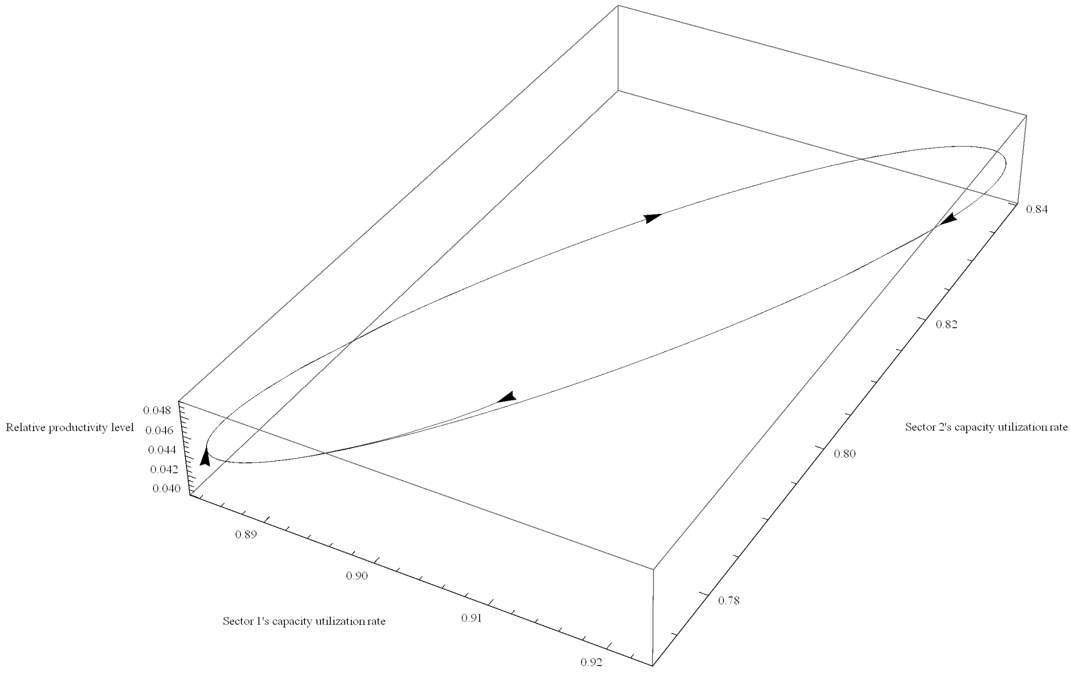

Figure 1: Behaviour of capacity utilization rates and the relative productivity level ($\phi_1 = 0.6$)

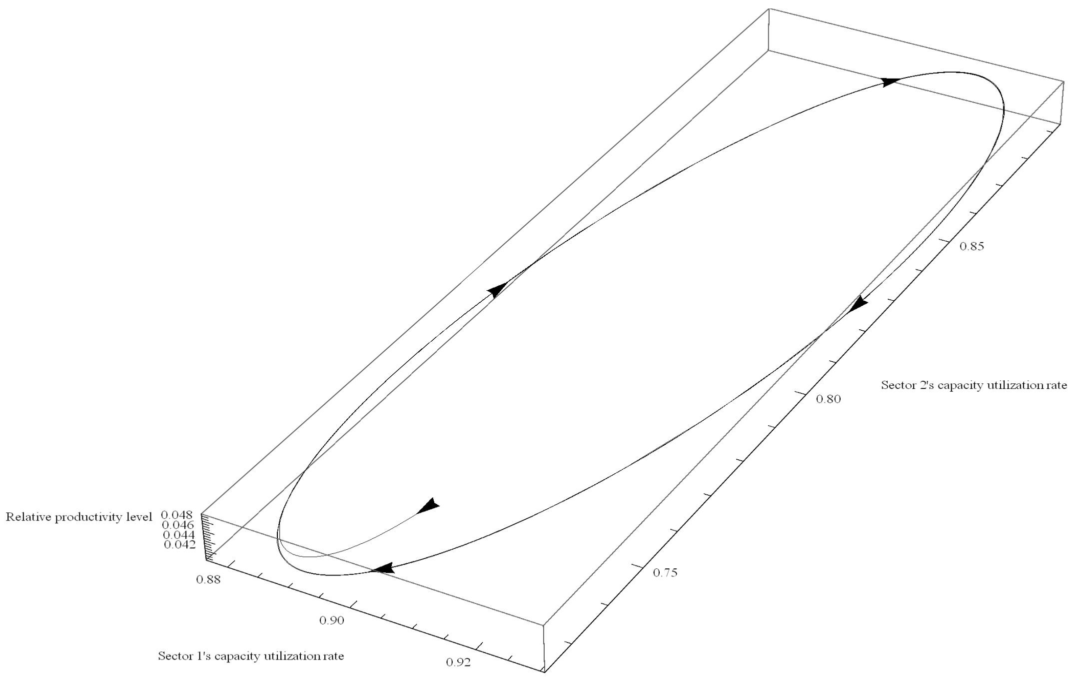

Figure 2: Behaviour of capacity utilization rates and the relative productivity level ($\phi_2 = 0.115$)

The determinant of the matrix $\boldsymbol{J_C}$ in the LHS is defined as follows:

$$\det \boldsymbol{J_C} = -\left(\frac{1-\pi_2}{k}\right) u_1^* \Theta_3 < 0. \tag{59}$$

A comparative statics analysis is conducted in the stable case. When the equilibrium of system is locally stable, the Jacobian matrix determinant is negative, implying that the sign of $\det \boldsymbol{J_C}$ is negative.

From Cramer's rule, the effect of a rise in sector 1's profit share on the steady-state values of capacity utilization rates is as follows.

$$\frac{du_1^*}{d\pi_1} = -\left(\frac{1+k}{\Theta_3}\right)(\gamma_2 q'_{1\pi} + \beta_1 q'_{2u}), \tag{60}$$

$$\frac{du_2^*}{d\pi_1} = -\frac{1}{\Theta_3}[(1+k)\beta_1 q'_{1u} + (1-(1+k)\gamma_1)q'_{1\pi}]. \tag{61}$$

The impact of a rise in sector 1 痴 profit share depends on the productivity growth regime $q'_{1\pi}$.

- If the productivity growth is stimulated by profit share (i.e. profit-led productivity regime: $q'_{1\pi} > 0$), it is obvious from Equation (60) that a rise in profit share in sector 1 necessarily decreases the capacity utilization rates in both sectors. That is, $du_1^*/d\pi_1 < 0$ and $du_2^*/d\pi_1 < 0$, and the WLO is realized in sectors 1 and 2.

- If the productivity growth is stimulated by wage share (i.e. wage-led productivity regime: $q'_{1\pi} < 0$), following cases would arise:

  1. If $-\dfrac{q'_{1\pi}}{\beta_1} > \dfrac{(1+k)q'_{1u}}{1-(1+k)\gamma_1}$, a rise in profit share in sector 1 will increase the capacity utilization rates in both sectors. That is, $du_1^*/d\pi_1 > 0$ and $du_2^*/d\pi_1 > 0$, and the PLO is realized in sectors 1 and 2.
  2. If $\dfrac{q'_{2u}}{\gamma_2} < -\dfrac{q'_{1\pi}}{\beta_1} < \dfrac{(1+k)q'_{1u}}{1-(1+k)\gamma_1}$, a rise in profit share in sector 1 will increase the capacity utilization rate in sector 1, whereas it will decrease the capacity utilization rate in sector 2. That is, $du_1^*/d\pi_1 > 0$ and $du_2^*/d\pi_1 < 0$, and the PLO is realized in sector 1, but the WLO is realized in sector 2.
  3. If $-\dfrac{q'_{1\pi}}{\beta_1} < \dfrac{q'_{2u}}{\gamma_2}$, a rise in profit share in sector 1 will decrease the capacity utilization rates in both sectors. That is, $du_1^*/d\pi_1 < 0$ and $du_2^*/d\pi_1 < 0$, and the WLO is realized in sectors 1 and 2.

In deriving these conditions, $\left|\frac{(1+k)q'_{1u}}{1-(1+k)\gamma_1}\right| > \left|\frac{q'_{2u}}{\gamma_2}\right|$ is established under the assumption that $\Theta_3 > 0$. Then, WLO will not be realized in sector 1 and PLO will not be realized in sector 2 because that would contradict $\Theta_3 > 0$.

When the capacity utilization rates are in the steady state, the rate of output expansion in both sectors will be equal to the capital accumulation rate at the steady state. Moreover, a rise in profit share will change the capacity utilization rates through Equations (60) and (61), with further impacts on the output growth rate. I also investigate the impacts of a rise in profit share in sector 1 on the output growth rate. By substituting them into differentiated Equation (13) with respect to π_1 and summarizing, the impact of a rise in sector 1's profit share becomes as follows:

$$\frac{\partial g^*}{\partial \pi_1} = -\frac{1}{\Theta_3}(\gamma_2 q'_{1\pi} + \beta_1 q'_{2u}). \tag{62}$$

From Equation (62), the two types of growth regimes can be discriminated. If the sign of $\frac{\partial g^*}{\partial \pi_1}$ is positive, the economy involves the PLG regime, whereas if the sign is negative, the economy involves the WLG regime. By elaborating Equation (62), the corresponding condition for them becomes reduced to the following inequality:

$$\frac{\partial g^*}{\partial \pi_1} \gtrless 0 \quad \Longleftrightarrow \quad -\frac{q'_{1\pi}}{\beta_1} \gtrless \frac{q'_{2u}}{\gamma_2}. \tag{63}$$

By combining the arguments above and the result in Equations (60) and (61), the impacts of a rise in sector 1's profit share on the capacity utilization rates and the output growth rate are summarized as in Table 1.

By the same token, the effect of a rise in sector 2's profit share on the steady-state values of capacity utilization rates are as follows.

$$\frac{du_1^*}{d\pi_2} = \left(\frac{1+k}{\Theta_3}\right)(\gamma_2 q'_{2\pi} - \beta_2 q'_{2u}), \tag{64}$$

$$\frac{du_2^*}{d\pi_2} = -\frac{1}{\Theta_3}\left[(1+k)\beta_2 q'_{1u} - (1-(1+k)\gamma_1)q'_{2\pi}\right]. \tag{65}$$

The impact of a rise in profit share in sector 2 depends on the productivity growth regime $q'_{2\pi}$.

- If the productivity growth in sector 2 is stimulated by its wage share (i.e. wage-led productivity regime: $q'_{2\pi} < 0$), it is obvious from Equation (64) that a rise in profit share in sector 1 necessarily decreases the capacity utilization rates in both sectors. That is, $du_1^*/d\pi_2 < 0$ and $du_2^*/d\pi_2 < 0$, and the WLO will be realized in sectors 1 and 2.

- If the productivity growth is stimulated by the profit share (i.e. profit-led productivity regime: $q'_{2\pi} > 0$), the following cases will arise:

  1. If $-\dfrac{q'_{2\pi}}{\beta_2} > -\dfrac{q'_{2u}}{\gamma_2}$, a rise in profit share in sector 2 will decrease the capacity utilization rates in both sectors. That is, $du_1^*/d\pi_2 < 0$ and $du_2^*/d\pi_2 < 0$, and the WLO will be realized in sectors 1 and 2.
  2. If $-\dfrac{(1+k)q'_{1u}}{1-(1+k)\gamma_1} < -\dfrac{q'_{2\pi}}{\beta_2} < -\dfrac{q'_{2u}}{\gamma_2}$, a rise in profit share in sector 1 will increase the capacity utilization rate in sector 1, whereas it will decrease the capacity utilization rate in sector 2. That is, $du_1^*/d\pi_2 > 0$ and $du_2^*/d\pi_2 < 0$, and the PLO will be realized in sector 1, and the WLO will be realized in 2.
  3. If $-\dfrac{q'_{2\pi}}{\beta_2} < -\dfrac{(1+k)q'_{1u}}{1-(1+k)\gamma_1}$, a rise in profit share in sector 2 will increase the capacity utilization rates in both sectors. That is, $du_1^*/d\pi_2 > 0$ and $du_2^*/d\pi_2 > 0$, and the PLO will be realized in sectors 1 and 2.

Note that the WLO will not be realized in sector 1 and the PLO will not be realized in sector 2 for the same reason explained above.

A rise in sector 2's profit share will change the capacity utilization rates through Equations (64) and (65), with further impacts on the output growth rate. I then investigate the impacts of a rise in profit share in sector 2 on the output growth rate. By substituting them into differentiated Equation (13) with respect to π_2 and summarizing, the impact of a rise in sector 2's profit share becomes as follows:

$$\frac{\partial g^*}{\partial \pi_2} = -\frac{1}{\Theta_3}(\beta_2 q'_{2u} - \gamma_2 q'_{2\pi}). \tag{66}$$

From Equation (66), both types of growth regimes are discriminated. If the sign of $\dfrac{\partial g^*}{\partial \pi_2}$ is positive, the economy involves the PLG regime, whereas if the sign is negative, the economy involves the WLG regime. By elaborating Equation (66), the corresponding condition for these will be as follows:

$$\frac{\partial g_1^*}{\partial \pi_2} \gtrless 0 \quad \Longleftrightarrow \quad -\frac{q'_{2\pi}}{\beta_2} \lessgtr -\frac{q'_{2u}}{\gamma_2}. \tag{67}$$

By combining the arguments in Equations (64) and (65) and the result in Equation (67), the impacts of a rise in sector 2's profit share on the capacity utilization rates and output growth rate are summarized as Table 1.

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