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# Multiple Long-Run Equilibria in a Free-Entry Mixed Oligopoly\*

Junichi Haraguchi<sup>†</sup> and Toshihiro Matsumura<sup>‡</sup>

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## Abstract

We investigate a free-entry mixed oligopoly with constant marginal costs. A privatization policy is implemented after private firms enter the market. We find that both full privatization and full nationalization are equilibrium policies, and the former is the worst privatization policy for welfare.

**JEL classification numbers:** D43, H44, L33

**Key words:** entry-then-privatization, constant marginal costs, profit-enhancing entry, two polar equilibrium privatization policies

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## Highlights

A free entry mixed oligopoly with constant marginal costs is examined.

The government chooses the degree of privatization after private firms' entries.

Firms' profits can increase with the new entry of a private firm.

Two polar policies, full privatization and nationalization, are equilibrium policies.

Full privatization is the worst policy for welfare.

# 1 Introduction

For more than 50 years, we observe a worldwide wave of privatization of state-owned public enterprises. Nevertheless, many public enterprises with significant government ownership remain active in strategic sectors and control large portions of the world's resources. According to an OECD report by Kowalski *et al.* (2013), over 10% of the 2,000 largest companies are public enterprises, with sales equivalent to approximately 6% of worldwide GDP. They are significant players in sectors such as transportation, telecommunications, energy, and finance in OECD countries. In planned and transitional countries, the presence of public enterprises is of further significance.

One classical rationale for public enterprises is to prevent private monopolies in natural monopolies where significant economies of scale are prevalent. Thus, many public enterprises existed or still exist in such national monopoly markets.<sup>1</sup> However, due to technological improvement, many markets that contain public enterprises are not always characterized by significant economies of scale. Indeed, a considerable number of public enterprises compete with private enterprises in a wide range of industries (mixed oligopolies).<sup>2</sup> The optimal privatization policies in these mixed oligopolies attracted extensive attention from economics researchers in such fields as industrial organization, public economics, financial economics, and development economics.<sup>3</sup>

Recent deregulation and liberalization significantly weakened entry restrictions in mixed oligopolies. Thus, private enterprises recently entered many mixed oligopolies such in the banking, insurance, telecommunications, energy, and transportation industries. The literature on mixed oligopolies contains many studies on optimal privatization policy in free entry markets. Matsumura and Kanda (2005) adopt Matsumura's (1998) partial privatization approach and show that the optimal degree of privatization is zero when private competitors are domestic, while Cato and Matsumura (2012) show that it is strictly positive when private competitors are foreign and that this is increasing

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<sup>1</sup>Nippon Telecom and Telecommunication (NTT) and Kokusai Denshin Denwa (KDD) are typical examples of public monopolists. These firms were monopolies until 1985.

<sup>2</sup>Examples include the United States Postal Service, Deutsche Post AG, Areva, NTT, Japan Tobacco (JT), Volkswagen, Renault, Electricite de France, Japan Postal Bank, Kampo, Korea Development Bank, and the Korea Investment Corporation.

<sup>3</sup>For examples of mixed oligopolies and recent developments in this field, see Colombo (2016), Chen (2017), Ishida and Matsushima (2009), and the works cited therein.

in the foreign ownership share in private firms. Chen (2017) revisits the problem by introducing the cost-reducing effect of privatization. Fujiwara (2007) find a non-monotonic (monotonic) relationship between the degree of product differentiation and optimal degree of privatization in a non-free-entry (free-entry) market. Cato and Matsumura (2015) discuss the relationship between optimal trade and privatization policies and show that a higher tariff rate reduces the optimal degree of privatization, and that the optimal tariff rate can be negative. Cato and Matsumura (2013) demonstrate the privatization neutrality theorem.<sup>4</sup>

These studies assume that the government chooses its privatization policies before private enterprises enter the market (privatization-then-entry model). Lee *et al.* (2018) adopt an alternative time line by formulating a model in which the government implements a privatization policy after private firms enter the market (entry-then-privatization model) and compare the equilibrium privatization policy with that of the privatization-then-entry model. They find that the entry-then-privatization model yields inefficient partial privatization. The privatization-then-entry model implicitly assumes that the government can commit to a privatization policy (can commit not to changing the privatization policy after observing private firms' entry). However, the commitment not to change the privatization policy over time is difficult (Sato and Matsumura, 2018). Therefore, the entry-then-privatization model is more realistic when the government cannot commit to maintaining the privatization policy.<sup>5</sup>

All of these studies assume increasing marginal costs. In the privatization-then-entry model, if we assume constant marginal costs, either the public monopoly in which no private firm enters the market, or the private oligopoly where only new entrants are active and the public (or privatized) firm's output is zero appear in equilibrium. In other words, there is no room to discuss mixed oligopolies under the assumption of constant marginal costs in the privatization-then-entry model.

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<sup>4</sup>White (1996) shows this theorem in his duopoly model.

<sup>5</sup>In the JT case, the Japanese government committed to holding a two-thirds share in JT until 2012 in its legislation; however, the law changed to commit to a one-third holding. The public ownership shares often changed over time. The Japanese government continued to sell shares in NTT, which was a state-owned public monopolist until 1985, from 1986 to 2016. In 2015, the Japanese government sold a minor share in the Japan Postal Bank, the largest bank in Japan, and Kamפו, a major life insurance company. The French government increased its ownership in Renault from 15% to 19.4% in 2015.

However, this is not true in the entry-then-privatization model.

In this study, we investigate a free entry mixed market with constant marginal costs. We show that even under the condition that the privatization-then-entry model yields a public monopoly, the entry-then-privatization yields multiple equilibria. Both full privatization and full nationalization appear as locally stable equilibria, and the former yields the lowest level of welfare among all (equilibrium and non-equilibrium) privatization policies. This result suggests that a non-committed flexible privatization policy may yield destructive results.

Our multiple equilibrium result is in sharp contrast to the studies mentioned above, because their models include a unique equilibrium. Our result implies that if private firms expect that the government will fully privatize (nationalize) public firms, private firms enter (do not enter) the market, and after observing private firms' entering (not entering), the government in fact fully privatizes (nationalizes) public firms. Therefore, both situations are self-fulfilling.

## 2 The Model

We consider a mixed oligopoly model in which one public firm (firm 0) competes with  $n$  private firms (firms 1, 2, ...,  $n$ ). These firms produce homogeneous products for which the inverse demand function is

$$p(Q) = a - Q,$$

where  $p$  denotes price,  $a$  is a positive constant, and  $Q := \sum_{i=0}^n q_i$  is the total output. We assume that all private firms have an identical cost function and marginal costs are constant. Let  $c_0$  be firm 0's marginal cost and  $c$  be the private firm's marginal cost. We assume that  $c < c_0$ ; that is, the public firm is less efficient than the private firm.<sup>6</sup> Let  $q_i$  be firm  $i$ 's output. When the private firm enters the market, it incurs an entry cost of  $F$ .

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<sup>6</sup>The assumptions of linear demand and constant marginal costs with the cost disadvantage for a public firm over private firms is popular in the literature on mixed oligopolies. See Pal (1998), Capuano and De Feo (2010), and Matsumura and Ogawa (2010). For a discussion on the endogenous cost disadvantage of public firms, see Matsumura and Matsushima (2004). If  $c \geq c_0$ , no private firm enters the market in equilibrium.

The social surplus  $W$  is

$$W = \int_0^Q p(q) dq - pQ + \sum_{i=0}^n \pi_i = \int_0^Q p(q) dq - c_0 q_0 - \sum_{i=1}^n c q_i - nF.$$

Following Matsumura (1998), the public firm's objective  $\Omega$  is a convex-combination of social surplus and their own profit,  $\Omega = \alpha\pi_0 + (1 - \alpha)W$ .  $\alpha \in [0, 1]$  represents the degree of privatization. In the case of full nationalization (i.e.,  $\alpha = 0$ ), firm 0 maximizes social welfare. In the case of full privatization (i.e.,  $\alpha = 1$ ), firm 0 maximizes its profit. Each private firm's objective is its profit.

The complete information game runs as follows. In the first stage, each firm chooses whether to enter the market. In the second stage, after observing  $n$ , the government chooses the degree of privatization  $\alpha$  to maximize social surplus. In the third stage, each firm simultaneously chooses its output to maximize its objective. We solve this game by backward induction and the equilibrium concept is the subgame perfect Nash equilibrium.

Throughout this study, we assume that  $c_0 < c + \sqrt{F}$ . Otherwise,  $q_0 = 0$  in equilibrium, regardless of  $\alpha$ , and thus, any  $\alpha$  is optimal (because  $\alpha$  never affects equilibrium outcomes).

### 3 Equilibrium

First, we solve the third stage game given  $\alpha$ . The first order-condition for each private firm is

$$p + p'q - c = 0. \tag{1}$$

The second-order condition is satisfied.

If  $n \geq \tilde{n} := (a - c_0)/(c_0 - c)$ , the solution to the third stage game is corner. That is,  $q_0 = 0$  regardless of  $\alpha$ , and the equilibrium outcome is characterized as a private oligopoly. The output of each private firm is  $(a - c)/(n + 1)$  and the equilibrium price is  $(a + nc)/(n + 1)$ . Note that  $n \geq \tilde{n}$  implies  $(a + nc)/(n + 1) \leq c_0$ . In this case, any degree of privatization is optimal because  $\alpha$  does not affect welfare. Henceforth, we restrict our attention to the case in which  $n < \tilde{n}$ . We can show that  $n < \tilde{n}$  in the free-entry equilibrium assuming  $c_0 < c + \sqrt{F}$ .

If  $n \in (0, \tilde{n})$ , the solution is interior. The first-order condition of the public firms is

$$p + \alpha p'q_0 - c_0 = 0. \tag{2}$$

The second-order condition is satisfied. These first-order conditions yield the following equilibrium quantities for the public and private firms in the third stage (given  $\alpha$  and  $n$ )

$$q_0^T(\alpha) = \frac{a - (n+1)c_0 + nc}{1 + (n+1)\alpha}, \quad (3)$$

$$q^T(\alpha) = \frac{\alpha(a-c) + c_0 - c}{1 + (n+1)\alpha}, \quad (4)$$

respectively (the superscript T indicates the third-stage subgame).

We obtain the following equilibrium total output, price, private firms' profit, and welfare:

$$Q^T(\alpha) = \frac{n(a-c)\alpha + a - c_0}{1 + (n+1)\alpha}, \quad (5)$$

$$p^T(\alpha) = \frac{(a+nc)\alpha + c_0}{1 + (n+1)\alpha}, \quad (6)$$

$$\pi^T(\alpha) = \left( \frac{\alpha(a-c) + c_0 - c}{1 + (n+1)\alpha} \right)^2 - F, \quad (7)$$

$$W^T(\alpha) = \frac{X_1}{2(1 + (n+1)\alpha)^2} - nF, \quad (8)$$

respectively, where  $X_1 := (a - c_0 + \alpha n(a - c))^2 + 2\alpha(a - (n+1)c_0 + nc)^2 + 2n(\alpha(a - c) + c_0 - c)^2$ .

Next, we discuss the government's welfare maximization problem in the second stage. Let  $\alpha^S$  be the equilibrium degree of privatization (the superscript S indicates the second-stage subgame). Let  $f(n) := n(c_0 - c) - a + (n+1)^2c_0 - (n+2)nc = (c_0 - c)n^2 + 3(c_0 - c)n - a + c_0$ . Because  $f(0) < 0$ , we obtain  $f(n) = 0$  has one positive and one negative solution. Let  $n^*$  be the positive solution to  $f(n) = 0$ . Because  $f(\tilde{n}) > 0$ , we obtain  $\tilde{n} > n^*$ .

**Lemma 1** (i) If  $n < n^*$ ,  $\alpha^S = \alpha^*$ , where

$$\alpha^* := \frac{n(c_0 - c)}{a - (n+1)^2c_0 + n(n+2)c}.$$

(ii) If  $n \in [n^*, \tilde{n}]$ ,  $\alpha^S = 1$ . (iii)  $\alpha^*$  is increasing in  $n$ . (iv)  $\alpha^*$  is decreasing in  $c$  and increasing in  $c_0$ .

**Proof** See the Appendix.

Lemma 1 (i,ii) implies that  $\alpha^S > 0$ , unless  $n = 0$ . Matsumura (1998) shows this for duopolies and Matsumura and Kanda (2005) shows this for oligopolies. Matsumura and Okamura (2015) demonstrate Lemma 1 (iii).



Lemma 1 (iv) states that as long as  $\alpha^S < 1$  (i.e., full privatization is not optimal), the optimal degree of privatization is decreasing with the cost of each private firm. An increase of the degree of privatization makes firm 0 less aggressive because it is less concerned with consumer surplus. Through the strategic interaction, firm 0's less aggressive behavior makes the private firms more aggressive. In other words, production substitution from the public firm to the private firms takes place. Because the marginal cost of the public firm is higher than that of each private firm, this production substitution improves welfare (welfare-improving effect).<sup>7</sup> However, because the total output is decreasing in  $\alpha$ , an increase in the degree of privatization reduces welfare (welfare-reducing effect). This trade-off determines the optimal degree of privatization. The higher (lower)  $c_0$  ( $c$ ) is, the stronger is the abovementioned welfare improving effect of production substitution. Therefore, the optimal degree of privatization is increasing in  $c_0$  and decreasing in  $c$ .

Let  $\pi^S(n)$  be the equilibrium profit in the second-stage subgame in which  $n$  is given exogenously.

Suppose that the solution to the second stage is interior (i.e.,  $\alpha^S < 1$ ). By substituting  $\alpha^*$  into  $\pi^T(\alpha)$ , we obtain the following equilibrium profit of the private firms:

$$\pi^S(n) = (n + 1)^2(c_0 - c)^2 - F. \quad (9)$$

Suppose that  $\alpha^S = 1$ . By substituting  $\alpha = 1$  into  $\pi^T(\alpha)$ , we obtain the following equilibrium profit of private firms:

$$\pi^S(n) = \left( \frac{a + c_0 - 2c}{n + 2} \right)^2 - F. \quad (10)$$

We now present an important result that directly leads to our main result (Proposition 2).

**Proposition 1** (i) *If the optimal privatization policy is not full privatization (i.e.,  $\alpha^S < 1$ ), private firm  $i$ 's profit is increasing in  $n$ .* (ii) *If the optimal privatization policy is full privatization (i.e.,  $\alpha^S = 1$ ), private firm  $i$ 's profit is decreasing in  $n$ .*

**Proof** See the Appendix.

Proposition 1 (ii) is intuitive. Thus, we explain the intuition behind Proposition 1 (i). An increase in  $n$  increases each private firm's profit as long as the solution is interior. An increase in  $n$

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<sup>7</sup>For an excellent discussion on welfare-improving production substitution, see Lahiri and Ono (1988).

strengthens the welfare-improving effect of production substitution from the public firm (firm 0) to private firms. Therefore, an increase in  $n$  strengthens the increases in the degree of privatization, which makes the public firm less aggressive and raises each private firm's profit.<sup>8</sup>

Because  $\pi^S(n)$  is increasing for  $n \in [0, n^*)$  and decreasing for  $n \geq n^*$ ,  $\pi^S(n)$  has an inverted U-shape.<sup>9</sup>

We now discuss the first stage. Let the superscript  $F$  denote the equilibrium outcome in the full game starting at the first stage. Each private firm enters the market as long as the profit is non-negative. Therefore, as long as  $n > 0$ ,

$$(p - c)q - F = 0. \tag{11}$$

From (9) and the assumption  $c_0 < c + \sqrt{F}$ , we obtain  $\lim_{n \rightarrow 0} \pi^S(n) = (c_0 - c)^2 - F < 0$ . Because  $\pi^S(n)$  is increasing for  $n < n^*$ ,  $\pi^S(n)$  is decreasing for  $n > n^*$ , and  $\pi^S(\tilde{n}) < 0$ , there are three equilibria, two of which are locally stable (See Figure 1). In one stable equilibrium  $\alpha = 0$  (and  $n = 0$ ), and in the other stable equilibrium,  $\alpha = 1$ . These lead to the following proposition.

**Proposition 2** *There are two locally stable equilibria. In one equilibrium, the degree of privatization is zero (full nationalization) and no private firm enters the market. In the other equilibrium, the degree of privatization is one and the number of private firms is strictly positive.*

Proposition 2 states that as long as  $n > 0$  in equilibrium, the equilibrium degree of privatization is one.

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<sup>8</sup>Matsumura and Sunada (2013) investigate a mixed oligopoly with misleading advertising competition. They show that the new entry of a private firm might increase the profits of incumbent private firms because it increases (decreases) the public (private) firm's advertising. Some studies on private oligopolies show that a new entry could increase the incumbents' profits. Mukherjee and Zhao (2009) consider an asymmetric Stackelberg setting in which there are two incumbent firms (leader) with different marginal costs and a potential entrant (follower) with a higher marginal cost. The authors then show that the existence of an inefficient follower can increase the profit of the more efficient leader. Ishida *et al.* (2011) consider a model in which a dominant firm competes with minor firms and show that an increase in the number of minor firms accelerates the dominant firm's R&D and profit. Chen and Riordan (2007) show that in a differentiated market, an increase in variety by a new entry might soften competition and increase the incumbent firms' profits. The driving force of our study thus differs from that in these studies.

<sup>9</sup>More precisely, an inverted V-shape. See Figure 1 below.

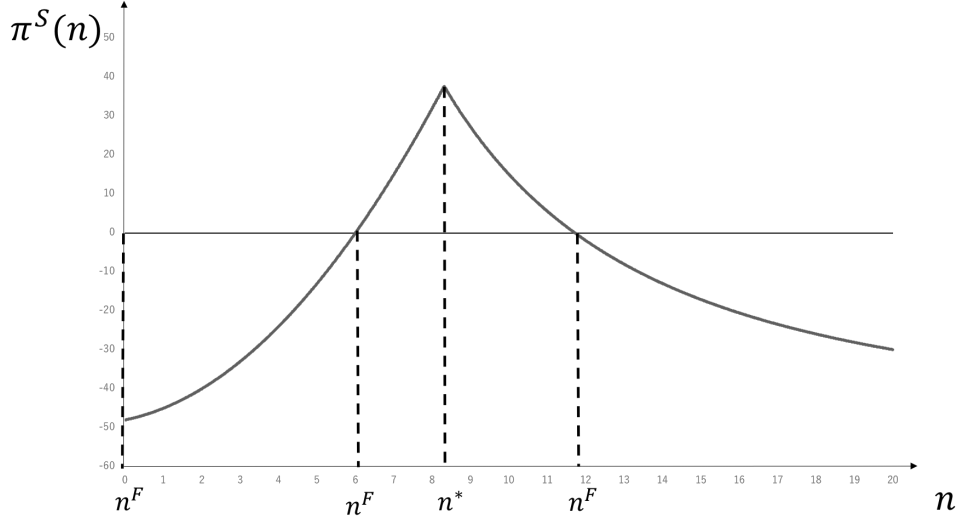


Figure 1: Multiple Equilibria

We now discuss the welfare implications. We consider the scenario where  $\alpha$  is given exogenously as a benchmark. In the first stage, each private firm chooses whether to enter the market given  $\alpha$ . In the second stage, firms face Cournot competition. Let  $n^B(\alpha)$  and  $W^B(\alpha)$  be the equilibrium number of entering private firms and the welfare of this benchmark case, respectively. We obtain

$$n^B = \begin{cases} \frac{a - (c + \sqrt{F})}{\sqrt{F}} - \frac{c + \sqrt{F} - c_0}{\alpha\sqrt{F}} & \text{if } (1 + \alpha)(c + \sqrt{F}) - \alpha a < c_0 < c + \sqrt{F} \quad (12a) \\ 0 & \text{if } c_0 \leq (1 + \alpha)(c + \sqrt{F}) - \alpha a \quad (12b) \end{cases}$$

and

$$W^B = \begin{cases} \frac{(c + \sqrt{F} - c_0)^2}{\alpha} + \frac{(a - c - \sqrt{F})^2}{2} & \text{if } (1 + \alpha)(c + \sqrt{F}) - \alpha a < c_0 < c + \sqrt{F} \quad (13a) \\ \frac{(1 + 2\alpha)(a - c_0)^2}{2(1 + \alpha)^2} & \text{if } c_0 \leq (1 + \alpha)(c + \sqrt{F}) - \alpha a \quad (13b) \end{cases}$$

From these, we obtain the following result.

**Proposition 3**  $W^B$  is non-increasing in  $\alpha$  and strictly decreasing in  $\alpha$  if  $n^B > 0$  or  $\alpha > 0$ .

**Proof** See the Appendix.

Proposition 3 states that the full privatization equilibrium in Proposition 2 minimizes the resulting welfare. In other words, full privatization is the worst privatization policy for welfare;

nevertheless, it is an equilibrium policy. This result suggests that a government may choose the worst privatization policy if it chooses the policy after private firms enter the market. Note that  $W^B(\alpha) = W^F$  if  $\alpha$  is equal to the equilibrium degree of privatization in the entry-then-privatization model.

## 4 Concluding remarks

In this study, we investigate a free-entry mixed oligopoly with constant marginal costs. Although constant marginal cost models are very popular in mixed oligopolies, discussions of such models in the literature are rare. We adopt the Lee *et al.*'s (2018) time line and show that multiple equilibria exist, and that both full nationalization and full privatization constitute equilibrium policies. We also show that full privatization is the worst privatization policy for welfare among all possible privatization policies (whether it is an equilibrium policy or not).

In this study, we assume that private firms are domestic. The literature on mixed oligopolies demonstrates that the nationality of the private firms often affects the behavior of a public firm and the optimal privatization policy. Extending our analysis in this direction is difficult work and remains for future research.<sup>10</sup>

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<sup>10</sup>Whether the private firm is domestic or foreign often yields contrasting results in the literature on mixed oligopolies. See Corneo and Jeanne (1994), Fjell and Pal (1996), Pal and White (1998), and Bárcena-Ruiz and Garzón (2005a, 2005b). The optimal degree of privatization is decreasing with the foreign ownership rate in private firms when the number of private firms is given exogenously (Lin and Matsumura, 2012), while it is increasing with the foreign ownership rate in private firms in free-entry markets (Cato and Matsumura, 2012).

## Appendix

### Proof of Lemma 1

From (8), we obtain

$$\frac{\partial W^T}{\partial \alpha} = - \frac{(a - (n+1)c_0 + nc)(\alpha(a - (n+1)^2c_0 + n(n+2)c) - n(c_0 - c))}{(1 + (n+1)\alpha)^3}. \quad (14)$$

When  $n \in [n^*, \tilde{n})$ ,  $a - (n+1)c_0 + nc > 0$  and  $\alpha(a - (n+1)^2c_0 + n(n+2)c) - n(c_0 - c) < 0$  for  $\alpha < 1$ . Thus, (14)  $> 0$ . This implies Lemma 1 (ii).

Suppose that  $n < n^*$ . By solving  $\partial W^T / \partial \alpha = 0$  with respect to  $\alpha$ , we obtain

$$\alpha^* = \frac{n(c_0 - c)}{a - (n+1)^2c_0 + n(n+2)c}. \quad (15)$$

The second-order condition

$$- \frac{(a - (n+1)^2c_0 + n(n+2)c)^4}{(a - (n+1)c_0 + nc)^2} < 0$$

is satisfied. Therefore, the optimal degree of privatization is  $\alpha^*$ . Note that  $\alpha^* < 1$  if  $n < n^*$ . This implies Lemma 1 (i).

From (15), we obtain

$$\begin{aligned} \frac{\partial \alpha^*}{\partial n} &= \frac{(c_0 - c)(a - c_0 + n^2(c_0 - c))}{(a - (n+1)^2c_0 + n(n+2)c)^2} > 0, \\ \frac{\partial \alpha^*}{\partial c} &= - \frac{a - (n+1)^2c_0 + n(n+2)c + n^2(n+2)(c_0 - c)}{(a - (n+1)^2c_0 + n(n+2)c)^2} < 0, \\ \frac{\partial \alpha^*}{\partial c_0} &= \frac{n(a - (n+1)^2c_0 + n(n+2)c) + n(n+1)^2(c_0 - c)}{(a - (n+1)^2c_0 + n(n+2)c)^2} > 0. \end{aligned}$$

These imply Lemma 1 (iii,iv). ■

### Proof of Proposition 1

Suppose that the solution to the second stage is interior (i.e.,  $\alpha^S < 1$ ). From (9), we obtain

$$\frac{\partial \pi^S(n)}{\partial n} = 2(n+1)(c_0 - c)^2 > 0.$$

This result implies Proposition 1 (i).

Suppose that  $\alpha^S = 1$ . From (10), we obtain

$$\frac{\partial \pi^S(n)}{\partial n} = -\frac{2(a + c_0 - 2c)^2}{(n + 2)^3} < 0.$$

This result implies Proposition 1 (ii). ■

### Proof of Proposition 3

If  $(1 + \alpha)(c + \sqrt{F}) - \alpha a < c_0 < c + \sqrt{F}$ , then a mixed oligopoly appears in equilibrium (i.e.,  $n_B > 0$ ). From (13a), we obtain

$$\frac{\partial W^B}{\partial \alpha} = -\left(\frac{c + \sqrt{F} - c_0}{\alpha}\right)^2 < 0.$$

Note that if  $\alpha = 0$ ,  $(1 + \alpha)(c + \sqrt{F}) - \alpha a < c_0 < c + \sqrt{F}$  is never satisfied.

If  $c_0 < (1 + \alpha)(c + \sqrt{F}) - \alpha a$ , a public monopoly appears in equilibrium (i.e.,  $n_B = 0$ ). From (13b), we obtain

$$\frac{\partial W^B}{\partial \alpha} = -\frac{\alpha(a - c_0)^2}{(1 + \alpha)^3} \leq 0.$$

The strict inequality in this equation holds if and only if  $\alpha > 0$ . These imply Proposition 3 (ii).

■

## References

- Bárcena-Ruiz, C. J., Garzón, B. M. (2005a). Economic integration and privatization under diseconomies of scale. *European Journal of Political Economy* 21(1):247–267.
- Bárcena-Ruiz, C. J., Garzón, B. M. (2005b). International trade and strategic privatization. *Review of Development Economics* 9(4):502–513.
- Capuano, C., De Feo, G. (2010). Privatization in oligopoly: the impact of the shadow cost of public funds. *Rivista Italiana Degli Economisti* 15(2):175–208.
- Cato, S., Matsumura, T. (2012). Long-run effects of foreign penetration on privatization policies. *Journal of Institutional and Theoretical Economics* 168(3):444–454.
- Cato, S., and T. Matsumura (2013), Long-run effects of tax policies in a mixed market. *FinanzArchiv* 69(2):215–240.
- Cato, S., Matsumura, T. (2015). Optimal privatization and trade policies with endogenous market structure. *Economic Record* 91(294):309–323.
- Chen, T. L. (2017). Privatization and efficiency: a mixed oligopoly approach. *Journal of Economics* 120(3):251–268.
- Chen, Y., Riordan, M. H. (2007). Price and variety in the spokes model. *Economic Journal*, 117(7):897–921.
- Colombo, S. (2016) Mixed oligopolies and collusion. *Journal of Economics* 118(2):167–184.
- Corneo, G., Jeanne, O. (1994). Oligopole mixte dans un marche commun. *Annales d’Economie et de Statistique* 33:73–90.
- Fjell, K., Pal, D. (1996). A mixed oligopoly in the presence of foreign private firms. *Canadian Journal of Economics* 29(3):737–743.
- Fujiwara, K. (2007). Partial privatization in a differentiated mixed oligopoly. *Journal of Economics* 92(1):51–65.
- Ishida, J., Matsumura T., Matsushima, N. (2011). Market competition, R&D and firm profits in asymmetric oligopoly. *Journal of Industrial Economics* 59(3):484–505.
- Ishida, J., Matsushima, N. (2009). Should civil servants be restricted in wage bargaining? A mixed-duopoly approach. *Journal of Public Economics* 93(3–4):634–646.
- Kowalski, P., Buge, M., Sztajerowska, M., Egeland, M. (2013). State-Owned Enterprises: Trade Effects and Policy Implications. *OECD Trade Policy Papers*.

- Lahiri, S., Ono, Y. (1988). Helping minor firms reduces welfare. *Economic Journal* 98:1199–1202.
- Lee, S.-H., Matsumura, T., Sato, S. (2018). An analysis of entry-then-privatization model: welfare and policy implications. *Journal of Economics* 123(1):71–88.
- Lin, M. H., Matsumura, T. (2012). Presence of foreign investors in privatized firms and privatization policy. *Journal of Economics* 107(1):71–80.
- Matsumura, T. (1998). Partial privatization in mixed duopoly. *Journal of Public Economics* 70(3):473–483.
- Matsumura, T., Kanda, O. (2005). Mixed oligopoly at free entry markets. *Journal of Economics* 84(1):27–48.
- Matsumura, T., Matsushima, N. (2004). Endogenous cost differentials between public and private enterprises: a mixed duopoly approach. *Economica* 71(284):671–688.
- Matsumura, T., Ogawa, A. (2010). On the robustness of private leadership in mixed duopoly. *Australian Economic Papers* 49(2):149–160.
- Matsumura, T., Okamura, M. (2015). Competition and privatization policies revisited: The payoff interdependence approach. *Journal of Economics* 116(2):137–150.
- Matsumura, T., Sunada, T. (2013). Advertising competition in a mixed oligopoly. *Economics Letters* 119(2):183–185.
- Mukherjee, A., Zhao, L. (2009). Profit raising entry. *Journal of Industrial Economics* 57(4):870–870.
- Pal, D. (1998). Endogenous timing in a mixed oligopoly. *Economics Letters* 61(2):181–185.
- Pal, D., White, M. D. (1998). Mixed oligopoly, privatization, and strategic trade policy. *Southern Economic Journal* 65(2):264–281.
- Sato, S., Matsumura, T. (2018). Dynamic privatization policy. forthcoming in *Manchester School*.  
<https://doi.org/10.1111/manc.12217>
- White, M. D. (1996). Mixed oligopoly, privatization and subsidization. *Economics Letters* 53(2):189–195.