Insider Trading With Different Risk Attitudes

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Abstract

This paper investigates the effect of different risk attitudes on the financial decisions of two insiders trading in the stock market. We consider a static version of the Kyle (1985) model with two insiders. Insider 1 is risk neutral while insider 2 is risk averse with negative exponential utility. First, we prove the existence of a unique linear equilibrium. Second, we obtain somewhat surprising results on how the risk attitudes affect the market liquidity, the price efficiency, when we carry out a comparative static analysis with respect to Tighe (1989) and Holden and Subrahmanyam(1994) models.

JEL classification: G14, D82

Keywords: Insider trading, Risk neutrality, Risk aversion, Exponential Utility, Market structure, Kyle model

1 Introduction

Investors' tolerance toward risk plays a central role in their investments decisions. Most of the research literature about the investors' risk tolerance, considered two types of risk tolerance: risk-neutral investors and risk-averse investors. In the market microstructure literature, and more specifically in

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Kyle (1985) model and its extensions, majority of the insiders were supposed to be risk-neutral.[1] However, there are considerable research papers, either theoretical or empirical, on the risk-aversion of the insiders in the financial markets.

Few extensions considered the risk-aversion case of the Kyle (1985) model. We can cite Subrahmanyam(1991), Holden and Subrahmanyam(1994), Vitale (1995), Zhang (2004) and Baruch (2004). Except Subrahmanyam(1991), in most of the discrete models, the comparative analysis studies were found numerically. Indeed, the introduction of the risk-aversion type of the insider makes the computational analysis quite complex. Subrahmanyam(1991) which extended the Kyle (1985) model to the case of partially informed insiders, succeeded to prove analytically, many of her comparative analysis with respect to the risk-neutral case. Holden and Subrahmanyam(1994) extended Kyle's (1985) multi-period auction model to include multiple risk-averse informed trader with long-lived information. Vitale (1995) generalized Kyle's (1985) model to the case in which the informed trader is risk-averse where the solution methods are based on LEQG dynamic programming problems. Tighe (1989) extended Kyle model to multiple informed traders, all risk-neutral. None of these extensions, studied the case with multiple informed traders with different risk attitudes.

This paper, is the first to our knowledge, to investigate the effect of risk attitudes on the financial decisions of two insiders trading in the stock market. We consider a static version of the Kyle (1985) model with two insiders. Insider 1 is risk neutral while insider 2 is risk averse with negative exponential utility. Like Subrahmanyam(1991), we analytically prove the existence of a linear Bayesian equilibrium and provide all the comparative statics analysis results with respect to the duopoly static model of Holden and Subrahmanyam(1994) and to the risk-neutral insiders duopoly case studied in Tighe (1989).

It should be pointed out to the reader that in the risk-neutral case, the normality distribution of the exogenous variables together with the linear structure of the stock price, simplifies the existence and the characterization of the linear Bayesian equilibrium. Thus, the comparative statics becomes straightforward. However, when the risk-aversion structure is introduced,

[1] For a detailed reference, the reader can check O'Hara (1995) or Ardalan (1998). Recent extensions papers which considered risk-neutrality of the insider, include Jain and Mirman (1999), Daher and Mirman(2007), Liang and Lin (2010), Wang, Wang and Ren (2009), Daher, Karam and Mirman (2012), and Daher, Mirman and Saleeby (2014).

the computation of the linear Bayesian equilibrium becomes difficult and the analysis turns to be more complex. In this paper we solve such complexity and provide exact result for the equilibrium outcomes as well as the comparative statics analysis.

Our findings reveal the impact of the risk attitudes on the equilibrium outcomes. First, we show that the risk-aversion coefficient A has a direct effect on the trading order of the risk-neutral insider. Second, the different types of risk attitudes, induce a non symmetric equilibrium trading orders. Third, we find that the market depth parameter λ in this paper is greater than the market depth parameter in Tighe (1989) model,i.e. when the two insiders are risk-neutral. However, the comparison between the market depth parameter λ in our paper and the market depth parameter in Holden and Subrahmanyam(1994) is not straightforward. We show how the the risk-aversion coefficient is a fundamental determinant of this comparison. Finally, we study the impact of different risk attitudes on the price revelation. We show that the equilibrium price in our model reveals more (less) information than the stock price in Holden and Subrahmanyam(1994) (Tighe (1989))

The paper is organized as follows: In section 2, we describe the model and characterize the unique linear Bayesian equilibrium of the model. In section 3, we conduct a comparative statics analysis of the equilibrium outcomes with respect to Holden and Subrahmanyam(1994) and Tighe (1989). All proofs are relegated to the Appendix.

2 The Model

We consider a static version of the Kyle (1985) model with two insiders. The economy consists of one financial asset. The underlying value of the asset is denoted $\tilde{z}$. The prior distribution of $\tilde{z}$ is normal with mean$\bar{z}$ (assumed to be positive) and variance σ_z^2. The two insiders exhibit different attitudes toward risk. We assume that insider 1 is risk neutral while insider 2 is risk averse with negative exponential utility and risk-aversion coefficient A expressed as:

$$U(x) = -e^{-Ax}.$$

The two insiders trade in the stock market based on their inside information. There are three types of agents. First, there are two rational insiders, each of whom knows the realization z of $\tilde{z}$. Second, there are the (nonrational) noise traders, representing small investors with no information on z. The aggregate noise trade is assumed to be a random variable $\tilde{u}$, which is normally distributed with mean zero and variance σ_u^2. Finally, there are $K(K \geq 2)$ risk-neutral market makers who act like Bertrand competitors. We assume, as in Kyle (1985), that the market makers observe the total order flow signal. We assume that $\tilde{z}$, and $\tilde{u}$ are independent.

Following Kyle (1985), the trading mechanism is organized in two steps. In step one, a linear pricing rule and optimal order rules are determined by the market makers and the insiders, respectively, as a Bayesian Nash equilibrium. The market makers determine a (linear) pricing rule p, based on their a priori beliefs, where p is a measurable function $p : \mathbb{R} \longrightarrow \mathbb{R}$. Each insider chooses a stock trade function $\tilde{x}_i = x_i(\tilde{z})$, where $x_i : \mathbb{R} \longrightarrow \mathbb{R}$ is a measurable function. In the second step, the insiders observe the realization z [2] of $\tilde{z}$ and submit their stock orders to the market makers based on the equilibrium stock trade functions. The market makers also receive orders from the noise traders, all these orders arrive as a total order flow signal $\tilde{y} = x_1(\tilde{z}) + x_2(\tilde{z}) + \tilde{u} = x(\tilde{z}) + \tilde{u}$. The order flow signal is used by the market makers to set the price $\tilde{p} = p(\tilde{y})$, based on the equilibrium price function, to clear the market. The insiders know only the value of $\tilde{z}$ and do not know the values of $\tilde{u}, \tilde{y}$ before their order flow decisions are made. Moreover, each market maker does not know the realization z of $\tilde{z}$ but only knows its distribution. Finally, the market makers cannot observe either x_1, x_2 or u.

The profits for each of the two rational traders are given, respectively, by

$$\Pi_1 := (\tilde{z} - p) \cdot \tilde{x}_1 \text{ and } \Pi_2 := (\tilde{z} - p) \cdot \tilde{x}_2$$

This is a game of incomplete information because the market makers unlike the insiders do not know the realization of $\tilde{z}$. Hence, we seek for a Bayesian-Nash equilibrium defined as follows,

Definition 1 *A Bayesian-Nash equilibrium is a vector of three functions $[x_1(.), x_2(.), p(.)]$ such that:*

(a) Profit maximization of the risk neutral insider, i.e. insider 1,

$$\begin{aligned} &E[(\tilde{z} - p(x_1(\tilde{z}) + x_2(\tilde{z}) + \tilde{u}))x_1(\tilde{z})] \\ &\qquad \geq E[(\tilde{z} - p(x_1'(\tilde{z}) + x_2(z) + \tilde{u}))x_1'(\tilde{z})] \end{aligned} \tag{1}$$

for any alternative trading strategy $x_1'(\tilde{z})$;

[2] Random variables are denoted with a tilde. Realized values lack the tilde. The mean of the random variable is denoted with bar.

(b) *Profit maximization of the risk-averse insider, i.e. insider* 2,

$$E[-e^{-A(\tilde{z} - p(x_1(\tilde{z}) + x_2(\tilde{z}) + \tilde{u}))x_2(\tilde{z})}] \\ \geq E[-e^{-A(\tilde{z} - p(x_1(\tilde{z}) + x_2'(\tilde{z}) + \tilde{u}))x_2'(\tilde{z})}] \quad (2)$$

for any alternative trading strategy $x_2'(\tilde{z})$*;*

(c) *Semi-Strong Market Efficiency: The pricing rule* $p(.)$ *satisfies,*

$$p(\tilde{y}) = E[\tilde{z}|\tilde{y}] \quad (3)$$

An equilibrium is linear if there exists constants μ and λ, such that,

$$\forall\ y,\quad p(y) = \mu + \lambda y. \quad (4)$$

Note that conditions (1), (2) define optimal strategies of the two insiders while condition (3) guarantees the zero expected profits for the market makers. The stock price, set by the market makers, is equal to the conditional expectation of the asset value given their information. We restrict our study to linear equilibrium. The normal distributions of the exogenous random variables, enable us to derive and to prove the existence of a unique linear equilibrium.

In the following Proposition, we characterize the unique linear equilibrium of the model.

Proposition 1 *In the presence of one risk neutral and one risk averse informed traders, a linear equilibrium exists and it is unique. It is characterized by,*

(i) $x_1(\tilde{z}) = \dfrac{(\tilde{z} - \mu)(1 + A\lambda^* \sigma_u^2)}{\lambda^*(3 + 2A\lambda^*\sigma_u^2)}$ and $x_2(\tilde{z}) = \dfrac{(\tilde{z} - \mu)}{\lambda^*(3 + 2A\lambda^*\sigma_u^2)}$

(ii) $p(\tilde{y}) = \mu + \lambda^* \tilde{y}$*, where* $\mu = \bar{z}$*, and* λ^* *is the unique strictly positive root of the following quartic equation:*

$$4A^2\sigma_u^4\lambda^{\mathbf{4}} + 12A\sigma_u^2\lambda^{\mathbf{3}} + (9 - A^2\sigma_u^2\sigma_z^2)\lambda^{\mathbf{2}} - 3A\sigma_z^2\lambda - 2\frac{\sigma_z^2}{\sigma_u^2} = 0.$$

Proof: See Appendix A.

Discussion of the equilibrium: First, note that the relationship between this paper, Holden and Subrahmanyam(1994) and Tighe (1989) should be clear. Indeed, Holden and Subrahmanyam (1994) considered the case of two risk-averse insiders. Moreover, Tighe (1989) considered the Kyle (1985) model with two risk neutral insiders. Hence, our model can be seen as a hybrid-model in comparison with Holden and Subrahmanyam(1994) and with Tighe (1989). Consequently, in this paper we will be able to show the effect of different risk-attitudes on equilibrium outcomes.

Second, it should be pointed out that our hybrid risk attitudes structure affects the equilibrium trading orders. Proposition 1 shows that they are not symmetric as in Holden and Subrahmanyam(1994) and in Tighe (1989). Specifically, the risk neutral insider (insider 1) trades more (in absolute value) than the risk-averse insider (insider 2). Although both insiders are fully informed about the realization z of the risky asset, we notice that the risk neutrality dominates the risk-aversion in term of trading.

Third, Proposition 1 reveals the role of the strategic behaviors of the two insiders. Indeed, the order of the risk-neutral insider (insider 1) depends on the risk-aversion coefficient A. Insider 1 takes into account all the possible decisions of insider 2 in her maximization problem.

Fourth, as in Holden and Subrahmanyam(1994), our model reduces to the case of two risk-neutral insiders, i.e. it converges to the Tighe (1989) model when the risk-aversion coefficient $A = 0$. However, Proposition 1 reveals very interesting results about the risk tolerance effect on the insiders trades, when the total order flow signal is too noisy (σ_u^2 is large). Indeed, in this case, the order of the risk-averse insider is almost close to zero while the order of risk neutral insider is close to the one in Kyle (1985). In other words, when the signal is too noisy, the risk-averse insider responds less aggressively to such increase and thus decides not to trade. Hence, the model is reduced to the single risk-neutral insider. Consequently, the insider order converges to the one in the risk-neutral monopolistic case (Kyle 1985).

Since in both Tighe (1989) and Holden and Subrahmanyam(1994) papers, insiders are either risk neutral or risk averse, the equilibrium trades are equal and thus the impact of the noise is the same on both insiders' trades. However, in our model, we see that the strategic behavior of the insiders together with their risk attitudes, drive the insider to take different positions of trades depending on their information.

Fifth, it is noteworthy to consider the case when insider 2 is too risk-averse (A is large). In this case, Proposition 1 shows that the risk-averse insider (insider 2) has no incentive to trade ($x_2(\tilde{z}) = 0$). Consequently the model converges to the one risk-neutral Kyle (1985) model, and the order of insider 1 is equal to the one in Kyle (1985).

Finally, it should be pointed out that the risk-averse property has a direct effect on the computation of the equilibrium outcomes and more specifically on the market depth parameter λ. Indeed, considering the static version of Kyle (1985) and its extensions when the insiders are risk neutral[3], we noticed that the market depth parameter is directly computed. On the other hand, when insiders are risk-averse, the market depth λ is the solution of a quartic equation.[4]

In the next section we develop the comparative static analysis. We will analyze the market depth and the stock price informativeness compared to their corresponding expressions in Holden and Subrahmanyam(1994) and Tighe (1989).

3 Comparative Statics

3.1 market depth parameter λ

In this section, we compare our market depth parameter λ to the ones in Holden and Subrahmanyam(1994) and Tighe (1989). Lemma 1 shows that market depth in Tighe (1989) is higher than our model. However, the relation with respect to Holden and Subrahmanyam(1994) is not straightforward. Indeed,

Lemma 1 *The market depth parameter λ^* in our model is greater than the market depth in Tighe (1989). However, the relation with respect to Holden and Subrahmanyam(1994) is ambiguous and given as follows.*

(i)

$$\lambda^T \leq \lambda^*$$

[3]See for example Tighe (1989),Jain and Mirman (1999), Daher and Mirman (2007), Wang et al (2009), Daher, Karam and Mirmam (2012).

[4]See for example Vitale (1995), Holden and Subrahmanyam(1994), Subrahmanyam(1991), Zhang (2004).

(ii) There exists a risk-aversion coefficient level A^* such that,

$$\begin{cases} \lambda^* > \lambda^{HS} \text{ forall } A < A^* \\ \\ \lambda^* < \lambda^{HS} \text{ forall } A > A^* \end{cases} \tag{5}$$

where λ^T and λ^{HS} refer to the market depth parameters in Tighe (1989) and Holden and Subrahmanyam(1994) respectively.

Proof: See Appendix B.

Note that Lemma 1 plays a central key in providing all the results in this section. In all the papers which extended the static Kyle (1985) models with risk neutral insiders, the parameter λ was explicitly characterized as a function of the exogenous variables of these models. However, in the presence of risk aversion, the market depth parameter is implicitly characterized as a solution of a quartic equation.[5] Thus, most of the comparative results in the literature, were made numerically. However, in our paper, the results of Lemma 1 are proved analytically and it will be used as foundation to subsequent results of this paper.

Lemma 1 shows the impact of different risk attitudes on the market depth parameter λ. The first part of the Lemma shows that the market depth parameter is greater in our model than in Tighe (1989). In other words, when the two insiders are risk neutral (Tighe 1989), the market is deeper (as defined by Kyle (1985) to be $\frac{1}{\lambda}$) than the market in the presence of one risk-averse insider (our model). Hence, the risk-averse insider together with the strategic behavior of the risk-neutral insider, drive them to trade more aggressively and thus reducing the market liquidity than in Tighe (1989).

However, the comparison between the market depth parameters in our paper and in Holden and Subrahmanyam(1994) model seems to be ambiguous and not intuitive. Indeed, one expects that the presence of the risk-neutral insider in our paper will increase the market liquidity. but the second part of Lemma 1 shows the existence of a unique risk aversion coefficient, A^*, before which the market depth parameter in our model is less than the market depth

[5]See Subrahmanyam(1991), Holden and Subrahmanyam(1994), Vitale (1995) and many related papers.

parameter in the presence of two risk averse insiders (Holden and Subrahmanyam(1994) model). Moreover, for the values of risk aversion coefficients greater than A^*, the market depth parameter in our model is greater than the market depth parameter in the presence of two risk averse insiders. Thus, the risk aversion coefficient is a principal determinant of market liquidity.

To better understand the effect of risk aversion on the market depth parameter, we decide to derive in Table 1, the λ values[6] in the following cases:
a- λ^V corresponds to market depth parameter in the case of a single risk-averse insider (Vitale 1995).
b- λ^{HS} corresponds to market depth parameter in the case of two risk-averse insiders both knowing the underlying value of the risky asset $\tilde{z}$ (Holden and Subrahmanyam(1994)).
c- λ^* corresponds to the market depth parameter in the case of one risk neutral and one risk averse insiders knowing the underlying value of the risky asset $\tilde{z}$ (our model).

Note that when $A = 0$, λ^V corresponds to λ in Kyle (1985). Similarly in our model and in Holden and Subrahmanyam(1994) model, when $A = 0$, we retrieve λ^T (Tighe 1989).

Table 1 shows that in the presence of one risk-averse insider (Vitale 1995), λ is always less than the one in the case of risk-neutral insider (Kyle 1985). This result is reversed when we compare our model's market depth parameter λ^* to λ^T (part (i) of Lemma 1). Consequently, the effect of risk-neutrality on the market depth dominates the effect of the risk-aversion on the market depth, when we add another risk neutral insider to Kyle 1985 model (Tighe 1989) and to Vitale 1995 model (our model).

Part (ii) of Lemma 1 shows how the effect of risk-aversion on the market depth parameter is crucial when we compare our hybrid duopoly model to the risk-aversion duopoly model of Holden and Subrahmanyam(1994). First note that going from the risk-aversion monopoly case (Vitale 1995) to the risk-aversion duopoly case (Holden and Subrahmanyam(1994)), Table 1 shows that the market depth parameter is not monotonic and more specifically, it is unimodal with respect to the risk-aversion coefficient. Thus the impact of risk-aversion on the market depth is the key of the relation.

[6]For a better comparison, we consider the same Table as in Vitale (1995) page 7, and add to it the case when the risk aversion coefficient $A = 4$ which is considered in Holden and Subrahmanyam(1994).

Table 1: Comparison of the market depth parameter λ

$\sigma_z^2 = 1, \quad \sigma_u^2 = 1$									
A	0	0.05	0.1	0.2	0.33	0.5	1	2	4
λ^V	0.5	0.4999	0.4998	0.4994	0.4986	0.4969	0.4902	0.4735	0.4416
λ^{HS}	0.4714	0.4732	0.4750	0.4782	0.4820	0.4862	0.4947	0.5	0.4905
λ^*	0.4714	0.4723	0.4732	0.4748	0.4767	0.4789	0.4839	0.4897	0.4948

$\sigma_z^2 = 1, \quad \sigma_u^2 = 2$									
A	0	0.05	0.1	0.2	0.33	0.5	1	2	4
λ^V	0.3536	0.3535	0.3533	0.3528	0.3516	0.3496	0.3419	0.3250	0.2969
λ^{HS}	0.3333	0.3351	0.3368	0.3399	0.3432	0.3468	0.3524	0.3520	0.3375
λ^*	0.3333	0.3342	0.3351	0.3366	0.3383	0.3403	0.3442	0.3482	0.3512

It should be pointed out that Holden and Subrahmanyam(1994) focused on the effect of the dynamic structure of trading and compared their results to the Kyle model. However, in the static case with 2 insiders, the relation between λ^{HS} and λ^T is quite similar to the relation between λ^{HS} and λ^*. In sum, the relation between the market depth parameters in the case of monopoly (Vitale 1994 and Kyle 1985) is monotonic but it does not hold when we consider the duopoly case (Tighe 1989 and Holden and Subrahmanyam 1994).[7]

Consequently, we show that the market liquidity is directly affected not only by the number of trading rounds (Holden and Subrahmanyam 1994) or the number of informed traders (Subrahmanyam(1991)), but also by the risk attitudes of the insiders.

3.2 Information Revelation

In this section we discuss the information revelation in our model and compare to Tighe (1989) and Holden and Subrahmanyam(1994). By adopting the same measure of information, i.e. the conditional variance of the liquidation asset value given the total order flow, we obtain the following result.

[7]Subrahmanyam(1991) found the same result with finite number of partially informed traders and analyzed the impact of the number of the insiders on the market depth parameter.

Proposition 2 *The equilibrium price reveals less information in the presence of 1 risk-neutral and 1 risk-averse insiders (our model) than in the presence of two risk-neutral insiders (Tighe 1989). However, with two risk-averse insiders (Holden and Subrahmanyam 1994), the equilibrium price is less revealing than in our model.*

Proposition 2 highlights the impact of risk attitude on price informativeness. First, it should be noted that the hybrid structure of the risk attitudes of the insiders does not alter the relation of the price informativeness. Indeed, Proposition 2 shows that risk-neutrality increase price revelation of information with respect to risk aversion. This results holds in Vitale (1995) model which considered the risk-averse monopolistic case and compared the price revelation to the result in Kyle (1985). For the duopoly case, Holden and Subrahmanyam (1994) also obtained the same result as well.

Similar to Subrahmanyam (1991), we found two common results related to price efficiency. First note that an increase in the variance of liquidity trading, decreases the price efficiency. This similarity shows that the presence of the risk-averse insider has more effect on the price efficiency than the risk neutral insider does. Second, it should be pointed out that price efficiency is decreasing in the risk-aversion coefficient. In other words, when insider 2, becomes more risk-averse (increasing the risk-aversion coefficient), her trades are more and more less aggressive. This effect has a direct increase on the market depth parameter λ (Lemma 1, part II) and thus reducing the price efficiency.

Finally, it is worth noting that our model highlights the impact of the risk attitudes on the price efficiency. Indeed, although one of the insiders is risk neutral, the price efficiency is directly affected by the risk-averse insider behavior.

Appendices

Appendix A: proof of Proposition 1

We begin by the maximization problem of the risk neutral insider, i.e. insider 1. The decision rule of insider 1 is the the function $x_1(\tilde{z})$. The expected profits after plugging the linear pricing function, become,

$$E[(\tilde{z} - p(x_1(\tilde{z}) + x_2(\tilde{z}) + \tilde{u}))x_1(\tilde{z})] = E[(\tilde{z} - \mu - \lambda(x_1(\tilde{z}) + x_2(\tilde{z}) + \tilde{u}))x_1(\tilde{z})]$$

The first and the second order conditions are

$$x_1(\tilde{z}) = \frac{z - \mu - \lambda x_2(\tilde{z})}{2\lambda} \qquad \text{and} \qquad \lambda > 0. \tag{6}$$

We move now to solve the maximization of the risk averse insider, i.e. insider 2. The decision rule of insider 2 is the the function $x_2(\tilde{z})$). The expected profits after plugging the linear pricing function, become,

$$E[-e^{-A(\tilde{z}-p(x_1(\tilde{z})+x_2(\tilde{z})+\tilde{u}))x_2(\tilde{z})}|\tilde{z}] = E[-e^{-A(\tilde{z}-\mu-\lambda(x_1(\tilde{z})+x_2(\tilde{z})+\tilde{u}))x_2(\tilde{z})}|\tilde{z}]$$

Using the normality and the independency of the noise traders'orders $\tilde{u}$, the first and the second order condition are

$$x_2(\tilde{z}) = \frac{z - \mu - \lambda x_1(\tilde{z})}{\lambda(2 + A\lambda\sigma_u^2)} \qquad \text{and} \qquad \lambda(2 + A\lambda\sigma_u^2) > 0. \tag{7}$$

Combining equations 6 and 7, we obtain

$$x_1(\tilde{z}) = \frac{(z-\mu)(1 + A\lambda\sigma_u^2)}{\lambda(3 + 2A\lambda\sigma_u^2)} \qquad \text{and} \qquad x_2(\tilde{z}) = \frac{(z-\mu)}{\lambda(3 + 2A\lambda\sigma_u^2)} \tag{8}$$

Regarding the price function coefficients, μ and λ, first note that the semi-strong market efficiency together with linear price function assumption lead to,

$$\mu + \lambda\tilde{r} = E[\tilde{z}|\tilde{r}] \tag{9}$$

Evaluating the expectation on both sides of equation 9 and then applying the law of iterated expectations, we obtain

$$\mu + \lambda\bar{r} = \bar{z} \tag{10}$$

Where $\bar{r} = \bar{x}_1 + \bar{x}_2 + \bar{u} = \bar{x}_1 + \bar{x}_2$. Using equation 8 to find the expression of $\bar{r}$ and plugging the result in equation 10, we obtain

$$\mu = \bar{z} \tag{11}$$

To complete the proof, it remains to find a unique value of the price function slope λ. Indeed, note that the linear expressions of the insiders strategies decisions, $\tilde{x}_1$ and $\tilde{x}_2$, induce the normality distribution of the total order flow $\tilde{r}$. Thus, by applying the projection theorem to equation 9, we have

$$\lambda = \frac{cov(\tilde{z}, \tilde{r})}{var(\tilde{r})} \tag{12}$$

Evaluating the right-hand side of equation 12 and after certain arrangement we find that λ is a root of the following quadric equation

$$4A^2\sigma_u^4\boldsymbol{\lambda^4} + 12A\sigma_u^2\boldsymbol{\lambda^3} + (9 - A^2\sigma_u^2\sigma_z^2)\boldsymbol{\lambda^2} - 3A\sigma_z^2\lambda - 2\frac{\sigma_z^2}{\sigma_u^2} = 0. \qquad (13)$$

By Descartes' rule of signs,[8] there is only one positive root satisfying the second order condition which ends the proof. ∎

Appendix B: proof of Lemma 1

First, recall that the market depth parameter in Tighe (1989) is given by $\lambda^T = \frac{\sqrt{2}}{3}\frac{\sigma_z}{\sigma_u}$ which corresponds to our market depth parameter when $A = 0$. Thus, if we show that λ^* is increasing in A, then the first part of the lemma will be proved. We begin by proving the following lemma

Lemma 2 $\lambda^* \in [\frac{\sqrt{2}}{3}\frac{\sigma_z}{\sigma_u}, \frac{1}{2}\frac{\sigma_z}{\sigma_u}]$

Proof: The Proof is divided in two parts.

Part I: In this part we show that $\lambda = \dfrac{1}{2}\dfrac{\sigma_z}{\sigma_u}$ is an asymptote in the (A, λ) plane. To find the asymptote, we view the quartic (equation 13)

$$f(\lambda, A) = 4A^2\sigma_u^4\lambda^4 + 12A\sigma_u^2\lambda^3 + \left(9 - A^2\sigma_u^2\sigma_z^2\right)\lambda^2 - 3A\sigma_z^2\lambda - 2\frac{\sigma_z^2}{\sigma_u^2} = 0, \quad (14)$$

as a plane algebraic curve.

Note that most of the curves represented by $f(\lambda, A; \sigma_z, \sigma_u)$ are irreducible. We work under this assumption. The projective curve corresponding to the affine curve $f = 0$ is

$$F(\lambda, A, Z) = 4A^2\sigma_u^4\lambda^4 + 12A\sigma_u^2\lambda^3Z^2 + \left(9Z^4 - A^2\sigma_u^2\sigma_z^2Z^2\right)\lambda^2 - 3A\sigma_z^2\lambda Z^4 - 2\frac{\sigma_z^2}{\sigma_u^2}Z^6 = 0. \quad (15)$$

[8] **(Theorem: Descartes'rule of signs)** If the terms of a single variable polynomial with real coefficients are ordered by descending variable exponent, then the number of **positive** roots of the polynomial is either equal to the number of sign differences between consecutive nonzero coefficients, or less than it by a multiple of two

It is not difficult to see that this projective curve has the singular points $(0:1:0)$ and $(1:0:0)$.
Now we consider the affine view $Z=1$. Put $Z=0$ into the equation (15) to get $A^2\lambda^4=0$. So the points $(1:0:0)$ and $(0:1:0)$ are at infinity. The second of these points is on the A-axis, and as noted above it is singular. Set $A=1$ in the equation of the projective curve (15), we thus obtain the affine curve

$$4\sigma_u^4\lambda^4+12\sigma_u^2\lambda^3Z^2+\left(9Z^4-\sigma_u^2\sigma_z^2Z^2\right)\lambda^2-3\sigma_z^2\lambda Z^4-2\frac{\sigma_z^2}{\sigma_u^2}Z^6=0.$$

The lower order terms $4\sigma_u^4\lambda^4-\sigma_u^2\sigma_z^2Z^2\lambda^2$ give us the distinct tangents $\lambda=0, \lambda=\frac{1}{2}\sigma_z\frac{Z}{\sigma_u}, \lambda=-\frac{1}{2}\sigma_z\frac{Z}{\sigma_u}$. Now dehomogenize to obtain, for $\lambda>0$, the affine asymptote $\lambda=\frac{\sigma_z}{2\sigma_u}$.

Next we show the the curve will not cross its asymptote. The equation of the affine curve above (14) can be written as

$$\left(4\sigma_u^2\lambda^2-\sigma_z^2\right)\left(\left(\sigma_u^2\lambda A+3\right)\lambda A+\frac{2}{\sigma_u^2}\right)=-\lambda^2<0.$$

This then implies that $4\sigma_u^2\lambda^2-\sigma_z^2<0$, and so $\lambda<\frac{\sigma_z}{2\sigma_u}$ for $\lambda>0$.

Part II: In this part of the proof, we show that λ is increasing with respect to A where σ_u and σ_z are treated as parameters. By implicit differentiation, we obtain

$$\frac{d\lambda}{dA}=-\frac{\frac{\partial f}{\partial A}}{\frac{\partial f}{\partial \lambda}},$$

where $\frac{\partial f}{\partial \lambda}$ must not be zero. It is then simple to find that

$$\frac{d\lambda}{dA}=\frac{-8A\sigma_u^4\lambda^4-12\sigma_u^2\lambda^3+2A\sigma_u^2\sigma_z^2\lambda^2+3\sigma_z^2\lambda}{16A^2\sigma_u^4\lambda^3+36A\sigma_u^2\lambda^2+2\left(9-A^2\sigma_u^2\sigma_z^2\right)\lambda-3A\sigma_z^2}.$$

Note that the numerator has one positive root - by Descartes' rule (since we are interested in $A\geq 0$). Now factor the expression, and so we have

$$\begin{aligned}\frac{d\lambda}{dA} &= \frac{-\lambda\left(2\sigma_u\lambda-\sigma_z\right)\left(2\sigma_u\lambda+\sigma_z\right)\left(2\lambda A\sigma_u^2+3\right)}{\left(2\lambda A\sigma_u^2+3\right)\left(8A\sigma_u^2\lambda^2+6\lambda-A\sigma_z^2\right)}\\ &= -\lambda\left(2\sigma_u\lambda-\sigma_z\right)\frac{2\sigma_u\lambda+\sigma_z}{8A\sigma_u^2\lambda^2+6\lambda-A\sigma_z^2}\end{aligned}$$

Set $\frac{d\lambda}{dA} = 0$. We then see that the only positive critical point is obtained when $\lambda = \frac{\sigma_z}{2\sigma_u}$, which is independent of A. Clearly, $\frac{d\lambda}{dA} > 0$ if $2\sigma_u\lambda - \sigma_z < 0$, or $\lambda < \frac{\sigma_z}{2\sigma_u}$; and provided that $8A\sigma_u^2\lambda^2 + 6\lambda - A\sigma_z^2 > 0$.
Now, note that the roots of this expression $8A\sigma_u^2\lambda^2 + 6\lambda - A\sigma_z^2 = 0$ are $\lambda = \frac{1}{8A\sigma_u^2}\left(-3 \pm \sqrt{(9 + 8A^2\sigma_u^2\sigma_z^2)}\right)$.
Discard the negative root, and then carry out a sign chart for $\lambda > 0$. Consider the upper bound (from part I) to see what is the value of $f(\lambda, A)$ for this value of λ. The evaluation gives that,

$$4A^2\sigma_u^4\left(\frac{\sigma_z}{2\sigma_u}\right)^4 + 12A\sigma_u^2\left(\frac{\sigma_z}{2\sigma_u}\right)^3 + (9 - A^2\sigma_u^2\sigma_z^2)\left(\frac{\sigma_z}{2\sigma_u}\right)^2 - 3A\sigma_z^2\left(\frac{\sigma_z}{2\sigma_u}\right) - 2\frac{\sigma_z^2}{\sigma_u^2} = \frac{1}{4}\frac{\sigma_z^2}{\sigma_u^2} > 0.$$

On the other hand, for $\lambda = \dfrac{\sqrt{2}\sigma_z}{3\sigma_u}$, we have

$$\begin{aligned} & 4A^2\sigma_u^4\left(\frac{\sqrt{2}\sigma_z}{3\sigma_u}\right)^4 + 12A\sigma_u^2\left(\frac{\sqrt{2}\sigma_z}{3\sigma_u}\right)^3 + (9 - A^2\sigma_u^2\sigma_z^2)\left(\frac{\sqrt{2}\sigma_z}{3\sigma_u}\right)^2 - 3A\sigma_z^2\left(\frac{\sqrt{2}\sigma_z}{3\sigma_u}\right) - 2\frac{\sigma_z^2}{\sigma_u^2} \\ = \; & -\frac{1}{81}\sigma_z^3\frac{A}{\sigma_u}\left(2A\sigma_z\sigma_u + 9\sqrt{2}\right) < 0. \end{aligned}$$

This shows that the unique root is bracketed between $\frac{\sqrt{2}\sigma_z}{3\sigma_u}$ and $\frac{\sigma_z}{2\sigma_u}$ (by the Intermediate Value Theorem).
To complete the proof, it remains to show that $\frac{\sqrt{2}\sigma_z}{3\sigma_u} - \frac{1}{8A\sigma_u^2}\left(-3 + \sqrt{(9 + 8A^2\sigma_u^2\sigma_z^2)}\right) > 0$ or equivalently

$$\frac{8\sqrt{2}A\sigma_z\sigma_u}{3} + 3 > \left(\sqrt{(9 + 8A^2\sigma_u^2\sigma_z^2)}\right).$$

By the positivity of the expressions, we have

$$\left(\frac{8\sqrt{2}A\sigma_z\sigma_u}{3} + 3\right)^2 - \left(\sqrt{(9 + 8A^2\sigma_u^2\sigma_z^2)}\right)^2 = \frac{56}{9}A^2\sigma_u^2\sigma_z^2 + 16\sqrt{2}A\sigma_z\sigma_u,$$

which is clearly > 0. This shows that the positive root in the quadratic $8A\sigma_u^2\lambda^2 + 6\lambda - A\sigma_z^2$ expression is lower than the lower bound $\frac{\sqrt{2}\sigma_z}{3\sigma_u}$ of λ^*, the solution $f(\lambda, A) = 0$. ∎

Next we show part (b) of Lemma 1. We begin by recalling the one-shot game equilibrium of the Holden-Subrahmanyam (1994) model.

Proposition 3 *(H.S. 1994) In the presence of two risk averse informed traders, a linear equilibrium exists and it is unique. It is characterized by,*

(i)

$$x_1(\tilde{z}) = x_2(\tilde{z}) = \frac{(\tilde{z} - \mu)}{\lambda^{HS}(3 + A\lambda^{HS}\sigma_u^2)} \tag{16}$$

(ii) $p(\tilde{y}) = \mu + \lambda^{HS}\tilde{y}$, *where* $\mu = \bar{z}$, *and* λ^{HS} *is the unique strictly positive root of the following quadric equation:*

$$A^2\sigma_u^4\lambda^{\mathbf{4}} + 6A\sigma_u^2\lambda^{\mathbf{3}} + 9\lambda^{\mathbf{2}} - 2A\sigma_z^2\lambda - 2\frac{\sigma_z^2}{\sigma_u^2} = 0. \tag{17}$$

We move now to prove the relation between λ^* and λ^{HS}. Note that (17) can also be considered as an algebraic curve $g\left(A, \lambda; \sigma_u, \sigma_z\right) = 0$, where σ_u and σ_z as viewed as parameters. By implicit differentiation, we obtain

$$\frac{d\lambda}{dA} = -\frac{\frac{\partial g}{\partial A}}{\frac{\partial g}{\partial \lambda}},$$

where $\frac{\partial g}{\partial \lambda}$ must not be zero. Thus, we have

$$\frac{d\lambda}{dA} = \frac{-\left(2A\sigma_u^4\lambda^4 + 6\sigma_u^2\lambda^3 - 2\sigma_z^2\lambda\right)}{4A^2\sigma_u^4\lambda^3 + 18A\sigma_u^2\lambda^2 + 18\lambda - 2A\sigma_z^2}.$$

Note that the numerator has one positive root - by Descartes' rule (since we are interested in $A \geq 0$); and we can write this equation as

$$\frac{d\lambda}{dA} = \frac{-\lambda\left(\lambda^2\sigma_u^2\left(\lambda A\sigma_u^2 + 3\right) - \sigma_z^2\right)}{\lambda\left(\lambda A\sigma_u^2 + 3\right)\left(2\lambda A\sigma_u^2 + 3\right) - A\sigma_z^2}.$$

Observe that $\frac{d\lambda}{dA} = 0$, implies that

$$\lambda\left(\lambda^2\sigma_u^2\left(\lambda A\sigma_u^2 + 3\right) - \sigma_z^2\right) = 0.$$

Solving this equation for A, gives $A = -\frac{1}{\lambda^3}\frac{3\lambda^2\sigma_u^2 - \sigma_z^2}{\sigma_u^4}$. Put this expression for A into the quartic to find λ, we then obtain $\lambda = \frac{1}{2}\frac{\sigma_z}{\sigma_u}$ or $\lambda = -\frac{1}{2}\frac{\sigma_z}{\sigma_u}$. Since $\lambda > 0$, we see that $g\left(A, \lambda\right) = 0$, for

$$(A, \lambda) = \left(-\frac{1}{\lambda^3}\frac{3\lambda^2\sigma_u^2 - \sigma_z^2}{\sigma_u^4}, \frac{1}{2}\frac{\sigma_z}{\sigma_u}\right) = \left(-\frac{1}{\left(\frac{1}{2}\frac{\sigma_z}{\sigma_u}\right)^3}\frac{3\left(\frac{1}{2}\frac{\sigma_z}{\sigma_u}\right)^2\sigma_u^2 - \sigma_z^2}{\sigma_u^4}, \frac{1}{2}\frac{\sigma_z}{\sigma_u}\right) = \left(\frac{2}{\sigma_z\sigma_u}, \frac{1}{2}\frac{\sigma_z}{\sigma_u}\right).$$

On the other hand, solve (17) for A to obtain $A = \frac{1}{2\lambda^3\sigma_u^4}\left(-6\lambda^2\sigma_u^2 + 2\sigma_z^2 \pm 2\sqrt{\left(-4\lambda^2\sigma_u^2\sigma_z^2 + \sigma_z^4\right)}\right)$.

In order to have real roots, we need $-4\lambda^2\sigma_u^2\sigma_z^2+\sigma_z^4 \geq 0$, that is $-4\lambda^2\sigma_u^2\sigma_z^2+\sigma_z^4$ $= -\sigma_z^2\left(2\lambda\sigma_u - \sigma_z\right)\left(2\lambda\sigma_u + \sigma_z\right) \geq 0$, or equivalently, $\left(2\lambda\sigma_u - \sigma_z\right)\left(2\lambda\sigma_u + \sigma_z\right) \leq 0$. This means that $\left(2\lambda\sigma_u - \sigma_z\right) \leq 0,$ that is $\lambda \leq \frac{1}{2}\frac{\sigma_z}{\sigma_u}$.
So this bound is the maximal value of λ. By the analysis above, this happens when $A = \frac{2}{\sigma_z\sigma_u}$. We now study the sign of $\frac{d\lambda}{dA}$. Evaluate $\frac{d\lambda}{dA}$ at $A = 0$. We get

$$\frac{d\lambda}{dA}|_{A=0} = \frac{-\left(6\sigma_u^2\lambda^3 - 2\sigma_z^2\lambda\right)}{18\lambda} = -\frac{1}{3}\lambda^2\sigma_u^2 + \frac{1}{9}\sigma_z^2.$$

Now when $A = 0$, $g\left(0, \lambda\right) = 9\lambda^2 - 2\frac{\sigma_z^2}{\sigma_u^2} = 0$, which has the solutions $\lambda = \pm\frac{1}{3}\sqrt{2}\frac{\sigma_z}{\sigma_u}$. Since $\lambda > 0$, we get that

$$\frac{d\lambda}{dA}|_{A=0} = \frac{-\left(6\sigma_u^2\lambda^3 - 2\sigma_z^2\lambda\right)}{18\lambda} = -\frac{1}{3}\left(\frac{1}{3}\sqrt{2}\frac{\sigma_z}{\sigma_u}\right)^2\sigma_u^2 + \frac{1}{9}\sigma_z^2 = \frac{1}{27}\sigma_z^2 > 0.$$

On the other hand, one can show that the $\frac{d\lambda}{dA}|_{A=\frac{3}{\sigma_z\sigma_u}} < 0$. Thus, the derivative at $A = 0$ is positive, and to the right of A at $A = \frac{3}{\sigma_z\sigma_u}$ is negative. This shows that the plane curve described by $g\left(A, \lambda\right) = 0$, has a max given by $\left(A, \lambda\right) = \left(\frac{2}{\sigma_z\sigma_u}, \frac{1}{2}\frac{\sigma_z}{\sigma_u}\right)$. So we can conclude that the algebraic curve $g = 0$ is unimodal. In fact, this can be verified by computing the second derivative of lambda with respect to A, which evaluates at $\left(\frac{2}{\sigma_u\sigma_z}, \frac{\sigma_z}{2\sigma_u}\right)$, is equal to $-\frac{1}{128}(\sigma_z)^3(\sigma_u) < 0$. This shows that it is concave down on the region containing the maximum point.

Now, computing the first derivative of λ^* with respect to A when $A = 0$, we obtain

$$\begin{aligned}
\frac{d\lambda^*}{dA}|_{A=0} &= -\frac{1}{6}\left(2\lambda\sigma_u - \sigma_z\right)\left(2\lambda\sigma_u + \sigma_z\right) \\
&= -\frac{1}{6}\left(2\left(\frac{1}{3}\sqrt{2}\frac{\sigma_z}{\sigma_u}\right)\sigma_u - \sigma_z\right)\left(2\left(\frac{1}{3}\sqrt{2}\frac{\sigma_z}{\sigma_u}\right)\sigma_u + \sigma_z\right) \\
&= -\frac{1}{54}\sigma_z^2\left(2\sqrt{2} - 3\right)\left(2\sqrt{2} + 3\right) = \frac{\sigma_z^2}{54} > 0
\end{aligned}$$

This show that that the initial slope in Holden and Subrahmanyam(1994) case is exactly twice of that obtained in our case.
Consider now the difference $h = g - f$, we get

$$h\left(A, \lambda\right) = \lambda A\left(-3A\lambda^3\sigma_u^4 - 6\lambda^2\sigma_u^2 + \sigma_z^2 + A\lambda\sigma_u^2\sigma_z^2\right)$$

So to find the intersection of f and g, h must vanish. Then either $\lambda A = 0$ or $-3A\lambda^3\sigma_u^4 - 6\lambda^2\sigma_u^2 + \sigma_z^2 + A\lambda\sigma_u^2\sigma_z^2 = 0$. The cubic polynomial has one sign change, and therefore, it has one positive root by Descarte's sign rule.

We show in the table below that at $A = \frac{3}{\sigma_z\sigma_u}$, $f < g$, but that at $A = \frac{4}{\sigma_z\sigma_u}$, $f > g$. It also shows the intersection point .

$\lambda \backslash A$	$\frac{3}{\sigma_z\sigma_u}$	$\frac{4}{\sigma_z\sigma_u}$	$\frac{3.5}{\sigma_z\sigma_u}$
λ from $f = 0$	$.492\,91\frac{\sigma_z}{\sigma_u}$	$.494\,81\frac{\sigma_z}{\sigma_u}$	$.493\,97\frac{\sigma_z}{\sigma_u}$
λ from $g = 0$	$.497\frac{\sigma_z}{\sigma_u}$	$.490\,51\frac{\sigma_z}{\sigma_u}$	$.494\,02\frac{\sigma_z}{\sigma_u}$

Note that the intersection point is roughly $(A^*, \lambda^*) = \left(\frac{3.5}{\sigma_z\sigma_u}, .494\,\frac{\sigma_z}{\sigma_u}\right)$.
Summarizing, we have that

1) the initial slope of g is bigger than the initial slope of f;

2) g is unimodal, it has one critical point which is a maximum occuring at $A = \frac{2}{\sigma_u\sigma_z}$;

3) the intersection point of f and g occurs to the right of the maximum of g, at roughly $A^* \simeq \frac{3.5}{\sigma_u\sigma_z}$;

4) $h = 0$ at two points.

Then it follows that there is one point (A^*, λ^*), where $\lambda < \lambda_{HS}$ for $A < A^*$, and $\lambda > \lambda_{HS}$ for $A > A^*$. ∎

Appendix C: proof of Proposition 2

Recall that the conditional variance in Tighe (1989) is given by

$$var(\tilde{z}|\tilde{y}) = \frac{1}{3}\sigma_z^2 \tag{18}$$

Computing the conditional variances in our model and in the Holden-Subrahmanyam(1994) model, we obtain respectively

$$var(\tilde{z}|\tilde{y}) = \frac{(3k^2 + 2k)\sigma_z^2 - \lambda^2(2k+1)^2\sigma_u^2}{(2k+1)^2}, \tag{19}$$

where $k = 1 + A\lambda\sigma_u^2$, and

$$var(\tilde{z}|\tilde{y}) = \frac{[(3 + A\lambda\sigma_u^2)^2 - 4]\sigma_z^2 - \lambda^2(3 + A\lambda\sigma_u^2)^2\sigma_u^2}{(3 + A\lambda\sigma_u^2)^2}. \tag{20}$$

We begin first by showing that (18) is less than (19). Indeed, combining (19) with (13) and after some simplifications, the problem becomes equivalent to showing that

$$\frac{1}{3} < \frac{3 + 5A\lambda\sigma_u^2 + 2\lambda^2 A^2 \sigma_u^4}{\left(2\left(1 + A\lambda\sigma_u^2\right) + 1\right)^2}. \tag{21}$$

Plugging the lower (upper) bound of λ^* found in Lemma 2 in the numerator (denominator) of the right hand side of (21), we obtain,

$$\frac{3 + 5A\sigma_u^2\left(\frac{\sqrt{2}}{3}\frac{\sigma_z}{\sigma_u}\right) + 2\left(\frac{\sqrt{2}}{3}\frac{\sigma_z}{\sigma_u}\right)^2 A^2\sigma_u^4}{\left(2\left(1 + A\left(\frac{\sigma_z}{2\sigma_u}\right)\sigma_u^2\right) + 1\right)^2} < \frac{3 + 5A\lambda\sigma_u^2 + 2\lambda^2 A^2\sigma_u^4}{\left(2\left(1 + A\lambda\sigma_u^2\right) + 1\right)^2}.$$

The left hand side simplifies to

$$\frac{1}{3} + \frac{\left(\left(15\sqrt{2} - 18\right) A\sigma_u\sigma_z + \sigma_z^2\sigma_u^2 A^2\right)}{9\left(3 + A\sigma_z\sigma_u\right)^2} > \frac{1}{3}.$$

This completes the first part of the proof. It remains to show that the conditional variance in our model is less than the conditional variance in Holden and Subrahmanyam model (1994). Indeed, combining (19) with (13) and combining (20) with (17), the problem reduces to showing that

$$\frac{1 + A\lambda^*\sigma_u^2}{3 + 2A\lambda^*\sigma_u^2} \leq \frac{A\lambda^{HS}\sigma_u^2 + 1}{A\lambda^{HS}\sigma_u^2 + 3}. \tag{22}$$

In order to complete the proof, we need to show first the following result.

Lemma 3 $\lambda^{HS} \in \left[\frac{\sigma_z}{4\sigma_u}, \frac{\sigma_z}{2\sigma_u}\right]$

Proof: Consider the equation

$$g(\lambda) = A^2\sigma_u^4\lambda^4 + 6A\sigma_u^2\lambda^3 + 9\lambda^2 - 2A\sigma_z^2\lambda - 2\frac{\sigma_z^2}{\sigma_u^2}, \tag{23}$$

such that $g(\lambda^{HS}) = 0$. Note that

$$\begin{aligned} g\left(\frac{\sigma_z}{4\sigma_u}\right) &= A^2\sigma_u^4\left(\frac{\sigma_z}{4\sigma_u}\right)^4 + 6A\sigma_u^2\left(\frac{\sigma_z}{4\sigma_u}\right)^3 + 9\left(\frac{\sigma_z}{4\sigma_u}\right)^2 - 2A\sigma_z^2\left(\frac{\sigma_z}{4\sigma_u}\right) - 2\frac{\sigma_z^2}{\sigma_u^2} \\ &= \frac{1}{256}A^2\sigma_z^4 - \frac{13}{32}\frac{A}{\sigma_u}\sigma_z^3 - \frac{23}{16}\frac{\sigma_z^2}{\sigma_u^2} = \frac{1}{256}\sigma_z^2\frac{A^2\sigma_z^2\sigma_u^2 - 104A\sigma_z\sigma_u - 368}{\sigma_u^2}. \end{aligned}$$

Consider the numerator $A^2\sigma_z^2\sigma_u^2 - 104A\sigma_z\sigma_u - 368 = 0$. It has the solutions $A = 4\frac{13+8\sqrt{3}}{\sigma_z\sigma_u} = \frac{107.43}{\sigma_z\sigma_u}$ or $A = 4\frac{13-8\sqrt{3}}{\sigma_z\sigma_u} = -\frac{3.4256}{\sigma_z\sigma_u}$. Now, the derivative

$$\frac{d\left(\frac{1}{256}\sigma_z^2\frac{A^2\sigma_z^2\sigma_u^2-104A\sigma_z\sigma_u-368}{\sigma_u^2}\right)}{dA} = \frac{1}{128}\frac{\sigma_z^3}{\sigma_u}\left(A\sigma_z\sigma_u - 52\right).$$

So for $0 \leq A < \frac{52}{\sigma_z\sigma_u}$, the derivative is negative and $g\left(\frac{\sigma_z}{4\sigma_u}\right)$ (at this value only) is decreasing in A for $0 \leq A < \frac{52}{\sigma_z\sigma_u}$. As also for $A = 0$, $g\left(\frac{\sigma_z}{4\sigma_u}\right) = 9\left(\frac{\sigma_z}{4\sigma_u}\right)^2 - 2\frac{\sigma_z^2}{\sigma_u^2} = -\frac{23}{16}\frac{\sigma_z^2}{\sigma_u^2}$ is negative, we can conclude that $\frac{\sigma_z}{4\sigma_u}$ is a lower bound for λ. Similarly, it follows from what we had before,

$$\begin{aligned}
& A^2\sigma_u^4\left(\frac{\sigma_z}{2\sigma_u}\right)^4 + 6A\sigma_u^2\left(\frac{\sigma_z}{2\sigma_u}\right)^3 + 9\left(\frac{\sigma_z}{2\sigma_u}\right)^2 - 2A\sigma_z^2\left(\frac{\sigma_z}{2\sigma_u}\right) - 2\frac{\sigma_z^2}{\sigma_u^2} \\
= \; & \frac{1}{16}\sigma_z^2\frac{A^2\sigma_z^2\sigma_u^2 - 4A\sigma_z\sigma_u + 4}{\sigma_u^2} = \frac{1}{16}\sigma_z^2\frac{\left(A\sigma_z\sigma_u - 2\right)^2}{\sigma_u^2} > 0.
\end{aligned}$$

Thus, by the Intermediate Value Theorem, the proof of Lemma 3 is complete.
■

We turn now to show that (22) holds. Since the expressions in (22) are both monotone increasing in λ, plugging the lower bound of Lemma 3 in the right hand side of (22) ; and plugging the upper bound of Lemma 3 in the left hand side of (22) , and then subtracting the resulting expressions from each others, we obtain

$$\frac{1}{2}\frac{2 + A\sigma_z\sigma_u}{3 + A\sigma_z\sigma_u} - \frac{A\sigma_z\sigma_u + 4}{A\sigma_z\sigma_u + 12} = -\frac{1}{2}\frac{\left(A\sigma_z\sigma_u\right)^2}{\left(3 + A\sigma_z\sigma_u\right)\left(A\sigma_z\sigma_u + 12\right)} < 0. \qquad (24)$$

This completes the proof of Proposition 2. ■

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