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A Dynamic Model of the Choice of Technology in Economic Development

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Abstract

In this overlapping-generations model, there is unemployment in the manufacturing sector. Manufacturing firms engage in oligopolistic competition and choose technologies to maximize profits. With capital as fixed costs of production, increasing returns in the manufacturing sector exist. In the unique steady state, first, when individuals become more patient, the saving rate increases while the level of income of an individual decreases. Second, an increase in population or percentage of income spent on the manufactured good does not change steady-state technology while decreases the level of income of an individual. Third, an increase in the wage rate leads a manufacturing firm to choose a more advanced technology and the steady-state capital stock increases. Finally, an increase in the level of subsidy to technology adoption does not change steady-state technology.

Keywords: choice of technology, overlapping-generations model, unemployment, economic development, increasing returns

JEL Classification Numbers: O10, D43

1. Introduction

For a developing country, one advantage of late development is that there is a large stock of existing technologies available for adoption. While modern capital intensive technologies are very productive, they may not be the best choices for developing countries with limited capital and widespread unemployment. The choice of technology is an interesting issue in economic development (Sen, 1960; Stewart, 1977; White, 1978; Otsuka, Ranis, and Saxonhouse, 1988). A country's choice of technology is likely to be affected by the level of capital stock, which is endogenously determined by the amount of saving. With the existence of high levels of fixed costs of production, modern technologies display significant degrees of increasing returns (Chandler, 1990). Thus, it is interesting to study a dynamic model of the choice of increasing technologies in which individuals choose saving optimally and unemployment exists.

This paper studies a dynamic general equilibrium model of technology choice in economic development in which manufacturing firms engage in oligopolistic competition and there is unemployment in the manufacturing sector. In this model, each individual lives for two

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periods. While an individual works only in the first period, he derives utility from the consumption of the agricultural good and the manufactured good in both periods. The agricultural good is produced by labor with constant returns. The manufactured good is produced by capital and labor. A manufacturing firm chooses from a continuum of technologies to maximize its profit. A more advanced technology is associated with a higher amount of capital as fixed costs of production and a lower amount of labor as marginal cost of production. With capital as fixed costs of production, there are increasing returns in the manufacturing sector.

Following Harris and Todaro (1970), we assume that the wage rate in the manufacturing sector is exogenously given. This assumption of a rigid manufacturing wage rate is used to capture the observation that labor markets in developing countries may not function well. One prominent feature for developing countries is that high percentages of workers are employed in the informal sector (Rauch, 1993, Frankel, 2005). Wage rigidity contributes to the existence of a significant informal sector. Wage rigidity could be the result of government regulations, the existence of labor unions, or adverse selection in the labor market (Bencivenga and Smith, 1997). Alternatively, the wage rate can be viewed as given in a Lewis type model in which a large amount of surplus labor exists.[1] The assumption that the wage rate can be stagnant for a long period of time is also supported by empirical research, such as Zhang, Yang, and Wang (2011). They study the wage rate during China's economic development and show that China's wage rate was stagnant before year 2000.

We show that there exists a unique steady state. In the steady state, an increase in the degree of patience of an individual increases the saving rate, but does not change the level of technology for a manufacturing firm. Actually, the employment rate in the manufacturing sector decreases and the level of utility of a consumer decreases. The reason is as follows. When an individual tries to save more, the demand for the manufactured good decreases and thus the derived demand for labor also decreases. An individual does not benefit from this increase in the saving rate because the level of income of an individual decreases.

In the steady state, an increase in population does not induce a manufacturing firm to adopt technologies using more labor. The reason is as follows. With the rigid wage rate, a

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[1] The wage rate will increase when surplus labor is absorbed. Zhang, Yang, and Wang (2011) show that China reached the Lewis turning point in about year 2000 and the wage rate began to rise since then.

higher population does not translate into a lower wage rate. As a result, the level of technology does not change with the size of the population. Since an increase in population decreases the employment rate in the manufacturing sector, a worker's income decreases with the size of the population. While population growth leads to many problems for developing countries, one potential benefit is that it could lead to a lower wage and helps the development of labor-intensive sectors. In this model, with the existence of a rigid wage rate, an increase in population does not lead to the development of labor intensive sectors and the potential benefit of population growth is not realized, as shown in Kotwal, Ramaswami, and Wadhwa (2011) for India.

Economic development is associated with structural changes such as a shift of spending from agricultural goods to manufactured goods. To address this shift of demand, we study the impact of an increase in the percentage of income spent on the manufactured good. In the steady state, the level of technology in the manufacturing sector does not change and the steady-state capital stock of this economy does not change. Also, the employment rate in the manufacturing sector does not change. Thus, an increase in the percentage of income spent on the manufactured good does not induce firms to choose more advanced technologies if manufacturing wage is rigid.

In the steady state, an increase in the manufacturing wage rate leads a firm to choose a more advanced technology. Also, the steady-state level of capital stock of this economy increases. However, the impact of an increase in the manufacturing wage rate on the employment rate in the manufacturing sector is ambiguous. As a result, the impact of a change in the manufacturing wage rate on the level of utility of a consumer is ambiguous. The impact of minimum wage legislation has been debated frequently in the literature. In this model, suppose the rigid wage rate is positively related to the minimum wage rate. Since an increase in the minimum wage rate increases the level of capital stock, minimum wage law legislation may not decrease a consumer's utility.

In this model, through a lump-sum tax on workers, the government may provide a subsidy (or tax) to manufacturing firms for their adoption of increasing returns technologies. In the steady state, we show that a change in the level of subsidy to technology adoption does not change steady-state technology.

In the literature, Drazen and Eckstein (1988) have studied an overlapping-generations model of economic development. There are some significant differences between their model and this one. First, the type of market structure in the manufacturing sector is different. In their model, manufacturing firms engage in perfect competition. In this model, manufacturing firms engage in oligopolistic competition. Second, the focus of their paper is different from that in this one. In their model, they focus on the impact of different types of the organization of the rural market on equilibrium variables such as the stock of capital and per capita consumption. In this model, we focus on how equilibrium variables such as the level of technology are affected by factors such as the size of the population and the level of government subsidy.

The choice of technology in a model of rural-urban migration is studied by Gang and Gangopadhyay (1987) and Zhou (2013, 2015). In Gang and Gangopadhyay (1987), there are constant returns in the manufacturing sector. In Zhou (2013, 2015), there are increasing returns in the manufacturing sector. While the above models are one-period models, this is a dynamic model with the amount of capital endogenously determined. This dynamic model is useful in addressing issues such as the size of the population on the amount of saving and thus the steady-state capital stock. While the size of the population can affect the choice of technology directly in a one-period model, the size of the population may affect the choice of technology and the level of employment in the manufacturing sector indirectly in a dynamic model through its influence on the capital stock.

The plan of the paper is as follows. Section 2 specifies the model. Section 3 studies the properties of the steady state through comparative statics. Section 4 addresses impact of government policies. Section 5 concludes.

2. Specification of the model

We study an overlapping-generations model. An individual lives for two periods: young and old. In each period, L young will be born and L old will die. Thus, the total size of the population does not change over time. There are two sectors: the agricultural sector and the manufacturing sector. In this model, subscripts are used to denote time periods and superscripts are used to denote sectors.

For the agricultural sector, the agricultural good is produced by labor with constant returns. Without further loss of generality, we assume that each individual employed in the

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agricultural sector produces one unit of the agricultural good. Thus, if the level of employment in the agricultural sector in period t is L_t^a, the level of the agricultural good produced is L_t^a. The agricultural good is used for consumption only.

For the manufacturing sector, the manufactured good can be used either for consumption or as investment to form capital. Each unit of the manufactured good is able to produce one unit of capital. For simplicity, we assume there is no depreciation of capital. The government provides a subsidy of g for each unit of capital used by a manufacturing firm (if g is negative, it is interpreted as a tax on the usage of capital).[2] To finance this subsidy, a lump-sum tax of b on each worker is imposed.

The wage rate in the manufacturing sector is exogenously given at $\overline{w}$. With the manufacturing wage rate higher than the labor market clearing wage rate, there is unemployment in the manufacturing sector. The employment rate in the manufacturing sector in period t is e_t. Here we interpret e_t as the percentage of time that an individual employed in the manufacturing sector has a job. With this interpretation, each individual in the manufacturing sector has a disposable income of $e_t \overline{w} - b$. This interpretation is useful so that all individuals in the manufacturing sector have positive consumption.

An individual is endowed with labor only in the first period. In each period, an individual derives utility from the consumption of the both goods. A consumer's utility is separable in the two periods. Let ρ denote the discount factor. For an individual born in period t, this individual's utility function is specified as

$$U(c_{t,1}^a, c_{t,1}^m, c_{t+1,2}^a, c_{t+1,2}^m) = \theta \ln c_{t,1}^a + (1-\theta) \ln c_{t,1}^m + \rho\theta \ln c_{t+1,2}^a + \rho(1-\theta) \ln c_{t+1,2}^m. \qquad (1)$$

In equation (1), $\theta \in (0,1)$, $c_{t,1}^a$ is the consumption of the agricultural good while $c_{t,1}^m$ is the consumption of the manufactured good when this individual is young, and $c_{t+1,2}^a$ is the consumption of the agricultural good while $c_{t+1,2}^m$ is the consumption of the manufactured good when this individual is old.

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[2] If g is specified as the percentage (rather than absolute level) of interest costs for a manufacturing firm, the result will be similar.

The interest rate in period $t+1$ is r_{t+1}. The price of the manufactured good in period t is p_t^m and the price of the agricultural good is p_t^a. This individual faces the following budget constraint:

$$p_t^a c_{t,1}^a + p_t^m c_{t,1}^m + \frac{p_{t+1}^a c_{t,2}^a}{1+r} + \frac{p_{t+1}^m c_{t,2}^m}{1+r} = e_t \overline{w} - b .\tag{2}$$

This individual chooses the amounts of consumption in the two periods $c_{t,1}^a$, $c_{t,1}^m$, $c_{t+1,2}^a$, and $c_{t+1,2}^m$ to maximize utility (1) subject to budget constraint (2). Utility maximization leads to

$$c_{t,1}^a = \frac{\theta}{(1+\rho)p_t^a}(e_t \overline{w} - b) ,\tag{3}$$

$$c_{t,1}^m = \frac{1-\theta}{(1+\rho)p_t^m}(e_t \overline{w} - b) ,\tag{4}$$

$$c_{t+1,2}^a = \frac{\rho(1+r)\theta}{(1+\rho)p_{t+1}^a}(e_t \overline{w} - b) ,\tag{5}$$

$$c_{t+1,2}^m = \frac{\rho(1+r)(1-\theta)}{(1+\rho)p_{t+1}^m}(e_t \overline{w} - b) .\tag{6}$$

From equations (4) and (6), the absolute value of this individual's elasticity of demand for the manufactured good is one. Also, this individual's total spending in the first period is

$$p_t^a c_{t,1}^a + p_t^m c_{t,1}^m = \frac{1}{1+\rho}(e_t \overline{w} - b) .\tag{7}$$

The amount of saving for a young individual is s_t. From equations (2) and (7), the amount of saving can be expressed as

$$s_t = s_t\left(e_t \overline{w}, b, \rho\right) = \frac{\rho}{1+\rho}(e_t \overline{w} - b) .\tag{8}$$

Partial differentiation of equation (8) leads to $\frac{\partial s_t}{\partial e_t} > 0$, $\frac{\partial s_t}{\partial w} > 0$, and $\frac{\partial s_t}{\partial \rho} > 0$. That is, the amount of saving of an individual increases when the level of income of this individual increases. Also, an individual saves a higher percentage of income when this individual becomes more patient.[3]

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[3] In this model, the amount of saving of an individual does not change with the interest rate. As discussed in Romer (2006, p. 80), when the interest rate increases, the income effect tends to decrease while the substitution effect tends

In each period, there are m_t identical firms producing the manufactured good, and $m_t > 0$. To produce the manufactured good, following Zhou (2009, 2013, 2015), we assume that there is a continuum of technologies indexed by the level of capital employed by a manufacturing firm. A technology with a higher amount of capital indicates a more advanced technology. For the manufacturing sector, capital is specified as the fixed cost and labor is specified as the marginal cost of production. When the amount of capital used by a manufacturing firm is k, the corresponding marginal cost in terms of labor units is $\beta(k)$. To capture the substitution between fixed and marginal costs of production, we assume that a technology with a higher amount of capital has a lower marginal cost of production.[4] That is, $\beta'(k) < 0$. In addition, we assume that $\beta''(k) > 0$. That is, when the amount of capital used by a manufacturing firm increases, the marginal cost decreases at a decreasing rate.

The level of output for a manufacturing firm is q_t. Since the manufactured good can be used as capital, the price of capital is equal to the price of the manufactured good. For a manufacturing firm, its revenue is $p_t^m q_t$, cost of labor is $\overline{w}\beta(k_t)q_t$, and cost of capital is $(1 + r_t - g)p_{t-1}^m k_t - p_t^m k_t$. Thus, its profit is equal to

$$p_t^m q_t - \overline{w}\beta(k_t)q_t + p_t^m k_t - (1 + r_t - g)p_{t-1}^m k_t. \tag{9}$$

Manufacturing firms engage in Cournot competition. A manufacturing firm takes the wage rate, the subsidy rate, and the interest rate as given and chooses the levels of output and technology to maximize its profit. The first order condition for a firm's optimal choice of output is

$$p_t^m + q_t \frac{\partial p_t^m}{\partial q_t} - \overline{w}\beta(k_t) = 0. \tag{10}$$

The first order condition for a firm's optimal choice of technology is

$$-\overline{w}\beta'(k_t)q_t + p_t^m - (1 + r_t - g)p_{t-1}^m = 0. \tag{11}$$

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to increase the amount of saving. With the logarithmic utility function, the two effects cancel out each other and the amount of saving does not change with the interest rate.

[4] For empirical evidence supporting the assumption that marginal cost of labor decreases when the amount of capital used increases, see the choices of technologies in three manufacturing sector in Prendergast (1990) and the choice of technology in the transportation sector in Levinson (2006).

Firms will enter the manufacturing sector until the profit of a firm is zero.[5] Zero profit for a manufacturing firm requires

$$p_t^m q_t - \overline{w}\beta(k_t)q_t + p_t^m k_t - (1 + r_t - g)p_{t-1}^m k_t = 0. \tag{12}$$

For the market for the manufactured good, the total demand is the sum of the amount used for capital accumulation and the demand for consumption. The amount of manufactured good used for capital accumulation is $K_{t+1} - K_t$. Consumption of the manufactured good by individuals born in period $t-1$ is $Lc_{t-1,2}^m$ and consumption by individuals born in period t is $Lc_{t,1}^m$. Thus, total demand for the manufactured good is $K_{t+1} - K_t + L(c_{t-1,2}^m + c_{t,1}^m)$. Each of the m_t firms supplies q_t units of the manufactured good and total supply of the manufactured good is $m_t q_t$. The clearance of the market for the manufactured good requires that

$$K_{t+1} - K_t + L\left(c_{t-1,2}^m + c_{t,1}^m\right) = m_t q_t. \tag{13}$$

Since a consumer has a constant elasticity of demand for the manufactured good and a firm takes the output of other firms as given when it chooses its level of output in a Cournot competition, partial differentiation of equation (13) leads to $\dfrac{\partial p_t^m}{\partial q_t} = \dfrac{p_t^m}{m_t q_t - (K_{t+1} - K_t)}$. Plugging this result into the condition for a manufacturing firm's optimal choice of output (10) leads to

$$p_t^m\left(1 - \frac{q_t}{m_t q_t - (K_{t+1} - K_t)}\right) - \overline{w}\beta(k_t) = 0. \tag{14}$$

For the market for the agricultural good, consumption of the agricultural good by individuals born in period $t-1$ is $Lc_{t-1,2}^a$ and consumption by individuals born in period t is $Lc_{t,1}^a$. Thus, total demand for the agricultural good is $L(c_{t-1,2}^a + c_{t,1}^a)$. The total supply of the agricultural good is L_t^a. The clearance of the market for the agricultural good requires that

$$L(c_{t-1,2}^a + c_{t,1}^a) = L_t^a. \tag{15}$$

The return for an individual employed in the manufacturing sector is $e_t \overline{w}$. Since each individual is able to produce one unit of the agricultural good, the return of an individual employed in the agricultural sector is p_t^a. Because an individual can move between the

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[5] For examples of models in which firms engaging in Cournot competition with zero profits, see Mankiw and Whinston (1986), Zhang (2007), and Chen and Shieh (2011).

agricultural sector and the manufacturing sector, the returns in the two sectors should be equal in equilibrium (Harris and Todaro, 1970; Zhang, 2002):

$$e_t \overline{w} - b = p_t^a - b .$$
(16)

For the labor market, total demand for labor is the sum of the demand from the agricultural sector L_t^a and the demand from the manufacturing sector $\beta(k_t)m_t q_t / e_t$. Thus, total demand for labor is $\dfrac{\beta(k_t)m_t q_t}{e_t} + L_t^a$. Total supply of labor is L. The clearance of the labor market requires that

$$\frac{\beta(k_t)m_t q_t}{e_t} + L_t^a = L .$$
(17)

For the market for capital, each of the m_t firms demands k_t units of capital and total demand for capital in this economy is $m_t k_t$. Total supply of capital in this economy is K_t. The clearance of the market for capital requires that

$$m_t k_t = K_t .$$
(18)

For the market for assets, total demand for assets is $Ls_t(e_t \overline{w}, b, \rho)$, which is equal to $\dfrac{L\rho}{1+\rho}(e_t \overline{w} - b)$. Total supply of assets is K_{t+1}. The clearance of the market for assets requires that

$$\frac{L\rho}{1+\rho}(e_t \overline{w} - b) = K_{t+1} .$$
(19)

The government's cost of subsidy is gK_t and its tax revenue is bL. For the government's budget to be balanced in each period, we need

$$gK_t = bL .$$
(20)

For given values of the initial capital stock K_0, the equilibrium path for this economy is solved by equations (11)-(20). For the rest of the paper, the manufactured good is used as the numeraire: $p_t^m \equiv 1$.

3. The steady state

In this section, we study the properties of the steady state. For variables associated with the steady state, we drop the time subscript.

From equations (11)-(20), the steady state of this economy is characterized by the following set of ten equations defining ten variables m, r, e, L^a, p^a, p^m, k, q, b, and K as functions of exogenous parameters:[6]

$$-\overline{w}\beta'(k)q - r + g = 0,\tag{11*}$$

$$q - \overline{w}\beta(k)q - (r - g)k = 0,\tag{12*}$$

$$(1-\theta)L(e\overline{w}-b)\left(\frac{1+\rho(1+r)}{1+\rho}\right) = mq,\tag{13*}$$

$$1 - \frac{1}{m} - \overline{w}\beta(k) = 0,\tag{14*}$$

$$\theta L(e\overline{w}-b)\left(\frac{1+\rho(1+r)}{1+\rho}\right) = p^a L^a,\tag{15*}$$

$$e\overline{w} = p^a,\tag{16*}$$

$$\frac{\beta(k)mq}{e} + L^a = L,\tag{17*}$$

$$mk = K,\tag{18*}$$

$$\frac{\rho}{1+\rho}L(e\overline{w}-b) = K,\tag{19*}$$

$$gK = bL.\tag{20*}$$

To derive properties of the steady state, we may log-linearize the system of equations (11*)-(20*) around the steady state (Perko, 2001, pp. 120-121). Here we use a different approach to derive properties of the steady state: we condense the number of ten equations charactering the steady state into a smaller number of equations so that the system is manageable and comparative statics can be conducted. It can be shown that the two approaches lead to the same qualitative results.

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[6] Closed form solutions for variables in the steady state are available if the level of marginal cost in the manufacturing sector is specified. For example, suppose that $\beta(k)=1/k$. With this specification of the marginal cost, solving the system of equations (11*)-(20*) leads to $e* = \frac{4(1+\rho+\rho g)}{\rho L}$, $m* = 2$, $k* = 2\overline{w}$, $K* = 4\overline{w}$, $p_a* = \frac{4(1+\rho+\rho g)}{\rho L}\overline{w}$, $L_a* = \frac{2\theta L}{1+\theta}$, $q* = \frac{4(1-\theta)(1+\rho+\rho g)\overline{w}}{\rho(1+\theta)}$, $b* = \frac{4g}{L}\overline{w}$, and $r* = g + \frac{(1-\theta)(1+\rho+\rho g)}{\rho(1+\theta)}$.

First, from equation (12*), a manufacturing firm's level of output can be expressed as

$$q = \frac{(r-g)k}{1-\beta \overline{w}}. \tag{21}$$

Plugging this value of output q into equation (11*) leads to

$$\overline{w}\beta'(k)k + 1 - \beta\,\overline{w} = 0. \tag{22}$$

Second, substituting the value of m from equation (14*) into equation (18*) and plugging the resulting value of K into equation (19*) leads to

$$\frac{\rho}{1+\rho}L(e\overline{w} - b) - \frac{k}{1-\beta\,\overline{w}} = 0. \tag{23}$$

Third, dividing equation (13*) by equation (15*) leads to $\dfrac{p^a L^a}{p^m m q} = \dfrac{\theta}{1-\theta}$. Plugging the value of q from equation (12*), the value of m from equation (14*), and the value of p^a from equation (16*) into this equation leads to $L^a = \dfrac{\theta(r-g)k}{e\overline{w}(1-\theta)(1-\beta\overline{w})^2}$. Plugging this value of L^a, the value of q from equation (12*), and the value of m from equation (14*) into equation (17*) leads to

$$\theta(r-g)k + (1-\theta)\beta(r-g)k\overline{w} - (1-\theta)eL\overline{w}\left(1-\beta\overline{w}\right)^2 = 0. \tag{24}$$

With the above manipulations, the system of ten equations is now reduced to the following set of three equations defining three variables r, e, and k as functions of exogenous parameters:

$$\Omega_1 \equiv \overline{w}\beta'(k)k + 1 - \beta\,\overline{w} = 0, \tag{22}$$

$$\Omega_2 \equiv \frac{\rho}{1+\rho}Le\overline{w} - \left(1 + \frac{\rho g}{1+\rho}\right)\frac{k}{1-\beta\,\overline{w}} = 0, \tag{23}$$

$$\Omega_3 \equiv \theta(r-g)k + (1-\theta)\beta(r-g)k\overline{w} - (1-\theta)eL\overline{w}\left(1-\beta\overline{w}\right)^2 = 0. \tag{24}$$

Partial differentiation of equations Ω_1, Ω_2, and Ω_3 with respect to k, r, e, ρ, L, θ, g, and $\overline{w}$ leads to[7]

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[7] Equation (22) is used to derive the result that $\partial\Omega_2 / \partial k = 0$.

$$\begin{pmatrix} \dfrac{\partial\Omega_1}{\partial k} & 0 & 0 \\[2mm] 0 & 0 & \dfrac{\partial\Omega_2}{\partial e} \\[2mm] \dfrac{\partial\Omega_3}{\partial k} & \dfrac{\partial\Omega_3}{\partial r} & \dfrac{\partial\Omega_3}{\partial e} \end{pmatrix} \begin{pmatrix} dk \\ dr \\ de \end{pmatrix} = -\begin{pmatrix} 0 \\ \dfrac{\partial\Omega_2}{\partial\rho} \\ 0 \end{pmatrix} d\rho - \begin{pmatrix} 0 \\ \dfrac{\partial\Omega_2}{\partial L} \\ \dfrac{\partial\Omega_3}{\partial L} \end{pmatrix} dL$$

$$-\begin{pmatrix} 0 \\ 0 \\ \dfrac{\partial\Omega_3}{\partial\theta} \end{pmatrix} d\theta - \begin{pmatrix} 0 \\ \dfrac{\partial\Omega_2}{\partial g} \\ \dfrac{\partial\Omega_3}{\partial g} \end{pmatrix} dg - \begin{pmatrix} \dfrac{\partial\Omega_1}{\partial \overline{w}} \\ \dfrac{\partial\Omega_2}{\partial \overline{\overline{w}}} \\ \dfrac{\partial\Omega_3}{\partial \overline{\overline{w}}} \end{pmatrix} d\overline{w} \,. \tag{25}$$

Let Δ denote the determinant of the coefficient matrix of endogenous variables in the system (25): $\Delta \equiv -\dfrac{\partial\Omega_1}{\partial k}\dfrac{\partial\Omega_2}{\partial e}\dfrac{\partial\Omega_3}{\partial r}$.   Partial differentiation of equations (22)-(24) leads to $\dfrac{\partial\Omega_1}{\partial k} > 0$, $\dfrac{\partial\Omega_2}{\partial e} > 0$, and $\dfrac{\partial\Omega_3}{\partial r} > 0$. Thus, $\Delta < 0$.

The following proposition establishes the existence of a unique local steady state.[8]

Proposition 1: There exists a unique local steady-state equilibrium.

Proof: With $\Delta < 0$ and thus nonsingular, there exists a unique equilibrium for the system (25). With k, r, and e determined in the system (25), the steady-state values of other variables can be determined correspondingly. First, from equation (14*), the value of m is determined. Second, from equation (16*), the value of p^a is determined. Third, from equation (18*), the value of K is determined. Fourth, from equation (12*), the value of q is determined. Fifth, with K determined, the value of b is determined by (20*). Finally, from equation (17*), the value of L^a is determined. ∎

With the existence of a unique steady state established, we proceed to conduct comparative statics on the properties of the steady state. The following proposition studies the impact of a change in the degree of patience of an individual.

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[8] Turnovsky (1977, chap. 2) provides conditions for the existence of a unique local equilibrium and a unique global equilibrium and shows that conditions for the existence of a unique global equilibrium are very strict. Thus, we focus on the existence of a unique local equilibrium.

Proposition 2: When the degree of patience of an individual increases, (i) the equilibrium technology of a manufacturing firm does not change, (ii) the interest rate decreases, (iii) the employment rate decreases, (iv) total capital stock does not change, (v) the price of the agricultural good decreases, (vi) the number of firms producing the manufactured good does not change, (vii) the level of output of a manufacturing firm decreases.[9]

Proof: An application of Cramer's rule on the system (25) leads to

$$\frac{dk}{d\rho} = 0,$$

$$\frac{dr}{d\rho} = -\frac{\partial \Omega_1}{\partial k} \frac{\partial \Omega_2}{\partial \rho} \frac{\partial \Omega_3}{\partial e} / \Delta < 0,$$

$$\frac{de}{d\rho} = \frac{\partial \Omega_1}{\partial k} \frac{\partial \Omega_2}{\partial \rho} \frac{\partial \Omega_3}{\partial R} / \Delta < 0,$$

$$\frac{dK}{d\rho} = \frac{d(mk)}{d\rho} = \frac{d\left(k / (1 - \beta \overline{w})\right)}{d\rho} = 0.$$

Since $\dfrac{de}{d\rho} < 0$, from equation (16*), we have $\dfrac{dp^a}{d\rho} < 0$.

Since $\dfrac{dk}{d\rho} = 0$, from equation (18*), we have $\dfrac{dm}{d\rho} = 0$.

Since $\dfrac{dk}{d\rho} = 0$ and $\dfrac{dr}{d\rho} < 0$, from equation (12*), we have $\dfrac{dq}{d\rho} < 0$. ∎

To understand why steady-state technology is not affected by a change in the degree of patience of individuals, it is helpful to study equation (22). From this equation, a firm's equilibrium technology is affected by the manufacturing wage rate only. The reason is as follows. Equation (11) is the first order condition for a firm's optimal choice of technology. In this equation, in additional to the manufacturing wage rate, a firm's choice of technology is affected by its output, interest rate, and the level of government subsidy. However, a firm's output and interest rate are endogenously determined. From equation (21), when interest rate and government subsidy change, a firm's output will change correspondingly. Since the impact

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[9] When the degree of patience of an individual increases, since the per capita consumption of the agricultural good does not change, the number of individuals employed in the agricultural sector does not change.

from a firm's output change will cancel out the impact from interest rate and government subsidy change, a firm's equilibrium technology is affected by the manufacturing wage rate only.

An individual's utility in a period is determined by the price of the agricultural good, the price of the manufactured good, the interest rate, and the level of income. The level of income is the product of the wage rate and the employment rate in the manufacturing sector. Remember that the price of the manufactured good is normalized to one. Since the employment rate and the price of the agricultural good change at the same rate, a decrease in the employment rate decreases a consumer's consumption of the manufactured good while the consumption of the agricultural good does not change. Thus, the level of utility of an individual in the first period decreases. Since the interest rate decreases when the degree of patience of a consumer increases, an individual's consumption of both goods in the second period decrease and the level of utility of an individual in the second period decreases.

The reason that capital stock does not increase with the degree of patience is as follows. While the saving rate increases with the degree of patience, the amount of income decreases when the degree of patience of an individual increases. The amount of income decreases because the employment rate decreases with the degree of patience of an individual: saving more means lower demand for consumption of the manufactured good and thus lower derived demand for labor. Since the amount of saving is the product of the saving rate and the level of income, the amount of saving may not change if the impact from the decrease in the level of income cancels the impact from the increase in the saving rate out. This is the case in this paper in which the amount of steady-state capital stock is not affected by the degree of patience of an individual.

In the literature, there are various studies on the role of saving on East Asian economic development. Does a higher saving rate lead to the takeoff of the East Asian economies? In this model the saving rate is endogenously determined. The saving rate increases with the degree of patience. Proposition 2 shows that an increase in the saving rate may not always be beneficial to an individual: while the saving rate increases, the level of income decreases. Overall, an individual could be worse off.

Will an increase in population lead a manufacturing firm to choose a technology using relatively more labor? The following proposition studies the impact of an increase in population.

Proposition 3: When the size of the population increases, (i) a manufacturing firm's equilibrium technology does not change, (ii) the interest rate does not change, (iii) the employment rate decreases, (iv) the number of manufacturing firms does not change, (v) total capital stock does not change, (vi) the price of the agricultural good decreases, (vii) the level of output of a manufacturing firm does not change.[10]

Proof: Partial differentiation of (23) and (24) leads to

$$\frac{\partial \Omega_2}{\partial e}\frac{\partial \Omega_3}{\partial L} - \frac{\partial \Omega_2}{\partial L}\frac{\partial \Omega_3}{\partial e} = 0 .$$

An application of Cramer's rule on the system (25) leads to

$$\frac{dk}{dL} = 0 ,$$

$$\frac{dr}{dL} = \frac{\partial \Omega_1}{\partial k}\left(\frac{\partial \Omega_2}{\partial e}\frac{\partial \Omega_3}{\partial L} - \frac{\partial \Omega_2}{\partial L}\frac{\partial \Omega_3}{\partial e}\right) / \Delta = 0 ,$$

$$\frac{de}{dL} = \frac{\partial \Omega_1}{\partial k}\frac{\partial \Omega_2}{\partial L}\frac{\partial \Omega_3}{\partial r} / \Delta < 0 ,$$

$$\frac{dm}{dL} = \frac{d\left(1/(1-\beta \overline{w})\right)}{dL} = 0 ,$$

$$\frac{dK}{dL} = \frac{d(mk)}{dL} = \frac{d\left(k/(1-\beta \overline{w})\right)}{dL} = 0 .$$

Since $\frac{d e}{d L} < 0$, from equation (16*), we have $\frac{d p^a}{d L} < 0$.

Since $\frac{dk}{dL} = 0$ and $\frac{dr}{dL} = 0$, from equation (12*), we have $\frac{dq}{dL} = 0$. ∎

Interestingly, the level of output of a manufacturing firm does not change when the size of the population increases. From Proposition 3, an increase in population does not lead to the adoption of technologies using more labor. The reason is that the wage rate in the manufacturing sector is rigid and does not change with population.

An increase in population leads to a decrease in the employment rate. Since the wage rate in the manufacturing sector does not change, the level of income of a worker decreases.

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[10] When the size of the population increases, the number of individuals in the manufacturing sector goes up. Since per capita consumption of the agricultural good does not change and the size of the population is larger, the level of employment in the agricultural sector increases.

Because the consumption of the agricultural good does not change while the consumption of the manufactured good decreases, the level of utility of an individual in the first period decreases. Since the interest rate does not change with population, an individual's income in the second period decreases because the amount of saving is lower.

During the process of economic development, the percentage of income spent on manufactured goods increases. The following proposition studies the impact of an increase in the percentage of income spent on the manufactured good.

Proposition 4: When the percentage of income spent on the manufactured good increases, (i) the level of technology in the manufacturing sector does not change, (ii) the interest rate increases, (iii) the employment rate in the manufacturing sector does not change, (iv) the level of output of a manufacturing firm increases, (v) the number of manufacturing firms does not change, (vi) the price of the agricultural good does not change, (vii) the total steady-state capital stock does not change.

Proof: An application of Cramer's rule on the system (25) leads to

$$\frac{dk}{d\theta} = 0,$$

$$\frac{dr}{d\theta} = \frac{\partial \Omega_1}{\partial k} \frac{\partial \Omega_2}{\partial e} \frac{\partial \Omega_3}{\partial \theta} / \Delta < 0,$$

$$\frac{de}{d\theta} = 0.$$

Since $\frac{dk}{d\theta} = 0$ and $\frac{dr}{d\theta} < 0$, from equation (12*), we have $\frac{dq}{d\theta} < 0$.

Since $\frac{dk}{d\theta} = 0$, from equation (18*), we have $\frac{dm}{d\theta} = 0$.

Since $\frac{de}{d\theta} = 0$, from equation (16*), we have $\frac{dp^a}{d\theta} = 0$.

Since $\frac{dk}{d\theta} = 0$ and $\frac{dm}{d\theta} = 0$, from equation (18*), we have $\frac{dK}{d\theta} = 0$. ∎

When the percentage of income spent on the manufactured good increases, the amount of individuals employed in the agricultural sector decreases. Since the employment rate in the manufacturing sector decreases when the percentage of income spent on the manufactured good

increases, a worker's income decreases. For an individual's consumption in the first period, because the ratio between the level of income and the price of the agricultural good does not change and this individual spends a lower percentage of income on the agricultural good, this individual's consumption of the agricultural good decreases. The impact on per capita consumption of the manufactured good is unclear because there are two effects working in opposite directions: a lower level of income, and a higher percentage of income spent on the manufactured good. The impact of an increase in the percentage of income spent on the manufactured good on the level of utility of an individual in the first period is unclear. Since the interest rate decreases and a worker's income in the first period also decreases, an individual's second period income also decreases when a higher percentage of income is spent on the manufactured good.

4. Impact of government policies

In this section, we study how a firm's equilibrium technology and total steady-state capital stock may be affected by government policies, such as minimum wage legislations and a change in the level of subsidy to technology adoption in the manufacturing sector.

The magnitude of the rigid manufacturing wage rate could be affected by minimum wage legislations. The following proposition studies the impact of a change in the manufacturing wage rate on the manufacturing sector.

Proposition 5: When the manufacturing wage rate increases, (i) a manufacturing firm chooses a more advanced technology, (ii) the total amount of capital stock increases.[11]

Proof: An application of Cramer's rule on the system (25) leads to

$$\frac{dk}{d\overline{w}} = \frac{\partial \Omega_1}{\partial \overline{w}} \frac{\partial \Omega_2}{\partial e} \frac{\partial \Omega_3}{\partial r} / \Delta > 0,$$

$$\frac{dK}{d\overline{w}} = \frac{d(mk)}{d\overline{w}} = \frac{d\left(k/(1-\beta\overline{w})\right)}{d\overline{w}}$$

$$= \frac{1}{\left(1-\beta\overline{w}\right)^2}\left[(1-\beta\overline{w})\frac{dk}{d\overline{w}} + k\left(\beta'\overline{w}\frac{dk}{d\overline{w}} + \beta\right)\right]$$

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$$= \frac{\beta k}{\left(1 - \beta \overline{w}\right)^2} > 0 . \quad \blacksquare$$

To understand Proposition 5, when the manufacturing wage rate increases, the wage cost of a manufacturing firm increases. This makes the adoption of a more advanced technology profitable because the saving on wage cost will be higher. While it is not clear how the number of firms producing the manufactured good changes, the total amount of capital stock always increases with the manufacturing wage rate. The reason is that the impact from the increase in the amount of capital of a firm always dominates the impact of the change in the number of firms.

When the manufacturing wage rate increases, interestingly, the level of employment in the manufacturing sector does not necessarily decrease. The reason is as follows. When the manufacturing wage rate increases, there are two effects affecting the level of employment working in opposite directions. First, to produce a given level of output, the demand for labor decreases. Second, the amount of steady-state capital increases. Since capital is fully employed, an increase in the amount of capital increases the level of output and this increases the demand for labor. Without imposing additional structure, it is not clear which effect dominates. Since whether the employment rate in the manufacturing sector will increase or decrease is unclear, the impact of a change in the manufacturing wage rate on the level of utility of a consumer is ambiguous.

If the level of subsidy to a firm's fixed costs of technology adoption increases, will firms adopt more advanced technologies? This question is addressed in the following proposition.

Proposition 6: When the level of subsidy increases, (i) a manufacturing firm's equilibrium technology does not change, (ii) the interest rate increases, (iii) the employment rate in the manufacturing sector increases, (iv) the level of output of a manufacturing firm increases, (v) the number of manufacturing firms does not change, (vi) the price of the agricultural good increases, (vii) the total steady-state capital stock does not change.

Proof: An application of Cramer's rule on the system (25) leads to

$$\frac{dk}{dg} = 0 ,$$

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$$\frac{dr}{dg} = \frac{\partial\Omega_1}{\partial k}\left(\frac{\partial\Omega_2}{\partial e}\frac{\partial\Omega_3}{\partial g} - \frac{\partial\Omega_2}{\partial g}\frac{\partial\Omega_3}{\partial e}\right)/\Delta > 0\,,$$

$$\frac{de}{dg} = \frac{\partial\Omega_1}{\partial k}\frac{\partial\Omega_2}{\partial g}\frac{\partial\Omega_3}{\partial r}/\Delta > 0\,.$$

Since $\overline{w}L > K$, $\dfrac{de}{dg} = -\dfrac{\partial\Omega_2}{\partial g}\Big/\dfrac{\partial\Omega_2}{\partial e} \propto \dfrac{K}{\overline{w}L} < 1$.

Since $\dfrac{dr}{dg} = 1 - \dfrac{\partial\Omega_1}{\partial k}\dfrac{\partial\Omega_2}{\partial g}\dfrac{\partial\Omega_3}{\partial e}/\Delta$ and $-\dfrac{\partial\Omega_1}{\partial k}\dfrac{\partial\Omega_2}{\partial g}\dfrac{\partial\Omega_3}{\partial e}/\Delta > 0$, $\dfrac{dr}{dg} > 1$. Thus, from

equation (21), we have $\dfrac{dq}{dg} > 0$.

Since $\dfrac{dk}{dg} = 0$, from equation (18*), we have $\dfrac{dm}{dg} = 0$.

Since $\dfrac{de}{dg} > 0$, from equation (16*), we have $\dfrac{dp^a}{dg} > 0$.

Since $\dfrac{dk}{dg} = 0$ and $\dfrac{dm}{dg} = 0$, from equation (18*), we have $\dfrac{dK}{dg} = 0$. ∎

From Proposition 6, a change in the level of subsidy to a firm's fixed costs of technology adoption does not change its steady-state technology. From equation (22), a firm's equilibrium technology is only affected by the manufacturing wage rate. The reason that a firm produces a higher level of output when the level of subsidy increases is as follows. From the proof of Proposition 6, an increase in subsidy leads to an even higher increase in the level of interest rate. A firm thus produces a higher level of output to break even. With a constant number of firms and each firm produces a higher level of output, the production of the manufactured good in each period increases with the level of subsidy. However, how an individual's utility changes with the level of subsidy is ambiguous.[12]

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[12] This ambiguity can be seen from the special case that closed form solutions are available as in footnote 6. From results in footnote 6, a subsidy does not change an individual's disposable income. From equations (3) and (5), since both the price of the agricultural goods and the interest rate increase, an individual's consumption of the agricultural good in the first period decreases and in the second period increases by the same amount. With diminishing marginal utility, this effect tends to decrease an individual's utility. From equations (4) and (6), since an individual's consumption of the manufactured good in the first period does not change and in the second period increases, this latter effect tends to increase an individual's utility. Without additional structure, it is not clear which effect dominates.

5. Conclusion

In this paper, we have studied an overlapping-generations model in which manufacturing firms engage in oligopolistic competition and choose increasing returns technologies to maximize profits and there is unemployment in the manufacturing sector. We have shown that there exists a unique steady state with the following properties. First, equilibrium technology of a manufacturing firm changes neither with the degree of patience of an individual, population, the percentage of income spent on the manufactured good, nor with the level of subsidy for technology adoption. Second, when the manufacturing wage rate increases, a manufacturing firm chooses a more advanced technology and the total amount of steady-state capital increases.

References:

Bencivenga, Valerie, and Bruce D. Smith. 1997. Unemployment, migration, and growth. *Journal of Political Economy* 105, 582-608.

Chandler, Alfred. 1990. *Scale and Scope: The Dynamics of Industrial Capitalism*. Cambridge, MA: Harvard University Press.

Chen, Ming Chieh, and Yeung-Nan Shieh. 2011. Specific commodity taxes, output and location decision under free entry oligopoly. *Annals of Regional Science* 47, 25-36.

Drazen, Allan, and Zvi Eckstein. 1988. On the organization of rural markets and the process of economic development. *American Economic Review* 78, 431-443.

Frankel, Francine. 2005. *India's Political Economy: 1947-2004*. New York, NY: Oxford University Press.

Gang, Ira, and Shubhashis Gangopadhyay. 1987. Employment, output and the choice of techniques: the tradeoff revisited. *Journal of Development Economics* 25, 321-327.

Harris, John, and Michael Todaro. 1970. Migration, unemployment, and development: a two-sector analysis. *American Economic Review* 60, 126-142.

Kotwal, Ashok, Bharat Ramaswami, and Wilima Wadhwa. 2011. Economic liberalization and India economic growth: What's the evidence? *Journal of Economic Literature* 49, 1152-1199.

Levinson, Marc. 2006. *The Box: How the Shipping Container Made the World Smaller and the World Economy Bigger*. Princeton, NJ: Princeton University Press.

Mankiw, N. Greogry, and Michael Whinston. 1986. Free Entry and Social Inefficiency. *Rand Journal of Economics* 17, 48-58.

Otsuka, Keijiro, Gustav Ranis, and Gary Saxonhouse. 1988. *Comparative Technology Choice in Economic Development: The Indian & Japanese Cotton Textile Industries*. New York: St. Martin's Press.

Perko, Lawrence. 2001. *Differential Equations and Dynamic Systems*. New York, NY: Springer.

Prendergast, Renee. 1990. Scale of production and choice of technique in the engineering industries in developing countries. *Journal of Development Studies* 27, 72-88.

Rauch, James. 1993. Economic development, urban underemployment, and income inequality. *Canadian Journal of Economics* 26, 901-918.

Romer, David. 2006. *Advanced Macroeconomics*, 3rd edition. New York, NY: McGraw-Hill Irwin.

Sen, A. K. 1960. *Choice of Techniques*. Oxford, UK: Basic Blackwell.

Stewart, Frances. 1977. *Technology and Underdevelopment*. Boulder, CO: Westview Press.

Turnovsky, Stephen. 1977. *Macroeconomic Analysis*. New York, NY: Cambridge University Press.

White, Lawrence. 1978. The evidence of appropriate factor proportions for manufacturing in less developed countries: a survey. *Economic Development and Cultural Change* 27, 27-59.

Zhang, Jie. 2002. Urbanization, population transition, and growth. *Oxford Economic Papers* 54, 94-117.

Zhang, Junxi. 2007. Endogenous markups, intensity of competition, and persistence of business cycles. *Southern Economic Journal* 74, 546-565.

Zhang, Xiaobo, Jin Yang, and Shenglin Wang. 2011. China has reached the Lewis turning point. *China Economic Review* 22, 542-554.

Zhou, Haiwen. 2009. Population growth and industrialization. *Economic Inquiry* 47, 247-265.

Zhou, Haiwen. 2013. The choice of technology and rural-urban migration in economic development. *Frontiers of Economics in China* 8, 337-361.

Zhou, Haiwen. 2015. Unemployment and economic integration for developing countries. *Frontiers of Economics in China* 10, 252-271.