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Abstract

In this paper an asymmetric autoregressive conditional heteroskedasticity (ARCH) model and a Levy-stable distribution are applied to some well-known financial indices (DAX30, FTSE20, FTSE100 and SP500), using a rolling sample of constant size, in order to investigate whether the values of the estimated parameters of the models change over time. Although, there are changes in the estimated parameters reflecting that structural properties and trading behaviour alter over time, the ARCH model adequately forecasts the one-day-ahead volatility. A simulation study is run to investigate whether the time variant attitude holds in the case of a generated ARCH data process revealing that even in that case the rolling-sampled parameters are time-varying.

Keywords: ARCH model, GED distribution, Leverage effect, Levy-stable distribution, Rolling sample, Spill over, Value at risk.

JEL codes: C32, C52, C53, G15.

1. Introduction

In the recent literature, regarding the description of the characteristics of financial markets, one can find a vast number of specifications of both ARCH and Stochastic Volatility (SV) processes that have been considered for. However, the SV models[1] are not as popular as the ARCH processes in applied studies. The purpose of the present study is to apply an asymmetric ARCH model to some well known financial indices, using a rolling sample of constant size, in order to observe the changes over time in the values of the estimated parameters. A thorough investigation is conducted by comparing the parameters of the full-sampled estimated model to the parameters of the rolling sub-sample estimated models. We conclude that the values of the estimated parameters change over time, indicating a data set that alters across time reflecting the information that financial markets reveal. The analysis is extended to simulated time series indicating that the time-varying estimated coefficients characterize the ARCH data generating process itself.

In ARCH modelling, the distribution of stock returns has fat tails with finite or infinite unconditional variance and time dependent conditional variance. Estimation of stable distributions is an alternative approach in modelling the unconditional distribution of returns. Thus, we adopt the estimation procedure of McCulloch (1986) and the parameters of the Levy-stable distribution are estimated at each of a sequence of points in time, using a rolling sample of constant size. The empirical findings suggest that the parameters of the unconditional distribution are also not constant over time.

Reviewing the relevant literature we notice absence of studies showing that although the parameters of a well-specified model vary significantly over time, their time varying attitude does not influence model's forecasting ability. The main object of our study is to provide evidence that model's parameters should be re-estimated on a frequent base in order to reflect any changes that have been occurred in the stock market and have been incorporated in the prices of assets.

The paper is divided in six sections. Section 2 lays out the asymmetric ARCH model that is applied in the FTSE20, DAX30, FTSE100 and SP500 stock indices. In section 3, the estimated rolling-sampled parameters of the asymmetric ARCH model are discussed. In section 4, a simulation study examines whether the parameters are time-varying in the case of a generated ARCH process. In section 5, the unconditional distribution of returns is estimated and the phenomenon of time-variant parameters is investigated in the Levy-stable distribution. Finally, in section 6 we summarize the main conclusions.

2. An asymmetric ARCH model

A wide range of proposed ARCH models is covered in surveys such as Andersen and Bollerslev (1998), Bera and Higgins (1993), Bollerslev et al. (1992), Bollerslev et al. (1994),

[1] The reader who is interested in SV models is referred to Barndorff-Nielsen et al. (2002), Chib et al. (1998), Ghysels et al. (1996), Jacquier et al. (1999), Shephard (2004), Taylor (1994).

Degiannakis and Xekalaki (2004) and Poon and Granger (2003). The Nobel price award to R.F. Engle for ARCH volatility modeling is the uncontested proof of the contribution of ARCH models in time series and econometric modelling (Diebold 2003). A plethora of studies applied ARCH models to predict future volatility by updating the available information set at each of a sequence of points in time. Among others, Balaban and Bayar (2005) tested in 14 countries the relationship between stock market returns and their forecast volatility, Blair et al. (2001) compared the information content of implied volatilities and intraday returns in the context of forecasting S&P100 volatility, Wei (2002) forecast China's weekly stock market volatility and Yu (2002) predicted stock price volatility using daily New Zealand data. Angelidis et al. (2004), Degiannakis (2004), Brooks and Persand (2003) and Giot and Laurent (2003) predicted Value-at-Risk (VaR) measures, while Degiannakis and Xekalaki (2001), Engle et al. (1997) and Noh et al. (1994) used rolling ARCH models to forecast volatility of options.

An ARCH process, $\varepsilon_t(\theta)$, can be presented as

$$\begin{gathered}\varepsilon_t(\theta) = z_t \sigma_t(\theta) \\ z_t \overset{i.i.d.}{\sim} f\left(E(z_t) = 0, V(z_t) = 1\right) \\ \sigma_t^2(\theta) = g\left(\sigma_{t-1}, \sigma_{t-2}, \ldots; \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots; \upsilon_{t-1}, \upsilon_{t-2}, \ldots\right),\end{gathered} \tag{1)}$$

where θ is a vector of unknown parameters, $f(.)$ is the density function of z_t, $g(.)$ is a linear or non-linear functional form and υ_t is a vector of predetermined variables included in information set I at time t. Since very few financial time series have a constant conditional mean of zero, an ARCH model can be presented in a regression form by letting ε_t be the unpredictable component of the conditional mean

$$y_{A,t} = E\left(y_{A,t} \mid I_{t-1}\right) + \varepsilon_t\,, \tag{2)}$$

where $y_{A,t} = \ln\left(P_{A,t} / P_{A,t-1}\right)$ denotes the continuously compound rate of return from time $t-1$ to t, and $P_{A,t}$ is the asset price A at time t. In order to investigate the characteristics of stock market A, we apply an ARCH model of the following form:

$$\begin{gathered}y_{A,t} = \mu_0 + \mu_1 \sigma_{A,t}^2 + \left(\mu_2 + \mu_3 e^{-\sigma_{A,t}^2 / \mu_4}\right) y_{A,t-1} + \varepsilon_t\,, \\ \varepsilon_t = z_t \sigma_{A,t}\,, \\ z_t \overset{i.i.d.}{\sim} GED(0,1;v)\,, \\ \ln\left(\sigma_{A,t}^2\right) = a_0 + \ln\left(1 + N_t \delta_0\right) + \frac{1}{\left(1 - \Delta_1 L\right)}\left(\Psi_1 L\left(\left|\frac{\varepsilon_t}{\sigma_{A,t}}\right| - E\left|\frac{\varepsilon_t}{\sigma_{A,t}}\right|\right) + \gamma L \frac{\varepsilon_t}{\sigma_{A,t}}\right),\end{gathered} \tag{3}$$

where $GED(0,1;v)$ denotes the generalized error distribution (GED), v is the tail thickness parameter of the GED, L is the lag operator and N_t is the number of non-trading days preceding the t^{th} day. The density function of a GED random variable is given by

$$f(z_t) = \frac{v e^{-2^{-1}\left|\frac{z_t}{\lambda}\right|^{v}}}{\lambda 2^{v+1/v}\,\Gamma\left(1/v\right)}, \qquad 4)$$

for $-\infty < z < \infty$, $0 < v \leq \infty$, where $\Gamma(.)$ denotes the gamma function and

$$\lambda \equiv \left(\frac{2^{-2v^{-1}}\Gamma\left(1/v\right)}{\Gamma\left(3/v\right)}\right)^{1/2}. \qquad 5)$$

The conditional variance specification has the form of the exponential GARCH, or EGARCH model, which is suggested by Nelson (1991). The EGARCH model captures the asymmetric effect exhibited in financial markets, as the conditional variance, σ_t^2, depends on both the magnitude and the sign of lagged innovations. Assuming GED distributed innovations with EGARCH specification for the conditional variance we take into account that i) the unconditional distribution of innovations is symmetric but with excess kurtosis and ii) their conditional distribution is asymmetric and leptokurtotic. Parameter γ allows for the leverage effect. The leverage effect, first noted by Black (1976), refers to the tendency of changes in stock returns to be negatively correlated with changes in returns volatility, i.e. volatility tends to rise in response to 'bad news' and to fall in response to 'good news'. Moreover, the logarithmic transformation ensures that the forecasts of the variance are non-negative. Parameter δ_0 allows us to explore the contribution of non-trading days to volatility. According to Fama (1965) and French and Roll (1986) information that accumulates when financial markets are closed is reflected in prices after the markets reopen. The conditional mean is modeled such as to capture the relationship between investors' expected return and risk[2] (μ_1), the non-synchronous trading effect[3] (μ_2), and the inverse relation between volatility and serial correlation[4] (μ_3).

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[2] The relationship between investors' expected return and risk was presented in an ARCH framework, by Engle et al. (1987). They introduced the ARCH in mean model where the conditional mean is an explicit function of the conditional variance.

[3] According to Campbell et al. (1997), 'The non-synchronous trading or non-trading effect arises when time series, usually asset prices, are taken to be recorded at time intervals of one length when in fact they are recorded at time intervals of other, possible irregular lengths.'

[4] LeBaron (1992) found a strong inverse relation between volatility and serial correlation for SP500, CRSP and Dow Jones returns. As LeBaron stated, it is difficult to estimate μ_4 in conjunction with μ_3 when using a gradient type of algorithm. So, μ_4 is set to the sample variance of the series.

Model (3) is expanded in order to take into account the phenomenon of volatility spill over from one market to the other[5]:

$$\ln\left(\sigma_{A,t}^{2}\right)=a_{0}+\ln\left(1+N_{t}\delta_{0}\right)+\frac{1}{\left(1-\Delta_{1}L\right)}\left(\Psi_{1}L\left(\left|\frac{\varepsilon_{t}}{\sigma_{A,t}}\right|-E\left|\frac{\varepsilon_{t}}{\sigma_{A,t}}\right|\right)+\gamma L\frac{\varepsilon_{t}}{\sigma_{A,t}}\right) \\ +\delta_{1}\ln\left(\sigma_{B,t-1}^{2}\right)+\delta_{2}\ln\left(\sigma_{C,t-1}^{2}\right). \quad (6)$$

where the parameters δ_1 and δ_2 account for the volatility spill over from B and C stock markets to the A stock market, respectively. In order to account for the volatility spill over effect from one market to the others, when (6) is estimated for stock market A, the daily conditional volatilities of stock markets B and C are regarded as exogenous variables that have been estimated according to framework (3)[6].

The data set used in this paper consists of the Financial Times Stock Exchange 20 (FTSE20) index for Greece, the Deutscher Aktien Index 30 (DAX30) for Germany, the Financial Times Stock Exchange 100 (FTSE100) index for U.K. and the Standard & Poor's 500 (SP500) index for U.S.A. The period covered for the FTSE20, DAX30, FTSE100 and SP500 is from January 3rd 1996, January 14th 1992, January 9th 1992 and January 7th 1992 to July 5th 2002, respectively. A thorough investigation is conducted by comparing the parameters of the full-sampled estimated model to the parameters of the rolling sub-sample estimated models. Maximum likelihood estimates of the parameters are obtained by numerical maximization of the log-likelihood function using the Marquardt (1963) algorithm.

INSERT TABLE 1 ABOUT HERE

Table 1 presents the estimated parameters of model (6) for each market separately. The standardized residuals, $\varepsilon_t\sigma_{A,t}^{-1}$, and their squared values, $\varepsilon_t^2\sigma_{A,t}^{-2}$, from all models obey the standard assumptions of autocorrelation and heteroskedasticity absence. Indicatively, we present the Ljung-Box *Q*-statistic for the null hypothesis that there is not autocorrelation up to 20th order computed on $\varepsilon_t\sigma_{A,t}^{-1}$ and $\varepsilon_t^2\sigma_{A,t}^{-2}$. Briefly discussing the values of the parameters, we note that i) the relation of the conditional variance with the risk premium, although positive, is statistically insignificant (coefficient μ_1), ii) the non-synchronous trading effect is not present in the estimated models (coefficient μ_2) and iii) concerning the cases of the FTSE20 and SP500 stock indices, the daily serial correlation is inversely related to its conditional volatility

[5] Engle et al. (1990) evaluated the role of the information arrival process in the determination of volatility in a multivariate framework providing a test of two hypotheses: heat waves and meteor showers. Using meteorological analogies, they supposed that information follows a process like a heat wave so that a hot day in New York is likely to be followed by another hot day in New York but not typically by a hot day in Tokyo. On the other hand, a meteor shower in New York, which rains down on the earth as it turns, will almost surely be followed by one in Tokyo. Thus, the heat wave hypothesis is that the volatility has only country specific autocorrelation, while the meteor shower hypothesis states that volatility in one market spills over to the next. See also Kanas (1998).

[6] For example, in the case of the FTSE20 index daily returns, the conditional variance of the DAX30 and SP500 returns were regarded as exogenous variables. In order to estimate the conditional variance of the DAX30 and SP500 indices, their daily returns were used for the period of January 1992 to July 2002, or 1000 trading days prior January 3rd, 1996.

(coefficient μ_3). Moreover, the leverage effect is not present in the Greek and German stock markets. On the contrary, for the SP500 and FTSE100 stock indices, the estimated value of parameter γ is statistically significant at 1% level of significance. The volatility spill over effect is statistically significant for the U.K. stock market. Regarding the SP500 index daily returns, there is evidence that volatility spillovers from Frankfurt to Chicago stock market. Finally, for the FTSE20, DAX30 and SP500 cases, parameter v is statistically different to the value of 2 at any level of significance, justifying the use of a thick-tailed distribution. The estimated value of δ_0 is about 0.187 and statistically significant only in the case of the Greek market indicating that a non-trading day contributes less than a fifth as much to volatility as a trading day.

3. Rolling-sampled parameters of the asymmetric ARCH model

Our purpose is to examine if the estimated parameters of the asymmetric ARCH model change over time and whether there is any impact of time-varying estimated parameters on volatility forecasting accuracy. The ARCH process is estimated, at each of a sequence of points in time, using a rolling sample of constant size equal to 1000 trading days, a sample size that is preferred[7] by the majority of applied studies.

We produce one-day-ahead conditional volatility predictions for the trading days of 11th January 2000 to 5th July 2002. Since the ARCH model is estimated at each point in time, we use the maximum likelihood estimates at time $t-1$ as starting values for the iterative maximization algorithm at time t. Figure 1 depicts the rolling-sampled estimated parameters for the FTSE20 index as well as the ± 2.06 times the conditional standard deviation confidence interval of the parameters estimated using the full data sample[8]. From visual inspection, the estimated rolling parameters are, clearly, out of the confidence interval bounds in many cases. Table 2 presents the percentage of rolling-sampled estimations, which are outside of the 95% confidence interval of the full-sampled parameters. Characteristic examples of the change in the parameter values are Ψ_1 and v for DAX30 as well as Δ_1 for FTSE20 and SP500. However, there are rolling parameters which do not change significantly across time, such as γ (leverage effect), and δ_0 (contribution of non-trading days to volatility). An important characteristic, which is extracted from the rolling-sampled estimated parameters, is the fact that the estimated values do not fluctuate in a mean reverting form but they change gradually. Sudden changes of the values of the rolling estimated parameters, which are characterized by a mean reverting form, should indicate an improperly maximum likelihood estimation procedure. On the other hand, gradual changes of the estimated

[7] Engle et al. (1993), Engle et al. (1997), Noh et al. (1994), Angelidis et al. (2004) note that the size of the rolling sample turns out to be rather important while Frey and Michaud (1997), Hoppe (1998) and Degiannakis and Xekalaki (2006) comment that the use of short sample sizes generates more accurate volatility forecasts, since it incorporates changes in trading behaviour more efficiently.

[8] Figures of the estimated rolling parameters for the DAX30, FTSE100 and SP500 indices, similar to Figure 1, are available upon request.

coefficients indicate a data set that alters from time to time, forcing us to believe that the values of the estimated parameters reflect the information that financial markets reveal.

INSERT FIGURE 1 ABOUT HERE

INSERT TABLE 2 ABOUT HERE

The percentage of estimated rolling parameters that are statistically different from the parameter values estimated using the full data sample, as presented in Table 3, is also indicative for the changes of the estimated values across time. There are four parameters, in the case of the Greek market, whose rolling-sampled estimators differ statistically significant from their full-sampled estimators in more than 10% of the trading days. Although, in the case of the FTSE100 index, only the rolling estimators of Δ_1 parameter differ statistically from their full data sample estimator, in the case of the SP500 index there are four parameters, which show a statistically significant difference from their full-sampled estimators in more than 20% of the trading days.

INSERT TABLE 3 ABOUT HERE

The values of the rolling parameters indicate that the characteristics of the markets change during the examined period. According to Table 4, which presents the percentage of trading days that the rolling parameters are statistically insignificant, there are parameters whose rolling-sampled estimations are statistically insignificant while their full-sampled estimations are significant. For example, parameters μ_3 and δ_1 for the SP500 index, as well as parameter γ for FTSE100 index, although they appear to be significant in the full sample, almost all their rolling-sampled estimations are insignificant at 5% level of significance. Therefore, in the full sample, an inverse relation between volatility and serial correlation characterizes FTSE20 index, but the values of rolling μ_3 are not different to zero in most of the cases. Of course, there are parameters whose estimations are statistically different to zero in both the full sample and the rolling samples (i.e. the parameter Δ_1 for the FTSE20, DAX30 and SP500 indices). Hence, we may infer that the values of the estimated parameters change across time, reflecting the individual features of particular periods that characterize financial markets.

INSERT TABLE 4 ABOUT HERE

However, although the estimated parameters are time varying, the in-sample and out-of-sample forecasting ability of the model is accurate. There are 31, 19, 17 and 29 cases, or 4.99%, 2.99%, 2.66% and 4.57%, observed returns outside the 95% confidence intervals for the FTSE20, DAX30, FTSE100 and SP500 indices, respectively. In Figure 2.a, the 95% in-sample confidence interval of the FTSE20 index of daily returns is plotted from 11th January 2000 to 5th July 2002. However, a model that uses a large number of parameters may exhibit an excellent in-sample fit but a poor out-of-sample performance. Studies such as Heynen and Kat (1994), Hol and Koopman (2000) and Pagan and Schwert (1990) examined a variety of volatility prediction models with in-sample and out-of-sample data sets. We investigate the possibility that model over-fitting can be occurred and evaluate the performance of the estimated ARCH model by computing the out-of-sample forecasts. In the sequel, the one-day-ahead 95% prediction intervals are constructed. Let us compute the one-day-ahead conditional mean, $y_{t+1|t} \equiv E\left(y_{t+1}\left(\theta^{(t)}\right)|\, I_t\right)$, and conditional variance, $\sigma^2_{t+1|t} \equiv E\left(\varepsilon^2_{t+1}\left(\theta^{(t)}\right)|\, I_t\right)$, using the following formulas:

$$y_{A,t+1|t} = \mu_0^{(t)} + \mu_1^{(t)}\sigma^2_{A,t+1|t} + \left(\mu_2^{(t)} + \mu_3^{(t)} e^{-\sigma^2_{A,t+1|t}/\mu_4^{(t)}}\right) y_{A,t}\,,$$

$$\ln\left(\sigma^2_{A,t+1|t}\right) = a_0^{(t)} + \ln\left(1 + N_{t+1}\delta_0^{(t)}\right) + \frac{1}{\left(1-\Delta_1^{(t)}L\right)}\left(\Psi_1^{(t)}\left(\left|\frac{\varepsilon_{t|t}}{\sigma_{A,t|t}}\right| - E\left|\frac{\varepsilon_t}{\sigma_{A,t}}\right|\right) + \gamma^{(t)}\frac{\varepsilon_{t|t}}{\sigma_{A,t|t}}\right) + \delta_1^{(t)}\ln\left(\sigma^2_{B,t|t}\right) + \delta_2^{(t)}\ln\left(\sigma^2_{C,t|t}\right), \quad (7)$$

where $\theta^{(t)} \equiv \left(\mu_0^{(t)}, \mu_1^{(t)}, \mu_2^{(t)}, \mu_3^{(t)}, a_0^{(t)}, \delta_0^{(t)}, \Psi_1^{(t)}, \Delta_1^{(t)}, \gamma^{(t)}, \delta_1^{(t)}, \delta_2^{(t)}, v^{(t)}\right)$ is the parameter vector that is estimated using the sample data set which is available at time t, $\varepsilon_{t|t} \equiv E\left(\varepsilon_t \mid I_t\right)$ denotes the prediction error conditional on the information set that is available at time t, and $\sigma_{A,t|t} \equiv \sqrt{E\left(\varepsilon_t^2 \mid I_t\right)}$ is the conditional standard deviation which is computed by the ARCH model, in equation (6), using the information set available at time t. Note that for $z_t \sim GED(0,1;v)$, the expected value of its absolute price is equal to $E\left|\varepsilon_t\sigma^{-1}_{A,t}\right| = \Gamma\left(2/v^{(t)}\right)\left(\Gamma\left(1/v^{(t)}\right)\Gamma\left(3/v^{(t)}\right)\right)^{-1/2}$.

Figure 2.b plots the one-day-ahead 95% prediction interval, which is constructed as the one-day-ahead conditional mean ± 2.06 times the conditional standard deviation, both measurable to I_t information set, or $y_{A,t+1|t} \pm GED\left(0,1;v^{(t)},0.025\right)\sigma_{A,t+1|t}$, where $GED\left(0,1;v^{(t)},a\right)$ is the $100(1-a)$ quantile of the GED distribution. Hence, each trading day, (t), the next trading day's, ($t+1$), prediction intervals are constructed, using only information available at current trading day, t. There are 29, 22, 21 and 32 observations or 4.67%, 3.46%, 3.29% and 5.04% for the FTSE20, DAX30, FTSE100 and SP500 indices, respectively, outside the 95% prediction intervals[9].

INSERT FIGURE 2 ABOUT HERE

For a more formal method of evaluating forecasting adequacy, we apply two hypotheses tests that measure the forecasting accuracy in a VaR framework. One-day-ahead VaR at a given probability level, a, is the next trading day's predicted amount of financial loss of a portfolio, or $VaR_{t+1|1}(1-a) = GED\left(0,1;v^{(t)},a\right)\sigma_{A,t+1|t}$. Kupiec (1995) introduced a likelihood

[9] Figures, similar to Figure 2, that depict the in-sample 95% confidence interval and the one-day-ahead 95% prediction intervals for the DAX30, FTSE100 and SP500 indices are also available upon request.

ratio statistic for testing the null hypothesis that the proportion of confidence interval violations is not larger than the VaR forecast. The test statistic, which is asymptotically X_1^2 distributed, is computed as $LR_K = 2[\ln((n/N)^n(1-n/N)^{N-n}) - \ln(p^n(1-p)^{N-n}]$, where $n \equiv \sum_{i=1}^{N} d\left(y_{t+1} < VaR_{t+1|t}(a/2)\right) + d\left(y_{t+1} > VaR_{t+1|t}(1-a/2)\right)$ is the number of trading days over the out-of-sample period N that a violation has occurred, for $d\left(y_{t+1} < VaR_{t+1|t}(a/2)\right) = 1$ if $y_{t+1} < VaR_{t+1|t}$ and $d\left(y_{t+1} < VaR_{t+1|t}(a/2)\right) = 0$ otherwise, and p is the expected frequency of violations. Christoffersen (1998) developed a likelihood ratio statistic that jointly investigates whether i) the proportion of violations is not larger than the VaR forecast and ii) the violations are independently distributed. The statistic is computed as $LR_C = -2\ln((1-p)^{N-n}p^n)$ $+ 2\ln((1-\pi_{01})^{n_{00}}\pi_{01}^{\ n_{01}}(1-\pi_{11})^{n_{10}}\pi_{11}^{\ n_{11}}))$, where $\pi_{ij} = n_{ij} \Big/ \sum_j n_{ij}$ and n_{ij} is the number of observations with value i followed by j, for $i, j = 0,1$. The values $i, j = 1$ denote that a violation has been made, while $i, j = 0$ indicate the opposite. Under the null hypothesis, the LR_C is asymptotically chi-squared distributed with two degrees of freedom. The main advantage of Christoffersen's test is that it can reject a VaR model that generates either too many or too few clustered violations. Both tests do not reject the null hypothesis of correct proportion of violations in all the cases, except for the 95%-VaR of the FTSE100 index. In the case of Kupiec's test the p-values are 70.28%, 6.08%, 3.45% and 96.37% for 95%-VaR and 8,15%, 13.63%, 56.56% and 52.70% for 99%-VaR, for the FTSE20, DAX30, FTSE100 and SP500 indices, respectively. Testing the null hypothesis of whether the violations are equal to the expected ones as well as if they are independent, we observe that the relative p-values are 40.03%, 16.42%, 0.15% and 95.19% in the 95%-VaR case and 17.98%, 32.51%, 7.10% and 73.92% in the 99%-VaR case, for the FTSE20, DAX30, FTSE100 and SP500 indices, respectively.

Despite the fact that the values of the estimated coefficients change over time, the model adequately forecasts the one-day-ahead volatility. Thus, changes in the values of the estimated parameters do not indicate inadequacy of the model in describing the data. On the contrary, model's parameters should be re-estimated on a daily base in order to reflect any changes that have been occurred in the stock market and have been incorporated in the prices of assets[10]**.**

[10] In order to investigate whether the phenomenon of time-variant values of estimated parameters is related to a specific structural characteristic of the model specification, we estimate another ARCH specification. Degiannakis (2004) and Giot and Laurent (2003) used an ARCH model with the APARCH volatility specification of Ding et al. (1993) and the skewed student-t distribution for the standardized innovations. We estimated such a model for our datasets and found similar qualitative results. The estimated parameters are time varying. We have also re-estimated model (6) using alternatively i) larger sample sizes of rolling parameters, ii) the BHHH algorithm (Berndt et al. 1974) instead of the Marquardt algorithm in estimating the maximum likelihood parameters and iii) the same starting values at each point in time, instead of the estimates at time $t-1$ as starting values for the

4. Rolling-sampled parameters from simulated processes

A simulation study could shed light in rolling-sampled estimated parameters' behaviour. A series of simulations is run in order to investigate if the time-variant attitude holds even in the case of an ARCH data generating process. We generate a series of 32000 values from the standard normal distribution, $z_t \overset{i.i.d.}{\sim} N(0,1)$. Then an AR(1)GARCH(1,1) process is created, $\{y_t\}_{t=1}^{32000}$, where $y_t = 0.0005 + 0.15 y_{t-1} + \varepsilon_t$, by multiplying the i.i.d. process with a specific conditional variance form $\varepsilon_t = z_t \sqrt{\sigma_t^2}$, for $\sigma_t^2 = 0.0005 + 0.05\varepsilon_{t-1}^2 + 0.90\sigma_{t-1}^2$. The AR(1)GARCH(1,1) model is applied on the $\{y_t\}_{t=1002}^{32000}$ generated data. Dropping out the first 1001 data, maximum likelihood rolling-sampled estimates of the parameters are obtained by numerical maximization of the log-likelihood function, using a rolling sample of constant size equal to 1000. According to Table 5, about 58% of the 30000 conditional variance rolling-sampled parameters are outside the 95% confidence interval of the parameters estimated using the whole sample set of the 30000 simulated data. The procedure is repeated for an AR(1)EGARCH(1,1) conditional variance form, $\ln(\sigma_t^2) = a_0 + a_1 \left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| - \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_1 \ln(\sigma_{t-1}^2)$, but the results are robust to the choice of the conditional variance specification.

A series of 32000 values from the first order autoregressive process are also produced. The AR(1) process is created as $y_t = 0.0001 + 0.12 y_{t-1} + z_t$, for $z_t \overset{i.i.d.}{\sim} N(0,1)$. Dropping out the first 1001 data, 30000 maximum likelihood rolling-sampled estimates of the parameters are also obtained. As far as the case of the AR(1) process is concerned, we infer that the rolling estimated parameters are time-invariant, as on average 5% of the estimated rolling parameters are outside the 95% confidence levels.

Both the AR(1)GARCH(1,1) and the AR(1) processes were simulated for various sets of parameters, but there are no qualitative differences to the fore mentioned conclusions. Moreover, a series of simulations were repeated i) for ARCH volatility forms without any conditional mean specification, ii) based on estimation procedures of the most well known packages, EVIEWS® 4.1 and OX-G@ARCH® 3.4, iii) for larger rolling samples of 5000 values, iv) for non-overlapping data samples, but there were no qualitative differences in any of these cases[11].

So, the simulation study provides evidence that the time-variant attitude of rolling-sampled parameters estimations characterizes not only the examined data sets but the ARCH data generating process itself as well.

INSERT TABLE 5 ABOUT HERE

likelihood algorithm at time t. Despite the slight changes occurred in each case, the rolling parameters are time-variant for all cases.

[11] All the simulation studies are available to the readers upon request.

5. Rolling-sampled parameters from a Levy-stable distribution

In this section, we investigate whether the phenomenon of parameter changing across time is related with the unconditional distribution of returns also. Mandelbrot (1963) and Fama (1965) made the first re-examination of the unconditional distribution of stock returns. Mandelbrot (1963) concluded that price changes can be characterized by a stable Paretian distribution with a characteristic exponent, a, less than two, thus exhibiting fat tails and infinite variance. Fama (1965) examined the distribution of thirty stocks of the Dow Jones Industrial Average; his results were consistent with Mandelbrot's. Thereafter, it has been accepted that the stock returns distributions are fat-tailed and peaked. In an attempt to model the unconditional distribution of stock returns several researchers have considered alternative approaches. See for example, Blattberg and Gonedes (1974), Bradley and Taqqu (2002), Clark (1973), Kon (1984), McDonald (1996), Mittnik and Rachev (1993), Panas (2001), Rachev and Mittnik (2000).[12]

The probability density function of a stable distribution cannot be described in a closed mathematical form. By definition, a univariate distribution function is stable if and only if its characteristic function has the form

$$\varphi(t) = \exp\left\{ i\delta t - \gamma |t|^a \left(1 - i\beta \left(\frac{t}{|t|} \right) \omega(t,a) \right) \right\}, \tag{8}$$

where $i = \sqrt{-1}$, $t \in R$, $\omega(t,a) = \tan\left(\frac{\pi\alpha}{2} \right)$ if $a \neq 1$ and $\omega(t,a) = \frac{-2}{\pi} \log|t|$ if $a = 1$. The particular distribution represented by its characteristic function is determined by the values of four parameters: a, β, γ and δ. The parameter a, $0 < \alpha \leq 2$, is called the characteristic exponent. It measures the thickness of the tails of a stable distribution. The smaller the value a, the higher the probability in the distribution tails. If $a < 2$ then we have thicker tails than the tails of normal distribution. Thus, stable distributions have thick tails and consequently increase the likelihood of the occurrence of large shocks. The skewness parameter β, $-1 \leq \beta \leq 1$, is a measure of the asymmetry of the distribution. The distribution is symmetric, if $\beta = 0$. As $|\beta|$ approaches one, the degree of skewness increases. The scale parameter γ, $\gamma > 0$, is a measure of the spread of the distribution. It is similar to the variance of the normal distribution, $\gamma = \sigma / \sqrt{2}$. However, the scale parameter γ is finite for all stable distributions, despite the fact that the variance is infinite for all $a < 2$. The location parameter δ, $-\infty < \delta < +\infty$, is the mean of the distribution, for $a > 1$, and the median for $0 < a \leq 1$. The

[12] De Vries (1991), Ghose and Kroner (1995) and Groenendijk et al. (1995) demonstrate that ARCH models share many of the properties of Levy-stable distribution but the true data generating process for an examined set of financial data is more likely ARCH than Levy-stable. A number of studies, such as Liu and Brorsen (1995), Mittnik et al. (1999), Panorska et al. (1995), Tsionas (2002), examined the properties of ARCH models with Levy-stable distributed innovations.

case of $a = 2$, $\beta = 0$ corresponds to the normal distribution, while $a = 1$, $\beta = 0$ corresponds to the Cauchy distribution.

In estimating the parameters of the stable distribution of index returns, we adopt the estimation procedure suggested by McCulloch (1986). The estimation procedure is a quantile method and works for $0.6 \le a \le 2$ and any value of the other parameters. Essentially, McCulloch suggests that if we have a random variable x, which follows a stable distribution and denotes the p^{th} quantile of this distribution by $x(p)$, then the population quantile can be estimated by the sample quantile $\hat{x}(p)$. McCulloch's estimator uses five quantiles to estimate a and β as $\hat{v}(\alpha) = \frac{\hat{x}(0.95) - \hat{x}(0.05)}{\hat{x}(0.75) - \hat{x}(0.25)}$ and $\hat{v}(\beta) = \frac{\hat{x}(0.95) + \hat{x}(0.05) - 2\hat{x}(0.50)}{\hat{x}(0.95) - \hat{x}(0.05)}$. Since $v(a)$ is monotonic in a and $v(\beta)$ is monotonic in β, we are able to find a and β by inverting $v(a)$ and $v(\beta)$, thus $\hat{a} = g_1(\hat{v}(a), \hat{v}(\beta))$ and $\hat{\beta} = g_2(\hat{v}(a), \hat{v}(\beta))$. McCulloch tabulated g_1 and g_2 for various values of $v(a)$ and $v(\beta)$. A similar procedure is also applied for the scale and location parameters. An alternative procedure to estimate the parameters of the stable distribution is the regression method proposed by Koutrouvelis (1980).

Following a procedure similar to that of ARCH modelling, the parameters of the stable distribution are estimated, at each of a sequence of points in time, using a rolling sample of constant size equal to 1000 trading days. The empirical findings, for the case of the Greek stock market, are graphically summarized in Figure 3, which plots the rolling-sampled estimates of parameters along with the 95% confidence interval of the parameters estimated using the full data sample. Inspection of Figure 3 shows that the estimates of a are less than two. The case of FTSE20 reveals that 92% of the a's rolling-sampled estimates are between 1.44 and 1.55. The parameter β is greater than zero, which implies skewness to the right. The rolling values of β are positive and range from 0.003 to 0.22 but there are not outside the 95% confidence interval for any case[13].

INSERT FIGURE 3 ABOUT HERE

In Table 6, we present the estimates of the parameters of stable distribution based on all data available as well as the standard deviation of the rolling-sampled estimated parameters. The estimates of a do not approach the value of two in any of the examined indices. However, there are estimated rolling parameters that are statistically different from the parameter values estimated using the full data sample. For example, the rolling-sampled estimates of the tail index (a) are statistically different to the full sample estimated parameter in the 51.46% of the trading days for the case of the SP500 index. The rolling estimates of

[13] Figures depicting the rolling-sampled estimates of the parameters for the DAX30, FTSE100 and SP500 indices are available upon request.

parameter β are statistically different to the relevant full-sampled values in 9.59% and 9.42% of the trading days for the DAX30 and FTSE100 indices, respectively, whereas the location (δ) parameters are time-variant in none of the cases. Another important parameter of the stable distribution, from the point of view of portfolio theory, is the scale parameter, γ. As far as the FTSE20 index is concerned, the rolling-sampled estimates of the scale parameter differ statistically from its full-sampled value in the 56.48% of the trading days. Hence, the parameter estimates, using the full data sample are statistically different from the parameter values estimated using the rolling samples of constant size for one parameter in each index.

INSERT TABLE 6 ABOUT HERE

6. Discussion

We estimated an asymmetric ARCH model using daily returns of the FTSE20, DAX30, FTSE100 and SP500 indices and concluded that although the estimated parameters of the model change over time, the model does not lose its ability to forecast the one-day-ahead volatility accurately. Furthermore, the rolling parameter analysis was applied to the unconditional distribution of returns. We observed the phenomenon of parameter changing across time for both the conditional (ARCH process) and the unconditional (Levy-stable) distribution of returns. Even in the case of a simulated ARCH process, the property of time varying rolling-sampled parameters holds. One possible reason for parameter instability might be that the behaviour of the market participants has undergone fundamental changes. Parameters instability indicates a change in market behavior but we can not determine the source of that change. The term *'a data set that alters'*, could incorporate a wide range of possible sources, i.e. financial legislation, market microstructure, market participants' perspective, technological revolution or even macroeconomic policy.

Gallant et al. (1991), Stock (1988), Lamoureux and Lastrapes (1990) and Schwert (1989) among others have aimed at explaining the economic interpretation of the ARCH process. As Engle et al. (1990) and Lamoureux and Lastrapes (1990) have noted, the explanation of the ARCH process must lie either in the arrival process of news or in market dynamics in response to the news. Based on some earlier work by Clark (1973) and Tauchen and Pitts (1983), Gallant et al. (1991) provided a theoretical interpretation of the ARCH effect. They assumed that the asset returns are defined by a stochastic number of intra-period price revisions and information flows into the market in an unknown rate. As the daily information does not come to the stock market in a constant and known rate, the estimation of the ARCH stochastic process that explains the dynamics of the stock market could be revised at regular time intervals. In our case the ARCH process is estimated using daily returns. Thus, the parameters of the model may be revised on a daily base, because of the observed phenomenon of changes in the estimated parameters. If we used data of higher frequency, i.e. ten-minutes intra-daily returns, the estimated model might be revised more frequent than on a daily base.

To the best of authors' knowledge, this is the first study that investigates the phenomenon of time varying estimated parameters either i) in real-world financial data or ii) in a simulated data generating process. A natural extension of this study would be to analyse the change and the relative economic interpretation of the estimated values of the parameters in intra-daily high-frequency data sets.

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Table 1. Parameter estimates for the FTSE20, DAX30, FTSE100 and SP500 index daily returns (January 3rd, 1996 to July 5th, 2002).

Parameter	Coefficient				Coefficient / Standard error			
	FTSE20	DAX30	FTSE100	SP500	FTSE20	DAX30	FTSE100	SP500
μ_0	-0.001	0.000	-0.000	-0.001	-1.898	0.596	-0.067	-1.027
μ_1	2.853	1.995	1.251	4.297	1.634	0.736	0.319	1.071
μ_2	0.053	0.024	0.005	-0.100	1.103	0.398	0.078	-1.620
μ_3	0.317[b]	-0.075	0.144	0.333[a]	2.809	-0.544	1.140	2.745
a_0	-6.833[a]	-9.858[a]	-1.326[b]	-4.059[a]	-6.341	-10.727	-2.538	-5.929
δ_0	0.187[a]	0.095	0.012	0.039	3.382	1.880	0.342	0.956
Ψ_1	0.394[a]	0.190[a]	0.056	0.060	19.79	27.847	0.892	1.378
Δ_1	0.920[a]	0.973[a]	-0.001	0.785[a]	38.34	73.455	-0.003	28.040
γ	-0.064	-0.068	-0.108[a]	-0.236[a]	-1.043	-0.856	-2.969	-2.975
δ_1	0.010	-0.008	0.694[a]	0.081[b]	0.415	-0.688	4.822	2.295
δ_2	0.002	0.004	0.201[b]	0.041	0.103	0.386	2.116	1.314
v	1.335[a]	1.735[a]	1.858	1.689	-15.540	-9.137	-1.495	-8.184
Q_{20}	20.065	22.597	23.913	24.090	[0.391]	[0.256]	[0.200]	[0.193]
Q_{20}^2	16.663	23.747	24.696	13.003	[0.615]	[0.206]	[0.171]	[0.838]

Notes: With v=1.335, v=1.735, v=1.858, v=1.689, the 97.5% point of the generalized error distribution are 2.06, 2.00, 1.98 and 2.00, respectively. With v=1.335, v=1.735, v=1.858, v=1.689, the 99.5% point of the generalized error distribution are 2.94, 2.70, 2.65 and 2.72, respectively. For the FTSE20 index, parameters δ_1 and δ_2 present the volatility spillover from the SP500 and DAX30 indices, respectively. For the DAX30 index, parameters δ_1 and δ_2 present the volatility spillover from the FTSE100 and SP500 indices, respectively. For the FTSE100 index, parameters δ_1 and δ_2 present the volatility spillover from the DAX30 and SP500 indices, respectively. For the SP500 index, parameters δ_1 and δ_2 present the volatility spillover from the DAX30 and FTSE100 indices, respectively. Q_{20} and Q_{20}^2 are the Q-statistics of order 20 computed on the standardized residuals and their squared values, respectively. The relative p-values are presented in brackets.

[a] Indicates that the coefficient is statistically significant at 1% level of significance.

[b] Indicates that the coefficient is statistically significant at 5% level of significance.

Table 2. Percentage of rolling-sampled estimated parameters that are outside the 95% confidence interval. (Values in parenthesis present the lower and upper bounds of the 95% confidence interval).

	FTSE20		DAX30	
μ_0	(-0.002 0.000)	56.48%	(-0.001 0.002)	33.18%
μ_1	(-1.780 7.485)	7.04%	(-4.989 8.978)	0.00%
μ_2	(-0.075 0.181)	0.00%	(-0.133 0.182)	0.00%
μ_3	(0.017 0.617)	0.32%	(-0.431 0.281)	0.00%
a_0	(-9.694 -3.972)	14.88%	(-12.227 -7.489)	3.20%
δ_0	(0.040 0.334)	1.12%	(-0.035 0.224)	0.00%
Ψ_1	(0.342 0.447)	13.12%	(0.172 0.207)	62.24%
Δ_1	(0.856 0.984)	54.40%	(0.939 1.007)	22.08%
γ	(-0.227 0.099)	0.00%	(-0.271 0.136)	0.00%
δ_1	(-0.056 0.076)	5.12%	(-0.038 0.022)	3.04%
δ_2	(-0.059 0.064)	32.16%	(-0.025 0.034)	1.60%
v	(1.222 1.449)	26.88%	(1.660 1.811)	46.72%
	FTSE100		SP500	
μ_0	(-0.001 0.001)	24.11%	(-0.002 0.001)	20.66%
μ_1	(-8.762 11.263)	0.80%	(-5.978 14.572)	16.48%
μ_2	(-0.148 0.157)	1.28%	(-0.258 0.058)	0.00%
μ_3	(-0.178 0.465)	12.32%	(0.022 0.644)	0.48%
a_0	(-2.659 -0.007)	16.64%	(-5.812 -2.306)	24.00%
δ_0	(-0.080 0.105)	0.00%	(-0.065 0.142)	0.00%
Ψ_1	(-0.104 0.215)	0.00%	(-0.052 0.173)	20.96%
Δ_1	(-0.472 0.471)	1.12%	(0.713 0.857)	60.48%
γ	(-0.201 -0.015)	1.12%	(-0.439 -0.033)	0.48%
δ_1	(0.327 1.062)	0.48%	(-0.009 0.171)	0.00%
δ_2	(-0.041 0.444)	0.00%	(-0.039 0.121)	35.36%
v	(1.616 2.100)	0.48%	(1.591 1.787)	9.44%

Table 3. Percentage of rolling-sampled estimated parameters that are statistically different from the parameter values estimated using the full data sample.

	FTSE20		DAX30		FTSE100		SP500	
Parameter	5% sign. Level	1% sign. Level	5% sign. Level	1% sign. Level	5% sign. Level	1% sign. Level	5% sign. Level	1% sign. Level
μ_0	21.86%	1.29%	13.67%	0.80%	4.02%	0.00%	14.15%	4.34%
μ_1	0.96%	0.00%	0.00%	0.00%	0.16%	0.00%	8.52%	0.64%
μ_2	0.00%	0.00%	0.00%	0.00%	1.13%	0.00%	0.00%	0.00%
μ_3	0.00%	0.00%	0.00%	0.00%	3.22%	0.64%	0.00%	0.00%
a_0	17.20%	3.86%	16.72%	7.40%	0.48%	0.00%	24.28%	6.59%
δ_0	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.73%	0.00%
Ψ_1	7.40%	0.00%	0.00%	0.00%	0.00%	0.00%	7.56%	0.00%
Δ_1	18.97%	10.13%	2.57%	0.00%	14.47%	5.79%	31.67%	3.54%
γ	0.00%	0.00%	5.14%	0.00%	4.50%	0.00%	36.17%	10.13%
δ_1	0.00%	0.00%	0.00%	0.00%	0.80%	0.32%	0.00%	0.00%
δ_2	12.54%	0.16%	0.00%	0.00%	0.16%	0.00%	24.92%	0.00%
v	1.29%	0.00%	16.72%	0.32%	0.00%	0.00%	0.00%	0.00%

Table 4. Percentage of the rolling-sampled estimated parameters that are statistically insignificant at 5% and 1% levels of significance.

	FTSE20		DAX30		FTSE100		SP500	
Parameter	5% sign. Level	1% sign. Level	5% sign. Level	1% sign. Level	5% sign. Level	1% sign. Level	5% sign. Level	1% sign. Level
μ_0	30.06%	76.21%	88.36%	99.37%	94.69%	100%	66.35%	84.28%
μ_1	32.80%	97.11%	93.87%	100%	99.22%	100%	57.08%	87.26%
μ_2	99.84%	100%	100%	100%	99.22%	100%	100%	100%
μ_3	65.11%	87.78%	100%	100%	79.69%	96.56%	92.77%	100%
a_0	0.00%	0.48%	0.00%	0.00%	17.81%	40.78%	1.57%	18.08%
δ_0	27.65%	57.07%	81.45%	100%	100%	100%	100%	100%

Ψ_1	0.00%	0.00%	0.00%	0.00%	100%	100%	100	100
Δ_1	0.00%	0.00%	0.00%	0.00%	31.25%	38.91%	0.00%	0.00%
γ	100%	100%	100%	100%	100%	100%	0.00%	44.18%
δ_1	100%	100%	100%	100%	0.00%	0.16%	94.03%	100%
δ_2	89.55%	99.84%	100%	100%	67.97%	96.56%	59.91%	91.19%
v	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

Table 5. AR(1)GARCH(1,1) simulated process. Percentage of rolling-sampled estimated parameters that are outside the 95% confidence interval.

$$y_t = \mu_0 + \mu_1 y_{t-1} + \varepsilon_t$$

$$\varepsilon_t = z_t \sqrt{\sigma_t^2}\ ,\ z_t \overset{i.i.d.}{\sim} N(0,1)$$

$$\sigma_t^2 = a_0 + a_1 \varepsilon_{t-1}^2 + a_2 \sigma_{t-1}^2$$

	μ_0	μ_1	a_0	a_1	a_2
Simulated Values	0.005	0.150	0.040	0.0500	0.900
Estimated Values (Full Data Sample)	-0.003	0.158	0.037	0.0138	0.895
Rolling parameters outside the 95% c.i.	11.70%	3.32%	73.17%	30.88%	72.17%

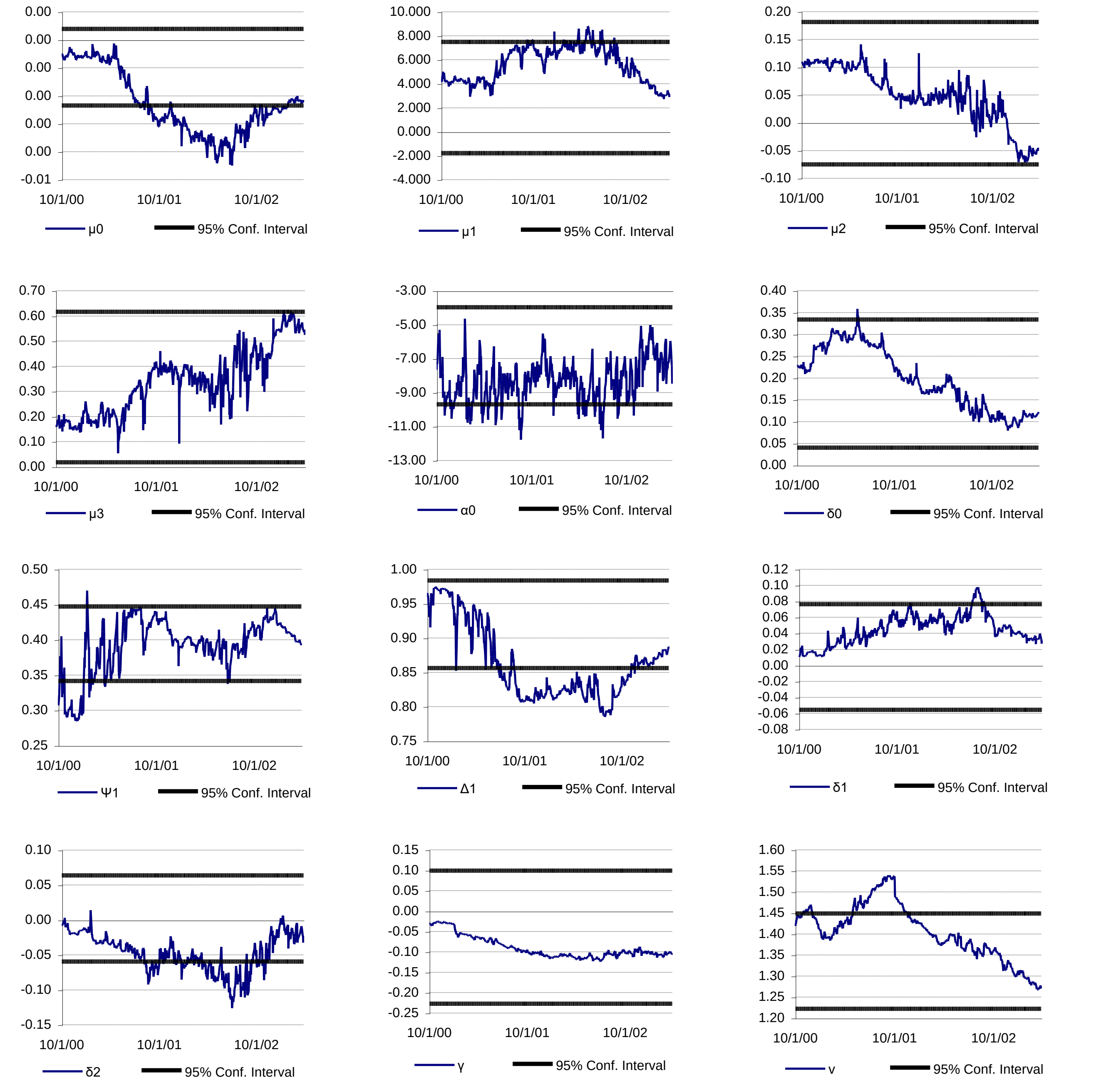
Table 6. Stable parameter estimates, using the full data sample, of the FTSE20, DAX30, FTSE100 and SP500 index daily returns, their standard errors and the percentage of rolling-sampled estimated parameters that are statistically different from the parameter values estimated using the full data sample at 5% level of significance.

	Tail index a	Skewness β	Location δ	Scale γ
	FTSE20			
Coefficient	1.48303	0.07799	-0.00033	0.01005
Standard error	0.05606	0.07965	0.00143	0.00081
5% sign. Level	0.32%	0.00%	0.00%	56.48%
	DAX30			

Coefficient	1.58306	-0.14798	0.00101	0.00754
Standard error	0.15725	0.18828	0.00069	0.00217
5% sign. Level	1.53%	9.59%	0.12%	0.00%
	FTSE100			
Coefficient	1.68238	-0.06489	0.00046	0.00591
Standard error	0.10944	0.25581	0.00039	0.00165
5% sign. Level	2.13%	9.42%	0.49%	0.00%
	SP500			
Coefficient	1.49172	-0.11841	0.0005	0.00525
Standard error	0.07160	0.09609	0.00052	0.00218
5% sign. Level	51.46%	5.00%	0.00%	0.00%

Notes: The standard error of parameter a is computed as the standard deviation of the rolling-sampled estimated parameters, $\hat{a}^{(t)}$, for $t = 1,...,T$ trading days, i.e. $\sqrt{(T-1)^{-1}\sum_{t=1}^{T}\left(\hat{a}^{(t)} - \overline{a}^{(T)}\right)^2}$, where $\overline{a}^{(T)} = T^{-1}\sum_{t=1}^{T}\hat{a}^{(t)}$.

Figure 1. The rolling-sampled estimated parameters of the ARCH model and the 95% confidence interval of the parameters estimated using the full data sample.



Notes: The 95% confidence interval is constructed as $\hat{\theta} \pm GED(0,1;1.335,0.025)\hat{S}_{\theta}\sqrt{1621/1000}$, where $\hat{\theta}$ denotes the parameter vector estimated using the full data sample, $\hat{S}_{\theta}$ is the standard deviation of $\hat{\theta}$ and $GED(0,1;v,a)$ is the $(1-a)$ percentile of the GED distribution, with v denoting the tail thickness parameter.

Figure 2.a. In-sample 95% confidence interval of the FTSE20 index daily returns for the ARCH model (11th January 2000 to 5th July 2002).

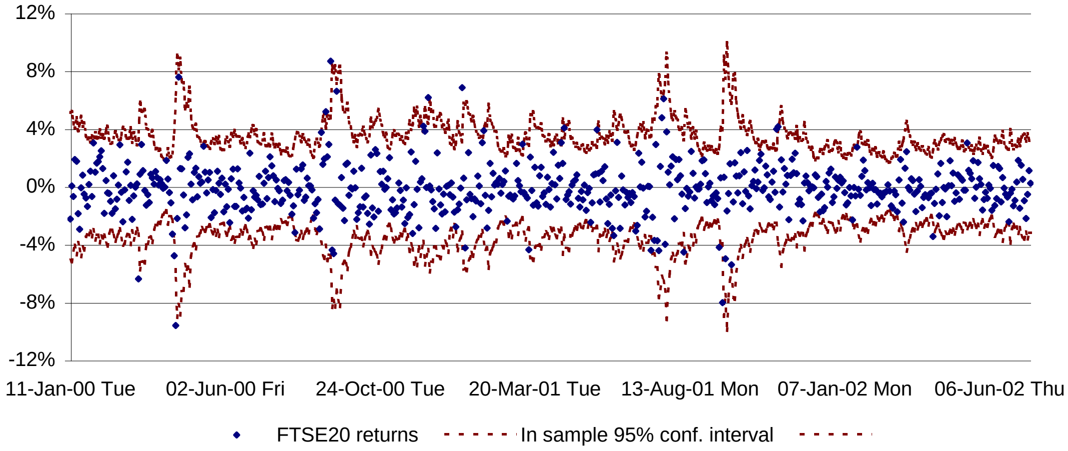


Figure 2.b. One-step-ahead 95% prediction interval of the FTSE20 index daily returns for the ARCH model (11th January 2000 to 5th July 2002).

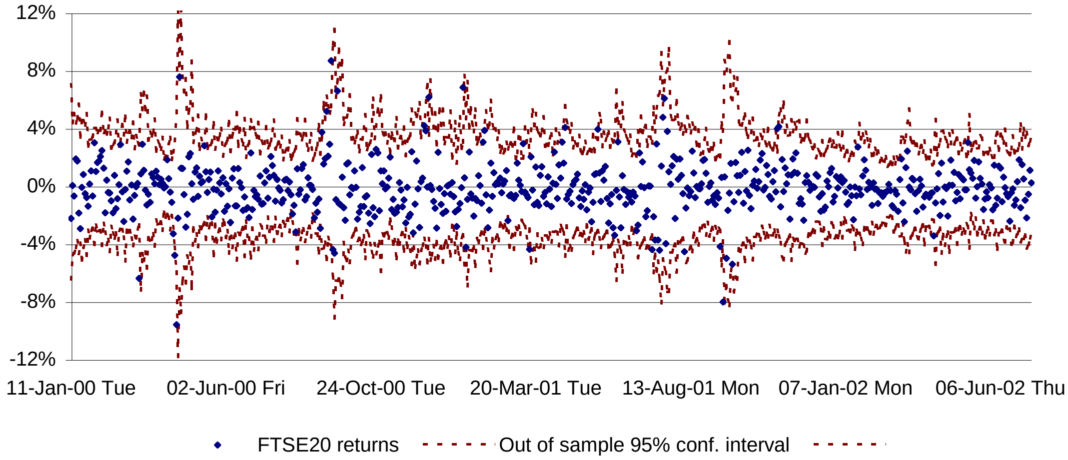


Figure 3. FTSE20 index daily returns. The rolling-sampled estimated parameters of the stable distribution and the 95% confidence interval of the parameters estimated using the full data sample.