Inequality and Production Elasticity

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Abstract

We address a contention regarding capital deepening when the labor share of income declines and the elasticity of substitution is above unity between Karabarbounis and Neiman (2013) and Elsby et. al (2013). We demonstrate the incentive for technical change, which increases inequality and how investments in new technology create temporal misalignment between a decrease in the labor share of income and capital deepening. We show how the decline in the saving rate that occurred during the 80's and 90's may resolve the contention regarding capital deepening. We find that elasticity of substitution below unity is less consistent with the decline in the labor share of income.

A second contention is whether the elasticity of substitution is above or below unity. We perform a time-varying state-space estimation of the evolution of elasticity using the unadjusted marginal product of labor and the Kalman Filter. We find that the elasticity between capital and labor has been fluctuating slightly above unity since 1980, which is consistent with our theoretical findings. We note that an elasticity of substitution above unity has important implications for balanced growth under capital augmentation.

Keywords: Inequality, Technical Change, Elasticity of Substitution, Labor Share

JEL: E1, E2, E3

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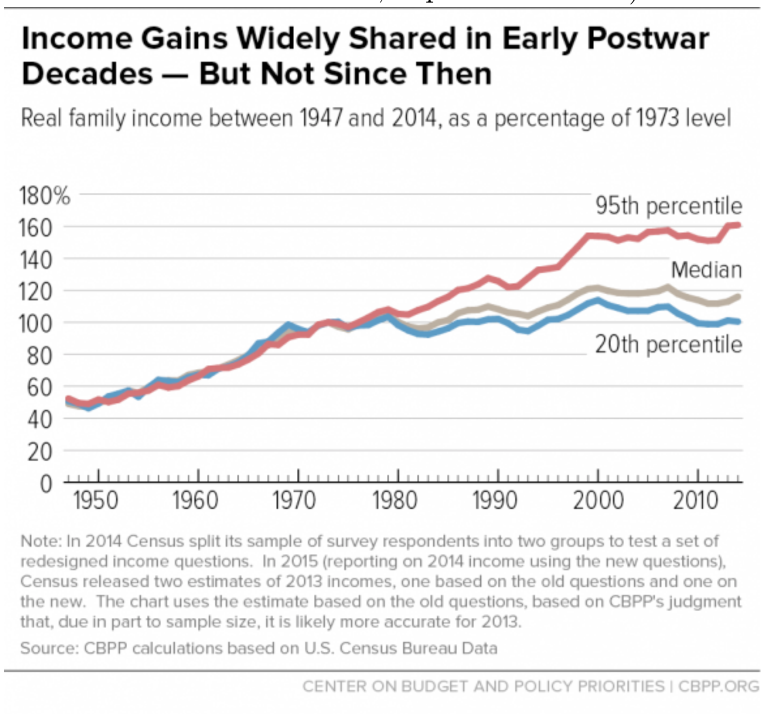
1 Introduction

"To determine the laws which regulate this (income) distribution is the principal problem in Political Economy" (Ricardo 1891 in reference to the distribution of income between land, capital and labor).



The census family income data, as compiled by CBPP (2015), suggest that since the late 70's the share of income gains between rich and poor have been diverging. This paper is focused on the determinants of income distribution between capital and labor since "wages make a small fraction of very top incomes, and trends in wage inequality can only explain a small fraction of the trends in very top income shares" (Piketty and Saez 2001).

Source: Karabarbounis and Neiman (2013)

Karabarbounis and Neiman (2013) note that global labor share of income has significantly declined since the early 1980s, with the decline occurring within the large majority of countries and industries. They further find that a reduction in the cost of investment goods relative to consumption goods explains half of the decrease in the labor share of income. Another key finding by Karabarbounis and Neiman (2013) is that under a CES production function the elasticity of substitution between capital and labor is 1.25, significantly above unity.

Two contentions arise from Karabarbounis and Neiman's (2013) that the current paper aims to address. The first contention is in regards to capital deepening and the second is in regards to whether the elasticity of substitution is above or below unity.

1.1 First Contention - Capital Deepening

Elsby et. al (2013) show that when the elasticity of substitution is above unity, a decrease in the labor share implies capital deepening. Such capital deepening did not occur and, therefore, they discount Karabarbounis and Neiman's findings and suggest, instead, that competition from import is responsible for the decline in the share of labor. However, Autor et. al (2017) note that the labor share has also declined in most non-traded sectors such as wholesale, retail and utilities, a pattern not readily explained by rising trade. They provide an explanation of "winner takes most" "superstar firms" that are disproportionally responsible for the decline in the labor share. However, their explanation does not solve the contention regarding the lack of capital deepening. Lawrence (2015) provides evidence that in the United States the elasticity of substitution between capital and labor is less than unity. Lawrence claims that given this low elasticity the cause of the decline in labor's share of income is the weakness of investment in the face of faster labor-augmenting technical change rather than more capital deepening. However, Koh et. al (2016) show that intellectual property products (IPP) entirely account for the observed decline of the US labor share. They also find that the elasticity of substitution implied from the most updated data that contains IPP capital (and income) is 1.866.

We use a Solow model with a representative firm that owns the capital and is controlled by the capitalists to demonstrate how investments in new technology together with the decline in the saving rate that occurred during the 80's and 90's resolve this contention and explain a decline in the labor share of income with no capital deepening.

1.2 Second Contention - Elasticity Above Unity

The first contention regarding capital deepening is centered around Karabarbounis and Neiman's (2013) estimate of elasticity of substitution above unity, which differs from most previous estimates of elasticity of substitution below unity.

The second contention is therefore whether the elasticity of substitution is above or below unity. We hypothesize that the inconsistency of elasticity estimates in the literature signify that the elasticity of substitution is an evolving parameter of the production technology.

Previous elasticity estimates were derived by linear regression, which produced fixed estimates for the elasticity of substitution. We employ a time-varying state-space technique to estimate the evolution of the elasticity. We find that the elasticity of substitution has been fluctuating slightly above unity since 1980.

1.3 Outline

In the following section we present a simple Solow model with a representative firm that owns the capital and is controlled by the capitalist agents in the economy. The heterogenous agents differ from the original model mainly in that they are not agnostic between capital and wage income. The more capital, or share of the firm, agents have the more they care about capital income. Since the capitalists control the representative firm they employ technical change in service of capital income growth.

In the third section we address the first contention regarding capital deepening by analyzing the dynamic behavior of the economy under different elasticity of substitution cases. In the fourth section we address the second contention regarding the elasticity of substitution using a time-varying state-space technique to estimate the evolution of elasticity. We conclude in the fifth section.

2 Model

We begin with a Solow model of the economy but we do not assume that capital is divided homogeneously between the agents in the economy. Instead, individual agents may be thought of as workers with random degrees of capital. This structure accommodates a continuum of classes rather than just one or two classes (O'connell 1995). We term agents whose capital income is larger than their wage income “capitalists”. All the capital in the economy is invested in a representative firm, which is controlled by the capitalists. Capitalists may divide their wealth between bonds and stocks but bonds, in this closed economy, sum to zero in aggregate. Hence, capital's share of income equals:

$$\pi = F(TK, EL) - W \cdot L$$

where π is capital income, K capital, T capital augmenting technology, L labor, E labor augmenting education, and W the wage. Education and labor are evolving exogenously while capital and production are endogenous.

We assume that markets are competitive and that the aggregate production function is concave with constant returns to scale in both capital and labor. From profit maximization, labor earns its marginal product $W = \frac{\partial F}{\partial L}$ and using Euler's rule, capital's share of income equals:

$$\pi = \frac{\partial F}{\partial K} \cdot K + \frac{\partial F}{\partial L} \cdot L - W \cdot L = \frac{\partial F}{\partial K} \cdot K \quad (1)$$

In this model the firm earns the marginal product of capital. This structure solves an age-old conundrum that “If all inputs were paid their value marginal product, the firm would suffer losses" (Romer 1989 attributing to Schumpeter 1942).

As in the original Solow model the saving rate s and depreciation rate δ are determined exogenously. The capital law of motion is therefore:

$$\dot{K} = sF(TK, EL) - \delta K$$

Capital per effective worker also remains as in the original Solow model:

$$k = \frac{K}{EL}$$

Accordingly, output per worker is:

$$f(Tk) = \frac{F(TK, EL)}{EL} = F(Tk, 1)$$

The important change from the Solow model is that neither the agents nor the firm are agnostic between wage and capital income. In the Solow model both factors earn their marginal product and the firm's profit is zero. Here, the more capital agents have, the stronger their interest in capital income over wage income. Moreover, capitalists control the representative firm, which earns the marginal product of its capital and therefore it employs technology with the sole purpose of growing capital income.

2.1 Technical Change

Capital with increased marginal productivity becomes available exogenously but its deployment is determined endogenously and requires investment that converts capital with lower marginal productivity to capital with higher marginal productivity. Capital with higher marginal productivity costs more than unity in terms of the lower marginal productivity capital. As a result, when the firm invests in new technology, it suffers a reduction in the amount of capital (Greenwood et. al 1997).

The firm deploys capital with higher marginal productivity only when capital income is increased as a result, which occurs when the increase in marginal productivity is higher than the decrease in capital: $\pi' = \frac{\partial F'}{\partial K} \cdot K' > \frac{\partial F}{\partial K} \cdot K$.

2.2 Inequality

Inequality θ is defined as the ratio of profit per worker, to wage (Ricardo 1891):

$$\theta = \frac{\pi/L}{W}$$

This definition of income inequality focuses on the disparity between wealth derived income and wage derived income. It does not make any assumptions regarding the concentration of wealth. Under the assumption of competitive markets both capital and labor earn their marginal product. Therefore, inequality in the model takes the form of capital to labor income ratio:

$$\theta = \frac{\frac{\partial F}{\partial K} \cdot K/L}{\frac{\partial F}{\partial L}} = \frac{\frac{\partial F}{\partial K} \cdot K}{\frac{\partial F}{\partial L} \cdot L} \tag{2}$$

3 Capital Deepening

Since the representative firm earns the marginal product of its capital, it employs technology with the sole purpose of growing capital income. In order to understand how technical change may be used for increasing capital income we consider three cases of a CES production function starting with the simple case of elasticity of substitution σ close to unity (Cobb-Douglas), then with elasticity larger than unity $\sigma > 1$ and finally with elasticity lower than unity $0 < \sigma < 1$.

3.1 Cobb-Douglass $\sigma = 1$

Consider the simple case of output when the elasticity of substitution equal to unity (Cobb-Douglas):

$$Y = (TK)^{\alpha}(EL)^{1-\alpha} = (Tk)^{\alpha}EL$$

Capital income is: $\pi = \frac{\partial F}{\partial K} \cdot K = \alpha Y = \alpha(Tk)^{\alpha}\bar{E}L$, which may be increased by an increase in capital intensity α, technology augmentation T or both, through capital conversion, taking augmented labor EL as given. Capital conversion that increases $\frac{\partial F}{\partial K}$ reduces total capital K and capital per worker k.

It is straight forward to see that capital income is increasing in capital augmentation T. For an increase in the intensity of capital α to have a positive effect on capital income the condition is a positive derivative:

$$\frac{\partial \pi}{\partial \alpha} = Y + \alpha Y \ln(Tk) = Y(1 + \alpha \ln Tk) > 0$$

$$(Tk)^{\alpha} > e^{-1} \tag{3}$$

As long as output per worker $y = (Tk)^{\alpha}$ is above $1/e$, an increase in capital intensity α has a positive effect on capital income. An increase in capital augmentation T is therefore synergetic with an increase in capital intensity α since it makes the condition (3) less binding. On the other hand, inequality sharply increases in capital intensity:

$$\theta = \frac{\alpha}{(1-\alpha)}$$

Increases in both capital intensity α and technology augmentation T affect output per worker and capital income. Such technical change requires investment in new technology through capital conversion. The investment reduces capital per worker k leading to a change in the dynamics of the model and to a new steady state. Using the capital law of motion $\dot{K} = sY - \delta K$, capital per worker $k = K/EL$ and the log derivation procedure while assigning corresponding growth rate terms for education and labor yields:

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{E}}{E} - \frac{\dot{L}}{L} = \frac{sY}{K} - \delta - g_E - g_L$$

$$\dot{k} = sy - k(g_E + g_L + \delta)$$

The steady state $\dot{k} = 0$ requires:

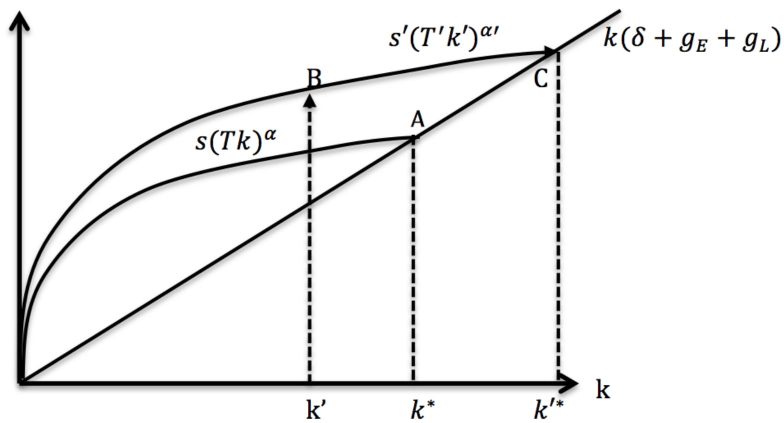
$$sy = k(g_E + g_L + \delta) \tag{4}$$

$$sT^{\alpha}k^{\alpha} = k(g_E + g_L + \delta)$$

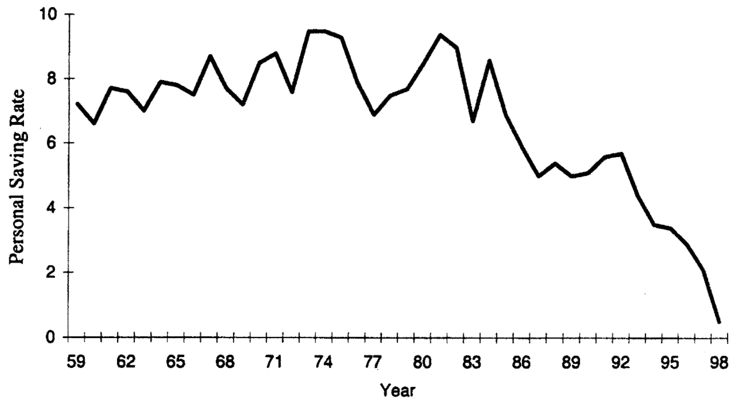
with a steady state capital per worker:

$$k_{ss} = \left(\frac{sT^{\alpha}}{g_E + g_L + \delta}\right)^{\frac{1}{1-\alpha}} \tag{5}$$

The following diagram depicts the effect of technical change on the dynamics of the economy beginning from the steady state before the change marked “A”. As a result of investment in new technology via capital conversion, the economy transitions to a higher production curve but lower capital per worker marked “B”, which is not a steady state. If the saving rate is unchanged $s = s'$, the economy ends up in a new steady state marked “C” where capital per worker is larger than before the change $k'^{*} > k^{*}$, meaning that technical change does result in capital deepening.



Elsby et al. define the income share of labor to be $\lambda = 1 - \alpha(k)$ where $\alpha(k) = \frac{f'(k)k}{f(k)}$. They conclude that because the income share of labor in the Cobb-Douglas case $\lambda = 1 - \frac{\alpha k^{\alpha}}{k^{\alpha}} = 1 - \alpha$ does not depend on k, there is no capital deepening that is implied by a decrease in the labor share. However, when the dynamics of the economy are taken into consideration, an increase in either T or α does result in capital deepening unless the saving rate s falls or, alternatively, that depreciation rate δ, labor force growth rate g_L, or education growth rate g_E increase. Parker (2000) shows evidence that the saving rate indeed dropped during the decline in labor share of income.



Source: Parker (2000)

The above analysis settles the first contention regarding capital deepening in the Cobb-Douglas case. First, the incentive for technical change that increases inequality is clearly demonstrated when the firm earns capital income. Second, the decline in saving rate provides a plausible explanation for a decreased labor share of income without capital deepening. Third, the concern of Elsby et al. regarding the temporal misalignment of capital deepening with the decrease in labor share is explained by the initial investment in new technology through capital conversion that temporarily reduces capital per worker.

3.2 Gross Substitution $\sigma > 1$

Next consider the CES production function with elasticity larger than unity in line with Karabarbounis and Neiman's estimate of $\sigma = 1.25$. The production function is: $Y = [\alpha(TK)^r + (1-\alpha)(EL)^r]^{1/r}$ where $r = \frac{\sigma-1}{\sigma}$, and capital income is: $\pi = \frac{\partial F}{\partial K} \cdot K = \alpha(TK)^r Y^{1-r}$. Capital income is increasing in α under the following condition:

$$\frac{\partial \pi}{\partial \alpha} = (TK)^r Y^{1-r} + \alpha(TK)^r(\frac{1-r}{r})Y^{1-2r}[(TK)^r - (EL)^r] > 0$$

$$(TK)^r Y^{1-2r}\{Y^r + \alpha(\frac{1-r}{r})[(TK)^r - (EL)^r]\} > 0$$

$$\alpha(TK)^r(1 + \frac{1-r}{r}) + (EL)^r - \alpha(EL)^r(1 + \frac{1-r}{r}) > 0$$

$$\frac{\alpha}{r}(TK)^r + (EL)^r > \frac{\alpha}{r}(EL)^r$$

$$(Tk)^r > 1 - \frac{r}{\alpha}$$

$$(Tk)^r > 1 - \frac{r}{\alpha} \tag{6}$$

When the elasticity σ is larger than unity, then $0 < r < 1$ and the expression on the right hand side of the inequality is close to zero or negative. Consequently, we conclude that an increase in capital intensity α has a positive effect on capital income. An increase in capital augmenting technology T increases both output and capital income. Similar to the Cobb-Douglas case, an increase in the marginal product of capital through increases in α, T or both would require an investment, which reduces capital K and capital per worker k. As a result, the economy's dynamics shift to a new steady state that requires:

$$sy = s[\alpha(Tk)^r + (1-\alpha)]^{1/r} = k(g_E + g_L + \delta)$$

which leads to a new capital per worker steady state:

$$k_{ss} = s\left(\frac{1-\alpha}{(g_E + g_L + \delta)^r - \alpha s^r T^r}\right)^{\frac{1}{r}} \tag{7}$$

In order to find out whether an increase in α results in capital deepening we check if:

$$\frac{\partial k_{ss}}{\partial \alpha} > 0$$

$$\frac{s}{r}\left(\frac{1-\alpha}{(g_E + g_L + \delta)^r - \alpha s^r T^r}\right)^{\frac{1}{r}-1}\left(\frac{-(g_E + g_L + \delta)^r + \alpha s^r T^r + (1-\alpha)s^r T^r}{[(g_E + g_L + \delta)^r - \alpha s^r T^r]^2}\right) > 0$$

Since $r > 0$ this condition requires that:

$$(g_E + g_L + \delta)^r > \alpha s^r T^r \bigcap s^r T^r > (g_E + g_L + \delta)^r$$

which is satisfied when:

$$s^r T^r > (g_E + g_L + \delta)^r > \alpha s^r T^r \tag{8}$$

Similarly, the steady state capital per worker k_{ss} increases in T under the following conditions:

$$\frac{\partial k_{ss}}{\partial T} > 0$$

$$\frac{s}{r}\left(\frac{1-\alpha}{(g_E + g_L + \delta)^r - \alpha s^r T^r}\right)^{\frac{1}{r}-1}\left(\frac{(1-\alpha) r \alpha s^r T^{r-1}}{[(g_E + g_L + \delta)^r - \alpha s^r T^r]^2}\right) > 0$$

which is satisfied when:

$$(g_E + g_L + \delta)^r > \alpha s^r T^r \tag{9}$$

Contrary to Elsby et al., a decrease in the labor share through increases in α, T or both would lead to capital deepening in steady state only under the above strict conditions (8) and (9) that may be violated by a high enough increases in α and T themselves as well as changes to the savings, depreciation or growth rates. On the other hand, inequality increases more sharply than in the Cobb-Douglass case due to increases in both capital augmentation T, and capital intensity α even without capital deepening:

$$\theta = \frac{\alpha (TK)^r Y^{1-r}}{(1-\alpha)(EL)^r Y^{1-r}} = \frac{\alpha (Tk)^r}{(1-\alpha)}$$

The above analysis settles the first contention regarding capital deepening in the gross substitution case. A decline in saving rate provides one plausible explanation for a decreased labor share without capital deepening. It is less clear whether there would be capital deepening to begin with, in the gross substitution case, due to strict conditions (8, 9) for capital deepening as a result of increases in α and T. On the other hand, the effects of these increases on capital share of income and inequality are stronger than in the Cobb-Douglas case.

3.3 Gross Complementarity $0 < \sigma < 1$

For elasticity between zero and unity $0 < \sigma < 1$, $r < 0$ and therefore inequality and capital share of income is increasing in α and decreasing in both T and k:

$$\theta = \frac{\alpha(Tk)^r}{1-\alpha} \tag{10}$$

Therefore, an increase in T is counterfactual and we rule it out since it reduces the capital share of income. The condition for a positive effect of an increase in α on capital income from (6) when $r < 0$ becomes:

$$(Tk)^r < 1 - \frac{r}{\alpha} \tag{11}$$

Since the right hand side is larger than unity, this condition is satisfied when $Tk > 1$. When (12) holds, an increase in α is pursued by the firm leading to capital deepening when:

$$\frac{\partial k_{ss}}{\partial \alpha} > 0$$

$$\frac{s}{r}\left(\frac{1-\alpha}{(g_E+g_L+\delta)^r - \alpha s^r T^r}\right)^{\frac{1}{r}-1}\left(\frac{-(g_E+g_L+\delta)^r + \alpha s^r T^r + (1-\alpha)s^r T^r}{[(g_E+g_L+\delta)^r - \alpha s^r T^r]^2}\right) > 0$$

Since $r < 0$ this condition requires that:

$$(g_E+g_L+\delta)^r > \alpha s^r T^r \bigcap s^r T^r < (g_E+g_L+\delta)^r$$

or

$$(g_E+g_L+\delta)^r < \alpha s^r T^r \bigcap s^r T^r > (g_E+g_L+\delta)^r$$

which are satisfied when:

$$(g_E+g_L+\delta)^r > s^r T^r > \alpha s^r T^r \tag{12}$$

or

$$(g_E+g_L+\delta)^r < \alpha s^r T^r < s^r T^r \tag{13}$$

Conditions (13) or (14) may well be met. Therefore, contrary to Elsby et al., an increase in α may well lead to capital deepening in steady state under gross complementarity. However, capital deepening would in turn reduce the capital share of income, which is counterfactual. Therefore, the decrease in labor share is less consistent with gross complementarity in the production function.

3.4 Summary

We note in passing that the choice by Elsby et al. of $k = \frac{TK}{EL}$ may result in disagreement with any measure of capital per worker that does not include capital augmentation. In addition, we have shown how considering initial capital investment in new technology may ease Elsby et al. concern over temporal misalignment between the declines in the labor share of income and capital deepening. We have shown how considering the dynamics of the economy solves the first contention regarding capital deepening through the reduction in the saving rate, which mitigates the effect of technical change on capital deepening. Finally, we have shown that a decrease in the labor share of income is less consistent with elasticity of substitution below unity.

4 Elasticity of Substitution

Most estimates of the elasticity of substitution in the economy under a CES production function are between 0.551 to 0.948 (Antras 2004) as opposed to Karabarbounis and Neiman's (2013) estimate of 1.25. The consequences of this contention go well beyond inequality and the decrease in the income share of labor. This contention relates to the feasibility of capital augmentation (Acemoglu 2003) and to the feasibility of balanced growth in the face of capital augmentation (Uzawa 1965, Grossman et al. 2017). Berndt (1976) as well as Chirinko (2008) show that estimates of the elasticity of substitution vary based on the estimation procedure, data processing and functional form estimated.

We hypothesize that the elasticity of substitution is an evolving parameter of a changing production technology, especially in light of the increase in the degree of automation and the disappearance of routine jobs (Orak 2017). Since previous elasticity estimates were derived by linear regression, they produced fixed estimates. We employ a time-varying state-space estimation technique to estimate the evolution of the elasticity of substitution. We begin from the equality of unadjusted marginal productivity of labor and wage :

$$\frac{\partial F}{\partial L} = Y^{1-r}(1-\alpha)E^r L^{r-1} = W$$

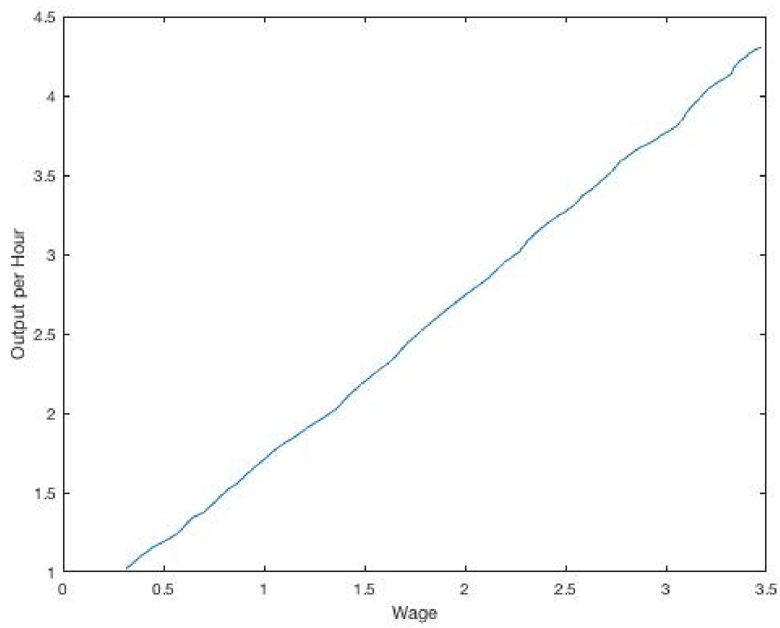
Taking logs and adding time subscripts we get the following observation equation:

$$(1 - r_t)\ln Y_t + \ln(1 - \alpha_t) + r_t \ln E_t + (r_t - 1)\ln L_t = \ln W_t$$

and letting: $y_t = \ln Y_t - \ln L_t$, $w_t = \ln W_t$, $\sigma_t = \frac{1}{1-r_t}$, which is the elasticity of substitution, and $c_t = \sigma_t[\ln(1 - \alpha_t) + r_t \ln E_t]$, produces a simplified observation equation:

$$y_t = \sigma_t w_t - c_t + \epsilon_t \tag{14}$$

Where the dependent variable y_t is log GDP per hour, the independent variable w_t is the log wage per hour and the slope σ_t is the elasticity of substitution. The relationship between the variables appears to be linear:



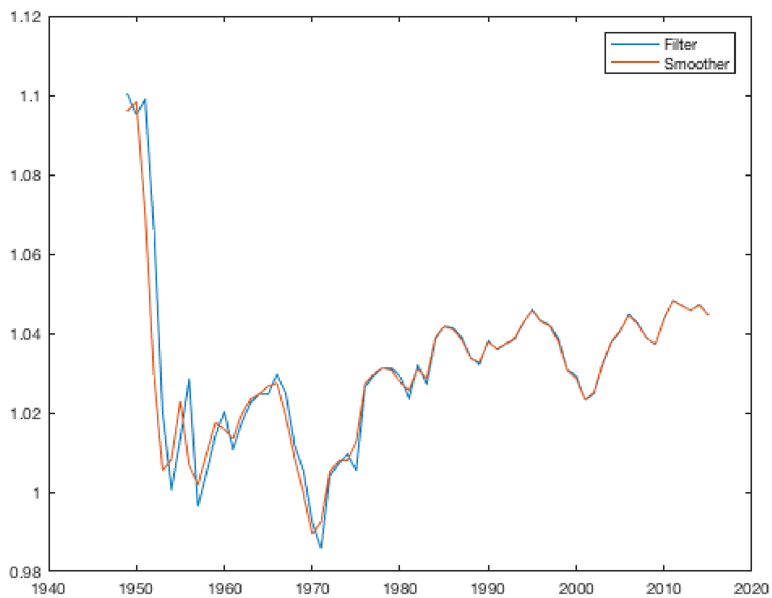
For a time-varying estimation we put the observation equation in a time varying regression form: $y_t = x_t'\beta_t + \epsilon_t$ where $x_t = \begin{bmatrix} w_t \\ -1 \end{bmatrix}$ and $\beta_t = \begin{bmatrix} \sigma_t \\ c_t \end{bmatrix}$. The time varying parameters β_t are assumed to evolve around their mean $\bar{\beta}$ according to a Markov process with Gaussian noise: $\beta_{t+1} - \bar{\beta} = F(\beta_t - \bar{\beta}) + \nu_t$. We define the state variable to be: $s_t = \beta_t - \bar{\beta}$, which evolves according to the state equation:

$$s_{t+1} = F s_t + \nu_t \tag{15}$$

Accordingly, the observation equation becomes:

$$y_t = x_t'\bar{\beta} + x_t's_t + \epsilon_t \tag{16}$$

We use annual data on output (GDP), hours worked, and wages (FRED 2017) for the sample period (1948-2015). In order to estimate $\bar{\beta}$ we use an OLS regression of the model $y_t = \sigma w_t - c + \epsilon_t$, which yields an elasticity estimate of 1.0505 with log-likelihood 167. Applying the Kalman Filter (Hamilton 1994) to the data, we maximize the likelihood of the model by optimizing the parameters $F, Var(\nu_t), Var(\epsilon_t)$. We report below the results of the Kalman Filter and Kalman Smoother algorithms for the time-varying estimation of the elasticity of substitution σ_t with log likelihood 1,144 (Matlab code is provided in appendix 2):



The estimated trajectory is sensitive to the choice of $\bar{\beta}$ until 1980 from which point it fluctuates at only slightly above unity and close to the OLS estimate. We did not find an increasing trend that we expected to find based on the increase in automation. Our finding that the elasticity of substitution is slightly above unity is consistent with our theoretical findings of the conditions for a decrease in the labor share of income.

5 Conclusions

We have clearly demonstrated the incentive for technical change that increases inequality when the capitalists control the representative firm, which earns capital income. We have shown how considering initial capital investment in new technology may ease Elsby et al. concern over temporal misalignment between the declines in the labor share of income and capital deepening. We have shown how considering the dynamics of the economy solves the contention regarding capital deepening through the reduction in the saving rate, which mitigates the effect of technical change on capital deepening. Finally, we have shown that a decrease in the labor share of income is less consistent with elasticity of substitution below unity.

We have addressed the second contention regarding whether the elasticity of substitution is above or below unity by performing a time-varying state-space estimation of the evolution of elasticity over time using the unadjusted marginal product of labor. We have shown that production elasticity between capital and labor has been fluctuating at slightly above unity since 1980, which is consistent with our theoretical findings of the conditions for a decrease in the labor share of income. However, an elasticity of substitution above or equal to unity is not consistent with theories that exclude capital augmentation (Acemoglu 2003) or that explain balanced growth under capital augmentation assuming gross complementarity (Grossman et al. 2017).

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