



Munich Personal RePEc Archive

Two conditions for the application of Lorenz curve and Gini coefficient to voting and allocated seats

Thomas Colignatus

Thomas Cool Consultancy & Econometrics

21 July 2017

Online at <https://mpra.ub.uni-muenchen.de/80297/>
MPRA Paper No. 80297, posted 22 July 2017 08:57 UTC

Two conditions for the application of Lorenz curve and Gini coefficient to voting and allocated seats

Thomas Colignatus
July 21 2017

JEL

A100 General Economics: General

D710 Social Choice; Clubs; Committees; Associations,

D720 Political Processes: Rent-seeking, Lobbying, Elections, Legislatures, and Voting Behavior

D630 Equity, Justice, Inequality, and Other Normative Criteria and Measurement

Abstract

The Lorenz curve and Gini coefficient are applied here to measure and graph disproportionality in outcomes for multiseat elections held in 2017. The discussion compares Proportional Representation (PR) in Holland (PR Gini 3.6%) with District Representation (DR) in France (41.6%), UK (15.6%) and Northern Ireland (NI) (36.7%). In France the first preferences of voters for political parties show from the first round in the two rounds run-off election. In UK and NI the first preferences of voters are masked because of strategic voting in the single round First Past the Post system. Thus the PR Gini values for UK and NI must be treated with caution. Some statements in the voting literature hold that the Lorenz and Gini statistics are complex to construct and calculate for voting. Instead, it appears that the application is actually straightforward. These statistics appear to enlighten the difference between PR and DR, and they highlight the disproportionality in the latter. Two conditions are advised to enhance the usefulness of the statistics and the comparability of results: (1) Order the political parties on the ratio (rather than the difference) of the share of seats to the share of votes, (2) Use turnout as the denominator for the shares, and thus include the invalid and wasted vote (no seats received) as a party of their own. The discussion also touches upon the consequences of disproportionality by DR. Quite likely Brexit derives from the UK system of DR and the discontent about (mis-) representation. Likely voting theorists from countries with DR have a bias towards DR and they are less familiar with the better democratic qualities of PR.

Keywords

General Economics, Social Choice, Social Welfare, Election, Majority Rule, Parliament, Legislative, Party System, Representation, Proportion, District, Voting, Seat, Equity, Inequality, Lorenz, Gini coefficient, Voting Paradox, Arrow's Impossibility Theorem

1. Introduction	2
1.1. Main statistical result	2
1.2. Two conditions.....	3
1.3. Seat allocation schemes.....	3
1.4. Strategic vote and masked outcomes	4
2. Lorenz curve and Gini coefficient in the voting literature.....	5
3. The Dutch General Election of 2017.....	6
3.1. PR with open party lists	6
3.2. Result on Lorenz curve and Gini	6
3.3. Data on turnout.....	7
3.4. Selection of coalition and prime minister	7
4. The UK General Elections of 2015 and 2017	8
4.1. District representation, one round with FPTP.....	8

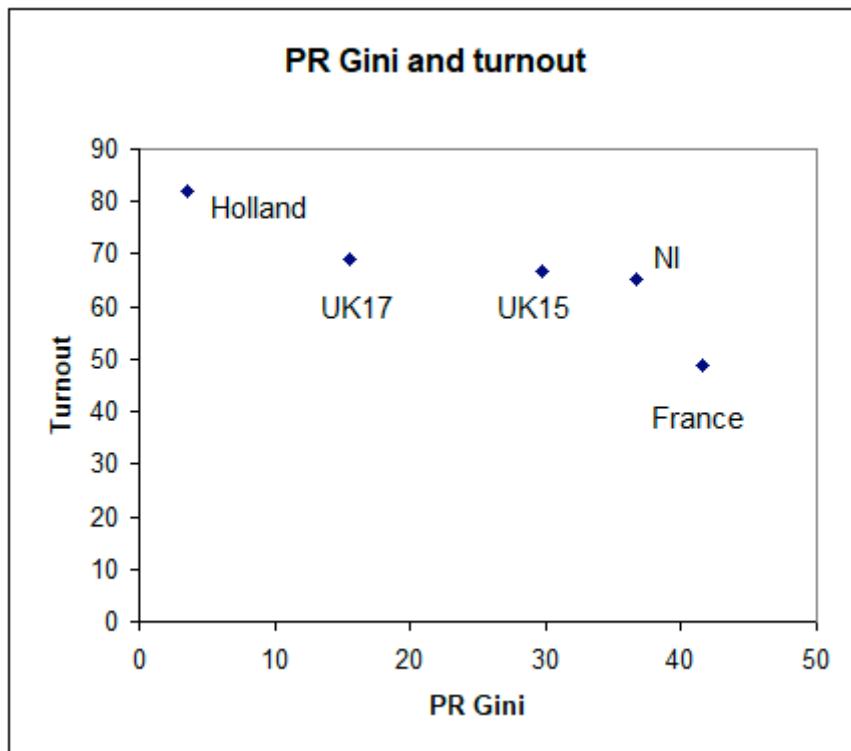
- 4.2. Result on Lorenz curve and Gini8
- 4.3. Data on turnout.....9
- 5. Northern Ireland in the UK General Election of 201710
 - 5.1. NI has 18 seats of the 650 seated UK House of Commons.....10
 - 5.2. Result on Lorenz curve and Gini10
 - 5.3. Data on turnout.....11
- 6. The French General Election of 2017.....12
 - 6.1. Two rounds run-off, strategic vote, majority swings12
 - 6.2. Two rounds mean two sets of data12
 - 6.3. Result on Lorenz curve and Gini12
 - 6.4. Data on turnout.....13
- 7. Risks of disproportionality, like Brexit14
 - 7.1. The combination of DR and populism14
 - 7.2. The situation in France14
 - 7.3. Brexit.....15
 - 7.4. Confusions on PR.....16
- 8. Conclusions17
- 9. References.....18

1. Introduction

1.1. Main statistical result

The main statistical result here is in Figure 1. The data are for general elections in 2017, and for the UK also 2015. The proposed statistic on the Gini for Proportional Representation (PR) - or the "PR Gini" - is on the horizontal axis. Voter turnout is on the vertical axis. A higher PR Gini indicates less proportional representation, and an expectation of such might contribute to less motivation to vote. This paper concerns the theory on the use of the Gini for voting.

Figure 1. Scatter of PR Gini and turnout



1.2. Two conditions

The Lorenz curve and Gini coefficient are well-known for measurement and graphing of income inequality. It is straightforward to apply these notions to other distributions. However, some applications might require some specific attention. The following looks at voting and the allocation of seats. It is suggested that two additional conditions seem rather sensible and that their use would make results (better) comparable.

The Lorenz curve for income requires the ordering of the population on income. Then the cumulated share of total income is related to the cumulated frequency share. A possible intermediate step in calculation - also for presenting the data - is to determine classes: counting the number of persons per class and summing the income per class.

In voting, such a class is commonly available as a party. We currently consider multiple seats elections, such that voters from the electorate vote for political parties and such that seats are allocated to the parties. The votes provide the frequencies. The seats would represent the "income". For District Representation (DR) the philosophy is that candidates may stand up as individuals, and then each such candidate may be taken as a party itself, or they might be grouped as "independents".

Let v_i be the votes shares for party i , and let s_i be the seat shares, for $i = 1, \dots, n$. The cumulated values would be $V_j = v_1 + \dots + v_j$ and $S_j = s_1 + \dots + s_j$. Since we are looking at shares, $S_n = V_n = 1$ or 100%. The Lorenz curve uses V_j on the horizontal axis and the diagonal, and S_j on the vertical axis. The Gini coefficient measures the area from the line connecting the S_j to the diagonal (which diagonal also has the value V_j). If $S_j = V_j$ then there is full proportional representation (PR). To prevent confusion with the Gini for income, this measure for voting may be called the Gini for Proportional Representation (PR), or PR Gini for short.

This article proposes these two conditions:

1. Order the parties from low to high according to the criterion s_i / v_i . The value $s_i / v_i - 1$ can be called the "mismatch". A negative mismatch means underrepresentation (less seats than expected), a zero mismatch means perfect PR, and a positive mismatch means overrepresentation (more seats than expected). The ratio s_i / v_i is more sensitive to disparities than the difference $s_i - v_i$, and thus better suited for an ordering that intends to measure inequality.
2. Use turnout as the base, which includes also the invalid votes and the wasted votes who receive no seats. The invalid and wasted votes thus would be like a party of their own. Common reports tend to express the vote shares as a percentage of the total for parties who got seats. Yet it is important to include the invalid and wasted votes since the very purpose of the exercise is to determine distribution inequality.

Potentially it may be considered to use the electorate as the base, so that the Don't Vote (DNV) are included as a party too. This might be relevant for a study on turnout though (and how turnout affects the proportionality of the outcome). This might be called the Electorate Gini. Here we only look at the (now baptised) PR Gini that takes turnout as the base.

The following discussion clarifies this theory by example results for The Netherlands (Holland), the UK, Northern Ireland and France 2017, and the UK 2015.

1.3. Seat allocation schemes

A high degree of disproportionality is not uncommon or necessarily unreasonable. The Senate in the USA e.g. has two seats per State while they have widely differing sizes of the electorates. The issue of proportionality arises for a House of Representatives (House of Commons), when the objective is to give attention to the sizes of electoral groups.

For electoral systems with district representation (DR), there is often a criterion like *First Past The Post* (FPTP) that determines who receives the seat. Candidates who are not affiliated with parties can be treated as parties by themselves or as a joint party of "independents".

For electoral systems with party-list proportional representation (PR), the seats are allocated according to some mechanism, like Jefferson / D'Hondt / Highest Averages or Webster / Sainte-Laguë / Highest Quotient. This already includes a notion of fair or equal distribution, so that the application of Lorenz and Gini seems less relevant. Yet such systems still have the wasted vote for parties that fail to meet the quota. Sometimes there are thresholds like 5% or 8% of the vote.

For proportional representation of the wasted vote W in total turnout T it would be possible to leave seats empty. When seats are filled then the common majority of $T/2$ would have to be covered by $T - W$, which would imply a qualified majority of $f = T/2 / (T - W)$ or $f = 1/2 / (1 - W/T)$.

We might also consider that the Don't Votes might require some representation. In that case the qualified majority would be $g = E/2 / (T - W)$ with E the electorate. This angle is less realistic, but it remains useful to be aware that comparisons of PR Gini outcomes might need to correct for turnout. Systems of DR might have lower turnout when voters have good reason to expect that their vote will be easily wasted.

Such electoral systems for DR and PR and the mechanisms for the allocation of remainder seats rely on notions of optimality, such that one might reason that there is no need for comparison with another measure on inequality, like the PR Gini. In the same way, one might reason that income is generated within some economic system with freedom and competition, so that there might be no need for additional criteria about inequality. However, it still seems a fair question to enquire about the effect of the method of allocation.

Lorenz curve and Gini coefficient can be used to compare the performance for a particular legislature over time. A major comparison is also between systems, e.g. of proportional representation (PR) and district representation (DR). The inclusion of the wasted vote is important because of the effect of quota or thresholds as well.

There is also the notion of the allocation of power. If a party has 49.9% of the vote and the mere technique of apportionment would generate a majority of 50.1% in the seats, then one might wonder whether this is merely technique, and not a major decision on content. Thus, one possible criterion is that (ruling) coalitions (combinations with a majority) within the electorate might be preserved amongst the seats. We will not consider this, other than above notion of qualified majority. (A simple resolution for such apportionment problems would be to drop the one-vote-per-seat paradigm, and assign a fractional vote, say, to the seat of the party list leader.)

1.4. Strategic vote and masked outcomes

There is the issue of strategic (UK: tactical) voting. A major effect within a system of DR is that voters in a district may not vote for their first preference, but vote to block a candidate who is regarded as worst. Voting outcomes in a system of DR (or for other relevant thresholds) thus cannot be seen as first preferences, and one should try to correct for strategy. When one has no information about strategy then the PR Lorenz and PR Gini better be called "masked", to prevent confusion about their real meaning. ("Masked PR Gini" seems preferable to "DR Gini".)

In a PR system there will be strategic voting too, for example w.r.t. the coalition government. This however doesn't seem a proper reason to adjust the PR Gini coefficient. Such considerations by voters are not caused by a disproportionality within the system, whence there is no reason to require a correction on this.

2. Lorenz curve and Gini coefficient in the voting literature

The voting theory literature contains statements that the Lorenz curve and Gini would be complex to calculate. This might explain why these clear and also graphically informative notions are used so little. However, the calculation is easy and the results are very informative.

The calculation of the Gini is straightforward. There is some advantage of using piecewise linearity here. Each step, from one party to the next one, generates a small trapezium, with the area $h(a + b) / 2$. The parallel sides a and b are the differences (on the left and right) between the diagonal and the cumulated seat curve. The height h is in this case the horizontal distance, given by the vote share v_j of the next party, as $v_j = V_j - V_{j-1}$. Summation of these areas gives a total A . The Gini is equal to $2A$, since the whole area of the square is 1.

The key differences between Proportional Representation (PR) and District Representation (DR) are discussed by Colignatus (2010) with a focus on didactics.

An accessible discussion on disproportionality is by Renwick (2015). It was this discussion that caused me to order the parties by the criterion s_i / v_i , while I had used the difference $s_i - v_i$ before.

Orit Kedar, Liran Harsgor and Raz A. Sheinerman (2013) refer on their page 5 to Taagepera and Shugart (1989) *Seats and Votes*. Let me reproduce the quote from the first authors quoting the second authors:

“They note that ‘an alternative [to the measure of deviation from PR which they use] is the Gini index of inequality, which has theoretical advantages but is more complex to calculate’ (p. 204). They add that ‘the Gini index is the most widespread index of inequality, and it does satisfy Dalton’s principle [of transfers]. The Gini index is useful for many purposes other than electoral studies (where it has been little used)’ (p.263).”

The calculation however is straightforward. It must be mentioned that Kedar et al. have a more complex analysis with districts though.

Anish Tailor and Nicolas Veron (2014) look at inequality in the European Parliament. Their problem is that Germany has 700,000 votes per seat while Malta has 70,000 votes per seat. They find a Gini of the UK of 6.3%, but this thus concerns another research question. If one would look at representation by parties then the EU Parliament might be less disproportional.

Kestelman (2005) also considers measures of apportionment and proportionality, and also refers to Taagepera and Shugart for the Gini (p14). He repeats that the Gini would be complicated to explain and calculate. Thus, curiously:

“Fortunately highly correlated with LHI [Loosmore-Hanby Index], the Gini Disproportionality Index (Gni) is rather complicated to explain and calculate (virtually necessitating computerisation).” (p16)

Kestelman also suggests that Single Transferable Vote (STV) would be a proportional method. This would tend to be correct if the nation were a single district. He might neglect that an application to districts causes disproportionality over the whole nation, as happens in (proposals for) the UK, see Colignatus (2017d).

Alexander Karpov (2008) gives a more analytical overview of the various measures. He also mentions the Gini, and then sums over the ratio of shares (s_i / v_i) but I do not see the rationale for this yet (and one would need a correction so that these sum to 1). The calculation above uses the ratio only for ordering, and further uses levels with their intuitive interpretation.

Karpov’s article got a comment from Michela Chessa (2012) who points to indices on power. Also Renwick (2015) points to the issue of power.

3. The Dutch General Election of 2017

3.1. PR with open party lists

Holland provides a useful point of reference for proportional representation.

- The Dutch electoral system uses PR open party lists, so that voters might also select particular candidates on the party lists. In 2017 four of 150 seats were thus selected by personal preferential vote.
- The system has the natural threshold of gaining a seat in a House of 150 seats, which means a threshold of 0.67% of the vote. This allows easy entry of new parties - which might appeal to economists who favour competition.
- The Dutch system officially still has 19 districts, remaining from 1917 when the country changed from DR to PR, which districts have mainly an administrative role. Parties may feature local candidates (and this might have an effect indeed) and voters might vote for those but few apparently do.

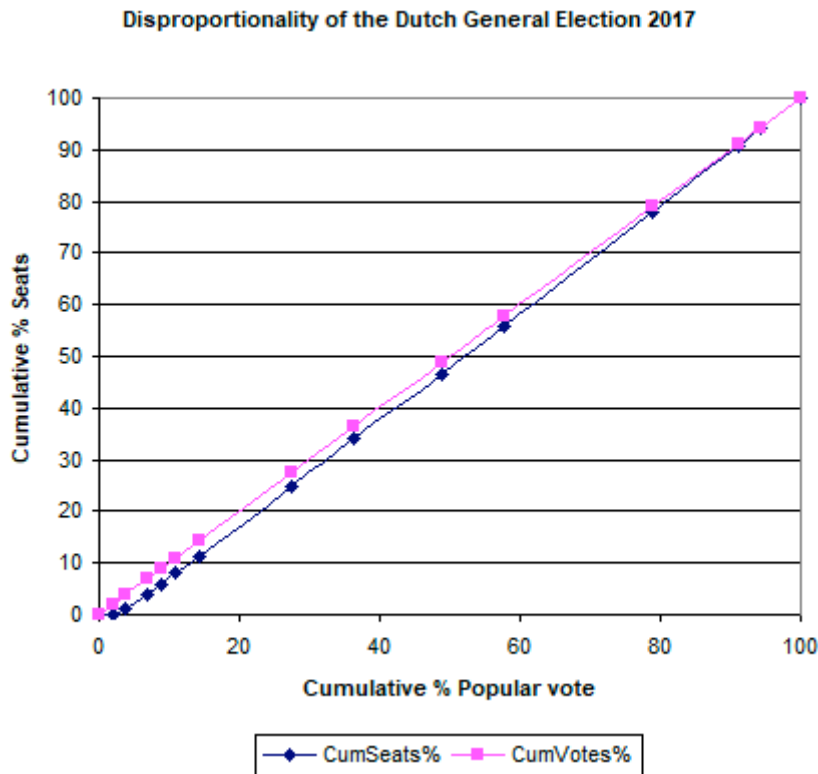
Colignatus (2010) already compared Holland and the UK on an earlier election. See Van Schendelen (1985) for Dutch conventions on "consociational democracy".

3.2. Result on Lorenz curve and Gini

Figure 2 gives the results for the Dutch General Elections of March 15 2017. The horizontal axis gives the cumulative percentage of the popular vote. The vertical axis gives the cumulative percentage of the seats. If there is proportional allocation of seats, then the blue line of the seats would cover the pink diagonal.

The Dutch general elections in 2017 generated the PR Gini of **3.6%**. The major impact in Holland are the 2% of voters for small parties who got no seats. The Dutch qualified majority is $f = 50\% / 98\% = 51\%$. Some more information is by Colignatus (2017a).

Figure 2. Lorenz curve for the Dutch General Election 2017



3.3. Data on turnout

The Dutch House of Commons has 150 seats. A coalition requires 50%+1 or 76 seats. Table 1 gives the data on turnout. This highlights alternative criteria for majorities.

- A first alternative focuses on turnout. With the turnout of 81.9% actually only 120 seats were fully taken. 27 Seats were lost to no-shows, 2 seats were lost to the dispersion of small parties and 1 seat was lost on blank or invalid votes. With such an apportionment of seats, it would be less easy for parties to form a coalition with 76 seats.
- A second alternative focuses on the electorate. When we divide the electorate by the actually used votes per seat, then the House would require 187 seats, and a majority would require 94 seats. One might still use all 150 seats but then use a qualified majority of 94 to make decisions (like on a ruling coalition).
- A third alternative focuses on the wasted vote. More realistic would be a qualified majority of 77 seats, to allow for the wasted vote. Instead, however, the elected parties take the 30 seats anyway and still apply the 76 seats majority rule.

Table 1. Turnout data for the Dutch General Election 2017

Dutch general election 2017			Elect. Seats	Seats
(1)	Elected in the House	10,354,714	120	150
(2)	Below threshold	161,327	2	0
(3)	Blank votes	15,876	0	0
(4)	Invalid votes	31,539	1	0
(5)	Votes	10,563,456	123	150
(6)	Did not vote	2,330,010	27	0
(7)	Electorate	12,893,466	150	150
	Turnout	81.93%		
	Wasted = $1 - (1) / (5)$	1.98%		
	Votes (1) for an elected seat	69,031	187	
	Threshold $((1)+(2)) / 150$	70,107		
	Elect. Threshold $(7) / 150$	85,956		
	Standard majority of elected		76	
	Qualified majority wasted vote of (5)		77	
	Qualified majority for electorate (7)		94	

3.4. Selection of coalition and prime minister

A common argument for DR is that it would generate majorities directly (no "hung Parliament") so that the selection of the ruling party and prime minister would be simple. PR is more complex because of the frequent need to form a coalition government. During elections parties need to emphasize their differences and after elections they need to focus on what they can agree on, and potentially many voters might see this as inconsistent.

Currently, Dutch politicians are showing the tediousness of coalition formation. The elections were on March 15 and this July after four months the main negotiators decided that the bargaining process will continue after the Summer break.

The current view in Dutch Parliament is that there must be a coalition agreement that decides on key issues for the next four years. This approach necessitates intensive bargaining within a short time window, and this will also kill much of the political discussion in the coming years because major issues would already have been agreed upon.

However, the selection of a coalition and prime minister within a system of PR need not be as tedious as the Dutch politicians are making it now. Colignatus (2017b) discusses the following notions:

- One could agree on an *agenda for discussion* rather than an *agenda for execution*. This would leave options open, and one can find majorities along the way, also differing majorities for different issues.
- The Executive may be chosen to reflect the Legislative almost in the same manner as the Legislative reflects the voters. Thus there is no real need (other than political opinion) to exclude parties from government, and it would in fact be advantageous that parties participate in the Executive, so that they can show their competence in governing and so that they have less appeal for the protest vote.
- The role of the prime minister need not be one of party-politician but rather would be one of a chairperson above party interests. The selection of this chairperson need not be part of the bargaining process itself, and one might use more advanced voting techniques that are impartial (except for the voting weights).

PM 1. The current Dutch coalition discussion is affected by the following phenomenon. The former coalition was created in 2012 by Conservative-liberal VVD (41 seats) and Labour PvdA (38 seats). Voters did not understand this coalition, since VVD and PvdA had been opposing each other strongly during the election of 2012. In the election of 2017 VVD lost 8 seats and received 33 seats, and PvdA lost 29 seats and is left with 9 seats now. PvdA leader Lodewijk Asscher now prefers not to take part in government, and "restore the party" (no quote) from the opposition benches. This reasoning seems part of a political notion that one loses seats in government and gains seats from opposition (e.g. from a protest vote). This view is dubious. The current voters for PvdA who generated 9 seats apparently supported the PvdA in government with VVD. For them the PvdA apparently could continue to partake in a government coalition. Thus the PvdA disappointed a major section of its voters of 2012 by joining an unlikely coalition and now disappoints its remaining voters of 2017 by refusing to consider government responsibility. Curiously, the Dutch system on coalition formation has not been able to resolve this PvdA conundrum (yet). Potentially Asscher and his PvdA have the Summer break to think about this.

PM 2. Holland had the option for parties for electoral alliances for "list combinations", such that the remainder seats in the Jefferson / D'Hondt allocation process were calculated on such combined base. This option however was (curiously) abolished in 2017.

4. The UK General Elections of 2015 and 2017

4.1. District representation, one round with FPTP

The UK has district representation (DR), with one round with First Past The Post. The UK thus tends to show disproportional results and masked outcomes due to strategic voting.

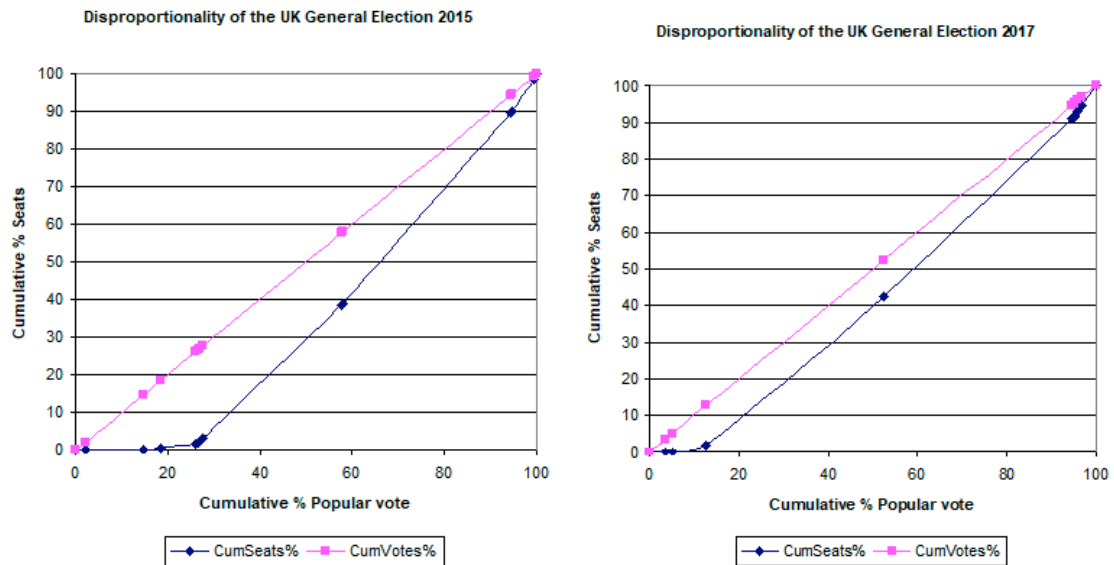
4.2. Result on Lorenz curve and Gini

Figure 3 presents the Lorenz graphs for the UK General Elections of 2015 and 2017. The masked PR Gini coefficients are **29.7%** in 2015 and **15.6%** in 2017.

In 2015, 2.1% of the votes went to parties who got no seats, which is a mismatch of -1. UKIP with 12.5% of the votes got only 0.2% of the seats, namely 1 seat for Nigel Farage. Their mismatch is $0.2 / 12.5 - 1 = -0.988$. LibDem got 7.8% of the votes but only 1.2% of the seats, a mismatch of -0.842.

The situation in 2017 has improved mainly because UKIP no longer really participated. The LibDem still got 7.3% of the vote and 1.8% of the seats, which is a marginal improvement. In 2017 the wasted vote was still 3.5%, and then $f = 50\% / 96.5\% = 51.8\%$. A representative majority in a full House of 650 seats then requires 337 seats, and not 325+1.

Figure 3. Lorenz curves for the UK General Elections of 2015 and 2017



In these graphs, one may recognise the positions of the Conservatives, within each graph on the right hand side. They had 36.4% of the votes and 50.8% of the seats in 2015, and have 42.2% of the votes and 48.8% of the seats in 2017. These percentage use turnout as the denominator (thus including the invalid and wasted votes). If the denominator is used which only sums votes that got seats in the House, then the Conservatives in 2017 got 43.8% of the vote.

The votes, graphs and coefficients are inaccurate because of strategic (or tactical, in UK parlance) voting as happens within DR. A voter in a district who favours a Conservative candidate but sees a loss against a Labour candidate might vote for the LibDem, if there would be a Conservative > LibDem > Labour preference ordering. There will be some averaging out over the districts, but the votes will likely not reflect the true proportions of the first choices.

The lecture by Butler (2014) on the British Electoral Studies (BES) is very informative, and he correctly asks what the elections have decided. Yet I find it remarkable that the issue of DR and its strategic vote apparently has received so little attention by UK researchers on voting. I have not been able to find data on what the UK first preferences might be.

4.3. Data on turnout

The system of DR may also cause a lower turnout. The UK has 68.9% while Holland has 81.9%, a difference of 13%. See Table 2 for the data.

Table 2. Turnout data for the UK General Election 2017

UK general election 2017			Elect. Seats	Seats
(1)	Elected in the House	31,135,797	432	650
(2)	Not in the House	1,068,344	15	0
(3)	Blank votes	0	0	0
(4)	Invalid votes	74,189	1	0
(5)	Votes	32,278,330	448	650
(6)	Did not vote	14,548,151	202	0
(7)	Electorate	46,826,481	650	650
	Turnout	68.93%		
	Wasted = $1 - (1) / (5)$	3.54%		
	Votes (1) for an elected seat	47,901	978	
	Threshold $((1)+(2)) / Seats$	49,545		
	Elect. Threshold $(7) / Seats$	72,041		
	Standard majority of the elected		326	
	Qualified majority wasted vote of (5)		337	
	Qualified majority for electorate (7)		489	

5. Northern Ireland in the UK General Election of 2017

5.1. NI has 18 seats of the 650 seated UK House of Commons

Within the UK House of Commons of 650 seats, there is a section with 18 seats for Northern Ireland. This election is also DR with one round of FPTP.

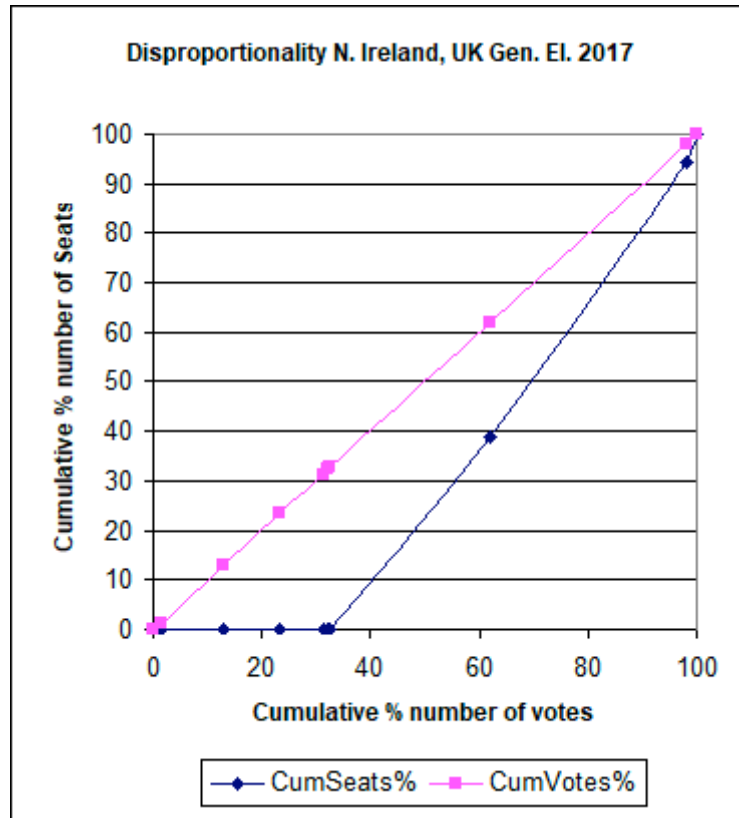
After the UK general election of 2017, Prime Minister May made a coalition agreement with the party DUP from Northern Ireland, including that DUP must respect the Brexit deal that May will achieve (or not) with the EU. The 317 Conservative seats and the 10 seats for DUP generate a majority of 327, just 1 above the common standard of 326.

The disproportionality in NI forms part of the UK disproportionality above. (For a decomposition analysis, the Theil index is better suited than the Gini.)

5.2. Result on Lorenz curve and Gini

The Lorenz curve for Northern Ireland (NI) looks quite like the unequal Lorenz curve of the UK in 2015, see Figure 4. There is a masked PR Gini of **36.7%**, while Holland has an unmasked PR Gini of 3.6. The major reason is the huge wasted vote of 32.6% of parties that lost out on the DR system.

Figure 4. Lorenz curve for Northern Ireland in the UK General Election 2017



5.3. Data on turnout

Turnout is also low at 65.4%. See Table 3 for the data on turnout. The data on blank or invalid votes are preliminary. A NI seat in the UK House of Commons required only 30,410 votes while the UK average is 47,901. At that rate NI should have 41 seats if the total electorate should be represented. Then a majority of 50% would require 21 seats, more than there actually are.

Table 3. Turnout data for Northern Ireland in the UK General Election 2017

Northern Ireland in UK general election 2017		Elect. Seats	Seats
(1)	Elected in the House	547,379	8
(2)	Not in the House	264,804	4
(3)	Blank votes	0	0
(4)	Invalid votes	0	0
(5)	Votes	812,183	12
(6)	Did not vote	430,515	6
(7)	Electorate	1,242,698	18
	Turnout	65.36%	
	Wasted = 1 - (1) / (5)	32.60%	
	Votes (1) for an elected seat	30,410	41
	Threshold ((1)+(2)) / Seats	45,122	
	Elect. Threshold (7) / Seats	69,039	
	Standard majority of the elected		10
	Qualified majority wasted vote of (5)		14
	Qualified majority for electorate (7)		21

6. The French General Election of 2017

This section summarizes findings from Colignatus (2017h).

6.1. Two rounds run-off, strategic vote, majority swings

The French general election for the Legislative was held on June 11 and 18 2017.

Both UK and France have district representation (DR) with a *First Past the Post* rule. In the UK this causes strategic voting, in which a voter may not vote for the candidate of first choice, but tries to block a candidate who might win but would be worst. France has elections in two rounds so that there is less need for such a strategy. The second round is a run-off between the two top candidates in the district. A voter thus has the limited strategy to try to get at least one good candidate in that position. If the candidate of first preference has little chance of getting elected, then one might vote for a second-best who has more chance, and who might not make it to the second round otherwise. If a second-best will be amongst the two top candidates anyway, then one might vote for the first preference. Or one might do the latter anyway, and see from there.

Proportional representation (PR) may allow a larger (but fairer) share of the seats for the more extreme parties, like the party of Geert Wilders in Holland, yet PR also allows more stability for the center. Thus PR tends to avoid the swings between extremes that might happen in systems of district representation (DR). While the UK had Nigel Farage as a challenger on the EU, France has Marine Le Pen.

The situation in France is also more complex because of the separate election of the President, who appoints the Prime Minister. The different elections cause different popular mandates, and this may be awkward when the political views do not match ("cohabitation"). The idea of separate election of the President likely derives from the need to balance the disproportionality of DR. In France, voters were asked to vote four times in 2017 for essentially the same issues. In general PR that selects a House that selects the Prime Minister would be more democratic (with the criteria of PR and the obvious respect for minority rights).

6.2. Two rounds mean two sets of data

The French system seems to make it more difficult to determine the Lorenz curve and Gini coefficient. There are two rounds, and thus there are two sets of data to choose from. However, the following choice suggests itself:

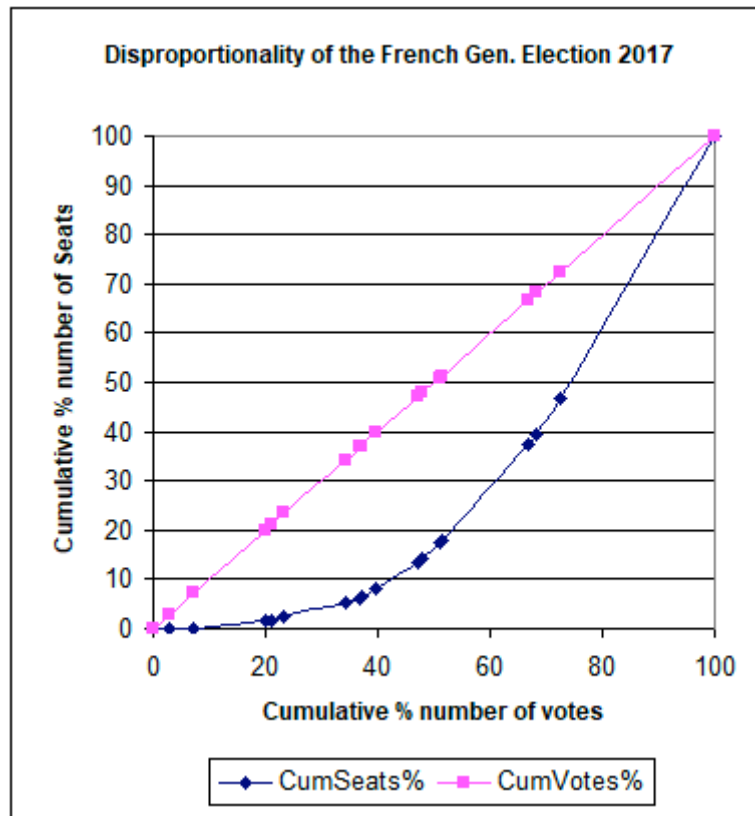
- The data of the first round provide the first preferences, and thus provide the votes.
- The data of both rounds provide the seats.

This choice finds support in the data. The first round had a turnout of 48.7% and 0.5 million invalid or blank votes. In the second round, more people remained at home, with a turnout of 42.6%, while those who voted produced almost 2 million invalid or blank votes, who apparently disapproved of the available candidates or the system itself. Thus the higher turnout and lower blanks in the first round suggest that these indeed reflect the first preferences (with some limited level of strategy). Obviously we are not sure of this, since this would require a system of PR, but it is the best assumption.

6.3. Result on Lorenz curve and Gini

The Lorenz curve in Figure 5 shows a rather surprising level of inequality, with a PR Gini of 41.6%. Compare the value of Holland with a PR Gini of 3.6%. If the blue line would cover the pink diagonal then there would be full proportionality.

Figure 5. Lorenz curve for the French General Election 2017



6.4. Data on turnout

Table 4 gives the data on turnout for the first round. The votes for "Elected in the House" is for parties that eventually got elected in the Legislative. The votes for "Not in the House" is for a radical leftist party that got votes in the first round but got no seat in none of the rounds.

The wasted vote for the first round consists of the invalid and blank votes and the latter "Not in the House", to a total of almost 3%. A standard majority would be 289 seats of a House of 577 seats. If one would keep account of the wasted vote, then one might leave seats empty, or use a qualified majority of 298 seats, thus 9 more than usual.

When we divide the electorate by the number of votes per seat, then the Legislative would require 1222 rather than 577 seats. A majority would require 611 seats, which is more than the actual number of seats used. If one would want to keep account of the voters who did not turn out, then 51.3% or 296 of the 577 seats would be empty, or one would use the 611 seats as a qualified majority.

Table 4. Turnout data for the French General Election 2017 (first round base)

French general election 2017				
	2017		Elect. Seats	Seats
(1)	Elected in the House	22,478,950	273	577
(2)	Not in the House	175,214	2	0
(3)	Blank votes	357,018	4	0
(4)	Invalid votes	156,326	2	0
(5)	Votes	23,167,508	281	577
(6)	Did not vote	24,403,480	296	0
(7)	Electorate	47,570,988	577	577
	Turnout	48.70%		
	Wasted = 1 - (1) / (5)	2.97%		
	Votes (1) for an elected seat	38,958	1222	
	Threshold ((1)+(2)) / Seats	39,262		
	Elect. Threshold (7) / Seats	82,446		
	Standard majority of the elected		289	
	Qualified majority wasted vote of (5)		298	
	Qualified majority for electorate (7)		611	

7. Risks of disproportionality, like Brexit

7.1. The combination of DR and populism

There is a remarkable brew of populism and District Representation (DR). One might think that countries with Proportional Representation (PR) would be most sensitive to populism, but it are rather the countries with DR. Countries with DR appear to be much more affected, like France, UK and USA that use First Past The Post (FPTP).

Countries with DR run the risk that the seats in the House of Commons do not reflect the popular vote, and then they might try to repair this with a referendum, that is proportional. In countries with PR there would be no need for referenda. The problem with referenda is that they are subject to the voting paradoxes that are so well known in voting theory. The UK and the 2016 Brexit referendum is a case in point.

A discussion of *PR versus DR & populism* is in an interview by Stavrou (2017).

7.2. The situation in France

The new French President Emmanuel Macron had the highest score in the first round of the Presidential election of 2017, namely 24% of the vote, with runner-up Marine Le Pen with 21.3%. Macron then won the second round with 66.1% (20.7 million) against Marine Le Pen with 33.9% (10.6 million).

Some time later for the Legislative in the first round, Macron's party REM got 27.6% while Le Pen's Front National (FN) got 12.9%.

Thus, for the Legislative Le Pen managed to get only 3 million votes, compared to the potential of 10.6 million at the presidential elections.

With both rounds REM got 308 seats and FN got 8 seats in the Legislative.

This disproportionality or political inequality doesn't bode well for the feelings amongst the French electorate about whether they are represented, while official ideology from "democracy" is that they would be. The low turnout seems to reflect dissatisfaction rather than satisfaction. Expressing one's first preference in the first round emphasizes the awareness about this first preference, which then is frustrated in three additional election rounds. Such dissatisfaction might also translate into a protest vote over four years, especially when Macron doesn't deliver.

The balance would tip, if Le Pen would manage to motivate the 10.6 million voters of the presidential race to also support her for the Legislative. If the other parties would have a divided vote then it would be Le Pen who would benefit from *First Past The Post*.

7.3. Brexit

The disproportionality in the UK may not without consequences.

- In 2015 UKIP threatened the marginal seats of the Conservatives. This likely caused David Cameron to call the Brexit referendum of 2016. A referendum seems like a PR correction within a DR system, but in fact a referendum is a populist measure for which there is little need within PR.
- Now in 2017 the disproportionality in Northern Ireland seems to facilitate the Hard Brexit that Theresa May seems to intend.

Brexit looks like a major confusion.

- Colignatus (2017e) shows that the Brexit referendum question was flawed in design.
- Colignatus (2017f) discusses preference rankings w.r.t. options for Brexit. Current UK voters tend to respect the Brexit referendum "outcome", but they do not discount for the fact that the Brexit referendum question was flawed in design.

In Holland, Geert Wilders had a similar percentage of the vote like UKIP, but remained rather inconsequential, due to the system of PR. He took part in a pact of tolerance on a minority coalition, but this collapsed, and his influence remains rather small apart from some presence in the media.

The UK Brexit referendum of June 23 2016 generated a relatively high turnout of 72.2%, with 51.9% Leave and 48.1% Remain. The Leave vote concerned only $72.2\% * 51.9\% = 37.5\%$ of the electorate. Normally one would require a majority of 50%, like for example a $2/3$ majority of a turnout of $3/4$ (as $2/3 * 3/4 = 1/2$).

The voter dynamics are a bit remarkable. Of the UK vote of about 32 million there is a stable core of only 26 million. The (surviving) non-voters at the referendum of 2016 amount to 12.3 million, of which 5 million or 41.4%, decide to vote at the general election in 2017, while the other 7.2 million do not show up at both occasions. The 26% Leavers who do not vote at the general election in 2017 amount to 4.4 million voters. The 15% Remainders who do not vote at the general election in 2017 amount to 2.3 million voters. See Table 5 for these voter dynamics in Great Britain, i.e. the UK minus Northern Ireland, and see Colignatus (2017g) for a discussion.

Table 5. Voter dynamics in Great Britain (UK – NI), 2016-2017

Great Britain (UK minus Northern Ireland)							
		Electorate	Did Not Vote	Voters	Turnout	Remain	Leave
(1)	Referendum 2016	45,239,046	12,477,212	32,761,834	72.4%	15,700,534	17,061,300
(2)	Deaths 2016-2017	596,637	164,556	432,081		207,067	225,014
(3)	Survivors	44,642,409	12,312,656	32,329,753		15,493,467	16,836,286
(4)	Attainers	339,734	146,086	193,648	57%	(YouGov)	
(5)	Net registration: (2) + (7) - (4)	601,640	60,164	541,476	90%	(Assumed)	
(6)	Election 2017	45,583,783	14,118,177	31,465,606	69.0%	(Voted for parties)	
(7)	Change: (6) - (1)	344,737	1,640,965	-1,296,228	-3.4%		
(8)	Voted in 2016 but not in 2017		20.7%			15%	26%
(9)	Surviving voters: (3) times (8)	32,329,753	6,701,454	25,628,299	79.3%	2,324,020	4,377,434
(10)	New voters: (6) - (9)	13,254,030	7,416,723	5,837,307	44.0%		
(11)	2016 non-voters: (10) - (4) - (5)	12,312,656	7,210,473	5,102,182	41.4%		

7.4. Confusions on PR

It seems to be a key problem for democracies like France, UK, USA and India (the latter two not discussed here) that they use district representation (DR) rather than proportional representation (PR). This problem is often increased by a separate election of the President, which causes clashes between the two different electoral mandates. Instead it would suffice to adopt a PR party list system, in which the voter selects the party of first choice (potentially like in Holland also in an open list in which one can vote for a particular candidate on a list), and that the professionals in Parliament select the prime minister, using both bargaining and more advanced voting techniques. We may wonder why researchers on voting theory do not generally advise the change from DR to PR.

Researchers on voting theory within a country with DR tend to do their research with the (often tacit) assumption of DR. One consequence of DR thus may be the development of a bias of such researchers on the supposed advantages of DR. When voting researchers have the attitude of informing the public about outcomes of DR as if that would be the natural state of the world, then there is little likelihood that the public is informed about its major disadvantages. Under such conditions there is little scope for advancement to a better democracy that uses PR.

In the English speaking world there are proposals for change from DR to PR, but there remains a preference for some kinds of districts, which however destroys the very purpose of PR.

- Mentioned might be *Alternative Vote* (AV, UK referendum of 2011) and *Single Transferable Vote* (STV) as proposed by the UK Electoral Reform Society (ERS), see the discussion in Colignatus (2010) and (2017di).
- Such proposals try to compromise between DR and PR, but this is like compromising between circles and squares. Logic requires a choice, like there are no square-circles.
- A hidden assumption amongst these proposals is that one wants the voter to use techniques of ranking options. Such techniques are rather complex and beriddled with voting paradoxes related to Arrow's Impossibility Theorem, see Colignatus (2014). The use of such techniques is better restricted to the professionals in Parliament, who have the resources to deal with such paradoxes. For voters it suffices to give their vote to a party, whence the resulting voting weights can be used for bargaining and voting techniques in Parliament.

This still leaves the option to do a little bit with districts. If V are the number of votes and S the number of seats, then $q = V / S$ is the natural electoral quota, or the number of votes required to cover a seat. If districts have size $2q$ then a candidate can be elected by passing 50% of the vote within the district, since 50% of $2q$ meets the quota q . A normal maximum of $V / (2q)$

= $S / 2$ seats might be allocated in such manner (unless votes are perfectly split on q versus q). The remaining votes can be used to allocate seats according to the party list system to guarantee overall PR. In such a manner one may have both districts and PR, but the key principle of design is PR, and the notion that one might create districts of size $2q$ is secondary. (In the UK one would have 325 districts of 100,000 voters, using the data on the present turnout.)

PM. Change of the role of the President in both France and the USA would tend to require a constitutional amendment, which might not be easy to achieve. It is an option that one keeps the current constitution but that the President decides to adopt a ceremonial role like the UK Queen (and perhaps the role of an Ombudsman).

8. Conclusions

Table 6 reviews the findings, and Figure 1 gives a scatter plot. The turnout statistics were already known and the PR Gini statistics are new. There are few data yet, but it stands to reason that a higher PR Gini would tend to contribute to a lower turnout.

Table 6. PR Gini and turnout

	<i>First preferences shown</i>	<i>First preferences masked</i>
<i>Proportional representation (open party lists)</i>	Holland, PR Gini 3.6%, turnout 81.9%	
<i>District representation</i>	France, PR Gini 41.6% and turnout 48.7%	UK 2015, Masked PR Gini 29.7% and turnout 66.9%. UK 2017, Masked PR Gini 15.6% and turnout 68.9%. Northern Ireland, Masked PR Gini 36.7% and turnout 65.4%

Conclusions are:

1. In 2017, France (PR Gini of 41.6%), the UK (Masked 15.6%) and Northern Ireland (Masked 36.7%) are at a problematic distance from proportional representation, e.g. compared to Holland (3.6%). For France we have information about first preferences from the first round, but for the UK and Northern Ireland we have no such information, and their PR Gini values are masked because of strategic voting in DR.
2. The PR Lorenz graph is useful to show (dis-) proportionality in political representation, and the PR Gini is a useful statistic to measure it. The graph and Gini coefficient are not difficult to make, contrary to some statements in the voting literature.
3. Two conditions appear to be very useful for proper measurement of disproportionality and comparability of results: (a) Order the parties on the ratio of seat share to vote share (and not the difference), (b) Use turnout as the base, so that the invalid and wasted votes are treated as a party of their own.
4. This paper hasn't focused on the detrimental consequences of disproportionality, yet some phenomena have been touched upon: (a) The ability within the political system to work together, compromise, form coalitions, and respect the opposition. (b) The possibility for smaller parties to partake in government and responsibility. (c) The entry of new parties and their ability to grow (i.e. political competition). (d) The idea among the electorate whether they are represented indeed (also shown by turnout). (e) The risk of swings to more extreme parties (with a protest vote). (f) The use of populist measures (like the referendum on Brexit) in trying to deal with such risks.
5. Researchers on voting might show a bias on PR and DR depending upon whether they come from countries that use PR or DR. Within voting theory there appears to be a line of research that tries to compromise PR with districts, but this neglects logic. Logic causes the need to choose between PR or DR, just like there is no compromise like square-

circles. PR would be the design principle for better democracy. The PR party list system tends to give the best PR outcome, from methods that have been tested in practice over a century. Within such a setup it might be possible to allow for some element of districts, but the latter then would be secondary and not be the design principle.

9. References

PM 1. Colignatus is the name in science of Thomas Cool, econometrician and teacher of mathematics, Scheveningen, Holland, <http://econpapers.repec.org/RAS/pco170.htm>

PM 2. This article is an edit of Colignatus (2017acgh).

PM 3. The excel sheet for a country / year contains an unsorted section and a section that sorts automatically. The data, calculations and charts are in:

- excl. of France <https://boycottholland.files.wordpress.com/2017/06/2015-2017-uk-holland-nireland-lorenz-gini.xls>
- for France: <https://boycottholland.files.wordpress.com/2017/07/2017-france-lorenz-gini1.xls>

Butler, D. (2014), "The Evolution of British Electoral Studies", lecture, <http://www.britishelectionstudy.com/bes-impact/the-evolution-of-british-electoral-studies-by-david-butler/>

Chessa, M. (2012), "A note on "Measurement of disproportionality in proportional representation systems", *Mathematical and Computer Modelling* Volume 55, Issues 3–4, February 2012, Pages 1655-1660, <http://www.sciencedirect.com/science/article/pii/S0895717711005589>

Colignatus, Th. (2010), "Single vote multiple seats elections. Didactics of district versus proportional representation, using the examples of the United Kingdom and The Netherlands", <https://mpra.ub.uni-muenchen.de/22782/>

Colignatus, Th. (2014), "Voting Theory for Democracy", Thomas Cool Consultancy & Econometrics, <https://zenodo.org/record/291985>

Colignatus, Th. (2017a), " The Dutch general elections of March 15 2017", <https://boycottholland.wordpress.com/2017/03/26/the-dutch-general-elections-of-march-15-2017/>

Colignatus, Th. (2017b), "The performance of four possible rules for selecting the Prime Minister after the Dutch Parliamentary elections of March 2017", <https://mpra.ub.uni-muenchen.de/77616/>

Colignatus, Th. (2017c), "Proportional representation, Lorenz diagram and Gini measure", <https://boycottholland.wordpress.com/2017/06/22/proportional-representation-lorenz-diagram-and-gini-measure/>

Colignatus, Th. (2017d), "Also Scotland doesn't understand democracy", <https://boycottholland.wordpress.com/2017/06/21/also-scotland-doesnt-understand-democracy/>

Colignatus, Th. (2017e), "Voting theory and the Brexit referendum question", RES Newsletter, issue 177, April, p14-15, <http://www.res.org.uk/view/art4Apr17Features.html>

Colignatus, Th. (2017f), "A puzzle on YouGov Brexit ranking data", <https://boycottholland.wordpress.com/2017/07/11/a-puzzle-on-yougov-brexit-ranking-data/>

Colignatus, Th. (2017g), "Brexit and the weird brew of populism and district representation", <https://boycottholland.wordpress.com/2017/06/29/brexit-and-the-weird-brew-of-populism-and-district-representation/>

Colignatus, Th. (2017h), "Lorenz en Gini for the French elections of 2017", <https://boycottholland.wordpress.com/2017/07/19/lorenz-en-gini-for-the-french-elections-of-2017/>

Colignatus, Th. (2017i), "Questioning Brexit doesn't mean questioning the UK Parliament mandate", <https://boycottholland.wordpress.com/2017/06/29/questioning-brexit-doesnt-mean-questioning-the-uk-parliament-mandate/>

Karpov, A. (2008), "Measurement of disproportionality in proportional representation systems", *Mathematical and Computer Modelling*, Volume 48, Issues 9–10, November 2008, Pages 1421-1438, <http://www.sciencedirect.com/science/article/pii/S0895717708001933>

Kedar, O., L. Harsgor and R.A. Sheinerman (2013), "Are Voters Equal under Proportional Representation?", <http://sticerd.lse.ac.uk/seminarpapers/pspe28112013.pdf>

Kestelman, P. (2005), "Apportionment and Proportionality: A Measured View", *Voting Matters*, 20, p 12-22, <http://www.votingmatters.org.uk/ISSUE20/I20P4.PDF>

Renwick, A. (2015), "Electoral Disproportionality: What Is It and How Should We Measure It?", <http://blogs.reading.ac.uk/readingpolitics/2015/06/29/electoral-disproportionality-what-is-it-and-how-should-we-measure-it/>

Schendelen, M.P.C.M. van (1985), "Consociational Democracy: The Views of Arend Lijphart and Collected Criticisms", *Political Science Reviewer*, Fall, 1985 - Vol. 15, No. 1, p143-183, https://isistatic.org/journal-archive/pr/15_01/schendelen.pdf

Stavrou, P. (2017), "Brexit and the choice between Parliaments or Populism with referenda. Interview with Thomas Colignatus", <https://protesilaos.com/colignatus-brexit-referenda-interview/>

Taagepera, R. and M.S. Shugart (1989), "Seats and Votes", Yale

Taylor, A. and N. Veron (2014), "How unequal is the European Parliament's representation?", <http://voxeu.org/article/inequality-european-parliament-representation>