Stock Market Indexes: A random walk test with ARCH (q) disturbances

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September 2014

International Journal of Innovation and Scientific Research
ISSN 2351-8014 Vol. 8 No. 2 Sep. 2014, pp. 305-316

http://www.ijisr.issr-journals.org/

Stock Market Indexes: A random walk test with ARCH (q) disturbances

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ABSTRACT: We will here study the stock market indexes, in the context of a random walk test with ARCH (q) disturbances. This model based on these theoretical predictions has been valuated from the Tunis Stock market data. The coherence of the parameters signs and the statistical relevance of the estimations are validating the choice of the conditionally heteroskedastic random walk model.

KEYWORDS: white noise, index, random walk, ARCH (or GARCH) model.

1 INTRODUCTION

We will, here, study the series of the stock market quotations. The choice of the topic is dictated by considerations originating, a priori, in the economic theory and the availability of the series of observations.

2 THE CONDITIONALLY HETEROSKEDASTIC RANDOM WALK MODEL

We will, here, achieve a random walk test. We will assume that the data-generating process is a random walk process, that is to say:

$$y_t = y_{t-1} + e_t$$

The disturbances e_t are checking the following hypotheses:

$$E[e_t] = 0$$

$$E[e_t e_{t-s}] = \begin{cases} \sigma^2 & \text{if} \quad s = 0 \\ 0 & \text{otherwise} \end{cases}$$

The first interpretation of the random walk hypothesis is dictated by the conditional expectation, that is to say:

$$E\left[y_t / y_{t-1} \right] = E\left[y_{t-1} + e_t / y_{t-1} \right]$$
$$= y_{t-1}$$

Then, the best prediction of the stock market quotations corresponds to its past value.

The choice of a random walk process means that in average, the change of quotation is zero.

This second interpretation is described in terms of the conditional expectation of the increases of the series:

$$E\left[y_t - y_{t-1} \big/ y_{t-1} \right] = E\left[e_t \big/ y_{t-1} \right]$$
$$= 0$$

We will achieve, as a first step, the conditionally heteroskedastic random walk test:

$$\begin{cases} y_t = y_{t-1} + e_t \\ e_t^2 = c_0 + \sum_{i=1}^{q} c_i e_{t-i}^2 + \omega_t \end{cases}$$

$$\text{Then, as, } y_t^* = y_t - y_{t-1}$$
$$= e_t :$$

We will, here, study the data-generating process as a conditionally heteroskedastic white noise model:

$$\begin{cases} y_t^* = e_t \\ e_t^2 = c_0 + \sum_{i=1}^{q} c_i e_{t-i}^2 + \omega_t \end{cases}$$

We will, here, achieve the white noise test in the series studied:

$$y_t^* = y_t - y_{t-1}$$
$$= e_t$$

$$\text{Then, } E\left[y_t^*\right] = E\left[e_t\right]$$
$$= 0$$

$$E\left[y_t^* y_{t-s}^*\right] = E\left[e_t e_{t-s}\right]$$
$$= \gamma(s)$$

$$\gamma(s) = E\left[y_t^* y_{t-s}^*\right]$$
$$= \begin{cases} \gamma(0) & \text{if} \quad s = 0 \\ 0 & \text{otherwise} \end{cases} .$$

Then:

$$\rho_s = \frac{\gamma(s)}{\gamma(0)}$$
$$= \begin{cases} 1 & \text{if} \quad s = 0 \\ 0 & \text{otherwise} \end{cases} .$$

y_t^* Is a white noise if $\rho_s = 0$, $s = 1,2,...$

Then, we will achieve a white noise test in the series y_t^* :

$$H_0^i : \rho_i = 0$$
$$H_a^i : \rho_i \neq 0$$

$i = 1,2,...$

Under $H_0^i : \rho_i = 0$, the data-generating process is a white noise.

This propriety of the process studied enables as to get interested to the future values of the stock market indexes series:

$$E\left[{y_{t+1}}/{y_t}\right] = y_t$$

As the matter of fact, the prediction to the time t of the process studied corresponds to the current value.

The statistics associated to this test are then:

$$t_{\hat{\rho}_i} = \frac{\hat{\rho}_i - \rho_i}{\hat{\sigma}(\hat{\rho}_i)}$$
$$= T^{1/2}\left[\hat{\rho}_i - \rho_i\right]$$

Under $H_0^i : \rho_i = 0$, $T^{1/2}\hat{\rho}_i \approx N(0,1)$

$$\hat{\rho}_i = \frac{\hat{\gamma}(i)}{\hat{\gamma}(0)}$$

$$\hat{\gamma}(i) = \frac{1}{T-i}\sum_t \hat{e}_t \hat{e}_{t-i} .$$

We will reject $H_0^i : \rho_i = 0$ if $\left|t_{\hat{\rho}_i}\right| > t_{\alpha/2}$.

If $\alpha = 0{,}05$, $t_{\alpha/2} = 1{,}96$.

The hypothesis $H_0^i : \rho_i = 0$ validates the white noise hypothesis.

We will, here, achieve a second white noise test:

$$H_0 : \rho_1 = \ldots = \rho_s = 0$$
$$H_a = H_0^c$$

The alternative hypothesis indicates the complementary of H_0.

The statistic associated to this test is then:

$$Q = T \sum_{i=1}^{s} \hat{\rho}_i^2 .$$

Under $H_0 : \rho_1 = \ldots = \rho_s = 0$, $Q \approx \chi_\alpha^2(s)$.

We will determine the value of the Ljung –Box statistic, Q^* :

$$Q^* = T(T+2)\sum_{i=1}^{s} \frac{1}{T-i}\hat{\rho}_i^2 \quad .$$

Q^* is following a Khi –square distribution with s degrees of freedom.

3 PROPRIETIES OF THE SERIES STUDIED-TESTS

3.1 PROPRIETIES OF THE SERIES STUDIED

The series of the stock market quotations is subject to an instantaneous variability.

We will study the series available, p_t, over the period, [1997, 2008].

The data constitute the daily frequencies.

i. Inference statistics

We will, at a first step, determine the moments of order one and two, the minimum and the maximum, the of skewness (s) and kurtosis (k) parameters …:

	p_t	y_t
m	1146,25	0,0005
Median	1031,85	0,0002
Minimum	449,64	-0,05
Maximum	2346,11	0,04
σ	473,10	0,0077
s	0,64	0,08
k	2,64	6,24
N	2742	2742

We will study, as a second step, in terms of rate increase, the stock market series:

$$p_t^* = Log\left(\frac{p_t}{p_{t-1}}\right).$$

We will compare the skewness (s) and the kurtosis (k) coefficients to the parameters values associated to a gaussian process: $s = 0$ and $k = 3$.

The values of these coefficients, corresponding to $s = 0{,}08$ and $k = 6{,}24$, do not belong to the reference values of a gaussian process.

As a matter of fact, the series of the stock market quotations is not following a normal distribution.

The value of kurtosis, which is quite high, expresses the leptokurtic aspect of the series studied.

The coefficient s does not belong to the proximity of zero. This hypothesis is significant of non-linearity.

ii. The graphic study:

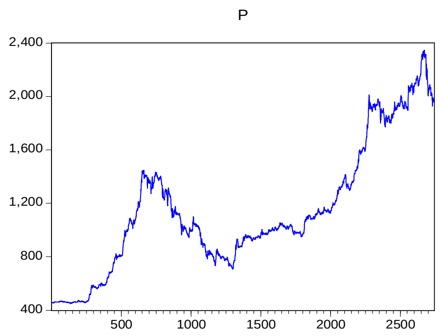
The series of the stock market quotations, p_t is not, a priori, stationary (Graph a).

In the other hand, the series p_t^* is stationary (Graph b).

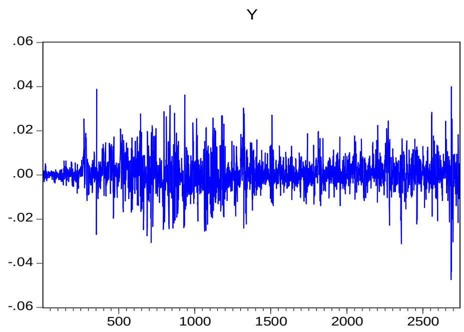
The graphic representation of this series is expressing the regroupings of volatility:

The values of this series, with unpredictable signs are followed by values with the same signs.

The variability of the series persists in time. These predictions are validating, a priori, of an ARCH model hypothesis, representative of a data-generating process.



Graph a



Graph b

3.2 TESTS

We will, here, achieve a whole range of tests: a stationarity test, a normality test,

i. Stationarity test

	ADF	$p-p$
p_t	$I(1)$	$I(1)$
p_t^*	$I(0)$	$I(0)$

The series of the stock market quotations p_t is not stationary.

The series p_t^* is stationary. Then, the series p_t is $I(1)$.

ii. Normality test

Test 1 : Jarque –Béra test:

We will, here, achieve the test: $H_0 : \begin{cases} s = 0 \\ k - 3 = 0 \end{cases}$

$H_a = H_0^c$.

The statistics associated to this test are then:

$$J - B = \frac{T}{6}\left[s^2 + \frac{1}{4}(k-3)^2 \right].$$

Under $H_0 : \begin{cases} s = 0 \\ k - 3 = 0 \end{cases} \quad : JB \approx \chi_\alpha^2(2)$.

s and k indicate the skewness and the kurtosis coefficients.

	Statistics of the test	p
p_t^*	**1197,01**	**0,000000**

As $p \le 0{,}05$, we will, here, reject:

$$H_0 : \begin{cases} s = k - 3 \\ \quad = 0 \end{cases}$$

The series y_t is not following a normal distribution.

Test 2 : Granger –Newbold test

We will, afterwards, achieve the normality test of Granger –Newbold.

We will determine the autocorrelation coefficients of order j, ρ_j :

j	$\rho_j(y_t)$	$\rho_j(y_t^2)$	$\dfrac{\rho_j^2(y_t)}{\rho_j(y_t^2)}$
1	**0,313**	**0,494**	**0,2**
2	**0,111**	**0,340**	**0,04**
3	**– 0,041**	**0,215**	**0,0080**
4	**– 0,024**	**0,199**	**0,0028**
⋮	⋮	⋮	⋮
23	**– 0,015**	**0,024**	**0,01**
24	**0,031**	**0,034**	**0,0282**

The ratio between $\rho_j^2(y_t)$ and $\rho_j(y_t^2)$ does not belong to the proximity of one the series studied is not following a normal distribution. It is constitutes the general characteristic of the financial series.

It is agreed to underline that: $y_t = p_t^*$

4 ESTIMATION

In the choice of model, we realize a whole class of tests**.**

4.1 TESTS ON THE MODEL

$$y_t = b_0 + e_t$$

i. White noise test

We will, here, achieve, on the series $\hat{e}_t^2$, the test:

$$H_0 : \rho_i = 0 \qquad (\text{or } H_0 : \rho_1 = \ldots = \rho_s = 0)$$
$$H_a = H_0^c \qquad .$$

We will reject, $H_0 : \rho_1 = 0$ (and $H_0 : \rho_1 = \rho_2 = 0$). Then, $\hat{e}_t^2$ is, a priori, following a second order autoregressive process: The disturbances e_t are following an ARCH (2) (or GARCH (1, 1)) model.

ii. Test

$$H_0 : c_1 = ... = c_q = 0$$

$$Ha = H_0^c$$

We will achieve, in the context of this hypothesis, the regression of de $\hat{e}_t^2$ on the values $\hat{e}_{t-i}^2$:

$$\hat{e}_t^2 = c_0 + \sum_{i=1}^{q} c_i \hat{e}_{t-i}^2 + \omega_t .$$

$ARCH(q)$	c_0	c_1	c_2	F	NR^2
$ARCH(1)$	$3{,}05\times10^{-5}$ $(12{,}30)$	0,48 $(29{,}25)$		255,56	652,34
$ARCH(2)$	$2{,}62\times10^{-5}$ $(10{,}38)$	0,42 $(22{,}15)$	0,14 $(7{,}44)$	463,71	693,40

The numbers in brackets are the t of Student.

	$\overline{R}^2$	$LogL$	AIC	SIC	HQC	DW
$ARCH(1)$	0,24	20872,52	$-15{,}24$	$-15{,}22$	$-15{,}24$	2,14
$ARCH(2)$	0,25	20891,86	$-15{,}25$	$-15{,}25$	$-15{,}25$	2,00

The values of $LogL$ and $\overline{R}^2$ and the information criteria validate the hypothesis of an $ARCH(2)$ model.

We will, then, achieve the test:

$$H_0 : c_2 = 0$$
$$H_a : c_2 \neq 0$$

We will, here, determine SCR_i :

$$SCR_0 = 3{,}88\times10^{-5}$$

$$SCR_a = 3{,}80\times10^{-5} \quad .$$

The statistics associated to this test is then:

$$F = \frac{(SCR_0 - SCR_a)/1}{SCR_a/(N-3)} \quad .$$

This statistics is following a Fisher distribution with 1 and $N-3$ degrees of freedom.

$$F = \frac{(3{,}88 - 3{,}80)\times10^5}{3{,}80\times10^{-5}}(N-3)$$
$$= 57{,}66$$

$F > F_\alpha(1, N-3)$: We will reject $H_0 : c_2 = 0$.

The test validates an $ARCH(2)$ process.

At the first time, the choice of model validates the following estimations:

$\hat{b}_0$	$t_{\hat{b}_0}$	p
0,000534	**3,62**	**0,0003**

$SCR = 0{,}163294$.

$LogL(y, b_0, \sigma^2) = 9443{,}30$

$DW = 1{,}37$

$AIC = -6{,}8896$

$SIC = -6{,}8875$

$HQC = -6{,}8888$

Nous réalisons le test :

$H_0 : b_0 = 0$
$H_a : b_0 \neq 0$

Under $H_0 : b_0 = 0$, $t_{\hat{b}_0} = \dfrac{\hat{b}_0}{\hat{\sigma}(\hat{b}_0)} = 3{,}62$

as $p < 0{,}05$, we reject $H_0 : b_0 = 0$.

Dans un second temps, nous réalisons le test d'un processus $ARCH(q)$:

$$\begin{cases} y_t = b_0 + e_t \\ \hat{e}_t^2 = c_0 + c_1 \hat{e}_{t-1}^2 + \omega_t \end{cases}$$

		p
$\hat{b}_0$	**0,000184** **(1,62)**	**0,1034**
c_0	$2{,}5 \times 10^{-5}$ **(36,14)**	**0,0000**
c_1	**0,62** **(15,90)**	**0,0000**

The parameter b_0 is not significantly different from zero.

4.2 RANDOM WALK WITH DISTURBANCES GARCH(P, Q)-TESTS

We will study the statistical pertinence of estimations of the two classes of models:

i- Random walk model with disturbances GARCH (p, q),

ii- Random walk model with disturbances TGARCH (p, q).

The estimations of these models are the following:

ARCH (1) model:

$$\begin{cases} y_t = y_{t-1} + e_t \\ \hat{e}_t^2 = c_0 + c_1 \hat{e}_{t-1}^2 + \omega_t \end{cases}$$

$$\underset{(52,10)}{\hat{e}_t^2 = 4{,}8\times10^{-5}} + \underset{(15,60)}{0{,}44\hat{e}_{t-1}^2}$$

ARCH (2) model:

$$- \begin{cases} y_t = y_{t-1} + e_t \\ \hat{e}_t^2 = c_0 + c_1 \hat{e}_{t-1}^2 + c_2 \hat{e}_{t-2}^2 + \omega_t \end{cases}$$

$$\underset{(38,85)}{\hat{e}_t^2 = 3{,}8\times10^{-5}} + \underset{(14,45)}{0{,}41\hat{e}_{t-1}^2} + \underset{(10,84)}{0{,}20\hat{e}_{t-2}^2}$$

ARCH (6) model:

$$- \begin{cases} y_t = y_{t-1} + e_t \\ \hat{e}_t^2 = c_0 + \sum_{i=1}^{6} c_i \hat{e}_{t-i}^2 + \omega_t \end{cases}$$

	$\hat{c}_i$	$t_{\hat{c}_i}$
c_0	$1{,}8\times10^{-5}$	20,84
c_1	0,41	14,52
c_2	0,20	11,11
c_3	0,08	8,10
c_4	0,14	8,12
c_5	0,10	6,48
c_6	0,04	4,84

GARCH (p, q) model:

$$- \begin{cases} y_t = y_{t-1} + e_t \\ \hat{e}_t^2 = c_0 + c_1 \hat{e}_{t-1}^2 + a_1 h_{t-1} + \omega_t \end{cases}$$

	c_0	c_1	a_1

$GARCH(1,1)$	$3,4\times10^{-6}$ $(11,72)$	$0,28$ $(19,72)$	$0,72$ $(61,82)$

TGARCH (p, q) model:

	c_0	c_1^+	c_1^-	c_2
$TARCH(1,1)$	$4,8\times10^{-5}$ $(51,32)$	$0,26$ $(6,76)$	$0,35$ $(5,42)$	
$TGARCH(1,1)$	$3,4\times10^{-6}$ $(11,30)$	$0,25$ $(10,45)$	$0,06$ $(2,12)$	$0,72$ $(57,56)$

The numbers in brackets are the t of Student.

The criterion of the maximum of likelihood and information leads us to the choice of a random walk model with disturbances ARCH (or TARCH). These criteria are determining bay the following values:

	$LogL(y,c,\sigma^2)$	AIC	SIC	HQC
$ARCH(1)$	9264,60	− 6,7602	− 6,7538	− 6,7580
$ARCH(2)$	9305,62	− 6,7895	− 6,7808	− 6,7864
$ARCH(6)$	9417,97	− 6,8715	− 6,8628	− 6,8684
$TARCH(1)$	9273,90	− 6,7664	− 6,7577	− 6,7632
$TGARCH(1,1)$	9419,6	− 6,8717	− 6,8608	− 6,8678
$GARCH(1,1)$	9417,97	− 6,8715	− 6,8628	− 6,8684
ARCH-M	9930,98	-7,2425	-7,2318	-7,2387

The series of stock market quotations is following a random walk process.

It is a random walk model with disturbances ARCH (or TARCH).

	$ARCH(2)$	$ARCH(6)$	$GARCH(1,1)$	$TARCH(1,1)$	$TGARCH(1,1)$

c_0	$3{,}8\times10^{-5}$ $(38{,}85)$	$1{,}8\times10^{-5}$ $(20{,}84)$	$3{,}4\times10^{-6}$ $(11{,}72)$	$4{,}8\times10^{-5}$ $(51{,}32)$	$3{,}4\times10^{-6}$ $(11{,}30)$
c_1	0,41 $(14{,}45)$	0,41 $(14{,}52)$	0,28 $(19{,}72)$		
c_2	0,20 $(10{,}85)$	0,20 $(11{,}11)$			
c_3		0,08 $(8{,}10)$			
c_4		0,14 $(8{,}12)$			
c_5		0,10 $(6{,}48)$			
c_6		0,04 $(4{,}84)$			
c_1^+				0,27 $(6{,}76)$	0,25 $(10{,}45)$
c_1^-				0,35 $(5{,}42)$	0,06 $(2{,}12)$
a_1			0,72 $(61{,}82)$		0,72 $(57{,}56)$

The value of the likelihood function and the information criteria validate the random walk hypothesis with disturbances $TGARCH(1,1)$.

5 CONCLUSION

The proprieties of the series studied are conditioning the inference methods:

- The series of stock market quotations, as any financial series, is subject to an instantaneous variability. The regrouping of volatility is described by the values of the series with unpredictable signs, which are followed by values with the same signs.
- The skewness (s) and the kurtosis (k) coefficients do not validate the hypothesis of a Gaussian process.
- In the tests on the residues, we will accept the hypothesis of an autoregressive conditionally heteroskedastic process.
- The choice of the ARCH model is dictated by the proprieties of the series studied.

REFERENCES

[1] Bollerslev T. [1986]: Generalized autoregressive conditional heteroskedasticity, Journal of Econometrics, 31, p. 307 -327.

[2] Bollerslev T., Chou R.Y. et Kroner K.F. [1992]: ARCH modeling in finance: a review of the theory and empirical evidence, Journal of Econometrics, 52, p. 5-60.

[3] Chan K. S et Tong H. [1986]: On estimating thresholds in autoregressive models, Journal of time series analysis, 7, p. 179 -190.

[4] Engle R. F. [1982]: Autoregressive conditional heteroskedasticity with estimates of the variance of united –Kingdom inflation, Econometrica, 50, p. 987 -1007.

[5] Engle R. F. [2002 a]: New frontiers for ARCH models, Journal of Applied Econometrics, 17, p 425 -446.

[6] Engle R. F. [2002 b]: Dynamic conditional correlation: A simple class of multivariate GARCH models, Journal of Business and Economic Statistics, 20, p. 339 -350.
[7] Florens J. P., Marimoutou V. et Peguin A. [2004] : Econométrie : modélisation et inférence.
[8] Geweke J. [1989] : Exact predictive densities for linear models with ARCH disturbances, Journal of Econometrics, 40, p. 63 -86.
[9] Gourieroux C. et Monfort A. [1992]: Qualitative threshold ARCH models, Journal of Econometrics, 52, p. 159 -200.
[10] Hsieh D. A. [1991]: Chaos and nonlinear dynamics: Application to financial markets, Journal of finance, XLVI (5), p. 1839 -1877.
[11] Pantula S. G. [1986]: Modeling the persistence of conditional variances: a comment, Econometric Reviews, 5, p. 71 -74.
[12] Weiss A. A. [1984]: ARMA models with ARCH errors, Journal of time series analysis, 5, p. 129 -143.