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The Stock-Bond Comovements and Cross-Market Trading*

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Abstract

We propose an asset pricing model with heterogeneous agents allocating capital to the stock and bond markets to optimize their portfolios, utilizing the dynamic interaction between the two markets. While some agents focus on the stock market and have more expertise in it, the others specialize in the bond market. Based on their comparative advantages in a particular market, heterogeneous agents constantly revise their investment portfolios by taking into account the time-varying stock-bond return comovements and the changing market conditions. Agents' collective investment behavior shapes the stock-bond interlinkage, which feeds back on their subsequent capital allocations. Using monthly US stock and bond data from January 1990 to June 2014, we estimate the vector autoregression model with threshold and Markov switching mechanisms. We find evidence in support of flight-to-quality and show that it is mainly driven by the technical traders who actively sell stocks and buy bonds during periods of high market uncertainty.

Keywords: Heterogeneity, Stock-Bond Comovement, Markov Switching VAR, Threshold VAR.

JEL Classification: G12, G15.

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1 Introduction

When the financial market is flooded with liquidity, there are greater capital flows to both the stock and bond markets, which moves the stock and bond prices up together. However, when the market is in panic, money flows from the stock market into the bond market as investors substitute risky investment in stock with safe heaven asset of bond. Such “flight-to-quality” phenomenon pushes down the stock price while bidding up the bond price, which induces a negative stock-bond return relation. The dynamic stock-bond return relation, in turn, affects the order flows as professional investors revise their asset allocation and risk management strategies according to the latest market environment. Motivated by these observations, this paper unifies behavioral heterogeneity in the heterogeneous agent model (HAM) literature and time-varying stock-bond return relation to study the interaction between cross-market trading behavior and joint price dynamics in the stock and bond markets.

This paper is closely related to the growing literature on estimating HAM. Chiarella et al. (2012) and Franke and Westerhoff (2012) document empirical evidence in support of behavioral heterogeneity in the stock market, namely the presence of fundamentalists, who trade on the belief that the asset price will mean revert towards its fundamental value, and chartists, who trade following the price trends. Menkhoff et al. (2009), Westerhoff and Reitz (2005), Frijns et al. (2010) and Chia et al. (2016) find similar evidence in the foreign exchange, commodity, option, and housing market respectively.¹ Accounting for behavioral heterogeneity can also improve the model’s performance in terms of in-sample estimation efficiency and out-of-sample forecasting precision (Chiarella et al. 2012; Lof 2015). These studies focus on a single asset in a single market. The interaction among different international markets are relatively underexplored with a few exceptions. For example, de Jong et al. (2009) consider two stock markets by extending the conventional HAM that consists of fundamentalists and chartists to account for international investors who trade based on foreign stock market and exchange rate. The trading behavior of international investors contributes to explain the financial linkage across interna-

¹Boswijk et al. (2007) find evidence of heterogeneous trading behavior of chartists in the stock market. See a detailed survey of relevant literature in Chia et al. (2014).

tional markets and the contagion effect during financial crisis. Schmitt and Westerhoff (2014) provides simulation results in support of such empirical evidence and highlights the role of speculators' trading behavior in driving the comovements of stock prices across different markets. No studies have yet ~~apply~~ HAM to explore the dynamic interaction among different asset classes.

By explicitly modelling the joint price dynamics in the stock and bond markets, we study how and to what extent the behavioral heterogeneity in both markets as well as the cross-market trading activities interact with the time-varying stock-bond comovements. Despite the importance of stock-bond comovements for asset allocation and risk management, no previous studies in this strand of literature have accounted for their impact in shaping agents' heterogeneous trading behavior. This paper seeks to fill this gap by introducing heterogeneous cross-market trading activities, where fundamentalists and chartists optimize asset allocation utilizing the time-varying stock-bond comovements. Agents first form dynamic expectations on the stock-bond comovements based on their analysis of the changing market environment. Taking into account the stock-bond market linkage, they then use their comparative advantages in a specific market (bond or stock) to forecast the price movements in the other market and place trading orders in each market accordingly. In this way, agents trading behavior in stock and bond market reflect how they respond to the linkage between stock and bond markets. Our model therefore not only captures how behavioral heterogeneity shapes the asset price comovements as in de Jong et al. (2009) and Schmitt and Westerhoff (2014) but also how the time-varying stock-bond comovements affect trading heterogeneity in these two markets.

The backbone of this paper is the time-varying stock-bond return relation. It tends to be positive when the stock market is normal and negative during periods of crisis when there is significant market uncertainty (Baele et al. 2010; Barsky 1989; Connolly et al. 2005; Fleming et al. 1998). Much recent effort has been made to document this time variation using macroeconomic variables such as productivity growth, inflation rate, macroeconomic news (Andersen et al. 2007; Barsky 1989; Campbell and Ammer 1993; Yang et al. 2009), market liquidity (Baele et al. 2010; Chordia et al. 2005), real interest rate (Shiller and Beltratti 1992; d'Addona and

Kind 2006) and aggregate market uncertainty (Connolly et al. 2005). Ultimately, these macro factors exert their influence on the stock-bond return relation through affecting investors' trading behavior, which directly shapes the dynamic interaction between stock and bond prices. In this paper we attempt to disentangle the underlying micro-behavioral mechanisms behind this time variation in stock-bond comovements.

Building upon the HAM literature, we model heterogeneous trading behavior that implicitly takes into account the various impact factors of asset prices (as both fundamentalists and chartists make their investment decisions based on their interpretations of both current and historical prices, which reflect, at least partially, the relevant information about the asset markets). To the best of our knowledge, this is the first paper that combines heterogeneous agents with time-varying stock-bond relation to explore the role of behavioral heterogeneity in determining the joint price dynamics in the stock and bond markets.

In our model, besides investing in the market in which they have expertise, in order to optimize their investment portfolios, agents also engage in cross-market trading activities based on their understanding of the time-varying stock-bond comovements. The two-stage portfolio optimization procedure and cross-market trading reduce the two-market HAM framework to a vector autoregression (VAR) model with time-varying coefficients. While it is not easy to forecast the stock-bond relation, Connolly et al. (2005) find that the time-varying relation can be reliably captured by a two-state regime-shifting approach. Motivated by their finding, we model the cross-market trading behavior conditional on the directly observable stock market volatility or some unobserved market states without directly estimating the stock-bond return relation. Specifically, we consider two types of nonlinear dynamics in the HAM framework, which are the threshold VAR (TVAR) and the Markov switching VAR (MSVAR).

By estimating both the TVAR and MSVAR models using the US stock and bond data, we find that, when the market volatility is high, the cross-market trading directs investments from the stock to bond market. Such empirical evidence is consistent with the notion of flight-to-quality during periods of extreme market uncertainty documented in much existing literature (Connolly et al. 2005; Fleming et al. 1998). Moreover, the result is mainly driven by the

trading behavior of chartists that explore the price trends in both stock and bond markets and less by the fundamentalists who trade based on the fundamental value of stock and bond. In addition, the state-dependent impulse responses suggest asymmetric responses of stock (bond) returns to bond (stock) market shocks between the low and high market states.

The remaining of the paper is organized as follows. Section 2 presents the model with heterogeneous agents that trade in both stock and bond markets. In Section 3 we discuss the estimation methodology. Section 4 describes the data and summary statistics. In section 5 we discuss the empirical estimation results. Section 6 focuses on the impulse responses analysis. Section 7 concludes.

2 Model

Our model generalizes the standard HAM setup to a multi-asset-class framework by allowing for cross-market trading. There are two risky assets, stock (s) and bond (b), and one risk-free asset in the market. All agents trade on stock, bond and risk-free asset for portfolio optimization. Each agent has developed an expertise in either stock or bond market based on either fundamental (f) or chartist (c) strategy. Instead of learning directly about the other market in which they are lack of experience, agents forecast the price movement in the other market, utilizing their comparative advantages in one market and the time-varying interdependence between the stock and bond prices. In such a setup, there are four types of agents trading in each market, namely fundamentalist with expertise in stock market (s -fundamentalist), chartist with expertise in stock market (s -chartist), fundamentalist with expertise in bond market (b -fundamentalist), and chartist with expertise in bond market (b -chartist). Figure 2 depicts the four types of agents and their respective presence in the stock and bond markets.

We assume that all agents share the same naive comovement expectation on the first-order and second-order moments of the stock and bond excess returns (the return that exceeds the

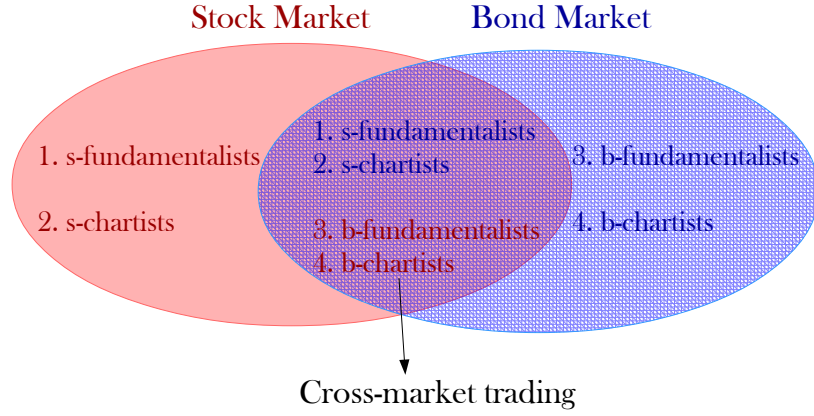


Figure 1: Heterogeneous Agents in the Stock and Bond Markets

Notes: *s*-fundamentalists and *b*-fundamentalists denote fundamentalists specializing in the stock and bond market respectively. *s*-chartists and *b*-chartists denote chartists specializing in the stock and bond market respectively.

risk-free interest rate), $r_{s,t}$ and $r_{b,t}$, at period t

$$E^h(r_{s,t}) = \tau_t E^h(r_{b,t}), \quad (1)$$

$$E^h(r_{s,t}^2) = \tau_t^2 E^h(r_{b,t}^2), \quad (2)$$

where $E(\cdot)$ is the expectation operator and $h \in \{sf, sc, bf, bc\}$ denotes respectively the *s*-fundamentalist, *s*-chartist, *b*-fundamentalist and *b*-chartist. The time-varying *comovement indicator* τ_t captures the state-dependent relation between stock and bond returns such that

$$\tau_t = \begin{cases} \tau_H & \text{if } I_t = 0 \\ \tau_L & \text{if } I_t = 1 \end{cases},$$

where I_t is an indicator function that equals to one in a low-volatility market state and zero otherwise. Such expectations on the stock-bond comovement are often not consistent with the real data generating process, which implies limited understanding on the market dynamics. Note however such a simple rule of thumb reflects agent's experience that is useful for making

quick trading decisions. As we show later with the empirical evidence that, despite the lack of accuracy, such a heuristic rule well captures the general comovement trend, which justifies agents' action based on such an assumption.

Let $\sigma_{s,t}$ and $\sigma_{b,t}$ denote the standard deviations of stock and bond returns, respectively. They can be interpreted as the standard deviations calculated based on all information available up to period t . Then Eqs. (1) and (2) imply

$$\sigma_{s,t} = \tau_t \sigma_{b,t}. \quad (3)$$

Based on Eqs. (1) and (3), we now turn to describe how heterogeneous agents with expertise in a particular market form expectations of asset price movements of stock and bond.

2.1 Heterogeneous Expectations

2.1.1 *s*-fundamentalist

We first describe how agents with expertise in stock market form their price expectations on stock. Fundamentalists believe in efficient market hypothesis so that they expect the price to reflect the fundamental value of the risky asset. The fundamentalists specializing in stock market, or *s*-fundamentalists (*sf*), expect the excess return to be a function of the log difference between the fundamental value and the price of the stock

$$\begin{aligned} E^{sf}(r_{s,t}) &= E^{sf}(p_{s,t} - p_{s,t-1} - r_{f,t}) \\ &= \eta_{s,t} (\theta_{s,t} - p_{s,t-1} - r_{f,t}), \end{aligned} \quad (4)$$

where $\theta_{s,t}$ and $p_{s,t}$ are, respectively, the logarithmic fundamental value and logarithmic price of the stock at period t , and $r_{f,t}$ is the risk-free interest rate. With $\eta_{s,t} > 0$, the fundamentalists believe in the mean-reverting of price, that is, they expect the price to increase (decrease) in the future if it is sufficiently lower (higher) than the fundamental value. Without loss of generality, we normalize the mean reversion parameter $\eta_{s,t}$ to 1.

For empirical purpose, the long term fundamental value of stock price $\theta_{s,t}$ is calculated from the static Gordon growth model (Gordon 1959) such that $\theta_{s,t} = d_{s,t}(1 + g)/(r - g)$, where $d_{s,t}$ is the dividend flow, g is the average growth rate of dividends and r is the discount rate. Following Fama and French (2002), r is assumed to equal to the sum of the average dividend yield \bar{y} and the average rate of capital gain \bar{x} , that is $r = \bar{y} + \bar{x}$. The Gordon model then implies that \bar{x} is equal to g . Consequently, the fundamental value of the stock is equal to the current dividend times a constant multiplier

$$\theta_{s,t} = d_{s,t} \frac{1 + g}{\bar{y}}.$$

Instead of forecasting the price movements in the bond market which they do not have expertise in, the s -fundamentalists utilize Eq. (1) and their comparative advantages in the stock market to form their expectations on the bond return. Substituting Eq. (4) into Eq. (1), we get s -fundamentalists' expected excess return on bond

$$E^{sf}(r_{b,t}) = (\theta_{s,t} - p_{s,t-1} - r_{f,t}) / \tau_t. \quad (5)$$

2.1.2 s -chartist

Chartists ignore the role of fundamental value and extrapolate the future price based on the price trend. The chartists specializing in stock market, or s -chartists (sc) expect the excess return of stock to follow its historical pattern

$$\begin{aligned} E^{sc}(r_{s,t}) &= E^{sc}(p_{s,t} - p_{s,t-1} - r_{f,t}) \\ &= \beta_{s,t}(p_{s,t-1} - p_{s,t-2} - r_{f,t-1}) \\ &= \beta_{s,t}r_{s,t-1}, \end{aligned} \quad (6)$$

where $\beta_{s,t}$ measures the trend patterns in stock market. When $\beta_{s,t} > 0$, the chartists expect the price trend to persist (bandwagon expectation), i.e. the price will continue to increase if it

increases in the previous period. On the other hand, when $\beta_{s,t} < 0$, the chartists expect the past price trend to reverse (contrarian expectation).

Similarly, s -chartists utilize their expertise in the stock market to forecast the price movements in the bond market by substituting Eq. (6) into Eq. (1)

$$E^{sc}(r_{b,t}) = \beta_{s,t} r_{s,t-1} / \tau_t. \quad (7)$$

2.1.3 b -fundamentalist

In a similar manner with s -fundamentalists, the fundamentalists specializing in bond market, or b -fundamentalists (bf), expect the excess return of the bond to be

$$E^{bf}(r_{b,t}) = \eta_{b,t} (\theta_{b,t} - p_{b,t-1} - r_{f,t}), \quad (8)$$

where $\theta_{b,t}$ is the fundamental value of bond calculated as the sum of all discounted cash flows (see Appendix A for the details). For simplicity, we assume the mean reversion parameter $\eta_{b,t}$ to be 1. Likewise, b -fundamentalists forecast the stock excess return based on their expertise in the bond market such that

$$E^{bf}(r_{s,t}) = (\theta_{b,t} - p_{b,t-1} - r_{f,t}) \tau_t. \quad (9)$$

2.1.4 b -chartist

In a similar manner with s -chartists, the chartists specializing in bond market, or b -chartists (bc) expect the excess return of bond to be

$$E^{bc}(r_{b,t}) = \beta_{b,t} r_{b,t-1}, \quad (10)$$

where $\beta_{b,t}$ measures the trend patterns in bond market. The expected excess return of stock by b -chartists is given by

$$E^{bc}(r_{s,t}) = \beta_{b,t} r_{b,t-1} \tau_t. \quad (11)$$

2.2 Portfolio Construction

According to the portfolio separation property, constructing an optimal complete portfolio can be separated into two independent steps (see for example, Ross (1978)). The first is to construct an optimal risky portfolio, which is the same for all agents regardless of their risk attitude. The second step is to allocate the capital between the risk-free asset and the optimal risky portfolio constructed in the first step to optimize the complete portfolio, the decision of which depends on individual risk preference.

2.2.1 Optimal Risky Portfolio

The first step is to construct an optimal risky portfolio (k) from the two risky assets, stock and bond. Denote $r_{k,t}$ as the excess return of the optimal risky portfolio at period t . Let ω_t be the weight of stock in the optimal risky portfolio, and the remainder, $1 - \omega_t$, is the weight of bond in this portfolio. The excess return on this portfolio, $r_{k,t}$, is a weighted average of the stock and bond excess returns

$$r_{k,t} = \omega_t r_{s,t} + (1 - \omega_t) r_{b,t}. \quad (12)$$

The variance of the optimal risky portfolio's excess return is

$$\sigma_{k,t}^2 = \omega_t^2 \sigma_{s,t}^2 + (1 - \omega_t)^2 \sigma_{b,t}^2 + 2\rho_t \omega_t (1 - \omega_t) \sigma_{s,t} \sigma_{b,t}, \quad (13)$$

where ρ_t is the correlation coefficient between the excess returns of stock and bond, $\sigma_{k,t}$ is the standard deviation of the excess returns on the risky portfolio.

Each type of agents, $h \in \{sf, sc, bf, bc\}$, seeks to maximize the Sharpe ratio of the optimal

risky portfolio by choosing ω_t^h , the weight of capital allocated to stock,

$$\max_{\omega_t^h} S_{k,t}^h = \frac{E^h(r_{k,t})}{\sigma_{k,t}},$$

where the risky portfolio's excess return $r_{k,t}$ and standard deviation $\sigma_{k,t}$ are defined in Eqs. (12) and (13). Solving for the optimization problem yields

$$\omega_t^h = \frac{\sigma_{b,t}^2 E^h(r_{s,t}) - \rho_t \sigma_{s,t} \sigma_{b,t} E^h(r_{b,t})}{\sigma_{b,t}^2 E^h(r_{s,t}) + \sigma_{s,t}^2 E^h(r_{b,t}) - \rho_t \sigma_{s,t} \sigma_{b,t} [E^h(r_{s,t}) + E^h(r_{b,t})]}.$$

Using the relation between the expected excess stock and bond returns and variances in Eqs. (1) and (3), the weight of stock in the optimal risky portfolio can be simplified to

$$\omega_t^h = \omega_t = \frac{1}{1 + \tau_t}. \quad (14)$$

Under the assumption on the stock-bond comovements in Eqs. (1) and (2), the weight of stock investment in the optimal risky portfolio ω_t^h is the same for all agent types and it depends only on the *comovement indicator* τ_t . Substituting Eqs. (4) – (11) and (14) into Eqs. (12) and (13) yields the simplified expected excess return and variance of the optimal risky portfolio

$$E^h(r_{k,t}) = \frac{2\tau_t}{1 + \tau_t} E^h(r_{b,t}) = \frac{2}{1 + \tau_t} E^h(r_{s,t}), \quad (15)$$

$$\sigma_{k,t}^2 = \frac{2\tau_t^2 (1 + \rho_t')}{(1 + \tau_t)^2} \sigma_{b,t}^2 = \frac{2(1 + \rho_t')}{(1 + \tau_t)^2} \sigma_{s,t}^2, \quad (16)$$

where

$$\rho_t' = \begin{cases} \rho_t & \text{if } \tau_t \geq 0 \\ -\rho_t & \text{if } \tau_t < 0 \end{cases}.$$

2.2.2 Optimal Complete Portfolio

Next, agents construct an optimal complete portfolio by allocating assets between the optimal risky portfolio and the risk-free asset to maximize their utility. All agents are assumed to share

the same constant absolute risk aversion (CARA) exponential utility function

$$U_t = -\exp(-\alpha W_t),$$

where $\alpha > 0$ is the absolute risk aversion coefficient, and W_t is the wealth at period t . Denote Y_{t-1}^h as the amount of capital allocated to the optimal risky portfolio at time $t-1$ by agent type $h \in \{sf, sc, bf, bc\}$. Then the expected utility of the agent type h becomes

$$\begin{aligned} E^h(U_t) &= -\exp\left\{-\alpha\left[E^h(W_t) - \frac{\alpha}{2}\text{Var}(W_t)\right]\right\} \\ &= -\exp\left\{-\alpha\left[W_{t-1}^h(1+r_{f,t}) + Y_{t-1}^h E^h(r_{k,t}) - \frac{\alpha}{2}(Y_{t-1}^h)^2 \sigma_{k,t}^2\right]\right\}. \end{aligned}$$

The maximization of the expected utility leads to the optimal investment in the optimal risky portfolio

$$Y_{t-1}^h = \frac{E^h(r_{k,t})}{\alpha \sigma_{k,t}^2}.$$

Recall that a proportion, ω_t , of the optimal risky portfolio is invested in the stock. The capital allocated to the stock by agent type h , denoted as $C_{s,t-1}^h$, is

$$\begin{aligned} C_{s,t-1}^h &= \omega_t Y_{t-1}^h = \omega_t \frac{E^h(r_{k,t})}{\alpha \sigma_{k,t}^2} \\ &= \frac{E^h(r_{b,t})}{\alpha \tau_t (1 + \rho'_t) \sigma_{b,t}^2} \end{aligned} \tag{17}$$

$$= \frac{E^h(r_{s,t})}{\alpha (1 + \rho'_t) \sigma_{s,t}^2}, \tag{18}$$

where the second and third lines are obtained by substituting ω_t , $E^h(r_{k,t})$ and $\sigma_{k,t}^2$ with Eqs. (14) – (16). Similarly, the capital allocated to the bond by agent type h , denoted as $C_{b,t-1}^h$, is

$$\begin{aligned} C_{b,t-1}^h &= (1 - \omega_t) Y_{t-1}^h \\ &= \frac{E^h(r_{b,t})}{\alpha(1 + \rho'_t) \sigma_{b,t}^2} \end{aligned} \quad (19)$$

$$= \frac{\tau_t E^h(r_{s,t})}{\alpha(1 + \rho'_t) \sigma_{s,t}^2}. \quad (20)$$

The optimal complete portfolio implies that $C_{b,t-1}^h = \tau_t C_{s,t-1}^h$, meaning that the cross-market demand of bond by an agent is proportional to their investment in the stock market. The comovement indicator τ_t essentially captures how agents respond to the linkages between the stock and bond markets when engaging in cross-market trading. If both $C_{b,t-1}^h$ and $C_{s,t-1}^h$ are positive (negative), that is, τ_t is positive, it means that the type- h agents increase (decrease) their investment positions in both stock and bond. In this case, the stock and bond prices will move in the same direction if these agents dominate the market. On the other hand, if $C_{b,t-1}^h$ is positive while $C_{s,t-1}^h$ is negative, it means that type- h agents are channeling their investment from the stock market to the bond market. Such a phenomenon reflects the notion of flight-to-quality with investors adjusting their investment portfolio to include safer assets like bond and fewer risky assets such as stock. Such trading behavior contributes to the movement of the stock and bond prices in the opposite direction. Note that in this setup, we do not in particular confine the direction of how the cross-market trading among different agent types responds to the stock-bond linkage, i.e., the sign of τ_t , but let the data determine the trading directions via the estimated parameters.

2.3 Heterogeneous Capital Allocation

While agents have the same risk attitude measured by α and share the same information on τ_t and ρ'_t , they have heterogeneous expectations on the excess returns of the stock and bond, which lead to heterogeneous capital allocation choices.

Substituting s -fundamentalists' expected excess return of stock defined by Eq. (4) into Eq. (18) yields s -fundamentalists' optimal capital allocation to the stock

$$C_{s,t-1}^{sf} = \frac{\theta_{s,t} - p_{s,t-1} - r_{f,t}}{\alpha(1 + \rho'_t) \sigma_{s,t}^2}. \quad (21)$$

Similarly, substituting the expected excess return and variance of stock by s -chartists, b -fundamentalists and b -chartists into Eq. (18) results in their optimal capital allocation to the stock

$$C_{s,t-1}^{sc} = \frac{\beta_{s,t} r_{s,t-1}}{\alpha(1 + \rho'_t) \sigma_{s,t}^2}, \quad (22)$$

$$C_{s,t-1}^{bf} = \frac{\theta_{b,t} - p_{b,t-1} - r_{f,t}}{\alpha(1 + \rho'_t) \tau_t \sigma_{b,t}^2}, \quad (23)$$

$$C_{s,t-1}^{bc} = \frac{\beta_{b,t} r_{b,t-1}}{\alpha(1 + \rho'_t) \tau_t \sigma_{b,t}^2}. \quad (24)$$

With similar analysis, we derive the optimal capital allocation to the bond by each type of agents by substituting their expected excess return and variance of bond into Eq. (19)

$$C_{b,t-1}^{sf} = \frac{\tau_t (\theta_{s,t} - p_{s,t-1} - r_{f,t})}{\alpha(1 + \rho'_t) \sigma_{s,t}^2} = \tau_t C_{s,t-1}^{sf}, \quad (25)$$

$$C_{b,t-1}^{sc} = \frac{\tau_t \beta_{s,t} r_{s,t-1}}{\alpha(1 + \rho'_t) \sigma_{s,t}^2} = \tau_t C_{s,t-1}^{sc}, \quad (26)$$

$$C_{b,t-1}^{bf} = \frac{\theta_{b,t} - p_{b,t-1} - r_{f,t}}{\alpha(1 + \rho'_t) \sigma_{b,t}^2} = \tau_t C_{s,t-1}^{bf}, \quad (27)$$

$$C_{b,t-1}^{bc} = \frac{\beta_{b,t} r_{b,t-1}}{\alpha(1 + \rho'_t) \sigma_{b,t}^2} = \tau_t C_{s,t-1}^{bc}. \quad (28)$$

Note that the optimal capital allocation to the bond can also be directly obtained based on the previous result that $C_{b,t-1}^h = \tau_t C_{s,t-1}^h$.

2.4 The Price Dynamics

In each of the stock and bond market, there is a market maker who adjusts the price up and down according to the latest capital flows. All agents submit their trading orders for asset type

$i \in \{s, b\}$ to the market maker in charge of market i , who adjusts the prices (in logarithmic value) according to

$$r_{i,t} = p_{i,t} - p_{i,t-1} - r_{f,t} = \gamma_i \sum_{h \in \{sf, sc, bf, bc\}} m_i^h C_{i,t-1}^h + \varepsilon_{i,t},$$

where $\gamma_i > 0$ measures the marginal impact of the aggregate capital flows on the price, $m_i^h \geq 0$ is the market fraction of the h type agents in market i , where $h \in \{sf, sc, bf, bc\}$ with sf, sc, bf and bc referring respectively to the s -fundamentalists, s -chartists, b -fundamentalists and b -chartists, and $\varepsilon_{i,t}$ is the noise term that captures the supply shock in market i . Moreover, the market fractions satisfy $\sum_{h \in \{sf, sc, bf, bc\}} m_i^h = 1$ for any $i \in \{s, b\}$.

Summarizing the price dynamics in the stock and bond markets based on the heterogeneous capital allocation decisions in Eqs. (21) – (28), we have the following VAR model

$$\begin{pmatrix} r_{s,t} \\ r_{b,t} \end{pmatrix} = A \begin{pmatrix} r_{s,t-1} \\ r_{b,t-1} \end{pmatrix} + B \begin{pmatrix} \tilde{r}_{s,t} \\ \tilde{r}_{b,t} \end{pmatrix} + \begin{pmatrix} \varepsilon_{s,t} \\ \varepsilon_{b,t} \end{pmatrix}, \quad (29)$$

where

$$\begin{aligned} \tilde{r}_{s,t} &= \theta_{s,t} - p_{s,t-1} - r_{f,t}, \\ \tilde{r}_{b,t} &= \theta_{b,t} - p_{b,t-1} - r_{f,t}. \end{aligned}$$

A and B are the coefficient matrices to be estimated that can be written as

$$A = \begin{pmatrix} \frac{\gamma_s m_s^{sc} \beta_{s,t}}{\alpha(1+\rho'_t) \sigma_{s,t}^2} & \frac{\gamma_s m_s^{bc} \beta_{b,t}}{\alpha(1+\rho'_t) \tau_t \sigma_{b,t}^2} \\ \frac{\gamma_b m_b^{sc} \tau_t \beta_{s,t}}{\alpha(1+\rho'_t) \sigma_{s,t}^2} & \frac{\gamma_b m_b^{bc} \beta_{b,t}}{\alpha(1+\rho'_t) \sigma_{b,t}^2} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix},$$

and

$$B = \begin{pmatrix} \frac{\gamma_s m_s^{sf}}{\alpha(1+\rho'_t) \sigma_{s,t}^2} & \frac{\gamma_s m_s^{bf}}{\alpha(1+\rho'_t) \tau_t \sigma_{b,t}^2} \\ \frac{\gamma_b m_b^{sf} \tau_t}{\alpha(1+\rho'_t) \sigma_{s,t}^2} & \frac{\gamma_b m_b^{bf}}{\alpha(1+\rho'_t) \sigma_{b,t}^2} \end{pmatrix} = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}.$$

While these coefficients are aggregates of several parameters, they are in fact measuring the

aggregate market trading power of each type of investors as suggested by the capital allocation functions in Section 2.3.² Since $\gamma_i > 0$, $m_i^h \geq 0$, $\alpha > 0$, $(1 + \rho'_i) \geq 0$ and $\sigma_{i,t}^2 > 0$ we can uncover the signs of $\beta_{i,t}$ and τ_t from the estimation results even though we cannot estimate these parameters separately due to identification issues.³ Knowing the signs of $\beta_{i,t}$ and τ_t can shed light on the trading behavior of the heterogeneous agents, and more importantly, their cross-market trading activities in each market. In the current set up, we assume the co-existence of fundamentalists and chartists in both the stock and bond markets. One could consider some immediately alternative models by restricting to one type of agents only, that is, there are only fundamentalists (matrix A is null) or only chartists (matrix B is null) in the markets. The estimation results for these restricted models are presented in Appendix C, which overall exhibit poorer performance than the full model and result in some inconsistent predictions for the comovement indicator.

3 Estimation Methodology

Agents take into account the latest market conditions and update their investment behavior accordingly. To capture the state-dependent behavioral heterogeneity and the joint price dynamics of stock and bond, we propose to estimate the VAR model under threshold and Markov switching frameworks. As a two-state regime-shifting model is found to be able to well capture the time-variation in behavioral heterogeneity (Chiarella et al. 2012) and stock-bond return correlation (Connolly et al. 2005), we focus on two market states in estimating Eq. (29).

3.1 Threshold VAR (TVAR)

We follow Connolly et al. (2005) to identify the market states according to VIX, Chicago Board Options Exchange Market Volatility Index, which is known as a gauge of aggregate market

²Similar empirical estimations are common in the estimating HAM literature, for instance, Boswijk et al. (2007), Chiarella et al. (2012) and Lof (2012). In all these papers, the estimated parameters are aggregates of several fundamental parameters describing investors' characteristics or market conditions due to under-specification issue.

³See Appendix B for more details on identification.

uncertainty. We employ the TVAR, which accounts for the regime-switching property in the nonlinear regression and splits the sample endogenously into two different regimes according to the value of VIX, so that within each regime the time series can be described by a linear model. The TVAR model allows for structural change in the behavioral heterogeneity when the threshold variable moves across certain boundary values. Specifically, the two-state TVAR model based on Eq. (29) can be written as

$$\begin{pmatrix} r_{s,t} \\ r_{b,t} \end{pmatrix} = A^1 \begin{pmatrix} r_{s,t-1} \\ r_{b,t-1} \end{pmatrix} I_t + B^1 \begin{pmatrix} \tilde{r}_{s,t} \\ \tilde{r}_{b,t} \end{pmatrix} I_t + A^2 \begin{pmatrix} r_{s,t-1} \\ r_{b,t-1} \end{pmatrix} (1 - I_t) + B^2 \begin{pmatrix} \tilde{r}_{s,t} \\ \tilde{r}_{b,t} \end{pmatrix} (1 - I_t) + \begin{pmatrix} \varepsilon_{s,t} \\ \varepsilon_{b,t} \end{pmatrix}, \quad (30)$$

where A^1 and B^1 correspond to the coefficient matrices in low volatility regime with $VIX_{t-q} \leq T$, while A^2 and B^2 correspond to those in the high volatility regime with $VIX_{t-q} > T$. In the TVAR framework, the market state variable I_t equals to one if the threshold variable VIX_{t-q} at lag order q (the delay parameter) is less than or equal to the threshold T , which correspond to a low-volatility state, and zero otherwise. The delay parameter implies that if the threshold variable VIX_{t-q} crosses the threshold value of T at time $t - q$, the dynamics actually change at time t . The difference between A^1 (B^1) and A^2 (B^2), if there is any, will shed light on how agents' trading behavior shifts conditional on different market conditions.

We follow the methodologies in Tsay (1998) and Balke (2000) to test, identify and estimate the TVAR model specified in Eq. (30). We first conduct a test of linear VAR against threshold alternative with VIX. As in Balke (2000), in order to test for threshold when the threshold value is unknown, the TVAR model is estimated for all possible threshold values. And then for each possible threshold value, we calculate the Wald statistics testing the hypotheses of no difference between the two regimes. The p -value of the Wald test is less than 1%, which supports the threshold effects based on VIX. Threshold nonlinearity test suggests using VIX without lag as the threshold variable, that is, the delay parameter $q = 0$. The threshold value is determined endogenously by a grid search over possible values of the VIX and the estimated threshold value is $T = 21.54$. After identifying q and T , the TVAR model can then be estimated

by least squares.

3.2 Markov-Switching VAR (MSVAR)

We next infer the unobserved market states I_t from the observed prices using MSVAR. All agents trade contingent on some unobserved market state I_t , which takes a discrete value of 0 or 1 so that $I_t \in S = \{1, 0\}$. The state I_t is modeled as a stationary ergodic two-state Markov chain on S with transition probabilities given by

$$P(I_t = j | I_{t-1} = k, I_{t-2} = l, \dots) = P(I_t = j | I_{t-1} = k) = P_{j,k},$$

for $j, k, l \in S$, where $P_{j,k}$ indicates the probability that state (regime) k transits to state j for $k, j \in \{1, 0\}$. The transition probabilities are constant and satisfy the conditions of $\sum_{j=1}^2 P_{j,k} = 1$ and $0 \leq P_{j,k} \leq 1$ for $k = 1, 0$. The state I_t is a random variable that is not directly observable. However, a filter estimate can be computed from the time series of stock and bond prices. Some filters, such as sequential filter, are capable of performing accurate inferences of I_t . It is therefore reasonable to assume that investment professionals can estimate the state with high precision.

The state-dependent coefficient matrices A and B in Eq. (29) can then be specified as

$$A = \begin{cases} A^0, & I_t = 0, \\ A^1, & I_t = 1. \end{cases} \quad \text{and } B = \begin{cases} B^0, & I_t = 0, \\ B^1, & I_t = 1. \end{cases} \quad (31)$$

The noise terms $\varepsilon_{i,t}$, $i \in \{s, b\}$ are assumed to be drawn from an $N(0, \sigma_{i,t}^2)$ distribution and $\sigma_{i,t}^2$ is regime-dependent, that is:

$$\varepsilon_{s,t} \sim \begin{cases} N(0, \sigma_{s,0}^2), & I_t = 0, \\ N(0, \sigma_{s,1}^2), & I_t = 1. \end{cases} \quad (32)$$

and

$$\varepsilon_{b,t} \sim \begin{cases} N(0, \sigma_{b,0}^2), & I_t = 0, \\ N(0, \sigma_{b,1}^2), & I_t = 1. \end{cases} \quad (33)$$

We then estimate the MSVAR model for the stock and bond markets specified by Eqs. (29) and (31) – (33) following the methods in Ehrmann et al. (2003). The estimation entails a joint estimation of all the parameters and the hidden Markov chain. The likelihood function has a recursive nature because the Markov chain is hidden. As a result, the model is estimated using the Expectations-Maximization (EM) algorithm (see Hamilton (1990) and Krolzig (1997)). The first expectations step optimally infers the hidden Markov-chain for a given set of parameters. The second maximization step then re-estimates the parameters for the inferred hidden Markov chain. These steps are continued until convergence.

4 Data and Summary Statistics

We use monthly US stock and bond prices data from January 1990 to June 2014.⁴ The stock price is based on the S&P 500 price index and the bond price is measured by the Bank of America Merrill Lynch US bond price index. The price and dividend of S&P 500 index is from Robert Shiller's website⁵ and the price, coupon payment and duration of the bond index is from Bloomberg. The risk free interest rate is measured by the 3-month US treasury bill rate. All measures are in nominal terms. Figure 2 plots the movements of the two price indexes over the sample period. The bond market has been through several boom-bust cycles while the stock market has two major boom-bust cycles followed by the most recent boom. While there are periods when the stock and bond prices move hand-in-hand (e.g., year 1996 to 1998), sometimes the two prices move in opposite directions (e.g., year 2003 to 2006). In particular, the main troughs (peaks) in the stock market are coinciding with periods of peaks (troughs) in the bond market, which take place in the years 2003 and 2009 (2000, 2007 and 2014).

In the following, we use log-transformations of all prices such that the first-lagged differ-

⁴The starting date is based on the availability of the bond data. There are $N = 294$ monthly observations.

⁵See http://www.econ.yale.edu/shiller/data/ie_data.xls

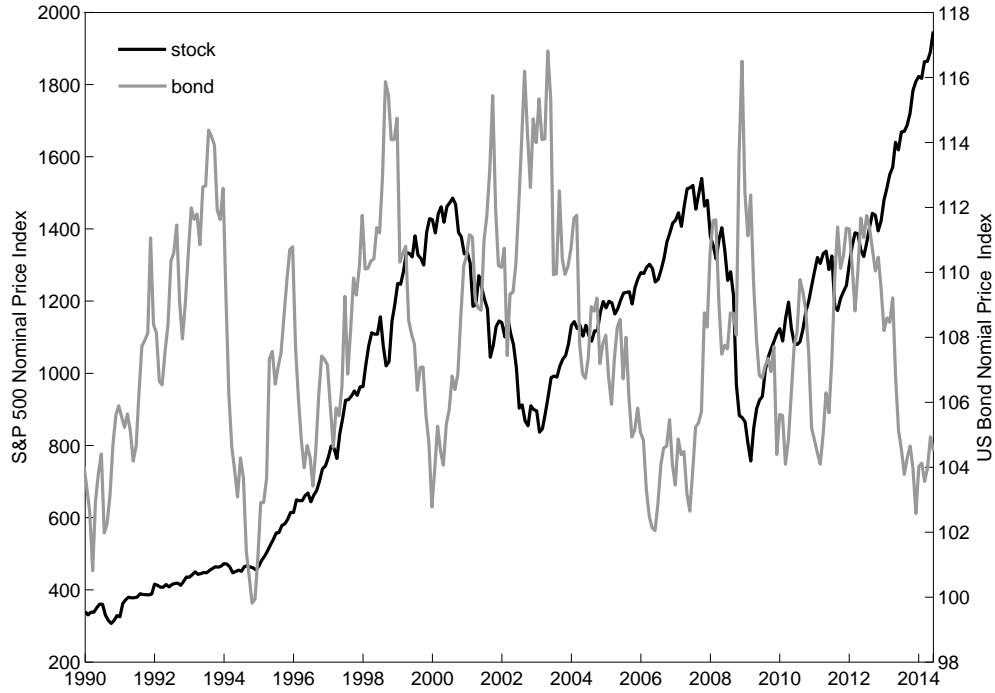



Figure 2: The S&P 500 Price Index and the US Bond Price Index

Table 1: Summary Statistics

Variable	Mean	Std. Dev.	Min	Max
$p_{s,t}$	6.816	0.486	5.727	7.574
$p_{b,t}$	4.232	0.180	3.887	4.556
$\theta_{s,t}$	6.769	0.322	6.275	7.486
$\theta_{b,t}$	4.229	0.182	3.878	4.557
$r_{f,t}$	0.257	0.186	0.001	0.650
$r_{s,t}$	0.328	3.658	-22.841	11.341
$r_{b,t}$	0.242	1.560	-4.223	5.769
$\tilde{r}_{s,t}$	-4.380	29.829	-63.298	62.193
$\tilde{r}_{b,t}$	-0.391	2.452	-6.146	6.835

Notes: Sample period is from January 1990 to June 2014. $N = 294$. $p_{s,t}$, $p_{b,t}$, $\theta_{s,t}$ and $\theta_{b,t}$ are in logarithmic values. $r_{f,t}$, $r_{s,t}$, $r_{b,t}$, $\tilde{r}_{s,t}$ and $\tilde{r}_{b,t}$ are expressed in percentage.

ences represent the price returns. Table 1 presents the summary statistics of our main variables. The stock price is more volatile than the bond price, as suggested by the standard deviations of 0.486 and 0.180 respectively. The fundamental value of the stock is also found to be more volatile than that of the bond. The mean of the stock price is very close to the mean of the fundamental value of the stock, suggesting the stock price fluctuates around its fundamental value. The standard deviation of the stock price is higher than that of the fundamental value, which is an evidence of excessive volatility that is commonly documented in the stock market (see for example uang et al. (2013)). During the sample period, the average risk free return is 0.257% with a standard deviation of 0.186%. The average excess return in the stock market is 0.328%, which is only moderately above that in the bond market of 0.242%. However, the stock market excess return exhibits much larger volatility than the bond market as suggested by the standard deviations of the respective excess returns of 3.658% and 1.560%. In particular, the excess return in the stock market ranges from -22.841% to 11.341% while that in the bond market is between -4.223% and 5.769% , which again suggests that the stock market is more volatile than the bond market. The difference between the deviation of the stock price from its fundamental and the risk free interest rate, $\tilde{r}_{s,t}$, is also much larger for the stock, ranging from -63.298% to 62.193% , as compared to that of the bond, $\tilde{r}_{b,t}$, which is only between -6.146% and 6.835% .

5 Estimation Results

At the beginning of estimation procedures, we conduct a lag selection test for the basic unrestricted VAR model. All three models selection criteria including the AIC (Akaike information criterion), BIC (Bayesian information criterion) and HQ (Hannan-Quinn information criterion), as shown in Table 2, suggest the optimal lag length of 1, which is consistent with our HAM specification. In the following, we use a lag length of 1 in both TVAR and MSVAR.

Table 2: VAR Lag Order Selection Criteria

Lag	0	1	2	3	4	5	6
AIC	-9.343	-9.443*	-9.395	-9.433	-9.411	-9.412	-9.425
BIC	-9.241	-9.261*	-9.192	-9.179	-9.105	-9.056	-9.018
HQ	-9.302	-9.352*	-9.314	-9.331	-9.288	-9.269	-9.262

Notes: * indicates lag order selected by the criteria. AIC: Akaike information criterion. BIC: Bayesian information criterion. HQ: Hannan-Quinn information criterion.

5.1 Basic Results

The TVAR estimation results are shown in columns (1) and (2) of Table 3, which correspond to low volatility state ($VIX \leq 21.54$) and high volatility state ($VIX > 21.54$) respectively. The MSVAR estimation results are shown in columns (3) and (4) of Table 3, which correspond respectively to two distinguished low and high volatility market states identified through the Markov transition process. The two sets of results are mostly consistent with each other as suggested by the signs and magnitudes of the coefficients under the two corresponding market regimes.

Both the TVAR and MSVAR estimation results provide evidence of cross-sectional and time-varying heterogeneity in agents' trading behavior, which is consistent with existing HAM literature. In both stock and bond markets, we find evidence of trading by fundamentalists, chartists and their cross-market trading activities in one market state or another, which supports the presence of cross-sectional behavioral heterogeneity. The estimated coefficients vary between the two states, which provides evidence of time-varying behavioral heterogeneity. In both low and high volatility states under TVAR, the estimation results ($A_{11} > 0, A_{22} > 0$) imply $\beta_s > 0$ and $\beta_b > 0$, which indicate momentum trading (or bandwagon expectations on price movements) for both s -chartists and b -chartists that specialize respectively in the stock and bond market. MSVAR finds similar evidence of momentum trading for s -chartists and b -chartists in both market states. Both the TVAR and MSVAR results suggest $B_{11} > 0$ and $B_{22} > 0$, which support our hypothesis that fundamentalists engage in trading activities that drive the price towards its fundamental value regardless of the market conditions.

Besides the different switching mechanism, the MSVAR incorporates an additional feature of the regime-dependent variances, which are highly significant in both markets as shown in the estimation results. In both low and high volatility market states, we have $\sigma_s > \sigma_b$, suggesting that the excess return volatility in the stock market is higher than that in the bond market, in consistency with the empirical facts documented in Table 1. Moreover, both σ_s and σ_b are larger in the high volatility state than in the low volatility state, which implies that both the stock and bond markets are more volatile in the high volatility states, which justify the identification of market states using MSVAR. The covariance is positive in the low volatility state while negative in the high volatility state, suggesting regime-dependent stock-bond comovements. The estimated transition probabilities indicate that the market continues staying in the low volatility state with a probability of 0.937 and transits from the high volatility state to the low volatility state with a probability of 0.155. The regimes are estimated to be very persistent with expected duration of $1/(1 - P_{1,1}) \approx 16$ months of low volatility state and $1/P_{1,2} \approx 6$ months of high volatility state.

Overall, comparisons of the model fitness statistics, including the log-likelihood, AIC and BIC as summarized in the lower part of Table 3, suggest that the MSVAR model provides a better fit to the joint price dynamics of stock and bond.

5.2 State-Dependent Cross-Market Trading

We now turn to study the cross-market trading behavior. To take a closer look into the trading activities of each type of investors, Table 4 reports the median excess returns and price deviations under different market regimes for both the TVAR and MSVAR, based on which investors form their capital allocation decisions for the stock and bond. Due to differences in market states classifications, there are some small variations in the signs and magnitudes of these statistics derived from the TVAR and MSVAR models.

We first explore how stock-market-based agents trade in the bond market. Conditional on the low volatility state, the coefficient B_{21} is statistically significant and positive (columns (1) and (3) of Panel B in Table 3) based on either TVAR or MSVAR, which provides evidence that

Table 3: TVAR and MSVAR Estimation Results

Coefficient	TVAR		MSVAR	
	Low (1)	High (2)	Low (3)	High (4)
<i>Panel A: Dependent Variable is $r_{s,t}$</i>				
A_{11}	0.172** (0.013)	0.265** (0.012)	0.299*** (0.000)	0.171 (0.328)
A_{12}	0.293** (0.033)	-0.080 (0.786)	0.228** (0.021)	-0.162 (0.716)
B_{11}	0.005 (0.444)	0.013 (0.367)	0.001 (0.852)	0.026 (0.271)
B_{12}	0.154** (0.017)	-0.035 (0.879)	0.085 (0.251)	-0.066 (0.871)
<i>Panel B: Dependent Variable is $r_{b,t}$</i>				
A_{21}	0.030 (0.400)	-0.121** (0.002)	0.012 (0.747)	-0.132** (0.002)
A_{22}	0.070 (0.333)	0.259** (0.018)	0.092 (0.268)	0.173 (0.185)
B_{21}	0.007* (0.065)	-0.004 (0.511)	0.008** (0.049)	-0.004 (0.636)
B_{22}	0.030 (0.378)	0.235*** (0.007)	0.027 (0.526)	0.209** (0.041)
σ_s	–	–	2.215*** (0.000)	5.464*** (0.000)
σ_b	–	–	1.330*** (0.000)	1.796*** (0.000)
$cov_{s,b}$	–	–	0.291 (0.261)	-4.028** (0.040)
$P_{1,1}$	–	–	–	0.937*** (0.000)
$P_{1,2}$	–	–	–	0.155** (0.040)
N	191	103	193	101
log-likelihood	-1376.023		-1276.467	
AIC	10.019		8.874	
BIC	10.148		9.225	

Notes: Low (High) refers to low (high) volatility state. The estimated TVAR model is specified by Eq. (30) and the MSVAR model is specified by Eqs. (29) and (31) – (33). *, ** and *** denote significance at 10%, 5% and 1% level, respectively. Numbers in the parentheses are p -values.

s -fundamentalists actively trade on the bond market. Note that the stock is overvalued with a negative median value of $\tilde{r}_{s,t}$ in the low uncertainty state (columns (1) and (3) of Table 4), we can infer that $C_{b,t-1}^{sf} = (\tau_t \tilde{r}_{s,t}) / (\alpha(1 + \rho'_t) \sigma_{s,t}^2) < 0$, that is, s -fundamentalists are selling bond (see Section 2.3 for the demand of stock and bond by each type of agent, the same for below). In the high volatility state, there is a lack of evidence that s -fundamentalists are trading in the bond market as B_{21} is statistically insignificant. The story is different for s -chartists. The coefficient A_{21} is statistically significant and negative in the high volatility state (columns (2) and (4) of Panel B in Table 3), but statistically insignificant in the low volatility state (columns (1) and (3) of Panel B in Table 3). It suggests that s -chartists with expertise in stock market actively trade in the bond market in the high volatility state but not in the low volatility state. Conditional on high volatility state, the stock price is declining with a negative median value of $r_{s,t}$ (columns (2) and (4) of Table 4), implying that $C_{b,t-1}^{sc} = (\tau_t \beta_{s,t} r_{s,t-1}) / (\alpha(1 + \rho'_t) \sigma_{s,t}^2) > 0$, that is, the s -chartists specializing in the stock market are buying the bond. Note that $C_{s,t-1}^{sc} = (\beta_{s,t} r_{s,t-1}) / (\alpha(1 + \rho'_t) \sigma_{s,t}^2) < 0$, the results suggest that s -chartists are selling the stock while buying the bond when the market volatility is high.

We next explore how bond-market-based agents trade in the stock market. The TVAR results suggest significant trading of b -fundamentalists in the stock market in low volatility state as indicated by the statistically significant and positive coefficient B_{12} (column (1) of Panel A in Table 3). No such evidence is found in MSVAR estimation (The coefficient B_{12} in columns (3) of Panel A in Table 3 is positive but statistically insignificant). In both TVAR and MSVAR estimation, the coefficient A_{12} is statistically significant and positive in the low volatility state (columns (1) and (3) of Panel A in Table 3), which indicates that b -chartists with expertise in bond market trade in the stock market. Note that in the low volatility state, the bond excess return $r_{b,t}$ is slightly negative (column (1) of Table 4) and $A_{12} > 0$, there is $C_{s,t-1}^{bc} = (\beta_{b,t} r_{b,t-1}) / (\alpha(1 + \rho'_t) \tau_t \sigma_{b,t}^2) < 0$, which suggests that b -chartists reduce their investment in the stock market. However, in MSVAR the b -chartists increase their investment in the stock market in the low volatility state because $r_{b,t} > 0$ in MSVAR (column (3) of Table 4) and $A_{12} > 0$. Such difference in b -chartists' investment in the stock market between TVAR and MSVAR

estimation is however not significant. In high volatility state, the cross-market trading of the b -fundamentalists and b -chartists becomes insignificant in both TVAR and MSVAR estimations.

Table 4: Regime-Dependent Sample Median of Excess Returns and Price Deviations

Variable	TVAR		MSVAR	
	Low (1)	High (2)	Low (3)	High (4)
$r_{s,t}$	1.016	-0.555	0.938	-0.469
$r_{b,t}$	-0.002	0.746	0.028	0.488
$\tilde{r}_{s,t}$	-3.938	-15.227	-5.107	-10.369
$\tilde{r}_{b,t}$	-0.303	-0.420	-0.645	-0.105
Observations	191	103	193	101

Notes: All numbers except for observations are expressed in percentage.

5.3 Implication on the Comovement Indicator

It is clear from Section 2.3 that the cross-market trading is mainly driven by the comovement indicator τ_t and the trading in the market where one has expertise in. Therefore the sign of τ_t critically determines in which direction the cross-market orders move the asset prices. Based on the estimated coefficients of A_{12} , A_{21} , B_{12} and B_{21} , we can uncover the sign of τ_t under different regimes because γ_i , α , $(1 + \rho'_i)$, $\sigma_{i,t}^2$ and m_i^h are all nonnegative by definition and the signs of $\beta_{i,t}$ has been uncovered in Section 5.1, where $i \in \{s, b\}$. The sign of τ_t helps us understand how cross-market trading by heterogeneous agents responds to the time-varying linkage between the stock and bond markets, which is not directly identified in the model. Note that all these four coefficients can individually and independently predict the sign of τ_t , therefore it is possible that the estimation results yield mixed implications about the sign of τ_t within each market state.⁶ Even though we only impose a simple and natural constraint that τ_t is the same for all types of agents in both stock and bond markets in the model, both the TVAR and MSVAR results yield consistent predictions on the signs of τ_t without any additional constraint in the

⁶In fact, there are in total $2 \times 2^4 = 32$ possible combinations of the predictions, out of which only $2 \times 2 = 4$ are consistent within each regime.

estimations. In particular, we find that $\tau_t > 0$ in the low volatility state while $\tau_t < 0$ in the high volatility state, regardless of which method is applied. This suggests that the cross-market trading directs order flows to (from) the stock and bond markets in the same direction when the market volatility is low, and channels the funds from one market to the other when the market volatility is in high, which are consistent with the findings in Connolly et al. (2005) and many other existing literature. It in turn implies that the specification of τ_t in our model well captures investors' time-varying responses to the stock-bond comovements.

5.4 Flight-to-Quality and Cross-Market Trading

Recall from the cross-trading activities of heterogeneous agents and the finding that $\tau_t > 0$ in low volatility state and $\tau_t < 0$ in high volatility state, we can uncover the general trading behavior of each type of agents in each market according to Tables 3 and 4. Agents' heterogeneous trading behavior under different market conditions corresponding to the TVAR and MSVAR estimation are summarized in Table 5. In the low volatility state, chartists are buying while fundamentalists are selling the stock and bond in general. The only exception is *b*-chartists' selling behavior in the low volatility state under TVAR. In the high volatility state, except for *b*-fundamentalists, all the other types of agents are selling the stock while buying the bond, although the magnitude of trading is different. The result is in line with the flight-to-quality argument, with investors substituting risky assets with safe haven alternatives when facing extreme uncertainty. However, the finding on the cross-market trading is only significant for *s*-chartists but not for the other types of agents. This suggests that the flight-to-quality phenomenon is mainly driven by the cross-market trading of *s*-chartists. By directing the investment from the stock market to the bond market during periods of high volatility, *s*-chartists drive up the bond price while distress the stock price, which contribute to a negative relation between the stock and bond excess returns.

Table 5: Summary of Trading Behavior in Different Market States

	TVAR				MSVAR			
	Low		High		Low		High	
Agents	Stock	Bond	Stock	Bond	Stock	Bond	Stock	Bond
<i>s</i> -chartists	buy*	buy	sell*	buy*	buy*	buy	sell	buy*
<i>b</i> -chartists	sell*	sell	sell	buy*	buy*	buy	sell	buy
<i>s</i> -fundamentalists	sell	sell*	sell	buy	sell	sell*	sell*	buy
<i>b</i> -fundamentalists	sell*	sell	buy	sell*	sell	sell	buy	sell*

Notes: * indicates that significant evidence on the designated trading activity.

5.5 Price Impact of Behavioral Heterogeneity

The TVAR and MSVAR results in Table 3 together with the summary statistics of regime-dependent median excess returns and price deviations in Table 4 provide more insights on how and to what extent the dynamic interactions among heterogeneous behavioral trading shape the joint stock-bond price dynamics.

We first analyze the price impact of behavioral heterogeneity in the low volatility state. The estimated coefficient B_{11} is insignificant and the absolute difference between the fundamental value deviation from the stock price and risk free interest rate, $\tilde{r}_{s,t}$, is much smaller in low volatility state as compared to high volatility state in absolute value. As a result, the impact of *s*-fundamentalists' aggregate trading on the stock price excess return, measured by $B_{11}\tilde{r}_{s,t}$ is trivial. On the other hand, the price impact of *s*-chartists is both economically and statistically significant. According to column (1) and (3) of Table 4, in a typical month with the stock price excess return increasing by 1.016% (TVAR) or 0.938% (MSVAR), the aggregate buying of *s*-chartists alone contributes to a 17 or 28 basis point increase in the subsequent stock price excess return ($A_{11}r_{s,t} = 0.172 \times 1.016\% = 0.174\%$ in TVAR or $0.299 \times 0.938\% = 0.281\%$ in MSVAR). Similarly, we can show that the cross-market trading of *b*-chartists contributes to increase the stock price excess return in the low volatility state under MSVAR while decreasing the stock price in the high volatility state of TVAR in general, both effects are however economically small. In the bond market, we find that the cross-market trading of *s*-fundamentalists

reduces the bond price excess return while that of s -chartists increases the bond price excess return most of the time. Although the price impact of b -fundamentalists' and b -chartists' trading are found to be mostly negative in the bond market exception of b -chartists in the low volatility state based on MSVAR, the results are not statistically significant.

We next look at the price impact of behavioral heterogeneity in the high volatility state. During the turbulent periods, the stock price is much higher than its fundamental value with $\tilde{r}_{s,t}$ being -15.227% (TVAR) or -10.369% (MSVAR), which triggers strong sell off by s -fundamentalists and exerts downward pressure on the stock price. Moreover, as $r_{s,t}$ is negative and A_{11} is positive, s -chartists are selling the stock in response to the declining stock price as well. Similarly, we can show that b -fundamentalists are buying while b -chartists are selling the stock when the market volatility is high. The aggregate selling by s -fundamentalists, s -chartists, and b -chartists dominates the buying of b -fundamentalists, which drives the stock price down subsequently. In the bond market, we observe that s -fundamentalists, s -chartists and b -chartists are buying while b -fundamentalists are selling the bond. However, as the buying power dominates the selling power significantly in the bond market, the aggregate trading of heterogeneous agent leads to the bond price boom when the market volatility is high. The trading behavior of heterogeneous agents therefore moves the stock and bond price into opposite directions and contribute to a negative relation between the stock and bond excess returns.

5.6 More on the Market States and Comovement Indicator

Both the TVAR and the MSVAR models have suggested $\tau_t > 0$ in one regime and $\tau_t < 0$ in the other, indicating time-varying responses to the interdependence between stock and bond excess returns. The shifting of τ_t across different market states are intuitive. To see whether τ_t is a good proxy for agents' response to the stock-bond return correlation, we compare ~~the~~ our identifications of market states with the realized stock-bond return correlations to examine whether they match well with the stylized facts. For each month in our sample, we calculate the 22-trading-day correlations between the stock and bond returns using daily data. Following Connolly et al. (2005), we assume the expected daily returns for both stocks and bonds are zero

instead of the each 22-day period sample mean. The summary statistics suggest substantial time-series variation in the calculated stock-bond return correlations. Over the sample period, the maximum correlation coefficient is 0.907 and the minimum is -0.896, with an average of -0.061 and a standard deviation of 0.476.

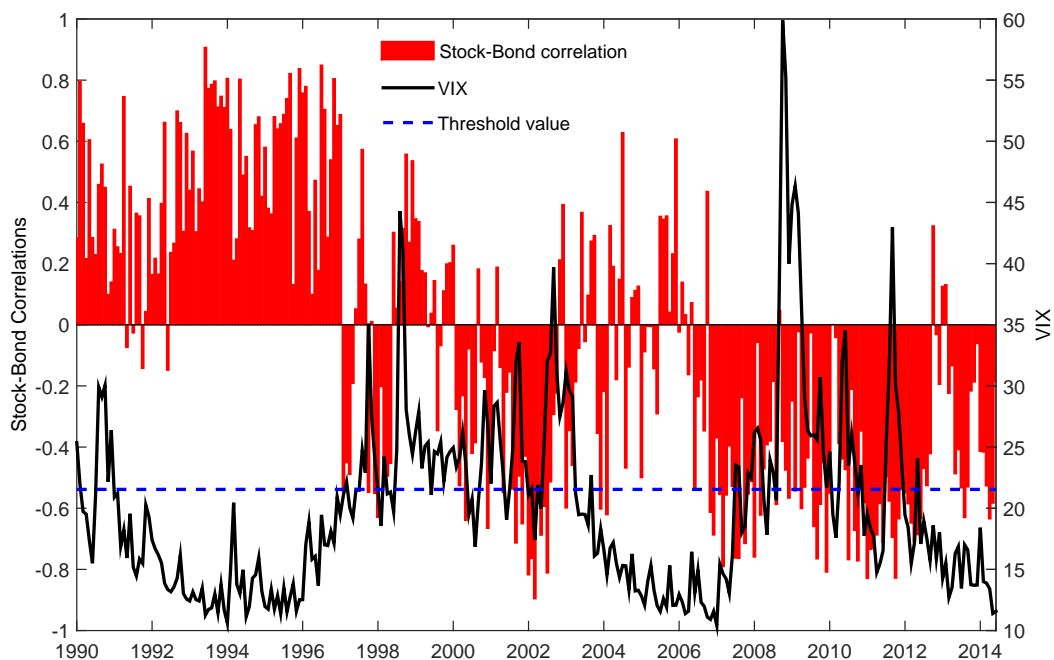


Figure 3: Stock-Bond Correlations, VIX and Threshold Value

The stock-bond return correlation coefficient and the VIX are plotted together in Figure 3. The dashed horizontal line indicates the threshold value (21.54) of the VIX estimated from the TVAR model. We can observe a clustering of the periods with a negative correlation or a positive correlation. Furthermore, eyeball observations suggest that periods of high (low) VIX and/or increasing (decreasing) VIX are associated with the periods of negative (positive) stock-bond return correlations. During periods with low volatility (VIX below the threshold value), e.g., 1991–1997 and 2004–2006, the empirically calculated correlation coefficients are mostly positive. On the other hand, in the periods with significantly high market volatility (VIX above the threshold value), e.g., 1997–2003 and 2007–2013, the correlation coefficients turn into negative for most of the times. These observations empirically validate the time-varying stock-bond comovement indicator estimated from the model.

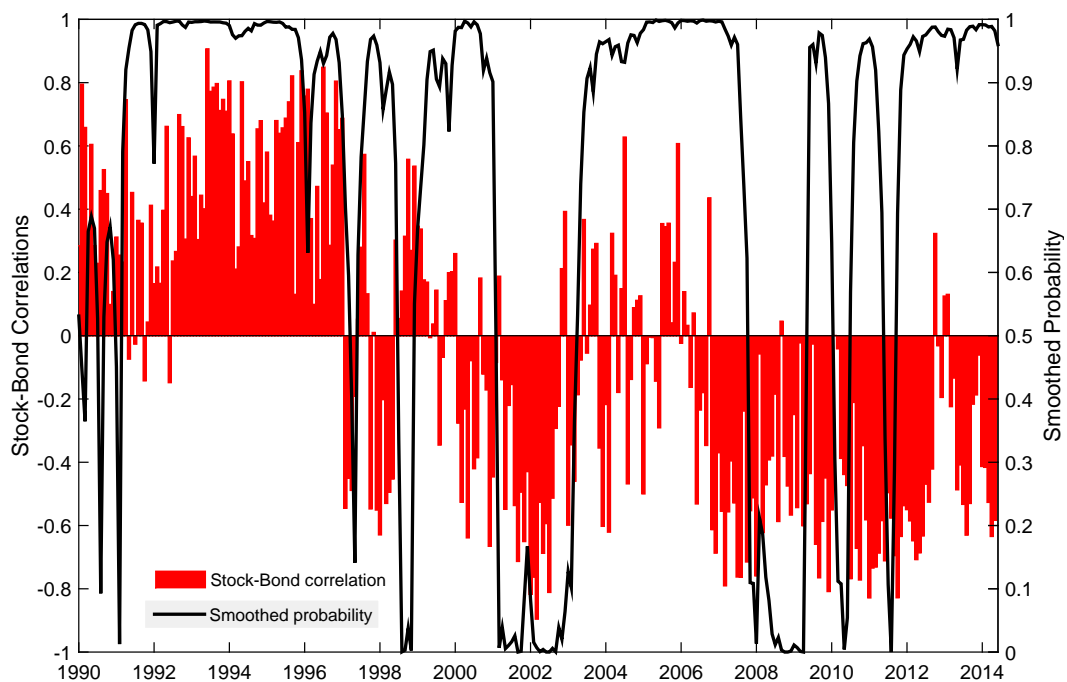


Figure 4: Stock-Bond Correlations and Smoothed Probabilities of Low Volatility State

Figure 4 plots the MSVAR estimated smoothed probabilities of the market being in low volatility state during the sample period together with the stock-bond correlations. The market is in low volatility state from 1991 up to 1998, then it evolves to high volatility state in late 1998 for a short period and again in 2001 to 2003. The market returned to low volatility state from 2003 to 2007. The regimes have been switching somewhat more frequently since late 2007. These results suggest two prolonged turbulent periods, one starting from the early 2001 and the other around the late 2007, which coincide with the two major financial crises, the dot-com bubble bursting and the recent global financial crisis. These are the periods associated with persistently negative stock-bond correlations. Therefore, the regime-dependent comovement indicator uncovered from the MSVAR estimation results coincides with the empirically observed time-varying stock-bond return correlations.

5.7 Robustness of the Comovement Indicator

Both the TVAR and MSVAR so far implicitly estimate τ_t as part of the aggregate market trading power parameters of each type of agents in a particular market, based on which we are able to

consistently uncover the sign of τ_t in different market states. While these results have already provided important evidence on state-dependent cross-market trading, we take a step further to test the robustness of our results by explicitly estimating τ_t first.⁷ Specifically, we use the 22-trading-day correlations between the stock and bond daily returns (Connolly et al. 2005) as a proxy for τ_t following the discussions in Section 5.6. In this way, τ_t is estimated and updated every month, which is therefore more general than the two regime-dependent values assumed previously. Given the estimated $\hat{\tau}_t$, then Eq. (29) becomes a more general system of equations as following

$$\begin{cases} r_{s,t} = A_{11}^* r_{s,t-1} + A_{12}^* (r_{b,t-1} / \hat{\tau}_t) + B_{11}^* \tilde{r}_{s,t} + B_{12}^* (\tilde{r}_{b,t} / \hat{\tau}_t), \\ r_{b,t} = A_{21}^* (r_{s,t-1} \hat{\tau}_t) + A_{22}^* r_{b,t-1} + B_{21}^* (\tilde{r}_{s,t} \hat{\tau}_t) + B_{22}^* \tilde{r}_{b,t}. \end{cases} \quad (34)$$

where we denote the corresponding coefficient matrices (taking τ_t out from A and B) as A^* and B^* . Similarly, we then estimate Eq. (34) under a threshold and Markov switching framework using seemingly unrelated regressions (SUR), which we refer to as TSUR and MSSUR, respectively. The estimation results are summarized in Table 6.

After explicitly accounting for the time-varying comovement, our results on both cross-sectional and time-varying heterogeneity remain robust. Results presented in Table 6 show that the estimated coefficients A_{12}^* , B_{12}^* , A_{21}^* and B_{21}^* are all positive in both market states under both TSUR and MSSUR. This is consistent with the previous results that separate out the effects of τ_t .⁸ The positive coefficient matrix A^* again consistently indicate the dominance of momentum beliefs among both the s -chartists and b -chartists. The rest estimated parameters are also mostly consistent with those in Table 3. In particular, the coefficients A_{11}^* and A_{21}^* are statistically significant and positive in either TSUR or MSSUR, suggesting that s -chartists actively sell stocks and buy bonds when the stock-bond correlations are negative (see Eq. (34)). Although similar evidence is found for s -fundamentalists, the results are not significant. To summarize,

⁷We are grateful to an anonymous referee who suggests the revisions in this section.

⁸Note that the previous findings in Section 5.3 suggest that $\tau_t > 0$ in low volatility state and $\tau_t < 0$ in high volatility state. Combining these with the estimates A_{12} , B_{12} , A_{21} and B_{21} in Table 3, we can see that the negative coefficients are all driven by $\tau_t < 0$ in high volatility state. It means the aggregate market trading power parameters after separating out the effects of τ_t are positive across different market states.

consistent with the previous findings, we find the trading by s -chartists to be the main driving force for the flight-to-quality phenomenon.

6 State-Dependent Impulse Responses

To gain further insights of the joint stock-bond price dynamics, we use impulse response functions (IRFs) to trace out how disturbances in the two markets affect the excess return movements of each other. The regime-dependent impulses conveniently summarize the information of parameters, variances and covariances of each regime. The main difference of the state-dependent IRFs from the conventional ones is that they are conditional on a given market state prevailing at the time of the disturbance and throughout the duration of the response. By comparing the state-dependent IRFs, we can observe potential asymmetries in terms of direction, magnitude, persistence and significance. We derive the state-dependent IRFs based on the MSVAR model specified by Eqs. (29) and (31) – (33) since it provides a better fit to the data and give more consistent estimations on the trading behavior of the heterogeneous agents under different market states.

Figure 5 plots in solid line the IRFs to 1% decrease in the stock price excess returns conditional on low volatility state (left panel) and high volatility state (right panel). The dashed lines plot one standard deviation confidence intervals from 1,000 bootstraps. In both market states, the shock to the stock price returns persists in the stock market for a few months before it decays. The bond return responds to the shock in the stock market asymmetrically in different states, especially in terms of the direction and the scale of the impulse responses. When the stock excess return decreases by 1%, the bond excess return decreases by about 0.05% in the low volatility state, the impact of which is however not statistically significant. The same shock increases the bond excess return immediately by 0.14% in the high volatility state. Such an impact is not only statistically significant but also persistent – the IRFs only decay to 0 in 5 months. The asymmetric response to the shock results in a positive stock-bond return relation in low volatility state and a negative one in high volatility state.

Table 6: Robustness Check: TSUR and MSSUR Estimation Results

Coefficient	TSUR		MSSUR	
	Low (1)	High (2)	Low (3)	High (4)
<i>Panel A: Dependent Variable is $r_{s,t}$</i>				
A_{11}^*	0.232*** (0.001)	0.292*** (0.001)	0.220*** (0.002)	0.233** (0.023)
A_{12}^*	0.281 (0.529)	0.069*** (0.001)	0.006 (0.868)	0.019** (0.015)
B_{11}^*	0.010 (0.176)	0.003 (0.804)	0.004 (0.639)	0.011 (0.486)
B_{12}^*	0.253 (0.141)	0.010 (0.727)	0.001 (0.826)	0.008 (0.121)
<i>Panel B: Dependent Variable is $r_{b,t}$</i>				
A_{21}^*	0.179** (0.021)	0.188** (0.014)	0.262*** (0.002)	0.135* (0.093)
A_{22}^*	0.043 (0.545)	0.236** (0.027)	0.133 (0.118)	0.212** (0.050)
B_{21}^*	0.003*** (0.004)	0.013 (0.328)	0.003 (0.689)	0.011 (0.453)
B_{22}^*	0.007 (0.839)	0.209** (0.020)	0.074** (0.046)	0.234** (0.011)
σ_s	–	–	2.195*** (0.000)	5.194*** (0.000)
σ_b	–	–	1.200*** (0.000)	1.905*** (0.000)
$cov_{s,b}$	–	–	0.316 (0.830)	-1.844** (0.000)
$P_{1,1}$	–	–		0.950*** (0.000)
$P_{1,2}$	–	–		0.080 (0.230)
N	191	103	189	105
log-likelihood	-1374.383		-1281.564	
AIC	10.027		8.881	
BIC	10.233		9.282	

Notes: Low (High) refers to low (high) volatility state. The model is specified as Eq. (34). *, ** and *** denote significance at 10%, 5% and 1% level, respectively. Numbers in the parentheses are p -values.

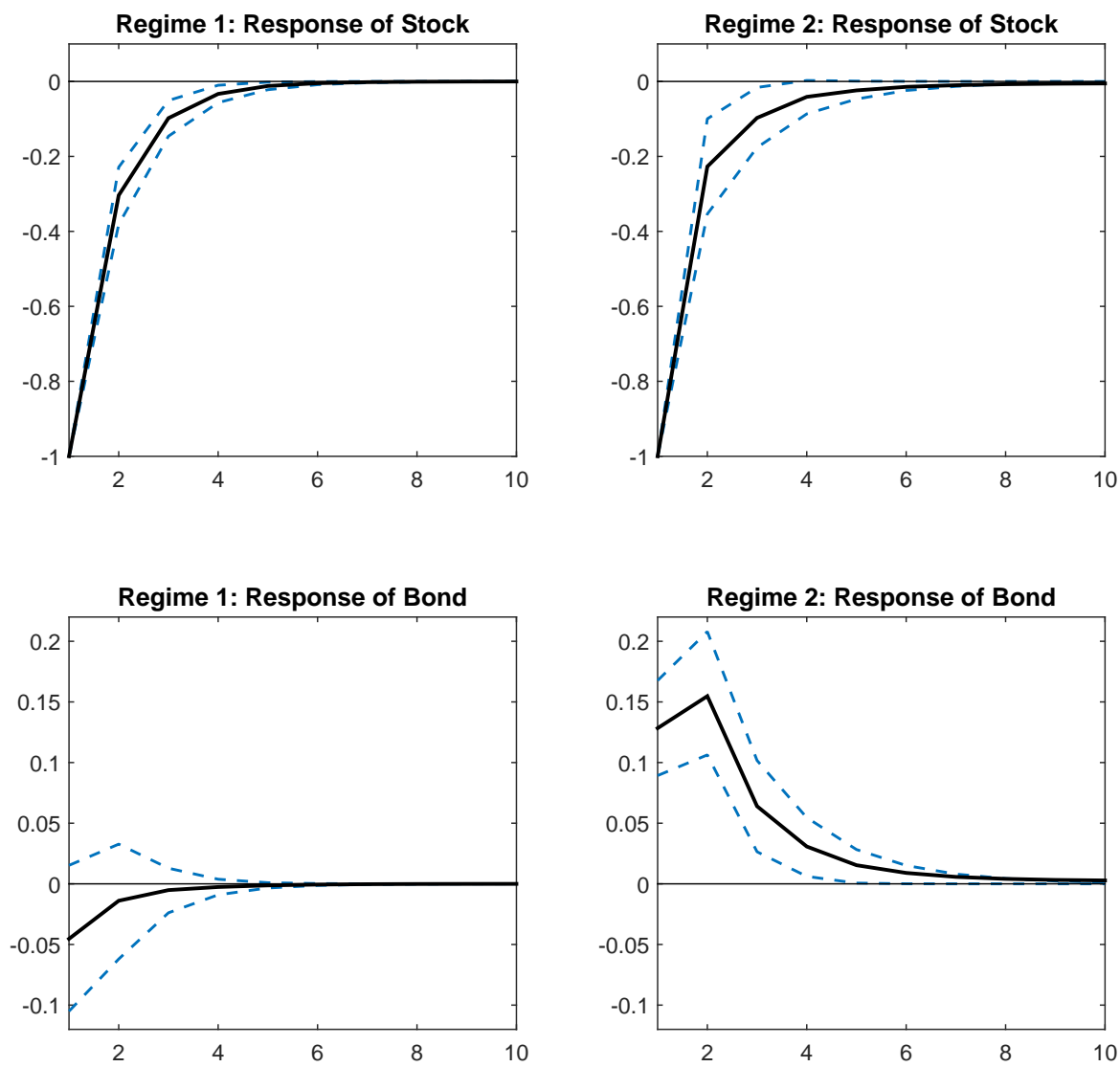


Figure 5: Responses to 1% Decline in the Stock Price Excess Returns

Notes: The left (right) panel corresponds to the low (high) volatility state.

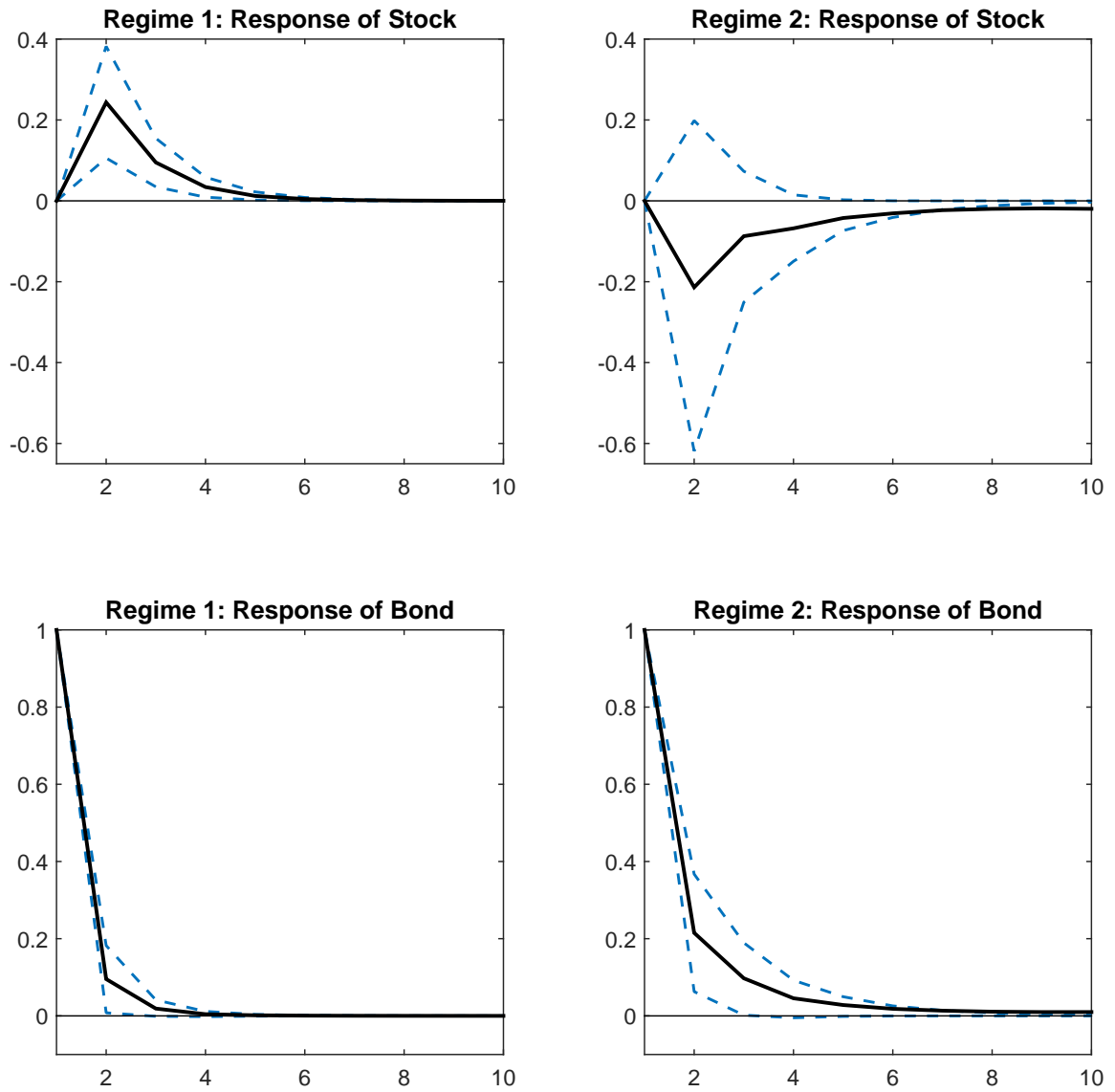


Figure 6: Responses to 1% Increase in the Bond Price Excess Returns

Notes: The left (right) panel corresponds to the low (high) volatility state.

Figure 6 plots the IRFs to 1% increase in the bond excess returns conditional on low volatility state (left panel) and high volatility state (right panel). The bond market shocks are less persistent compared to the stock market shocks. With a 1% increase in the bond market excess return, the stock excess return increases by as much as 0.21% in the low volatility state while it decreases by 0.21% in the high volatility state. However, the decrease in the high volatility state is not statistically significant. Similarly, these asymmetric responses lead to a positive stock-bond return relation when the market is relatively tranquil and a negative one when the market is relatively turbulent.

Overall, the impulse responses suggest asymmetric responses of excess returns in one market to shocks in the other market under different market states by explicitly depicting the time-varying stock-bond comovements.

7 Conclusion

Heterogeneous agents driven by various needs take into consideration the interdependence between stock and bond excess returns when optimizing their portfolio allocations. This in turn shapes the joint dynamics of stock and bond prices. We develop a behavioral asset pricing model in which agents allocate their capital among stock, bond and risk-free asset to optimize their portfolios, taking into account ~~of~~ the dynamic stock-bond return comovements. Agents have comparative advantages in either the stock or the bond market. They constantly revise their investment portfolios by taking into account the time-varying stock-bond return comovements and the changing market conditions. Agents' collective investment behavior shapes the stock-bond interlinkage, which feedbacks on their subsequent capital allocation decisions. The two-market HAM framework is then estimated with threshold VAR and Markov switching VAR.

We find significant evidence that heterogeneous agents trade across markets, taking into account the time-varying linkage between the stock and bond excess returns as well as the evolving market states. In particular, we find that agents tend to direct new investment flows

into both stock and bond markets when the market volatility is low, whereas they tend to move the money out of the stock market and into the bond market when the market volatility is high. Moreover, we find that chartists with comparative advantage in the stock market play a dominant role in driving the flight-to-quality phenomenon. Behavioral heterogeneity that is conditional on the market states thus provides an additional channel to understand the time-varying stock-bond return relations.

Our model setup is kept as simple as possible to keep it empirically testable. It can be extended in various dimensions to provide more sophisticated theoretical and numerical results on the stock and bond joint price dynamics. First, one can allow agents to actively switch between different expectation rules and/or market states. Second, one may consider different switching mechanisms and evaluate their profitability in both stock and bond markets. The third extension is to explore the stability conditions for the joint price dynamics in a more complex nonlinear framework. Finally, one may relax the assumptions on comovement expectations by heterogeneous agents and enable more disaggregated trading behavior.⁹

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⁹We thank an anonymous referee for suggesting interesting dimensions of future research.

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A Fundamental Value of the Bond Price Index

The Bloomberg data terminal reports the gross and clean price of the bond index. We derive the fundamental gross price based on the price index. The essential idea is based on the present value analysis where the fundamental bond price is determined by discounting its expected cash flows, which can be traced back to Fisher (1930).¹⁰

The *Accrued Interest* is the accumulated coupon payment that has accrued since the most recent coupon payment but has not been realized yet. It is calculated as

$$\text{Accrued Interest} = \text{coupon} \times \frac{\text{Days since last coupon payment}}{\text{Days between the previous and forthcoming coupon payments}},$$

where *coupon* is the semiannual coupon payment. The *Accrued Interest* is a proportion of coupon payment and is independent of bond price fluctuations.

The *gross price* include while the *clean price* exclude the accrued interest such that

$$\text{Gross Price} = \text{Clean Price} + \text{Accrued Interest}. \quad (35)$$

At the issuing date, we have $\text{Gross Price} = \text{Clean Price} = P_0$. Dividing Eq. (35) by P_0 yields

$$\begin{aligned} \frac{\text{Gross Price}_t}{P_0} &= \frac{\text{Clean Price}_t}{P_0} + \frac{\text{Accrued Interest}_t}{P_0} \\ \frac{\text{Gross Price}_t}{P_0} \times 100 &= \frac{\text{Clean Price}_t}{P_0} \times 100 + \frac{\text{Accrued Interest}_t}{P_0} \times 100. \end{aligned} \quad (36)$$

Note that $\frac{\text{Gross Price}_t}{P_0} \times 100$ and $\frac{\text{Clean Price}_t}{P_0} \times 100$ are the gross price and clean price of the bond index (with initial value of 100), which are both directly available from the data terminal.

The fundamental value of the clean price equals to the present value of all future cash flows such that

$$\text{Fundamental Clean Price} = \sum_{t=1}^{t=2n-1} \frac{\text{Coupon}}{2(1+r)^{2t}} + \frac{\text{Coupon} + \text{Principal}}{(1+r)^{2n}},$$

¹⁰Refer to Brigham and Houston (1997) Chapter 7 among many others for a general introduction to bond valuation.

where *Principal* is the par value that is paid upon maturity ($t = 2n$) and r is the yield to maturity. Based on the average coupon, average life, and the average yield to maturity provided by the data terminal, we can calculate the *Fundamental Clean Price* directly.

As *Accrued Interest* is independent of the pricing of bond, the fundamental value of the gross price is

$$\text{Fundamental Gross Price} = \text{Fundamental Clean Price} + \text{Accrued Interest}. \quad (37)$$

Multiplying Eq. (37) with $\frac{100}{P_0}$ yields

$$\frac{\text{Fundamental Gross Price}_t}{P_0} \times 100 = \frac{\text{Fundamental Clean Price}_t}{P_0} \times 100 + \frac{\text{Accrued Interest}}{P_0} \times 100. \quad (38)$$

Assuming the initial pricing of the bond is efficient so that *Fundamental Clean Price* = *Clean Price* = P_0 at the issuing date. As $\frac{\text{Accrued Interest}}{P_0} \times 100$ can be obtained from Eq. (36), and *Fundamental Clean Price* _{t} and P_0 are known, we can calculate $\frac{\text{Fundamental Gross Price}_t}{P_0} \times 100$, the fundamental gross price of the bond index.

B Identification

Without loss of generality, we discuss identification issues using the estimation results from MSVAR under the low volatility state as an example. The equations are summarized as following, where (39) – (46) are the estimated values, and (47) – (48) are constraints from the model setup

$$\frac{\gamma_s m_s^{sc} \beta_{s,t}}{\alpha (1 + \rho'_t) \sigma_{s,t}^2} = 0.299, \quad (39)$$

$$\frac{\gamma_s m_s^{bc} \beta_{b,t}}{\alpha (1 + \rho'_t) \tau_t \sigma_{b,t}^2} = 0.228, \quad (40)$$

$$\frac{\gamma_s m_s^{sf}}{\alpha (1 + \rho'_t) \sigma_{s,t}^2} = 0.001, \quad (41)$$

$$\frac{\gamma_s m_s^{bf}}{\alpha (1 + \rho'_t) \tau_t \sigma_{b,t}^2} = 0.085, \quad (42)$$

$$\frac{\gamma_b m_b^{sc} \tau_t \beta_{s,t}}{\alpha (1 + \rho'_t) \sigma_{s,t}^2} = 0.012, \quad (43)$$

$$\frac{\gamma_b m_b^{bc} \beta_{b,t}}{\alpha (1 + \rho'_t) \sigma_{b,t}^2} = 0.092, \quad (44)$$

$$\frac{\gamma_b m_b^{sf} \tau_t}{\alpha (1 + \rho'_t) \sigma_{s,t}^2} = 0.008, \quad (45)$$

$$\frac{\gamma_b m_b^{bf}}{\alpha (1 + \rho'_t) \sigma_{b,t}^2} = 0.027, \quad (46)$$

$$m_s^{sc} + m_s^{sf} + m_s^{bc} + m_s^{bf} = 1, \quad (47)$$

$$m_b^{bc} + m_b^{bf} + m_b^{sc} + m_b^{sf} = 1. \quad (48)$$

By combining (40)&(44), (42)&(46), (39)&(43), (41)&(45), we can eliminate $\beta_{i,t}$, $\alpha(1 + \rho'_t)$, $\sigma_{i,t}^2$ and therefore get the following system of equations

$$\begin{aligned}
0.092\gamma_s m_s^{bc} - 0.228\gamma_b m_b^{bc} \tau_t &= 0, \\
0.027\gamma_s m_s^{bf} - 0.085\gamma_b m_b^{bf} \tau_t &= 0, \\
0.012\gamma_s m_s^{sc} - 0.299\gamma_b m_b^{sc} \tau_t &= 0, \\
0.008\gamma_s m_s^{sf} - 0.001\gamma_b m_b^{sf} \tau_t &= 0, \\
m_s^{sc} + m_s^{sf} + m_s^{bc} + m_s^{bf} &= 1, \\
m_b^{bc} + m_b^{bf} + m_b^{sc} + m_b^{sf} &= 1,
\end{aligned}$$

which can be written in matrix form as

$$\begin{pmatrix}
0 & 0 & 0.092\gamma_s & 0 & -0.228\gamma_b \tau_t & 0 & 0 & 0 \\
0 & 0 & 0 & 0.027\gamma_s & 0 & -0.085\gamma_b \tau_t & 0 & 0 \\
0.012\gamma_s & 0 & 0 & 0 & 0 & 0 & -0.299\gamma_b \tau_t & 0 \\
0 & 0.008\gamma_s & 0 & 0 & 0 & 0 & 0 & -0.001\gamma_b \tau_t \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1
\end{pmatrix}
\begin{pmatrix}
m_s^{sc} \\
m_s^{sf} \\
m_s^{bc} \\
m_s^{bf} \\
m_b^{bc} \\
m_b^{bf} \\
m_b^{sc} \\
m_b^{sf}
\end{pmatrix}
=
\begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
1 \\
1 \\
1 \\
1
\end{pmatrix}.$$

Even if for a simpler case where γ_s , γ_b and τ_t are known, we are not able to solve the linear system because there are eight parameters with only six equations. Therefore, the model is under-identified, which makes it impossible to identify each individual parameter separately based on the estimation results and the model constraints.

C Estimation Results with One Type of Agents Only

In this appendix, we estimate alternative models with one type of agents only by imposing restrictions on the coefficients of Eq. (29). The model with fundamentalists only corresponds to the coefficient matrix A being null while the model with chartists only corresponds to the coefficient matrix B being null. Similarly, we estimate the restricted models under threshold and Markov switching frameworks.

Table 7 presents the estimation results with fundamentalists only. In the threshold switching model, the overall estimation significance level for coefficient matrix B is similar to that in the full specification model. However, the estimates for B are less significant in the Markov switching model compared to that in the full specification model. The threshold switching results show that $B_{21} > 0$ in the low volatility state while $B_{21} < 0$ in the high volatility state, implying $\tau_t > 0$ in the low volatility state while $\tau_t < 0$ in the high volatility state. These predications are consistent with the flight-to-quality phenomenon during high-volatility market state. However, $B_{12} < 0$ in the low volatility state while $B_{12} > 0$ in the high volatility state (in both the threshold and Markov switching models) suggest $\tau_t < 0$ in the low volatility state while $\tau_t > 0$ in the high volatility state, giving rise to contractions. The contradictory prediction on the sign of $\tau_t > 0$ implies such a model may have missed some key elements. The model fitness measure statistics (the log-likelihood, AIC and BIC) suggest the restricted model with fundamentalists does not fit the real data as well as the full model.

Table 8 presents the estimation results with chartists only. In both the threshold and Markov switching results, the overall estimation significance level for coefficient matrix A is similar to that in the full model. The threshold switching estimates for A_{11} and A_{21} in two market states imply $\tau_t > 0$ in the low volatility state while $\tau_t < 0$ in the high volatility state. However, estimates for A_{12} and A_{22} in the threshold switching model suggest $\tau_t < 0$ in both market states. These two sets of predications for τ_t therefore are inconsistent with each other. As for the Markov switching model, it gives consistent predictions for τ_t , but the overall fitness of the model is not as good as the full model in terms of the log-likelihood, AIC and BIC statistics.

Table 7: Estimation Results with Fundamentalists Only

Coefficient	Threshold Switching		Markov Switching	
	Low (1)	High (2)	Low (3)	High (4)
<i>Panel A: Dependent Variable is $r_{s,t}$</i>				
B_{11}	0.013* (0.089)	0.005 (0.733)	0.006 (0.372)	0.018 (0.338)
B_{12}	-0.231*** (0.002)	0.125 (0.589)	-0.141 (0.112)	0.138 (0.695)
<i>Panel B: Dependent Variable is $r_{b,t}$</i>				
B_{21}	0.011*** (0.002)	-0.001 (0.835)	0.007* (0.091)	0.004 (0.544)
B_{22}	0.082 (0.323)	0.235 (0.373)	0.019 (0.655)	0.123 (0.221)
σ_s	–	–	2.326*** (0.000)	5.381*** (0.000)
σ_b	–	–	1.282*** (0.000)	1.997*** (0.000)
$cov_{s,b}$	–	–	0.025 (0.920)	-3.805*** (0.003)
$P_{1,1}$	–	–		0.963*** (0.000)
$P_{1,2}$	–	–		0.078* (0.063)
N	191	103	210	84
log-likelihood	-1397.544		-1299.110	
AIC	10.306		8.946	
BIC	10.409		9.547	

Notes: Low (High) refers to low (high) volatility state. The threshold switching model is specified by Eq. (30) with matrices A^1 and A^2 set to null and the Markov switching model is specified by Eqs. (29) and (31) – (33) with matrix A set to null. *, ** and *** denote significance at 10%, 5% and 1% level, respectively. Numbers in the parentheses are p -values.

Table 8: Estimation Results with Chartists Only

Coefficient	Threshold Switching		Markov Switching	
	Low (1)	High (2)	Low (3)	High (4)
<i>Panel A: Dependent Variable is $r_{s,t}$</i>				
A_{11}	0.216*** (0.003)	0.270*** (0.007)	0.299*** (0.000)	0.197 (0.258)
A_{12}	0.369*** (0.007)	-0.069 (0.795)	0.308** (0.021)	-0.123 (0.728)
<i>Panel B: Dependent Variable is $r_{b,t}$</i>				
A_{21}	0.065* (0.079)	-0.117*** (0.002)	0.025 (0.511)	-0.129** (0.001)
A_{22}	-0.069 (0.314)	0.148 (0.145)	0.117 (0.139)	0.067 (0.566)
σ_s	—	—	2.236*** (0.000)	5.429*** (0.000)
σ_b	—	—	1.351*** (0.000)	1.836*** (0.000)
$cov_{s,b}$	—	—	0.148 (0.552)	-3.744** (0.020)
$P_{1,1}$	—	—	—	0.937*** (0.000)
$P_{1,2}$	—	—	—	0.155** (0.040)
N	191	103	206	88
log-likelihood	-1389.405		-1289.287	
AIC	10.140		8.880	
BIC	10.243		9.380	

Notes: Low (High) refers to low (high) volatility state. The threshold switching model is specified by Eq. (30) with matrices B^1 and B^2 set to null and the Markov switching model is specified by Eqs. (29) and (31) – (33) with matrix B set to null. *, ** and *** denote significance at 10%, 5% and 1% level, respectively. Numbers in the parentheses are p -values.