Assessing portfolio market risk in the BRICS economies: use of multivariate GARCH models

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Abstract

This paper compares the performance of the different models used to estimate portfolio value-at-risk (VaR) in the BRICS economies. Portfolio VaR is estimated with three different multivariate risk models, namely the constant conditional correlation (CCC), the dynamic conditional correlation (DCC) and asymmetric DCC (ADCC) GARCH models. Risk performance measures such as the average deviations, quadratic probability function score and the root mean square error are used to back-test the performance of the models at 90%. The results indicate that portfolios with more weight to currency and less to equities prove to be the best way of minimizing loses in BRICS.

Keywords: portfolio value-at-risk, multivariate GARCH, risk performance measures, BRICS

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1. Introduction

Over the years the economies of the BRICS grouping have grown tremendously. The group has been predicted by a number of economists to overtake the US and EU by 2050 in term of real GDP (O'Neill, 2001). In the previous decade, returns from the BRICs equities grew by more than four times in the Standard and Poor's Index and the average economic growth in these countries was as much as four times higher than the US's (Patterson and Chen, 2011). This reality has made the BRICS countries an attractive investment destination for asset managers and investors in search of high yield and opportunities for portfolio diversification. In spite of being an attractive investment destination, the volatile BRICS environment is associated with high risk and as a result investors are always cautious about such risk and the consequences thereof. Thus, risk management is an important requirement for investors who are willing to invest in emerging markets in general and in the BRICS countries in particular.

Value at risk (VaR) is used by investors to measure and control the level of risk that they undertake. It is the responsibility of investors to ensure that the risks undertaken are not beyond the level at which they can absorb the losses of a probable worst outcome (Bonga-Bonga and Mutema, 2009). VaR attempts to measure how much can be a lost on an investment over a target horizon within a given confidence interval.

The estimation and the valuation of VaR as well as the possibility of reducing risk when investingin the BRICS countries should be one of the priorities and concerns of investors and asset managers. Jorion (1996) suggests that VaR is an important method for controlling institutional investor's risk exposures and as such, any investor considering investing in the BRICS or any other country or region for that matter should consider determining the level of VaR for any investment exposure. In addition, an investor that intends to invest in different asset classes needs to determine the level of weights for each asset that minimizes the portfolio VaR.

There are three methods of quantifying VaR, namely historical simulation, Monte Carlo simulation and the variance-covariance method (see Cabedo and Moya, 2003; Glasserman et al., 2000; Berkowitz and O'Brien, 2002; Bonga-Bonga and Mutema, 2009. Hendricks (1996) suggests that the best method to apply when estimating VaR depends on the task at hand. This suggestion implies that a study that focuses on assessing the best volatility model to be used for VaR estimation should naturally rely on the variance-covariance method. Thus, this paper chooses to estimate the VaR for BRICS countries by making use the variance-covariance method based on the different families of multivariate GARCH models.

A substantial number of studies have concentrated on market risk modelling using multivariate GARCH models (Lee, Chiou & Lin (2006), Hsu Ku & Wang (2008), Santo et al. (2013) and Nyssanov & Agren (2013), among many others) but few of these studies, to the best of our knowledge, has applied this technique on emerging markets in general and on BRICS data, in particular. Moreover none of these studies have analyzed the effects of different portfolio weights on the VaR for BRICS economies. The high volatile nature of emerging markets data raises a particular interest for VaR estimation based on GARCH models and for portfolio selection. As such our paper is the first to estimate VaR by using multivariate GARCH models and accounting for the effect of different portfolio weights on the VaR within BRICS economies. Thus, this paper compares the performance of three multivariate GARCH risk models, the DCC, ADCC and CCC, in estimating portfolio VaR for each of the five BRICS countries (Brazil, Russia, India, China and South Africa). In addition, this paper investigates the effect of changing portfolio weights on our VaR estimation. We construct three different portfolios for each country and each portfolio is made up of two assets: equities and currencies. The first portfolio considers equal weighting between currency and equity, the second portfolio gives more weight to equities (80%) and less weight to currencies (20%) and the third portfolio provides less weight to equities (20%) and more weight to currencies (80%). Although the weights assigned were provided arbitrarily, nonetheless they provide information as to how different weights of the two assets within a portfolio that is constituted of equity and currency will affect the performance of the VaR measure. The performance of these models is compared with the aid of a back-testing process by making use of the quadratic probability score (QPS) function, the root mean square error (RMSE), the number of exceptions / prediction failures and average deviations (AD) between the VaR and the realized return series as previously employed by Hsu Ku and Wang (2008) and Aniunas, Nedzveckas and Krusinskas (2015). As stated earlier, no study has ever attempted to estimate the VaR of a portfolio that is constituted of equity and currency in order to uncover the optimal weight of the two assets that minimizes the portfolio risk.

It is important to note that a portfolio that combines equity and currency not only has the ability to minimize the risk (exchange rate risk) of investing in an emerging market, but this combination of assets also provides investors with some safety to conserve the real value of their investment in the equity market. The findings of this paper will be beneficial for asset managers and investors that seek to hedge their equity exposure in the BRICS markets.

The rest of the paper is structured as follows: section 2 presents a review of literature of selected studies that focus on the estimation of VaR. Section 3 chapter explains how value-at-risk is estimated based on the variance-covariance method, with a focus on the different multivariate volatility models used in the paper. Section 4 presents the data used in the paper, the estimation of VaR for the different BRICS countries and discussion of the results obtained. Section 5 concludes the paper.

2. Literature review

Accurate estimation of covariance matrices and correlations between assets is essential for optimal portfolio construction, asset allocation and risk management, and therefore numerous studies have been devoted to obtaining reliable correlation estimates. The dynamic nature of correlations between assets has been the motivation for the use of number of multivariate models.

Multivariate GARCH models have received a lot attention recently and new models have been proposed. Bollerslev (1990) proposed a constant conditional correlations model with time-varying conditional variances and covariances. Recent studies such as that of Engle (2002) have shown that the assumption of constant correlations between financial assets is too limiting and not realistic in practice and as such new correlation models that take into account time-varying correlations have been proposed. For example, Engle (2002) proposed the DCC, which has the flexibility of univariate GARCH models and can be estimated very simply. According to this method, correlations are estimated in two steps and the fact that the number of parameters to be estimated does not depend on the number of the series to be correlated is one of the DCC's computational advantages over other multivariate GARCH models.

As already shown by studies such as that of Engle (2002), correlations between financial assets are not constant, as is usually assumed. Silvennoinen and Teräsvirta (2005) used daily returns of Standard and Poor's 500 (S&P 500) futures index and a 10-year bond futures index to investigate the relationship between stocks and bonds. The BEKK, GOF, DCC, DSTCC and SPCC GARCH models were used to estimate conditional correlations. The authors found that correlations vary most of the time. Furthermore, Hsu Ku (2008) used the DCC-GARCH-t and the CCC-GARCH-t models for the computation of correlation coefficients among major equity and currency markets in the US, Japan and the UK, and all correlation coefficients were found to be time varying.

Generally, there are very few studies on market risk modelling that have used multivariate GARCH models as compared to univariate models. For example, Lee, Chiou and Lin (2006) made use of the DCC-GARCH, simple moving average (SMA) and exponentially weighted moving averages (EWMA) models to estimate the portfolio VaR of the G7 countries (US, UK, Japan, Germany, France, Canada and Italy). The Kupiec proportion of failure test and the RMSE were applied to measure the accuracy and efficiency of the models. The authors found that the DCC-GARCH (1, 1)-t outperformed all the other models in measuring VaR followed by the DCC-GARCH (1, 1), then lastly the SMA.

Different methods for testing the performance of VaR have been used. For example, Hsu Ku and Wang (2008) compared the performance of the different GARCH models in forecasting the VaR of the usd/gbp, usd/jpy and the usd/eur exchange rates. The authors used two tests, namely the number of prediction failures and the average deviation between VaR and the realized returns, to back-test the VaR. They evaluated the performance of the DCC, BEKK and the CCC. The authors found that the BEKK outperformed the other models according to average deviations and the DCC was best according to the number of failures, but they found that the number of failures criterion reveals stronger ranking and as a result the DCC performed better.

Nyssanov and Agren (2013) evaluated the performance of GARCH models and classical approaches and compared these models in one-step-ahead forecasts of VaR. The authors made use of four tests, namely the violation ratio, Kupiec test, Christoffersen's test and joint tests for the evaluation of the methods. The asset returns of the seven largest copper companies, namely Codelco, Freeport McMoRan, BHP Billiton, Xstrata, Anglo American Pic, Rio Tinto and Kazakhmys, were used in the estimation of 99% and 95% VaR estimates. Four portfolios were constructed for the calculation of VaR values. The historical simulation, unconditional parametric, RiskMetrics, DCC-GARCH and GO-GARCH VaR estimation methods were employed. 99% VaR forecasts showed that the historical simulation method gives better results, while 95% VaR forecasts on the other hand showed that the DCC- and GO-GARCH VaR-based models outperformed the other models.

Very few studies have incorporated different portfolio weights in the estimation of portfolio VaR. Rombouts and Verbeek (2009) compared parametric (normal and student-t distributions) and semi-parametric distribution of innovation in the estimation of VaR of a portfolio with arbitrary weights. Three MGARCH models, the diagonal VEC, the DCC of Tse and Tsui (2002) and the DCC of Engle (2002) were used to estimate VaR of a portfolio made up of the Standard and Poor's 500 (S&P 500) and NASDAQ indexes. Unlike our study, the authors did not assess the effect of using different weights on the performance of the different VaR models. The Kupiec likelihood ratio test was used to compare the different methods. The authors found that the semi-parametric distribution improves VaR estimates when compared to the normal and t distributions.

A few studies have made comparisons of multivariate GARCH and univariate GARCH models in the estimation of portfolio VaR. In addition, the distribution of the GARCH model also matters when evaluating which model fits the data well. Morimoto and Kawasaki (2008) generated a regular time series from irregularly spaced data to evaluate intraday value at risk by comparing the forecasting performance of five univariate models and five multivariate GARCH models. The univariate models used in the study include the normal, normal GARCH, student, student GARCH and the RiskMetrics and the multivariate GARCH models include the VECH, BEKK, diagonal, CCC and the DCC. As in the Ku and Wang (2008) study, the DCC was found to be the best forecasting model.

Santo, Nogales and Ruiz (2013) also conducted a study to compare the performance of multivariate GARCH models with univariate models. Three multivariate GARCH models were used in the forecasting of VaR: these included DCC-GARCH, CCC-GARCH and Asymmetric DCC-GARCH. Three real market portfolios of daily returns were used: the first portfolio was made up of returns of 48 US industry portfolios, the second was composed of returns of 25 portfolios of stocks formed on the basis of size and book-to-market and the third portfolio was made up of returns of all stocks of the S&P 100 index. Models were compared by making use of back-testing and the CPA test. Results showed that the DCC-GARCH-t is the most appropriate specification when used in the estimation of portfolio VaR. Multivariate student-t models, except for the CCC, gave the lowest number of violations as compared to the normal distribution models. The DCC and asymmetric DCC GARCH models outperformed the CCC, thus proving that conditional correlations are dynamic rather than constant.

While there seems to be a consensus on the preeminence of the DCC-GARCH model over other conditional correlation GARCH models in estimating VaR, this should not be seen as a stylized fact and should be left as a matter of empirical analysis. Thus, this paper will add to the literature on portfolio market risk estimation by comparing a family of conditional correlation GARCH models, the CCC, DCC and ADCC-GARCH in estimating the VaR.

3. Methodology

3.1 Value-at-risk methods: The variance-covariance method

Value-at-risk (VaR) is a measure of potential loss in value of a risky asset or portfolio over a defined period for a given confidence level. From equation 4.1, given that *c* is the confidence level and L is the loss, Jorion (2007) defines the VaR as the smallest loss in absolute value such that:

$$P(L > VaR) \leq 1 - c \qquad (4.1)$$

As already mentioned, there are three methods of quantifying VaR, namely the historical simulation, Monte Carlo simulation and the variance-covariance method. Our study employs the variance-covariance method from different portfolios made up of equity and foreign exchange assets and constructed with different weights of each asset.

According to Jorion (2007), the portfolio rate of return is given by:

$$R_{p,t+1} = \sum_{i=1}^{N} w_{i,t} R_{i,t+1} \qquad (4.2)$$

Where $w_{i,t}$ is the portfolio weight. The portfolio variance is given by:

$$\sigma^2(R_{p,t+1}) = w_t' \sum_{t+1} w_t \qquad (4.3)$$

Where Σ_{t+1} is the forecast of the covariance matrix.

We use the 90% confidence level for all our VaR calculations. Based on the conditional volatility (covariance matrix) obtained from the three MGARCH models, the portfolio VaR is then given by:

$$VaR = E(R) - \gamma\sqrt{w_t' \Sigma_{t+1} w_t}$$

Where γ corresponds to a parametric distribution used in the study, either the normal distribution or the student-t distribution and $E(R)$ is the expected return of a portfolio. It is important to that E(R) is often approximated to zero. Thus, equation 4.4 becomes

$$VaR = -\gamma\sqrt{w_t' \Sigma_{t+1} w_t} \qquad (4.4)$$

As stated earlier, one of the aims of this study is to find the most appropriate portfolio weights and the multivariate volatility model that will minimize the estimated VaR.

3.2 Multivariate volatility models

Tsay (2010) states that in order to understand the dynamic structure of the global finance, financial markets must be considered to be related, as they are dependent on each other. Hsu Ku (2008) adds that the level of interaction among major financial markets has increased, which leads to transmission effects, and these effects should not be overlooked in portfolio construction. Thus, it is essential to account for asset interdependence when estimating VaR using covariance-variance method. It is in this context that this study makes use of the multivariate DCC GARCH model to account for time-varying correlation among assets in a given portfolio. Taking into account time-varying correlations is useful in finance as evidence shows that correlation coefficients change over time in real applications (Engle, 2002).

Our study focuses on the calculation of portfolio VaR; therefore we will employ simple methods for modelling the dynamic relationship between volatility processes of multiple asset returns. Thus we model the conditional covariance matrix of multiple asset returns, which is essential for the computation of value-at-risk of a position made up of multiple returns to take into account co-movements in financial returns.

3.2.1 Multivariate GARCH models

If we let $\{\boldsymbol{r_t}\}$ be a multivariate return series, we can rewrite it as:

$$r_t = \mu_t + \varepsilon_t \qquad (4.5)$$

Where $\mu_t = E(r_t|F_{t-1})$, is the conditional expectation of r_t given the past information F_{t-1} and

$\varepsilon_t = (\ \varepsilon_{1t}, \dots, \varepsilon_{kt})'$, is the shock of the series at time t and is given by:

$$\varepsilon_t = H_t^{1/2} z_t \qquad (4.6)$$

Such that $Cov(\varepsilon_t|F_{t-1}) = Cov(r_t|F_{t-1}) = H_t$

Where H_t is a N x N positive definite conditional covariance matrix of portfolio returns and z_t is a N x 1 independently and identically distributed random vector with mean zero and identity covariance matrix;

$$z_t \sim (0, I_N)$$

Where I_N is the identity matrix of order N.

Different specifications of H_t related to the classes of multivariate conditional correlation GARCH models, namely the constant conditional correlation (CCC) GARCH model of Bollerslev (1990), the dynamic conditional correlation (DCC) GARCH model of Engle (2002) and asymmetric dynamic conditional correlation (ADCC) GARCH model of Cappiello (2006), will be reviewed in the following subsections.

3.2.2 The constant conditional correlation (CCC) GARCH model

Bollerslev (1990) proposed the CCC GARCH model with constant conditional correlations. The CCC GARCH model can be estimated in two steps: firstly univariate GARCH models are employed to estimate the volatility of each series, and in the second step, standardized residuals from the first step are employed to construct the conditional correlation matrix. The CCC GARCH is defined as:

$$H_t = D_t R D_t \qquad (4.7)$$

Where $D_t = \begin{bmatrix} \sqrt{h_{11,t}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sqrt{h_{kk,t}} \end{bmatrix}$ and R $= \begin{bmatrix} 1 & \cdots & \rho_{1k} \\ \vdots & \ddots & \vdots \\ \rho_{1k} & \cdots & 1 \end{bmatrix}$ (4.8)

and the variance equation for the CCC is given by:

$$h_{i,t} = C_{ii} + a_{ii}\varepsilon_{i,t-1}^2 + b_{ii}h_{i,t}, \quad h_{ij,t} = \rho_{ij}\sqrt{h_{i,t}}\sqrt{h_{j,t}} \qquad (4.9)$$

Where ρ_{ij} is the constant unconditional correlation and $i = 1,2$ with 1 representing the foreign exchange market and 2 the equity market.

In other words, where $D_t = diag\,(h_{t1}^{1/2} \ldots h_{tN}^{1/2})$ with diag (.) being the operator that transforms a Nx1 vector into a NxN diagonal matrix and h_{tj} follows any univariate GARCH model. R is a symmetric positive definite conditional correlation matrix with elements $\rho_{ij,t}$ where $\rho_{ii,t} = 1$ and contains constant conditional correlations ρ_{ij}.

3.2.3 The dynamic conditional correlation (DCC) model

Given the limits of the CCC GARCH models, Engle (2002) proposed the dynamic conditional correlation (DCC) model, which has the flexibility of univariate GARCH but not the complexity of other MGARCH models with the advantage that the number of parameters to be estimated in the correlation does not depend on the number of series to be correlated. The DCC came as an extension/generalization of the constant conditional correlation (CCC) model of Bollerslev (1990), and it assumes that the conditional correlation matrix is time-dependent. The DCC GARCH model is defined as:

$$H_t = D_t R_t D_t \qquad (4.10)$$

Where D_t is defined as equation 4.8 above and

$$R_t = diag\left(Q_t^{-1/2}\right) Q_t diag\left(Q_t^{-1/2}\right)$$

$$Q_t = \left(q_{ij,t}\right)$$

$$\left(diag(Q_t)\right)^{-1/2} = \left(diag\frac{1}{\sqrt{q_{11,t}}}, \ldots, \frac{1}{\sqrt{q_{nn,t}}}\right) \qquad (4.11)$$

Where Q_t is the unconditional covariance of standardized residuals from the univariate GARCH models and diag Q_t is a diagonal matrix that contains diagonal elements of an N x N positive definite matrix Q_t. The elements of Q_t are given by:

$Q_t = (1 - \alpha - \beta)\bar{Q} + \alpha\epsilon_{t-1}\epsilon'_{t-1} + \beta Q_{t-1}$ and can be reduced to:

$$q_{ij,t} = \bar{\rho}_{ij} + \alpha\left(\epsilon_{t-1}\epsilon'_{t-1} - \bar{\rho}_{ij}\right) + \beta\left(q_{ij,t-1} - \bar{\rho}_{ij}\right) \quad (4.12)$$

Where ϵ_{it} is the standardized innovation vector with elements $\epsilon_{it} = {\varepsilon_{it}}/{\sqrt{\sigma_{ii,t}}}$, $\bar{Q}$ is the N x N unconditional covariance matrix of ϵ_t and α and β are non-negative scalar parameters that ensure that $\alpha + \beta < 1$.

The variance equation for the DCC model is then given by:

$$h_{1,t} = C_{11} + a_{11}\varepsilon^2_{1,t-1} + b_{11}h_{1,t-1} \qquad h_{2,t} = C_{22} + a_{22}\varepsilon^2_{1,t-1} + b_{22}h_{1,t-1}$$

$$q_{ij,t} = \bar{\rho}_{ij} + \alpha\left(\epsilon_{t-1}\epsilon'_{t-1} - \bar{\rho}_{ij}\right) + \beta\left(q_{ij,t-1} - \bar{\rho}_{ij}\right) \qquad h_{ij,t} = \rho_{i,j,t}\sqrt{h_{i,t}}\sqrt{h_{j,t}}$$

(4.13)

3.2.4 The asymmetric dynamic conditional correlation (AsyDCC) model

This model is an extension of the DCC and was proposed by Cappiello et al. (2006). It takes into account asymmetry in conditional correlations, and in this model Q_t is given by:

$$Q_t = (\bar{Q} - \alpha\bar{Q} - \beta\bar{Q} - \delta\bar{\Gamma}) + \alpha\epsilon_{t-1}\epsilon'_{t-1} + \beta Q_{t-1} + \delta n_{t-1}n'_{t-1}$$

(4.14)

Where $n_t = I(\epsilon_t < 0) \odot \epsilon_t \, and \, \bar{\Gamma} = E[n_t n_t']$

It is assumed that for Q_t to be positive definite $\alpha + \beta + \lambda\delta < 1$ should hold where λ is the maximum eigenvalue of $\bar{Q}^{-1/2}\bar{N}\bar{Q}^{-1/2}$.

3.3 Portfolio construction

With regard to the construction of different portfolios, this study makes use of three different sets of arbitrarily chosen portfolio weights for each of the BRICS countries. For portfolio 1 (PF 1) we give equal weighting to equities and currencies (0.5, 0.5). More weight is assigned to equities and less to currencies (0.8, 0.2) for the construction of PF 2 and for the construction for PF 3 we give less weight to equities and more to currencies (0.2, 0.8). Through this arbitrary weight allocation, the study aimed at assessing which of the portfolio is less risky given the composition of assets (equity and foreign exchange) within each of the BRICS countries.

3.4 Evaluation methods

A substantial number of studies have used the Kupiec (1995) test and the Christofferson (1998) test to back-test VaR models, but these tests are suitable only when one wants to evaluate the performance of an individual model. To compare the VaR forecasting performance of the different multivariate GARCH models and to back-test these models, this study makes use of efficacious ranking methods such as the Quadratic Probability Score function (QPS), the root mean square error (RMSE) and average deviations (AD).

3.4.1 The quadratic probability score function (QPS)

According to Lopez (1997), the quadratic probability score function is expressed as:

$$QPS = \frac{2}{n}\sum_{t=1}^{n}(C_t - p)^2 \qquad (4.15)$$

Where n is the number of trading days in the testing period, p is the expected probability of exceptions, C_t is a predetermined binary loss function reflecting the interest of users and L_t is denoted as the actual losses. Thus C_t is an indicator function that equals one if the specified event happens and zero otherwise. C_t is given by:

$$C_t = \begin{cases} 1 & L_t > VaR_t \\ 0 & L_t \leq VaR \end{cases} \quad (4.16)$$

The QPS ranges between zero and two, and according to Lopez (1997) the best-performing VaR model produces the lowest score.

3.4.2 The root mean squared error (RMSE)

The RMSE is expressed as:

$$RMSE = \sqrt{E[(VaR_t - L_t)^2]} = \sqrt{\frac{1}{n}\sum(VaR_t - L_t)^2} \quad (4.17)$$

This measure is applied only during non-violation days – thus when actual losses are less than or equal to the VaR and the smallest RMSE is preferred.

Finally we use the number of exceptions/prediction failures, which is the number of times the actual returns are less than the estimated VaR and average deviations (AD). Average deviation is the average absolute difference between the VaR and the realized return series and is given by:

$$AD = \frac{1}{m}\sum_{t=1}^{m}(|VaR_t| - |r_{t|})^{+} \quad (4.18)$$

Where m is the number of days in the testing period, r_t is the realized return series and the superscript (+) denotes that the AD computation considers only situations where $|VaR_t| \geq |r_t|$ and sound risk management requires lower levels of AD (Hsu Ku & Wang, 2008).

4. Data and estimation of results

4.1 Data

Data description

Given the objective of the study, which consists of estimating and evaluating the performance of the VaR of the different portfolios constructed by combining positions in the foreign exchange and equity market of the different BRICS countries, we make use of daily data for the foreign exchange (currency) and equity markets from Brazil, India, China and South Africa (weekly data is used for Russia because of the unavailability of daily data). The equity market indexes from the five countries used in the study are: the Brazilian Ibovespa, Brasil Sao Paulo Stock Exchange Index (IBOV), Russian MICEX index, Indian S&P BSE SENSEX Index (SENSEX), Chinese Shanghai Stock Exchange Composite Index (SHCOMP) and the South African Johannesburg All Share Index (ALSI). In addition, the Brazilian real/USD (BRL), Russian ruble/USD (RUB), Indian rupee/USD (INR), renminbi/USD (CYN) and the rand/USD (Zar) exchange rates were employed in the study. The dataset covers the periods from 4 January 2005 to 10 September 2014 for the four countries and the periods from 5 July 1998 to 28 December 2014 for Russia.[1] The data was sourced from I-net Bridge. It is important to note that for both daily data and weekly data the last 252 observations are used for VaR estimation and the back-testing exercise.

The table below shows the different portfolios constructed for each country.

[Insert Table 1]

According to Table 1 above, to construct portfolio 1 (PF 1) we give equal weighting to both equities and currencies (0.5, 0.5). More weight is assigned to equities and less to currencies (0.8, 0.2) for the construction of PF 2. For the construction for PF 3 we give less weight to equities and more to currencies (0.2, 0.8). Figure 1 below depicts the log returns series for all the assets used in the study, and from the figure it is evident that all the series depict volatility clustering and heteroscedasticity. Increased volatility is observed in all the foreign exchange and equity markets, especially from the end of 2007 to 2010, and this occurrence is ascribed to the panic in the markets caused by the global financial crisis. The Indian Sensex had the largest jump in volatility during this crisis period, while on the other hand both the Chinese SHCOMP and the cyn seemed to be the least affected by the crisis.

[1] A long weekly sample aimed at increasing the number of observations.

Table 2 presents the descriptive statistics of BRICS foreign exchange and equity markets daily log returns. The table shows that micex has the highest return and the highest risk with a standard deviation of 6.860, while the cyn had the lowest return and the lowest risk, as shown by a standard deviation of 0.122. In addition, the table shows that the average daily return for both brl and the cyn are negative, while the rest of the returns had positive average daily returns. Furthermore, it is evident from the results reported in Table 2 that all equity returns are more volatile than foreign exchange returns. Lastly, all the series are fat-tailed with kurtosis of greater than 3.

4.2 VaR estimation

In order to estimate the variance-covariance VaR for the four BRICS countries we make use of the CCC, DCC and ADCC GARCH models for volatility model. The following steps were taken:

1. Portfolio weights were chosen arbitrarily (shown in Table 1 above) with the aim of assessing how different weights of the two assets in a portfolio will affect the performance of the VaR measure.
2. Then we estimated volatility models, namely the CCC, DCC and the ADCC.
3. Lastly, using portfolio weights from step 1 and the standard deviations from step 2, the VaR was estimated as per equation 4.4.

[Insert Table 2]

[Insert Figure 1]

VaR estimates for different BRICS countries

We follow the steps described above to estimate the VaR of each of the portfolios in specific BRICS countries. We make use of the last 252 observations to forecast the one-day 90% VaR of these portfolios. Figures A1 to A15 in the appendix display the estimated VaR obtained from the different GARCH models, namely the CCC-, DCC- and ADCC-GARCH models,[2] against the returns of each of the portfolios. For example, Figure A1 displays the VaR obtained from the CCC-GARCH models with normal (CCC normal) and student-t (CCC t) distributions against the returns of the different portfolios, namely PF1, PF2 and PF3, respectively. It is worth noting that the number of exceptions, i.e. the number of times the negative return or loss is greater in absolute value than the VaR estimates, are deduced for these figures. Tables 3 to 7 below summarise the

[2] The estimation of these volatility models can be provided on request.

number of exceptions obtained by comparing the VaR obtained from each of the GARCH models and the given portfolio returns for each BRICS country. For example, Table 3 shows that in South Africa the VaR for PF 3 has the least exceptions compared to the VaR of the rest of the portfolios. Moreover, the results reported in Table 3 show that in terms of the GARCH models used to estimate portfolio VaR, the DCC_t fares better than all the other models in SA. The CCC fares the worst in two of the three portfolios. Similarly, the results reported in Table 4 show the better performance of a dynamic conditional GARCH model in estimating the VaR in china. The results reported in Table 4 show that the ADCC_t model outperforms all the other models with zero exceptions, followed by the DCC_t. The CCC fares the worst in all three portfolios with the highest number of exceptions.

Table 5 shows that in India, PF 2 has the least exceptions and the DCC_norm, DCC_t and ADCC_norm models outperform all the other models with zero exceptions, followed by the ADCC_t. The CCC_t and CCC_norm perform the worst in two portfolios (1 and 2) with the highest number of exceptions. The results reported in Table 6 show that in Brazil PF 3 has the least exceptions and the DCC_norm fares better than all the other models. All the other models perform the same with the same number of exceptions. Table 7 shows that in Russia, PF 2 has the least exceptions across all models with zero exceptions. However, PF3 has the highest number of exceptions for Russia.

It is important to note that exception criteria cannot be considered as the only benchmarks for selecting the best VaR model or the best portfolio, in the case of this paper. One needs to apply performance evaluation methods to establish which portfolio provides the least VaR (Hsu Ku and Wang, 2008).

[Insert Table 3 to Table 7]

4.3 Performance evaluation of each VaR model

In this section we employ the AD, the QPS function and the RMSE measures to evaluate the performance of the models with the aid of the above exceptions. Thus we employ the three methods to back-test our models as shown by equations 4.18, 4.15 and 4.17. Tables A1 to A5 in the appendix report the back-testing results according to the AD, the QPS and the RMSE for the five BRICS countries. When making use of the average deviations as a measure of accuracy of VaR models, low deviations are favourable as they represent close to perfect risk management. In addition, as already mentioned in the previous chapter, the QPS ranges between zero and two, and, according to Lopez (1999), the best-performing VaR model produces the lowest score.

Table A1 shows that in South Africa, the DCC norm VaR has the least deviations across all three portfolios. Moreover, PF 1 has the least average deviations among all portfolios. In terms of the QPS measure, the DCC_t VaR and ADCC_t VaR perform well in two out of the three portfolios and both PF2 and PF3 have the lowest QPS measure. According to the RMSE the DCC_norm VaR fares the best in comparison to the rest of the models as a GARCH model for VaR estimation, and PF1 has the lowest RMSE overall.

Table A2 shows a different outcome in China. The CCC_norm VaR outperforms all its counterparts across all three portfolios according to the AD and the RMSE, and PF 3 has the least deviations and RMSE when compared to the other portfolios. The ADCC_t VaR outperforms all its counterparts in terms of the QPS with the lowest QPS across all 3 portfolios. As shown in Table A3, the CCC_norm VaR outperforms all its counterparts across all three portfolios according to the AD. According to the same criteria, PF 3 has the least deviations when compared to the other portfolios in India. According to the QPS, both the DCC_norm VaR and DCC_t VaR and the ADCC_norm VaR outperform all the other dynamic correlation models, and PF2 has the least QPS. The ADCC_norm VaR fares the best in terms of the RMSE, and PF3 has the lowest RMSE.

Table A4 shows that according to the AD, the CCC_norm VaR and ADCC_t VaR outperformed the other models in Brazil. In addition, PF 3 had the least average deviations. The DCC_norm VaR outperforms all the other models according to the QPS, and PF2 has the least QPS while the CCC is the worst-performing model. In terms of the RMSE, the ADCC_t VaR outperforms all the other models, with PF3 performing better than the other two portfolios. Table A5 shows that according to average deviations and the RMSE, the DCC and ADCC_norm VaR outperformed its counterparts and PF 3 has the lowest AD and RMSE in Russia. In terms of the QPS, PF2 has the lowest QPS across all models and all models fared the same. Table 8 below gives a summary of the performance evaluation results.

Given that the aim of this paper is to assess the best GARCH models for VaR estimation and the best portfolio, in combining currency and equity indices, that minimizes loses in each of the BRICS countries, Table 8 provides a further treatment of the above reported results. The results reported in Table 8 show that, in SA in terms of AD and the RMSE, PF1 outperforms the other portfolios, while according to the QPS both PF 2 and 3 performed well. Nevertheless, when PF3 dominates the other portfolios, it dominates it by a higher amount than when the other portfolios dominate. For example in SA according to the AD, PF 1 dominates PF2 by 13.82%= (0.560-0.492)/0.492, while PF1 dominates PF3 by 25% = (0.615-0.492)/0.492. In terms of the QPS, PF2 dominates PF1 by 18.18%= (0.039-0.033)/0.033, while PF3 dominates PF1 by 36.36% = (0.045-0.033)/0.033. In terms of the RMSE, PF1 dominates PF2 by 14.13% = (1.171-1.026)/1.026, while PF1 dominates PF3 by 20.27% = (1.234-1.026)/1.026. Therefore where PF3 dominates, it dominates by a value (36.36%) more than when another portfolio is dominating. In addition, according to all four performance evaluation methods, the DCC_norm VaR outperforms all the other models according to the same ranking order. In China PF 3 fared well according to AD and the RMSE, yet according to the QPS all portfolios fared the same. Across all performance measures, PF3 dominates the other portfolios. In addition, the ADCC_t VaR performs better than the other models. In India, PF3 showed the best performance according to the AD and the RMSE. In addition, a different case is observed in India: all the models except for the CCC_t VaR perform well according to the different evaluation methods. The ADCC_norm and DCC_t perform better than the rest of the models. In Brazil PF3 outperforms its counterparts according to the AD and the RMSE, while PF2 fares well according to the QPS. PF3 dominates all the other portfolios. The DCC_norm outperforms the rest of the models in Brazil. In Russia PF2 fares well according to the QPS, while PF3 fared well according to the AD and RMSE, and according to our ranking order PF3 dominates the other portfolios. In addition, the DCC and ADCC_norm fare better than the other models.

Across all five countries, the DCC performs best, followed by the ADCC, while the CCC comes last. Thus most methods are in support of the dynamic correlation models (DCC and the ADCC) and thus models of dynamic correlation perform better than the CCC. This indicates that dynamic correlations between assets are essential for portfolio risk management in the BRICS. In addition, these results also indicate the importance of the use of models that account for asymmetries in both asset returns and correlations for appropriate VaR forecasts.

In terms of portfolio performance, of all three of our portfolios, PF3 (which gives more weight to foreign currency market (80%) and less weight to equities (20%)) performs better across all performance measures in all five BRICS countries. The portfolio dominates both PF1 and PF2 in all the BRICS countries. This suggests that giving more weight to the foreign exchange market and less to equities proves to be the best way of minimizing loses in BRICS when holding a portfolio made up of foreign exchanges and equities. This is probably due to the fact that each position in equity is often balanced by a position in the currency market that hedges against foreign exchange risk. In addition, the foreign exchange markets of emerging economies attract speculators, hedgers and arbitrageurs, so there are high investment potentials/opportunities, which do not necessarily require a counterpart investment in the equity market. Thus more weight is given to currency markets than equities. Furthermore, Bonga-Bonga and Hoveni (2013) found that the size of the equity and foreign exchange market is disproportionate: for instance, the daily average turnover of the foreign exchange market was estimated at US$9 billion in 2010, yet the average daily equity trading was estimated at US$2 billion, thus contributing to a higher participation in the foreign exchange market. Lastly, investment opportunities such as carry trade, where an investor borrows money at low interest rates, usually in developed economies, and invests in emerging markets where interest rates are high, for example, also leads to a higher participation in the foreign exchange market. Thus allocating more weight to forex and less to equities results in fewer exceptions, lower AD, QPS and RMSE than when more is allocated to equities and when the two assets are allocated equally.

[Insert Table 8]

5 Conclusion

The aim of this paper was to compare the performance of three multivariate GARCH models, the DCC, ADCC and CCC GARCH models, in estimating portfolio VaR for each of the five BRICS countries (Brazil, Russia, India, China and South Africa). In addition, different performance metrics for the evaluation of the estimated VaR were discussed. Three different portfolios made up of different combinations of equity index and foreign exchange (forex) assets were constructed for each BRICS country. The data used is drawn from stock market indices and foreign exchange market data from the five countries used in the paper. In order to assess the performance of the VaR estimation, this paper uses performance metrics such as the quadratic probability score (QPS) function, the root mean square error (RMSE), the number of exceptions / prediction failures and average deviations (AD). Both the normal and student t distributions were assumed for the innovations.

Our findings show that across all five countries, the DCC performed best, followed by the ADCC, while the CCC came last. Thus most methods are in support of the dynamic correlation models (DCC and the ADCC), and thus models of dynamic correlation perform better than the CCC. This indicates that dynamic correlations between assets are essential for portfolio risk management in the BRICS. In addition, these results also indicate the importance of the use of models that allow for asymmetries in both asset returns and correlations for appropriate VaR forecasts. In terms of portfolio weights, of all three of our portfolios, PF3 (which gives more weight to foreign exchanges (80%) and less weight to equities (20%)) showed a better performance across all four models in all five countries covered in our paper. Therefore giving more weight to forex and less to equities proves to be the best way of minimizing loses in BRICS when holding a portfolio made up of forex and equities. Our results are consistent with those of previous studies such as Morimoto and Kawasaki (2008), Hsu Ku and Wang (2008) and Santo, Nogales and Ruiz (2013) with regard to the best model, in that models of dynamic correlation like the DCC outperform the constant correlation model (CCC) in forecasting VaR.

The findings of this paper will provide investors looking into investing in BRICS' countries with a guideline on how to combine positions in the currency and equity markets in order to constitute a portfolio that minimizes loses. Investors need to make informed decisions with regard to portfolio selection and market risk measurement. However, we suggest that for further research, portfolios with more than two assets be considered, as they allow for better diversification. With the use of more than two assets in a portfolio, the mean variance method, copulas, the Black-Litterman method and other portfolio optimization methods can be used to obtain more reliable portfolio weights. In addition we suggest the estimation of VaR for a portfolio of BRICS countries combined together.

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Appendix

[Insert Table A1 to Table A5]

[Insert Figures A1 to A15]

Table 1: Portfolio construction

Country	Portfolio Assets	PF1 weights	PF2 weights	PF3 weights
Brazil	brl/usd, IBOV	0.5, 0.5	0.2, 0.8	0.8, 0.2
Russia	rub/usd, MICEX	0.5, 0.5	0.2, 0.8	0.8, 0.2
India	inr/usd, SENSEX	0.5, 0.5	0.2, 0.8	0.8, 0.2
China	cyn/usd, SHCOMP	0.5, 0.5	0.2, 0.8	0.8, 0.2
South Africa	zar/usd, ALSI	0.5, 0.5	0.2, 0.8	0.8, 0.2

Note: the numbers show the weight of currency and equity, respectively, in each portfolio.

Table 2: Descriptive statistics of log returns of currency and equity

Statistic	BRL	IBOVESPA	RUB	MICEX	INR	BSE	CYN	SHCOMP	ZAR	ALSI
Mean	-0.007	0.034	0.107	0.255	0.013	0.056	-0.012	0.025	0.024	0.055
Standard deviation	1.133	1.736	2.806	6.860	0.535	1.52	0.122	1.609	1.105	1.24
Minimum	-7.259	-12.096	-45.183	-28.768	-3.551	-11.604	-2.019	-9.278	-7.475	-7.581
Maximum	12.047	13.678	55.595	35.667	4.091	15.99	0.858	9.034	15.913	6.834
Kurtosis	12.469	6.624	10.378	69.201	6.305	8.704	36.879	4.162	20.288	4.301
Skewness	0.952	-0.039	-0.045	4.240	0.275	-0.008	-1.748	-0.342	1.373	-0.22

Table 3: Exceptions/Violations for South Africa

	PF 1	PF 2	PF 3
weights	0.5, 0.5	0.2, 0.8	0.8, 0.2
CCC_norm	4	9	4
CCC_t	4	8	4
DCC_norm	5	9	4
DCC_t	3	2	3
ADCC_norm	4	8	4
ADCC_t	4	6	2

Table 4: Exceptions/Violations for China

	PF 1	PF 2	PF 3
weights	0.5, 0.5	0.2, 0.8	0.8, 0.2
CCC_norm	11	10	7
CCC_ t	2	3	3
DCC_norm	1	1	2
DCC_t	1	1	1
ADCC_norm	1	1	2
ADCC_t	0	0	0

Table 5: Exceptions/Violations for India

	PF 1	PF 2	PF 3
weights	0.5, 0.5	0.2, 0.8	0.8, 0.2
CCC_norm	7	4	6
CCC_t	7	4	6
DCC_norm	1	0	6
DCC_t	1	0	4
ADCC_norm	1	0	8
ADCC_t	2	1	7

Table 6: Exceptions/Violations for Brazil

	PF 1	PF 2	PF 3
weights	0.5, 0.5	0.2, 0.8	0.8, 0.2
CCC_norm	10	9	7
CCC_t	10	9	7
DCC_norm	5	4	5
DCC_t	11	9	7
ADCC_norm	9	9	7
ADCC_t	9	8	7

Table 7: Exceptions for Russia

	PF 1	PF 2	PF 3
weights	0.5, 0.5	0.2, 0.8	0.8, 0.2
CCC_norm	7	0	52
CCC_t	6	0	53
DCC_norm	14	0	61
DCC_t	10	0	60
ADCC_norm	14	0	61
ADCC_t	2	0	46

Table 8: Summary of performance evaluation results for each model

	AD	QPS	RMSE	Best model according to relative ranking order	Best portfolio in terms of AD	Best portfolio in terms of QPS	Best portfolio in terms of the RMSE	Best portfolio according to relative ranking order
SA	DCC_norm	DCC_t & ADCC_t	DCC_norm	DCC_t	PF1	PF2 & PF 3	PF1	PF3
China	CCC_norm	ADCC_t	CCC_norm	ADCC_t	PF3	PF1, 2 & 3	PF3	PF3
India	CCC_norm	DCC_norm & t, ADCC_norm	ADCC_norm	ADCC_norm &DCC_t	PF3	PF2	PF3	PF3
Brazil	ADCC_t & CCC_norm	DCC_norm	DCC_norm	DCC_norm	PF3	PF2	PF3	PF3
Russia	DCC & ADCC_norm	N/A	DCC &ADCC_norm	ADCC_norm & DCC_norm	PF3	PF2	PF3	PF3

Note 1: this table reports the best volatility model according to specific performance evaluation criteria as well as the optimal portfolio according to the same criteria for each of the BRICS countries.

Note 2: N/A – none of the models outperformed its counterparts; i.e. all the models fared the same

Appendix

Table A1: South Africa back-testing results

South Africa	AD			QPS			RMSE		
VaR Model	**PF1**	**PF2**	**PF3**	**PF1**	**PF2**	**PF3**	**PF1**	**PF2**	**PF3**
CCC_norm	0.508	0.568	0.624	0.045	0.077	0.045	1.042	1.178	1.243
CCC_t	0.514	0.576	0.625	0.045	0.071	0.045	1.046	1.186	1.241
DCC_norm	0.492	0.560	0.615	0.052	0.077	0.045	1.026	1.171	1.234
DCC_t	0.613	0.782	0.683	0.039	0.033	0.039	1.137	1.374	1.293
ADCC_norm	0.505	0.574	0.628	0.046	0.071	0.046	1.039	1.191	1.244
ADCC_t	0.574	0.639	0.721	0.045	0.058	0.033	1.100	1.244	1.325

Table A2: China back-testing results

China	AD			QPS			RMSE		
VaR model	**PF1**	**PF2**	**PF3**	**PF1**	**PF2**	**PF3**	**PF1**	**PF2**	**PF3**
CCC_norm	0.499	0.802	0.214	0.090	0.084	0.065	0.919	1.462	0.407
CCC_t	0.677	1.080	0.296	0.033	0.039	0.039	1.088	1.728	0.489
DCC_norm	0.834	1.314	0.377	0.026	0.026	0.033	1.239	1.956	0.558
DCC_t	1.152	1.850	0.455	0.026	0.026	0.026	1.544	2.468	0.633
ADCC_norm	0.835	1.329	0.381	0.026	0.026	0.033	1.247	1.980	0.564
ADCC_t	1.360	2.175	0.566	0.020	0.020	0.020	1.743	2.780	0.746

Table A3: India back-testing results

India	AD			QPS			RMSE		
VaR Model	PF1	PF2	PF3	PF1	PF2	PF3	PF1	PF2	PF3
CCC_norm	0.552	0.766	0.491	0.065	0.045	0.058	1.074	1.488	0.959
CCC_t	0.573	0.802	0.500	0.065	0.045	0.058	1.094	1.522	0.971
DCC_norm	1.111	1.763	0.593	0.026	0.020	0.058	1.595	2.430	1.018
DCC_t	1.179	1.816	0.727	0.026	0.020	0.045	1.661	2.484	1.149
ADCC_norm	0.920	1.460	0.498	0.026	0.020	0.071	1.412	2.138	0.930
ADCC_t	0.845	1.297	0.557	0.033	0.026	0.065	1.364	2.020	0.985

Table A4: Brazil back-testing results

Brazil	AD			QPS			RMSE		
VaR Model	PF1	PF2	PF3	PF1	PF2	PF3	PF1	PF2	PF3
CCC_norm	0.757	1.002	0.683	0.084	0.077	0.065	1.433	1.964	1.330
CCC_t	0.755	1.002	0.700	0.084	0.077	0.065	1.431	1.964	1.326
DCC_norm	0.985	1.321	0.942	0.052	0.045	0.052	1.659	2.286	1.562
DCC_t	0.744	1.013	0.690	0.090	0.077	0.065	1.420	1.973	1.317
ADCC_norm	0.721	0.971	0.690	0.077	0.077	0.065	1.404	1.940	1.316
ADCC_t	0.729	0.990	0.683	0.077	0.071	0.065	1.412	1.958	1.311

Table A5: Russia back-testing results

Russia	AD			QPS			RMSE		
Model	PF1	PF2	PF3	PF1	PF2	PF3	PF1	PF2	PF3
CCC norm	2.373	4.883	0.809	0.065	0.020	0.351	4.299	6.385	3.858
CCC t	2.526	5.239	0.798	0.058	0.020	0.358	4.460	6.763	3.846
DCC norm	1.986	4.697	0.602	0.109	0.020	0.409	3.888	6.182	3.550
DCC t	2.180	5.027	0.618	0.084	0.020	0.402	4.090	6.533	3.572
ADCC norm	1.986	4.697	0.602	0.109	0.020	0.409	3.888	6.182	3.550
ADCC t	3.304	7.057	1.054	0.033	0.020	0.313	5.276	8.783	4.123

Figure 1: Log returns of the BRICS equities and currencies

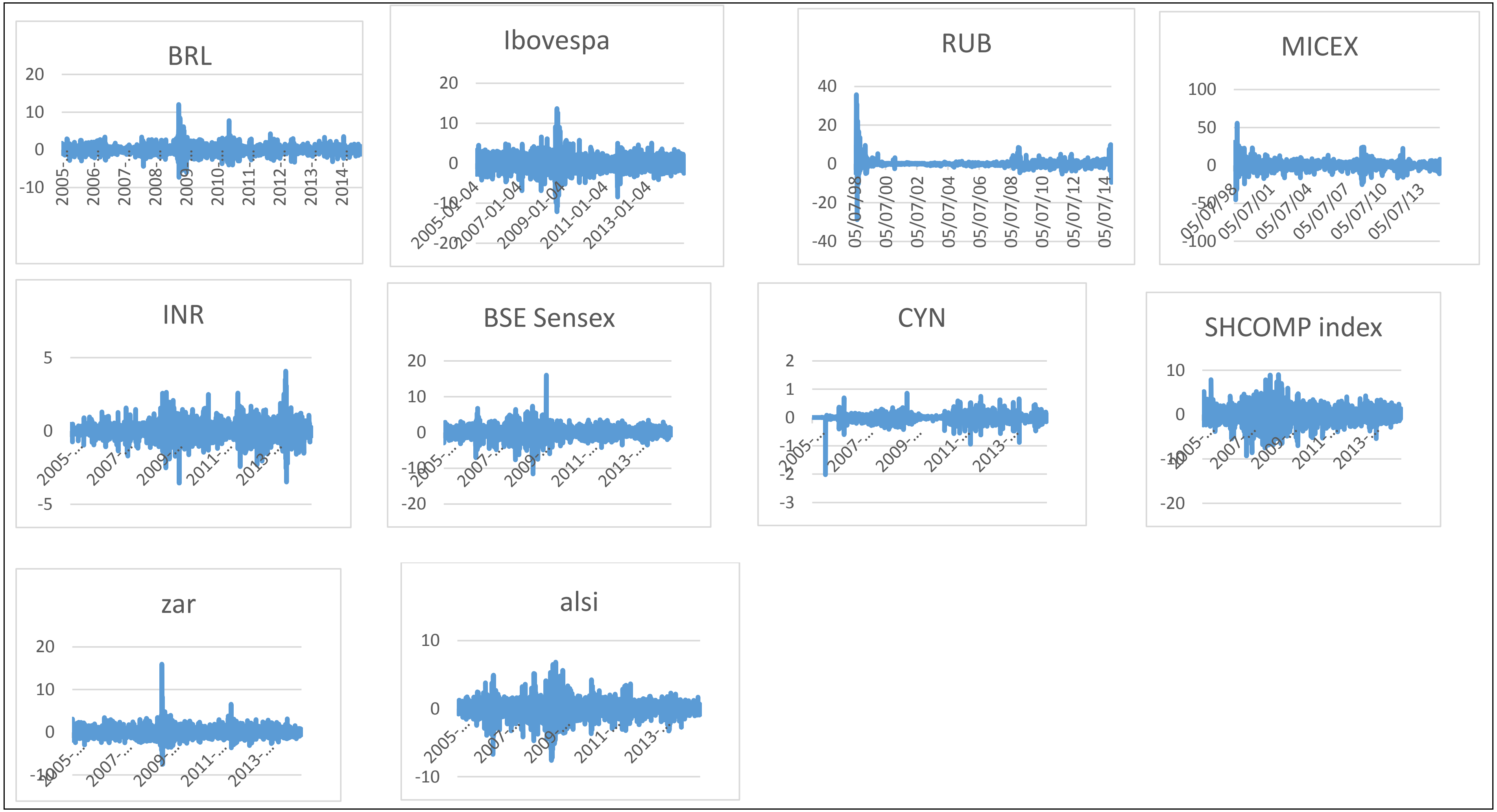

Figure A1: Different VaR forecasts from CCC-GARCH model against realized returns for Brazil

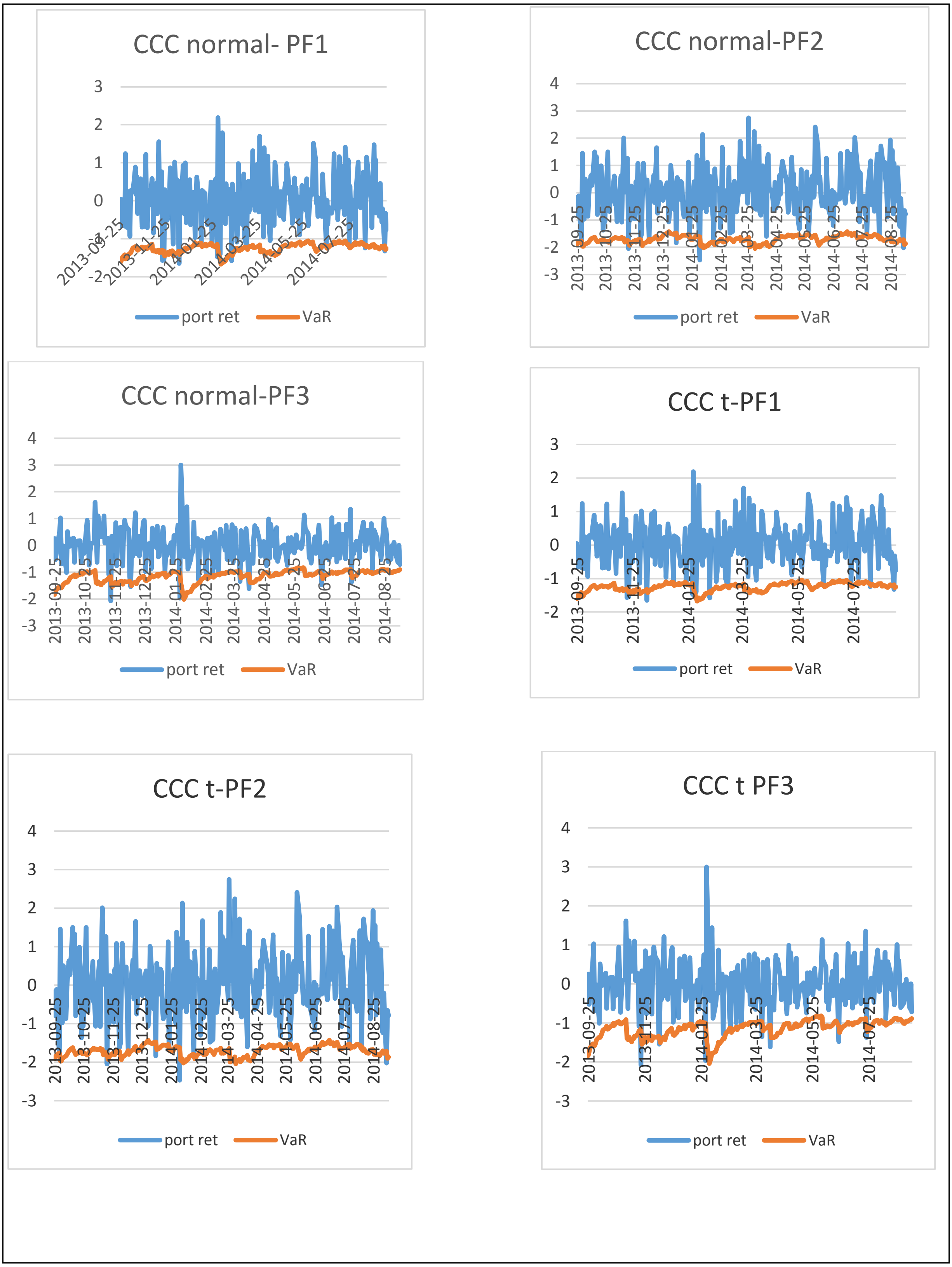

Figure A2: Different VaR forecasts for DCC-GARCH model against realized returns for Brazil

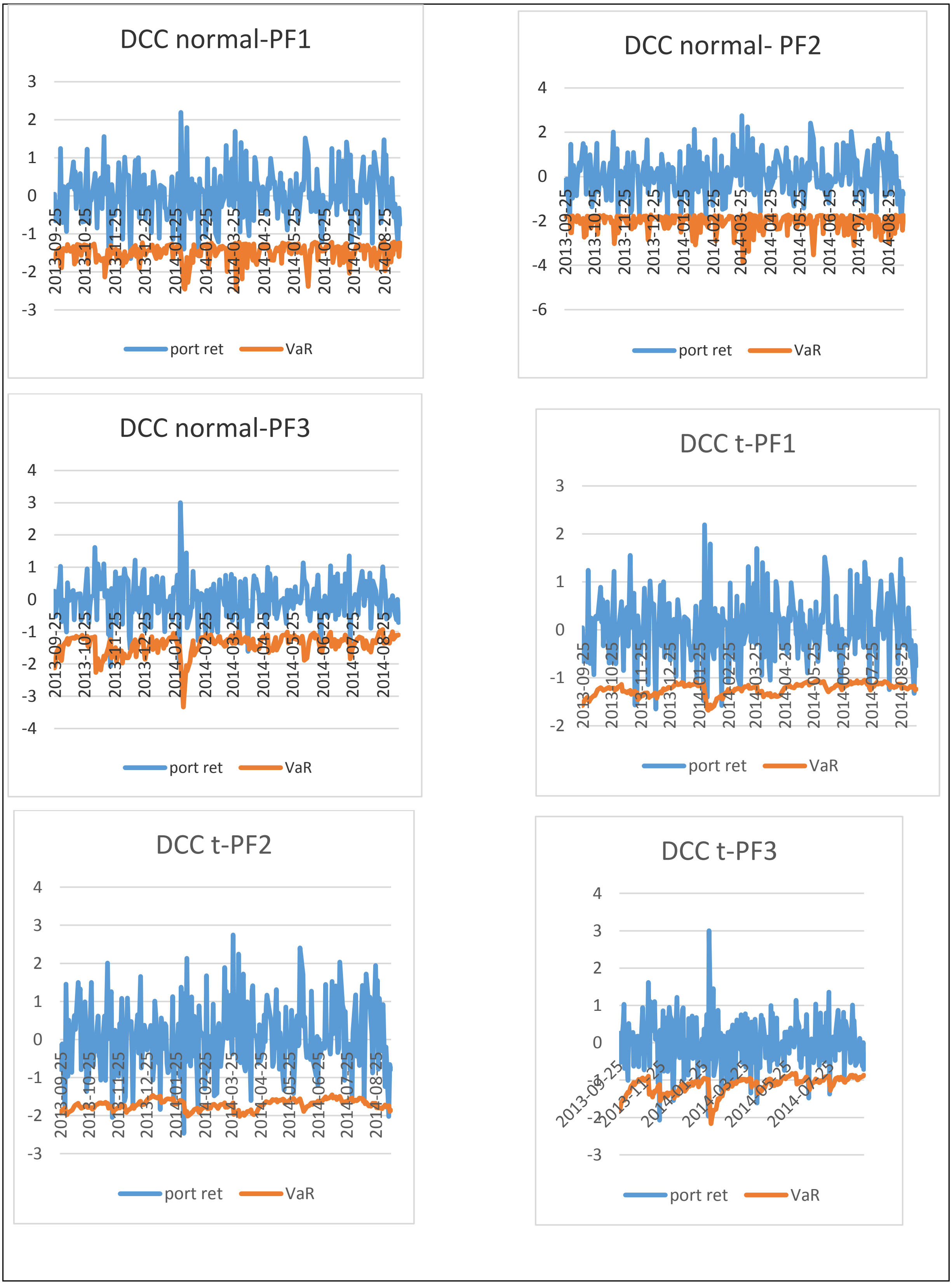

Figure A3: different VaR forecasts for ADCC-GARCH model against realized returns for Brazil

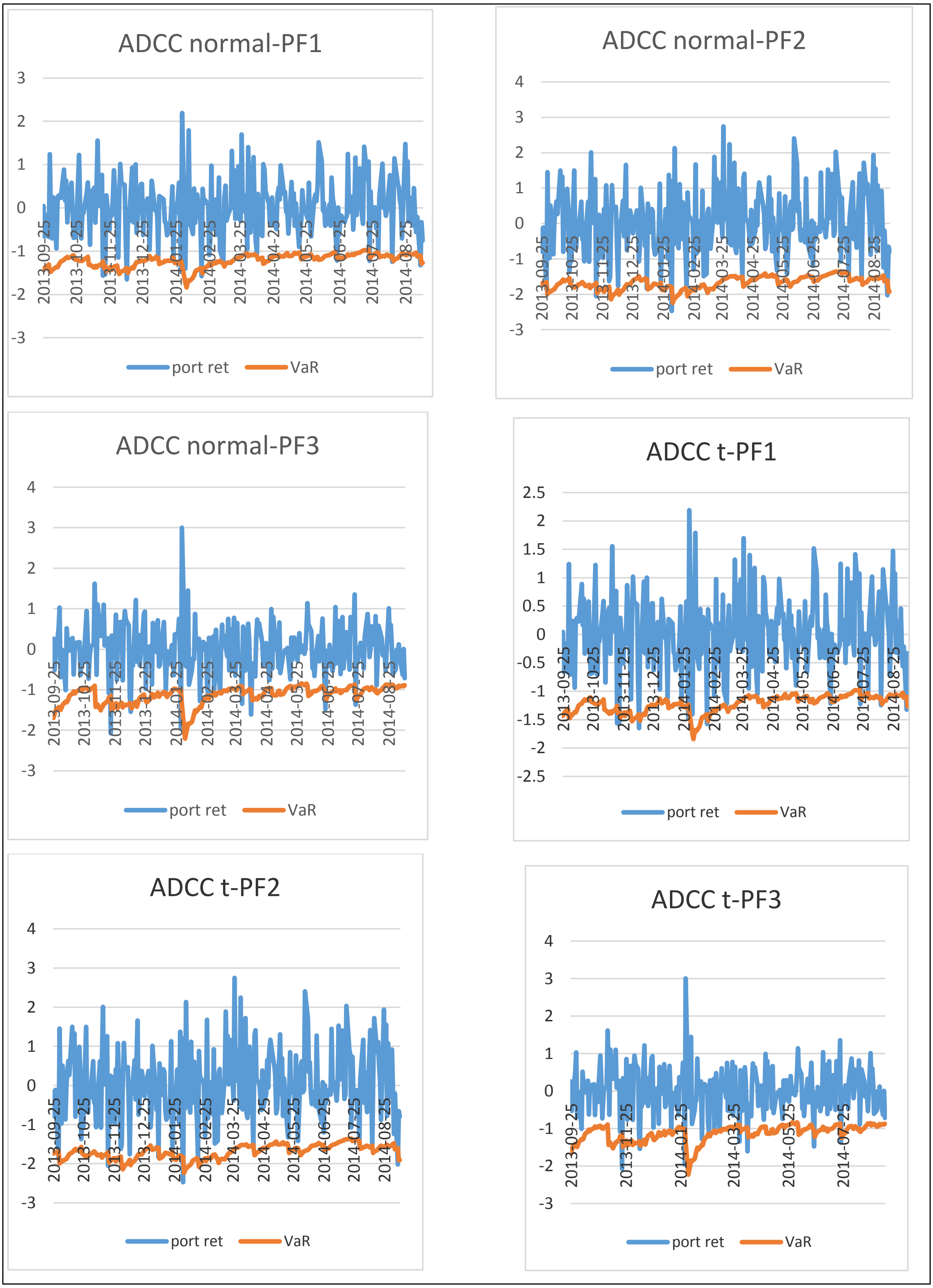

Figure A4: different VaR forecasts for CCC-GARCH model against realized returns for China

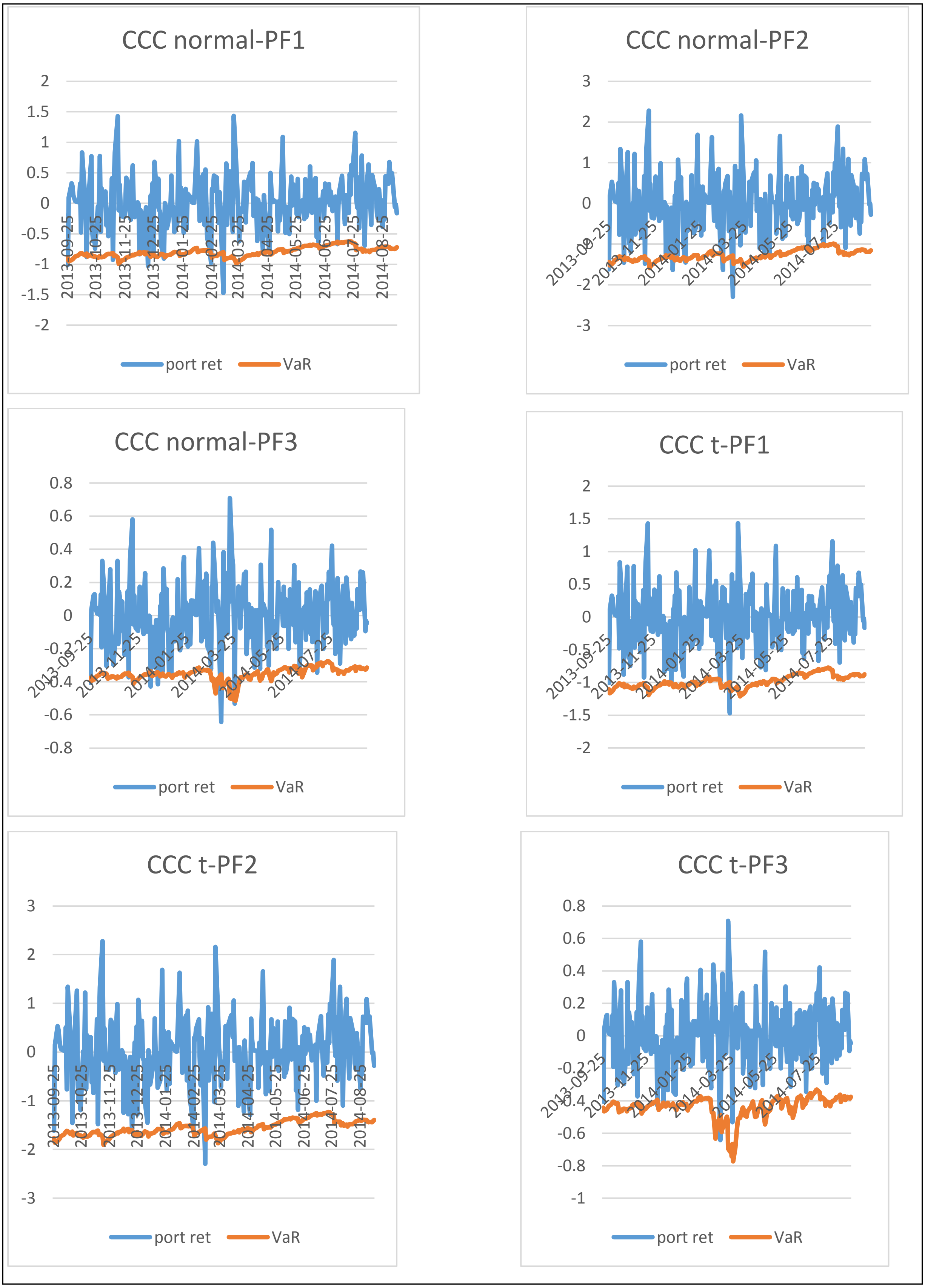

Figure A5: Different VaR forecasts for DCC-GARCH model against realized returns for China

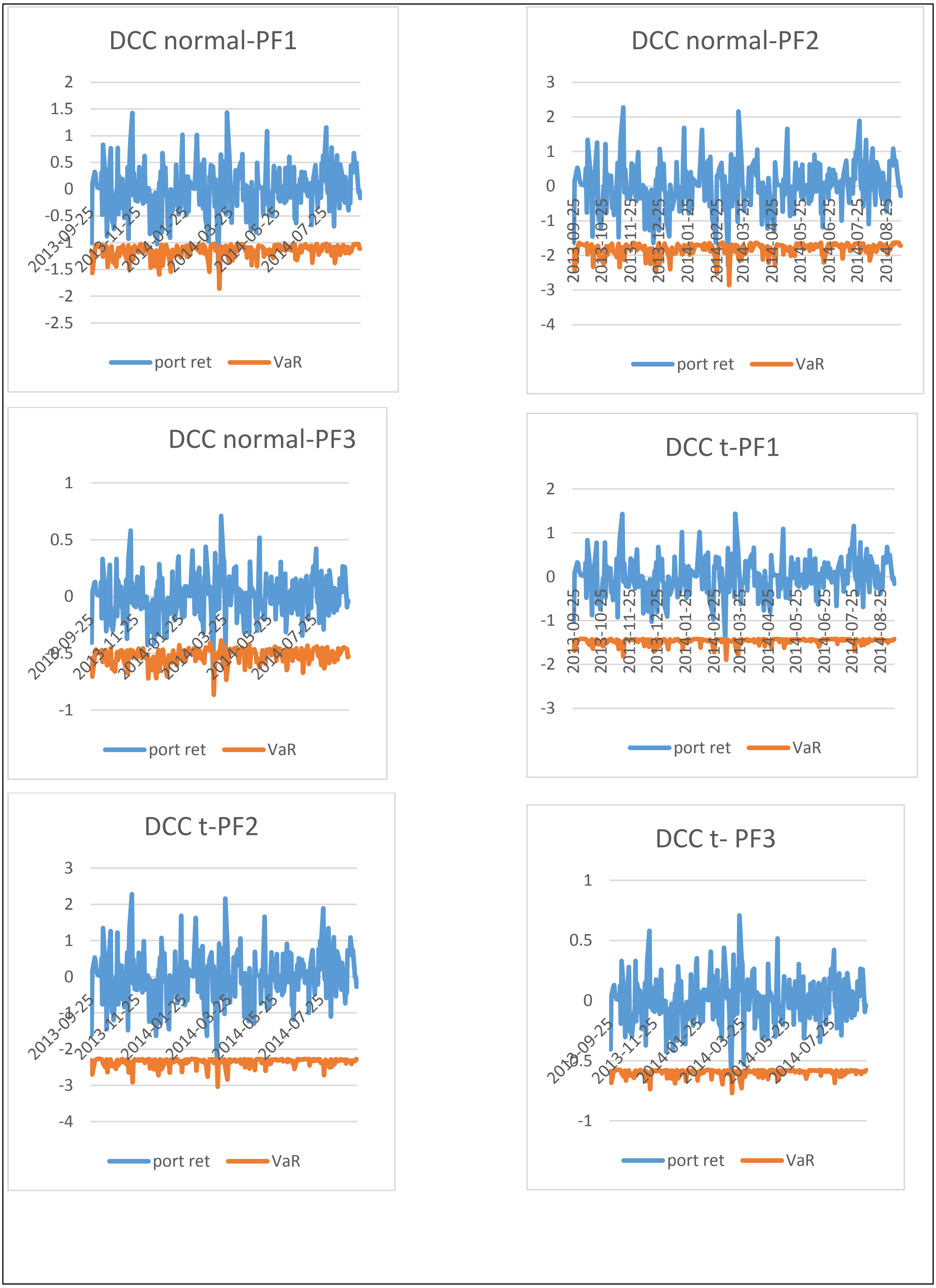

Figure A6: Different VaR forecasts for ADCC-GARCH model against realized returns for China

Figure A7: Different VaR forecasts from CCC-GARCH model against realized returns for India

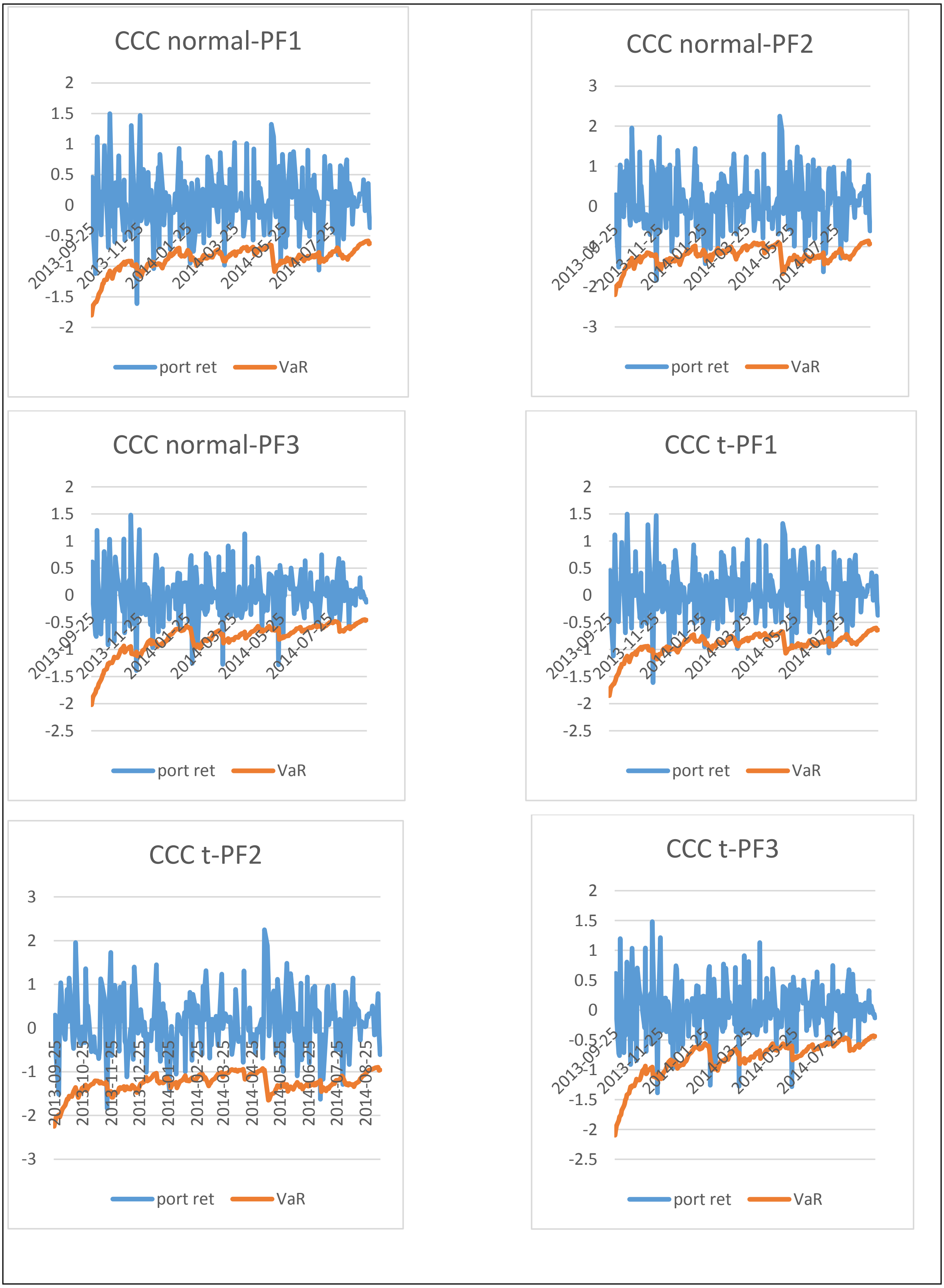

Figure A8: Different VaR forecasts for DCC-GARCH model against realized returns for India

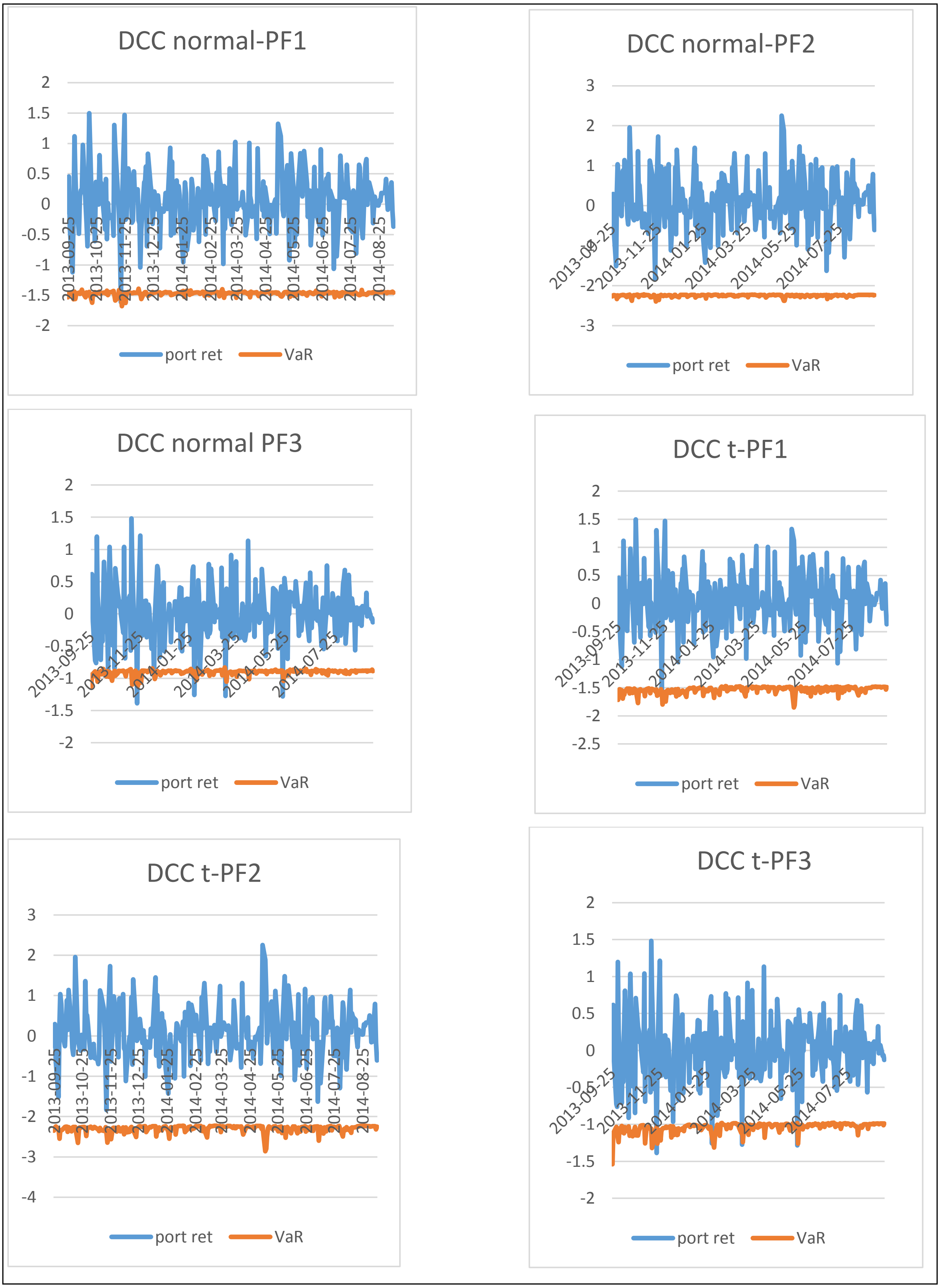

Figure A9: Different VaR forecasts from ADCC-GARCH model against realized returns for India

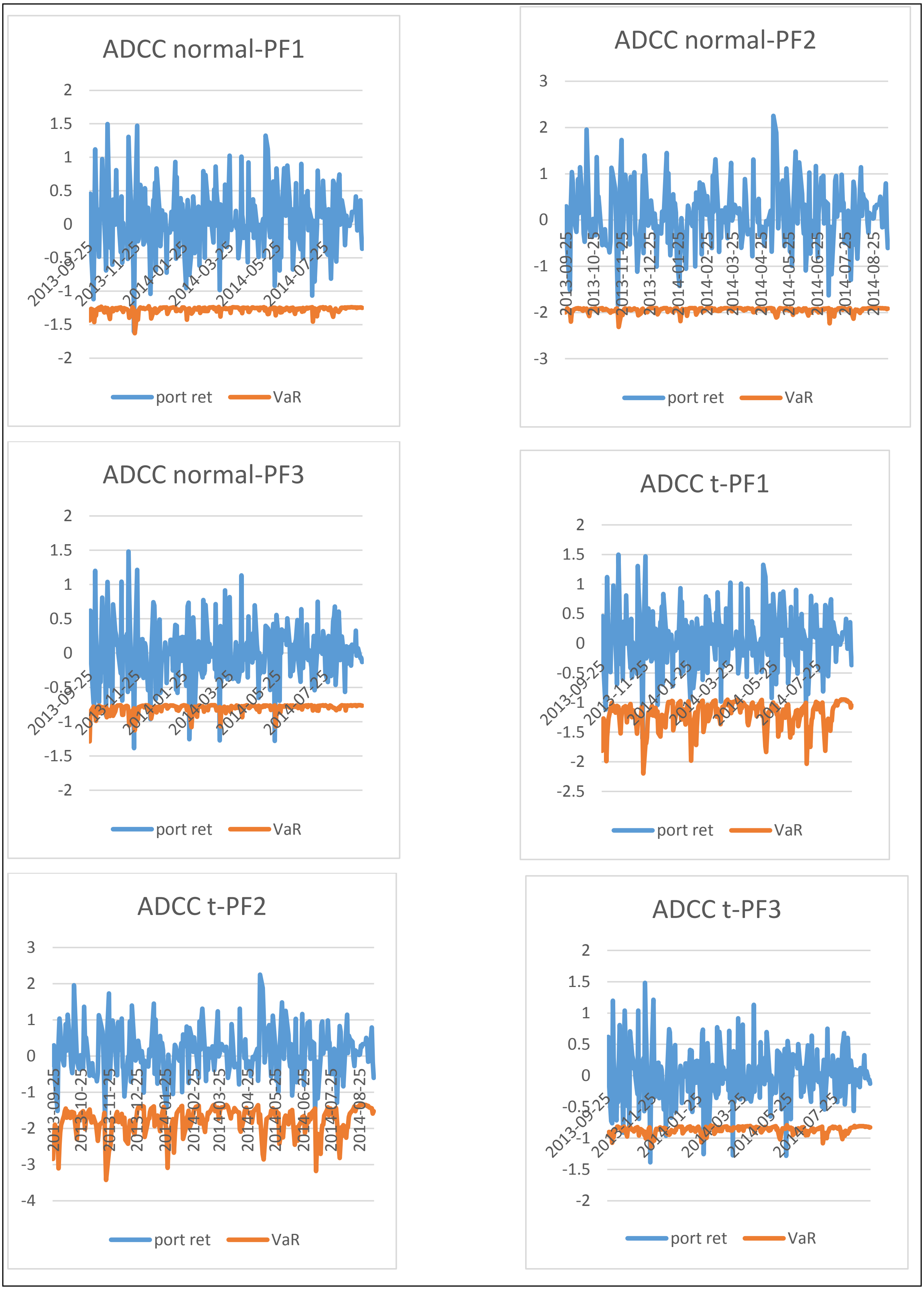

Figure A10: Different VaR forecasts from CCC-GARCH model against realized returns for South Africa

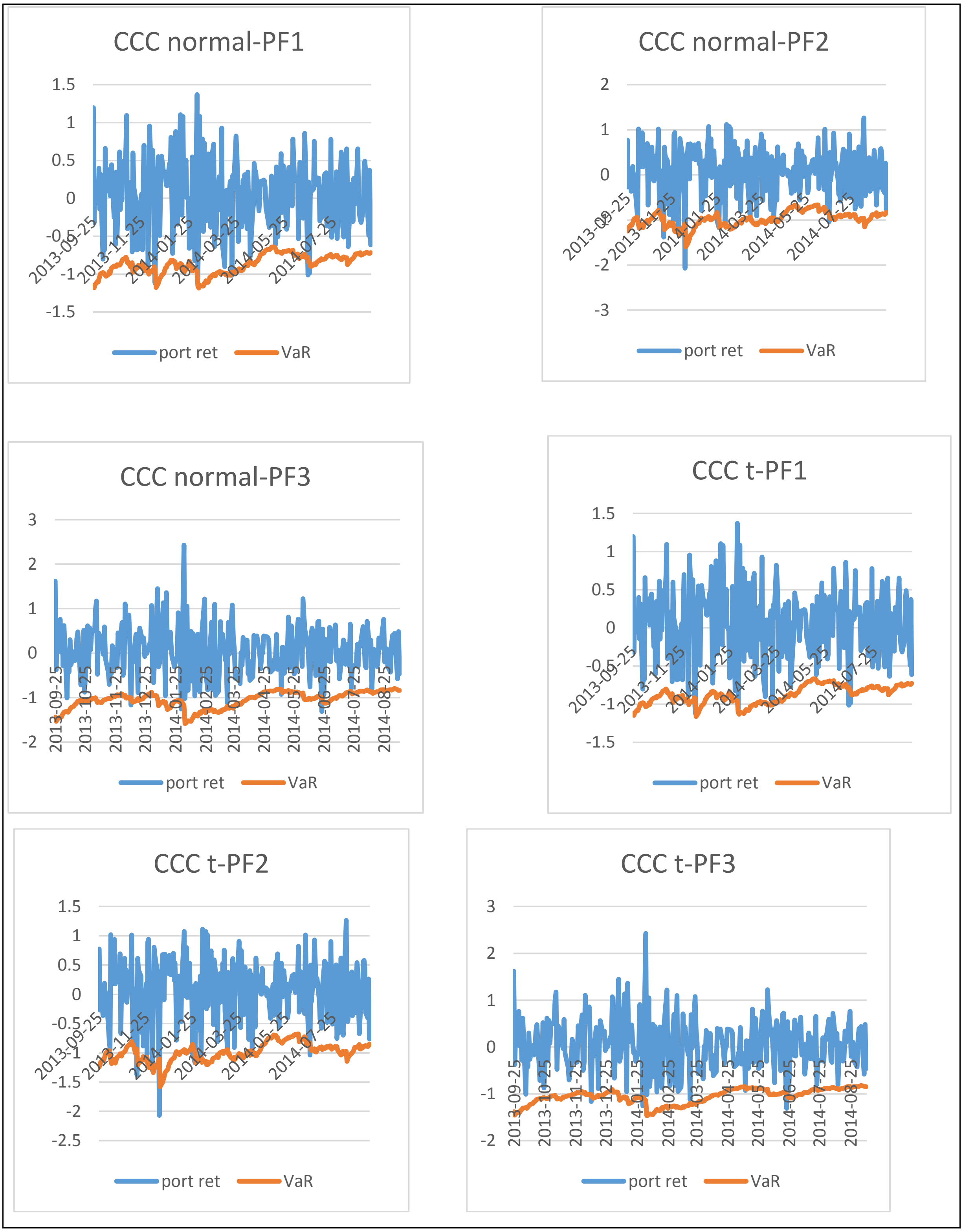

Figure A11: different VaR forecasts from DCC-GARCH model against realized returns for South Africa

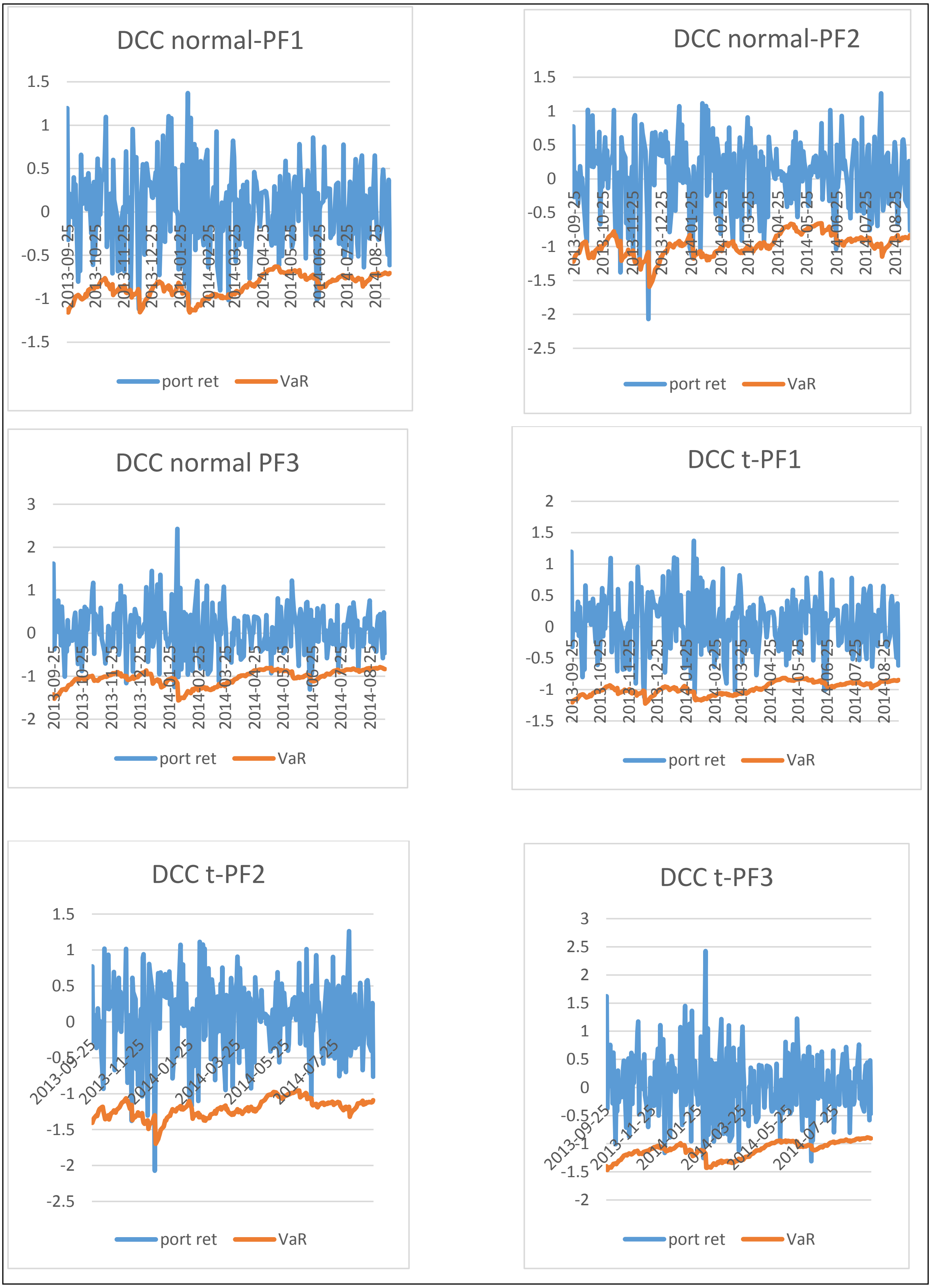

Figure A12: Different VaR forecasts from ADCC-GARCH model against realized returns for South Africa

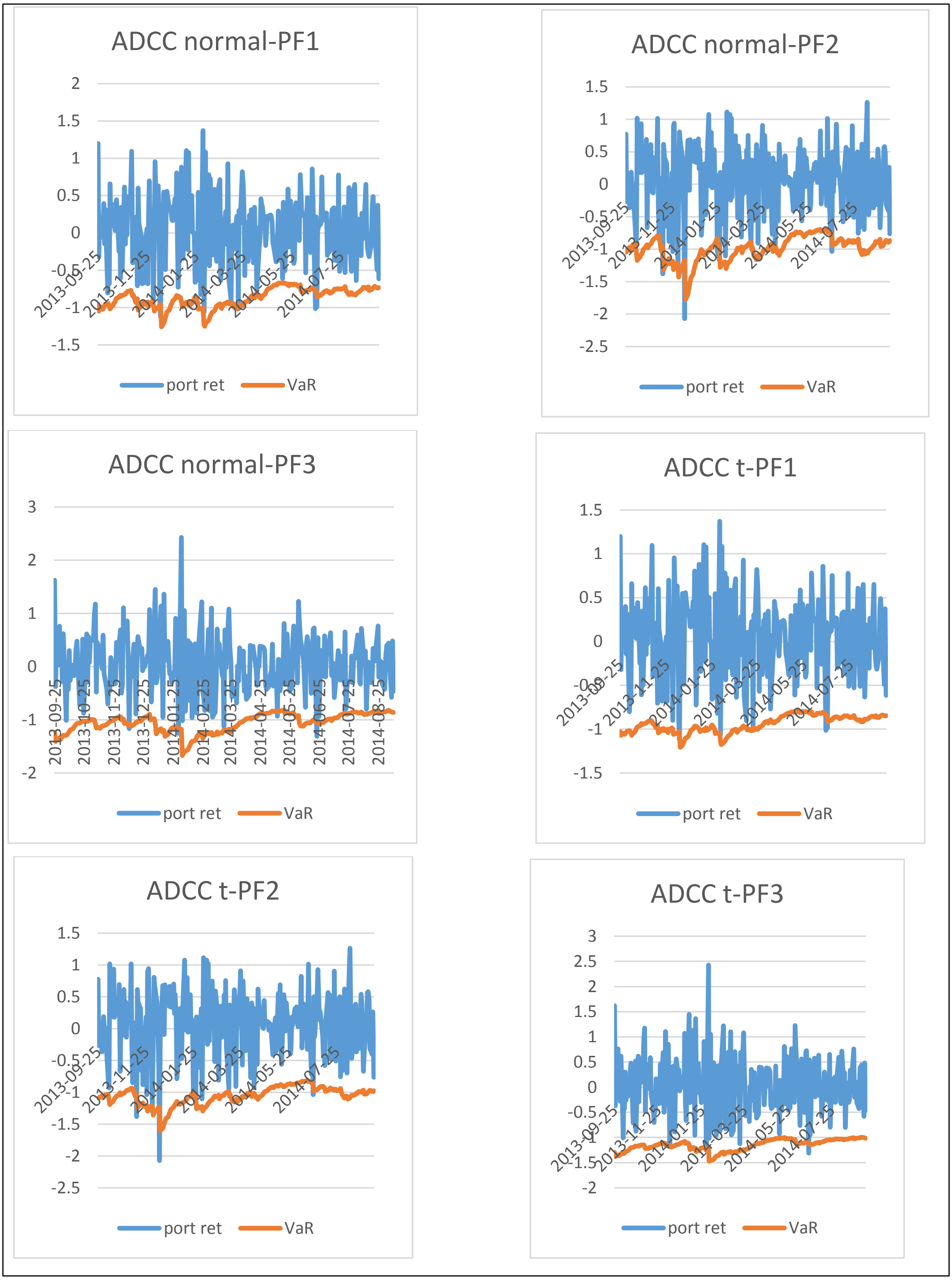

Figure A13: Different VaR forecasts from CCC-GARCH model against realized returns for Russia

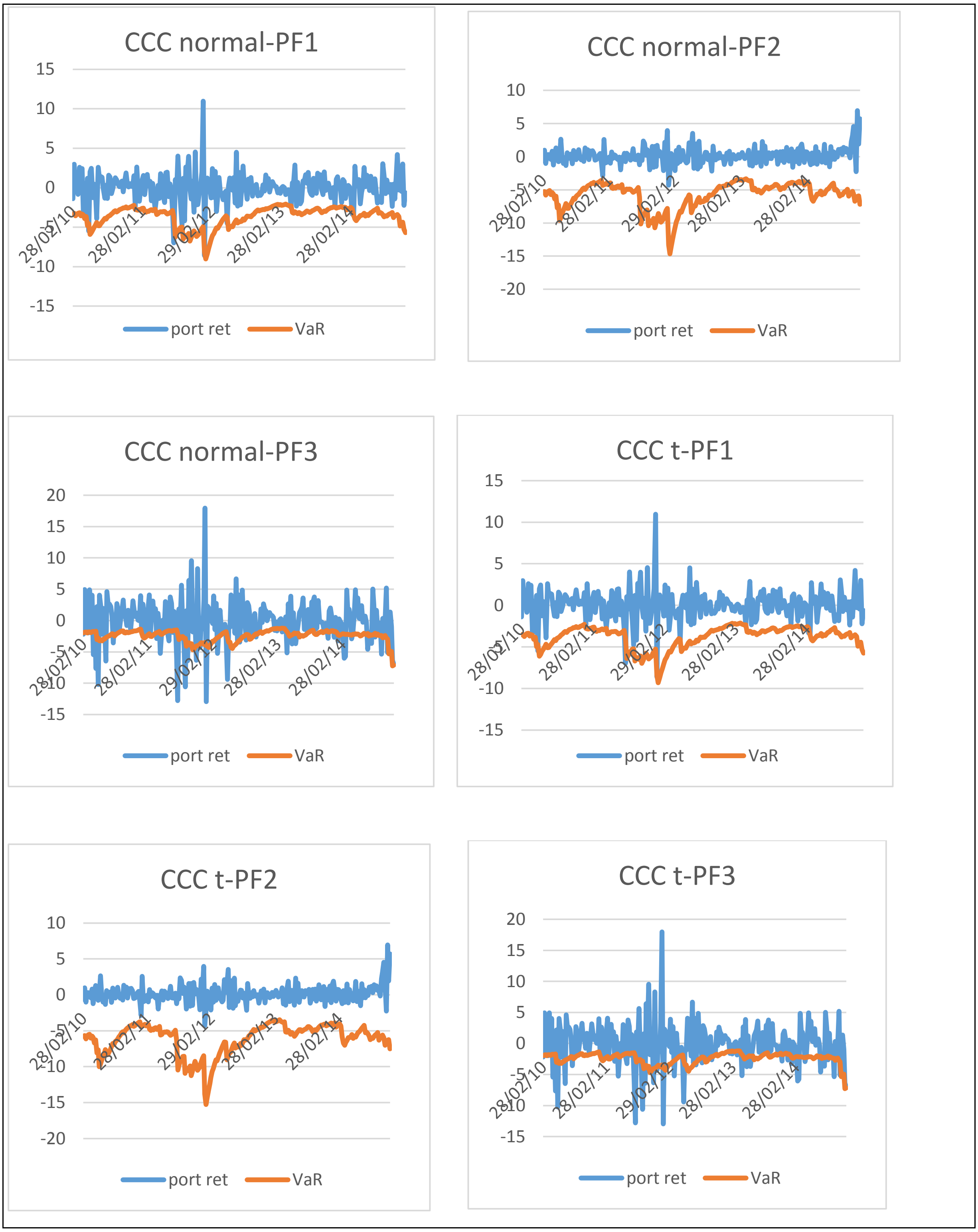

Figure A14: Different VaR forecasts from DCC-GARCH model against realized returns for Russia

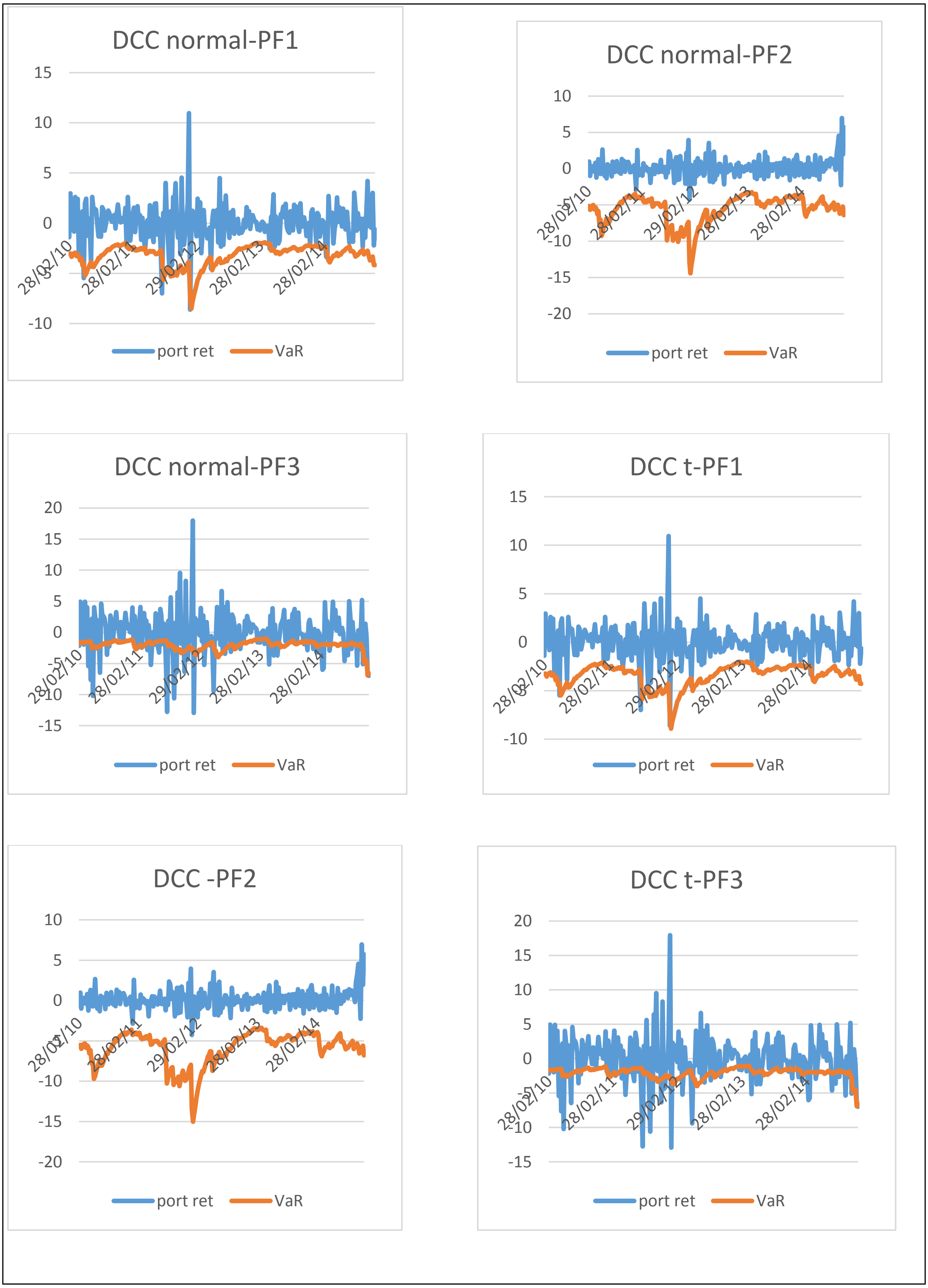

Figure A15: Different VaR forecasts from ADCC-GARCH model against realized returns for Russia