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25 January 2016

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Abstract

The results of an experiment with simulated data show that using multiple positive lineal aggregators of the same inputs instead of the original variables increases the accuracy of the Data Envelopment Analysis (DEA) technical efficiency estimator in data sets beset by dimensionality problems. Aggregation of the inputs achieves more than the mere reduction of the number of variables, since replacement of the original inputs with an equal number of aggregates improves DEA performance in a wide range of cases.

Key words Technical efficiency, Aggregation bias, Monte Carlo, DEA Estimator accuracy

JEL classification C14 C61 D20

1. Introduction

Data Envelopment Analysis (DEA) is one of the most widely-used non-parametric frontier models for evaluating the technical efficiency of Decision Making Units (DMUs) in multiple input/output scenarios. However, while statistically consistent (see Simar and Wilson 2015, for a summary of DEA properties), the DEA radial technical efficiency estimator is also, like many other non-parametric estimators, prey to the curse of dimensionality, which means that its rate of convergence to true efficiency diminishes as more inputs and outputs are added. DEA technical efficiency scores estimated from data sets containing large numbers of inputs and outputs and a small number of observations are well known to be upwardly biased (Banker 1993; Simar and Wilson 2008).

The aggregation of inputs or outputs has been widely used to reduce number of variables (Podinovski and Thanassoulis 2007) in DEA applications. A frequent procedure in many studies is to collapse the set of inputs or outputs into a single linear aggregate using prior information such as prices, unit pollutant output coefficients, energy coefficients, etc. It is very common, for example, to transform groups of inputs or outputs into a single cost or income variable. The relation between the DEA aggregated models and the DEA disaggregated based models is well known: estimates obtained from aggregated value models can be downwardly biased by price allocative inefficiency with respect to those obtained from the fully-disaggregated DEA efficiency model (Primont 1993; Thomas and Tauer, 1994; Färe and Zelenyuk 2002; Färe et al. 2004; Banker et al. 2007). Then, aggregation of the value of inputs or outputs expands the DEA production possibility set towards the cost possibility set or the revenue possibility set (Banker et al. 2007; Hougaard and Tind 2009; Primont 1993), thereby improving the accuracy of the DEA efficiency estimator in studies with data sets beset by dimensionality problems.

This expansion could, however, lead to the loss of relevant technological information of the efficiency frontier: strong efficient units might classified as inefficient after aggregation and full dimension efficient facets might be blurred. Olesen and Petersen (1996) emphasize the negative consequences of this loss of information in research oriented towards the identification of substitution ratios. Less attention has been given to its impact on the consistency of the efficiency estimator. This has potential relevance, given that one of the main aspects of the problem of dimensionality in DEA estimation is that a large proportion of the DMUs might not naturally lie within the strongly efficient frontier (Bessent et al. 1988). This suggests that the aggregation technique could be improved by identifying an input or output aggregation procedure that would reduce the size of the data set while preserving the maximum amount of relevant information of the strong efficient frontier.

In this respect, Aldanondo and Casasnovas (2015) have shown that, if the DEA model with a single linear aggregate of each input is extended to one using several linear aggregates of the same inputs has two effects: 1º) the aggregation bias with respect the fully disaggregated model diminishes, and 2º) the two models coincide for the set observations that are canonical combinations of allocatively efficient units under either/any aggregation criterion. Then the estimated best-practice frontier using multiple aggregates is an envelope that preserves a greater amount of information about the original frontier of the fully disaggregated DEA model than the one estimated from the single-aggregate model. There is no proof that this necessarily leads to a more accurate efficiency estimate, however.

The precise aim of this study is to explore the extent to which the use of models with multiple linear aggregates of the same inputs affects the accuracy of the DEA estimator. The basic idea is to use an input-aggregation approach that will preserve the maximum amount information about the efficient frontier in order to improve the sufficiency of the estimator.[1] By means of Monte Carlo simulation, we compare results across aggregated-input technical efficiency models in order to show how they vary as a function of sample size and inefficiency distribution. In this experiment we generate a set of input prices and one output price for each DMU to simulate input demand and randomly choose a set of several DMU input prices to create input aggregates. Through this experiment, we find that the DEA technical efficiency estimator improves as the number of aggregators is increased.

This note is organized as follows: section two describes the Monte Carlo design and the methodology used to analyze the DEA model; section three presents the results of the analysis of aggregation bias in technical efficiency when using multiple aggregate criteria. The paper ends with some conclusions from the research.

2. Experimental design

We use a Monte Carlo experiment to compare the performance of constant return DEA radial technical efficiency models with different numbers of linear aggregates of the same inputs. The accuracy of the models, including the baseline model, is determined by comparing the simulated true efficiency value with the DEA efficiency estimates.[2]

[1] Another approach would be incorporate statistics to improve the efficiency of the variable aggregation process in DEA. Simar and Wilson (2001), for example, use "aggregation bias" as a statistic to test the potential of the aggregation of variables. Dario and Simar (2007) use the index of correlation between variables to construct a single ad hoc aggregate set of inputs or outputs. Models based on Principal Component Analysis DEA (Alder and Golany 2001, 2002) or Independent Component Analysis DEA (Kao et al. 2011) include a single or multiple linear aggregates (components) of the same inputs, in accordance with the variance explained. These components do not need to be positive linear combinations as used in this study. A comparison of the different procedures that can be used to improve the aggregation of variables in DEA is beyond the scope of this note.

[2] Spearman rank correlation coefficients estimated to measure the accuracy of the estimator in no way alter the conclusions from this paper. They are omitted for reasons of space but are available from the authors upon request.

In our experiment, the different aggregation criteria are represented by different price sets. Thus, the procedure is similar to that reported in Tauer (2001).[3] In particular, we generate random input price and output quantity data for each DMU and use them to simulate factor demands for a constant-returns-to-scale Cobb-Douglas production function. This procedure ensures that the aggregate coefficients represent marginal rates of substitution on the frontier for the data production set (Varian 1984; Banker and Maindiratta 1988).

All comparisons are carried out for different numbers of observations n € (10, 50, 100, 500, 1000, 2000 and 5000). Samples larger than 500 units are not very common in empirical research, but we have simulated samples of this size in order to analyze convergence of estimators (Simar and Wilson 2008). There are five inputs x_{ik} (k=1,…,5) for each observation i and a single efficient output y_i^e. Our choice of efficiency simulation procedure was guided by that of Simar and Wilson (2000, 2001): the efficient output of each DMU was multiplied by an inefficient term with no *a priori* assumptions regarding the distribution of the DMUs along the efficiency frontier. Based on these general criteria, the experiment consists of 1000 replications of the following procedure.

1) Five parameters α_k are generated from a uniform distribution [0.1, 1] and each α_k is divided by the sum of the five selected α_k, such that the coefficients add up to 1.

2) 5,000 observations of a single efficient output, y_i^e, are generated from a uniform distribution [0.1, 100], and five w_{ik} input prices, from independent random variables with uniform distribution [0.1, 5]. The quantity of inputs of each DMU is computed by means of the factor demand function: $x_{ik} = \left(\prod \alpha_k^{\alpha_k}\right)^{-1} \alpha_k y_i^e w_{ik}^{-1} \left(\prod w_{ik}^{\alpha_k}\right)$.

3) Inefficiency is simulated by multiplying the output of each unit y_i^e by the technical inefficiency coefficient $A_i = \exp(-u_i)$, where u_i is a random value drawn from a normal distribution

[3] In other DEA experimental studies, such as Principal Components (Adler and Yazhemsky 2010) or Independent Components (Kao et al. 2011), the aggregation criteria are based on input covariance. The procedure consists of the generation of random observations of inputs which are then used to simulate a production function.

$N(0,\sigma_u)$. σ_u takes the values 0.2 and 0.3. Then, the observed output value of each unit *i* is computed as $y_i = A_i y_i^e$.

4) Four different linear aggregates of the same four inputs are created for each DMU. The aggregator coefficients are the same for all the DMUs included in the sample.[4] By arbitrary selection , the prices of these inputs for the first four units of the sample are taken as weights:[5] respectively, $C_i^j = \sum_{k=1}^{4} w_{jk} x_{ik}$ (*i*=1,…,*n* ; *j*=1,…,4) where C_i^j denotes the aggregate of the *k* inputs of unit *i*, weighted by the w_{jk} input prices of unit *j*.

5) From this initial 5000-unit population, we take subsamples of the first 10, 50, 100, 500, 1000 and 2000 observations in order to obtain smaller samples. Thus, this study simulates change in sample size as successive enlargements up to population size, thereby maintaining the same technology and the same aggregate weights for different-sized samples in each replication.

6) The linear programming in Equation (1) is used to compute radial technical input efficiency $\hat{A}_i^h$ with constant returns to scale (Charnes et al. 1978) for unit *i*, with models with different numbers of aggregates (*h*=0,1,2,3,4). The baseline model $\hat{A}_i^0$ computes the efficiency scores obtained with the five original inputs. The other models include one or several aggregates (*h*=1,2,3,4) of the first four inputs and the fifth original input.[6]

[4] In contrast to this procedure, Tauer (2001) uses different prices for each DMU.

[5] The prices of any DMU can be chosen indistinctly.

[6] Thus, the aggregate models have different numbers of composite inputs and one original input. We have replicated this experiment with different numbers of original and aggregated inputs. We chose to present this case because it is highly illustrative. Results of other simulations, similar to the case presented here, are available from the authors upon request.

$$
\begin{aligned}
&\hat{A}_i^h(y_i, C_i^j, x_{ik}) = \min\ \beta \\
&\text{subject to} \\
&\sum_i z_i y_i \geq y_i \\
&\sum_i z_i C_i^j \delta^h \leq \beta C_i^j \delta^h \quad j = 1,\ldots,h;\ \delta^h = 0 \text{ if } \ \mathrm{h} = 0 \text{ and } \delta^h = 1 \text{ if } \ \mathrm{h} > 0 \\
&\sum_i z_i x_{ik} \leq \beta x_{ik} \quad k = 1,\ldots,5 \text{ if } \ h = 0 \text{ and } k = 5 \text{ if } \ h > 0 \\
&z_i \geq 0 \quad i = 1,\ldots,n
\end{aligned}
\qquad (1)
$$

All the efficiency scores are computed using FEAR software (Wilson 2008) for platform R.

7) For every replication, we compute average technical efficiency scores and mean absolute error (MAE) (absolute difference between estimated efficiency and true or simulated efficiency),

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|\hat{A}_i^h - A_i\right| .$$

8) Finally, we compute the average over 1000 replications to obtain average technical efficiency and the MAE values reported in the tables below.

3. Results and discussion

Tables 1 and 2 give the estimates for standard deviations of inefficiency of 0.2 and 0.3, respectively. For the sake of clarity, both tables include average technical efficiency and mean absolute error (*MAE*). The results are discussed in blocks, starting with the average technical efficiency scores, which are the indicators most widely discussed in the literature cited above. This is followed by an analysis of the *MAE* in each model.

The average technical efficiency scores uphold some known theoretical and experimental findings reported by Färe et al. (2004) and Thomas and Tauer (1994). Firstly, the DEA average efficiency score for the baseline model, using the five original inputs, is well above the true average efficiency score for small sample sizes, converging towards true efficiency with growing sample size. This can be checked by looking at the average efficiency trends displayed in Table 1 and Table 2. In Table 1, for example, the DEA baseline model average efficiency score decreases from 0.993 (n=10) to 0.890 (n=5000) for a true average efficiency score of 0.858. Secondly, the average efficiency score obtained using models with input aggregators is lower than that given by the model with fully disaggregated inputs. This difference decreases as more aggregates of the same inputs are added. Thirdly, the average efficiency score for models with composite inputs falls below true efficiency as the sample size increases. Again, Table 1 shows that the average efficiency score drops to 0.706 (*n*=5000) for the single-aggregate model and to 0.817 (*n*=5000) for the four-aggregate model. This has been reported by Tauer (2001) as evidence of the inconsistency of DEA technical efficiency estimation using aggregates, since the computed average efficiency score does not converge towards true average efficiency.

The true error values, that is, the *MAE* scores, confirm some of the above observations while also providing new findings. With respect to aggregation bias, the trend of the *MAE* as a function of sample size confirms the inconsistency of DEA efficiency estimators when using aggregates for non-additive inputs. As the two tables show, the *MAE* does not converge towards zero with larger sample size in any of the aggregated models, while it always decreases with larger simple in the baseline non-aggregated DEA estimator model. By way of example, with four aggregates of four inputs and σ_u =0.2, the *MAE* score gradually decreases from 0.104 to 0.049 as sample size grows from 10 to 1000 units, and increases slightly to 0.051 when sample size reaches 5000 units. The stabilization or increase in *MAE* appears in all the aggregated models reported in Tables 1 and 2, thereby highlighting the fact that an increase in sample size does not correct the true error or bias in a misspecification of the variables. However, aggregation bias is lower in models with more aggregators of the same inputs.

The *MAE* performance does, however, suggest that it is better to use aggregates when faced with dimensionality problems. Indeed, although the aggregate models contain some bias and do not converge towards true efficiency, they may, in certain empirical contexts, have greater estimation accuracy than the baseline model. As can be seen in Table 2 for the case of *n*=10-unit, for example, the *MAE* for the single-aggregate model is 0.111, which is lower than in the baseline model (*MAE*=0.211) and all the other models. Conversely, using the same table, the lowest *MAE* for *n*=50 is found in the three-aggregate model and, for *n*=100, in the four-aggregate model. Overall, the results show that models with fewer aggregates produce better estimates with small sample size and high standard deviation of efficiency; while the accuracy of estimators using a larger number of linear aggregates improves as sample size grows. For sufficient sample size , the basic non-aggregated DEA estimator gives the best performance.

As far as we are aware, there is no evidence from experimental studies that the DEA efficiency estimator obtained using a number of aggregates equal to the original number of inputs could give a better result than the model using fully disaggregated inputs, even when both programs have the same dimension.[7] It can be seen, for example, that the four-aggregate model has a lower *MAE* than the model with the five original inputs when applied to samples of 1000 units or less for a σ_u =0.2 (Table 1) or to samples of 2000 units or less for a σ_u =0.3 (Table 2). This holds even for a production function without additive inputs, like that specified in this study.

Thus, input aggregation achieves more than the mere reduction of the number of variables in the DEA program. Although it is not immediately obvious from the results of this study how aggregates work to improve the estimates, the literature provides some possible interpretations. Charnes et al. (1990) and Podinovski and Thanassoulis (2007), for example, suggest that, in very particular circumstances,[8] replacing the inputs with linear aggregates in the primal DEA program is equivalent to constraining input weights in the corresponding multiplicative DEA model. In accordance with this theory, it can be assumed that the introduction of aggregates intervenes in a similar way to that of weight constraints: by expanding the DEA technical efficiency envelope, thereby preventing the occurrence of non-zero slacks in the DEA solution (Allen et al. 1997; Podinovski 2004; Podinovski 2005) while preserving part of the information provided by the production efficient frontier data set (Aldanondo and Casasnovas 2015). Thus, the radial technical efficiency score more accurately reflects all the excess in inputs and shortfall in outputs. To our knowledge, however, no definite relationship between the two methods (relative weight

[7] In contrast to Principal Component - DEA studies, when the number of aggregates is equal to the number of inputs or outputs (explaining the total variance of the sample), the DEA efficiency estimator gives the same result with or without aggregates (Adler and Golany 2001).

[8] Charnes et al. (1990) demonstrate the mathematical equivalence of constraining relative input weights (assurance region) and aggregating inputs, for the very simple case of a data set with two inputs and one output. Podinovski and Thanassoulis (2007) suggest the equivalence of aggregating inputs or outputs in the baseline DEA and constraining their relative weights in the dual form of the DEA model. This, however, would hold whenever the constraint on the respective aggregate input (output) in the DEA model holds.

constraints and the use of input or output aggregates) under all types of conditions has ever been confirmed.[9] In conclusion, all we are able to say is that our results suggest that multiple-aggregate DEA models could work in two ways: firstly, the higher the degree of aggregation, the fewer slacks (or zero weights) will appear (Olesen and Petersen 1996; Førsund 2013); and, secondly, the larger the number of aggregators, the closer the DEA envelope approaches the true efficiency frontier.

4. Conclusions

The main conclusion from this research is that the use of multiple linear aggregates of the same inputs has a positive impact on the performance of the radial DEA efficiency estimator in the presence of dimensionality problems. Our results show that this positive effect outweighs the known effect of reducing the number of variables in the DEA program. Indeed, in many cases, the mean absolute error (*MAE*) of the model with four linear aggregates of the same four inputs is lower than the *MAE* of the program using the original inputs. Assuming no dimensionality problems and no additive inputs, DEA technical efficiency models with fully disaggregated inputs are the most appropriate method.

Estimators with multiple aggregates of the same inputs perform better overall than those with a single aggregate, except when applied to very small samples with high standard deviation of inefficiency. These results have major implications for DEA efficiency estimation. The use of multiple, rather than a single linear aggregate of the same inputs can improve the performance of the radial DEA efficiency estimator while also ensuring coherence between technical efficiency measures and multiple criteria of overall efficiency.

[9] See Førsund (2013) for a recent review of the research on weight constraints.

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Table 1. Computed average technical efficiency, mean aggregation bias and mean absolute error (from random production data and σ_u=0.2)

	DEA Basic	1 agg.	2 agg.	3 agg.	4 agg.
			Efficiency scores		
n=10	0.993	0.864	0.920	0.942	0.953
	(0.020)*	(0.140)	(0.108)	(0.089)	(0.077)
50	0.967	0.788	0.852	0.878	0.893
	(0.066)	(0.147)	(0.129)	(0.118)	(0.111)
100	0.953	0.765	0.831	0.857	0.873
	(0.077)	(0.145)	(0.130)	(0.121)	(0.115)
500	0.922	0.729	0.797	0.824	0.841
	(0.093)	(0.141)	(0.129)	(0.122)	(0.117)
1,000	0.910	0.719	0.787	0.815	0.832
	(0.096)	(0.140)	(0.128)	(0.122)	(0.117)
2,000	0.901	0.710	0.779	0.807	0.824
	(0.097)	(0.139)	(0.128)	(0.121)	(0.116)
5,000	0.890	0.706	0.772	0.801	0.817
	(0.099)	(0.137)	(0.127)	(0.120)	(0.116)
			Mean Absolute Error (*MAE*)		
n=10	0.135	0.104	0.096	0.100	0.104
50	0.108	0.108	0.074	0.068	0.067
100	0.094	0.116	0.073	0.062	0.058
500	0.063	0.136	0.078	0.059	0.050
1,000	0.052	0.144	0.082	0.061	0.049
2,000	0.042	0.151	0.086	0.063	0.050
5,000	0.031	0.153	0.090	0.064	0.051

Standard deviation of computed efficiency in parentheses. True mean efficiency is 0.858 with standard deviation 0.097 for all sample sizes.

Table 2. Computed average technical efficiency, mean aggregation bias and mean absolute error (from random production data and σ_u=0.3)

	DEA Basic	1 agg.	2 agg.	3 agg.	4 agg.
			Efficiency scores		
n=10	0.979	0.837	0.892	0.913	0.924
	(0.049)*	(0.163)	(0.135)	(0.121)	(0.111)
50	0.935	0.751	0.810	0.835	0.850
	(0.107)	(0.166)	(0.154)	(0.148)	(0.143)
100	0.914	0.726	0.786	0.811	0.827
	(0.119)	(0.164)	(0.155)	(0.149)	(0.146)
500	0.873	0.687	0.749	0.775	0.790
	(0.132)	(0.157)	(0.152)	(0.148)	(0.145)
1,000	0.859	0.675	0.738	0.764	0.780
	(0.135)	(0.155)	(0.150)	(0.147)	(0.145)
2,000	0.847	0.666	0.730	0.756	0.772
	(0.136)	(0.154)	(0.149)	(0.146)	(0.144)
5,000	0.835	0.661	0.722	0.749	0.764
	(0.136)	(0.152)	(0.148)	(0.145)	(0.143)
			Mean Absolute Error (*MAE*)		
n=10	0.211	0.111	0.116	0.124	0.130
50	0.136	0.099	0.076	0.075	0.076
100	0.115	0.104	0.071	0.064	0.063
500	0.074	0.122	0.072	0.056	0.049
1,000	0.060	0.130	0.075	0.056	0.047
2,000	0.048	0.137	0.078	0.057	0.046
5,000	0.035	0.140	0.082	0.058	0.047

Standard deviation of computed efficiency in parentheses. True mean efficiency is 0.799 with standard deviation 0.133 for all sample sizes.