The term structure of psychological discount rate: characteristics and functional forms

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ABSTRACT

The discounted utility theory is a cornerstone of financial theory, particularly in inter-temporal asset pricing and portfolio management. This theory questioning has opened a whole field of research in psychology, economics, and management, and has undergone several enhancements recently. Its violation seems widely established and opened the way for the building of a more efficient framework to understand the individual time preferences. One improvement is related to the refining of the knowledge on the psychological discount function that underlies inter-temporal choices. In fact, an individual's time preferences may be characterized by various discount functions such as the exponential, Hernstein, Harvey, proportional, Laibson, Rachlin, hyperbolic and generalized hyperbolic discount functions. Empirical validation of these proposed psychological discount rate term structures to explain individual preferences and the distribution of their parameters in a given population has been the subject of a number of recent studies. These researches have insufficiently question the problem of the validity of the proposed function on a given population.

The aim of this research is to empirically study the shape and parameters of psychological discount functions that characterized a given population. Based on the data collected through an experimental study, the violation of the discounted utility theory is confirmed, which means that time preferences could not be characterized by an exponential discount function. This finding is consistent with other empirical studies and shows that the population is characterized by a decreasing impatience. In addition, it shows that the population is characterized by a heterogeneity of the psychological discount functions.

JEL Classification: C91, C81, D11
Key words: discounted utility theory, psychological discount function, experimental study

1 Introduction

Time plays an important role in individual behavior. It is an essential input for all human activities and also a decision parameter. Indeed, many human decisions are based on a trade-off between costs and benefits in terms of welfare gains and losses at different points in time. In this situation, dentifying a decision criterion is crucial, especially in economics and management. Following Rae (1834), von Bohm-Bawerk (1889), Fisher (1930) and Samuelson (1937) was the first to propose a decision framework. In its framework, Samuelson stated that when faced with an inter-temporal trade-off situation, the preferred option is given by the maximization of the sum of the discounted value of the welfare associated with each stream. Formally, the chosen option is the one that maximizes the following function:

$$U(C)=\sum_{t=1}^{\infty}\delta^{t-1}u(c_t) \text{ with } 1 \leq \delta$$

This prescriptive approach has benefited from the work of Koopmans (1960) and Bleichrodt et al. (2007), who built a body of axioms that justifies the existence of such a function as support for any individual preference. This framework forms the discounted utility theory. It gives a decision criteria for choosing between two or more options, characterized by a sequence of cash flows which don't occur at the same time.

This theory has been applied in several disciplines, specifically in finance, in which it serves as a cornerstone of many inter-temporal asset pricing (Detemple & Zapatero, 1991; Campbell & Cochrane, 2000; Wachter, 2002; Lengwiler, 2005; Mulligan, 2007; Tran & Zeckhauser, 2011; Durbin et al., 2012) and portfolio (Merton, 1971; Merton, 1973; Breeden, 1979; Cox et al., 1985) models and market equilibrium theory.

This theory of inter-temporal decision making received extensive theoretical, empirical, and econometric contributions. However, some issues need to be addressed. One of the issues is related to the effective decision-making scheme adopted by an individual in various life situations. Indeed, following the work that has challenged the constant of the subjective discount rate, several authors have proposed different a discount function shape to characterize individual time preferences. These authors and the discounting shape which they proposed include, among others, Herrnstein (1961)[1], Harvey (1986), proportional (Harvey, 1995), Rachlin (2006), quasi-hyperbolic (Laibson, 1997), hyperbolic (Loewenstein & Prelec, 1992), and generalized hyperbolic discount function (Scholten & Read, 2010).

[1] Cited by Bleichrodt et al. (2009), p. 8.

These revisions of the time preference framework have some implications for theoretical and empirical asset pricing and portfolio management models. These proposed psychological discount functions have been insufficiently subjected to empirical validation.

The purpose of this paper is to test the empirical validity of these proposed psychological discount functions and to contribute to the improvement of the knowledge on this time preferences component. It addresses the following question°: what is the form and parameters of the psychological discount function that best characterizes an individual's inter-temporal decision making? This research is also focused on the question of whether a given individual applies the same inter-temporal analytical framework in all life domains.

The analysis is based on data collected through an experimental study which involved 130 students of the African Center for Advanced Studies in Management Master's degree Programs. This study took place in Dakar (Senegal) during the first semester of 2013.

The analysis confirms the hyperbolic shape of the psychological price of time that has been established in previous work. Moreover, it shows that time preferences are consistent from one situation to another. However, the analysis of the underlying discount functions shows that one individual's inter-temporal decision making can be characterized by two or more different psychological discount rate shapes, depending on the domain.

The remainder of the paper is organized as follows: The first section is dedicated to a literature review on time preference characteristic. Data collection methodology is presented in the second section. The third section presents the empirical analysis. The fourth section presents results. Finally, the paper ends with a conclusion.

2 Literature review

The psychological discount rate can be defined as the ratio between welfare associated with the immediate enjoyment of income and the welfare associated with its use in the future (Mises, 1949)[2]. According to the discounted utility theory, the psychological discount rate is a key decision parameter in an inter-temporal situation. Moreover, it assumes that this parameter is specific to each person.

A person with a low discount rate tends to give a relatively high weight to welfare attached to an amount of goods expected in the future. This person will sacrifice today's welfare in anticipation of a future gain. In contrast, an individual with a high psychological discount rate gives significant weight to the present situation and today's welfare. When making a decision, this person privileges that pay an immediate welfare or short-term welfare, ignoring the future price of this choice. In this context, an in-depth analysis of an individual or a company

[2] Cited by Mulligan (2007), p. 27.

discount rate is crucial to understand their choices in every life situation. In particular, it will be helpful to understand the trade-off between current consumption and saving and between the various assets in legacy management. It determines the equilibrium between supply and demand that prevails in the financial system.

The discounted utility theory provides a comprehensible framework for understanding individual time preference. Even so, its background and conclusions have not withstood the empirical tests, as shown in the literature review proposed by Frederick et al. (2002). Based on a meta-analysis of approximately 40 empirical tests conducted between the late 1970s and early 2000s, the authors draw three main conclusions. The first relates to the extreme variability of estimated psychological discount rate. The second, meanwhile, establishes that a longitudinal analysis does not show an improvement of estimated psychological discount rate. This indicates that the employed inquiry methods were not refined over the years. The third finding is that, in much of the research, the estimated psychological discount rates were relatively high.

Moreover, Frederick et al. (2002) proposed an explanation of the possible causes of this discrepancy in the estimations. Three factors could explain this discrepancy: the psychological discount rate estimation method used, the inquiry tools used to allow subjects to reveal their psychological discount rate, and an inaccuracy in the psychological discount rate construct.

In addition to these conclusions, the results of these studies tend to question the validity of the discounted utility theory as a framework to understand an individual's time preferences. This theory faces 12 anomalies, widely documented by Thaler (1981), Benzion et al. (1989), Loewenstein and Thaler (1989), Loewenstein and Prelec (1992), Green et al. (1997), Frederick et al. (2002), Petry (2001), and supplemented by Tsukayama and Duckworth (2010) and Scholten and Read (2010). These anomalies are related to hyperbolic decreasing of the discount rate (‘‘hyperbolic discounting’’), the sensitivity of the psychological discount rate to a common difference (‘‘common difference effect’’), the influence of the reward size (‘‘magnitude effect’’), the influence of the sign of the expected flux on the psychological discount rate (‘‘sign effect’’), the psychological discount rate difference by trade-off media (‘‘domain effect’’), the difference of psychological discount rate between delaying or speeding up (’’delay – speed up asymmetry’’), the preference for an increasing sequence (‘‘preference for improving sequences’’), the violation of the independence assumption (‘‘violations of independence and preference spread’’), strong sub-additivity (‘‘strong sub-additivity’’), and the separability (‘‘separability’’).

These discounted utility theory anomalies sparked research on the time preferences characteristics and their implications for public policy driving. The authors conducted these studies in three directions:

- ✓ the improvement of the basic model by changing the instantaneous utility function (Leigh, 1986; Prelec & Loewenstein, 1991; Ahlbrecht & Weber, 1997; Yaari, 1965[3]; Bommier, 2006 ; Detemple & Zapatero, 1991; Constantinides, 1990; Wathieu, 1997; Wachter, 2002; Gomesa et al, 2003; Loewenstein et al., 2003; Loewenstein & Prelec, 1992; Eswaran & Oxoby, 2008; Loewenstein, 1987 ; Caplin & Leahy, 2001);
- ✓ the improvement of the basic model by changing the psychological discount rate function (Harvey, 1986 [4]; Mazur, 1987[5]; Laibson, 1997; Loewenstein & Prelec, 1992; Barry et al., 1996; Scholten & Read, 2006); and
- ✓ the questioning of the original framework and the establishment of a more suitable framework than discounted utility theory (Read et al., 1999; Thaler & Shefrin, 1981; Shui & Ausubel, 2005; Gul & Pesendorfer, 2003; Killeen, 2009; Scholten & Read, 2010).

The purpose of these studies was to provide a better description of time preference characteristics and provide an explanation of behavior that are not consistent with the basic version of discounted utility theory.

In the first two categories of studies, the researchers targeted the modification of the inter-temporal utility function in order to explain behaviors identified as anomalies by Samuelson’s (1937) discounted utility theory. Thus, an individual decision is based on the inter-temporal utility function optimization defined as follows:

$$U(C) = \sum_{s=1}^{\infty} D(t,s)u(C)$$

The first proposals were focused on modifying the instantaneous utility function u(C) and introduce habit formation, projection bias, reference point, Veblen function, and expectations utility models. Their adoption has made relevant contributions in anomalies rationalization and were applied in economics and management, including a habit formation model that helped to rationalize the equity puzzle.

[3] Bommier, (2006), p. 1223.
[4] Cited by Scholten and Read (2006).
[5] Cited by Petry, (2001), p. 244.

In the second category, some researchers aimed to modify the psychological discount function to better fit evidence provided by empirical tests. To date, there have been eight major contributions in this area (see Table 1, below).

Table 1 : List of psychological discount term structure forms

Term structure form	Formula	Parameters restriction*
Hernstein	$D(t,s) = \frac{1}{1+s}$	α = β = λ = υ = 1
Proportional	$D(t,s) = \frac{1}{1+\alpha s}$	α = β = 1
Exponential	$D(t,s) = \frac{1}{(1+\alpha)^s}$	α = β λ = 1 υ = 0
Harvey	$D(t,s) = \frac{1}{(1+s)^\beta}$	α = λ = υ = 1
Laibson	$D(t,s) = \begin{cases} 1 & if \quad s=1 \\ \frac{\beta}{(1+\alpha)^s} & if \quad s>1 \end{cases}$	α = β λ = 1 υ = 0
Rachlin	$D(t,s) = \frac{1}{1+\alpha s^\upsilon}$	α = β λ = 1
Hyperbolic	$D(t,s) = \left[\frac{1}{1+\alpha s}\right]^{\frac{\beta}{\alpha}}$	λ = υ = 1
Generalized Hyperbolic	$D(t,s) = \left[\frac{1}{1+\alpha\left((t+s)^\lambda - t^\lambda\right)^\upsilon}\right]^{\beta/\alpha}$	

t: departure date, s: maturity
*** Parameters restriction is relative to the derivation of the given form from the generalized hyperbolic discount function**
Source : Scholten and Read (2006)

Each psychological discount function proposal aims to take into account the hyperbolic decreasing of the psychological discount rate observed in empirical tests of Samuelson's (1937) proposition. Note that the generalized hyperbolic discount function is the most successful form that aims to rationalize the violation of inter-temporal choices additivity property.

The discounted utility theory faced more radical proposition in the questioning of this paradigm, especially from a model with scheduling bias (Read et al., 1999), with multiple self (Thaler & Shefrin, 1981; Shui & Ausubel, 2005), with preference for temptation (Gul & Pesendorfer, 2003), discounted utility with additive discount factor (Killeen, 2009), and discounting by intervals (Scholten & Read, 2010) model.

All these proposals aim at improving our knowledge of an individuals' time preferences specific characteristics. In addition, they give rise to the questioning and improvement of empirical tools used for the validation of discounted utility theory and its alternatives.

Among others, the finding of psychological discount rate's divergence by domain enriched the empirical test by introducing the use of multiple trade-off medias to control the uncertainties associated with each of them (Petry, 2001; Reuben et al., 2008, 2010; Tsukayama & Duckworth, 2010) and the introduction of various trade-off media such as free time (La Bruslerie, 2015).

One of these developments concerns the improvement of the theory inquiry tools (Reuben et al., 2007; Zauberman et al., 2009; Coller & Williams; 1999; Harrison et al., 2005; Andersen et al., 2006) and the methods of estimation of psychological discount rate (Andreoni & Sprenger, 2010; Harrison et al., 2005; Andersen et al., 2008; Andreoni & Sprenger, 2010).

Finally, researchers who studied discounted utility theory identified the psychological discount rate's determinants in order to determine why some individuals have a higher psychological discount rate than others. It is, therefore, established that the psychological discount rate is specific to each individual and seems to be determined by gender, age, level of education, cognitive abilities, religion, personality traits, culture, level of income, health status, addiction and frustration, and economic environment (See Mulligan, 2007 for a literature review). However, the empirical tests imperfections require an integrated framework in order to confirm the results and establish additional conclusions about time preference determinants.

3 Experimental design and data

Data were collected through an experimental study which involved 130 students at the African Center for Advanced Studies in Management Master's degree Programs, located in Dakar (Senegal).

Each participant answered a series of 18 inter-temporal trade-off questions divided into three trade-off media: free time, monetary, and tokens. The choice of free time as trade-off media followed the contribution of La Bruslerie (2015). He has introduced this trade-off media as a best tool to measure pure time preference. Money is included as a benchmark trade-off media in order to compare the time preference parameters estimated in basic researches by relying on money and those estimated with free time. In addition, the researcher added a token as a neutral trade-off media stripped of any affection or economic value to qualify the results achieved on the other two media.

Some researchers reject the use of money as a media for measuring the time preference because its use creates an explicit or implicit reference to the market interest rate and causes a bias in the assessment of time preference parameters. Thus, the literature on discounted utility theory validation has improved with the use of other trade-off media such as alcohol (Petry, 2001), chocolate (Reuben et al., 2010), beer, chips, candies (Tsukayama & Duckworth, 2010), and free time (La Bruslerie, 2015).
Moreover, the use of three different trade-off medias allows researchers to test the consistency of the time preferences (domain by domain) and to question the assumption of uniqueness of the psychological discount function that underlies time preferences in various life circumstances.

For a given trade-off media, each participant faced six inter-temporal choices, each of which was characterized by an expected amount (M_t), an initial date (t), and a proposed delay (s). The purpose of the experiment was to identify the minimum amount required by a participant to accept the proposed postponement. The researchers chose the combinations of dates and intervals to allow testing ‘’present bias’’, super additive, sub additive, and separability of time preferences. Researchers has consequently chosen the combinations of initial date and maturity. The initial date was extended between the -current time and 12 months (0 months, 3 months, 6 months, and 12 months). The researchers spread out the proposed postponement duration to between 3 and 24 months (3 months, 6 months, 12 months, and 24 months). This combination allowed researchers to analyze the influence of maturity on the psychological discount rate in order to identify the term structure of the psychological discount rate. The association of many initial dates with a given delay is used to study the influence of the trade-off initial date and to test the present bias introduced by Laibson (1997) in modeling time preferences. For example, for a delay of 3 months, we use two initial date today and three months.
For each combination, is given to each participant a table (market price list) which shows the choice between receiving an amount expected at a given initial date and another amount promised on the final date. Table 2 (below) is similar to the table that researchers presented participants with.

Table 2 : Monetary, free-time, and token trade-off framework

Free time trade-off framework

You have 6 hours of free time today, I propose you to choose between the alternatives below. You must choose for each line, the option you prefer by checking box A or box B in the last column.

N°	Option A : Enjoy of … today	Option B : Enjoy of … in three month	Chosen option
1	6.00	6.30	A B
2	6.00	7.00	A B
3	6.00	7.30	A B
4	6.00	8.00	A B
5	6.00	8.30	A B
6	6.00	9.00	A B
7	6.00	9.30	A B
8	6.00	10.00	A B
9	6.00	10.30	A B
10	6.00	11.00	A B

Monetary trade-off framework

You are expected to receive an amount of 3,000 XOF today. The person who owe you the amount propose to delay the collection of this amount and offers you the following alternatives.
In the following table, you must choose for each line, the option you prefer by checking box A or box B in the last column.

N°	Option A : Receive … XOF today	Option B : Receive … XOF in 6 month	Chosen Option
1	3,000	3,100	A B
2	3,000	3,200	A B
3	3,000	3,300	A B
4	3,000	3,400	A B
5	3,000	3,500	A B
6	3,000	3,600	A B
7	3,000	3,700	A B
8	3,000	3,800	A B
9	3,000	3,900	A B
10	3,000	4,000	A B

Tokens trade-off framework

You have 6 hours of free time today, I propose you to choose between the alternatives below. You must choose for each line, the option you prefer by checking box A or box B in the last column.

N°	Option A : Keep your … tokens today	Option B : Receive in exchange … tokens in three month	Chosen option
1	100	100	A B
2	100	101	A B
3	100	102	A B
4	100	103	A B
5	100	104	A B
6	100	105	A B
7	100	110	A B
8	100	120	A B
9	100	130	A B
10	100	140	A B

The time trade-off treatments are part of a global experimental study which included treatments for the study of other behavior like risk and social interaction. The description of the entire experiment is available on request. The researchers build all the experiment materials in Z-tree platform (Zurich Toolbox for Readymade Economic Experiments), which was developed by Fischbacher (2007).

In summary, the various choices allowed us to collect information on the minimum quantities required to accept the postponement of the expected quantities of free time, money, or tokens. The researchers used the data collected to calculate the psychological price of time for each individual, each media, and different combinations of starting date and maturity.

Recall that if M_t is an expected quantity of goods on a given date "t," "s" is a proposed postponement maturity, and N_{t+s} is the minimum quantity of goods required to accept the postponement of the benefice of the quantity, the psychological price of time, denoted by SIR, is defined by the expression:

$$SIR_{t,s} = \left(\frac{N_{t+s}}{M_t} \right)^{\frac{1}{s}} - 1$$

Notice here that delay ''s'' is expressed in years.

The interest of the psychological price of time for the study of time preference is twofold. This indicator has the advantage of allowing a comparative analysis of one media to another. This is an indicator which is closed to the notion of interest used in finance and provides an intuitive interpretation. Note that it is the tool that has been privileged in conventional time preference analysis. This choice ensures a comparison of the results of this study with the previous.

4 Methodology

Furthermore, to allow identification of the discount function that underlies the choices made by each individual, a data modelling framework is necessary.

It is assumed that each individual is characterized by two functions: U(.) and D(., .). Function D(., .) is the psychological discount function that translated the implicit value of time associated to the delaying of goods collection from the expected date to a later one. The function U(.) is a utility function which reflects welfare associated with monetary, free time, or token quantities. These functions are such that when faced with an inter-temporal trade-off decision (money, free time, or tokens), the preferred option is dictated by maximizing the following discounted utility function:

$$U(t, s, M) = D(t, s)\, u(M) \qquad \text{(I)}$$

The operationalization of this model requires the specification of the expressions of these different functions.

For the psychological discount function, it is assumed that it has the general form, originated from the work of Scholten and Read (2006):

$$D(t,s) = \left[\frac{1}{1+\alpha\left((t+s)^{\tau} - t^{\tau}\right)^{\upsilon}}\right]^{\delta/\alpha}$$

With t the initial date, and s the delay of the proposed postponement.

The utility function is assumed to be as CRRA (Constant Relative Risk Aversion) type and defined as[6]:

$$u(M) = \frac{M^{1-\gamma}}{1-\gamma} \quad \text{avec } 0 < \gamma < 1$$

Based on these notations, when a person is faced with a choice between receiving a quantity M_t at time t or a quantity N_{t+s} at time t + s , he/she prefers to receive[7] :

- ☞ the quantity M_t expected at time t if $u(M_t) > D(t, s)\, u(N_{t+s})$;
- ☞ the quantity N_{t+s} promised at time t if $u(M_t) < D(t, s)\, u(N_{t+s})$.

The minimum quantity that he/she requires to accept the postponement of the collection of the quantity M_t expected at time t is noted as N^*_{t+s}. This quantity is characterized by the following relation:

$$u(M_t) = D(t, s)\, u(N^*_{t+s})$$

By applying the neperian logarithm to each member of the equality and integrating an error parameter μ[8], the amount of goods that makes the individual indifferent is determined by the expression:

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[6] Notice that alternatives choice can be made for the utility function; particularly it can be the exponential Constant Relative Risk Aversion (CRRA) utility function. In this case, it is necessary to replace the utility function by $u(M) = \frac{1}{\theta}\left[1 - \exp\left(-\theta M^{\gamma}\right)\right]$. However, the analysis is limited to the case of a Constant Relative Risk Aversion (CRRA) utility function for decision parameters estimation.

[7] The retained approach here is that of Scholten and Read (2006), which posits that the comparison of two amounts M_t (expected at time t) and N_{t+s} (expected at time t + s) is done through the comparison of their present value. The present value of the expected amount N_{t+s} at the latest date t + s is done in two steps. It is firstly discounted at time t; thereafter at the decision date. In this case, he prefers :

- ☞ the quantity M_t expected at time t if $D(0, t)\, u(M_t) > D(0, t)\, D(t, s)\, u(N_{t+s})$;
- ☞ the quantity N_{t+s} promised at time t if $D(0, t)\, u(M_t) < D(0, t)\, D(t, s)\, u(N_{t+s})$.

The content of the presentation stems from the simplification of the last two lines.

$$\ln\left[\frac{N^*_{t,s}}{M_t}\right]=\frac{\delta}{\alpha(1-\gamma)}\ln\left[1+\alpha\left((t+s)^{\tau}-t^{\tau}\right)^{\upsilon}\right]+\mu$$

This expression defines the link between this amount and an individual's time preference parameters. Notice that these parameters represent the psychological discount function characteristic (δ, α, τ, υ), the utility function (γ), and the error term (μ).

Moreover, these parameters are assumed to depend on the individual and the trade-off media (monetary, free time, or token), so that we can refine the previous equation as follows:

$$\ln\left(\frac{N_{t,s}}{M_t}\right)=\frac{\delta_{i,k}}{\alpha_{i,k}\left(1-\gamma_{i,k}\right)}\ln\left[1+\alpha_{i,k}\left((t+s)^{\tau_{i,k}}-t^{\tau_{i,k}}\right)^{\upsilon_{i,k}}\right]+\mu_{i,k,j} \qquad \text{(II)}$$

In addition, the parameters can be decomposed according to the following system [9]:

$$\begin{array}{llllllll} Ln(\alpha_{i,k}) & = & \alpha & + & \alpha_k & + & \alpha_i & & (II.1)\\ Ln(\delta_{i,k}) & = & \delta & + & \delta_k & + & \delta_i & & (II.2)\\ Ln(\tau_{i,k}) & = & \tau & + & \tau_k & + & \tau_i & & (II.3)\\ Ln(\upsilon_{i,k}) & = & \upsilon & + & \upsilon_k & + & \upsilon_i & & (II.4)\\ Inv.Logit(\gamma_{i,k}) & = & \gamma & + & \gamma_k & + & \gamma_i & & (II.5)\\ \mu_{i,k,j} & = & \mu & + & \mu_k & + & \mu_i & + \ \varepsilon_{i,k,j} & (II.6) \end{array}$$

In this system, the following notations are used:

- i represents the individual (i = 1, …, N) ;
- k represents the trade-off media (k = 1, …, K) ;
- j represents, for a given support, the trade-off line (j = 1, …, J) characterized by an initial date t (t= 1, …, T), the expected amount (or a quantity) M_t and a proposed delay maturity s (s = 1, …, S).

In this system of equations, each time preference parameter (δ, α, τ, υ) and the coefficient of risk aversion (γ) is split up into three components: global, trade-off media, and decision maker. It captures the specificity of each participant[10].

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[8] Hey and Orme (1994) in their work on individual risk behavior parameter introduced this error term to capture errors committed by experimental study participants regardless of their intrinsic time preference parameters.

[9] InvLogit function which is involved in the parameter γ definition is the reciprocal bijection of the logistic function defined by the expression $\log it(x)=\frac{e^x}{1+e^x}=\frac{1}{1+e^{-x}}$. The explicit expression of the InvLogit function is given by $Inv\log it(x)=\log\left(\frac{x}{1-x}\right)$. By using it, this parameter is always between 0 and 1 as constrained by the Constant Relative Risk Aversion (CRRA) utility function definition.

[10] Note that this component is likely to depend on the individual characteristics of the respondent. This dependence will be tested when studying the time preference influencing factors. It will be done by studying the link between this component and, among others, personality traits, gender, age and religious commitment.

The estimation method is expected to test the constancy or lack thereof of each component. In addition, to provide estimations of the other psychological discount function (exponential, Herrnstein, Harvey, proportional, Laibson, Rachlin, hyperbolic, generalized hyperbolic) form, the relation between the time preference parameters and the time trade-off characteristics is established. These variants of equation (II) are given in Table 3 (below).

Table 3 : Relation between required compensatory amount and time preferences parameters

Discount function	Formula
Hernstein	$\ln\left(\frac{N_{t,s}}{M_t}\right) = \mu_{i,k,j} + s\frac{\ln(1+\delta_{i,k})}{1-\gamma_{i,k}}$
Proportional	$\ln\left(\frac{N_{t,s}}{M_t}\right) = \mu_{i,k,j} + \frac{1}{1-\gamma_{i,k}}\ln(1+s)$
Exponential	$\ln\left(\frac{N_{t,s}}{M_t}\right) = \mu_{i,k,j} + \frac{\ln(1+\alpha_{i,k}s)}{1-\gamma_{i,k}}$
Harvey	$\ln\left(\frac{N_{t,s}}{M_t}\right) = \mu_{i,k,j} + \delta_{i,k}\frac{\ln(1+s)}{1-\gamma_{i,k}}$
Laibson	$\ln\left(\frac{N_{t,s}}{M_t}\right) = \mu_{i,k,j} - \frac{\beta_{i,k}}{1-\gamma_{i,k}}I_{\{s>0\}} + s\frac{\ln(1+\delta_{i,k})}{1-\gamma_{i,k}}$
Rachlin	$\ln\left(\frac{N_{t,s}}{M_t}\right) = \mu_{i,k,j,l} + \frac{\ln(1+\alpha_{i,k}s^{\upsilon_{i,k}})}{1-\gamma_{i,k}}$
Hyperbolic	$\ln\left(\frac{N_{t,s}}{M_t}\right) = \mu_{i,k,j,l} + \frac{\delta_{i,k}}{\alpha_{i,k}(1-\gamma_{i,k})}\ln(1+\alpha_{i,k}s)$

This table presents the link between required compensatory amount (N_{t+s}), the expected amount of goods (M_t), the date on which it is expected (t), the proposed length of the delay (s) for the alternatives modelling framework and the time preference parameters of the decision maker ; especially, for exponential, Hernstein, Harvey, proportional, Laibson, Rachlin and hyperbolic discount function. These equations complete the equation (II).

Equation (II) is that of a non-linear mixed model [11] proposed by Tran (2003) [12]. It is characterized by parameters that are likely to be different from one person to another, from one trade-off media to another. However, the link between the variable and the parameters of interest is nonlinear. This situation adds a complexity in the estimation method choice. In the current case, the estimation should be made under the R software using the Non Linear Mixed-Effect Models (nlmer) package developed by Pinheiro and Bates (1995) based on Lindstrom and Bates (1990)[13]. However, because of an inability to obtain the results from the platform, the researchers used the package “BB,” developed by Varadhan and Gilbert (2009) and available in R, for parameter estimation using the maximum likelihood method.

[11] Note that contrarily to simple linear model, the model used here is called mixed because the parameters of the linear model are not constant for the whole population but are themselves distributed according to a log-normal distribution whose determinants (mean and standard deviation) vary from one individual to another.
[12] Cited by Hole (2007).
[13] Cited by Pinheiro and Bates (1995)

Moreover, due to a possible problem of over identification, the researchers chose to estimate the model by a group of persons. In this sense, the researchers adopted an approach consisting of dividing individuals into categories. In each category, individuals were the most similar possible according to time preference and the most dissimilar possible from one category to another. To achieve this, the researchers performed a classification on the factors of a multiple factor analysis (MFA) with psychological prices of free time, money, and token as active variables. The groups of variables were modeled on different trade-off media. This approach help the researchers to obtain a group of persons. The researchers estimated the psychological discount function by class.

For a class of people, the researchers estimated the time preference parameters by assuming that a person's preferences were characterized by each psychological discount function. Thereafter, the discount function corresponding to the model that minimizes the Akaike information criterion[14] was attributed to that person.

The application of this methodology provided the characterized term structure of psychological discount rate and parameters estimation for each participant.

This framework is more complete than traditional analysis of the term structure of psychological discount rate and reflects the recent developments of research in the area, including Benhabib et al. (2010) and Coller et al. (2012). This improvement is obtained with a greater formalization complexity and estimation procedures, which is the price of refining researchers' knowledge of time preference specificity.

5 Results

The researchers present the results of the time preference analysis by relying on the psychological price of time and the estimated parameters. The researchers present these results in two sub-sections. The researchers first present the characteristics of psychological price of time. Then the researchers present the results of the analysis of the term structure of psychological price of time.

5.1 Characteristics of the psychological price of time

The researchers conducted the analysis of the psychological price of time through options offered (trade-off media, the initial date, and the maturity) to the participant. The results indicate that, on average, and on an annual basis, a participant requires approximately 2.9 times of the expected quantity of free time, money, or token to accept a one-year delay from

[14] Note that this criterion seems the well-adapted one because the term structures of psychological discount rate do not have the same number of parameters. The Akaike criterion is more efficient than the others. In particular, it is more appropriate than an assignment based on the sum of squared errors or, equivalently, on the R Square.

the initial date. This price, however, is characterized by a high degree of heterogeneity in the population and in terms of characteristics of the submitted trade-off.

The standard deviation of psychological price of time is relatively high; the coefficient of variation is approximately 1.54. This high heterogeneity is translated in the range (difference between the highest and the lowest discount rate), which is approximately 1499%. This high heterogeneity seems due to the person’s behavior changing, depending on the trade-off media, the initial date, and the maturity.

5.1.1 Time preference consistency

The estimated psychological price of time shows that it is, on average, lower for monetary and higher for the token. The mean comparison test confirms that the observed differences are statistically significant.

Indeed, the results of the Welch mean comparison test supported by a variance comparison test established that in its inter-temporal decisions, an individual preferred option depended on the purpose (free time, monetary, or tokens) of the decision. It is important to understand the nature of the difference. Indeed, if the difference lies only in the mean, then this result does not question the consistency of time preference. Contrarily, if the differences intervene in the direction of the preference, then this would call into question the existence of a discounting behavior. This would be the case of a person who discounts (would require a positive compensation) the welfare associated with a particular good and, in contrast, would capitalize (disposed to pay for) for other goods.

This result must be supplemented by an analysis of the relationship between the psychological price of time applied to each trade-off media.

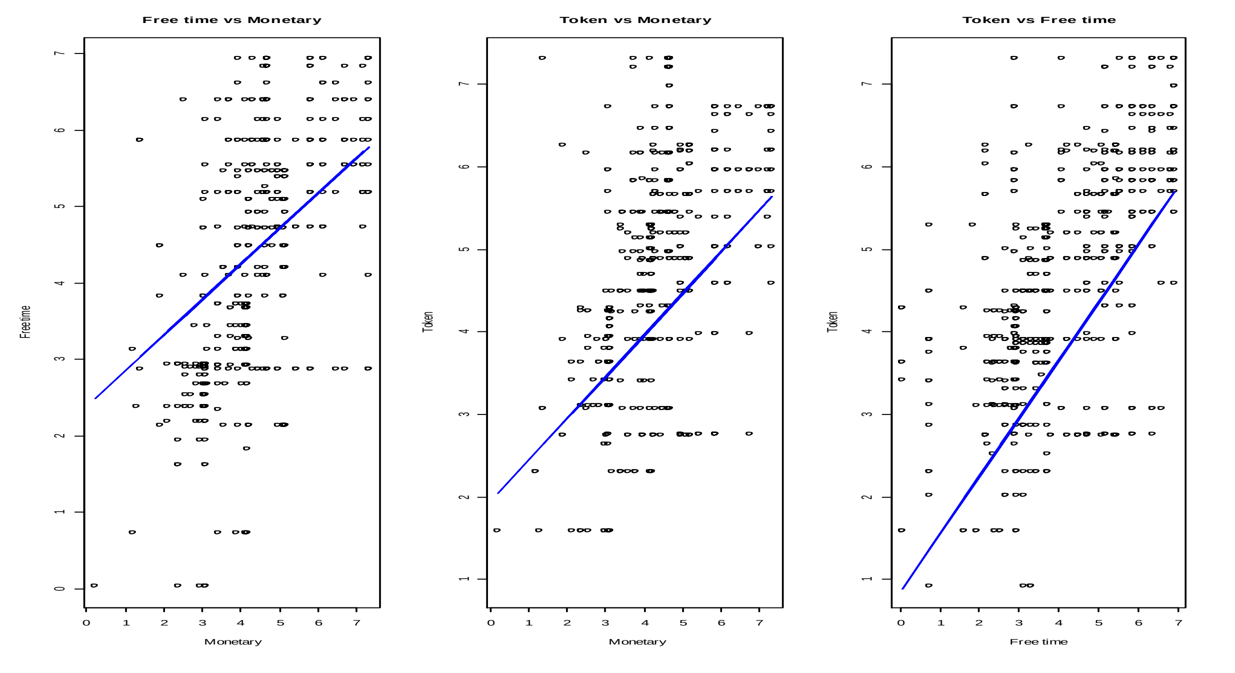
The time preference consistency implies that a person who has a strong time preference for free time should have a strong monetary or token time preference.

To investigate the consistency of time preference, the researchers analyzed the correlations between psychological prices calculated for the different trade-off media. The results are shown in Figure 1 (below).

It is clear from this figure that there is a positive and statistically significant correlation between psychological prices of time calculated for each of the trade-off media. This result confirms the existence of a consistent time preference that governs time trade-off decision in every life situation. The linear relationship between these three psychological prices of time is shown in Figure 1 (below).

The pairwise graph in each plan shows an elongation, which is the translation of a linear relationship between these different psychological prices of time.

Figure 1 : Psychological price of time by trade-off media



ρ = 0.5900 p value = 0.0000 | ρ = 0.5800 p value = 0.0000 | ρ = 0.5500 p value = 0.0000

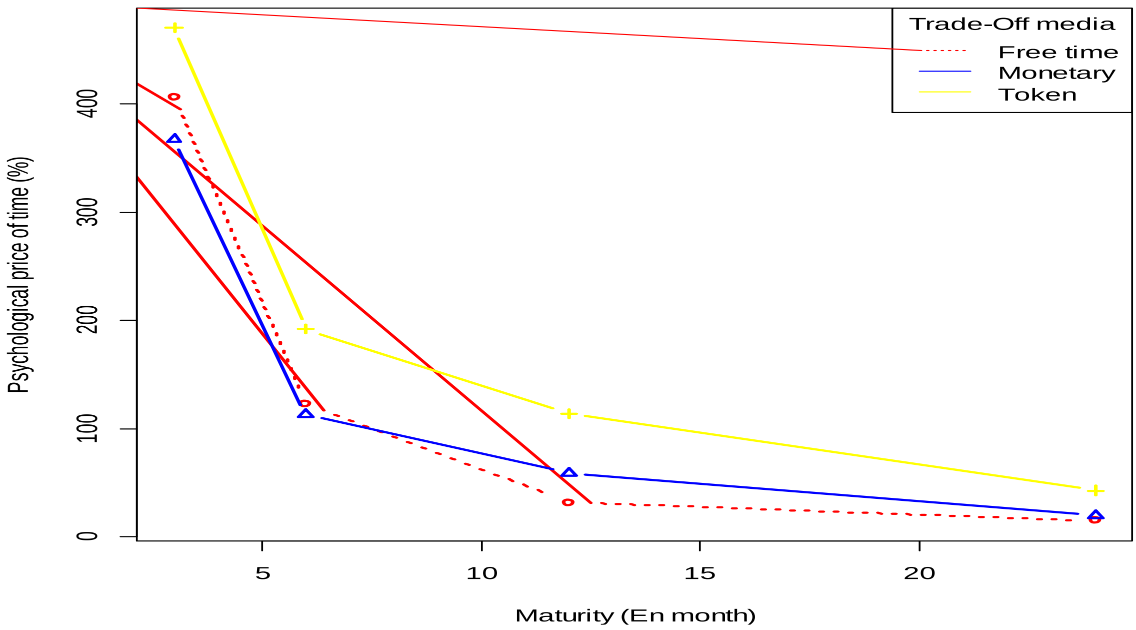
The figure shows pairwise graph with on horizontal axis the psychological price of time of a given trade-off media and at the vertical axis that of the others. On each of the component, the legend shows the trade-off media pair which is shown. In addition, a regression line is added. Correlations are calculated between the logarithms of the variables.

Note that the coefficient of correlation between the logarithms of the variables is higher than that computed between the absolute variables. A logarithmic relationship between psychological prices of time is adopted. Also, it is worth noting that even though it is statistically significant, the correlation coefficient is relatively low. It is an average equal to 0.58. This weakness reflects the fact that some people have, all things being equal, a lower psychological price of time than the mean in the free time trade-off and a higher psychological price of time than the mean for the two other trade-off media. These results indicate that researchers must further and deeply explore the specificity of time preference. The analysis of psychological price of time determinants is expected to enhance researchers' knowledge in this field.

5.1.2 Term structure of psychological price of time

The psychological price of time term structure is its relation with the length of the proposed postponement. Figure 2 (below) shows the relationship between these two variables. This graph shows that the relationship is hyperbolic (decreasing) in nature, which is consistent with previous studies that established a similar relationship between the psychological price of time and delay maturity.

Figure 2 : Term structure of psychological price of time by trade-off media



The figure is based on the average price of time for a given initial date and maturity.

This relationship shows the degree of impatience of participants in experiments. Indeed, they are more likely to accept a low compensation to defer the benefit of particular goods when it is expected for a long maturity, but when the collection date approaches, the compensatory amount required to accept the postponement increases exponentially. This preference reflects a decreasing impatience.

Particular emphasis should be placed on the results of free time's the term structure psychological price of time. Indeed, it follows the same logic as other goods (primary, ...) that were used as trade-off media such as chocolate (Reuben et al., 2010), beer, chips and candy (Tsukayama & Duckworth, 2010) or alcohol (Petry, 2001). Notice that this hyperbolic configuration has inspired the study of linear coefficient of correlation between logarithms of psychological prices introduced in the previous section.

To allow the researchers to formalize this relationship, they regressed psychological price of time on the maturity assuming two possible forms:

$$SIR_{t,s} = Ae^{-bs} \qquad SIR_{t,s} = a + \frac{b}{s} \qquad \text{with s the maturity}$$

These models are supplemented by the linear model as a benchmark. Although it is presumably of no interest, because of the form of Figure 2, it seems interesting to include it as a candidate model. The researchers estimated these equations through analysis of variance (ANOVA) with mixed effects[15]. The implementation of the model must take into account

[15] This technique has been proposed by Laird and Ware (1982) and help to estimates the equations by taking into account the specific behavior in relation to different trade-off media (free time, monetary and tokens) and each

whether the effect attributed to a trade-off media is fixed or random. For each function, the model admits four variants depending on the retained assumption of the trade-off media influence. It is essential to identify the model that best fits the data. Following the methodology of mixed effect ANOVA, the researchers compared the estimated models by maximum likelihood according to Akaike or Schwartz information criteria. The most suitable model to describe the relationship between the psychological price of time and maturity is the exponential function ($SIR_{t,s} = Ae^{-bs}$). Indeed, for each formulation of the random effect model, this model is better than its competitors (linear and inverse). The second step is the comparison of different formulations of fixed and random effects of the exponential model.

The psychological price of time is linked to the maturity by an exponential function, and the use of different trade-off media does not bring uncertainty in the estimation of the psychological discount function. This conclusion comes from the fact that the incorporation of the trade-off media as fixed or random form in the model is not statistically relevant. Indeed, Akaike and Bayesian information criteria are minimal for this function.

Even so, the results of statistical tests on the estimation residuals, including Grubbs outliers and normality test, indicated that the chosen model must be improved because its statistical properties are unsatisfactory. Indeed, the residue is not distributed according to a Gaussian distribution and contains outliers.

The researchers propose an improved model by introducing the initial date as an indicator variable that is equal to 1 when the original date is today, and 0 otherwise. The introduction of this variable permits the test of the present bias introduced by Laibson (1997) in the term structure of the psychological discount rate model.

The researchers again estimated three models by introducing this variable. The results of these estimates indicated that there was a present bias in time preference because the new variable was significant in each model and helped to significantly improve the explanatory power of each model. Among the improved models, the exponential model with present bias was the best according to the information criterion. Thus, the results of the final model are as follows:

individual. In our case, the adoption of this technique is to assume that the relationship between the psychological price of time and maturity can be translated by an equation of the form:

$$SIR_{t,s} = f(s) + a_k + a_i + a_{i,k} + u_k + u_i + u_{i,k}$$

with i individual, k the trade-off media, $u_k \approx N(0, \sigma_k)$, $u_i \approx N(0, \sigma_i)$ et $u_{i,j} \approx N(0, \sigma_{i,j})$ independent.

Terms a_k, a_i, $a_{i,k}$ are constant. This form help to take into account the specific discount rate of each participant and each trade-off media. Remind that the suitable equation form depend on the data and it can be limited to some relatively simple cases; for example, a form without random component can be suitable for a given trade-off media.

$$\log(SIR_{i,k,t,s}) = \underset{(0.0765)}{5.4165} - \underset{(0.0022)}{0.1480s} + \underset{(0.0351)}{0.1568I_{\{Monétaire\}}} + \underset{(0.0344)}{0.2028I_{\{t=0\}}} + \underset{(0.7833)}{u_i} + \underset{(0.7503)}{u_{i,k,t,s}}$$

$$\text{with } u_i \approx N(0,\sigma_i) \text{ and } u_{i,k,t,s} \approx N(0,\sigma)$$

i: the individual; k: the trade-off media; t: initial date; s: maturity of the postpone

These results are consistent with the hyperbolic discount function, as well as those of La Bruslerie (2015).

These results call for three interpretations. First, they confirm the hyperbolic relationship between the psychological price and the proposed maturity, as indicated by the negative sign of the maturity coefficient. Moreover, there is "present bias " with a positive sign. Thus, when the proposed trade-off is for an amount (quantity) expected today, the psychological price is higher than if the same delay and amount is proposed to be delayed from an initial date different from today. These results are consistent with Laibson's (1997), who proposed a discount function which takes this effect into account. Finally, the difference between the psychological price of time by trade-off media takes place between the monetary and the other trade-off media.

We find in the final model an effect attributable to the preference of each individual in the various trade-offs. This component can be used to study the determinants of time preference and would allow researchers to identify individual characteristics (demographic, social, personal, and social position) that significantly influence this component.

Overall, the analysis of psychological price of time shows a hyperbolic relationship between the psychological price of time, as well as a "present bias”. Furthermore, in terms of level, only monetary preferences exhibit a significant difference compared with other trade-off media. Notice also the absence of an adverse effect on each trade-off media.

5.2 Form and parameters of the term structure of psychological discounting rate

Recall that the psychological discount rate measure the loss of well-being resulting from the adjournment of the collection of an expected amount of goods expected at a given date, or, equivalently, the gain in welfare resulting from adjournment of the acquittal of an invoice or the registery of a loss. The objective of this section is to present the psychological discount functions (or combination) that best explains the inter-temporal choice behavior observed in the experiments. Recall that there are eight candidate discount functions which correspond to eight models that specify, for each one, the relationship between choices and decision parameters.

The researchers divide the results of the identification of the psychological discount function which governs the decisions made by each participant in the inter-temporal trade-offs into four sub-sections. The researchers first discuss the validity of each psychological discount function for the studied population. Next, the researchers present the allocation of psychological discount function to each classes of time preferences. Therefore, the researchers discuss the distribution of the population according to the characterized psychological discount function. The researchers conclude the section with a discussion on the homogeneity of time preference by trade-off media.

5.2.1 Comparison between psychological discount functions parameters estimations

Recall that the researchers adopted an approach consisting in dividing individuals into categories. In each category, individuals were the most similar possible according to time preferences and the most dissimilar possible from one category to another. To achieve this, the researchers performed a classification on the factors of a multiple factor analysis (MFA) with psychological prices of free time, money, and token as active variables. Due to space limitation, the researchers do not present the detailed results of this categorization here.

The researchers grouped individuals into five categories. The use of traditional criteria suggests a grouping into three classes. The choice of five classes responds to the researchers' desire to have more specific groups in terms of time preference specificities.

For each class, the researchers assumed that individuals classified therein were homogenous by time preference characteristics. On this basis, the researchers estimated the different discounted functions parameter for each group. Table 4 (below) summarizes the estimations.

Before discussing the allocation of discount functions to each category, it is necessary to question the validity of the estimated parameters for each of these functions.

To achieve this, the researchers referred to Scholten and Read (2006), who discussed the admissible range of psychological discount functions parameters. Scholten and Read (2006) established that the parameter delta (δ) of the exponential function, the parameters alpha (α) and delta (δ) and hyperbolic functions of generalized hyperbolic discounting psychological must be positive, and the vega parameter (υ) of the latter must be greater than 1 and the parameter tau (τ) is expected to be between 0 and 1. Following Scholten and Read (2006), we established similar requirements for alpha parameters (α), delta (δ), and beta (β) of Mazur, Harvey, and Laibson discount functions which must be positive. The researchers can also recall that the parameters alpha (α) and vega (υ) of Rachlin discount function must be positive. For each of these frameworks, the gamma (γ), which characterizes the instantaneous utility function, must be between 0 and 1.

Table 4: Synthesis of pychological discount function parameters estimations

		Class 01			Class 02			Class 03			Class 04			Class 05		
		Free time	Monetary	Token	Free time	Monetary	Token	Free time	Monetary	Token	Free time	Monetary	Token	Free time	Monetary	Token
Exponential	δ	0,10	0,51	0,01	0,05	0,03	0,19	0,14	0,40	0,01	0,08	0,04	0,41	0,06	0,04	0,32
	γ	0,01	0,01	0,98	0,50	0,81	0,17	0,01	0,01	0,98	0,57	0,83	0,20	0,53	0,83	0,19
Herrnstein	γ	0,09	0,01	0,43	0,06	0,02	0,08	**0,02**	**0,04**	**0,23**	**0,02**	**0,03**	**0,27**	0,01	0,21	0,04
Harvey	α	0,10	0,51	0,01	0,05	0,04	0,19	0,06	0,03	0,32	0,08	0,04	0,42	0,22	0,58	0,01
	γ	0,01	0,01	0,98	0,50	0,81	0,17	0,52	0,81	0,19	0,57	0,83	0,20	0,01	0,01	0,98
Proportional	α	0,01	0,04	0,96	0,14	0,42	0,01	0,15	0,44	0,01	0,18	0,60	0,01	0,21	0,56	0,01
	γ	0,50	0,74	0,22	0,01	0,01	0,98	0,01	0,01	0,99	0,01	0,01	0,99	0,01	0,01	0,99
Laibson	β	0,92	0,01	1,56	**0,08**	**0,01**	**0,58**	0,04	0,00	0,02	0,04	0,01	0,02	0,04	0,01	0,02
	δ	4,33	0,87	6,53	**0,02**	**0,08**	**4,82**	0,01	0,01	0,04	0,01	0,01	0,05	0,01	0,01	0,05
	γ	0,04	0,04	0,28	**0,08**	**0,05**	**0,15**	0,42	0,98	0,94	0,42	0,98	0,95	0,42	0,98	0,95
Rachlin	α	0,70	0,01	2,15	0,08	0,00	0,01	0,07	0,00	0,01	0,10	0,00	0,01	0,09	0,01	0,01
	υ	2,23	0,99	7,15	0,01	0,01	0,13	0,01	0,01	0,13	0,01	0,01	0,12	0,01	0,01	0,13
	γ	0,04	0,04	0,28	0,49	0,98	0,97	0,47	0,98	0,97	0,51	0,98	0,98	0,50	0,99	0,98
Hyperbolic	α	0,03	6,24	0,01	3,03	21,87	131,55	0,11	0,01	0,02	0,11	0,01	0,02	0,11	0,01	0,02
	δ	0,22	0,25	0,45	0,13	0,04	0,04	0,01	0,01	0,03	0,01	0,01	0,04	0,01	0,01	0,04
	γ	0,05	0,18	0,24	0,10	0,01	0,09	0,54	0,98	0,93	0,54	0,98	0,94	0,54	0,98	0,94
Generalized Hyperbolic	α	**2,09**	**0,05**	**0,84**	2,09	0,05	0,84	14,26	0,13	3,17	14,47	0,14	2,95	**14,31**	**0,14**	**3,19**
	δ	**0,83**	**6,07**	**0,36**	0,83	6,07	0,36	0,01	0,05	0,47	0,01	0,04	0,54	**0,01**	**0,04**	**0,51**
	τ	**0,18**	**0,26**	**0,01**	0,18	0,26	0,01	0,57	0,21	15,56	0,57	0,20	15,73	**0,57**	**0,20**	**14,32**
	υ	**0,03**	**0,22**	**0,02**	0,03	0,22	0,02	0,72	1,94	0,97	0,73	1,91	0,98	**0,72**	**1,92**	**0,89**
	γ	**0,45**	**0,01**	**0,27**	0,45	0,01	0,27	0,08	0,78	0,03	0,08	0,76	0,03	**0,08**	**0,77**	**0,03**

We present in this table a synthesis of the eight psychological discount function parameters estimations

These results indicate that the overall parameters obtained for the groups are within the intended range for each discount function.

The researchers were surprised by some of these results. For example, the alpha coefficient (α) of the free time's hyperbolic discount function for Class 5 individuals seemed extremely high. Even if the estimation was consistent with theoretical range, its value seems unrealistic. It is difficult to interpret it as a decision parameter since it assumes a discount rate of 1,400%.

5.2.2 Allocation of psychological discount functions to classes

Table 5 (below) presents the criteria used to assess the relevance of each of the discount functions for different classes of people. It appears from this table that the Herrnstein discount function can be assigned as time preferences of classes 03 and 04 individual. Classes 01 and 05 individuals are characterized by the generalized hyperbolic discount function. The preferences of class 02 individuals, in turn, can be characterized by the Laibson discount function.

Table 5 : Psychological discount function attribution criteria

			Discount function							
			Expo.	**Herrn.**	**Harvey**	**Propor.**	**Laibson**	**Rachlin**	**Hyper.**	**Gen. Hyper.**
# of parameters			**2**	**1**	**2**	**2**	**3**	**3**	**3**	**5**
Class 01	**Obs : 2**	**LL**	-36	-41	-36	-29	-33	-30	-28	-72
		AIC	-161	-173	-162	-149	-156	-149	-146	**-233**
Class 02	**Obs : 10**	**LL**	-16	-25	-13	-16	-238	-19	-19	-360
		AIC	31 828	12 477	31 835	31 827	**-65 277**	-64 839	-64 839	-10 287
Class 03	**Obs : 28**	**LL**	-92	-111	-92	-92	-89	-86	-86	-124
		AIC	363	**311**	364	364	383	390	390	343
Class 04	**Obs : 12**	**LL**	-27	-38	-27	-26	-29	-24	-30	-303
		AIC	1 958	**1 757**	1 958	1 960	2 164	2 173	2 163	2 164
Class 05	**Obs : 78**	**LL**	-259	-275	-258	-248	-261	-211	-248	-334
		AIC	-113	-153	-111	-90	-109	-9	-83	**-239**

LL: Logarithm of the model likelihood; AIC: Akaike criteria

This allocation is based on the minimization of the Akaike criterion associated with each model.

Based on these results, three discount functions—Herrnstein, Laibson, and generalized hyperbolic—characterize the time preferences of the studied population.

No preference was characterized by exponential discount function. These results confirm the violation of the assumption of the theory of discounted utility with exponential discount factor.

5.2.3 Distribution of the population by the psychological function discount

Analysis of the data in Table 6 (below) allowed the researchers for this study to establish that the population was distributed on three psychological discount functions: Herrnstein, generalized hyperbolic, and Laibson. Individuals characterized by generalized hyperbolic discount function represented fewer than two-thirds of the population.

These results led the researchers to conclude that the studied population was characterized by homogeneity in relation to the psychological discount function that governs individual preferences. The researchers also found that none of the individuals was characterized by the exponential discount function. This finding was consistent with previous work that documented the violation of the exponential discount function psychological premise (Loewenstein and Thaler, 1989; Loewenstein and Prelec, 1992).

Table 6: Population distribution according to psychological discount function characteristics

Term structure	Frequency	Percent (%)
Exponential	0	0,0
Harvey	0	0,0
Herrnstein	40	30,8
Hyperbolic	0	0,0
Generalized Hyperbolic	80	61,5
Laibson	10	7,7
Proportional	0	0,0
Rachlin	0	0,0
Ensemble	130	100

Before discussing the implications of these results, the researchers consider the problem of the consistency of time preference depending on the purpose of the trade-off.

5.2.4 Homogeneity of time preference trade-off media

To study the homogeneity of individual time preferences, the researchers for this study compared the characteristics of global psychological discount functions with those depicted in monetary, free time, and token trade-offs. To achieve this, the researchers estimated the term structure of the psychological discount rate implicit to preference by trade-off media.

Thereafter, the researchers used the same principle of psychological discount function allocation used on the global level. Once a psychological discount function had been assigned to each trade-off media, the researchers compared the discount function assigned on the global level to the one depicted by limiting themselves to the decisions related to each trade-off media. The consistency of the inter-temporal trade-off rationale should induce the same shape of the discount function for all media.

The researchers present the synthesis of the results of this work in Table 7 (below), which presents the overall discount function in each column and the discount function for trade-off media by row. Indeed, as at the global level, the researchers conducted a grouping of individuals into categories of homogeneous time preferences determined by the classification of the factors of Principal Component Analysis (PCA) on psychological price associated with the given medium.

Table 7: Comparative analysis of psychological discount functions characteristics by trade-off media

		All trade-off media			Ensemble
	Model	**Herrnstein**	**Hyper. Gen.**	**Laibson**	
Free time trade-off	**Herrnstein**	**28**	19	9	56
		70,0%	*23,8%*	*90,0%*	*43,1%*
	Hyper.	12	61	1	74
		30,0%	*76,3%*	*10,0%*	*56,9%*
Monetary trade-off	**Herrnstein**	**7**	3	5	15
		17,5%	*3,8%*	*50,0%*	*11,5%*
	Gen. Hyper.	3	**0**	1	4
		7,5%	***0,0%***	*10,0%*	*3,1%*
	Propor.	30	77	4	111
		75,0%	*96,3%*	*40,0%*	*85,4%*
Token trade-off	**Herrnstein**	**35**	41	10	86
		87,5	*51,3*	*100,0*	*66,2*
	Hyper.	5	39	0	44
		12,5%	*48,8%*	*0,0%*	*33,8%*
Ensemble		40	40	80	10

The table shows the distribution of individuals by psychological discount function assigned after parameters estimation. In Column, we represent the discount function assigned taking into account all trade-off media. In line, we represent the distribution of individuals according to the assigned term structure when limited respectively to monetary, free time and token trade-off decisions. For each trade-off media (in line), the distribution is made according to two lines. The first is the frequency and the second is the percent.

The researchers found that the psychological discount functions which underlie the inter-temporal trade-off decisions were not the same from one trade-off media to another.

These results comes from individuals whose decisions on all media taken together were characterized by generalized hyperbolic discount function or the Laibson discount function.

For these individuals, the analysis trade-off media by trade-off media led to the identification of another psychological discount function. Indeed, none of the individuals characterized by Laibson discount function in the global analysis was characterized by this function in monetary, free time, or token trade-off.

These results indicate that time preference is characterized by a specificity by domain. An individual can adopt an inter-temporal trade-off framework characterized by a hyperbolic discount function for a given area of interest (money, free time, ...) and another discount according to another center of interest. Thus, taking into account individual heterogeneity in understanding time preference, it is necessary to take into account the heterogeneity by domain in a multidisciplinary research.

After estimating the shape and parameters of psychological discount functions that characterized inter-temporal trade-off decisions observed in the current experiments, the researchers can recall that the studied population was characterized by heterogeneity on an individual's psychological discount functions. The researchers found that in this case, individuals were divided between Herrnstein, Laibson, and generalized hyperbolic discount functions.

In addition, the researchers established that an individual does not apply the same framework (represented by the discount function) in all situations that he/she faces, since they found that the time trade-off discount function may differ from one trade-off media to another. These last results are new and have not been sufficiently documented in the relevant literature.

6 Conclusion

The questioning of the discounted utility theory opened a whole field of research in economics and management and received several enhancements in recent years. The results for the violation of this theory seem largely established independently of the cultural context and open the way to building a more efficient framework for understanding inter-temporal trade-off behavior.

In conducting this study, the researchers investigated the characteristics of time preferences. The results were based on the data collected on 130 students from the master cycles of the African Center for Advanced Studies in Management (CESAG) during the months of February and March 2013 through an experimental study. The choice of this testing ground was dictated by the necessity to explore the validity of this theory in other cultural environments.

The results of the analysis indicated that regardless of the cultural context, time preferences are characterized by the violation of the theory of discounted utility function with exponential discount function. Furthermore, the psychological value of time is related to the delay's maturity by a hyperbolic function with present bias, demonstrating the decrease of impatience of these individuals. The identification of the shape and parameters of psychological discount function which underlies inter-temporal decisions observed in the researchers' experiments, the researchers can recall that the studied population was characterized by a heterogeneity of individual psychological discount functions. The researchers found that in this case, individuals were divided into hyperbolic, Laibson, and Mazur discount functions.

Furthermore, an individual does not apply the same time trade-off framework (represented by the discount function) in all situations that he/she faces, given that the researchers found that the psychological discount function may differ from trade-off media to another. This last result was new and has been sufficiently documented in the relevant literature.

These results question the validity of the conclusions of asset pricing and inter-temporal portfolio management models built on the basis of an exponential discount function. More specifically, these findings call for research on the involvement of the questioning of the empirical validity of the exponential discount function and the homogeneity of the discount based on the theories of asset valuation and portfolio management.

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