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An Extended Nonlinear Hicks Model of the Cycle for a Semi-Open Economy: Application to the USA (1960-2008)

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***Abstract*—This paper presents both an extended nonlinear Hicks model of the cycle for a semi-open economy and a method for deriving estimators based on Nonlinear Least Squares as a Numerical Optimization Problem. Hicks thought that fluctuations in investment, caused by nonlinear changes in autonomous investment and the acceleration principle governing induced investment, led to an adjustment process taking place throughout many periods. An empirical application for the US economy (1960-2008), demonstrates the validity of the model and its almost ideal fit to real world data.**

***Keywords*—Nonlinear; Hicks model; dynamics; US economy**

I. INTRODUCTION

Modern economics is often regarded as a mathematical science and draws heavily on the tools of nonlinear mathematics [1]. These tools are considered as promising ways towards overcoming the problems associated with the traditional approaches and have developed through different strands of thought and across diverse disciplines ([2]-[5]).

A seminal contribution in the economics literature on Business Cycles (BC) was Hicks's famous *A Contribution to the theory of the Trade Cycle* (1950), where the author developed his own endogenous nonlinear model of the cycle. Although, there is a plethora of models on BC, the proposed model with its generality, conformity with theory and simplicity of structure is an appropriate vehicle for testing, expanding and improving conventional BC theory in empirical applications. However, so far, it has found no applications in the relevant literature because of the lack of an appropriate method for its empirical estimation. In other words, first, we propose a modified nonlinear Hicks model for a semi-open economy and, second, we propose a method for its empirical application. In this context, we apply it to the USA to test its validity, using real world data for the time period 1960-2008.

II. THE MODEL

According to Hicks, Consumption (C_t) is a linear function of Y_{t-1}

$$C_t = (1-s)Y_{t-1} \tag{1}$$

where 0 <*1-s*< 1 is the so-called marginal propensity to consume, $1/s$ is the so-called multiplier and Y_{t-1} denotes output one period back.

Meanwhile, Hicks thought that fluctuations in investment are caused by (i) nonlinear changes in autonomous investment and (ii) the acceleration principle governing induced investment. Analytically, Hicks though that autonomous investment expenditure may be growing exponentially at a constant rate *g*:

$$A_t = A_0(1+g)^t$$

where A_0>0 is the autonomous investment.

Also, there is the induced part of investment which responds to changes in output. This part of investment is assumed to be proportional to the changes in output, or:

$$IN_t = u(Y_{t-1} - Y_{t-2})$$

where IN_t denotes induced investment in time period *t*, Y_{t-1} and Y_{t-2} output one and two periods back, respectively, and *u* (>0) is the so-called accelerator. Thus:

$$I_t = A_0(1+g)^t + u(Y_{t-1} - Y_{t-2}) \tag{2}$$

where I_t denotes total investment in time period *t*.

Hicks worked in a Keynesian framework. In this context:

$$Y_t = C_t + I_t . \tag{3}$$

Consider now a consumption function with constant term:

$$C_t = C_0 + (1-s)Y_{t-1} + v_t \tag{4}$$

where C_0>0 is the constant term and v_t is the random error term, independent and identically distributed (*i.i.d.*). This formulation is consistent with economic theory since C_0 expresses the so-called autonomous consumption.

Also, consider the following investment function:

$$I_t = A_1 + A_0(1+g)^t + u(Y_{t-1} - Y_{t-2}) + e_t \tag{5}$$

where A_1>0 is the constant term and e_t the random error (*i.i.d.*). Once again, the constant term expresses that part of investment which does not depend on output and does not grow exogenously. Both constant terms introduced, namely C_0 and A_1, are convenient for the econometric estimation.

By substituting (4) and (5) into (3) and rearranging we get the following second order difference equation

$$Y_t - (1-s+u)Y_{t-1} + uY_{t-2} = A_0(1+g)^t + A_1 + C_0 \, . \quad (6)$$

Also, λ_1 and λ_2 are the roots of its characteristic equation

$$\lambda_1, \lambda_2 = 0.5\left[1 - s + u \pm \sqrt{(1-s+u)^2 - 4u}\right]. \quad (7)$$

The complete solution for (6) is

$$Y(t) = Y_c(t) + Y_e(t) \quad (8)$$

where $Y_c(t)$ is the complementary function and $Y_e(t)$ is the particular integral. It is easy to show that the particular integral is equal to the following "moving equilibrium" expression

$$Y_e(t) = Y_p(t) = [(A_0 m^2)m^t]/[m^2 - (1-s-u)m + u] + [(A_1 + C_0)/s] \, . \quad (9)$$

The solution depends on the discriminant $\Delta = (1-s+u)^2 - 4u$. Analytically:

(a) When $\Delta > 0$, i.e. λ_1 and λ_2 are both real and unequal, the complementary function takes the form:

$$Y_c(t) = K_1\lambda_1^t + K_2\lambda_2^t \quad (10)$$

(b) When $\Delta = 0$, i.e. λ_1 and λ_2 are both real and equal ($\lambda_1 = \lambda_2 = \lambda$ where λ is real), the complementary function takes the form:

$$Y(t) = (K_1 + K_2 t)\lambda^t \quad (11)$$

(c) When $\Delta < 0$, i.e. λ_1 and λ_2 are both complex ($\lambda_1 = \lambda$ and $\lambda_2 = \lambda$, where λ is complex) the complementary function takes the form:

$$Y_c(t) = r^t(B_1 \cos \vartheta t + B_2 \sin \vartheta t) \quad (12)$$

where:

$$\lambda_{1,2} = \alpha \pm bi \quad (13)$$

$$r = \sqrt{a^2 + b^2} \quad (14)$$

$$\vartheta = \tan^{-1}(b/a) \quad (15)$$

Finally, stability conditions are $|\lambda_i| < 1$ for i=1,2 in the real root case ($\Delta > 0$ or $\Delta = 0$) and $|r| < 1$ in the complex root case ($\Delta < 0$), implying that the solution is periodically convergent, i.e. stable.

III. Econometric estimation

It is clear that the proposed model should be confronted with real world data, not only to assess the model's ability to replicate the behavior of observed output but also to allow formal inference for parameters and functions of interest.

The estimation of the consumption function (4) is straightforward using 2 Stages Least Squares (2SLS) relevant for the estimation of multiplier–accelerator systems given the structure of the problem [6].

In what follows, the proposed method estimates the modified nonlinear investment function. It is the case that the Least Squares (LS) estimation principle applies as a method for deriving estimators, i.e. the nonlinear least squares (NLS). Unlike Ordinary Least Squares (OLS) and 2SLS, NLS estimators cannot be obtained analytically as closed form expressions.

However, the minimization of the Sum of Squared Residuals (SSR) is a well-defined optimization problem that can be solved numerically by iterating on a solution. The algorithm begins with some initial guess for the coefficient (starting value), and then proceeds by a series of steps. We provide the routine with starting values that are very good guesses of the coefficient, given that we have good knowledge of the nature of the economic problem being studied and this suggests plausible coefficient values. Consider the following procedure:

Step 1: Let $g_{(i)} \in [\alpha, \beta], \alpha, \beta \in \Re, i = 1,..,I$ be drawn from a uniform distribution.

Step 2: For $i = 1$ and $g = g_{(i)} = g_{(1)}$ estimate A_1, A_0 and u in the following (intrinsically) linear equation using 2SLS:

$$I_t = A_1 + A_0(1+g)^t + u(Y_{t-1} - Y_{t-2}) + e_t \, . \quad (5)$$

Step 3: Compute the Sum of Squared Residuals $SSR_{(i)}$ for equation (5), for $i = 1$.

Step 4: Repeat for $i = 2,...,I$ and select the value $\bar{g}$ of $g_{(i)}$ that yields the minimum $SSR_{(i)}$ subject to A_1 A_0 and u being statistically significant.

Step 5: Given the value of $\bar{g}$ estimated in the previous step, keep the estimates of A_1, A_0 and u.

Since g expresses the economy's growth rate in (autonomous) investment it should, normally, be positive in the long run (i.e. over several decades) and range between 0% and 20%. This relatively small range of plausible coefficient values makes it possible to iterate on each value - with reasonable accuracy - and to reach, thus, a global minimum.

It should be made clear that once the parameter g takes on a certain value (even if not the 'optimal' value of g), the investment function becomes intrinsically linear and its estimation is straightforward employing 2SLS.

IV. Empirical analysis

In order to apply the model to explain US output data, we define Consumption (C_t) to include private, government as well as consumption from abroad. The data for the US economy come from AMECO and cover the time period 1960-2008, at constant prices.

The estimation of (4) employing 2SLS yields

$$C_t = 430.71 + 0.76Y_{t-1}$$

$$(10.3) \quad (113.0)$$

R^2 =0.996, SSE=102.02.

Next, following the procedure described above we find that for g =0.04, the estimation of (5) yields the minimum value of SSR (see Fig. 1).

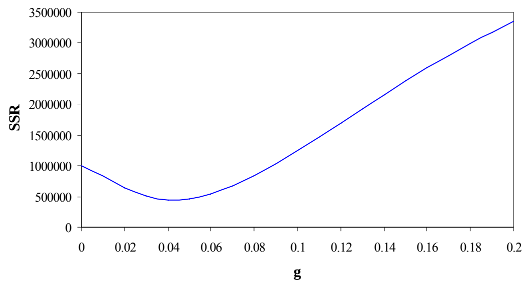

Figure 1. Calculated SSR for the values of *g*

The model could also be selected using the R^2 goodness-of-fit criterion, according to which one should select the value of parameter *g* that maximizes R^2. Other criteria that could be used for the selection of parameter *g* include the minimization of SIC [7] and AIC [8].

Figs. 2-4 illustrate the value of *g* that optimizes the aforementioned criteria.

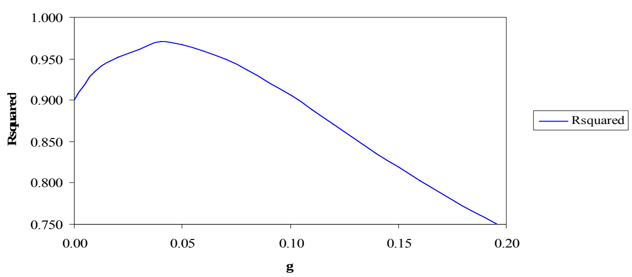

Figure 2. Calculated R^2 for the values of *g*

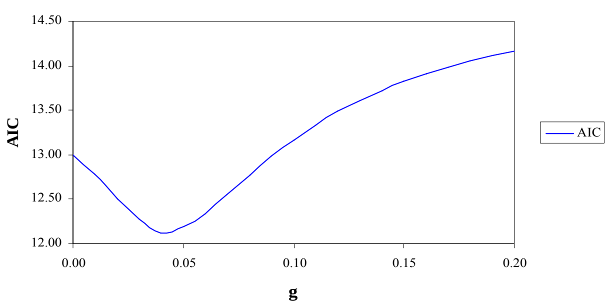

Figure 3. Calculated AIC for the values of *g*

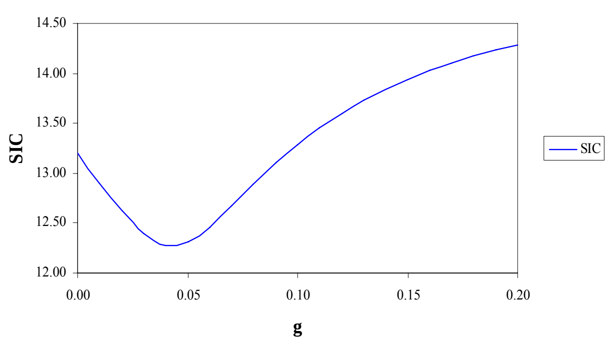

Figure 4. Calculated SIC for the values of *g*

Thus, for $\overline{g}$=0.04 all the aforementioned criteria (SSR, R^2, AIC, SIC) are optimized. So, for $\overline{g}$=0.04, the estimation of the intrinsically linear equation (5) using 2SLS yields:

$$I_t = \underset{(1.99)}{72.32} + \underset{(2.35)}{322.39}(1+0.04)^t + \underset{(30.13)}{0.38}(Y_{t-1} - Y_{t-2}) + e_t$$

R^2 =0.96, SSE=104.03

The values in parentheses are *t*-statistics which imply that all estimated parameters in both equations are statistically significant. Also, we note that the proposed model provides an excellent fit to the data, as expressed through R^2 for both equations. Finally, our findings are consistent with economic theory given that $C_0 > 0$, $A_1 > 0$, $A_0 > 0$, $u > 0$ and $0 < 1\text{-}s < 1$.

In Fig. 5 below we illustrate the estimated values (Iestimated) of Investment along with its real values (Ireal).

The calculated correlation coefficient ($r_{correlation} = 0.98$) is another indication of the almost ideal fit of the model.

V. Solutions and Stability

Substitution in (9) yields the particular integral

$$Y_e(t) = 1282.87(1.04)^t + 2132.35$$

Substitution in (7), given that $\Delta < 0$, yields

$$\lambda_1 = 0.571 + 0.228i \text{ and } \lambda_2 = 0.571 - 0.228i$$

Also, based on (14)-(15), equation (12) yields:

$$Y_c(t) = 0.61^t(B_1 \cos 0.38t + B_2 \sin 0.38t)$$

and

$$Y_c(t) = 0.61^t(B_1 \cos 0.38t + B_2 \sin 0.38t) + 1282.87(1.04)^t + 2132.35.$$

Finally, given the two initial conditions (i.e. actual values for Y(0) and Y(1)) we get the values for the arbitrary constants

$$B_1 = -1026.7341 \text{ and } B_2 = -1906.65.$$

Conclusively, the analytical solution for $Y(t)$ is:

$$Y(t) = 0.61^t[-1026.73\cos(0.38t) - 1906.65\sin(0.38t)] + 1282.87(1.04)^t + 2132.35$$

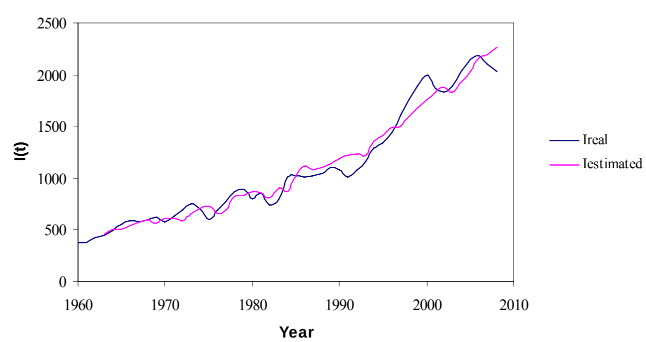

Figure 5. Real VS estimated Investment for the US economy (1960-2008)

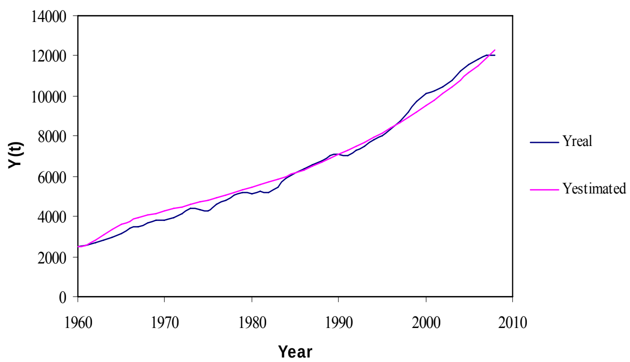

Figure 6. Real VS estimated GDP for the US economy (1960-2008)

By substituting the values of *t* we get the estimated values (Yestimated) of $Y(t)$ which are illustrated in Fig. 6 above along with the real output values (Yreal).

The calculated correlation coefficient ($r_{correlation} = 0.99$) is another indication of the almost ideal fit of the model. Also, since $|r| < 1$, given that 0.61<1, the solution of the complementary function is periodically convergent, i.e. stable.

VI. Conclusions

The proposed approach for a semi-open economy yielded very satisfactory results fitted to data for the US economy (1960-2008). The results of this paper suggest that the proposed model with its generality, conformity with theory and simplicity of structure is an appropriate vehicle for testing, expanding and improving conventional BC theory and predictions. Clearly, future research would be of great interest.

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