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Two-Stage Contests with Preferences over Style*

Todd R. Kaplan† and David Wettstein‡

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Abstract

Many grant applications have a preliminary stage where only a select group are invited to submit a full application. Similarly, procurement contracts by governments are often awarded through a two-stage procedure. We model and analyze such environments where the designer cares about the style of the application as well as its quality. The designer has the option of choosing an initial stage, where contestants can enter and learn about their desirability while the designer learns about their style. We determine closed form solutions for equilibrium outcomes and designer payoffs and use this to analyze design questions regarding whether or not a second stage is desirable, different rules for deciding who will advance, as with whether or not to communicate the number of contestants that qualify for the second stage.

Keywords: contests, innovation, all-pay auctions, mechanism design.

JEL codes: C70, D44, L12, O32

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1 Introduction

There are many examples of contests run in two stages. Often grant applications have a preliminary stage where only some of the applications advance to the second stage. The Leverhulme Foundation has several funding schemes that require two stages (for instance for a research project grant): an outline application and a detailed application.[1] In 2012, there were 908 applications to this scheme approximately 50% made it to the second round and 40% of those received funding (20% of the original received funding). This two-stage process is common for large grants of several UK funding agencies (NERC, ESRC, etc.) as well as used in the establishment of centers for research excellence (I-CORE) in Israel. The latter had 26 out of 67 applications advance to the second stage of which 12 were chosen for funding.

In architecture, it is common to hold a contest for determining a building design. One of the earliest examples was a contest for a design to rebuild the Houses of Parliament in 1836 after a fire. A recent prestigious example of such a contest is when the Mumbai city museum ran a design competition for a $45 million for an additional wing (using Malcolm Reading Consultants to run the competition). Expressions of interest were received from 104 architects worldwide with 8 teams shortlisted. The jury, which consisted of 11 distinguished members, chose New York-based Steven Holl as the winner. There are also a plethora of smaller architecture contests using two stages.[2]

We also note that such a practice is common in advertising. There is a call for a request

[1] The applications to the two stages do not differ in the general idea: there cannot be substantial differences in the intentions, aims, objectives, personnel or budget between the two applications.

[2] The Garden Museum in South London sought an architect to take forward plans to extend the museum in a second phase of renovation. In a two-stage contest, the value of the contract to the architects was estimated between £380,000 and £420,000 and expressions of interest was due by January 20, 2013. It was announced that up to five practices would be shortlisted for the job. In another advertisement, the Tricycle Theatre in Kilburn, North London sought an architect for its £2.4 million refurbishment. There was no mention of how many would be shortlisted. See http://www.bdonline.co.uk/home/competitions for other examples.

for a proposal (RFP) sent to half a dozen ad agencies that asks not only background questions and who will be on the core team, but creative questions about approach. After seeing the RFP responses, the top two or three are invited to present the ideas (perhaps after feedback).[3] Other examples include government procurement, talent show contests, and television series pilots.

There are two characteristics about these contests that are worth noting: (i) This practice appears to be most common in areas where a particular preference or style might be a major factor in selecting the winner. (ii) It appears that sometimes (and sometimes not) the proposed number of finalists is announced.

We find that (i) may be because the contestants are not aware of the preferences of the designer, who may favor some contestants over others. This can be thought of the economics grant committee either preferring theoretical research over empirical research or vice-versa. The reason for (ii) is less obvious and may simply be a feature of the optimal contest design.

We suppose that style is an exogenous feature of a proposal, but quality is a function of effort. For simplicity, we assume that the designer's preference over style is dichotomous: either preferred or not. We also assume for simplicity that the first stage is limited in scope such that a contestant can put in the effort required for the designer to determine his or her type or not put in the effort. This matches many real world contests where the first stage is meant to weed out those with an inappropriate style. It may not be feasible to put in extra effort or at least have that extra effort visible. Such may be the case in a two-stage grant proposal where the first stage is limited to a 1000 word summary.

We model and analyze such environments where the designer wants to maximize the

[3] From personal correspondence with Rachel Greene, a Public Relations & Media Relations Consultant.

best overall effort (as opposed to the total effort of the contestants) by a preferred style. In designing the contest, the designer may choose between a one-stage contest and a two-stage contest. The advantage of the two-stage contest is that the designer learns the type of a contestant if that player puts forth some minimal effort. The disadvantage is that this minimal effort does not contribute to the efforts in the second stage. If the designer chooses a two-stage contest, the designer also has other options. He may choose to advance only a specific number of contestants that satisfy his preferences or anyone that satisfies his preference. He may also choose whether or not to announce how many made it to the second stage before it starts. We analyze these under the condition as to whether or not the stage one effort needs to be redone in the second stage.

Here we use a framework where information is symmetric among contestants and there is complete information about the value of winning the contest (see Baye et al., 1996, and more recently Kaplan et al., 2003, and Siegel, 2009). Recently there has been a number of papers on multi-stage contests (see Sela, 2011, 2012, Segev and Sela, 2014a, 2014b as well as experiments comparing one-stage to two-stage (see Sheremeta 2010). There has also been research where the designer has preferences over style (see Kaplan, 2012). Also related to our paper is research on entry in contests where there is potentially an unknown number of entrants (see Fu et al., 2011, 2014, 2015, and Chen et al., 2015) as well as auctions with a unknown number of bidders (see McAfee and McMillan, 1987). The contribution of this paper is adding the possibility of a two-stage design to a contest where the designer cares about style as well as quality.

The paper proceeds as follows. In the next section, we present the model, followed by the equilibrium analysis in Section 3. We compare the various possibilities in Section 4 and conclude in Section 5.

2 The Contest Environment

There are N players competing in a contest for a prize of value V. Style is exogenous and each contestant independently has a probability p of having a style that the designer desires and $(1-p)$ of not having a desired style. The contest can be run in one or two stages. Contestants decide how much effort to exert in each stage. The designer of the contest cares only about the highest effort exerted in the last stage by a contestant of a style that he desires. The designer is only able to determine the style of a contestant if that contestant's effort is m or above. Furthermore, awarding the prize to a contestant with a non-desired style is prohibitively costly.

As a benchmark, we study a one-stage contest with a minimum effort of m and compare this benchmark with several two-stage contest designs. In all the two-stage contests, the first stage requires that contestants put in effort m in order to have the possibility of advancing. Thus, each contestant doing so has his/her style revealed by the end of stage one. We note that putting in more effort than m does not increase one's chances of advancing. Thus, the first stage is really about screening contestants. To focus the paper on this point, we assume that V is large enough to ensure that all contestants will choose to enter in the first stage.

The two-stage contests differ in the following three aspects: whether there is minimal effort required in the second stage, the criteria to qualify for the second stage, and information revealed to qualifying contestants. The minimal effort in the second stage can be m (*a* $2m$ *environment*) or zero (*an* m *environment*). (Note that we use this nomenclature since doing so counts the aggregate minimal effort needed to participate in the second stage.) Whether it is a $2m$ or m environment may at times not be a choice of the designer, but an exogenous feature of the environment. The designer can choose between two qualifying rules: (i) all those that are discovered to have a desired style are asked back (*all pass*), or (ii) of the contestants eligible to move to the second stage, two are randomly asked back (if there are indeed two) (*random two*). Finally, the designer can choose to inform or not the contestants about the number of contestants advancing to second stage (*inform* or *not inform*).

3 Equilibrium Analysis

In this section, we derive the equilibrium strategies and outcomes for several possible contest designs. We start with the benchmark case of a one-stage contest and proceed to analyze several families of two-stage contests.

3.1 Benchmark Case: One Stage

Here we examine the symmetric equilibrium where all contestants choose effort according to a distribution function F. To ensure entry by all contestants, V must satisfy, $(1-p)^{N-1}pV-m \geq 0$. For F to be part of an equilibrium, it must satisfy:

$$p[pF(x) + (1-p)]^{N-1}V - x = (1-p)^{N-1}pV - m. \tag{1}$$

The RHS of (1) is the expected profit of putting in effort m, in which case it costs the contestant m and that contestant wins only when he is the only contestant with the preferred style (the probability of which is $(1-p)^{N-1}p$). The LHS of (1) is the expected profit of putting in $x \geq m$. The probability of having the preferred style is p and, given this, the probability of winning is that each other contestant either does not have the preferred style (with probability $1-p$) or has the preferred style but puts in less effort (with probability $pF(x)$). Both sides are equal since in a mixed-strategy equilibrium, expected payoffs are constant over the support of the equilibrium strategies.

Solving (1) for $F(x)$ yields:

$$
\begin{aligned}
F(x) &= \frac{\left[\frac{x-m}{pV}+(1-p)^{N-1}\right]^{\frac{1}{N-1}}-(1-p)}{p} \\
&= \left[\left(\frac{1-p}{p}\right)^{N-1}+\frac{x-m}{p^N V}\right]^{\frac{1}{N-1}}-\frac{(1-p)}{p}
\end{aligned}
$$

with support $[m, pV\left[1-(1-p)^{N-1}\right]+m]$. For example, if $N=3$, we have $F(x)=\frac{\left[\frac{x-m}{pV}+(1-p)^2\right]^{\frac{1}{2}}-(1-p)}{p}$ with support $[m, pV\left[1-(1-p)^2\right]+m]$.

To determine the designer's one-stage profits Π^{one} we proceed to evaluate the expected value of the highest effort put forth by a contestant with a preferred style. We define a distribution $G(x)$ by $G(x)=pF(x)+(1-p)$. The function G represents the distribution of the effort that is preferred. Here, we replace the case where effort is not useful by an atom of size $(1-p)$ at zero. Now $\Pi^{one}=\int_0^{\overline{x}} x dG^N$ where $G(x)=\left[(1-p)^{N-1}+\frac{x-m}{pV}\right]^{\frac{1}{N-1}}$ and $\overline{x}=pV\left[1-(1-p)^{N-1}\right]+m$. Hence, $\Pi^{one}=m+\frac{NpV}{2N-1}+\frac{(N-1)pV(1-p)^{2N-1}}{2N-1}-(1-p)^{N-1}(m(1-p)+pV)$.

Next, we consider the first of several two-stage contests.

3.2 Two Stages - All Pass

With two stages and all pass (denoted by AP), all the contestants that put forward effort m in stage one and have the preferred style pass to the second stage. A contestant that makes it to the second stage learns that he has the preferred style. Also, depending upon the information condition, the contestant may or may not know how many other contestants also have a preferred style. In the latter case since all that have the preferred style make it to the second stage, making it to the second stage does not affect a contestant's estimate about how many *other* contestants with a preferred style are competing in the second stage.

3.2.1 All pass: 2m, not inform

For full participation in stage one, we require

$$p(1-p)^{N-1}V \geq m(1+p). \tag{2}$$

The LHS of (2) is the expected probability of being alone in the second stage times the prize, while the RHS of (2) is the expected minimum effort needed if a contestant puts in effort m in stage one and again in stage two if advancing. Since a contestant is not informed in stage two about how many others advanced, the contestant would always put forth at least effort m.

Again we look for a symmetric equilibrium with a distribution function F that represents effort in the second stage. For F to be part of an equilibrium, the corresponding G distribution function must satisfy:

$$G(x)^{N-1}V - x = (1-p)^{N-1}V - m. \tag{3}$$

The RHS of (3) is the expected profit of putting in effort m and winning with probability of $(1-p)^{N-1}$. The LHS of (3) is the expected profit of putting in $x \geq m$. Note that as opposed to (1), here at the second stage, each contestant already knows he has the preferred style.

Thus, moving from one stage to two stages effectively increases the prize from pV to V.

Solving (3) for $F(x)$ yields (by first solving for $G(x)$):

$$
\begin{aligned}
F(x) &= \frac{\left[\frac{(1-p)^{N-1}V+x-m}{V}\right]^{\frac{1}{N-1}}-(1-p)}{p} = \\
&= \left[(\frac{1-p}{p})^{N-1}+\frac{x-m}{p^{N-1}V}\right]^{\frac{1}{N-1}}-\frac{(1-p)}{p}
\end{aligned}
$$

with support $[m, V\left[1-(1-p)^{N-1}\right]+m]$.

Proceeding similarly to the one-stage environment, we can calculate two-stage profits, when contestants are not informed and must pay at least m in the second stage. Here, $\Pi^{AP}_{2m,NI}=\int_m^{\overline{x}} x dG^N$ where $G(x)=\left[(1-p)^{N-1}+\frac{x-m}{V}\right]^{\frac{1}{N-1}}$ and $\overline{x}=V\left[1-(1-p)^{N-1}\right]+m$. Hence, $\Pi^{AP}_{2m,NI}=m-m(1-p)^N+\frac{NV}{2N-1}+(1-p)^{N-2}V\left[\frac{(N-1)(1-p)^{N+1}}{2N-1}-(1-p)\right]$.

3.2.2 All pass: m, not inform

This environment is identical to the previous one except for the fact there is no minimal bid m required in the second stage. For full participation in stage one, we now require

$$p(1-p)^{N-1}V \geq m(1+p). \tag{4}$$

Again we look for a symmetric equilibrium with a distribution function F that represents effort in the second stage. Looking at the second stage, for F to be part of an equilibrium, the corresponding G distribution function must satisfy:

$$G(x)^{N-1}V - x = (1-p)^{N-1}V. \tag{5}$$

The RHS of (5) differs from the RHS of (3) in that m need not be expended in the second stage. The LHS of (5) is identical to the LHS of (3).

Solving (5) for $F(x)$ yields:

$$\begin{aligned} F(x) &= \frac{\left[\frac{(1-p)^{N-1}V+x}{V}\right]^{\frac{1}{N-1}} - (1-p)}{p} = \\ &= \left[(\frac{1-p}{p})^{N-1} + \frac{x}{p^{N-1}V}\right]^{\frac{1}{N-1}} - \frac{(1-p)}{p} \end{aligned}$$

with support $[0, V(1-(1-p)^{N-1})]$.

To determine the designer's profits we proceed similarly to before, to obtain $\Pi^{AP}_{m,NI} = \int_0^{\overline{x}} x dG^N$ where now $G(x) = \left[(1-p)^{N-1} + \frac{x}{V}\right]^{\frac{1}{N-1}}$ and $\overline{x} = V(1-(1-p)^{N-1})$. Hence $\Pi^{AP}_{m,NI} = \frac{(N-1)(1-p)^{2N-1} - (2N-1)(1-p)^{N-1} + N}{2N-1} V$.

3.2.3 All pass: 2m, inform

For full participation in stage one, we now require

$$p(1-p)^{N-1}V \geq 2m. \tag{6}$$

With probability $Np(1-p)^{N-1}$, only one contestant will participate in the second stage. Since the contestant knows this, the designer will get m. For $i \geq 2$, there will be i contestants in the second stage with probability $\binom{N}{i} p^i (1-p)^{N-i}$, and the equilibrium must satisfy $F(x)^{i-1}V - x =$

0 for all $x \geq m$. Hence, each contestant bids according to the distribution function:

$$F_i(x) = \begin{cases} \left(\frac{x}{V}\right)^{\frac{1}{i-1}} & \text{if } x \geq m, \\ \left(\frac{m}{V}\right)^{\frac{1}{i-1}} & x < m. \end{cases}$$

The designer's profits are then given by $\Pi^{AP}_{2m,I} = \sum_{i=2}^{N} \binom{N}{i} p^i (1-p)^{N-i} \int_m^V xd\left(\frac{x}{V}\right)^{\frac{i}{i-1}} + N \cdot p(1-p)^{N-1}m = \sum_{i=2}^{N} \binom{N}{i} p^i (1-p)^{N-i} \frac{i}{2i-1}(V - m(\frac{m}{V})^{\frac{i}{i-1}}) + N \cdot p(1-p)^{N-1}m$.

3.2.4 All pass: m, inform.

For full participation in stage one, we require $Vp(1-p)^{N-1} > m$.

With probability $Np(1-p)^{N-1}$, only one contestant will participate in the second stage. Since the contestant knows this, the designer will get 0. With probability $\binom{N}{i}p^i(1-p)^{N-i}$, there will be $i \geq 2$ contestants in the second stage, the equilibrium distribution function F must then satisfy $F(x)^{i-1}V - x = 0$.

Hence, in equilibrium each contestant bids according to the distribution function:

$$F_i(x) = \left(\frac{x}{V}\right)^{\frac{1}{i-1}}$$

on the interval $[0, V]$.

This leads to the following payoff to the designer: $\Pi^{AP}_{m,I} = \sum_{i=2}^{N} \binom{N}{i} p^i (1-p)^{N-i} \int_0^V xd\left(\frac{x}{V}\right)^{\frac{i}{i-1}} = V \sum_{i=2}^{N} \binom{N}{i} p^i (1-p)^{N-i} \frac{i}{2i-1}$.

3.3 Two Stages - Random-Two Pass

We now look at where the designer randomly chooses two contestants among those that have the preferred style. With random-two pass (*ran2*), a contestant advancing to the second stage learns something about the other contestants. The fact that a contestant was selected means that he is more likely to be the only one with the preferred style (by Bayes' rule).

Consider for example the case where $N = 3$ and $p = 0.5$. In *all-pass*, if a contestant makes it to the second stage, the probability that he is the only one that advanced is 25%. In *ran2*, if a contestant is preferred, then there is a 25% probability that he is the only one with a preferred style, a 50% probability that there is one other contestant with a preferred style, and a 25% probability that there are two others with a preferred style. When there are two others with a preferred style, he advances with a 2/3 probability. In the other cases, he would be always advance. Thus, the probability of being the only one that advanced given that one advanced is $\frac{25}{25+50+\frac{2}{3}\cdot 25} \approx 27\%$.

3.3.1 Random Two: 2m, not inform

Denote by p_a the probability of being the only remaining contestant given that a contestant advances. We have $p_a = \frac{(1-p)^{N-1}}{(1-p)^{N-1}+\sum_{i=1}^{N-1}\binom{N-1}{i}p^i(1-p)^{N-1-i}\frac{2}{i+1}} = \frac{Np}{\frac{2}{(1-p)^{N-1}}-Np-2(1-p)}$ (using the same logic as above for $N = 3$).

In equilibrium, in stage 2, $F(x)$ satisfies:

$$[(1-p_a)F(x) + p_a]V - x = p_aV - m.$$

Solving for $F(x)$ yields:

$$F(x) = \frac{x - m}{(1 - p_a)V}.$$

The designer's payoff is then:

$$\Pi^{ran2}_{2m,NI} = Np(1-p)^{N-1} \int_m^{\overline{x}} x dF + (1 - (1-p)^N - Np(1-p)^{N-1}) \int_m^{\overline{x}} x dF^2$$

$$= Np(1-p)^{N-1}(m + \frac{1}{2}V(1-p_a)) + (1 - (1-p)^N - Np(1-p)^{N-1})(m + \frac{2}{3}V(1-p_a)).$$

3.3.2 Random Two: m, not inform

In equilibrium, in stage 2, $F(x)$ satisfies:

$$[(1-p_a)F(x) + p_a]V - x = p_a V.$$

Solving for $F(x)$ yields: $F(x) = \frac{x}{(1-p_a)V}$. The designer's payoff is then given by $\Pi^{ran2}_{m,NI} = Np(1-p)^{N-1} \int_0^{\overline{x}} x dF + (1 - (1-p)^N - Np(1-p)^{N-1}) \int_0^{\overline{x}} x dF^2 = Np(1-p)^{N-1}(\frac{1}{2}V(1-p_a)) + (1 - (1-p)^N - Np(1-p)^{N-1})(\frac{2}{3}V(1-p_a))$.

3.3.3 Random Two: 2m, inform

If only one contestant participates in the second stage, then the contestant knows this and the designer will get m (which is the minimum effort). If there are two contestants in the second stage and this is commonly known, the equilibrium must satisfy $F(x)V - x = 0$ for all $x \geq m$. Thus, the overall expected profits are given by $\Pi^{ran2}_{2m,I} = Np(1-p)^{N-1}m + (1 - (1 - p)^N - Np(1-p)^{N-1}) \int_m^V x dF^2 = Np(1-p)^{N-1}m + (1 - (1-p)^N - Np(1-p)^{N-1})(\frac{2V}{3} - \frac{2m^3}{3V^2})$.

3.3.4 Random Two: m, inform

If only one contestant participates in the second stage, then when in the inform design the contestant knows he is the only contestant that advanced and the designer will get 0. When there are two contestants in the second stage, the equilibrium must satisfy $F(x)V - x = 0$ for all $V \geq x \geq 0$. The designer's expected profits are then $\Pi_{m,I}^{ran2} = (1-(1-p)^N - Np(1-p)^{N-1}) \int_0^V x dF^2 = (1-(1-p)^N - Np(1-p)^{N-1})(\frac{2V}{3})$.

4 Ranking the Designs

We now proceed to compare the various designs from the point of view of the designer. The designer is interested in the expected highest effort by contestant with a preferred style. We start by comparing the two qualification rules.

4.1 Random two pass versus all pass

Proposition 1 *For $N > 2$, in any of the four two-stage designs, random two pass generates higher revenue than all pass, that is, $\Pi_{2m,NI}^{ran2} > \Pi_{2m,NI}^{AP}$, $\Pi_{m,NI}^{ran2} > \Pi_{m,NI}^{AP}$, $\Pi_{2m,I}^{ran2} > \Pi_{2m,I}^{AP}$ and $\Pi_{m,I}^{ran2} > \Pi_{m,I}^{AP}$.*

Proof. The latter two can be shown by directly looking at the differences: $\Pi_{2m,I}^{ran2} - \Pi_{2m,I}^{AP} = (1-(1-p)^N - Np(1-p)^{N-1})(\frac{2V}{3} - \frac{2m^3}{3V^2}) - [\sum_{i=2}^{N} \binom{N}{i} p^i (1-p)^{N-i} \frac{i}{2i-1}(V - m(\frac{m}{V})^{\frac{i}{i-1}})] > (1-(1-p)^N - Np(1-p)^{N-1})(\frac{2V}{3} - \frac{2m^3}{3V^2}) - [\sum_{i=2}^{N} \binom{N}{i} p^i (1-p)^{N-i} \frac{2}{3}(V - m(\frac{m}{V})^2)] = 0$

and

$\Pi_{m,I}^{ran2} - \Pi_{m,I}^{AP} = (1-(1-p)^N - Np(1-p)^{N-1})(\frac{2V}{3}) - V \sum_{i=2}^{N} \binom{N}{i} p^i (1-p)^{N-i} \frac{i}{2i-1} >$
$(1-(1-p)^N - Np(1-p)^{N-1})(\frac{2V}{3}) - V \sum_{i=2}^{N} \binom{N}{i} p^i (1-p)^{N-i} \frac{2}{3} = 0.$

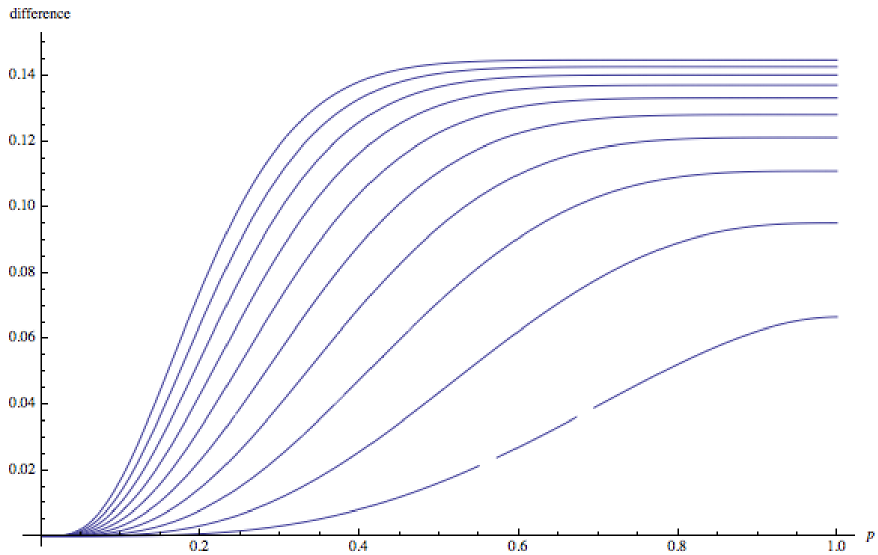

Figure 1: Difference of $\Pi^{ran2}_{2m,NI} - \Pi^{AP}_{2m,NI}$ (divided by V) versus p when n varies from 3 to 12. Higher curve corresponds to higher n.

Intuition is that with all pass the profit is strictly lower when three or more contestants have a preferred style. We show here the first two for the case of $N = 3$. The difference for the $2m$ case is $\Pi^{ran2}_{2m,NI} - \Pi^{AP}_{2m,NI} = V[\frac{2p^4(3p^3-15p^2+26p-15)}{15(p^2-3)}]$. At $p = 1/2$, this is strictly positive. There are real roots at 0 and 1.28. Thus, the difference for $2m$ is strictly positive for any $p \in (0,1)$. Likewise, the difference for the m case is $\Pi^{ran2}_{m,NI} - \Pi^{AP}_{m,NI} = V[\frac{p^2(3p^3-15p^2+26p-15)}{15(p^2-3)}]$. At $p = 1/2$, this is strictly positive. There are real roots at 0 and 1.58. Again, this shows that the difference for m is strictly positive for any $p \in (0,1)$.

For $N > 3$, the exercise is similar. By plotting the difference for both cases in Figures 1 and 2, we see that the difference is increasing in N and hence always positive.[4]

■

This supports the use of random two rather than all pass, yet we often see the all pass design. This might be due to considerations outside the scope of our model. First, when

[4] We have also done this up to N=200.

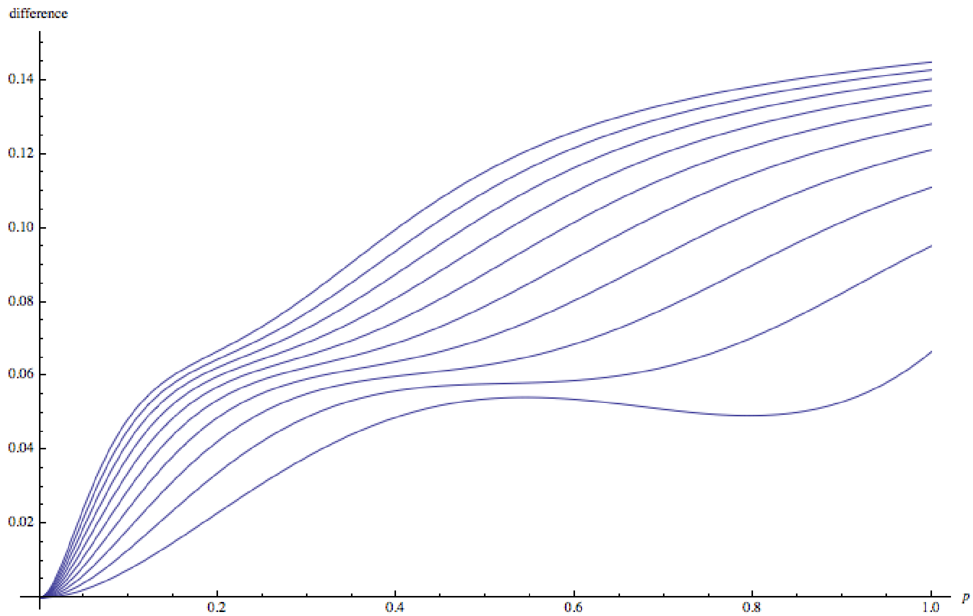

Figure 2: Difference of $\Pi^{ran2}_{m,NI} - \Pi^{AP}_{m,NI}$ (divided by V) versus p when n varies from 3 to 12. Higher curve corresponds to higher n.

just two contestants pass, cooperation and manipulations on part of the two, leading to reduced payoff for the designer, is more likely than when there are several more contestants. Second, there might be public outcry against the arbitrary decision, due to possible concerns regarding possible discrimination and favoritism on part of the designer. Next we compare the desirability of requiring a minimum effort in the second stage as well.

4.2 Minimum effort m versus $2m$

Here we find that in the case where contestants are not informed, it is better to have a minimum effort in both stages. This is shown in the following two propositions.

Proposition 2 $\Pi^{AP}_{m,NI} < \Pi^{AP}_{2m,NI}$.

Proof. Note that $\Pi^{AP}_{2m,NI} = \int_m^{V\left[1-(1-p)^{N-1}\right]+m} xd\left[(1-p)^{N-1} + \frac{x-m}{V}\right]^{\frac{N}{N-1}}$ and $\Pi^{AP}_{m,NI} = \int_0^{V\left[1-(1-p)^{N-1}\right]} xd\left[(1-p)^{N-1} + \frac{x}{V}\right]^{\frac{N}{N-1}}$. We perform a change of variables $z = x - m$ to obtain

$$\begin{aligned}
\Pi^{AP}_{2m,NI} &= \int_0^{V\left[1-(1-p)^{N-1}\right]} (z+m) d\left[(1-p)^{N-1}+\frac{z}{V}\right]^{\frac{N}{N-1}}, \\
&= \Pi^{AP}_{m,NI} + \int_0^{V\left[1-(1-p)^{N-1}\right]} md\left[(1-p)^{N-1}+\frac{z}{V}\right]^{\frac{N}{N-1}} > \Pi^{AP}_{m,NI}.
\end{aligned}$$

We now make the comparison for the case of $ran2$. ■

Proposition 3 $\Pi^{ran2}_{m,NI} < \Pi^{ran2}_{2m,NI}$.

Proof. The difference of profits is $\Pi^{ran2}_{2m,NI} - \Pi^{ran2}_{m,NI} = (1-(1-p)^N)m > 0$. ■

In the case where contestants are informed regarding the number of contestants who qualified, the ranking depends on p. We see this in the next two propositions.

Proposition 4 *(i)* $\Pi^{ran2}_{m,I} < \Pi^{ran2}_{2m,I}$ *if* p *is close to 0; (ii)* $\Pi^{ran2}_{m,I} > \Pi^{ran2}_{2m,I}$ *if* p *is close to 1.*

Proof. Note that $\Pi^{ran2}_{2m,I} - \Pi^{ran2}_{m,I} = Np(1-p)^{N-1}m + (1-(1-p)^N - Np(1-p)^{N-1})(\frac{2m^3}{3V^2})$. When $p = 1$, this expression is negative. When $p = 0$, the expression is 0. The derivative of the expression w.r.t. p at 0 equals Nm, which is strictly positive. ■

Proposition 5 *(i)* $\Pi^{AP}_{m,I} < \Pi^{AP}_{2m,I}$ *if* p *is close to 0; (ii)* $\Pi^{AP}_{m,I} > \Pi^{AP}_{2m,I}$ *if* p *is close to 1.*

Proof. We have $\Pi^{AP}_{2m,I} - \Pi^{AP}_{m,I} = -\sum_{i=2}^{N}\binom{N}{i}p^i(1-p)^{N-i}\frac{i}{2i-1}(m(\frac{m}{V})^{\frac{i}{i-1}}) + N\cdot p(1-p)^{N-1}m$. The proof is similar to that of the previous proposition. When $p = 1$, this expression is negative. When $p = 0$, the expression is 0. The derivative of the difference w.r.t. p at 0 equals Nm, which is strictly positive. ■

The intuition for both the above propositions is as follows. If $p \sim 0$, then anytime one enters they are likely to be alone and choose efforts close to the minimum effort. If $p \sim 1$, the minimum effort causes contestants to sometimes drop out when two or more advance.

We remark that it is plausible that in some types of contests the designer cannot choose between the two environments of m and $2m$. For instance, the designer may prefer a $2m$ design, but politically it would be difficult not to award a contract when there is a contestant with a suitable style and the designer knows this.

We now proceed to compare informing and not informing contestants.

4.3 Informing or not informing

Here we answer the question of whether or not the designer should let contestants know how many advance to the second stage. McAfee and McMillan (1987) show that with standard auctions and risk-neutrality there is no difference in revenue between informing and not informing, but with constant absolute risk-aversion, not informing is superior. As opposed to the auction literature, in our setup all the contestants pay their costs. With contests unlike auctions (where effort is only expended by the winner), there is a distinction between the objective of maximizing the highest effort and the objective of maximizing the total effort. Serena (2016) also looks at information revelation in contests but with the objective of maximizing total effort and the information is about the rival's types.

We note that not informing requires a policy of committing to not making announcements. Otherwise, there would be an incentive to state when there is a relatively large number of contestants in the second stage. This would then allow the contestants to deduce the state based upon what is and what is not revealed. For example, if $N = 2$, when both make it to the second stage it is worthwhile to say so. If only one, then it makes sense to stay quiet. We now proceed to rank the inform and not inform policies for the various scenarios and start with the all-pass design.

Proposition 6 *(i)* $\Pi^{AP}_{m,I} < \Pi^{AP}_{m,NI}$ *: for* $N = 2$ *or* $N > 2$ *and small* p*. (ii)* $\Pi^{AP}_{m,I} > \Pi^{AP}_{m,NI}$ *: for* $N > 2$ *and large* p*.*

Proof. We note that for $N > 2$, we have $\Pi^{AP}_{m,I} = \Pi^{AP}_{m,NI}$ for $p = 0$ and $p = 1$. Furthermore at $p = 1$, the derivative of the difference $(\Pi^{AP}_{m,NI} - \Pi^{AP}_{m,I})$ is positive and at $p = 0$, the derivative of the difference is 0 with a positive second derivative. ■

Proposition 7 *(i)* $\Pi^{AP}_{2m,I} < \Pi^{AP}_{2m,NI}$ *for* $N = 2$ *or* $N > 2$ *and small* p*.*

(ii) It is possible that $\Pi^{AP}_{2m,I} > \Pi^{AP}_{2m,NI}$ *for large* N *and small* m*.*

Proof. For $N > 2$, when $p = 0$, we have $\Pi^{AP}_{2m,NI} = \Pi^{AP}_{2m,I} = 0$ and the derivative of $\Pi^{AP}_{2m,NI} - \Pi^{AP}_{2m,I}$ is positive. Also, at $p = 1$, the difference is positive for $m > 0$. However, this inequality is not satisfied for all parameter values. ■

As we see in the above propositions, that the ranking is ambiguous when $N > 2$ in that it depends upon p. However, as we now see in the random-two pass environment, the ranking is unequivocal in favor of not informing. We see this in the following proposition.

Proposition 8 $\Pi^{ran2}_{m,NI} > \Pi^{ran2}_{m,I}$ *and* $\Pi^{ran2}_{2m,NI} > \Pi^{ran2}_{2m,I}$*.*

Proof. Looking at the difference $\Pi^{ran2}_{m,NI} - \Pi^{ran2}_{m,I} =$

$$Np(1-p)^{N-1}(\tfrac{1}{2}V(1-p_a)) + (1-(1-p)^N - Np(1-p)^{N-1})(\tfrac{2}{3}V(-p_a))$$

$$= V\frac{Np(2(1-p) + (1-p)^N(1 - 3(\frac{1}{1-p})^{N-1} + p(N-1))}{3(1-p)(2 - 2(\frac{1}{1-p})^{N-1} + p(N-2))}$$

For $N = 2$, this equals V times $p^2(1-p)/3$. For $N = 3$, this equals V times

$$\frac{(1-p)^2p^2(3-2p)}{3-p^2}.$$

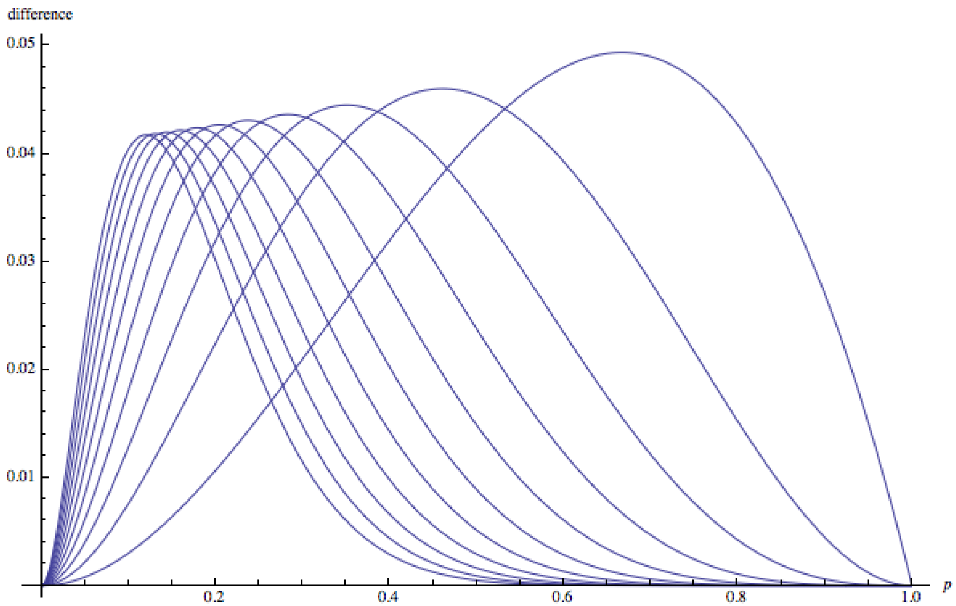

Figure 3: Difference of $\Pi^{ran2}_{m,NI} - \Pi^{ran2}_{m,I}$ (divided by V) versus p when n varies from 2 to 12. Higher n corresponds to a left shift.

For $N = 4$, this equals V times

$$\frac{2(1-p)^3 p^2 (6 - 8p + 3p^2)}{3(2 - p^2(2-p))}.$$

These expressions (for $N = 2, 3, 4$) are strictly positive for $0 < p < 1$. For $N > 4$, the exercise is similar. By plotting the difference in Figure 3, we see that the difference is positive.[5] We also have $\Pi^{ran2}_{2m,NI} - \Pi^{ran2}_{2m,I} =$

$Np(1-p)^{N-1}(\frac{1}{2}V(1-p_a)) + (1-(1-p)^N - Np(1-p)^{N-1})(m - \frac{2}{3}V(p_a) - \frac{2m^3}{3V^2}) =$

$(\Pi^{ran2}_{m,NI} - \Pi^{ran2}_{m,I}) + (1-(1-p)^N - Np(1-p)^{N-1})(m - \frac{2m^3}{3V^2}) > \Pi^{ran2}_{m,NI} - \Pi^{ran2}_{m,I} > 0$ (for $0 < p < 1$). ■

[5] We have also done this up to $N = 200$.

4.4 One stage versus two stages

We finally address the basic design question of whether or not the contest should be a one or two stage competition.

Proposition 9 $\Pi^{one} < \Pi^{AP}_{2m,NI}$.

Proof. We note that $\Pi^{one} = \int_0^{pV\left[1-(1-p)^{N-1}\right]+m} xd\left[(1-p)^{N-1} + \frac{x-m}{pV}\right]^{\frac{N}{N-1}}$ whereas $\Pi^{AP}_{2,2m,NI} = \int_m^{V\left[1-(1-p)^{N-1}\right]+m} xd\left[(1-p)^{N-1} + \frac{x-m}{V}\right]^{\frac{N}{N-1}}$. Since $p < 1$, the inequality follows by first-order stochastic dominance. ■

Profits of $\Pi^{AP}_{m,NI}$ and Π^{one} cannot be unequivocally ranked.

Proposition 10 $\Pi^{one} > \Pi^{AP}_{m,NI}$ *for large* p*,* $\Pi^{one} < \Pi^{AP}_{m,NI}$ *for small* p *and* m*.*

Proof. For p close to 1, we have $\Pi^{one} \approx \Pi^{AP}_{2m,NI}$, thus $\Pi^{one} > \Pi^{AP}_{m,NI}$. For small p and small m, the effective prize is higher in two stages without the impact of m, thus we have a higher $\Pi^{AP}_{m,NI}$. ■

Intuitively, a small p favors two stages by increasing the effective prize. A higher m makes contestants more aggressive if the prize is large. Which effect is higher, determines the higher profit.

Proposition 11 $\Pi^{one} < \Pi^{ran2}_{2m,NI}$.

Proof. Since $\Pi^{ran2}_{2m,NI} > \Pi^{AP}_{2m,NI}$ and $\Pi^{one} < \Pi^{AP}_{2m,NI}$. ■

We note that while $\Pi^{one} < \Pi^{ran2}_{2m,NI}$, it could still be the case that $\Pi^{one} > \Pi^{ran2}_{m,NI}$ for large p (and small N). We see this for $N = 2$ since then $\Pi^{AP}_{m,NI} = \Pi^{ran2}_{m,NI}$.

The above findings from all the sections show that $\Pi^{ran2}_{2m,NI}$ is the largest profit the designer can obtain. However, to obtain this profit will require the designer to be able to commit to both not inform contestants about the number making it to the second stage as well as being able to commit to a minimum bid in the second stage.

5 Conclusion

We analyzed contest environments where the designer cares about the style and quality of the winning effort. We considered four design issues: whether to use one stage or two, requiring a minimum bid in both stages, whether to advance all qualified contestants or to place a limit, and whether or not to inform contestants about the number of contestants that advance to the second stage. These, for the most part, can be observed in actual contests run.

We found closed form solutions for the equilibrium strategies and expected designer's profits for the various contest designs. We then examined rankings between several design options. While some can be unequivocally ranked, other rankings were dependent upon the parameters defining the environment. Overall, we found that the design maximizing the highest effort is a random two design where there is no information given about the number that make it to the second stage and there is a minimum bid in each stage. However, there may be many reasons why such a design cannot be used. For instance, limiting the number of contestants that advance may cause concern about favoritism on part of the designer. Requiring a minimum standard in the second stage as well as the first stage could be problematic since not awarding a contract to a company that one knows is suitable may not be a credible threat. Hence, there is a need to understand rankings beyond the first-best design.

There are many ways to expand our work. While the purpose of this work is to focus on screening by having a two-stage mechanism, it is possible to look at parameters where the contestants' willingness to enter in the first stage is not ensured. It is also worthwhile to have contestants with heterogeneous abilities. In this case, it may be superior to allow more than two contestants to advance. Also, one can examine social welfare issues and different objective functions of the designer (such as total effort). Finally, one can change the model to a scenario where the chance of having a desired style depends upon the effort put forth in stage one.

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