# Patentability, R&D direction, and cumulative innovation

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**Abstract:** We present a model of cumulative innovation where firms can conduct R&D in both a safe and a risky direction. Innovations in the risky direction produce quality improvements with higher expected sizes and variances. As patentability standards rise, an innovation in the risky direction is less likely to receive a patent that replaces the current technology, which decreases the static incentive for new entrants to conduct risky R&D, but increases their dynamic incentive because of the longer duration—and hence higher reward—for incumbency. These, together with a strategic substitution and a market structure effect, result in an inverted-U shape in the risky direction but a U shape in the safe direction for the relationship between R&D intensity and patentability standards. There exists a patentability standard that induces the efficient innovation direction, whereas R&D is biased towards (against) the risky direction under lower (higher) standards. The optimal patentability standard may distort the R&D direction to increase the industry innovation rate that is socially deficient.

**Keywords**: cumulative innovation, patentability standards, R&D intensity, R&D direction, rate of innovation, innovation direction

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## 1. INTRODUCTION

A central issue in the economics of innovation is how patent policy may affect innovative activities. The recent literature has examined this issue in the context of *cumulative* innovation, where discoveries build on each other, under a standard assumption that firms pursue innovations along a single research direction. In many industries, however, firms can conduct R&D in multiple directions to achieve a specific goal, as, for example, the development of a next generation color copier in the early 1990s by Fuji Xerox, of a new mobile system by Ericsson in the mid-1990s, and of an X Terminal workstation by the Hewlett Packard in the late 1980s (Birkinshaw and Lingblad, 2001). The purpose of this paper is to inquire how patent policy, specifically patentability standards, may affect the rate and direction of cumulative innovation in an industry where firms can conduct R&D in multiple directions.

We consider a situation where there are two research directions, $A$ and $B$, for a sequence of innovations (or new products) that deliver higher product qualities over time. The stochastic quality improvement of an innovation in direction $B$ has a higher variance than that in direction $A$. Hence, if an innovation is patentable only when its quality improvement (or innovation size) is sufficiently large, as for instance implied by the requirement of a minimum inventive step, there will exist a range of quality thresholds or patentability standards ($S$), under which an innovation is certainly patentable in direction $A$ but not in direction $B$. We will focus on patentability standards in the interior of such a range, and call $A$ the *safe* direction while $B$ the *risky* direction.

If innovation is a one-time activity that ends with the successful introduction of a new product, a (marginally) higher patentability standard would discourage R&D in $B$ by making it harder to obtain a patent and the rents associated with it through this direction, which we shall call the *threshold* effect, whereas it would have no impact on R&D in $A$, provided that there are no (dis)economies of scope in R&D and that the return to a successful patentable discovery in one direction is not diminished by that in the other. In this static setting, a higher $S$ reduces industry R&D through the threshold effect, and it also allocates relatively more resources in direction $A$ than in direction $B$, which can reduce the expected size of innovation if, as we shall assume, a successful innovation through $B$ has a higher expected quality improvement than that through $A$.

The issue is more complex if innovations are cumulative, as we assume in this paper. Specifically, we consider the following model that builds on and extends Hunt (2004) by having two research directions: Suppose that $n+1$ firms have entered an industry. At any time, one of them is the leader and the other $n$ firms are challengers. The challengers are in a patent race to develop a new product that improves upon the current leader's. When a challenger succeeds in a patentable innovation, it becomes the new leader to replace the current one, who then joins the rank of challengers; and this process repeats itself indefinitely. In this dynamic setting, a marginal increase in the patentability standard will increase the value of being a leader because it will take longer before the leader is replaced by a successful challenger. This *incumbency-prolonging* effect can potentially increase the incentive for R&D in both $A$ and $B$, even though the threshold effect from a higher $S$ will still have a negative impact on the incentive for R&D in direction $B$.

Moreover, the changes in the R&D incentives in the two different directions will interact with each other, giving rise to a dynamic *strategic substitution* effect between the two directions. In particular, an increase of R&D in direction $B$ motivated by the incumbency-prolonging effect could reduce the incentive for R&D in direction $A$, and vice versa. This turns out to be the crucial new force that leads to new effects of patentability standards under multiple research directions.

Finally, as we shall assume, a firm needs to incur a fixed cost to enter the market in order to conduct R&D and innovate. Therefore, patentability standards, by impacting the expected return to R&D in each direction, also affects the number of entrants in the free entry equilibrium. Our analysis will examine how this *market-structure* effect interacts with the other forces in the model.

We find that as patentability standards rise, R&D intensity in direction $B$ first rises and then falls, exhibiting an inverted-U shape, whereas R&D intensity in direction $A$ is U-shaped, initially decreasing and then increasing. Thus, the incumbency-prolonging effect is the dominating force in direction $B$ when $S$ is low, but it is dominated by the negative threshold effect when $S$ is high. More surprising is that despite the positive impact from the incumbency-prolonging effect, increases in $S$ initially lower R&D in direction $A$, due to the strategic substitution effect.

We also find that as $S$ increases, the industry rate of innovation initially goes up and eventually falls down, reaching its maximum at some intermediate level. The market-structure effect plays a balancing role: there will be more firms when the expected return from R&D investment is higher, which moderates the effects of patentability standards on R&D intensities both for each firm and for the industry.

We further compare the market equilibrium with the solutions that maximize social welfare. First, in relation to the first-best innovation rate, we show that R&D intensities and the number of entrants in the free entry equilibrium are deficient. This is due to the familiar intuition that a firm's private innovation incentive does not internalize the positive externalities to consumers.[1] Second, compared to the first-best innovation direction, we find that there exists a critical value of patentability standard, $\hat{S}$, such that the equilibrium R&D direction coincides with the first best when $S = \hat{S}$, and it is biased towards (against) the risky direction when $S$ is below (above) $\hat{S}$.[2] For the second-best social welfare maximization problem, in which a hypothetical social planner can only set the patentability standard but not the R&D and entry activities of firms, the optimal $S$ balances the two goals of moving

[1] Under competition, there can also be a business-stealing effect that potentially results in excessive R&D and entry. In our model, as in Hunt (2004), the externality to consumers dominates.

[2] Intuitively, when $S$ is low, innovations in direction $B$—the risky direction—are patentable even when the quality improvement is small, which motives socially excessive R&D in $B$, relative to $A$. And the opposite is true when $S$ is high. Since we measure innovation or R&D direction by the ratio of R&D intensities in the two directions, R&D can be efficient in both $A$ and $B$ and yet biased towards one direction.

towards the socially optimal innovation rate and towards the socially optimal innovation direction. Thus, in general, the second best patentability standard will be different both from $\hat{S}$ and from the $S$ that maximizes the number of innovating firms.

Our paper is related to the existing theoretical literature on patents and cumulative innovation, which has studied models with R&D along a single direction and offered mixed findings on the effects of patent protection. For example, O'Donoghue (1998) and O'Donoghue et al. (1998) suggest that stronger patent protection has positive effects on the rate of innovation, provided that ex-ante agreement or contracting between innovators is efficient, whereas Bessen and Maskin (2009) and Segal and Whinston (2007) find cases where the effects are negative. Horowitz and Lai (1996) consider a model in which longer patents increase the size but decrease the frequency of the innovation. They show that the patent length that maximizes the rate of innovation is finite (or intermediate).[3] As we mentioned earlier, our model is most closely related to Hunt (2004), who studies patentability and cumulative innovation in a model with R&D only in one direction that corresponds to our $B$. By allowing multiple R&D directions, we introduce the important dynamic substitution effect and offer several new insights. In particular, in contrast to the result in Hunt that the patentability standard affects innovation only through a market structure effect, with no impact on each innovating firm's R&D intensity, we show that it also affects innovation through its impact on R&D intensities, in ways that are non-monotonic and somewhat unexpected. Thus, in our model, patentability standards affect industry innovation through both the extensive margin (number of entrants) and the intensive margin (R&D intensities). Moreover, our results on innovation (or R&D) direction are novel in this literature.

Our paper is also related to the literature on R&D portfolio, which has focused on the issue of how competition may affect the choice between safe and risky research projects for

[3] Chen et al. (2014) find that stronger patent protection can affect cumulative innovation either positively or negatively, and the effect is generally non-monotonic. Empirically, some recent studies on cumulative innovation (Murray et al., 2008; Furman and Stern, 2011; Galasso and Schankerman, 2013; Williams, 2013; Sampat and Williams, 2014) find no evidence of a relationship.

a stand-alone innovation.[4] Some authors have found, under the assumption of winner-take-all, that competition leads to over-investment in risky R&D projects because it magnifies the negative externality of investment by one firm on other firms' probability to win the patent (e.g., Bhattacharya and Mookherjee, 1986; Klette and de Meza, 1986; Dasgupta and Maskin, 1987). Others, however, have argued that investment in risky R&D project decreases with the strength of competition, because the negative externality of the risky R&D becomes small when competition strengthens, if each firm pursues multiple patents (Cabral, 1994; Kwon, 2010).[5] Anderson and Cabral (2007) study a game where firms choose the variance of a stochastic innovation outcome. They find that the level of equilibrium variance may be greater, smaller, or equal to the social optimum. Our paper contributes to this literature by examining the effect of patent policy on R&D portfolio and by considering cumulative innovations.

In the rest of the paper, we describe our model and its equilibrium in Section 2. In Section 3, we establish our results on how the patentability standard affects the rates of innovation, as measured by the R&D intensities of each firm in the two directions and by the overall R&D intensity of the industry, and how it affects the direction of innovation, as measured by the ratio of the innovation rates in the two directions. Section 4 contains our welfare results, comparing the equilibrium rate and direction of innovation with the social optimum, and discussing optimal patentability policy as the second best. Section 5 concludes. The main theoretical results are illustrated through a numerical example, and proofs that are more technical in nature are relegated to the appendix.

[4] Relatedly, Choi and Gerlach (2014) study the R&D choice between easy and difficult projects that are complementary for the production of a final product. They find that firms tend to invest excessively on the easy innovation due to hold-up problems.

[5] Unlike the other papers, Kwon (2010) considers complementary projects—basic and applied research.

## 2. THE MODEL

Time is continuous and is divided into periods, $t = 0, 1, 2, ...$, between stochastic discoveries by innovating firms. There are $n + 1$ firms in the industry, one of whom is the incumbent and the others are challengers in each period. At period $t$, the incumbent, through a patented innovation at an earlier period, can produce a product that has quality $q_t$. Each of the challengers conducts R&D to further improve the product quality.

There are two possible research directions for the challengers, $A$ and $B$. A successful innovation through direction $A$ will result in a certain quality improvement, $\Delta_A$.[6] A successful innovation through direction $B$ will yield an uncertain quality improvement, $\Delta_B$, which is a random variable with cumulative distribution function $G(\cdot)$ and continuous density $g(\cdot)$ on support $\left[\underline{\Delta}_B, \overline{\Delta}_B\right]$. As we pointed out before, this formulation follows closely Hunt (2004), with the main difference being that he considers R&D only along a single uncertain direction corresponding to $B$ here.

A challenger decides on a R&D portfolio by choosing the R&D intensity in each of the two directions. We assume that each innovation occurs according to a Poisson process. The cost for a challenger to maintain an arrival rate $\lambda_z$ in research direction $z \in \{A, B\}$ is $C(\lambda_z)$, which is strictly increasing and twice continuously differentiable, with $C(0) = C'(0) = 0$, $C''(\cdot) > 0$, and $\lim_{\lambda_z \to \infty} C'(\lambda_z) = \infty$.[7] We shall also refer to $\lambda_z$ as the R&D intensity in direction $z$.

The statutory life of a patent is assumed to be infinite, even though the patent life effectively ends when the next patentable invention occurs. To be awarded a patent, the quality improvement from an invention needs to meet a minimum improvement size, or the patentability standard, $S$. In practice, the patentability standard (or requirement) can

[6] We can allow $\Delta_A$ to be stochastic, as long as its variance is sufficiently small. For convenience, we assume $\Delta_A$ to be a constant.

[7] Notice that we allow the "corner" case where each firm chooses to conduct R&D only in one direction among the alternatives available. Under our assumptions on the cost function, however, the equilibrium will be interior.

correspond to the requirement of non-obviousness in the American patent code, or of the inventive step in Europe. For the purpose of this paper, we assume that $S \in [\underline{\Delta}_B, \bar{\Delta}_B]$ and $S < \Delta_A$. Thus, an innovation achieved through direction $A$ is always patentable, whereas $\theta \equiv 1 - G(S)$ is the probability that an innovation in direction $B$ is granted a patent. When an innovation is not protected by a patent, it becomes freely available to the public, in which case we assume that competition drives the profit from marketing the product to zero. Notice that the more stringent the patentability requirement, other things equal, the smaller the probability that the challenger can profitably market her innovation achieved through direction $B$.

We assume that at the beginning of period $t = 0$, there is a large number of firms, each deciding whether to pay a one-time fixed investment cost $k$ to enter the market. Thus, the number of challengers, $n$, is endogenously determined by the free-entry condition. If a challenger wins the race for a patentable innovation, it becomes the incumbent in the next period, and the previous incumbent becomes a challenger. If a challenger succeeds in an innovation that does not meet the patentability standard, then the incumbent maintains its leader position, and all $n + 1$ firms enter into a new period of patent race. The innovation arrival rates and the costs to achieve them remain the same after any discovery, whether patentable or not. Therefore, in either case, the relative positions of the $n + 1$ firms in the market are the same, and hence the choice problem for any firm in the market is stationary. We denote the discount rate, common for all firms, by $r$.

The market contains a representative consumer, who demands one unit of the product per period. The consumer's valuation for a product is equal to its quality. The marginal cost of production for any firm is normalized to zero. The incumbent and the challengers engage in price competition. Thus, when the incumbent's product quality exceeds the next closest quality by $\Delta$, it's flow profit is exactly $\Delta$ until the arrival of a new patentable innovation. The challengers earn no flow profit.

As in Hunt (2004) and other studies in this literature, we shall focus on an equilibrium where only challengers, but not the incumbent, will invest in R&D. Incumbents tend to have lower incentive to invest in R&D than entrants due to their existing profit. The assumption that they make no investment is more extreme, and it is motivated mainly for analytical tractability. Notice that in our model, players rotate their roles as the incumbent and the challengers over time, so a firm may only temporarily stop investing. In our analysis that follows, by construction, the strategies by the challengers and the incumbent will constitute a stationary Markov Perfect Equilibrium (MPE).[8]

We shall maintain the following assumption throughout the paper:

$$\underline{\Delta}_B < rk < \Delta_A < E\left[\Delta_B\right] \equiv \int_{\underline{\Delta}_B}^{\overline{\Delta}_B} \Delta_B dG\left(\Delta_B\right), \tag{A1}$$

because of two considerations: First, we are interested in situations where a successful innovation in the risky direction yields a higher expected quality improvement, which is captured by $\Delta_A < E\left[\Delta_B\right]$. Second, we wish to ensure that a positive number of firms will be willing to enter the market to pursue innovation in each direction, which will require $\underline{\Delta}_B < rk < \Delta_A$.

If a challenger innovates through direction $A$, she becomes an incumbent and receives a profit flow of

$$\pi_A = \Delta_A \tag{1}$$

until she is replaced by a future challenger. If the challenger succeeds in innovation direction $B$, the expected profit flow (conditional on the innovation being patentable) is

$$\pi_B = \frac{\int_S^{\overline{\Delta}_B} \Delta_B dG\left(\Delta_B\right)}{1 - G\left(S\right)}. \tag{2}$$

[8] There could potentially be another equilibrium where an incumbent from direction $B$ may conduct R&D if the realized $\Delta_B > S$ is relatively small, but its analysis appears to be untractable.

Notice that

$$\frac{\partial \pi_B}{\partial S} = \frac{-Sg(S)[1-G(S)] + g(S)\int_S^{\overline{\Delta}_B} \Delta_B dG(\Delta_B)}{[1-G(S)]^2} = \frac{g(S)}{1-G(S)}(\pi_B - S) \geq 0,$$

where $\pi_B \geq S$ because

$$\int_S^{\overline{\Delta}_B} \Delta_B dG(\Delta_B) \geq \int_S^{\overline{\Delta}_B} SdG(\Delta_B) = S[1-G(S)].$$

It follows that

$$\pi_B \equiv \pi_B(S) \geq \pi_B(\underline{\Delta}_B) = E[\Delta_B].$$

This, together with (1) and assumption (A1), implies that $\pi_B > \pi_A > rk > \underline{\Delta}_B$. Thus, entry to pursue innovation in each direction can be potentially profitable. The equilibrium number of entrants in the market will be determined simultaneously as the arrival rate of innovation in each direction, as we show next.

At a stationary MPE, let $V_z^I$ be the value of being an incumbent through type-$z$ innovation and $V^E$ the value of being a challenger, all of which are evaluated at the beginning of a period. Then $V_A^I$, $V_B^I$ and $V^E$ satisfy[9]:

$$rV_A^I = \pi_A + n(\lambda_A + \theta\lambda_B)\left(V^E - V_A^I\right), \tag{3}$$

$$rV_B^I = \pi_B + n(\lambda_A + \theta\lambda_B)\left(V^E - V_B^I\right), \tag{4}$$

and

$$rV^E = \lambda_A\left(V_A^I - V^E\right) - C_A(\lambda_A) + \theta\lambda_B\left(V_B^I - V^E\right) - C_B(\lambda_B). \tag{5}$$

Equations (3), (4) and (5) suggest that the value of being an incumbent depends on the

[9]Notice that the probability that any two innovations succeed simultaneously is zero. Because we are constructing an equilibrium in which the incumbent does not invest, no matter what its realized quality improvement is, the value function $V_B^I$ below is not contingent on the realization of $\Delta_B$.

type of innovation that has led to the incumbency.[10]

From (5), the challenger chooses optimal $\lambda_A$ and $\lambda_B$, which respectively satisfy the first-order conditions:[11]

$$C_A'(\lambda_A) = V_A^I - V^E, \tag{6}$$

and

$$C_B'(\lambda_B) = \theta\left(V_B^I - V^E\right). \tag{7}$$

The free entry condition implies

$$V^E = k. \tag{8}$$

From (3), (6) and (8), we find

$$V_A^I - V^E = \frac{\pi_A - rk}{r + n(\lambda_A + \theta\lambda_B)} = C_A'(\lambda_A). \tag{9}$$

Similarly, from (4), (7) and (8), we have

$$V_B^I - V^E = \frac{\pi_B - rk}{r + n(\lambda_A + \theta\lambda_B)} = \frac{C_B'(\lambda_B)}{\theta}. \tag{10}$$

Substituting (9) and (10) into (5) yields

$$\lambda_A C_A'(\lambda_A) + \lambda_B C_B'(\lambda_B) - C_A(\lambda_A) - C_B(\lambda_B) - rk = 0. \tag{11}$$

The system of equations, (9), (10) and (11), determine the three equilibrium values $\lambda_A^*$, $\lambda_B^*$ and $n^*$, which we assume to exist uniquely. In particular, from (9) and (10), the equilibrium

[10] Note that the size of quality improvement appears with the corresponding probability, even though the incumbent knows its exact value after innovation is successful. Hence $\pi_B$ is shown in the right hand of (4).

[11] The properties of the cost functions ensure that the second-order conditions are satisfied.

number of challengers can be expressed as

$$n^* = \left[\frac{\pi_A - rk}{C_A'(\lambda_A^*)} - r\right] \cdot \frac{1}{\lambda_A^* + \theta\lambda_B^*} = \left[\frac{\theta(\pi_B - rk)}{C_B'(\lambda_B^*)} - r\right] \cdot \frac{1}{\lambda_A^* + \theta\lambda_B^*}. \tag{12}$$

We illustrate the equilibrium of the model with the following example:

**Example 1**. Suppose that $\Delta_B$ follows the uniform distribution on $[0,1]$, while $C_A(\lambda_A) = \frac{1}{2}\lambda_A^2$ and $C_B(\lambda_B) = \frac{1}{2}\lambda_B^2$. Then, from (9), (10) and (11):

$$\lambda_A^* = \frac{(\Delta_A - rk)}{(1-S)\left(\frac{1+S}{2} - rk\right)}\sqrt{\frac{2rk}{\left(\frac{(\Delta_A - rk)}{(1-S)\left(\frac{1+S}{2}-rk\right)}\right)^2 + 1}}, \quad \lambda_B^* = \sqrt{\frac{2rk}{\left(\frac{(\Delta_A - rk)}{(1-S)\left(\frac{1+S}{2}-rk\right)}\right)^2 + 1}},$$

and

$$n^* = \left[\frac{2(\Delta_A - rk)}{\lambda_A} - r\right]\left[\frac{1}{\lambda_B + (1-S)\lambda_A}\right].$$

Further assuming $\Delta_A = 0.3$, $r = 0.05$, $k = 0.9$, and $S = 0.1$, we have $\lambda_A^* = 0.147$, $\lambda_B^* = 0.261\,63$, and $n^* = 8.68$.

We shall continue with this example to also illustrate results in the following sections.

## 3. THE RATES AND DIRECTION OF INNOVATION

We are now in a position to examine how the patentability standard, $S$, may affect the rates and direction of innovation. We first consider the effects of $S$ on the equilibrium R&D intensities, $\lambda_A^*$ and $\lambda_B^*$, which can be viewed as each entrant's innovation rates in directions $A$ and $B$, respectively. Recall that $\lambda_A^*$, $\lambda_B^*$ and $n^*$ are determined by (9), (10) and (11). In the appendix, we show the following by using the Cramer's rule:

$$\frac{\partial\lambda_A^*}{\partial S} = \frac{g(S)\lambda_B^*(\lambda_A^* + \theta\lambda_B^*)\left[C_A'(\lambda_A^*)\right]^2 C_B''(\lambda_B^*)(rk - S)}{|M|} \tag{13}$$

and

$$\frac{\partial \lambda_B^*}{\partial S} = \frac{-g(S)\lambda_A^*(\lambda_A^* + \theta\lambda_B^*)\left[C_A'(\lambda_A^*)\right]^2 C_A''(\lambda_A^*)(rk - S)}{|M|}, \tag{14}$$

where

$$|M| = -(\lambda_A^* + \theta\lambda_B^*)C'(\lambda_B^*)\,C''(\lambda_A^*)\,C''(\lambda_B^*)\left[\lambda_A^*(\pi_A - rk) + \theta\lambda_B^*(\pi_B - rk)\right] < 0$$

since $\pi_B > \pi_A > rk$. Thus, if $S < rk$, then $\frac{\partial \lambda_A^*}{\partial S} < 0$ and $\frac{\partial \lambda_B^*}{\partial S} > 0$; while if $S > rk$, then $\frac{\partial \lambda_A^*}{\partial S} > 0$ and $\frac{\partial \lambda_B^*}{\partial S} < 0$. This leads to the following result, where we define

$$d(S) \equiv \frac{\lambda_B^*}{\lambda_A^*} \tag{15}$$

as the innovation direction.

**Proposition 1** *As $S$ increases, $\lambda_B^*$ first increases and then decreases, whereas $\lambda_A^*$ first decreases and then increases, reaching the maximum and the minimum, respectively, at $S \equiv rk$. Moreover, innovation direction $d(S)$ has an inverted-U shape, maximized at $S = rk$.*

Interestingly, R&D intensities in both directions vary non-monotonic with $S$, in contrast to the result in Hunt (2004) that R&D intensity is invariant with the patentability standard. As we discussed in the introduction, the presence of two research directions in our model introduced a strategic substitution effect. When the R&D intensity in one direction becomes higher (or lower), it exerts an opposite force on the R&D intensity in the other direction. Thus, in the free entry equilibrium of our model, patentability standards impact industry R&D not only through the number of firms (the extensive margin), but also through changes in the R&D intensities in different directions (the intensive margin). Notice that in (14), if $\lambda_A^* = 0$, then $\lambda_B^*$ would be independent of $S$, and our results would be the same as Hunt's.[12]

[12] See the equation after (A.7) on pp. 421 in Hunt (2004), except that, in Hunt, (i) there is an industry-specific productivity parameter, which we assume to be 1; and (ii) the reservation value of the product is the level of its quality multiplied by $p$, and we assume $p = 1$.

In our model, innovation direction, $d(S)$, is measured by each challenger's R&D intensity in the risky direction relative to that in the safe direction, which determines the relative rates of innovation achieved through the two directions. The inverted-U shaped $d(S)$, with its maximum attained at $S = rk$, follows directly from the shapes of $\lambda_B^*(S)$ and $\lambda_A^*(S)$. Since the expected size of each innovation is higher in direction $B$ than in direction $A$, one might think that it would be desirable to choose $S = rk$. However, the overall expected innovation rate of each challenger,

$$\rho \equiv \lambda_A^* \Delta_A + \lambda_B^* E[\Delta_B],$$

depends also on how different $\lambda_B^*(S)$ and $\lambda_A^*(S)$ are. Hence, $\rho$ may not be maximized at $rk$. For the industry innovation rate, we need to further consider the number of entrants in equilibrium ($n^*$), which is also a function of $S$.

The equilibrium overall innovation rate of the industry can be defined as:

$$R \equiv n^* \rho. \tag{16}$$

The result below indicates that the shape of $R \equiv R(S)$ is consistent with that of $d(S)$.

**Proposition 2** *As $S$ rises, $R$ initially increases and eventually decreases, reaching its maximum when $S$ is at some intermediate level.*

**Proof.** See the appendix. ■

As in Hunt (2004), the industry rate of innovation ($R$) is maximized when the patentability standard is neither too high nor too low. However, the channels through which $S$ affects $R$ differ in the two models. In Hunt, as $S$ increases, the equilibrium number of firms to conduct R&D in the market first increases and then decreases, whereas the equilibrium R&D intensity remains unchanged. Our model entails a second channel: the changes in the

R&D intensities, since

$$\frac{\partial R}{\partial S} = n^* \frac{\partial \rho}{\partial S} + \rho \frac{\partial n^*}{\partial S},$$

and in our model, $\lambda_A^*$ and $\lambda_B^*$—and hence also $\rho$—in general vary with $S$.

Define $S_R$ as the patentability requirement that maximizes the innovation rate of the industry $R = R(S)$:

$$S_R = \arg\max \{R(S)\}.$$

In the appendix, we show that

$$\frac{\partial R}{\partial S}|_{S=rk} > 0, \tag{17}$$

which immediately leads to:

**Remark 1** *If $R$ is a single-peaked function of $S$, then $S_R > rk$.*

Therefore, the patentability standard that maximizes the overall rate of innovation in the industry is higher than $S = rk$, which maximizes $\lambda_B$, provided that $R(S)$ is single-peaked.

Continuing with Example 1, we next illustrate Proposition 1 by showing how $\lambda_A^*$ and $\lambda_B^*$ vary with $S$:

**Example 1 (Continue)**. Assume that the parameter values are as assumed in Example 1, except now $S$ is allowed to vary. Then:

$$\begin{aligned} \lambda_A^* &= 0.153\sqrt{\frac{1}{S^4 - 0.18S^3 - 1.811\,9S^2 + 0.163\,8S + 1.088\,2}}, \\ \lambda_B^* &= 0.3\,(S + 0.91)\,(1 - S)\sqrt{\frac{1}{S^4 - 0.18S^3 - 1.811\,9S^2 + 0.163\,8S + 1.088\,2}}. \end{aligned}$$

Figures 1 and 2 below illustrate $\lambda_A^*$ and $\lambda_B^*$ as functions of $S$, where $rk = 0.045$.

Figure 1:

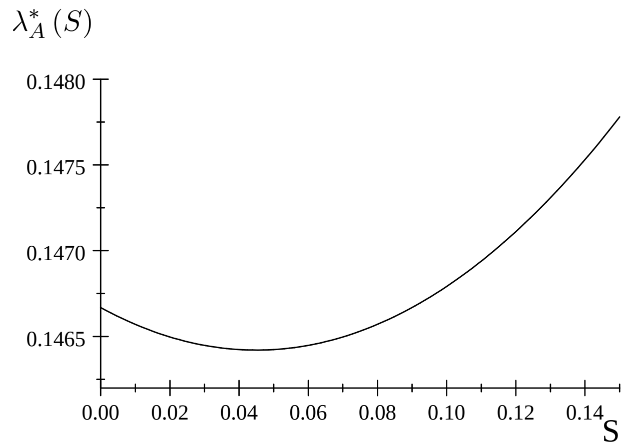
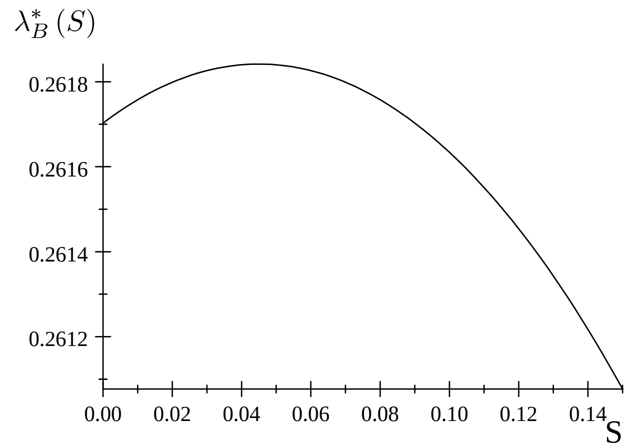



Figure 2:




## 4. EFFICIENT INNOVATION INCENTIVE AND OPTIMAL $S$

In this section, we compare the equilibrium and the efficient incentives for cumulative innovation, where "efficient" means welfare-maximizing or the first-best; and we study how to choose $S$ optimally at the market equilibrium. Specifically, we seek to answer two questions. First, if one could directly choose the number of entrants and the R&D intensities to maximize social welfare, what would be these choices and how would they differ from those in the market equilibrium? Second, if policy can choose patentability standards, but not firms' innovative activities, what should be the optimal $S$? Subsection 4.1 addresses the two questions in terms of the values for $\lambda_A$, $\lambda_B$, and $n$, while subsection 4.2 considers the questions from the perspective of innovation directions.

### 4.1 Comparing R&D Intensities and the Number of Entrants

When there are $n$ challengers, each choosing R&D intensities $\lambda_A$ and $\lambda_B$ in directions $A$ and $B$, respectively, total welfare is

$$W = \frac{n}{r}\left[\lambda_A \frac{\Delta_A}{r} - C_A(\lambda_A) + \lambda_B \frac{E[\Delta_B]}{r} - C_B(\lambda_B) - rk\right], \tag{18}$$

where $\frac{\Delta_A}{r}$ and $\frac{E[\Delta_B]}{r}$ are the expected social values of innovations generated by one innovating firm through directions $A$ and $B$, respectively. The expression inside the square brackets in (18) is thus the instantaneous social benefit from one innovating firm, and there are $n$ independent innovating firms for the industry, multiplied by $\frac{1}{r}$ to account for the discounted sum of the instantaneous benefits.

At the first best where a hypothetical social planner directly chooses $\lambda_A$, $\lambda_B$ and $n$ to maximize $W$, the welfare-maximizing $\lambda_A^o$ and $\lambda_B^o$ satisfy the following first-order conditions:

$$C'(\lambda_A^o) = \frac{\Delta_A}{r} \qquad \text{and} \qquad C'(\lambda_B^o) = \frac{E[\Delta_B]}{r}. \tag{19}$$

Notice that the efficient R&D intensities equate their marginal social benefits and costs. Comparing (19) to (9) and (10) and noticing that

$$\theta(\pi_B - rk) = \int_S^{\overline{\Delta}_B} \Delta_B dG(\Delta_B) - \theta rk < E[\Delta_B], \tag{20}$$

we find that the efficient R&D intensities are higher than those in the free entry equilibrium: $\lambda_z^o > \lambda_z^*$, for $z = A, B$. Intuitively, this is because when choosing R&D intensities in the market equilibrium, a firm does not internalize the positive externalities of its innovation to consumers.

Moreover, since $\lambda_z C_z'(\lambda_z) - C(\lambda_z)$ increases in $\lambda_z$ and $\lambda_z^o > \lambda_z^*$ for $z = A, B$, utilizing

(19) and (11), we have

$$\begin{aligned} & \lambda_A^o \frac{\Delta_A}{r} - C_A\left(\lambda_A^o\right) + \lambda_B^o \frac{E\left[\Delta_B\right]}{r} - C_B\left(\lambda_B^o\right) - rk \\ = \ & \lambda_A^o C_A'\left(\lambda_A^o\right) - C_A\left(\lambda_A^o\right) + \lambda_B^o C_B'\left(\lambda_B^o\right) - C_B\left(\lambda_B^o\right) - rk \\ > \ & \lambda_A^* C_A'\left(\lambda_A^*\right) - C_A\left(\lambda_A^*\right) + \lambda_B^* C_B'\left(\lambda_B^*\right) - C_B\left(\lambda_B^*\right) - rk = 0. \end{aligned} \quad (21)$$

Hence, as in Hunt (2004), the efficient number of firms is $n^o = \infty > n^*$.

Summarizing the discussions above, we have:

**Proposition 3** *Compared to the first-best, R&D intensities and the number of entrants are deficient under the free entry equilibrium.*

When policy can choose the patentability standard whereas firms choose R&D intensities under free entry to maximize their private benefits, the optimal choice of $S$ is also called the second-best problem. Let $W(S)$ be the welfare in equilibrium at the second best. Then, from (18),

$$\begin{aligned} \frac{\partial W(S)}{\partial S} = \ & \frac{1}{r}\frac{\partial n^*}{\partial S}\left[\lambda_A^* \frac{\Delta_A}{r} - C_A\left(\lambda_A^*\right) + \lambda_B^* \frac{E\left[\Delta_B\right]}{r} - C_B\left(\lambda_B^*\right) - rk\right] \\ & + \frac{n^*}{r}\left\{\left[\frac{\Delta_A}{r} - C_A'\left(\lambda_A^*\right)\right]\frac{\partial \lambda_A^*}{\partial S} + \left[\frac{E\left[\Delta_B\right]}{r} - C_B'\left(\lambda_B^*\right)\right]\frac{\partial \lambda_B^*}{\partial S}\right\}. \end{aligned} \quad (22)$$

The optimal patentability standard, denoted by $S^*$, coincides with the one that maximizes the number of entrants in Hunt (2004) if $\lambda_A^* \equiv 0$. To see this, notice that if $\lambda_A^* \equiv 0$, our model reduces to that in Hunt (2004), implying $\frac{\partial \lambda_A^*}{\partial S} = 0$, and by (14) $\frac{\partial \lambda_B^*}{\partial S} = 0$. Hence, $\frac{\partial W(S)}{\partial S} = 0$ implies $\frac{\partial n^*}{\partial S} = 0$. However, in our model

$$\left\{\left[\frac{\Delta_A}{r} - C_A'\left(\lambda_A^*\right)\right]\frac{\partial \lambda_A^*}{\partial S} + \left[\frac{E\left[\Delta_B\right]}{r} - C_B'\left(\lambda_B^*\right)\right]\frac{\partial \lambda_B^*}{\partial S}\right\}$$

is generally not zero when $\frac{\partial n^*}{\partial S} = 0$, and thus the optimal patentability standard generally differs from the one that maximizes $n^*$.

From Remark 1, the $S$ that maximizes the industry innovation rate ($R$) exceeds $rk$, provided that $R$ is a single-peaked function of $S$. If $W$ is a single-peaked function of $S$, then $S^*$ also exceeds $rk$. To see this, note that, from (9), (10) and (20),

$$C_A'(\lambda_A^*) < \frac{\Delta_A}{r} \qquad \text{and} \qquad C_B'(\lambda_B^*) < \frac{\theta(\pi_B - rk)}{r} < \frac{E[\Delta_B]}{r}.$$

Thus, noticing $\frac{\partial \lambda_A^*}{\partial S}|_{S=rk} = \frac{\partial \lambda_B^*}{\partial S}|_{S=rk} = 0$ and $\frac{\partial n^*}{\partial S}|_{S=rk} > 0$, we have

$$\begin{aligned}
\frac{\partial W}{\partial S}|_{S=rk} &= \frac{1}{r}\frac{\partial n^*}{\partial S}|_{S=rk}\left[\lambda_A^*\frac{\Delta_A}{r} - C_A(\lambda_A^*) + \lambda_B^*\frac{E\Delta_B}{r} - C_B(\lambda_B^*) - rk\right] \\
&> \frac{1}{r}\frac{\partial n^*}{\partial S}|_{S=rk}\left[\lambda_A^* C_A'(\lambda_A^*) - C_A(\lambda_A^*) + \lambda_B^* C_B'(\lambda_B^*) - C_B(\lambda_B^*) - rk\right] \\
&= 0,
\end{aligned}$$

where the equality follows from (11).

Summarizing the above discussion, we have:

**Remark 2** *As a second-best, the patentability standard that maximizes $W \equiv W(S)$, $S^*$, generally does not maximize the number of firms in the industry. Furthermore, if $W(S)$ is single-peaked, then $S^* > rk$.*

Therefore, even though the expected quality improvement from an innovation is higher in direction $B$ than in direction $A$, under the single-peak condition, the welfare-maximizing $S$ does not maximize innovation in direction $B$. This is because by raising $S$ above $rk$, industry innovation can be increased.

Notice that for $S^*$ to be a valid solution to the maximization problem for $W(S)$, we have implicitly assumed that $S^* \leq \Delta_A$. If this constraint is binding, then we would have $S^* = \Delta_A$. This is because if $S > \Delta_A$, then no entrant would conduct R&D in direction $A$, so that $\lambda_A = 0$ and the problem is the same as if $B$ were the only research direction. But since $C_A'(0) = 0$ by assumption, it is socially desirable to have strictly positive R&D investment in direction $A$. This implies that $W(S)$ would jump down at $S = \Delta_A$. Therefore, it is likely that $S^* \leq \Delta_A$ even if we allow $S$ to be larger than $\Delta_A$.

### 4.2 Comparing the Innovation Directions

We now compare the equilibrium innovation direction $d(S)$ with the innovation direction that maximizes social welfare, $d^o$. From (19), we have

$$d^o = \frac{\lambda_B^o}{\lambda_A^o}, \text{ where } \frac{C'(\lambda_B^o)}{C'(\lambda_A^o)} = \frac{E[\Delta_B]}{\Delta_A}.$$

Hence, at the welfare-maximizing innovation direction, the ratio of the marginal costs equals the ratio of the marginal benefits of innovations in the two directions.

Recall from Proposition 1 that the equilibrium innovation direction $d(S)$ is maximized at $S = rk$. The result below states that firms are biased towards (against) innovation in direction $B$ when $S$ is below (above) some threshold. A sketch of the proof is as follows: If $S = \underline{\Delta}_B$, we have $d(S) > d^o$, with a bias towards $B$. As $S$ increases but is smaller than $rk$, innovation is even more biased towards $B$ because, from Proposition 1, $\lambda_B^*/\lambda_A^*$ increases in $S$ if $S < rk$. As $S$ further increases and surpasses $rk$, $\lambda_B^*$ starts to decrease and $\lambda_A^*$ to increase, and thus $d(S)$ becomes smaller but can still be larger than $d^o$. When $S > \hat{S}$, the threshold value of $S$, $d(S)$ falls below $d^o$ and monotonically decreases, so that innovation direction is biased towards $A$. Formally:

**Proposition 4** *There exists $\hat{S} \in [\underline{\Delta}_B, \overline{\Delta}_B]$ such that $d(\hat{S}) = d^o$, with $d(S) > d^o$ if $S < \hat{S}$ but $d(S) < d^o$ if $S > \hat{S}$. Moreover, $\hat{S} > rk$.*

**Proof.** From (19), under the social optimum,

$$\frac{C_B'(\lambda_B^o)}{C_A'(\lambda_A^o)} = \frac{E\Delta_B}{\Delta_A}.$$

From (9) and (10), given $S$, in the free-entry equilibrium

$$\frac{C_B'(\lambda_B^*(S))}{C_A'(\lambda_A^*(S))} = \frac{\theta(S)[\pi_B(S) - rk]}{\Delta_A - rk}.$$

Thus,

$$\delta(S) \equiv \frac{\frac{C_B'\left(\lambda_B^*(S)\right)}{C_A'\left(\lambda_A^*(S)\right)}}{\frac{C_B'\left(\lambda_B^o\right)}{C_A'\left(\lambda_A^o\right)}} = \frac{\theta(S)[\pi_B(S) - rk]}{E[\Delta_B]\frac{\Delta_A - rk}{\Delta_A}}.$$

As $S$ increases,

$$\Phi(S) \equiv \theta(S)[\pi_B(S) - rk]$$

first increases and then decreases, reaching its maximum at $S = rk$, because, since $\theta(S) = 1 - G(S)$,

$$\begin{aligned}
\frac{\partial \Phi(S)}{\partial S} &= \frac{\partial \theta}{\partial S}[\pi_B(S) - rk] + \theta(S)\frac{\partial \pi_B(S)}{\partial S} \\
&= -g(S)[\pi_B(S) - rk] + \theta(S)\frac{g(S)[\pi_B(S) - S]}{1 - G(S)} \\
&= -g(S)(S - rk).
\end{aligned}$$

Moreover, $\Phi(\underline{\Delta}_B) = E[\Delta_B] - rk > 0$ and $\Phi\left(\overline{\Delta}_B\right) = 0$. Therefore, there exists a unique $\hat{S} > rk$, determined by

$$\Phi\left(\hat{S}\right) = E[\Delta_B]\frac{\Delta_A - rk}{\Delta_A} < E[\Delta_B],$$

such that $\delta\left(\hat{S}\right) = 1$, with

$$d\left(\hat{S}\right) = \frac{\lambda_B^*\left(\hat{S}\right)}{\lambda_A^*\left(\hat{S}\right)} = \frac{\lambda_B^o}{\lambda_A^o} = d^o.$$

Moreover, since

$$\delta\left(\underline{\Delta}_B\right) = \frac{\Phi\left(\underline{\Delta}_B\right)}{E\left[\Delta_B\right]\frac{\Delta_A - rk}{\Delta_A}} = \frac{E\left[\Delta_B\right] - rk}{E\left[\Delta_B\right]\frac{\Delta_A - rk}{\Delta_A}} = \frac{1 - \frac{rk}{E[\Delta_B]}}{1 - \frac{rk}{\Delta_A}} > 1$$

and $\frac{\lambda_B^*(S)}{\lambda_A^*(S)}$—hence $\delta(S)$—increases for $S \in [\underline{\Delta}_B, rk)$ but decreases for $S \in \left(rk, \hat{S}\right)$, we have $\delta(S) > 1$ and $\frac{\lambda_B^*(S)}{\lambda_A^*(S)} > d^o$ if $S < \hat{S}$. Also, since $\lambda_B^*(S)$ decreases in $S$ while $\lambda_A^*(S)$ increases in $S$ for $S > rk$, we have $\delta(S) < 1$ and $\frac{\lambda_B^*(S)}{\lambda_A^*(S)} < d^o$ if $S > \hat{S}$. ■

Therefore, $\hat{S}$ implements the welfare-maximizing innovation direction, provided that $\hat{S} \leq \Delta_A$; innovation is biased towards $B$ when $S < \hat{S}$, whereas it is biased towards $A$ when $S > \hat{S}$. Intuitively, when the patentability standard is relatively low, the risky research direction with uncertain innovation size is likely to yield a patent even when the quality improvement is small, which motivates firms to conduct R&D in that direction excessively relative to the direction with a certain innovation size. Conversely, when the patentability standard is high enough, the direction with uncertain innovation size is unlikely to receive a patent even when the quality improvement is relatively large, which unduly discourages R&D in that direction.

Notice that while $\hat{S}$ leads to the efficient choice of research direction, it need not be the welfare-maximizing $S$ for $W(S)$ in the second-best problem. This is because $S$ also affects $W(S)$ through $n^*(S)$, as can be seen from (22), and thus $\hat{S}$ need not maximize $W(S)$. Intuitively, the second-best choice of $S$, $S^*$, will generally involve a trade off between two policy goals: moving towards the efficient R&D direction ($d^o$) and towards the efficient number of entrants ($n^o$). When $S$ achieves the efficient R&D direction, as $\hat{S}$ does, it does not optimally balance the two goals, and hence in general $\hat{S}$ does not maximize $W(S)$ (i.e. $\hat{S} \neq S^*$).

### 4.3 Example

To illustrate the results in this section, we continuing with example 1:

**Example 1 (Continue)**. First, as an illustration of Remark 2. we find that $W(S)$ is maximized at $S = S^* = 0.272$, whereas $n^*(S)$ is maximized at $S = 0.269$.

Next,

$$\lambda_A^o = \frac{\Delta_A}{r} = \frac{0.3}{0.05} = 6, \qquad \lambda_B^o = \frac{E[\Delta_B]}{r} = \frac{0.5}{0.05} = 10, \quad d^o = \frac{5}{3} = 1.67.$$

Recall from Section 3 that $\lambda_A^* = 0.147$, $\lambda_B^* = 0.261\,63$, and $n^* = 8.\,68$. Therefore $\lambda_A^* < \lambda_A^o$, $\lambda_B^* < \lambda_B^o$, and $n^* < n^o = \infty$, illustrating Proposition 3.

Notice that

$$\theta(S)[\pi_B(S) - rk] = (1-S)\left(\frac{\int_S^1 xdx}{1-S} - \frac{45}{1000}\right) = \frac{1}{200}(100S+91)(1-S),$$

$$E[\Delta_B]\frac{\Delta_A - rk}{\Delta_A} = 0.5\frac{0.3-0.045}{0.3} = 0.425.$$

Thus, from

$$\frac{1}{200}(100S+91)(1-S) = 0.5\frac{0.3-0.045}{0.3},$$

we find $\hat{S} = 0.294$, such that $d(S) > d^o$ if $S < \hat{S}$ but $d(S) < d^o$ if $S > \hat{S}$. Notice that in this example, to maximize $W(S)$, $S^* = 0.272$, which is lower than $\hat{S}$. Figure 3 displays the curve for $d(S)$. As predicted in Proposition 1, $d(S)$ initially increases in $S$, reaching its maximum at $rk = 0.045$, and decreases thereafter.

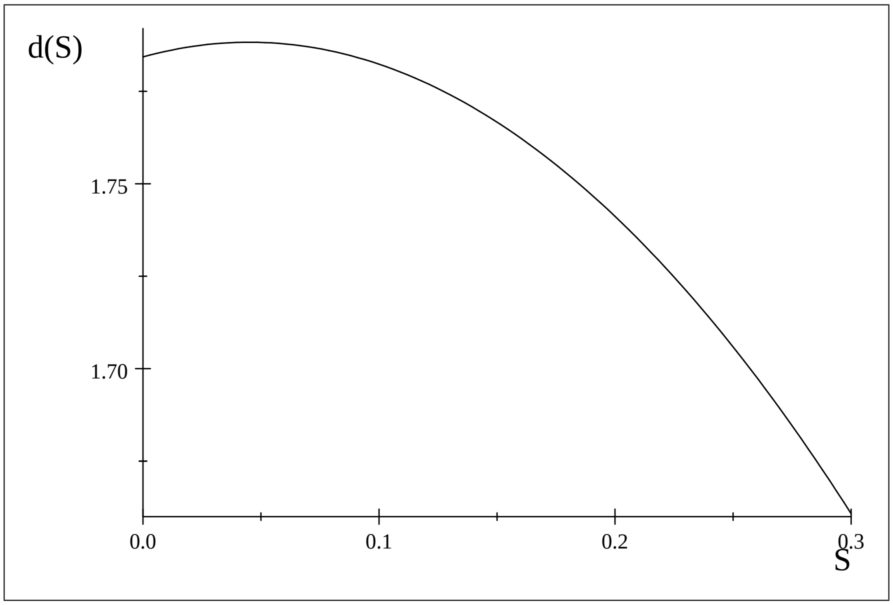



Figure 3: Innovation Direction $d(S)$

## 5. CONCLUSION

This paper has provided a first look at how patent policy may impact the rate and direction of cumulative innovation when firms can conduct R&D in multiple directions. We have three main findings: (i) Patentability standards affect the rate of industry innovation through both the number of entrants and their R&D intensities in the free entry equilibrium. As $S$ rises, the rate of industry innovation initially increases and eventually decreases. (ii) Compared to the social optimum, market incentives for cumulative innovation are deficient for both R&D intensities and the number of entrants. (iii) There exists a critical level of patentability standard ($\hat{S}$) under which the innovation direction is efficient, whereas R&D is biased towards (against) the risky direction when $S$ is below (above) $\hat{S}$. However, if $S$ is the only policy variable available, then the optimal policy, which balances the trade-off between the rate and direction of innovation, will in general be different from $\hat{S}$.

Discussions about patent policy and the patent system have frequently surrounded the issue of patentability standards. It has been argued that patentability standards in the

U.S. are too low, leading to excessive incentives for small-size innovations (e.g., Hunt, 2004; Jaffe, 2000). Our results suggest that raising patentability standards may indeed improve innovation direction, with two caveats: first, the effect of a higher $S$ on innovation direction may be non-monotonic, and a small increase in $S$ can either alleviate or exacerbate possible direction biases depending on the starting point; second, in our model, the risky direction may lead to more small-size innovations but to a higher expected size than the safe direction. Hence, even when raising $S$ reduces the patenting of small-size innovations, it may not raise the expected innovation size.

In our model, the fixed setup cost for R&D (adjusted by $r$), $rk$, plays important roles in determining the innovation incentives and the optimal patentability standards. This cost generally differs for different industries. For instance, it is likely much larger in the pharmaceutical industry than in the software industry. Thus, it would be desirable that patentability standards differ for different industries, depending (indirectly) on the setup cost for R&D projects. Moreover, innovations in developing countries tend to be much below the world technology frontier and require lower setup cost $rk$ than those in developed countries. Then, the desirable patentability requirement could be lower in developing countries in order to promote innovation.[13]

For tractability, we have studied a highly stylized model. It would be desirable for future research to extend our analysis to more general situations, especially to settings where incumbents and challengers both make R&D investments. It would also be interesting to consider other patent policy instruments, including patent strength aimed at preventing imitations. Our framework of cumulative innovation with multiple R&D directions can be a useful starting point for these and other studies.

[13] Chen and Puttitanun (2005) shows how intellectual property rights (IPRs) affect innovations in developing countries and how the optimal IPRs policy may vary with a country's level of development.

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## APPENDIX

The appendix contains proofs for Proposition 1, Proposition 2, and inequality (17).

**Proof of Proposition 1.** From (9), (11), and (10), the equilibrium $\lambda_A^*, \lambda_B^*$ and $n^*$ solve the system of equations below:

$$M \equiv \begin{array}{l} M^1 \equiv C_A'(\lambda_A)[r + n(\lambda_A + \theta\lambda_B)] - (\pi_A - rk) \\ M^2 \equiv \lambda_A C_A'(\lambda_A) + \lambda_B C_B'(\lambda_B) - C_A(\lambda_A) - C_B(\lambda_B) - rk \\ M^3 \equiv \theta C_A'(\lambda_A)(\pi_B - rk) - C_B'(\lambda_B)(\pi_A - rk) \end{array} = 0$$

Let $M_j^i \equiv \frac{\partial M^i}{\partial j}$, for $i = 1, 2, 3$ and $j = \lambda_A, \lambda_B, S, n$. Define $|M_{AS}|, |M_{BS}|, |M_{nS}|$ and $|M|$ as the determinants of matrix $M_{AS}$, $M_{BS}$, $M_{nS}$ and $M$, respectively:

$$|M_{AS}| = \begin{vmatrix} M_S^1 & M_{\lambda_B}^1 & M_n^1 \\ M_S^2 & M_{\lambda_B}^2 & M_n^2 \\ M_S^3 & M_{\lambda_B}^3 & M_n^3 \end{vmatrix}, \qquad |M_{BS}| = \begin{vmatrix} M_{\lambda_A}^1 & M_S^1 & M_n^1 \\ M_{\lambda_A}^2 & M_S^2 & M_n^2 \\ M_{\lambda_A}^3 & M_S^3 & M_n^3 \end{vmatrix},$$

$$|M_{nS}| = \begin{vmatrix} M_{\lambda_A}^1 & M_{\lambda_B}^1 & M_S^1 \\ M_{\lambda_A}^2 & M_{\lambda_B}^2 & M_S^2 \\ M_{\lambda_A}^3 & M_{\lambda_B}^3 & M_S^3 \end{vmatrix}, \quad \text{and} \quad |M| = \begin{vmatrix} M_{\lambda_A}^1 & M_{\lambda_B}^1 & M_n^1 \\ M_{\lambda_A}^2 & M_{\lambda_B}^2 & M_n^2 \\ M_{\lambda_A}^3 & M_{\lambda_B}^3 & M_n^3 \end{vmatrix}.$$

By Cramer's rule, we have

$$\frac{\partial \lambda_A^*}{\partial S} = -\frac{|M_{AS}|}{|M|} \qquad \text{and} \qquad \frac{\partial \lambda_B^*}{\partial S} = -\frac{|M_{BS}|}{|M|}.$$

We next compute the relevant derivatives:

$$\begin{aligned} M_{\lambda_A}^1 &= [r + n(\lambda_A + \theta\lambda_B)]C_A''(\lambda_A) + nC_A'(\lambda_A), \quad M_{\lambda_B}^1 = n\theta C_A'(\lambda_A), \\ M_n^1 &= (\lambda_A + \theta\lambda_B)C_A'(\lambda_A), \quad M_S^1 = -g(S)n\lambda_B C_A'(\lambda_A); \end{aligned}$$

$$M^2_{\lambda_A} = \lambda_A C''_A(\lambda_A), \quad M^2_{\lambda_B} = \lambda_B C''_B(\lambda_B), \quad M^2_n = 0, \quad M^2_S = 0;$$

$$\begin{aligned} M^3_{\lambda_A} &= \theta(\pi_B - rk)C''_A(\lambda_A), \quad M^3_{\lambda_B} = -(\pi_A - rk)C''_B(\lambda_B), \\ M^3_n &= 0, \quad M^3_S = g(S)C'_A(\lambda_A)(rk - lS). \end{aligned}$$

Thus,

$$\begin{aligned} |M_{AS}| &= M^1_S M^2_{\lambda_B} M^3_n + M^1_{\lambda_B} M^2_n M^3_S + M^1_n M^2_S M^3_{\lambda_B} - M^1_n M^2_{\lambda_B} M^3_S - M^1_{\lambda_B} M^2_S M^3_n - M^1_S M^2_n M^3_{\lambda_B} \\ &= -g(S)\lambda_B(\lambda_A + \theta\lambda_B)\left(C'_A\right)^2 C''_B(rk - S), \end{aligned}$$

$$\begin{aligned} |M_{BS}| &= M^1_{\lambda_A} M^2_S M^3_n + M^1_S M^2_n M^3_{\lambda_A} + M^1_n M^2_{\lambda_A} M^3_S - M^1_n M^2_S M^3_{\lambda_A} - M^1_S M^2_{\lambda_A} M^3_n - M^1_{\lambda_A} M^2_n M^3_S \\ &= g(S)\lambda_A(\lambda_A + \theta\lambda_B)\left(C'_A\right)^2 C''_A(rk - S), \end{aligned}$$

and

$$\begin{aligned} |M| &= M^1_{\lambda_A} M^2_{\lambda_B} M^3_n + M^1_{\lambda_B} M^2_n M^3_{\lambda_A} + M^1_n M^2_{\lambda_A} M^3_{\lambda_B} - M^1_n M^2_{\lambda_B} M^3_{\lambda_A} - M^1_{\lambda_B} M^2_{\lambda_A} M^3_n - M^1_{\lambda_A} M^2_n M^3_{\lambda_B} \\ &= -(\lambda_A + \theta\lambda_B)C'_B C''_A C''_B[\lambda_A(\pi_A - rk) + \theta\lambda_B(\pi_B - rk)] < 0. \end{aligned}$$

Therefore,

$$\frac{\partial \lambda^*_A}{\partial S} = -\frac{|M_{AS}|}{|M|} = \frac{g(S)\lambda^*_B(\lambda^*_A + \theta\lambda^*_B)\left(C'_A\right)^2 C''_B(rk - S)}{|M|},$$

and

$$\frac{\partial \lambda^*_B}{\partial S} = -\frac{|M_{BS}|}{|M|} = \frac{-g(S)\lambda^*_A(\lambda^*_A + \theta\lambda^*_B)\left(C'_A\right)^2 C''_A(rk - S)}{|M|}.$$

It follows that $\frac{\partial \lambda^*_B}{\partial S} > 0$ and $\frac{\partial \lambda^*_A}{\partial S} < 0$ if $S < rk$ and $\frac{\partial \lambda^*_B}{\partial S} < 0$ and $\frac{\partial \lambda^*_A}{\partial S} > 0$ if $rk < S$. This further implies that $\lambda^*_B/\lambda^*_A$, same as $\lambda^*_B$, is an inverted-U function of $S$, maximized at $S = rk$. ■

**Proof of Proposition 2.** By Cramer's rule,

$$\frac{\partial n}{\partial S} = -\frac{|M_{nS}|}{|M|}.$$

where

$$\begin{aligned} |M_{nS}| &= g(S)C_A' C_A'' C_B'' \left\{ \begin{array}{c} n\lambda_B[\lambda_A(\pi_A - rk) + \theta\lambda_B(\pi_B - rk)] \\ +\lambda_B(rk - S)[r + n(\lambda_A + \theta\lambda_B)] \end{array} \right\} \\ &\quad + g(S)n\left(C_A'\right)^2 \left(\lambda_B C_B'' - \theta\lambda_A C_A''\right)](rk - S). \end{aligned} \tag{23}$$

From (13), (14) and (23), we can show that, after substitution and simplification,

$$\begin{aligned} \frac{\partial R}{\partial S} &= (\lambda_A\Delta_A + \lambda_B E\Delta_B)\frac{\partial n}{\partial S} + \Delta_A\frac{\partial \lambda_A}{\partial S} + E\Delta_B\frac{\partial \lambda_B}{\partial S} \\ &= \frac{-g(S)C_A' \left\{ \begin{array}{c} (\lambda_A\Delta_A + \lambda_B E\Delta_B)C_A'' C_B'' \lambda_B[n\lambda_A(\pi_A - S) \\ +n\theta\lambda_B(\pi_B - S) + r(rk - S)] \\ +nC_A'(E\Delta_B - \theta\Delta_A)\left(\lambda_A^2 C_A'' + \lambda_B^2 C_B''\right)(rk - S) \end{array} \right\}}{|M|}. \end{aligned} \tag{24}$$

Note that $E\Delta_B > \theta\Delta_A$ since $\theta \le 1$. If $S = \underline{\Delta}_B$, then $S < \min\{\pi_A, \pi_B, rk\}$ and $E\Delta_B - \theta\Delta_A > 0$. It follows that $\frac{\partial R}{\partial S}|_{S=\underline{\Delta}_B} > 0$. If $S = \overline{\Delta}_B$, then $S > \max\{\pi_A, \pi_B, rk\}$ and we have $\frac{\partial R}{\partial S}|_{S=\overline{\Delta}_B} < 0$. ■

**Proof of (17).** From (24),

$$\frac{\partial R}{\partial S}|_{S=rk} = \frac{-g(S)C_A'}{|M|}(\lambda_A\Delta_A + \lambda_B E\Delta_B)C_A'' C_B'' \lambda_B[n\lambda_A(\pi_A - rk) + n\theta\lambda_B(\pi_B - rk)] > 0.$$

■