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Completing incomplete preferences*

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Abstract

In this paper we propose a model for individuals who have incomplete preferences and attempt to complete them. We show that two empirical puzzles - the willingness-to-pay and willingness-to-ask (WTP-WTA) gap as well as the present bias - arise naturally in the process of completing incomplete preferences. Based on the model, an incentive-compatible mechanism to measure the incompleteness in preferences is developed. An experimental implementation of the measurement mechanism provides results consistent with our model.

Keywords: incomplete preferences, incentive-compatible measurement mechanism, experiments, the incompleteness in preferences

JEL Classification B40, C91, D81

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1 Introduction

That individuals' preferences might be incomplete is an old idea. The concept has already been suggested in, e.g., Von Neumann and Morgenstern (1944, pp 19-20), Armstrong (1950), Quandt (1956), and Aumann (1962). Decision makers may often find it difficult to compare two alternatives over which there is limited information. Aumann (1962) argues that the completeness of the preferences is neither realistic nor normative. Von Neumann and Morgenstern (1944) suggest that it is conceivable and in a way more realistic to allow for incomplete preferences. Much advance has been made on incomplete preferences. Among others, see e.g., Dubra et al. (2004); Ok et al. (2012); Galaabaatar and Karni (2013) for recent developments.

Yet, a question remains: what would people do when they are forced to make a choice in the face of incomplete preferences? People are often confronted with such decision situations. For example, a young PhD graduate receives two job offers from universities she is not entirely faimilair with, or a young family sees a few acceptable houses needs to decide which one to buy. People do make a decision ultimately in those situations, i.e., individuals (are forced to) complete incomplete preferences in the end. Then, what are the behavioral consequences in the process of completing incomplete preferences? In this paper we go beyond incomplete preferences per se and provide an answer for the above questions. We propose a model for individuals who have incomplete preferences and attempt to complete them. Based on the model, we then discuss two behavioral implications - the willingness-to-pay and willingness-to-ask (WTP-WTA) gap as well as the present bias - that might arise naturally in the attempt of completing incomplete preferences. Finally, we develop an incentive-compatible mechanism to measure the incompleteness in preferences and implement a first experimental test of the model.

Concretely, we capture incomplete preferences by individuals having not a single but a set of utility functions. This idea is consistent with the definition of incomplete preferences in, e.g., Dubra et al. (2004) and Ok et al. (2012). Given any specific utility function in this set, individuals have complete preferences and perform standard expected utility calculations. Unlike decision models under ambiguity, we define the set of states to be the set of individual's utility functions, with each utility function corresponding to a potential state. Individuals' ultimate decision utility is obtained by aggregating across utility functions. The aggregation process is done by taking a *subjective* expectation of *the concavely transformed* standard expected utilities with respect to the set of utility functions.

Based on the model, we discuss the WTP-WTA gap rigorously. We show that the WTP-WTA gap can be due to a cautious attitude when one attempts to complete incomplete preferences. Intuitively, a cautious completion of incomplete preferences leads to cautious reasoning. Individuals' cautious reasoning takes the form of "what if the alternative is not worth that much" when reporting WTP, and this cautious thinking lowers WTP; whereas individuals' cautious reasoning takes the form of "what if the alternative is worth more" when reporting WTA, and this cautious thinking increases WTA. Together, the two instances of cautious reasoning create the WTP-WTA gap. We also identify the present bias in intertemporal choice (see e.g., Frederick et al., 2002) and show that this bias can be a natural consequence of a cautious completion of incomplete preferences.

Finally, we develop an incentive-compatible mechanism to measure the incompleteness in preferences. In the mechanism, individuals face a series of tasks. In each of these tasks, individuals face two alternatives: an alternative x over which we are interested in knowing the incompleteness in preferences, and a sure payment y, over which individuals have (more) complete preferences. Instead of choosing one option out of the two, as in typical pairwise choice tasks, individuals are allowed to choose a $\lambda \in [0, 1]$ and build a simple lottery $(\lambda x, (1 - \lambda)y)$. When individuals' preferences are incomplete and their behavior is in line with our model, we show that (1) there exists a unique optimal λ^* that maximizes each individual's decision utility; (2) the value of the optimal λ^* can be used to calculate the incompleteness in preferences over alternative x. In particular, the value of λ can be interpreted as the choice probability in the stochastic choice models (e.g., Machina, 1985; Harless and Camerer, 1994b), has a striking similarity to the matching law in operant conditioning (Herrnstein, 1961; McDowell, 2005), and reflects a taste for flexibility (Cerreia-Vioglio, 2009). We ran an experiment to check the performance of our measurement mechanism. Option x was a payment of 20 euro in one month's time, while Option y consisted of an immediate payment ranging from 11 euro to 20 euro with 1 euro increments. Our subjects exhibited a strong preference for mixing Option x with Option y. The median λs assigned to Option x were, respectively, 1.00 when Option y was 15 euro or lower, 0.955 when Option y was 16 euro, 0.87 when Option y was 17 euro, 0.65 when Option y was 18 euro, 0.45 when Option y was 19 euro, 0.00 when Option y was 20 euro. Such a preference for a deliberate randomization has also been observed in some recent experiments. Without an explicit model, Agranov and Ortoleva (2015) in one treatment explicitly tell subjects that choices are repeated three times. They find that a large majority of subjects deliberately choose one option in some choices and another in other choices. Dwenger et al. (2014) find that decision makers, when facing two risky alternatives, sometimes prefer delegating the decision to an external random device, e.g., a coin flip, to making the choice themselves. They attribute this preference to responsibility aversion. Dean and McNeil (2014) find that 48% of subjects exhibit strict preference for flexibility, and they attribute the taste for flexibility to subjects being uncertain about their future preferences.

Our paper is related to a recent paper by Cerreia-Vioglio et al. (2015). They start with standard axioms - including the compleness axiom - and replace only the independence axiom with a weaker axiom which they call Negative Certainty Independence. They characterize utility representations for all preferences that satisfy Negative Certainty Independence and other basic rationality postulates. In their representation individuals behave as if they have a set of utility functions and evaluate alternatives according to the utility function giving the lowest certainty equivalent; other utility functions or the incompleteness in preferences play no role. They show beautifully that the representation can be used to complete an incomplete preference relation. Our model starts with incomplete preferences and aims to complete them. The model is built on a different set of assumptions, and the representation is characterized by smoothness. In the completion process individuals consider all utility functions in the set, and the incompleteness in preferences plays a central role. More importantly, due to the smoothness of the representation an incentive-compatible measure of the incompleteness in preferences can be developed.

There is a close parallel between our model and ambiguity models of multiple priors. Indeed, in ambiguity models of multiple priors, an individual faces an ambiguous scenario and she has a number of probability measures, i.e., multiple priors. In our model, an individual is uncertain which utility functions she should use to evaluate an alternative. Note, however, that the two lines of models are conceptually different. In models of multiple priors, the individual has a unique utility function, and the focus is on the aggregation across different probability measures. In the current model, the individual faces a simple lottery, and the difficulty arises in the aggregation across types. In the development of assumptions and obtaining the representation theorem, we borrow the modeling technique of multiple priors models, the smooth ambiguity model of Klibanoff et al. (2005) in particular, and apply it to incomplete preferences. We show that, with some minor departures from the standard expected utility theory, our model explains a broad range of anomalies, e.g., the WTP-WTA gap, the present bias, stochastic choices, and the matching law in operant conditioning.

Our paper is related to probabilistic choice models (e.g., McFadden, 1973; Harless and Camerer, 1994b; Hey and Orme, 1994; Loomes and Sugden, 1995), and the literature on choice of menus (e.g., Ahn and Sarver, 2013; Karni and Safra, 2014). Our model is fundamentally different from those models. There is no error term, and choices from a given menu are deterministic. Our preferences are over a single lottery instead of over menus. We postpone a more detailed discussion and comparison of the related literature to Section 4.

The paper proceeds as follows. Section 2 presents a model of valuation under incomplete preferences and discusses the assumptions of the model. Based on the model, Section 3 provides an incentive-compatible measure of the incompleteness in preferences. After presenting and discussing our model, we compare it to the most closely related literature in Section 4. Section 5 concludes.

2 Completing incomplete preferences: the model

We capture an individual's incomplete preferences by assuming that she has not a single but a set of utility functions. The set of individual's utility functions is defined as the state space in the sense of Anscombe and Aumann (1963), with each utility function corresponding to a potential state. As a concrete example, consider an individual sometimes prefers wine to beer, i.e., the utility over wine is higher than the utility over beer when he is in a wine mood, and in other times he prefers beer to wine, i.e., the utility over beer is higher than the utility over wine when he is in a beer mood. The individual needs to decide whether to attend a wine tasting event or a beer festival one month from now. The individual may find this decision difficult to make, i.e., she has incomplete preferences. To make a decision the individual needs to find a way to complete her preferences. Below we propose such a model.

Let C be a closed and bounded outcome space, and $c \in C$ be an outcome. The set of outcomes includes all possible aspects of a decision that affect the decision maker's well-being. Thus, the outcome space is not limited to monetary outcomes. It also includes, for example, a one-week trip to Paris, an increased safety of a car, or an improvement in air quality. A risky lottery $l \in L$ is then a cumulative probability measure over C. The model is mainly interested in $\succeq$, an individual's preference over L. In a standard expected utility framework, the individual's preference is captured by a single utility function, u, $u : L \to R$, such that for any risky lotteries l_1 and $l_2 \in L$, $l_1 \succeq l_2 \Longleftrightarrow \int_C u(c)\, dl_1 \geq \int_C u(c)\, dl_2$. When $\succeq$ is potentially incomplete, Dubra et al. (2004) suggest the following representation: There exists a set $\{u_\tau\}_{\tau \in \Gamma}$ of real functions on L such that, for all lotteries l_1 and l_2,

$$l_1 \succeq l_2 \Longleftrightarrow \int_C u_\tau(c)\, dl_1 \geq \int_C u_\tau(c)\, dl_2 \; \nabla\tau \in \Gamma.$$

When the set $\{u_\tau\}_{\tau \in \Gamma}$ is of a singleton, we are back to the standard expected utility theory with complete preferences. When an individual's preference over L is incomplete, difficulties arise. In this case, it is unclear how an individual makes a decision. We define the set of the individual's possible preference orderings as the type space. Let $\tau \in \Gamma$ be

The deciding individual			
Type 1	Type 2	...	Type n
l	l	...	l

The deciding individual			
$ce_1(l)$	$ce_2(l)$	...	$ce_n(l)$

Table 1: The left panel depicts the decision problem $D(l)$ when the individual faces l and has n types. The right panel depicts the decision problem $D(f)$, where $ce_\tau(l)$ is the certainty equivalent of l for type τ.

one potential type and Γ denote the type space. Given a type τ, an individual's preference over the set of lotteries is clear and complete. Let $\succeq_\tau$ denote this preference.

Assumption 1. *Expected Utility Over Risky Lotteries Given a Type. Given a type, τ, there exists a unique utility function, u_τ, continuous, strictly increasing, and normalized so that for some c_1, c_2, $u_\tau(c_1) = 0$ and $u_\tau(c_2) = 1$ such that for all l_1 and $l_2 \in L$, $l_1 \succeq_\tau l_2$ if and only if $\int_C u_\tau(c)\, dl_1 \geq \int_C u_\tau(c)\, dl_2$.*

An individual's preference of type τ can then be captured by utility function $u_\tau(\cdot)$, $\forall \tau \in \Gamma$. These utility functions are an artificial construct; one cannot directly observe them. However, the existence of such utility functions should not be surprising. For example, an individual is a financial expert and has accumulated extensive experiences with financial products. With these experiences, her preference over financial products could become sufficiently complete to be represented by a utility function. Given the above setup, the individual essentially faces the situation depicted in the left panel of Table 1. Let $D(l)$ denote the decision problem when the individual faces l.

Let $ce_\tau(l)$ denote the certainty equivalent of risky lottery l given type τ. For any risky lottery l, let $EU_\tau(l) = \int_C u_\tau(c)\, dl$. From Assumption 1, we know that $u_\tau(ce_\tau(l)) = EU_\tau(l)$. The difficulty remains: how does an individual aggregate across types? Aggregation of similar forms has been discussed extensively in the social welfare literature (see e.g., Hicks, 1939). It is generally agreed that aggregating across different individuals is extremely difficult or even meaningless. Cerreia-Vioglio et al. (2015) do not attempt to aggregate across types. Instead, by invoking the negative certainty independence axiom, they obtain a representation similar to the Rawlsian scheme: the alternative is evaluated by the type who gives the lowest certainty equivalent. We believe more can be done. Recall in Assumption 1 that the utility function, u_τ, is normalized so that for some c_1, c_2, $u_\tau(c_1) = 0$ and

$u_\tau(c_2) = 1$. Thus, utility functions, although different across types, are based on the same metric. Comparison among types is thus not comparing "apples" and "oranges." After all, the aggregation across types is performed for one individual. We believe, the idea that for the same individual there exists a common scale of utility across types is a plausible one. For example, Binmore (1998, chapter 4, p.259) concludes that intrapersonal comparisons of oneself in different roles in a society are completely acceptable. This observation motivates the next assumption. But before stating it, a new object needs to be defined.

Definition 1. An act, $f \in F$, is a function $f : \Gamma \to C$ that assigns each type to an outcome.

The act f is defined over the type space instead of the state space, thus it is different from the act defined in, e.g., Gilboa and Schmeidler (1989); Klibanoff et al. (2005). Yet, it is consistent with the Savage act where the states of the world is the type space. For this reason we still call f an act. Note that $f(\tau) \in C$ is an outcome given a type τ. For example, an individual faces an act when facing a sure outcome, e.g., a payoff of 100 euro in one year, a one-week trip to Paris, or an improvement in air quality. Let $\succeq^f$ denote the individual's preference over the acts.We can now state the second assumption.

Assumption 2. *Aggregation in the Form of Subjective Expected Utility (SEU) over Acts. There exists a countably additive probability measure $\pi \in \Pi$ and a continuous and strictly increasing function $v : C \to \mathbb{R}$ such that for all $f_1, f_2 \in F$,*

$$f_1 \succeq^f f_2 \iff \int_\Gamma v[f_1(\tau)] d\pi \geq \int_\Gamma v[f_2(\tau)] d\pi.$$

The subjective probability distribution π captures the individual's subjective assessment of the relevance of the types in evaluating l. Assumption 2 is crucial. It defines how aggregation across types takes place. Aggregation in the form of SEU in Assumption 2 takes place in the following manner: for each type τ the lottery l is replaced by an outcome $f(\tau) \in C$. As evidenced by the use of τ, the $f(\tau)$ have already reflected the difference in types. For example, $f(\tau)$ could be the certainty equivalent of l given u_τ. Thus, as depicted in the right panel of Table 1, the individual essentially faces an act f.

These $f(\tau)$s are evaluated by the function $v(\cdot)$ and then weighted by π, the individual's subjective assessment of the relevance of the types in evaluating l. Let $D(f)$ denote the decision problem when the individual faces f.

Given a type, τ, by Assumption 1, lottery l is evaluated with the utility function u_τ according to the expected utility theory. An individual should then be indifferent between facing lottery l as in $D(l)$ or facing f that produces $ce_\tau(l)$ for each type τ as in $D(f)$. This property motivates the following definition:

Definition 2. Given $l \in L$, $f^l \in F$ denotes an act reduced from l. The reduced act of l, f^l, is defined as

$f^l(\tau) = ce_\tau(l)$ for $\forall \tau \in \Gamma$.

The final assumption relates the preference ordering of lotteries $l \in L$ to the preference ordering of their reduced acts $f^l \in F$:

Assumption 3. *Consistency with Preferences over Reduced Acts. Given $l_1, l_2 \in L$ and their reduced acts, $f_1^l, f_2^l \in F$,*

$l_1 \succeq l_2 \Longleftrightarrow f_1^l \succeq^f f_2^l$.

Assumption 3 essentially states that the individual regards $D(f)$ and $D(l)$ equivalent. Assumption 3 suggests how preferences over acts $\succeq^f$ are related to preferences over lotteries $\succeq$. Based on assumption 1, 2, and 3, it can then be shown that

Theorem 1. *Given Assumption 1, 2, and 3, there exists a continuous and strictly increasing $\phi : R \to R$ subjecting to positive affine transformation such that $\succeq$ is represented by the preference functional $V : L \to \mathbb{R}$ given by*

$$V(l) = \int_\Gamma \phi\left[EU_\tau(l)\right] d\pi. \qquad (1)$$

Please see Appendix 1 for the proof. In Equation 1 π measures the incompleteness in preferences and $\phi(\cdot)$ captures attitudes toward the incomplete preferences. In principle, function

$\phi(\cdot)$ can be concave, linear, or convex, and the curvature of $\phi(\cdot)$ captures an individual's attitudes toward incomplete preferences. A concave $\phi(\cdot)$ implies aversion to incomplete preferences; a linear $\phi(\cdot)$ implies a neutral attitude toward incomplete preferences; and a convex $\phi(\cdot)$ implies incomplete preferences seeking. Below we assume that individuals are averse to incomplete preferences and, hence, $\phi(\cdot)$ is concave. Note that to arrive at a single value when one has many types is, in essence, similar to situations where a group of people with different opinions tries to reach a consensus. The more strongly members disagree with each other, the more difficult it is for the group to make compromises and agree on a single opinion. Aversion to the incompleteness in preferences can then be interpreted as the cost of forcing different types to agree on a single value.

2.1 Two implications of the model

Below we discuss two anomalies in light of the model. One anomaly, the WTP-WTA gap, has been intuitively linked to ideas closely related to the incompleteness in preferences (Dubourg et al., 1994), and here we provide a formal demonstration.[1] The linkage of incomplete preferences and the other anomaly - the present bias in intertemporal choice - is new.

2.1.1 The WTP-WTA gap

In this section, we show that when individuals behave cautiously when completing incomplete preferences, they display a WTP-WTA gap. Suppose an individual is thinking of buying alternative x. To illustrate the idea intuitively, we first consider a situation where $-\frac{\phi''(\cdot)}{-\phi'(\cdot)}$ approaches to infinity, i.e., the individual is extremely cautious and considers only the worst scenario. In this case, the worst scenario is the alternative has a value of $min_{\tau\in\Gamma}\, EU_\tau(x)$ when she is prompted to buy, and thus $WTP = u_{\tau*}^{-1}\left(min_{\tau\in\Gamma}\, EU_\tau(x)\right),$ where $\tau*$ is the type with the minimum expected utility. The worst scenario is the alternative has a value of $max_{\tau\in\Gamma}\, EU_\tau(x)$ when she is asked to sell, and thus $WTP =$

[1]Dubourg et al. (1994) discuss the WTP-WTA gap in a framework similar to incomplete preferences, the so-called âĂIJthe imprecision in preferences".

$u_{\tau}^{-1}\left(max_{\tau\in\Gamma} EU_{\tau}(x)\right)$, where $\tau*$ is the type with the maximum expected utility.

More formally, the individual cares about the utility surplus she obtains from buying alternative x. Given a utility function u_τ and the expected utility $EU_\tau(x)$ of alternative x, the consumer's utility when she pays p is: $EU_\tau(x) - u_\tau(p)$. An incentive-compatible mechanism should result in WTP such that the individual's decision utility of buying alternative x at the price of WTP is equal to zero:

$$\int_{\Gamma} \phi[EU_\tau(x) - u_\tau(WTP)]d\pi = 0. \tag{2}$$

Similarly, when an individual is selling alternative x, she cares about the utility surplus she obtains from selling alternative x. Given a utility function u_τ and the expected utility $EU_\tau(x)$ of alternative x, the consumer's utility when receiving p is: $u_\tau(p) - EU_\tau(x)$. An incentive-compatible mechanism should result in WTA such that the individual's decision utility of selling alternative x at the price of WTA is equal to zero:

$$\int_{\Gamma} \phi[u_\tau(WTA) - EU_\tau(x)]d\pi = 0. \tag{3}$$

The above formulation is consistent with the inertia assumption in Bewley (1986), where he suggests that an individual moves away from the status quo - buying or selling an alternative here - only if the alternative option is sufficiently attractive (in the sense of dominance). When an individual has a unique stable utility function, the two above conditions reduce to $EU(x) - u(WTP) = 0$ and $u(WTA) - EU(x) = 0$, and, hence, $WTP = WTA$. However, when an individual has a set of utility functions and $\phi(\cdot)$ is concave, we have $WTP < WTA$.

Proposition 1. When an individual has incomplete preferences over an alternative and she is strictly averse to incomplete preferences, i.e., $\phi(\cdot)$ is strictly concave, we have $WTP < WTA$.

The proof of Proposition 1 can be found in Appendix 2.

The following numerical example provides some intuitive ideas about the impact of concavity of function $\phi(x)$ on the $WTP - WTA$ gap. An individual has two utility functions, u_τ, $\tau = 1, 2$, $u_1(x) = 1$, and $u_2(x) = 0$ for option x, and she considers both utility functions equally likely. To simplify calculation, assume for the sure payoff p $u_\tau(p) = p, \tau = 1, 2$. Buying alternative x at the price of p gives the individual a utility surplus of $u_1(x) - p = 1 - p$ under $u_1$and a utility surplus of $u_2(x) = 0 - p$ under u_2. Selling alternative x at the price of p gives the individual a utility surplus of $p - u_1(x) - p = p - 1$ under u_1 and a utility surplus of $p - u_2(x) = p$ under u_2. According to conditions 2 and 3, we have:

$0.5\phi[u_1(x) - WTP] + 0.5\phi[u_2(x) - WTP] = 0.5\phi(1 - WTP) + 0.5\phi(0 - WTP) = 0.5(1 - e^{-(1-WTP)}) + 0.5(1 - e^{-(0-WTP)}) = 1 - e^{-0} = 0,$

$0.5\phi[WTA - u_1(x)] + 0.5\phi[WTA - u_2(x)] = 0.5\phi(WTA - 1) + 0.5\phi(WTA) = 0.5(1 - e^{-(WTA-1)}) + 0.5(1 - e^{-WTA}) = 1 - e^{-0} = 0.$

Solving for WTP and WTA, we obtain $WTP = 0.4$ and $WTA = 0.6$. Since $WTA = 0.6 > WTP = 0.4$, there is a WTP-WTA gap.

Intuitively, an individual who is averse to incomplete preferences behaves cautiously when considering to buy or sell an alternative. Specifically, she may think: "what if the alternative is not worth that much" when considering to buy an alternative, and this cautious thinking lowers the price she is willing to pay; whereas cautious thinking would take the form of "what if the alternative is worth more" when the individual is considering to sell an alternative, and this cautious thinking increases the price she is willing to accept.

The above demonstration also shows that to produce a WTP-WTA gap, one condition must be met: individuals' preferences must be sufficiently incomplete. Plott and Zeiler's (2005) findings have produced a substantial influence in the literature that explains the WTP-WTA gap. The model we propose adds insights to their findings. Plott and Zeiler (2005) attributed the WTP-WTA gap to inappropriate experimental elicitation procedures. According to our model, unclear experimental procedures produce subjects' misconceptions and, hence, causes preferences to be less complete. The high level of incompleteness subsequently leads to the WTP-WTA gap. The Plott and Zeiler procedure makes clear the relationship between choices and their consequences under the evaluation mechanism and reduces the degree of the incompleteness in preferences. This leads to a smaller WTP-WTA gap. Inconsistent with Plott and Zeiler (2005), our model predicts that the Plott and Zeiler procedure cannot eliminate the WTP-WTA gap if the preference over a good at consideration is sufficiently incomplete. Indeed, Isoni et al. (2011a) find a significant and persistent WTP-WTA gap when the same experimental procedures are applied to lotteries, an evidence supporting our model.

2.1.2 The present bias

In Samuelson's (1937) discounted utility model on intertemporal choice, discount rates are constant. Abundant empirical studies have found, however, that discount rates decline over time (see, e.g., Table 1 in Frederick et al., 2002). The empirical anomaly of declining discount rates is often referred to as the present bias. There have been some explanations on the psychological motives underlying the present bias, for example, models of habit formation (see, e.g., Ryder and Heal, 1973), models of utility from anticipation (Loewenstein, 1987), reference-point models (Loewenstein and Prelec, 1992), and visceral influences (Loewenstein, 2000). Here, we offer a new explanation based on the incompleteness in preferences.

The present bias is illustrated in Figure 1. Individuals need to estimate the present utilities of a certain amount of money at time 0, time 1, ..., time 4. For the ease of demonstration, suppose the present utility of the amount of money at time 0 is 100. Letting subjects be time consistent and δ denote the discount factor, we have the present utility of 100 at time t to be $100\delta^t$. This is captured by the squares in the figure. Empirical studies typically found that individuals do not discount future rewards consistently (see, e.g., Frederick et al., 2002, and the references therein). Instead, individuals' discounting behavior is characterized by a relatively high discount rate over short horizons and a relatively low discount rate over long horizons. Laibson (1997) offers a quasi-hyperbolic discounting model that captures such behavior. In his model, the utility of 100 at time t discounted to present would be $100\beta\delta^t$, where parameter β captures the present bias. The solid dots in Figure 1 represent the values calculated according to the quasi-hyperbolic discounting model of Laibson (1997) .

To explain the present with incomplete preferences, note that individuals' preferences over a sure payment now can be certain but may become incomplete in the future. In many models of intertemporal decisions, individuals' utilities from consumption – their tastes – change over time, and to predict the exact future taste is difficult (Loewenstein et al., 2003). Kreps (1979) attributes a preference for flexibility to an anticipation of uncertain future tastes. As a consequence of the uncertainty in tastes, the set of possible types becomes larger when one projects into the future, i.e., the set of utility function becomes larger. The incompleteness in preferences over the money at time t is captured by the vertical bars. As one can see, instead of having a single point value, the discounted present utility of a future payment at time t lies in a certain range. Since individuals are averse to incomplete preferences, they report a value smaller than the mean of the discounted present utilities, e.g., the solid dots in Figure 1. Furthermore, it seems reasonable to assume that the marginal change of the incompleteness in preferences is much larger when one moves from present to the future than that when one moves from the future to a more distant future. Consequently, individuals discount more when a future date payoff is compared to a present date payoff than when a future date payoff is compared to a a more distant future date payoff . Specifically, discount rates decline over time.

Our model offers an explanation for an interesting observation in Sutter et al. (2013), who find that students with high intellectual capacity are also more patient. In our framework, highly intellectual students have more complete preferences over future payoffs. As a result, they discount future payoffs less and behave more patiently.

3 An incentive-compatible measurement of incomplete preferences

As the discussion in the introduction highlights, there have been a number of papers on the existence and importance of incomplete preferences. A natural question to ask, then,

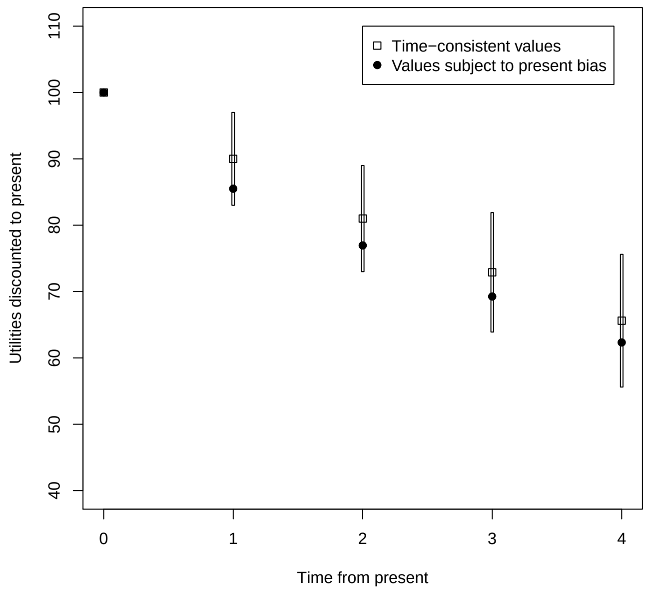

Figure 1: The present bias and the incompleteness in preferences. The x-axis represents time from present, and time 0 denotes present. The y-axis represents the utilities discounted to present.

given an alternative, is how we measure the incompleteness in preferences? According to Equation 1, an individual's incompleteness in preferences is captured by the subjective distribution f over u_τ. Consistent with the literature in decision making under risk, a natural candidate for the measurement of the incompleteness in preferences is the standard deviation of the subjective distribution π over u_τ (σ_π^2). Hence, if we could somehow measure σ_π^2, we would obtain a proxy for the incompleteness in preferences. Below we propose such a measurement mechanism. As it will be seen shortly, it is incentive-compatible and easy to implement.

More specifically, the mechanism works as follows. An individual faces two options. Denote these two options by x and y. Option y is a yardstick, and we are interested in measuring the individual's incompleteness in preferences over Option x. In most preceding studies, an individual would be asked to choose between two options, Option x and Option y (see e.g., Holt, 1986). In the so-called outcome matching method, an individual is asked to compare option x with a list of increasing sure payoffs y.[2] To be efficient and avoid inconsistent choices, a frequently used approach is to explicitly require the individual to indicate a single switching point where the preference between options x and y reverses. As argued by Butler et al. (2014), however, straightforward choices yield only limited dichotomous information. In particular, there is no room for individuals to express their incompleteness in preferences.

In the current mechanism, we proceed differently: we ask the individual, instead of choosing one option out of the two, to choose a $\lambda \in [0,1]$ and build a simple lottery: $(\lambda x, (1-\lambda)y)$. The meaning of the lottery $(\lambda x, (1-\lambda)y)$ is easier to understand for decisions with a finite set of outcomes. Let x be $(x_1, p_1; ...; x_n, p_n)$ and y be $(y_1, q1; ...; y_n, q_n)$, then $(\lambda x, (1-\lambda)y) = (x_1, \lambda p_1; ...; x_n, \lambda p_n; y, (1-\lambda)q_1; ...; y_n, (1-\lambda)q_n)$. Below we show that, when the individual behaves according to our model, there exist some values of y inducing the individual to strictly prefer the lottery $(\lambda x, (1-\lambda)y)$, with $0 < \lambda < 1$, over Option x *and* Option y.

By Assumption 1, the individual's preference over the lottery $(\lambda x, (1-\lambda)y)$ satisfies the expected utility theory for any type τ. Explicitly, given a type τ, we have $EU_\tau[\lambda x + (1-\lambda)y] = \lambda EU_\tau(x) + (1-\lambda)EU_\tau(y)$. The individual's decision is then to maximize her utility by choosing $0 \leq \lambda \leq 1$ properly:

$$Max_\lambda \; V[\lambda x + (1-\lambda)y] = \int_\Gamma \phi[\lambda EU_\tau(x) + (1-\lambda)EU_\tau(y)] \, d\pi.$$

Taking first order derivative of the above equation gives[3]

[2] In discussion below Option y is often an alternative offering a sure amount of payment y. When no confusion is possible we abuse the use of notations slightly and identify y with Option y.

[3] The second-order derivative is

$$\frac{d^2V[\lambda x + (1-\lambda y)]}{d\lambda^2} = \int_\Gamma \phi''[\lambda EU_\tau(x) + (1-\lambda)EU_\tau(y)] \times [EU_\tau(x) - EU_\tau(y)]^2 \, d\pi.$$

Since $\phi(\cdot)$ is concave, $\phi''(\cdot)$ is negative. We are interested in situations where options x and y are not the same, i.e., $EU_\tau(x) \neq EU_\tau(y)$ for some $\tau \in \Gamma$. Together we have $\phi''[\lambda EU_\tau(x) + (1-\lambda)EU_\tau(y)] \times [EU_\tau(x) - EU_\tau(y)]^2 \leq 0$, and the inequality is strict for some $\tau \in \Gamma$. Consequently, $\frac{d^2V[\lambda x+(1-\lambda y)]}{d\lambda^2} = \int_\Gamma \phi''[\lambda EU_\tau(x) + (1-\lambda)EU_\tau(y)] \times [EU_\tau(x) - EU_\tau(y)]^2 \, d\pi < 0$. This ensures we are indeed seeking for

$$\frac{dV\left[\lambda x+(1-\lambda)y\right]}{d\lambda}=\int_{\Gamma}\phi'\left[\lambda EU_{\tau}(x)+(1-\lambda)EU_{\tau}(y)\right]\times\left[EU_{\tau}(x)-EU_{\tau}(y)\right]d\pi=0.$$

In some cases, preferences of x over y can be straightforward, e.g., when options can be ordered by some dominance rules. For example, when options x and y are risky lotteries and option x first degree stochastically dominates option y, it seems natural that individuals have $EU_{\tau}(x)>EU_{\tau}(y)$, for $\forall\tau\in\Gamma$. Since $\phi'\left[\lambda EU_{\tau}(x)+(1-\lambda)EU_{\tau}(y)\right]>0$, this leads to a positive first order condition and, hence, $\lambda=1$. Unfortunately, two options cannot in general be ordered via simple dominance rules, such as first degree stochastic dominance. When options x and y cannot be ordered by simple dominance rules , different u_{τ} might offer a different ordering of the two options. In such situations the choice of λ would give insights on the incompleteness in preferences over option x.

Note that $EU_{\tau}(x)$ and $EU_{\tau}(y)$ are random variables governed by the probability distribution π. Let $X=EU_{\tau}(x)$ and $Y=EU_{\tau}(y)$, and $\Delta_{\tau}=X-Y$. With these notations, we have

$$\phi'\left[\lambda EU_{\tau}(x)+(1-\lambda)EU_{\tau}(y)\right]=\phi'\left[Y+\lambda\Delta_{\tau}\right].$$

We are mostly interested in the scenario where the individual finds choosing out of Option x and Option y difficult, i.e., when the two options are similar. Specifically, we are interested in those situations where Δ_{τ} is small relative to X and Y. When this is the case, we can use the Taylor expansion and obtain

$$\phi'\left[Y+\lambda\Delta_{\tau}\right]=\phi'(Y)+\phi''(Y)\lambda\Delta_{\tau}+O\left(\lambda\Delta_{\tau}\right)\approx\phi'(Y)+\phi''(Y)\lambda\Delta_{\tau},$$

where $O\left(\lambda\Delta_{\tau}\right)$ is the sum of the terms that have $\lambda\Delta_{\tau}$ with a power of two or higher. The above first order condition can then be written as

the maximum.

$$
\begin{aligned}
\frac{dV[\lambda x+(1-\lambda)y]}{d\lambda} &= \int_{\Gamma} \phi' \left[Y + \lambda\Delta_\tau\right] \Delta_\tau d\pi, \\
&\approx \int_{\Gamma} \left[\phi'(Y) + \phi''(Y)\lambda\Delta_\tau\right] \Delta_\tau d\pi \\
&= E_\tau \left[\phi'(Y)\Delta_\tau\right] + \lambda E_\tau \left[\phi''(Y)\Delta_\tau^2\right] = 0,
\end{aligned}
$$

where $E_\tau(\cdot)$ is the expectation operator with respect to the distribution π . Solving for λ, and we have

$$
\lambda^* = \begin{cases} 0 & -\frac{E_\tau[\phi'(Y)\Delta_\tau]}{E_\tau[\phi''(Y)\Delta_\tau^2]} \leq 0, \\ -\frac{E_\tau[\phi'(Y)\Delta_\tau]}{E_\tau[\phi''(Y)\Delta_\tau^2]} & 0<-\frac{E_\tau[\phi'(Y)\Delta_\tau]}{E_\tau[\phi''(Y)\Delta_\tau^2]} < 1, \\ 1 & -\frac{E_\tau[\phi'(Y)\Delta_\tau]}{E_\tau[\phi''(Y)\Delta_\tau^2]} \geq 1. \end{cases}
$$

It seems reasonable to assume that the individual's preference over a sure payment today is complete. Such an assumption is similar to but weaker than the requirement that preferences are complete over constant acts (see e.g., Ok et al., 2012).[4] Specifically, when option y is a sure payment, Y would be a constant. It follows then that $E_\tau[\phi'(Y)\Delta_\tau] = \phi'(Y)E_\tau[\Delta_\tau]$, and $E_\tau[\phi''(Y)\Delta_\tau^2] = \phi''(Y)E_\tau[\Delta_\tau^2]$. Similar to decision making under risk, let $-\frac{\phi''(Y)}{\phi'(Y)}$ denote a metric of attitudes toward the incompleteness in preferences, and the optimal value of λ becomes:

$$
\lambda^* = \begin{cases} 0 & \lambda^* \leq 0, \\ \frac{1}{-\frac{\phi''(Y)}{\phi'(Y)}} \times \frac{E_\tau[\Delta_\tau]}{E_\tau[\Delta_\tau^2]} & 0<\lambda^* < 1, \\ 1 & \lambda^* \geq 1. \end{cases} \tag{4}
$$

Recall $\Delta_\tau = X - Y$. We then have $E_\tau\left[\Delta_\tau^2\right] = E_\tau\left[(X-Y)^2\right] = \sigma_x^2 + [E_\tau(X) - Y]^2$ and $E_\tau\left[\Delta_\tau\right] = E_\tau(X) - Y$. Thus, when $0<\lambda^* < 1$, the optimal value of λ^* decreases with the incompleteness in preferences over option x and the individual's attitudes toward the incompleteness in preferences. When $\sigma_x^2 > [E_\tau(X) - Y]^2$, i.e., the incompleteness in preferences is sufficiently large, λ^* increases with Δ_τ.

[4] In those papers acts are defined over the external state space, not over the type space.

Equation 4 can be used to estimate δ_x^2. Y is the utility over sure outcomes and is relatively easy to estimate. The metric for incompleteness attitudes $-\frac{\phi''(Y)}{\phi'(Y)}$ should not change substantially with small variation of Y. With Y at hand, only three variables in Equation 4 remain unknown: a measure of the incompleteness in preferences that we want to estimate σ_x^2, the metric for incompleteness attitudes $-\frac{\phi''(Y)}{\phi'(Y)}$, and $E_\tau(X)$. Note that there is an optimal λ^* for each Y. With three Ys and accordingly three λ^*, one can easily calculate σ_x^2.

As a concrete illustration, consider the following numerical example: suppose the individual has two types $\tau = 1, 2$, and $Prob(u_1) = 0.5$ and $Prob(u_2) = 0.5$. There are two options, Option x and Option y. Option x is a lottery, and $EU_1(x) = 1$ and $EU_2(x) = 0$. Option y is a sure payment, and the individual's preference over y is precise, i.e., $EU_1(y) = EU_2(y)$. For the ease of illustration, assume further that $EU_1(y) = EU_2(y) = y$. The function $\phi(\cdot)$ takes the form of $\phi(EU_\tau) = 1 - e^{-EU_\tau}$. The decision utility of choosing option x is then $V(x) = 0.5(1-e^{-1}) + 0.5 \times 0 = 0.316$, and the decision utility of choosing option y is $V(y) = 1 - e^{-y}$. It can be easily shown that when $\frac{e}{1+e} < y < \frac{1}{2}$, the individual is better off by opting for the lottery$(\lambda x; (1-\lambda)y)$ instead of choosing option x or option y. The decision utility of such a lottery is: $V(\lambda x + (1-\lambda)y) = 0.5\left[1 - e^{-[\lambda \times 1 + (1-\lambda)\times y]}\right] + 0.5 \times \left[1 - e^{-[\lambda \times 0 + (1-\lambda)\times y]}\right]$. Simple calculation shows that the optimal λ:

$$\lambda^* = \begin{cases} 0 & y \geq \frac{1}{2}, \\ -ln(\frac{y}{1-y}) & \frac{e}{1+e} < y < \frac{1}{2}, \\ 1 & y \leq \frac{e}{1+e}. \end{cases}$$

Figure 2 provides the optimal λ for the value of y. As one can see, when Option y becomes more attractive, the value of λ^* decreases. Moreover, λ^* approaches to 0.5 when the two options become similar in terms of their decision utilities ($V(y) = 0.314$ versus $V(x) = 0.316$).

Note that in the current setup, λ^* is constructed as the ratio according to which the individual is paid with Option x. As a first interpretation of λ^*, note that the value of λ^* increases with the utility difference Δ_τ of Option x over Option y. This feature of λ^*

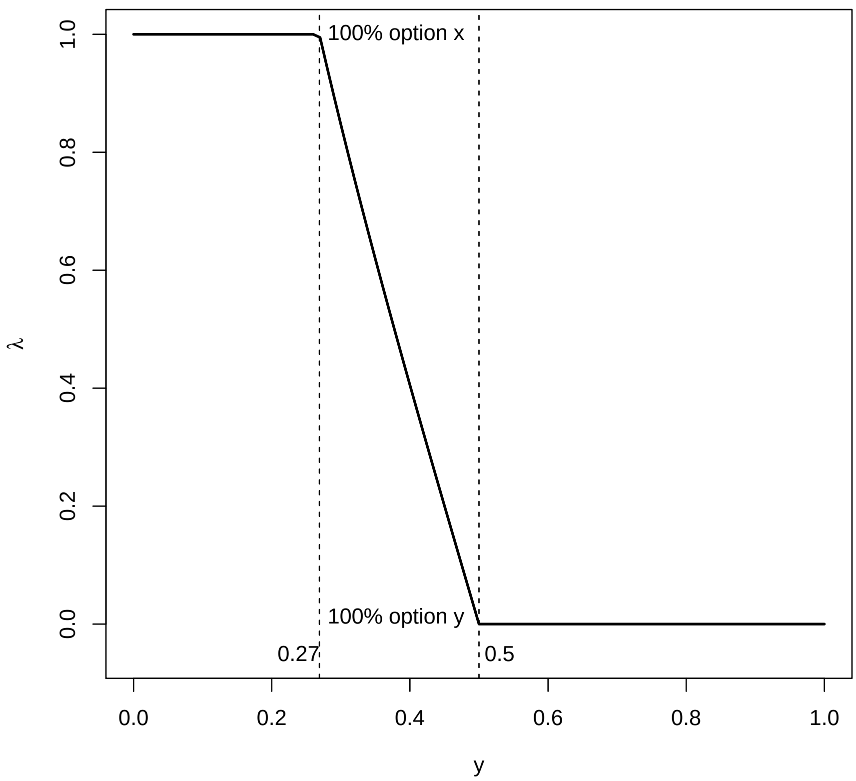

Figure 2: The optimal λ^* depending on the value of y. Figure is produced by assuming $\phi(EU_\tau) = 1 - e^{-EU_\tau}$, $Prob(u_1) = 0.5$, $Prob(u_2) = 0.5$, $EU_1(x) = 1$,$EU_2(x) = 0$, and $EU_1(y) = EU_2(y) = y$.

is closely related to the error term in stochastic choice models, where the probability of choosing the more attractive option is increasing with the utility difference of the more attractive over the less attractive option (see, e.g., Loomes and Sugden, 1998). In this sense, the value of λ^* can be interpreted as the choice probability in stochastic choice models. When preferences are incomplete, the choice probability is moderated by the incompleteness in preferences. An increase in σ_x^2, i.e., when the individual's preference over Option x versus Option y becomes less complete, decreases its probability of being chosen. As a second interpretation, the mechanism through which λ^* is obtained and the interpretation of λ^* have a striking similarity to an old idea in psychology: the matching law in operant conditioning (Herrnstein, 1961; McDowell, 2005). This states that the probability of an alternative being chosen is based on the relative attractiveness of options. It is observed in both animals and human agents and is considered as a clear violation of rational choices. However, Loewenstein et al. (2009) show that the matching law can be made consistent with rational choices if we regard the choices of a single subject as being made up of a sequence of multiple selves - one for each instant of time. Although their result is obtained in an entirely different context, the fundamental idea is surprisingly similar. Last, due to the incompleteness in preferences, the individual orders Option x better than Option y with some utility functions and shows the reverse preference ordering with others, and she considers all utility functions relevant. By choosing a combination ratio λ rather than either of the two options, she exhibits a preference for convexity. Cerreia-Vioglio (2009) interprets the preference for convexity as a preference for hedging and links it to uncertainty about future tastes.

Among others, Dwenger et al. (2014) and Agranov and Ortoleva (2015) experimentally examine individuals' preferences for deliberate randomization. Compared to their methods, our setting has three advantages. First, we need only one choice to reveal preferences for deliberate randomization and the extent of randomization, whereas Agranov and Ortoleva (2015) need a number of repeated choices to do so. Second, our randomization emerges endogenously and the extent of randomization varies with choice pairs, whereas in Dwenger et al. (2014) the randomization is exogenously fixed by a random device. Third, our measure is continuous.

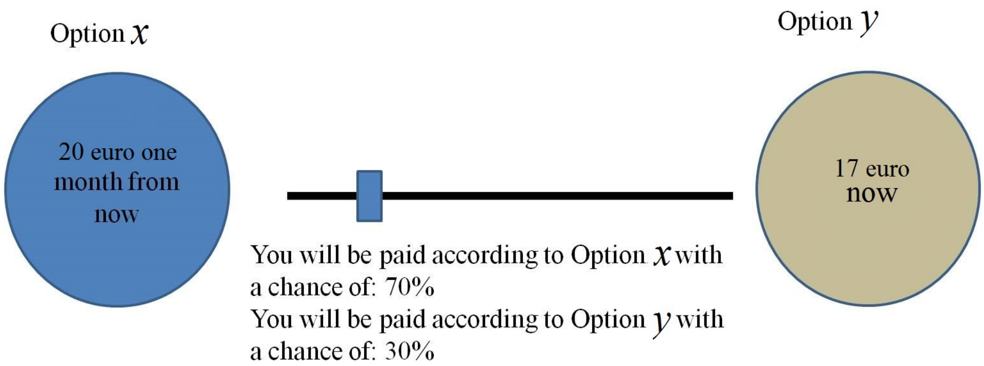

Figure 3: The experimental implementation of the elicitation method for the incompleteness in preferences. Subjects move the slider to decide their optimal λ^*.

3.1 An experimental implementation of the measurement mechanism

To test our model and check the performance of our measure of the incompleteness in preferences, we ran an experiment in which subjects faced a table consisting of ten choice pairs: Option x versus Option y. Option x was a payment of 20 euro in one month's time, while Option y consisted of an immediate payment ranging from 11 euro to 20 euro with 1 euro increments. Note that both Option x and Option y are sure payments. This allows us to interpret the combination ratio λ as the probability according to which subjects are paid with Option x. For example, subjects could choose to be paid accordingly to option x with a probability of 0.75, while being paid according to option y with a probability of 0.25. The increment of this payment probability was specified at 0.01. Figure 3.1 illustrates the elicitation method. There is no existing method for the measurement of the incompleteness in preferences. There are related measures in a literature closely related to incomplete preferences, the so-called imprecise preferences. In Butler and Loomes (2007), for example, subjects were asked to make a straight choice and indicate how confident they felt about their choice. They could state their confidence in five steps: surely Option x, probably Option x, unsure, probably Option y, and surely Option y. We use this measure as a robustness check for our measure.

The experiment was conducted online with a random sample of students from Radboud University between Oct. 21st to Oct. 30th, 2015. It was programmed with OTree (Chen et al., 2015). The experiment contained several other tasks which will be reported in sep-

Figure 4: A simulation of optimal λ given different values of the sure payment in Option y. The present value of Option x is assumed to be 18. The parametric form of the probability weighting function is $\frac{\delta p^{\gamma}}{(\delta p^{\gamma}+(1-p)^{\gamma})^{1/\gamma}}$, and the value function is $u(x) = x^{\alpha}$, with $\delta = 0.76, \gamma = 0.69$ and $\alpha = 0.88$. The values of δ, γ and α are chosen according to the estimates in Tversky and Fox (1995).

arate papers. The whole experiment lasted on average 15 minutes. In total, 92 students participated in the experiment. Each student received 2.5 euro participation fee. Additionally, 20% of students were chosen for real payment. The average payment was 22.50 euro among the chosen students.

Before presenting the experimental results, we offer the predictions under the two most popular theories of decision making under risk and uncertainty: the expected utility theory (EUT) and cumulative prospect theory (CPT).

Predictions under EUT: $\lambda \in (0, 1)$ occurs only when subjects are indifferent between Option x and Option y. This results follows directly from the independence axiom. To see this, suppose subjects prefer Option x over Option y, then by the independence axiom λOption $x+(1-\lambda)$Option x is preferred to λOption $x+(1-\lambda)$Option y, and thus a mixing of Option x with Option y is not optimal. The other case obtains similarly.

Predictions under CPT: $\lambda \in (0, 1)$ is possible with some values of the sure payment of y, but the relationship between the value of λ and the sure payment of y is not smooth; there exhibits a jump: subjects start with choosing $\lambda = 1$ and then suddenly choose a λ close to 0. The above result is based on simulations. Intuitively, the violation of betweenness comes from the overweighting of small probabilities. Thus, the optimal λ is chosen such that the probability for the less attractive option is strongly overweighted. With most popular probability weighting functions, this occurs when λ is around 0.1. Figure 3.1 depicts a simulation. As it can be seen, the optimal λ starts with 1 when the sure payment of y is low, jumps to 0.09 when the value of Option y is close to the value of Option x, and ends up with 0 when the sure payment of y is sufficiently attractive.

Predictions under Cerreia-Vioglio et al.'s (2015) model:

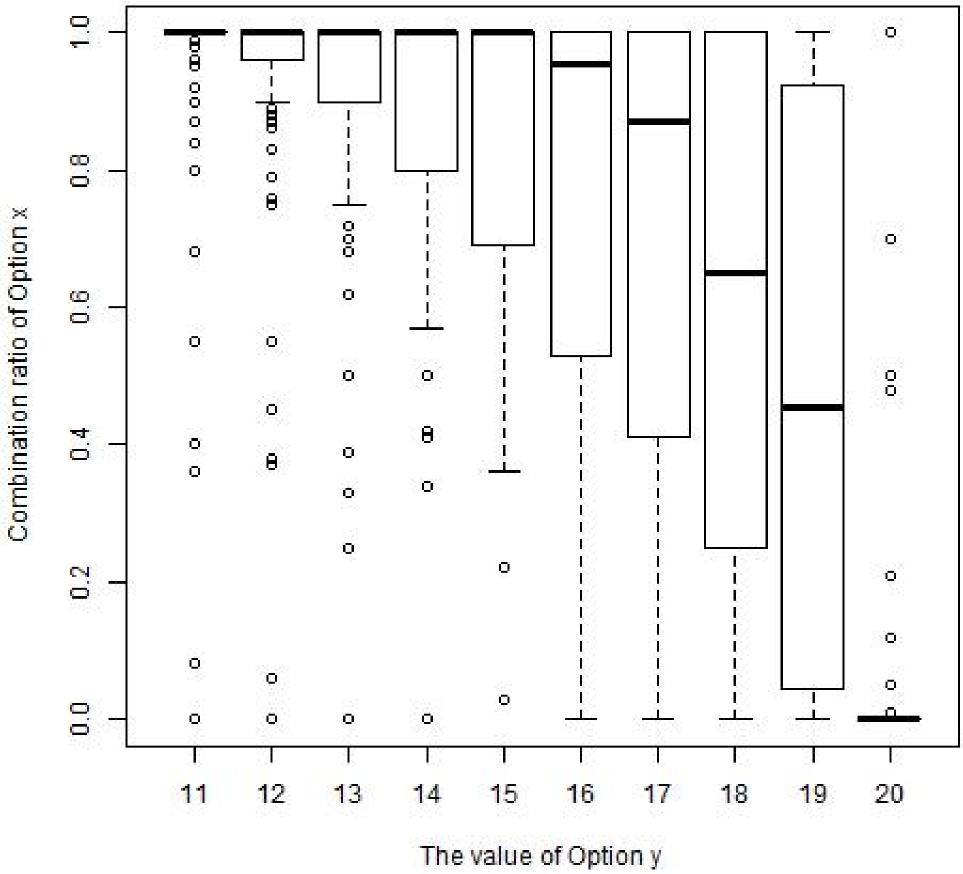

Figure 5: A boxplot of the payment probabilities that subjects assign to Option x, as a function of the value of Option y. The thick lines are median, the upper and lower bars are 1st and 3rd quartiles, respectively.

Experimental results are summarized in Figure 5. There are mainly two important features of Figure 5. First, a majority of subjects assigned positive payment probabilities to Option y when Option y was sufficiently attractive. Only 16 out of 92 students could be identified as having complete preferences: they first assigned $\lambda = 1$, and then assigned $\lambda = 0$ when Option y was sufficiently attractive (for 14 out of 16 students this occurred when Option y was 20 euro). Second, the λ assigned to Option x decreased with the value of the Option y . The median payment λs assigned to Option x were, respectively, 1.00 when Option y was 15 euro or lower, 0.955 when Option y was 16 euro, 0.87 when Option y was 17 euro, 0.65 when Option y was 18 euro, 0.45 when Option y was 19 euro, 0.00 when Option y was 20 euro. Therefore, subjects exhibited a clear preference for mixing Option x with Option y. Such a preference for mixing is consistent with our model but stands in sharp contrast with the expected utility theory. The relationship between λ and the value of Option y is smooth, which cannot be captured by CPT.

The measures of confidence statements in, e.g., Butler and Loomes (2007) provide valuable insights beyond dichotomous choices. One possible criticism regarding those measures could be, however, that confidence statements are self-reports and they could mean differently for different people. Below we link the confidence statements to our quantitative measure. We find, firstly, that the same confidence statement could indeed correspond to different λs. As one can observe from Figure 6, there is a significant heterogeneity of combination ratios in each confidence statement, in particular "Probably x", "Unsure", "Probably y ", and "Surely y ".

Secondly, we observe that choosing "surely x" approximately corresponds to assigning option x to a median payment probability of 1.00; choosing "probably x" approximately corresponds to assigning option x to a median payment probability of 0.90; choosing "unsure" roughly corresponds to assigning option x to a median payment probability of 0.70; choosing "probably y" approximately corresponds to assigning option x to a median payment probability of 0.56; choosing "surely y" corresponds to assigning option x to a median payment probability of 0.35. The above result reveals a clear asymmetry: given the same qualitative statements, i.e., "surely" or "probably", subjects used a much higher combination ratio of Option x than Option y. A possible interpretation of the above asymmetry

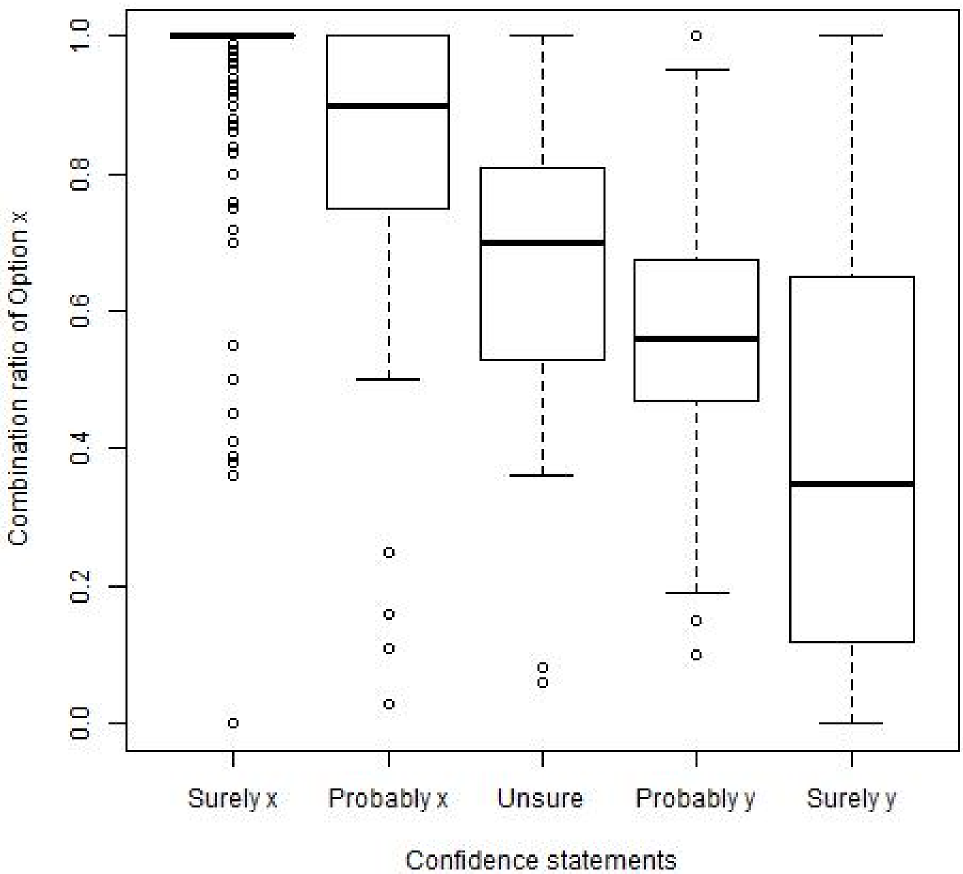

Figure 6: A boxplot of the combination ratio of Option x, given each confidence statements. The thick line is median, the upper and lower bars are 2nd and 3rd quartiles, respectively.

is that subjects made qualitative statements such as "surely" or "probably" based on the $E_\tau[\Delta_\tau]$, the expected utility difference between Option x and Option y, while the optimal combination ratio λ depended additionally on the imprecision over Option y , as revealed by Equation 4.

4 Discussion

Our model is related to the literature of probabilistic choices. Models of probabilistic choices (see, e.g., Harless and Camerer, 1994b; Hey and Orme, 1994) assume that a true preference exists and is deterministic. This serves as the core of individuals' preferences. However, individuals' choices are probabilistic due to errors such as mistakes, carelessness, slips, inattentiveness, etc. Our model is fundamentally different from these models. It assumes no error term, and the choices are deterministic. Most importantly, individuals do not have a unique preference; instead, they have a set of utility functions. In models of random utility (McFadden, 1973; McFadden and Train, 2000), an individual's utility function is subject to random shocks. Ex ante, the individual can thus be seen as facing a set of utility functions. A similar model is the so-called random preferences model by Loomes and Sugden (1995). This model assumes that an individual's preferences are imprecise and may vary from one moment to the next, and/or she may not know them with absolute precision. This idea is developed further by Karni and Safra (2014). In their model, individuals have a set of utility functions – "states of mind" in their paper – and their actual choice is determined by the state of mind that obtains. The paper shares some features with models on choice over menus (e.g., Ahn and Sarver, 2013; Karni and Safra, 2014). Ahn and Sarver (2013) assume that individuals are uncertain about their future preferences. The uncertainty in preferences leads to (1) a preference for menus with more options, i.e., a preference for flexibility; and (2) probabilistic choices from a given menu. The uncertainty in preferences is captured by individuals having a set of potential future preferences. Our model shares this notion in that it assumes an individual has multiple utility functions. There is one crucial difference, though. In models of random utility, an individual makes a decision based on a single utility function at any particular moment, although her preferences may vary over time or are not known with certainty. The utility function she uses might change over time, leading to preference reversals. In our model, the individual has a set of preference relationships at any particular moment, and she uses all of them when making a decision. Furthermore, when aggregating across preference relationships, the individual is averse to incomplete preferences. In the random preferences model, the aversion to incomplete preferences plays no role.

Our model is closely connected to an idea called “imprecise preferences”, in which it is suggested that individuals might have difficulty to pin down a precise value of an alternative (Cubitt et al., 2015). The measurement of the imprecision in preferences has been tricky. Dubourg et al. (1994) and Dubourg et al. (1997), in addition to standard choices, use un-incentivised responses to capture imprecise preferences. Subjects are considered to have imprecise preferences when they report that they are uncertain about their choices. Similarly, Butler and Loomes (2007) use two levels of responses to capture imprecise preferences: definitely preferred and probably preferred. Butler et al. (2014) introduce a continuous measurement of imprecise preferences, which they call the “strength of preferences.” Cubitt et al. (2015) allow subjects comparing a risky lottery and a sure payment to report that they are not sure about their preferences. The above measures of imprecise preferences provide valuable additional information on individuals’ decision making, which cannot be captured by choices. However, the above elicitation mechanisms are based more on intuition than on concrete theories, and the interpretation of these measures is not entirely clear. For example, the measure of strength of preference is meant to capture the perceived relative degree of preference difference between the two options. But, as acknowledged by Butler et al. (2014), some participants might have instead reported their confidence on their decision. Our measurement mechanism is developed from a rigorous model and is incentive compatible. Of course, our model has a different focus from that of the imprecise preferences; it aims to capture the process of completing an incomplete preference relation. But we believe the measurement mechanism can be used as a supplementary tool for the measurement of the imprecision in preferences. As we demonstrated in Section 3.1, the value of λ can be intuitively linked to the confidence of choices.

Finally, the idea of individuals having a set of utility functions is related to the concept of multiple selves. Fudenberg and Levine (2006) propose an intertemporal dual-self model where individuals possess two selves at the same time: a long-term and a short-term self, and the two selves might conflict with each other. Kalai et al. (2002) argue that one might use more than one rationale - with one rationale corresponding to one self - in making choices. In a similar spirit, Kalai et al. (2002), Manzini and Mariotti (2007) propose a decision procedure in which the decision maker sequentially uses two selves to make choices. The above models assume that the active preference relationship, i.e., the specific self that is activated, depends on the set of alternatives. However, given a particular set of alternatives, an individual activates only one self, and the active self has a clear preference over the alternatives. In the current model, incomplete preferences arise when an individual is uncertain about her relevant utility function over a given set of alternatives. Different utility functions offer a different ordering of the same set of alternatives.

5 Conclusion

We have developed a model of completing incomplete preferences with an axiomatic foundation. Incomplete preferences are captured by individuals having not a single but a set of utility functions. Individuals perform standard expected utility calculations, given any specific utility function, and have an subjective expectation of the *transformed* standard expected utilities with respect to a set of utility functions. We show that two empirical puzzles - the WTP-WTA gap and the present bias - are natural consequences of incomplete preferences.

Based on this model, we propose an incentive-compatible mechanism to measure the degree of incomplete preferences. In the mechanism, individuals face two alternatives. Instead of choosing one alternative, as in typical pairwise choice tasks, individuals are allowed to allocate the probabilities according to which they are paid with the alternative or the sure payment. When individuals' preferences are incomplete and their behavior is in line with our model, we show that the value assigned to the alternative provides a proxy for the degree of the incompleteness in preferences over the alternative. The obtained measure can be interpreted as the choice probability in stochastic choice models and has a striking similarity to the matching law in operant conditioning. An experimental implementation of the measure provides results consistent with our model but challenges standard utility theory.

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Appendix 1: The proof of Theorem

By assumption 3, $l_1 \succeq l_2 \Longleftrightarrow f_1 \succeq^f f_2$. By assumption 2, $f_1 \succeq^f f_2 \Longleftrightarrow \int_\Gamma v[f_1(\tau)]d\pi \geq \int_\Gamma v[f_2(\tau)]d\pi$. Definition 2 states that $f(\tau) = ce_\tau(l)$ for all $\tau \in \Gamma$. Thus, we have $f_1 \succeq^f f_2 \Longleftrightarrow \int_\Gamma v[ce_\tau(l_1)]d\pi \geq \int_\Gamma v[ce_\tau(l_2)]d\pi$. Together it gives:

$$l_1 \succeq l_2 \Longleftrightarrow \int_\Gamma v[ce_\tau(l_1)]d\pi \geq \int_\Gamma v[ce_\tau(l_2)]d\pi.$$

Since v and u_τ are strictly increasing, $v[ce_\tau(l_1)] = \phi\{u_\tau[ce_\tau(l_1)]\}$ for some strictly increasing $\phi(\cdot)$. Substituting $v[ce_\tau(l_1)]$ with $\phi\{u_\tau[ce_\tau(l_1)]\}$, we have

$$l_1 \succeq l_2 \Longleftrightarrow \int_\Gamma \phi\{u[ce_\tau(l_1)]\}\, d\pi \geq \int_\Gamma \phi\{u_\tau[ce_\tau(l_2)]\}\, d\pi.$$

By Assumption 1, $u_\tau[ce_\tau(l)] = EU_\tau(l)$, and finally we have

$$l_1 \succeq l_2 \Longleftrightarrow \int_\Gamma \phi[EU_\tau(l_1)]\, d\pi \geq \int_\Gamma \phi[EU_\tau(l_2)]\, d\pi. \; Q.E.D.$$

Appendix 2: The proof of Proposition

Proof: Without loss of generality assume $\phi(0) = 0$. Since $\phi(\cdot)$ is strictly concave, we have

$$0 = \int_\Gamma \phi[EU_\tau(x) - u_\tau(WTP)]d\pi < \phi\left\{\int_\Gamma [EU_\tau(x) - u_\tau(WTP)]d\pi\right\},$$

and

$$0 = \int_\Gamma \phi[u_\tau(WTA) - EU_\tau(x)]d\pi < \phi\left\{\int_\Gamma [u_\tau(WTA) - EU_\tau(x)]d\pi\right\}.$$

Together we have

$$0 < \int_\Gamma EU_\tau(x)d\pi - \int_\Gamma u_\tau(WTP)]d\pi,$$

and

$$0 < \int_\Gamma u_\tau(WTA)d\pi - \int_\Gamma EU_\tau(x)]d\pi.$$

Thus,

$$\int_\Gamma u_\tau(WTP)]d\pi < \int_\Gamma EU_\tau(x)d\pi < \int_\Gamma [u_\tau(WTA)d\pi.$$

Finally, note that $\int_\Gamma u_\tau(x)]d\pi$ is monotonically increasing in x, one obtains $WTP < WTA$. $Q.E.D.$