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Abstract

We quantitatively evaluate the various types of working capital loans affected by borrowing constraints using a simple real business cycle model. We explore which borrowing constraints generate persistence and/or amplified output responses to productivity and financial shocks. We find that limiting investment on account of borrowing constraints generates a persistent response to a one-time transitory shock. This finding implies that investment wedge plays an important role in generating persistence. There is a trade-off relationship between persistence and amplification among models and the working capital loan channel does not always generate amplification.

Keywords: Financing frictions, Business cycle propagation, Persistence, Business cycle accounting,

JEL Classification: E32, E37, E44, G01

1 Introduction

The standard dynamic stochastic general equilibrium (DSGE) models used during the Great Moderation, such as those of Christiano, Eichenbaum and Evans (1999) and Smets and Wouters (2003, 2007), do not employ financial friction, with some exceptions.¹ These models assume that the financial market is perfect. However, after the Great Recession, financial market imperfection has become a major topic of discussion in macroeconomics, monetary

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¹For example, Christiano, Motto and Rostagno (2003) introduce a Bernanke, Gertler and Gilchrist (1999) borrowing constraint in their medium-scale DSGE model.

economics, and finance. Moreover, to understand the mechanism of a financial crisis, an important assumption in the DSGE model is financial market imperfection. Particularly, many authors introduce financial friction to study financial crisis.

The recent macroeconomic studies considering financial friction can roughly be classified under two frameworks. The first is the costly state verification framework, developed by Townsend (1979), Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), and Bernanke et al. (1999). In this framework, there exists asymmetric information between borrowers and lenders and lenders are required to incur monitoring costs to verify the borrower's realized return on capital. The borrowing constraint is derived from the optimal contract between the borrower and lender. Many DSGE studies have introduced the Bernanke et al. (1999) type of friction; for example, see Christiano et al. (2003), Gilchrist and Saito (2008), von Hedecken (2009), Del Negro and Schorfheide (2013), Muto, Sudo and Yoneyama (2013), Iiboshi, Matsumae and Nishiyama (2014), and Christiano, Motto and Rostagno (2014). This type of financial friction can generate a sufficiently large amplification of monetary policy shocks but not of productivity shocks.

The second framework is the borrowing constraint due to lack of commitment, à la Kiyotaki and Moore (1997). Hart and Moore (1994) developed an incomplete contract model, assuming that the borrower can threaten repudiation and renegotiate repayment to the lender. In other words, borrowers do not commit to repay their debt. This assumption sets the upper limit of borrowing. Kiyotaki and Moore (1997) introduce a borrowing constraint that is a special case of Hart and Moore (1994), in the general equilibrium model. Besides Kiyotaki and Moore (1997), many applications use the Hart and Moore (1994) framework; for example, Albuquerque and Hopenhayn (2004), Iacoviello (2005), Hirano and Yanagawa (2010), Kiyotaki and Moore (2012), and Perri and Quadrini (2014). Following Hart and Moore (1994), numerous models with the borrowing constraint assume lack of commitment.

Our study focuses on the borrowing constraints due to lack of commitment. We summarize the various types of borrowing constraint and investigate them quantitatively using the Jermann and Quadrini (2012a) (JQ henceforth) model, which is a real business cycle (RBC) model with financial friction. This economy has perfect information, but there is no heterogeneity between agents, debt contracts are incomplete, and firms can default on their debt due to lack of commitment; thus, firms face the borrowing constraint.² We characterize and summarize various types of borrowing constraints quantitatively with this simple DSGE

²We do not cover household credit market imperfections, such as Chatterjee, Corbae, Nakajima and Ríos-Rull (2007), Nakajima and Ríos-Rull (2014), and Huo and Rios-Rull (2015). This issue is also important and we will consider it in a future work.

model. In particular, we consider combinations of the variables financed as working capital loans before production takes place. Working capital loans are used for several purposes: wage payments, investment, equity payout, and debt repurchase. We consider various combinations of these expenditures that are affected by the borrowing constraint. These various types of borrowing constraints are derived from lack of commitment.

In this study, we specifically focus on the persistence and amplification of output. Following the Great Recession, many studies are now interested in the adverse long-term effects of financial crisis. However, the results of many of these models crucially depend on the persistence of shocks. It is not clear as to why these shocks are so persistent. In this study, we show how only financial friction can explain the facts of business cycles and financial crises. Recently, some studies have for the first time considered the mechanism of persistence recession more deeply. Kobayashi and Shirai (2016b) shows that a one-time exogenous shock can replicate productivity slowdown, such as during Japan’s “lost two decades.” Following an exogenous shock, a proportion of firms own substantial debt, but there is no change in structural parameters. Substantial debt tightens the borrowing constraint persistently. This situation is considered a financial crisis.

We find that when an investment is affected by the borrowing constraint, the model generates a persistent output response to shock. When the borrowing constraint is tight and binding, the firm may want to relax the borrowing constraint, but it will be difficult to do so. There are only two ways to relax the borrowing constraint: increase the net worth, and decrease the working capital loans. To relax the borrowing constraint by accumulating capital stock (net worth), more working capital finance would be required, but this borrowing would be difficult because the borrowing constraint will become more tight. In addition, if the payment of wages and investment are affected by the borrowing constraint, the persistence of output would become more strong. When the payment of wages and investment are constrained, it affects the interaction between them through the borrowing constraint and this propagates the shock.

We also consider the hump-shaped response of output in this study. We find that for the hump-shaped response, both persistence of shock and the countercyclical response of wedges are required. Previous studies argue that the hump-shaped response requires some additional propagation mechanisms. In addition, our finding implies that the hump-shaped response of output also requires the countercyclical response of wedges.

We also find that the working capital channel does not always generate strong amplification. This finding contradicts the results of Inaba and Kobayashi (2009) and Mendoza (2010). They show that working capital loans are affected by the borrowing constraint and

that the working capital loan channel generates amplification. Our result implies that some other additional mechanism is needed for the working capital loans channel to generate strong amplification.

Some excellent surveys of financial frictions can be found in the literature. For example, see Gertler and Kiyotaki (2010), Quadrini (2011), Brunnermeier, Eisenbach and Sannikov (2013), and Gertler, Kiyotaki and Prestipino (2016). Quadrini (2011) and Brunnermeier et al. (2013) are deeply related to our study. Quadrini (2011) summarizes theoretically the literature since the work of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) using a simple two periods model. Brunnermeier et al. (2013) surveys a wider range of the macroeconomic implications of financial frictions that include some fields of macroeconomics, finance, and general equilibrium theory. Our work complements these works with quantitative experiments of various types of borrowing constraints.

The paper is organized as follows: Section 2 outlines the benchmark model. This model is identical to the basic JQ model. Section 3 introduces various types of borrowing constraints. Using these borrowing constraints, Section 4 examines the persistence and amplification of output. Section 5 concludes the paper.

2 Model

We use a JQ model that is in fact an RBC model with financial friction. This model is identical to the basic JQ model. Time is discrete and continues from 0 to infinity: $t = 0, 1, 2, \dots, \infty$. There are two agents in this model: firms and households. Firms face the borrowing constraint. Our paper's aim is to consider various types of borrowing constraints and compare them quantitatively. In this section, we introduce the basic JQ model. In the next section, we introduce some other types of borrowing constraints.

2.1 Firms

Consider a unit mass of homogeneous firms. A firm borrows intra-period loans, l , and inter-period loans, b , subject to the borrowing constraint. Suppose that the firm holding capital stock k and owing debt b at the beginning of period borrows an intra-period loan. The intra-period loan is used as working capital that is required at the beginning of the period for payments before the realization of revenue and is repaid with no interest at the end of the period. In the basic JQ model, working capital loans are used for new capital investment, wage payments, equity payouts, and repayment of debt prior to production. In this case, the intra-period loan is equal to the firm's revenue (production), $l = F(z, k, n)$.

The firm also borrows intra-period debt b at the effective gross interest rate, $R = 1 + r(1 - \tau)$, where r is the interest rate and τ is the tax benefit. Debt is preferred to equity because of tax benefits.

As Hart and Moore (1994) assume, a firm can counterfactually default repayment of intra-period debts after the realization of revenue but before repaying the intra-period loan. Once the firm defaults, the lender can seize the capital stock with probability ξ . Thus, the lender imposes an enforcement constraint on the firm,

$$\xi \left(k' - \frac{b'}{1+r} \right) \geq l = F(z, k, n), \quad (1)$$

where n is the labor input and ξ follows the AR(1) process $\ln \xi' = \rho_\xi \ln \xi + (1 - \rho_\xi) \ln \xi^* + e_\xi$; here, $0 \leq \rho_\xi \leq 1$, ξ^* is the steady-state value of ξ and e_ξ is i.i.d. with standard deviations σ_ξ . In this study, we call these stochastic innovations, e_ξ , “financial shocks.”

In this economy, there are two macro shocks, productivity shocks (e_z) and financial shocks (e_ξ). Since there is no idiosyncratic shock, no heterogeneity exists between firms and we only consider symmetric equilibrium. The optimization problem of a firm is

$$V(\mathbf{s}; k, b) = \max_{d, n, k', b'} \{d + Em'V(\mathbf{s}'; k', b')\},$$

subject to the borrowing constraint (1) and budget constraint

$$(1 - \delta)k + F(z, k, n) - wn + \frac{b'}{R} = b + \varphi(d) + k', \quad (2)$$

where $V(\mathbf{s}; k, b)$ is the cum-dividend market value of the firm, $\mathbf{s} = (z, \xi)$ is the aggregate state, δ is the depreciation rate of capital, $F(z, k, n) = y = zk^\theta n^{1-\theta}$ is the production (revenue), z is a productivity that follows an AR(1) process, $\ln z' = \rho_z \ln z + (1 - \rho_z) \ln z^* + e_z$, $0 \leq \rho_z \leq 1$, z^* is the steady-state value of z , e_z is the productivity shock i.i.d. with standard deviations σ_z , w is the wage rate, $m' \equiv \frac{\beta C'}{C}$ is the stochastic discount factor derived from household optimization, $\varphi(d) \equiv d + \kappa(d - \bar{d})^2$ is the gross dividend consisting of the equity payout d and adjustment cost $\kappa(d - \bar{d})^2$, and the prime denotes next-period value. This adjustment cost is the deadweight loss equal to zero in the steady state. In the special case when $\kappa = 0$, this economy is equivalent to a frictionless economy. Here, firms can costlessly issue new equities that deny the role of financial friction. We compare this frictionless case to our financial friction models quantitatively.

The first-order conditions (FOCs) for the firm's problem are

$$\begin{aligned}
d : \quad & \omega = \frac{1}{\varphi_d(d)}, \\
n : \quad & (1 - \theta) \frac{y}{n} = w \left(\frac{1}{1 - \mu \varphi_d(d)} \right), \\
k' : \quad & Em' \frac{\varphi_d(d)}{\varphi_{d'}(d')} \left\{ 1 - \delta + [1 - \mu' \varphi_{d'}(d')] \frac{\theta y'}{k'} \right\} + \xi \mu \varphi_d(d) = 1, \\
b' : \quad & RE m' \frac{\varphi_d(d)}{\varphi_{d'}(d')} + \xi \mu \varphi_d(d) \frac{R}{1+r} = 1, \tag{3}
\end{aligned}$$

where ω and μ are the Lagrange multiplier for flow of fund and the borrowing constraint, respectively.

In the steady state, solving (3) for μ^* yields

$$\mu^* = \frac{1}{\xi^*} \left(\frac{1}{\beta R^*} - 1 \right),$$

where $R = 1 + r(1 - \tau)$ and the asterisk (*) denotes steady-state variables. If and only if $\tau = 0$, the borrowing constraint is not binding. We assume throughout that $\tau > 0$ and the borrowing constraint is always binding.

2.2 Households

A unit mass of households consume, provide labor, and save to maximize the utility function

$$\max_{c, n, b', s'} E_0 \sum_{t=0}^{\infty} \beta^t [\ln c + \alpha \ln(1 - n)],$$

subject to the budget constraint

$$wn + b + s(d + p) = \frac{b'}{1+r} + s'p + c + T,$$

where β is the discount factor, c is consumption, n is labor supply, b is a one-period bond, s represents equity shares, p is the market price of shares, $T = \frac{B}{R} - \frac{B'}{1+r}$ is the lump-sum tax financing the tax benefits of firm debts, and B is the aggregated inter-period debt b .

The FOCs are

$$\begin{aligned}
c : \quad & \lambda = \frac{1}{c}, \\
n : \quad & w = \frac{\alpha c}{1 - n}, \\
b' : \quad & \frac{1}{c(1+r)} = E_0 \frac{\beta}{c'}, \\
s' : \quad & \frac{p}{c} = \beta E_0 \left(\frac{d' + p'}{c'} \right),
\end{aligned}$$

where λ is the Lagrange multiplier of budget constraint. We summarize the system of dynamics in Appendix 8.1.

2.3 Business cycle accounting

We also measure the efficiency wedge (EW), labor wedge (LW), and investment wedge (IW) by applying the business cycle accounting (BCA) method of Chari, Kehoe and McGrattan (2007). BCAs decompose output fluctuations into wedges that are distortions caused by taxes, frictions, and policies. In Appendix 7, we define the three wedges using a prototype RBC model. The EW is identical to TFP and TFP follows an exogenous process in our model. Thus, we mainly focus on LW and IW.

Chari et al. (2007) investigate the Great Depression using BCAs and conclude that a sharp decline in output during the Great Depression can be largely accounted by LW and EW deteriorations. IW is a minor source of business cycles and financial crises. Kobayashi and Inaba (2006) and Otsu (2011) also find that the lost two decades of the Japanese economy can be explained by LW deterioration and TFP slowdown. Brinca (2014) shows that EW and LW can replicate business cycle fluctuations for many OECD countries. A sharp decline in LW was also observed in the US economy during the 2008–2010 global financial crisis (see Pescatori and Tasci, 2011).

BCA can be used for the criteria by which a model is better to explain facts. If we follow this criteria by using BCA, the borrowing constraints of Carlstrom and Fuerst (1997) and Bernanke et al. (1999) are not good models because these models do not generate LW deterioration and IW deterioration leads to a decreasing output. This result is inconsistent with data. However, Inaba and Nutahara (2009) point out a problem with these criteria. They conclude that IW is not the main driving force behind output but leads to the persistence of output. Output persistency is one of the important features of empirical business cycles. BCA does not capture this role. In this paper, we consider numerical experiments with the following result in mind.

3 Financial friction

In this section, we quantitatively compare the models by introducing various types of borrowing constraints into the JQ model. These borrowing constraints can be derived as non-default conditions, as in JQ. The differences between these borrowing constraints are based on the working capital loan settings and liquidation value. The JQ model assumes that a firm's working capital loans are equal to its revenue (output) and constrained by the liquidation value of

the physical capital remaining after the deduction of long-term debt stock. We modify this assumption and consider the borrowing constraint on financing only the portion of revenue and other specifications of liquidation value. The borrowing constraints are constructed as follows:

$$\text{Borrowing constraint: } \underbrace{\text{liquidation value}}_{\left\{ \xi \left(k' - \frac{b'}{1+r} \right), \xi Em'V' \right\}} \geq \underbrace{\text{working capital loan}}_{\left\{ \begin{array}{cccc} wn, & i, & \varphi, & b - \frac{b'}{R} \\ \langle 1 \rangle & \langle 2 \rangle & \langle 3 \rangle & \langle 4 \rangle \end{array} \right\}}$$

We consider the combinations by which the q expenditure items from the set of four expenditure items,

$\{\langle 1 \rangle wn, \langle 2 \rangle i, \langle 3 \rangle \varphi, \langle 4 \rangle b - b'/R\}$, are constrained by borrowing constraints, where q is the number of constrained variables. The total number of combinations of constrained expenditure as working capital loans is $\sum_{q=1}^4 4!/[q!(4-q)!] = 15$. In addition, we also consider Kiyotaki and Moore (1997)'s specification as a special case in our model. Thus, we consider 16 types of working capital loans. In addition, the liquidation value can be considered in various ways, such as capital stocks, net worth, firm's value, and output. In this paper, we consider two types of liquidation values, $\xi \left(k' - \frac{b'}{1+r} \right)$ and $\xi Em'V'$. For example, the latter case is considered by a working paper version of JQ (Jermann and Quadrini, 2009) and Albuquerque and Hopenhayn (2004). Therefore, the total number of borrowing constraint combinations is 32. Tables 1–2 list out the borrowing constraints. “Model No.” in the tables identifies each model. When the liquidation value is $\xi \left(k' - \frac{b'}{1+r} \right)$, the first letter of “Model No.” is labeled as “m.” We call this group of models “m-models.” When the liquidation value is $\xi Em'V'$, the first letter of “Model No.” is labeled as “v.” We call this group of models “v-models.” For example, the borrowing constraint of m5 is $\xi \{ k' - b'/(1+r) \} \geq wn + i$ and means that the working capital loans for wage payment and investment are constrained by the liquidation net value of physical capital. The borrowing constraint of m15 is the same as with equation (1). The FOCs and the steady state of all models are provided in Appendix 8.

In Appendix 9, we investigate the models by their ability to replicate the main statistical features of US business cycles. In these results, the m04, v04, m07, v07, m09, m10, v10, m13, v13, m16, and v16 models do not have properties that are needed for business cycle models.³ Especially, financial shocks of the model-generated series in these models do not explain the actual financial conditions. In addition, the second moments of the simulated series also fail to match the actuals. These models show too much volatility and are negatively correlated

³As explained in Section 3.3, we also exclude models m03 and v03.

Combination	Model No.	Borrowing constraint
$\langle 1 \rangle$	m01	$\xi \{k' - b'/(1+r)\} \geq wn$
$\langle 2 \rangle$	m02	$\xi \{k' - b'/(1+r)\} \geq i$
$\langle 3 \rangle$	m03	$\xi \{k' - b'/(1+r)\} \geq \varphi$
$\langle 4 \rangle$	m04	$\xi \{k' - b'/(1+r)\} \geq b - b'/R$
$\langle 1 \rangle + \langle 2 \rangle$	m05	$\xi \{k' - b'/(1+r)\} \geq wn + i$
$\langle 1 \rangle + \langle 3 \rangle$	m06	$\xi \{k' - b'/(1+r)\} \geq wn + \varphi$
$\langle 1 \rangle + \langle 4 \rangle$	m07	$\xi \{k' - b'/(1+r)\} \geq wn + b - b'/R$
$\langle 2 \rangle + \langle 3 \rangle$	m08	$\xi \{k' - b'/(1+r)\} \geq i + \varphi$
$\langle 2 \rangle + \langle 4 \rangle$	m09	$\xi \{k' - b'/(1+r)\} \geq i + b - b'/R$
$\langle 3 \rangle + \langle 4 \rangle$	m10	$\xi \{k' - b'/(1+r)\} \geq \varphi + b - b'/R$
$\langle 1 \rangle + \langle 2 \rangle + \langle 3 \rangle$	m11	$\xi \{k' - b'/(1+r)\} \geq wn + i + \varphi$
$\langle 1 \rangle + \langle 2 \rangle + \langle 4 \rangle$	m12	$\xi \{k' - b'/(1+r)\} \geq wn + i + b - b'/R$
$\langle 1 \rangle + \langle 3 \rangle + \langle 4 \rangle$	m13	$\xi \{k' - b'/(1+r)\} \geq wn + \varphi + b - b'/R$
$\langle 2 \rangle + \langle 3 \rangle + \langle 4 \rangle$	m14	$\xi \{k' - b'/(1+r)\} \geq i + \varphi + b - b'/R$
$\langle 1 \rangle + \langle 2 \rangle + \langle 3 \rangle + \langle 4 \rangle$	m15	$\xi \{k' - b'/(1+r)\} \geq wn + i + \varphi + b - b'/R$
	m16	$\xi k' \geq \frac{b'}{1+r}$

Table 1: Borrowing constraints, liquidation value $\equiv \xi(k' - b'/(1+r))$

with the GDP for consumption and investment. However, m16 and v16 do not have suitable properties for business cycle model, and so we continue to consider the m16 and v16 models for comparison. Therefore, we conclude that models m04, v04, m07, v07, m09, m10, v10, m13, and v13 can be excluded from further analysis.

Before conducting numerical experiments, we discuss each constrained variable.

3.1 Working capital loans for wage payments

In this case, we assume that the firm needs to borrow working capital loans for wage payment l from intra-period loans. The firm can default in repayment of intra-period loan wn and retain a part of the revenue equal to the value of wage payment wn when the firm decides to default. This setting is also considered in JQ, Section III-C. Inaba and Kobayashi (2009), Kobayashi, Nakajima and Inaba (2012), Nutahara (2015), and Kobayashi and Shirai (2016a,b) also assume that working capital loans are used for wage payments. The borrowing constraint on financing wage payments is important when considering business cycles and financial crises. This financial friction distorts the labor market and may lead to LW deterioration when a

Combination	Model No.	Borrowing constraint
$\langle 1 \rangle$	v01	$\xi Em'V' \geq wn$
$\langle 2 \rangle$	v02	$\xi Em'V' \geq i$
$\langle 3 \rangle$	v03	$\xi Em'V' \geq \varphi$
$\langle 4 \rangle$	v04	$\xi Em'V' \geq b - b'/R$
$\langle 1 \rangle + \langle 2 \rangle$	v05	$\xi Em'V' \geq wn + i$
$\langle 1 \rangle + \langle 3 \rangle$	v06	$\xi Em'V' \geq wn + \varphi$
$\langle 1 \rangle + \langle 4 \rangle$	v07	$\xi Em'V' \geq wn + b - b'/R$
$\langle 2 \rangle + \langle 3 \rangle$	v08	$\xi Em'V' \geq i + \varphi$
$\langle 2 \rangle + \langle 4 \rangle$	v09	$\xi Em'V' \geq i + b - b'/R$
$\langle 3 \rangle + \langle 4 \rangle$	v10	$\xi Em'V' \geq \varphi + b - b'/R$
$\langle 1 \rangle + \langle 2 \rangle + \langle 3 \rangle$	v11	$\xi Em'V' \geq wn + i + \varphi$
$\langle 1 \rangle + \langle 2 \rangle + \langle 4 \rangle$	v12	$\xi Em'V' \geq wn + i + b - b'/R$
$\langle 1 \rangle + \langle 3 \rangle + \langle 4 \rangle$	v13	$\xi Em'V' \geq wn + \varphi + b - b'/R$
$\langle 2 \rangle + \langle 3 \rangle + \langle 4 \rangle$	v14	$\xi Em'V' \geq i + \varphi + b - b'/R$
$\langle 1 \rangle + \langle 2 \rangle + \langle 3 \rangle + \langle 4 \rangle$	v15	$\xi Em'V' \geq wn + i + \varphi + b - b'/R$
	v16	$\xi Em'V' \geq \frac{b'}{1+r}$

Table 2: Borrowing constraints, liquidation value $\equiv \xi Em'V'$

negative shock hits the economy.

3.2 Working capital loans for investment

This constraint limits investment by the firm and distorts capital allocation. The collateral constraint of Buera (2009), Buera and Shin (2013), Moll (2014), Buera and Moll (2015), and Buera, Fattal-Jaef and Shin (2015) limits the amount of capital rental by ratio of firms' net worth. These studies employ the same form of borrowing constraint and assume that households invest and hold capital stock, firms accumulate net worth, and rent capital stock is affected by the borrowing constraint. This type of borrowing constraint relates to the situation in which investment is affected by the borrowing constraint. These borrowing constraints deteriorate IW. These studies' concern is long-term economic development, and they explain some stylized facts of economic development that are not explained by standard growth models. Their idea is built by empirical evidence of capital misallocation, for example, see Hsieh and Klenow (2009).

3.3 Working capital loans for equity payout

Working capital loans for equity payout is an uncommon setting in this literature except for Jermann and Quadrini (2012a). The most simplest models in our settings are the m03 and v03 models whose working capital loans are only for equity payout. The borrowing constraint for model v03 is

$$\xi Em'V' \geq \varphi.$$

In the steady state, this constraint can be written as⁴

$$\xi^* = \frac{1}{\beta} - 1.$$

This result implies that d and b cannot be identified in equilibrium. Thus, we cannot conduct numerical simulation for this model.

Equity payout is a required positive value, because the firm's value becomes negative when its equity payout is negative in the steady state. In model m03, proposition 1 implies that condition $d^*/y^* > 0$ is difficult to be satisfied.

Proposition 1. *Suppose the borrowing constraint is $\xi \left(k' - \frac{b'}{1+r} \right) \geq \varphi$. If and only if $\xi^* < \frac{(1-\beta)(1-\tau)}{\beta[(\beta-1)\tau+1]}$, $\frac{d^*}{y^*} > 0$.*

Proof. See Appendix 8.4. □

In the standard setting,⁵ $\xi^* = 0.162$ is not satisfied for condition $\frac{(1-\beta)(1-\tau)}{\beta[(\beta-1)\tau+1]} = 0.0116 > \xi^*$. In addition, the Blanchard–Kahn conditions are not satisfied and there is no stable equilibrium even when $\xi^* < \frac{(1-\beta)(1-\tau)}{\beta[(\beta-1)\tau+1]}$. We thus cannot conduct a numerical experiment for model m03.

Therefore, we will not consider models v03 and m03 in our further analysis. However, we do not conclude that working capital loans for equity payout is not a good modeling. We will show that the combinations of equity payout and other factor(s) are not worse compared to other combinations.

⁴The derivation of the borrowing constraint in the steady state is as follows:

$$\begin{aligned} \xi^* \beta V^* &= d^*, \\ \xi^* \beta \frac{d^*}{1-\beta} &= d^*, \\ \xi^* &= \frac{1}{\beta} - 1. \end{aligned}$$

⁵See Section 4 for parameter settings.

3.4 Working capital loans for debt repurchase

Debt repurchase, $b - b'/R$, is also introduced in the borrowing constraint by Kobayashi and Shirai (2016b). In this setting, if debt repurchase is required as a working capital loan, the firm borrows debt repurchase as a working capital loan to pay the lender of intra-period debt in order to fix the next-period inter-period debt b'/R . Intuitively, intertemporal debt, b , can be interpreted as corporate bond. When a firm redeems her corporate bond from a bondholder, it would require working capital loans for debt repurchase from a lender to finance her before production takes place.

3.5 Debt stock affected by borrowing constraint

We also consider the borrowing constraint on financing only intertemporal loan. This setting is the same as Kiyotaki and Moore (1997) and Kiyotaki (1998). The borrowing constraint is derived as follows:

$$\left. \begin{array}{l} \xi k' \\ \xi Em'V' \end{array} \right\} \geq \frac{b'}{1+r}.$$

The intertemporal loan is limited by the capital of collateral. In this setting, ξ represents the ratio of collateral.

4 Persistence and amplification

In this section, we compare persistence and amplification, which are important features of the business cycle, for each model. In particular, we investigate which specification of the borrowing constraint can generate persistence and/or amplified responses to a transitory shock. We consider two aggregate shocks, “productivity shocks” and “financial shocks,” and calculate stochastic simulations using log-linearized models.

4.1 Calibration

We conduct numerical simulations using two parameter sets, (i) common parameters among models and (ii) model-specific calibrated parameters. We conduct model-specific calibration to also investigate for robustness of our results. First, except for the parameters of shock processes, the parameter values follow JQ, and are given in Table 4.1. To compare them quantitatively, we assume that the parameter settings are equal among models. α is chosen to match the steady-state working hours equal to 0.3, and this value is between 1.75 and 1.99 in each model. Except for a parameter in m02, we use all parameters commonly in numerical

Parameter	Economic interpretation	value
β	the subjective discount factor	0.9825
δ	the depreciation rate of capital	0.025
κ	the inverse of the adjustment cost of dividends	0.146
τ	the tax advantage for debt	0.35
θ	the share of capital in production	0.36
ρ_z	the persistence of productivity shock	0
ρ_ξ	the persistence of financial shock	0
n^*	labor supply in the steady state	0.3
ξ^*	the collateral ratio in the steady state	0.162
z^*	TFP in the steady state	1

Table 3: Exogenous values in the steady state

simulation in this section. We set $\xi^* = 0.15$ in m02 to be a positive value for equity payout in the steady state.

For TFP and financial shocks, we assume no persistency, $\rho_z = \rho_\xi = 0$, to consider persistence and amplification that only depend on the structural model, and not on the shock process. The standard deviation of each shock is assumed to be a 1% impulse in variable when calculating the impulse response functions.

4.2 The method of stochastic simulation

In this subsection, we conduct stochastic simulation. The log-linearized dynamic system can be represented as the state-space representation:

$$\mathbf{S}_t = \mathbf{F}\mathbf{S}_{t-1} + \mathbf{Q}\mathbf{e}_t, \quad t = 0, 1, \dots, T, \quad (4)$$

$$\mathbf{x}_t = \mathbf{H}\mathbf{S}_t \quad (5)$$

where \mathbf{S}_t is the vector of state variables; \mathbf{x}_t is the vector of control variables; \mathbf{F} , \mathbf{Q} , and \mathbf{H} are functions of structural parameters; \mathbf{e}_t is the vector of productivity and financial shocks, $\mathbf{e}_t = [e_{z,t} \ e_{\xi,t}]'$; and T is the number of observations. By substituting (4) for (5),

$$\mathbf{x}_t = \mathbf{H}\mathbf{F}\mathbf{S}_{t-1} + \mathbf{H}\mathbf{Q}\mathbf{e}_t. \quad (6)$$

(4) and (6) can be combined with the single matrix equation,

$$\begin{bmatrix} \mathbf{S}_t \\ \mathbf{x}_t \end{bmatrix} = \begin{bmatrix} \mathbf{F} \\ \mathbf{HF} \end{bmatrix} \mathbf{S}_{t-1} + \begin{bmatrix} \mathbf{Q} \\ \mathbf{HQ} \end{bmatrix} \mathbf{e}_t.$$

We assume that the economy is in the steady state when $t = 0$. \mathbf{e}_t can be generated by normal distribution independently.

We define the persistency of how the periods in an impulse response of output to a shock converge to the steady state. The value of \mathbf{e}_t is chosen such that the output in period 1 is lower than the steady state by 1%. We judge convergence when the deviation is less than 0.01%. Amplification is measured by the elasticity of output with respect to a shock, such as Cordoba and Ripoll (2004), by calculating the impulse response functions, that is,

$$\text{amplification} \equiv \frac{\min \tilde{y}_t \text{ or } \max \tilde{y}_t}{\text{size of a negative shock (\% deviation from the steady state)'}}$$

where $\tilde{y}_t \equiv \ln(y_t)/\ln(y^*)$ is the percentage deviation from the steady state of output and y^* is the steady-state value of output. We consider a negative shock, but the response of output to a negative shock is not always negative and depends on the structure of the model. When the response of output is positive, we calculate amplification using the max operator.

4.3 Simulation results

To compare the simulation results quantitatively, we introduce a frictionless RBC economy as reference for a special case of our model with parameters, the adjustment cost of dividend $\kappa = 0$, and the tax advantage of debt $\tau = 0$. In this case, firms do not issue debt because there is no tax advantage and the borrowing constraint is not binding any more. This model is identical to the standard RBC model without any adjustment costs and friction. However, the model does not work when $\kappa = 0$ and $\tau = 0$ because of indeterminacy between b and d . Therefore, we omit b in the frictionless RBC model. The frictionless RBC model is described in Appendix 6.

Table 4 shows persistence of output and amplification.⁶ The first column gives the “Model No.,” which is already defined in Table 1-2. We label the frictionless RBC model as “RBC.” The second column gives the “constrained variables”; these comprise the necessary working capital finance before production begins in each model and are affected by the borrowing constraint.

From this table, all classes of borrowing constraints do not generate a strong amplified response to a shock relative to the standard RBC model.⁷ Roughly speaking, m-models and

⁶We also apply stochastic simulations to persistence shocks and set the persistence parameter values of shocks equal to 0.5; that is, $\rho_z = \rho_\xi = 0.5$. However, this modification has little effect on amplification and does not change our conclusion.

⁷In an extreme parameter settings, model m14 can generate strong amplification. This result is based on a particular combination of parameters at the edge of the parameter space. This result is similar to the finding of Cordoba and Ripoll (2004).

		Persistence		Amplification	
		e_z	e_ξ	e_z	e_ξ
	constrained variables				
RBC		15	NaN	1.717	NaN
m01	wn	14	6	1.096	0.396
m02	i	39	32	1.579	0.245
m05	$wn + i$	40	19	0.622	0.360
m06	$wn + \varphi$	12	13	1.570	0.107
m08	$i + \varphi$	35	50	1.487	0.041
m11	$wn + i + \varphi$	38	31	1.281	0.168
m12	$wn + i + b - \frac{b'}{R}$	12	38	1.207	0.308
m14	$i + \varphi + b - \frac{b'}{R}$	7	16	1.903	-0.111
m15	$wn + i + \varphi + b - \frac{b'}{R}$	33	7	0.293	0.881
m16	$b'/(1+r)$	16	10	1.574	-0.545

		Persistence		Amplification	
		e_z	e_ξ	e_z	e_ξ
	constrained variables				
RBC		15	NaN	1.717	NaN
v01	wn	15	6	1.072	0.406
v02	i	27	27	1.257	0.071
v05	$wn + i$	31	19	0.992	0.239
v06	$wn + \varphi$	12	12	1.568	0.110
v08	$i + \varphi$	22	40	1.527	0.035
v11	$wn + i + \varphi$	28	36	1.319	0.151
v12	$wn + i + b - \frac{b'}{R}$	8	35	1.259	0.265
v14	$i + \varphi + b - \frac{b'}{R}$	22	29	1.452	0.178
v15	$wn + i + \varphi + b - \frac{b'}{R}$	38	6	0.191	0.932
v16	$b'/(1+r)$	15	3	1.610	-0.165

Table 4: Persistence and amplification of output: $\rho = 0$, common parameters among models

Note: Persistence is defined as the number of convergence periods in response to a shock that decreases output by 1% from the steady state. We judge convergence when the deviation is less than 0.01%. Amplification is defined as the elasticity of output to a shock.

result is also similar to the finding of Cordoba and Ripoll (2004) of a trade-off relationship in output in the parameter space using the generalized Kiyotaki and Moore (1997) model.⁸

In the case of a simple investment-constrained model (m02 and v02), output shows a relatively strong persistence response to productivity shock without decreasing amplification. Figure 2 shows an impulse response function to productivity and financial shocks. The transitory negative productivity shock decreases output and relaxes the borrowing constraint, because the demand of working capital loans decrease. Tightness of the borrowing constraint is shown by the Lagrange multiplier of the borrowing constraint μ . In period 1, μ decreases in response to a negative productivity shock, implying relaxation of the borrowing constraint. To show this, the borrowing constraint of m02 is rewritten as follows using (2):

$$\xi \left(k' - \frac{b'}{1+r} \right) \geq zk^\theta n^{1-\theta} - wn - \varphi + \frac{b'}{R} - b.$$

The right-hand side of z decreases in response to a negative productivity shock and the borrowing constraint loosens. The firm borrows more debt for investment following relaxation of the borrowing constraint, although the debt cannot decrease immediately even after z returns to the pre-shock level, and the borrowing constraint persistently tightens. From this result, the firm cannot obtain sufficient working capital for investment and the problem of insufficient capital stock continues. In this result, the output response to productivity shock is persistent. Next, we evaluate this mechanism through the lens of BCA; the Euler equations for capital stock is

$$Em' \frac{\varphi_d(d)}{\varphi_{d'}(d')} \left\{ (1-\delta) [1 + \varphi_{d'}(d')\mu] + \frac{\theta y'}{k'} \right\} = 1 + (1-\xi)\varphi_d(d)\mu,$$

where the Lagrange multiplier of the borrowing constraint μ and the term $1 + \varphi_d(d)\mu$ determine the IW. The persistence response of output is generated by IW. This finding is consistent with Inaba and Nutahara (2009). They find that the role of IW is to delay the propagation of productivity shocks in the Carlstrom and Fuerst (1997) economy; the borrowing constraint of their model also limits investment. Without relying on the structure of the shock process, financial friction can generate persistency.

This finding relates to Buera and Shin (2013). They quantitatively analyze the development dynamics using a heterogeneous agents model with financial friction. Their borrowing constraint limits the finance capital stock. They find that slower transition dynamics, which are the observed growth experiences of economic miracles, can be explained by financial friction. Such persistence partially depends on IW deterioration.

⁸Pintus (2011) finds that a low-risk aversion can cause amplification and persistence. However, their results cannot generate strong amplification and the elasticity of output with respect to TFP shock is not greater than 1.

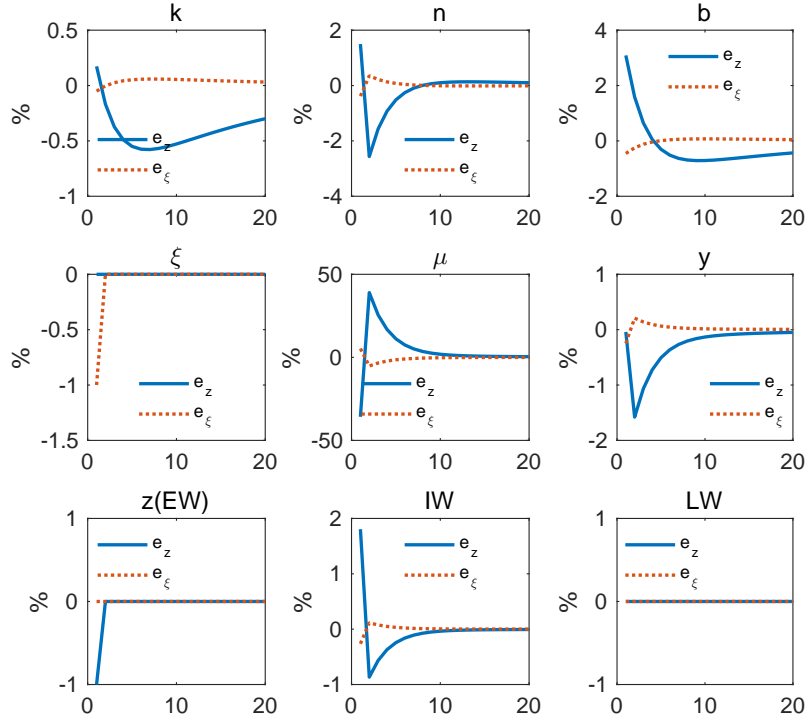


Figure 2: Impulse responses to one-time productivity and financial shocks for investment-constrained model (m02)

Note: The standard deviation of each shock is assumed to be a 1% impulse in the variable.

A negative financial shock also leads to persistence of output, but the response of output inverts the sign from negative to positive after period 2. In period 1, a negative financial shock hits the economy, the borrowing constraint becomes more tight, and investment, debt, and output decrease. In the next period, the financial condition returns to the steady state and the borrowing constraint is relaxed due to decreasing debt. In this result, the increase in investment and accumulating capital stock relax the borrowing constraint more. Thus, a negative financial shock decreases output temporarily, but after some periods, the output increases persistently.⁹ This persistence is also generated by IW.

The wage payment-constrained models (m01 and v01) show nearly the same persistence with a frictionless RBC model. However, the combination of wage payment and investment (m06 and v06) increases the persistence slightly. The other combination depresses the persistence, except for v15. From Figure 3, the main force of the persistence response of output to a productivity shock comes from LW. IW still contributes to persistence, but it is quantita-

⁹The positive response of output to a negative financial shock after some periods disappears, when the shock process becomes highly persistent.

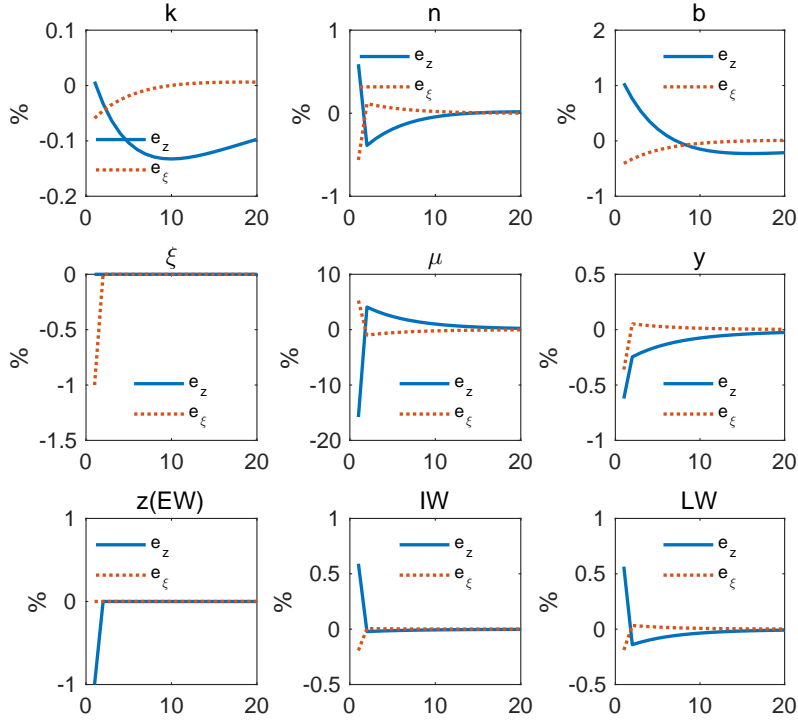


Figure 3: Impulse responses to one-time productivity and financial shocks for wage payment and investment-constrained model (m05)

Note: The standard deviation of each shock is assumed to be a 1% impulse in the variable.

tively very small. This result implies that the combination of wage payment and investment has a complementary relationship and IW causes the spillover effect of LW in the m06 and v06 model. This figure also shows why only financial friction does not amplify the shock. In the aftermath of a productivity shock, wedges counterfactually respond to the shock due to borrowing constraints and partially cancel out the effect of the shock.¹⁰ If wage payment, investment, and equity payout are constrained (m11 and v11), amplification becomes more strong than in the m05 and v05 models, although persistence becomes a little weak.

4.4 Robustness and the hump-shaped response of output

In this subsection, we assess the robustness of the results by computing the same numerical simulations using model-specific calibrated parameters. Table 5 gives the calibrated parameters. More details of the calibration strategy are shown in Appendix 9. Table 6 gives the simulation results and is basically identical to Table 4. Investment-constrained and wage payment-constrained models are still important to generate persistence. We confirm that

¹⁰In the case of financial shock, the shock does not directly affect output.

	m01	m02	m05	m06	m08	m11	m12	m14	m15	m16
α	1.856	1.889	1.868	1.875	1.806	1.776	1.871	1.921	1.883	1.920
ξ^*	0.092	0.041	0.140	0.109	0.089	0.248	0.145	0.064	0.162	0.371
κ	0.065	0.105	0.125	0.000	0.000	0.044	0.000	0.290	0.052	0.000
ρ_ξ	0.981	0.927	0.969	0.974	0.945	0.977	0.975	0.881	0.973	0.989
σ_ξ	0.012	0.047	0.017	0.009	0.026	0.014	0.010	0.020	0.009	0.010

	v01	v02	v05	v06	v08	v09	v11	v12	v14	v15	v16
α	1.857	1.881	1.864	1.876	1.912	1.902	1.877	1.871	1.927	1.883	1.920
ξ^*	0.093	0.032	0.131	0.117	0.056	0.042	0.155	0.140	0.067	0.164	0.593
κ	0.061	0.087	0.094	0.000	0.000	0.000	0.062	0.000	0.087	0.051	0.000
ρ_ξ	0.981	0.927	0.970	0.973	0.926	0.855	0.966	0.975	0.884	0.973	0.989
σ_ξ	0.012	0.061	0.018	0.009	0.033	0.028	0.015	0.010	0.020	0.009	0.010

Table 5: Parameter settings

our results are robust. Amplification becomes relatively weak and inversely relates to persistence. Kiyotaki–Moore-type borrowing constraints (m16 and v16) generate highly persistent response to financial shock but nearly zero amplification.

This table also gives the hump-shaped response of output. We measure the hump-shaped response as when the impulse response to a negative shock reaches the bottom. Shirai (2014) shows that the hump-shaped response of output requires a highly persistent shock process in the frictionless RBC model. The persistence of productivity shock is calibrated as $\rho_z = 0.9378$; this parameter is too small when the standard RBC model generates the hump-shaped output response. As is well known, the standard RBC model has weak endogenous propagation mechanisms,¹¹ and an additional mechanism is required.¹² We examine whether financial friction has propagation mechanisms. Interestingly, although the most persistent models are investment-constrained ones (m02 and v02), these models do not generate the hump-shaped response. On the other hand, a combination with investment and other variable(s) (m05, v05, m12, v12, v14, m15, and v15) generate the hump-shaped response. This hump-shaped response is generated by the countercyclical response of IW and LW that are enhanced by the spillover effect. Figure 4 shows the impulse response functions for wage payment, investment, and the debt repurchase-constrained model (m12). The hump-shaped response of output

¹¹See Cogley and Nason (1995).

¹²For example, the habit formation of consumption and human capital accumulation enhance the propagation mechanism. See Fuhrer (2000) and Chang, Gomes and Schorfheide (2002).

		Persistence		Amplification		Hump-shape	
		e_z	e_ξ	e_z	e_ξ	e_z	e_ξ
	constrained variables						
m01	wn	94	138	1.057	0.344	3	1
m02	i	114	88	0.927	0.103	1	1
m05	$wn + i$	103	103	0.866	0.310	6	1
m06	$wn + \varphi$	85	190	1.505	0.028	1	1
m08	$i + \varphi$	101	127	1.452	-0.002	1	5
m11	$wn + i + \varphi$	105	142	1.299	0.067	1	1
m12	$wn + i + b - \frac{b'}{R}$	107	97	0.659	0.882	7	1
m14	$i + \varphi + b - \frac{b'}{R}$	88	63	1.728	-0.196	1	1
m15	$wn + i + \varphi + b - \frac{b'}{R}$	94	83	1.118	0.612	5	1
m16	$b'/(1+r)$	87	444	1.508	0.028	1	18

		Persistence		Amplification		Hump-shape	
		e_z	e_ξ	e_z	e_ξ	e_z	e_ξ
	constrained variables						
v01	wn	94	145	1.068	0.296	3	1
v02	i	118	106	1.025	0.082	1	1
v05	$wn + i$	102	125	0.964	0.206	2	1
v06	$wn + \varphi$	85	183	1.504	0.029	1	1
v08	$i + \varphi$	95	100	1.473	0.029	1	1
v09	$i + b - \frac{b'}{R}$	97	134	1.291	0.086	1	1
v11	$wn + i + \varphi$	91	77	1.310	0.092	1	1
v12	$wn + i + b - \frac{b'}{R}$	101	110	0.882	0.573	2	1
v14	$i + \varphi + b - \frac{b'}{R}$	102	18	1.167	0.273	3	1
v15	$wn + i + \varphi + b - \frac{b'}{R}$	94	91	1.105	0.499	5	1
v16	$b'/(1+r)$	87	444	1.508	0.018	1	18

Table 6: Persistence and amplification of output: model-specific calibrated parameters

Note: Persistence is defined as the number of convergence periods in response to a shock that decreases output by 1% from the steady state. We judge convergence when the deviation is less than 0.01%. Amplification is defined as the elasticity of output to a shock. Hump shape is measured as the period of bottom on impulse response after the shock is realized.

to a productivity shock is generated by the countercyclical response of IW and LW. In this model, the borrowing constraint limits not only investment but also wage payment and debt repurchase and these strengthen the countercyclicity of IW and LW.

The response to financial shock does not cause the hump-shaped response of output. Models m08, m16, and v16 generate the hump-shaped response, but these amplifications are nearly zero. The big difference between the model-specific calibrated results in this subsection and the results with common parameters of models in the previous subsection is the financial shock. In this subsection, the response of output to a financial shock monotonically converges to the steady state. This response crucially depends on the persistence parameter of financial shock ρ_ξ .

We already showed that the main driving forces of the financial crisis are TFP and LW. However, these figures show that it is difficult for a negative productivity shock to replicate LW deterioration. In period 1, when the shock realizes, LW responds countercyclically to a productivity shock, but after some periods, LW becomes pro-cyclical. However, a sufficiently persistent financial shock can replicate LW deterioration soon after a shock is realized. According to our models, financial crises are considered to be due to both negative productivity shocks and negative persistent financial shocks.

5 Conclusion

After the Great Recession, the global economy continues to suffer from stagnation. To explain this, we need to understand the mechanism of financial crisis. In this paper, we considered various forms of borrowing constraints to investigate which of the borrowing constraints generate persistent and/or amplified output responses to a productivity and financial shock using a simple business cycle model with financial friction. Specifically, we consider how financial friction can replicate business cycle factors without multiple friction or shocks.

We find a trade-off relationship between persistence and amplification in the models. When a relatively strong persistence is generated, amplification becomes weak. In addition, financial friction alone does not generate a stronger amplification than the standard RBC model. However, even if a transitory shock is assumed, some models can generate persistence. In particular, investments affected by the borrowing constraint model have an important role. When inefficient capital stock continues, it leads to the tightened borrowing constraint and generates persistence of output. IW is caused by the borrowing constraint limiting the working capital finance for investment. In addition, the interaction between IW and LW strengthens the propagation mechanism. We can conclude that IW and LW have an important role in

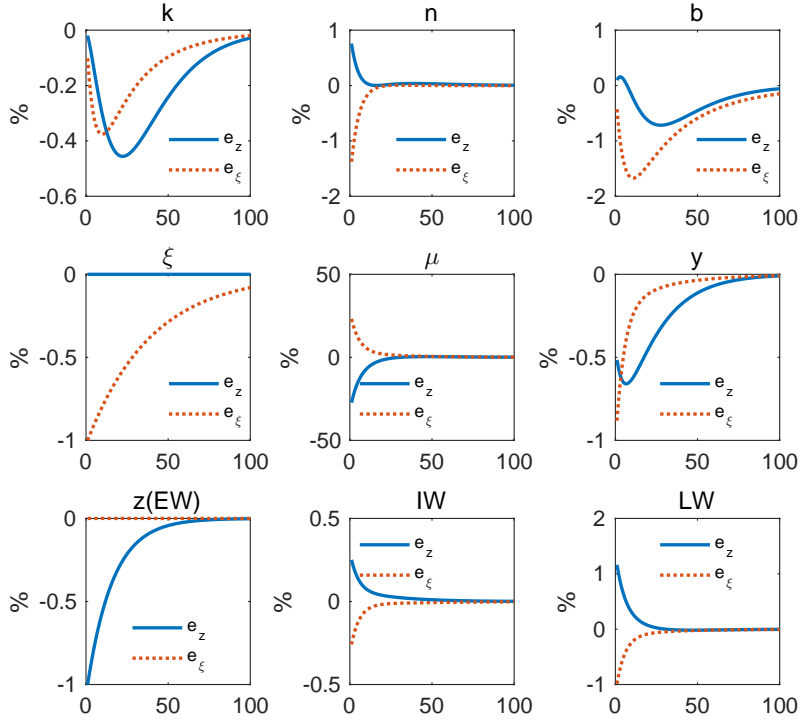


Figure 4: Impulse responses to one-time productivity and financial shocks for wage payment, investment, and debt repurchase-constrained model (m12)

Note: The standard deviation of each shock is assumed to be a 1% impulse in the variable.

generating persistence and the propagation mechanism.

Appendices

6 Appendix: Frictionless model economy

When we assume that $\kappa = 0$ and $\tau = 0$, b and d cannot be identified in equilibrium. We modify the model by excluding b to solve the model.

A firm's problem is

$$\begin{aligned}
 V &= \max \{d + Em'V'\}, \\
 \text{s.t.} \quad &(1 - \delta) + y - wn = d + k'.
 \end{aligned}$$

The FOCs are

$$\begin{aligned}
 k : \quad &Em' \left(1 - \delta + \theta \frac{y'}{k'}\right) = 1, \\
 n : \quad &w = (1 - \theta) \frac{y}{n}.
 \end{aligned}$$

A household's problem is

$$\begin{aligned} & \max_{c, n, b', s'} E_0 \sum_{t=0}^{\infty} \beta^t [\ln c + \alpha \ln(1 - n)], \\ \text{s.t.} \quad & wn + s(d + p) = s'p + c. \end{aligned}$$

The FOCs are

$$\begin{aligned} c & : \quad \lambda = \frac{1}{c}, \\ n & : \quad w = \frac{\alpha c}{1 - n}. \end{aligned}$$

This model is identical to the standard RBC model.

7 Appendix: BCA for the JQ economy

In this appendix, we describe the prototype model to measure the wedges.

The representative household solves

$$\max_{c, n, b', s'} E_0 \sum_{t=0}^{\infty} \beta^t [\ln c + \alpha \ln(1 - n)],$$

subject to

$$(1 - \tau_n)wn + s(d + p) = s'p + c,$$

where $(1 - \tau_n)$ is LW. The FOCs are

$$\begin{aligned} c & : \quad \lambda = \frac{1}{c}, \\ n & : \quad (1 - \tau_n)w = \frac{\alpha c}{1 - n}. \end{aligned}$$

The firm solves

$$V(\mathbf{s}; k) = \max_{d, n, k'} \{d + Em'V(\mathbf{s}'; k')\},$$

subject to

$$F(z, k, n) - wn = \varphi(d) + (1 + \tau_x)[k' - (1 - \delta)k],$$

where $1/(1 + \tau_x)$ is IW. The FOCs are

$$\begin{aligned} n & : \quad w = (1 - \theta) \frac{y}{n}, \\ k' & : \quad Em' \frac{\varphi_d(d)}{\varphi_{d'}(d')} \{(1 - \delta)(1 + \tau'_x) + F_{k'}(z', k', n')\} = 1 + \tau_x. \end{aligned}$$

We summarize the three wedges below:

$$\begin{aligned}
LW &: (1 - \tau_n) = \frac{MRS}{MRL} = \frac{\frac{\alpha c}{1-n}}{(1-\theta)\frac{y}{n}}, \\
EW &: z = \frac{y}{k^\theta n^{1-\theta}}, \\
IW &: \frac{1}{1 + \tau_x} = E \frac{\varphi_{d'}(d')}{m' \varphi_d(d)} \left\{ \frac{\frac{1}{1+\tau'_x}}{1 - \delta + \frac{F_{k'}(z', k', n')}{1+\tau'_x}} \right\}.
\end{aligned}$$

Since EW follows an exogenous shock process, we mainly focus on LW and IW.

8 Appendix: System of dynamics and the steady state

8.1 The benchmark case: m15

8.1.1 System of dynamics

The system of dynamics of the benchmark model is summarized below:

Households

$$w = \frac{\alpha c}{1-n}, \quad (7)$$

$$\frac{1}{c(1+r)} = E_0 \frac{\beta}{c'}. \quad (8)$$

Firms

$$\xi \left(k' - \frac{b'}{1+r} \right) \geq y, \quad (9)$$

$$w = \frac{(1-\theta)(1-\mu\varphi_d(d))y}{n}, \quad (10)$$

$$Em' \frac{\varphi_d(d)}{\varphi_{d'}(d')} \left\{ 1 - \delta + [1 - \mu' \varphi_{d'}(d')] \frac{\theta y'}{k'} \right\} + \xi \mu \varphi_d(d) = 1, \quad (11)$$

$$REm' \frac{\varphi_d(d)}{\varphi_{d'}(d')} + \xi \mu \varphi_d(d) \frac{R}{1+r} = 1, \quad (12)$$

$$V = d + Em'V', \quad (13)$$

$$(1-\delta)k + y - wn + \frac{b'}{R} = b + \varphi(d) + k', \quad (14)$$

$$y = F(z, k, n) = zk^\theta n^{1-\theta}, \quad (15)$$

$$\varphi(d) = d + \kappa(d - \bar{d})^2, \quad \varphi_d(d) = 1 + 2\kappa(d - \bar{d}), \quad (16)$$

$$m' = \frac{\beta C}{C'}, \quad (17)$$

$$R = 1 + r(1 - \tau). \quad (18)$$

Shocks

$$\ln z' = \rho_z \ln z + (1 - \rho_z) \ln \bar{z} + e_z, \quad (19)$$

$$\ln \xi' = \rho_\xi \ln \xi + (1 - \rho_\xi) \ln \bar{\xi} + e_\xi. \quad (20)$$

The resource constraint

$$(1 - \delta)K + Y - K' - C = \varphi(D) - D. \quad (21)$$

8.1.2 Steady state

The steady-state values of the benchmark model are obtained by

$$\begin{aligned} n^* &= 0.3, \\ (19) : z^* &= 1, \\ (20) : \xi^* &= \bar{\xi}, \\ (8) : r^* &= \frac{1}{\beta} - 1, \\ (18) : R^* &= 1 + r^*(1 - \tau), \\ (12) : \mu^* &= \frac{1}{\xi^*} \left(\frac{1}{\beta R^*} - 1 \right), \\ (17) : m^* &= \beta = \frac{1}{1 + r^*}, \\ (11) : k^* &= \left[\frac{(1 - \mu^*)\theta\beta z^*}{1 - \xi^*\mu^* - \beta(1 - \delta)} \right]^{\frac{1}{1-\theta}} n^*, \\ (15) : y^* &= z^* k^{*\theta} n^{*1-\theta}, \\ (21) : c^* &= y^* - \delta k^*, \\ (7) : \alpha &= \frac{w^*(1 - n^*)}{c^*}, \\ (10) : w^* &= \frac{(1 - \theta)(1 - \mu^*)y^*}{n^*}, \\ (9) : b^* &= \frac{1}{\beta} \left(k^* - \frac{y^*}{\xi^*} \right), \\ (14) : d^* &= y^* - \delta k^* - w^* n^* + b^* \left(\frac{1}{R^*} - 1 \right), \\ (16) : \varphi(d^*) &= d^*, \quad \varphi_d(d^*) = 1, \\ (13) : V^* &= \frac{d^*}{1 - \beta}. \end{aligned}$$

8.2 m01: $\xi \left(k' - \frac{b'}{1+r} \right) \geq wn$

The borrowing constraint is

$$\xi \left(k' - \frac{b'}{1+r} \right) \geq wn,$$

The FOCs of a firm

$$\begin{aligned} d : \quad & \omega = \frac{1}{\varphi_d}, \\ n : \quad & w = \frac{1 - \theta}{1 + \frac{\mu}{\omega} n} y, \\ k : \quad & Em' \frac{\omega'}{\omega} \left[1 - \delta + \frac{\theta y'}{k'} \right] + \frac{\mu}{\omega} \xi = 1, \\ b : \quad & REM' \frac{\omega'}{\omega} + \xi \frac{\mu}{\omega} \frac{R}{1+r} = 1. \end{aligned}$$

8.3 m02: $\xi \left(k' - \frac{b'}{1+r} \right) \geq k' - (1 - \delta)k$

The borrowing constraint is

$$\xi \left(k' - \frac{b'}{1+r} \right) \geq i,$$

where $i \equiv k' - (1 - \delta)k$.

The FOCs of a firm

$$\begin{aligned} d : \quad & \omega = \frac{1}{\varphi_d}, \\ n : \quad & w = (1 - \theta) \frac{y}{n}, \\ k : \quad & Em' \frac{\omega'}{\omega} \left[(1 - \delta) \left(1 + \frac{\mu'}{\omega'} \right) + \frac{\theta y'}{k'} \right] - (1 - \xi) \frac{\mu}{\omega} = 1, \\ b : \quad & REM' \frac{\omega'}{\omega} + \xi \frac{\mu}{\omega} \frac{R}{1+r} = 1. \end{aligned}$$

8.4 m03: $\xi \left(k' - \frac{b'}{1+r} \right) \geq \varphi$

The borrowing constraint is

$$\xi \left(k' - \frac{b'}{1+r} \right) \geq \varphi.$$

The FOCs of a firm:

$$\begin{aligned}
d : \quad & \omega = \frac{1}{\varphi_d} - \mu, \\
n : \quad & w = (1 - \theta) \frac{y}{n}, \\
k : \quad & Em' \frac{\omega'}{\omega} \left[1 - \delta + \frac{\theta y'}{k'} \right] + \frac{\mu}{\omega} \xi = 1, \\
b : \quad & REM' \frac{\omega'}{\omega} + \xi \frac{\mu}{\omega} \frac{R}{(1+r)} = 1.
\end{aligned}$$

8.4.1 Steady state

$$\begin{aligned}
\text{BC} : \quad & \xi^* (k^* - \beta b^*) = d^* \\
d : \quad & \omega^* = 1 - \mu^*, \\
n : \quad & w^* = (1 - \theta) \frac{y^*}{n^*}, \\
k : \quad & k^* = \left[\frac{(1 - \mu^*) \theta \beta z^*}{1 - \mu^* (1 + \xi^*) - \beta (1 - \delta) (1 - \mu^*)} \right]^{\frac{1}{1-\theta}} n^* = \left[\frac{\beta^2 \theta R^* z^*}{2R^* \beta - \beta^2 R (1 - \delta) - 1} \right]^{\frac{1}{1-\theta}} (1 - \theta n^*) \\
b : \quad & \mu^* = \frac{1 - \beta R^*}{\beta R^* (\xi^* - 1) + 1} \iff \frac{\xi^* \mu^*}{1 - \mu^*} = \frac{1}{\beta R^*} - 1
\end{aligned}$$

Proposition 1:

Proof. In the steady state, the borrowing constraint can be rewritten:

$$\frac{b^*}{y^*} = \frac{\xi^* k^* - d^*}{\beta \xi^* y^*}.$$

A firm's flow of funds is:

$$d^* = y^* - \delta k^* - w^* n^* + \left(\frac{1 - R^*}{R^*} \right) b^*.$$

Using the steady state relationship and divide by y^* , the flow of funds can be rewritten:

$$\frac{d^*}{y^*} = \underbrace{\left[\theta - \frac{k^*}{y^*} \left(\delta - \frac{1 - R^*}{R^* \beta} \right) \right]}_{\ominus} \left(\frac{\beta \xi^* R^*}{\beta \xi^* R^* + 1 - R^*} \right) > 0.$$

ξ^* is the only free parameter and used to adjust the steady state value to fit the observational data. ξ does not appear in the first square bracket on the right hand side. This term is negative constant and can be ignored to consider the condition $d^*/y^* > 0$. The sign of the second bracket is determined by ξ^* in the denominator. Hence, the condition of $d^*/y^* > 0$ is determined by

$$\xi < \frac{R^* - 1}{\beta R^*} = \frac{(1 - \beta)(1 - \tau)}{\beta [(\beta - 1)\tau + 1]},$$

where $R^* = 1 + r^*(1 - \tau)$ and $r^* = 1/\beta - 1$. □

8.5 m04: $\xi \left(k' - \frac{b'}{1+r} \right) \geq b - \frac{b'}{R}$

The borrowing constraint is

$$\xi \left(k' - \frac{b'}{1+r} \right) \geq b - \frac{b'}{R}.$$

The FOCs of a firm:

$$\begin{aligned} d : \quad & \omega = \frac{1}{\varphi_d}, \\ n : \quad & w = (1 - \theta) \frac{y}{n}, \\ k : \quad & Em' \frac{\omega'}{\omega} \left[1 - \delta + \frac{\theta y'}{k'} \right] + \frac{\mu}{\omega} \xi = 1, \\ b : \quad & REM' \frac{\omega'}{\omega} \left(1 + \frac{\mu'}{\omega'} \right) + \frac{\mu}{\omega} \left(\frac{\xi R}{1+r} - 1 \right) = 1. \end{aligned}$$

8.6 m05: $\xi \left(k' - \frac{b'}{1+r} \right) \geq wn + k' - (1 - \delta)k$

The borrowing constraint is

$$\xi \left(k' - \frac{b'}{1+r} \right) \geq wn + k' - (1 - \delta)k.$$

The FOCs of a firm:

$$\begin{aligned} d : \quad & \omega = \frac{1}{\varphi_d} \\ n : \quad & w = \frac{1 - \theta}{1 + \frac{\mu}{\omega}} \frac{y}{n}, \\ k : \quad & Em' \frac{\omega'}{\omega} \left[(1 - \delta) \left(1 + \frac{\mu'}{\varphi_d} \right) + \frac{\theta y'}{k'} \right] + \frac{\mu}{\omega} (\xi - 1) = 1, \\ b : \quad & REM' \frac{\omega'}{\omega} + \xi \frac{\mu}{\omega} \frac{R}{(1+r)} = 1. \end{aligned}$$

8.7 m06: $\xi \left(k' - \frac{b'}{1+r} \right) \geq wn + \varphi$

The borrowing constraint is

$$\xi \left(k' - \frac{b'}{1+r} \right) \geq wn + \varphi.$$

The FOCs of a firm:

$$\begin{aligned}
d : \quad & \omega = \frac{1}{\varphi_d} - \mu, \\
n : \quad & w = \frac{1 - \theta}{1 + \frac{\mu}{\omega} n} y, \\
k : \quad & Em' \frac{\omega'}{\omega} \left[1 - \delta + \frac{\theta y'}{k'} \right] + \frac{\mu}{\omega} \xi = 1, \\
b : \quad & REM' \frac{\omega'}{\omega} + \xi \frac{\mu}{\omega} \frac{R}{(1+r)} = 1.
\end{aligned}$$

8.8 m07: $\xi \left(k' - \frac{b'}{1+r} \right) \geq wn + b - \frac{b'}{R}$

The borrowing constraint is

$$\xi \left(k' - \frac{b'}{1+r} \right) \geq wn + b - \frac{b'}{R}.$$

The FOCs of a firm:

$$\begin{aligned}
d : \quad & \omega = \frac{1}{\varphi_d}, \\
n : \quad & w = \frac{1 - \theta}{1 + \frac{\mu}{\omega} n} y, \\
k : \quad & Em' \frac{\omega'}{\omega} \left[1 - \delta + \frac{\theta y'}{k'} \right] + \frac{\mu}{\omega} \xi = 1, \\
b : \quad & REM' \frac{\omega'}{\omega} \left(1 + \frac{\mu'}{\omega'} \right) + \frac{\mu}{\omega} \left(\frac{\xi R}{1+r} - 1 \right) = 1.
\end{aligned}$$

8.9 m08: $\xi \left(k' - \frac{b'}{1+r} \right) \geq k' - (1 - \delta)k + \varphi$

The borrowing constraint is

$$\xi \left(k' - \frac{b'}{1+r} \right) \geq i + \varphi.$$

The FOCs of a firm

$$\begin{aligned}
d : \quad & \omega = \frac{1}{\varphi_d}, \\
n : \quad & w = (1 - \theta) \frac{y}{n}, \\
k : \quad & Em' \frac{\omega'}{\omega} \left[1 - \delta + \left(1 - \frac{\mu'}{\omega'} \right) \frac{\theta y'}{k'} \right] - \frac{\mu}{\omega} \xi = 1, \\
b : \quad & (1 - \xi \mu) REM' \frac{\omega'}{\omega} + \frac{\mu}{\omega} + \frac{\mu \xi R}{\omega(1+r)} = 1.
\end{aligned}$$

8.10 m09: $\xi \left(k' - \frac{b'}{1+r} \right) \geq k' - (1 - \delta)k + b - \frac{b'}{R}$

The borrowing constraint is

$$\xi \left(k' - \frac{b'}{1+r} \right) \geq i + b - \frac{b'}{R}.$$

The FOCs of a firm:

$$\begin{aligned} d : \quad & \omega = \frac{1}{\varphi_d}, \\ n : \quad & w = (1 - \theta) \frac{y}{n}, \\ k : \quad & Em' \frac{\omega'}{\omega} \left[(1 - \delta) \left(1 + \frac{\mu'}{\varphi_d} \right) + \frac{\theta y'}{k'} \right] + \frac{\mu}{\omega} (\xi - 1) = 1, \\ b : \quad & REM' \frac{\omega'}{\omega} \left(1 + \frac{\mu'}{\omega'} \right) + \frac{\mu}{\omega} \left(\frac{\xi R}{1+r} - 1 \right) = 1. \end{aligned}$$

8.11 m10: $\xi \left(k' - \frac{b'}{1+r} \right) \geq \varphi + b - \frac{b'}{R}$

The borrowing constraint is

$$\xi \left(k' - \frac{b'}{1+r} \right) \geq \varphi + b - \frac{b'}{R}.$$

The FOCs of a firm:

$$\begin{aligned} d : \quad & \omega = \frac{1}{\varphi_d} - \mu, \\ n : \quad & w = (1 - \theta) \frac{y}{n}, \\ k : \quad & Em' \frac{\omega'}{\omega} \left[1 - \delta + \frac{\theta y'}{k'} \right] + \frac{\mu}{\omega} \xi = 1, \\ b : \quad & REM' \frac{\omega'}{\omega} \left(1 + \frac{\mu'}{\omega'} \right) + \frac{\mu}{\omega} \left(\frac{\xi R}{1+r} - 1 \right) = 1. \end{aligned}$$

8.12 m11: $\xi \left(k' - \frac{b'}{1+r} \right) \geq wn + k' - (1 - \delta)k + \varphi$

The borrowing constraint is

$$\xi \left(k' - \frac{b'}{1+r} \right) \geq wn + i + \varphi,$$

where $i \equiv k' - (1 - \delta)k$.

The FOCs of a firm

$$\begin{aligned}
d : \quad & \omega = \frac{1}{\varphi_d}, \\
n : \quad & w = \frac{1 - \theta}{1 + \frac{\mu}{\omega}} \frac{y}{n}, \\
k : \quad & Em' \frac{\omega'}{\omega} \left[(1 - \delta) \left(1 + \frac{\mu'}{\omega'} \right) + \frac{\theta y'}{k'} \right] - \frac{\mu}{\omega} \xi = 1, \\
b : \quad & (1 + \xi \mu) RE m' \frac{\omega'}{\omega} = 1.
\end{aligned}$$

8.13 m12: $\xi \left(k' - \frac{b'}{1+r} \right) \geq wn + k' - (1 - \delta)k + b - \frac{b'}{R}$

The borrowing constraint is

$$\xi \left(k' - \frac{b'}{1+r} \right) \geq wn + i + b - \frac{b'}{R}.$$

The FOCs of a firm:

$$\begin{aligned}
d : \quad & \omega = \frac{1}{\varphi_d} \\
n : \quad & w = (1 - \theta) \left(1 - \frac{\mu}{\omega} \right) \frac{y}{n}, \\
k : \quad & Em' \frac{\omega'}{\omega} \left[\left(1 - \delta + \frac{\theta y'}{k'} \right) \left(1 - \frac{\mu'}{\omega'} \right) \right] + (1 - \xi) \frac{\mu}{\omega} = 1, \\
b : \quad & RE m' \frac{\omega'}{\omega} + \xi \frac{\mu}{\omega} \frac{R}{(1+r)} = 1.
\end{aligned}$$

8.14 m13: $\xi \left(k' - \frac{b'}{1+r} \right) \geq wn + \varphi + b - \frac{b'}{R}$

The borrowing constraint is

$$\xi \left(k' - \frac{b'}{1+r} \right) \geq wn + \varphi + b - \frac{b'}{R}$$

The FOCs of a firm:

$$\begin{aligned}
d : \quad & \omega = \frac{1}{\varphi_d} - \mu, \\
n : \quad & w = \frac{1 - \theta}{1 + \frac{\mu}{\omega}} \frac{y}{n}, \\
k : \quad & Em' \frac{\omega'}{\omega} \left[1 - \delta + \frac{\theta y'}{k'} \right] + \frac{\mu}{\omega} \xi = 1, \\
b : \quad & RE m' \frac{\omega'}{\omega} \left(1 + \frac{\mu'}{\omega'} \right) + \frac{\mu}{\omega} \left(\frac{\xi R}{1+r} - 1 \right) = 1.
\end{aligned}$$

8.15 m14: $\xi \left(k' - \frac{b'}{1+r} \right) \geq k' - (1 - \delta)k + \varphi + b - \frac{b'}{R}$

The borrowing constraint is

$$\xi \left(k' - \frac{b'}{1+r} \right) \geq i + \varphi + b - \frac{b'}{R}.$$

The FOCs of a firm:

$$\begin{aligned} d : \quad & \omega = \frac{1}{\varphi_d}, \\ n : \quad & w = (1 - \theta) \frac{y}{n}, \\ k : \quad & Em' \frac{\omega'}{\omega} \left[1 - \delta + \left(1 - \frac{\mu'}{\varphi_d} \right) \frac{\theta y'}{k'} \right] + \frac{\mu}{\omega} \xi = 1, \\ b : \quad & REM' \frac{\omega'}{\omega} + \xi \frac{\mu}{\omega} \frac{R}{(1+r)} = 1. \end{aligned}$$

8.16 m15: $\xi \left(k' - \frac{b'}{1+r} \right) \geq y$

The borrowing constraint is

$$\xi \left(k' - \frac{b'}{1+r} \right) \geq y.$$

The FOCs of a firm:

$$\begin{aligned} d : \quad & \omega = \frac{1}{\varphi_d(d)}, \\ n : \quad & w = \frac{(1 - \theta) \left(1 - \frac{\mu}{\omega} \right) y}{n}, \\ k : \quad & Em' \frac{\omega'}{\omega} \left[1 - \delta + \left(1 - \frac{\mu'}{\omega'} \right) \frac{\theta y'}{k'} \right] + \xi \frac{\mu}{\omega} = 1, \\ b : \quad & REM' \frac{\omega'}{\omega} + \xi \frac{\mu}{\omega} \frac{R}{(1+r)} = 1. \end{aligned}$$

8.17 m16: $\xi k' \geq \frac{b'}{1+r}$

The borrowing constraint is

$$\xi k' \geq \frac{b'}{1+r}.$$

The FOCs of a firm:

$$\begin{aligned}
d : \quad & \omega = \frac{1}{\varphi_d(d)}, \\
n : \quad & w = \frac{(1-\theta)y}{n}, \\
k : \quad & Em' \frac{\omega'}{\omega} \left(1 - \delta + \frac{\theta y'}{k'} \right) + \xi \frac{\mu}{\omega} = 1, \\
b : \quad & REM' \frac{\omega'}{\omega} + \frac{\mu}{\omega} \frac{R}{(1+r)} = 1.
\end{aligned}$$

8.18 v1: $\xi Em'V' \geq wn$

The borrowing constraint is

$$\xi Em'V' \geq wn,$$

The FOCs of a firm

$$\begin{aligned}
d : \quad & \omega = \frac{1}{\varphi_d}, \\
n : \quad & w = \frac{1-\theta}{1+\frac{\mu}{\omega}n} \frac{y}{n}, \\
k : \quad & (1+\mu\xi)Em' \frac{\omega'}{\omega} \left[1 - \delta + \frac{\theta y'}{k'} \right] = 1, \\
b : \quad & (1+\mu\xi)REM' \frac{\omega'}{\omega} = 1.
\end{aligned}$$

8.19 v2: $\xi Em'V' \geq k' - (1-\delta)k$

The borrowing constraint is

$$\xi Em'V' \geq i,$$

where $i \equiv k' - (1-\delta)k$.

The FOCs of a firm

$$\begin{aligned}
d : \quad & \omega = \frac{1}{\varphi_d}, \\
n : \quad & w = (1-\theta) \frac{y}{n}, \\
k : \quad & (1+\mu\xi)Em' \frac{\omega'}{\omega} \left[(1-\delta) \left(1 + \frac{\mu'}{\omega'} \right) + \frac{\theta y'}{k'} \right] - \frac{\mu}{\omega} = 1, \\
b : \quad & (1+\mu\xi)REM' \frac{\omega'}{\omega} = 1.
\end{aligned}$$

8.20 v3: $\xi Em'V' \geq \varphi$

The borrowing constraint is

$$\xi Em'V' \geq \varphi.$$

The FOCs of a firm:

$$\begin{aligned} d : \quad & \omega = \frac{1}{\varphi_d} - \mu, \\ n : \quad & w = (1 - \theta) \frac{y}{n}, \\ k : \quad & (1 + \mu\xi) Em' \frac{\omega'}{\omega} \left[1 - \delta + \frac{\theta y'}{k'} \right] = 1, \\ b : \quad & (1 + \mu\xi) REM' \frac{\omega'}{\omega} = 1. \end{aligned}$$

8.20.1 Steady state

$$\begin{aligned} \text{BC} : \quad & \xi = \frac{1 - \beta}{\beta} = r^* \\ d : \quad & \omega^* = 1 - \mu^*, \\ n : \quad & w^* = (1 - \theta) \frac{y^*}{n^*}, \\ k : \quad & k^* = \left[\frac{(1 + \mu^* \xi^*) \theta \beta z^*}{1 - \beta(1 - \delta)(1 + \mu^* \xi^*)} \right]^{\frac{1}{1 - \theta}} n^*, \\ b : \quad & \mu^* = \frac{1}{\xi^*} \left(\frac{1}{\beta R^*} - 1 \right). \end{aligned}$$

In this model, b and d can not be identified.

8.21 v4: $\xi Em'V' \geq b - \frac{b'}{R}$

The borrowing constraint is

$$\xi Em'V' \geq b - \frac{b'}{R}.$$

The FOCs of a firm:

$$\begin{aligned} d : \quad & \omega = \frac{1}{\varphi_d}, \\ n : \quad & w = (1 - \theta) \frac{y}{n}, \\ k : \quad & (1 + \mu\xi) Em' \frac{\omega'}{\omega} \left[1 - \delta + \frac{\theta y'}{k'} \right] = 1, \\ b : \quad & (1 + \mu\xi) REM' \frac{\omega'}{\omega} \left(1 + \frac{\mu'}{\omega'} \right) - \frac{\mu}{\omega} = 1. \end{aligned}$$

8.22 v5: $\xi Em'V' \geq wn + i$

The borrowing constraint is

$$\xi Em'V' \geq wn + k' - (1 - \delta)k.$$

The FOCs of a firm:

$$\begin{aligned} d : \quad & \omega = \frac{1}{\varphi_d} \\ n : \quad & w = \frac{1 - \theta y}{1 + \frac{\mu}{\omega} n}, \\ k : \quad & (1 + \mu\xi)Em'\frac{\omega'}{\omega} \left[(1 - \delta) \left(1 + \frac{\mu'}{\varphi_d} \right) + \frac{\theta y'}{k'} \right] + \frac{\mu}{\omega} = 1, \\ b : \quad & (1 + \mu\xi)REm'\frac{\omega'}{\omega} = 1. \end{aligned}$$

8.23 v6: $\xi Em'V' \geq wn + \varphi$

The borrowing constraint is

$$\xi Em'V' \geq wn + \varphi.$$

The FOCs of a firm:

$$\begin{aligned} d : \quad & \omega = \frac{1}{\varphi_d} - \mu, \\ n : \quad & w = \frac{1 - \theta y}{1 + \frac{\mu}{\omega} n}, \\ k : \quad & (1 + \mu\xi)Em'\frac{\omega'}{\omega} \left[1 - \delta + \frac{\theta y'}{k'} \right] = 1, \\ b : \quad & (1 + \mu\xi)REm'\frac{\omega'}{\omega} = 1. \end{aligned}$$

8.24 v7: $\xi Em'V' \geq wn + b - \frac{b'}{R}$

The borrowing constraint is

$$\xi Em'V' \geq wn + b - \frac{b'}{R}.$$

The FOCs of a firm:

$$\begin{aligned}
d : \quad & \omega = \frac{1}{\varphi_d}, \\
n : \quad & w = \frac{1 - \theta}{1 + \frac{\mu}{\omega}} \frac{y}{n}, \\
k : \quad & (1 + \mu\xi)Em' \frac{\omega'}{\omega} \left[1 - \delta + \frac{\theta y'}{k'} \right] = 1, \\
b : \quad & (1 + \mu\xi)REm' \frac{\omega'}{\omega} \left(1 + \frac{\mu'}{\omega'} \right) - \frac{\mu}{\omega} = 1.
\end{aligned}$$

8.25 v8: $\xi Em'V' \geq i + \varphi$

The borrowing constraint is

$$\xi Em'V' \geq i + \varphi.$$

The FOCs of a firm

$$\begin{aligned}
d : \quad & \omega = \frac{1}{\varphi_d} - \mu, \\
n : \quad & w = (1 - \theta) \frac{y}{n}, \\
k : \quad & (1 + \mu\xi)Em' \frac{\omega'}{\omega} \left[(1 - \delta) \left(1 - \frac{\mu'}{\omega'} \right) + \frac{\theta y'}{k'} \right] - \frac{\mu}{\omega} = 1, \\
b : \quad & (1 + \mu\xi)REm' \frac{\omega'}{\omega} = 1.
\end{aligned}$$

8.26 v9: $\xi Em'V' \geq i + b - \frac{b'}{R}$

The borrowing constraint is

$$\xi Em'V' \geq i + b - \frac{b'}{R}.$$

The FOCs of a firm:

$$\begin{aligned}
d : \quad & \omega = \frac{1}{\varphi_d}, \\
n : \quad & w = (1 - \theta) \frac{y}{n}, \\
k : \quad & (1 + \mu\xi)Em' \frac{\omega'}{\omega} \left[(1 - \delta) \left(1 + \frac{\mu'}{\varphi_d} \right) + \frac{\theta y'}{k'} \right] + \frac{\mu}{\omega} = 1, \\
b : \quad & (1 + \mu\xi)REm' \frac{\omega'}{\omega} \left(1 + \frac{\mu'}{\omega'} \right) - \frac{\mu}{\omega} = 1.
\end{aligned}$$

8.27 v10: $\xi Em'V' \geq \varphi + b - \frac{b'}{R}$

The borrowing constraint is

$$\xi Em'V' \geq \varphi + b - \frac{b'}{R}.$$

The FOCs of a firm:

$$\begin{aligned} d : \quad & \omega = \frac{1}{\varphi_d} - \mu, \\ n : \quad & w = (1 - \theta) \frac{y}{n}, \\ k : \quad & (1 + \mu\xi) Em' \frac{\omega'}{\omega} \left[1 - \delta + \frac{\theta y'}{k'} \right] = 1, \\ b : \quad & (1 + \mu\xi) REM' \frac{\omega'}{\omega} - \frac{\mu}{\omega} = 1. \end{aligned}$$

8.28 v11: $\xi Em'V' \geq wn + i + \varphi$

The borrowing constraint is

$$\xi Em'V' \geq wn + i + \varphi,$$

The FOCs of a firm

$$\begin{aligned} d : \quad & \omega = \frac{1}{\varphi_d} - \mu, \\ n : \quad & w = \frac{1 - \theta}{1 + \frac{\mu}{\omega}} \frac{y}{n}, \\ k : \quad & (1 + \mu\xi) Em' \frac{\omega'}{\omega} \left[(1 - \delta) \left(1 - \frac{\mu'}{\omega'} \right) + \frac{\theta y'}{k'} \right] - \frac{\mu}{\omega} = 1, \\ b : \quad & (1 + \xi\mu) REM' \frac{\omega'}{\omega} = 1. \end{aligned}$$

8.29 v12: $\xi Em'V' \geq wn + i + b - \frac{b'}{R}$

The borrowing constraint is

$$\xi Em'V' \geq wn + i + b - \frac{b'}{R}.$$

The FOCs of a firm:

$$\begin{aligned}
d : \quad & \omega = \frac{1}{\varphi_d} \\
n : \quad & w = \frac{1 - \theta}{1 + \frac{\mu}{\omega} n} y \\
k : \quad & (1 + \mu\xi)Em' \frac{\omega'}{\omega} \left[(1 - \delta) \left(1 + \frac{\mu'}{\varphi_d} \right) + \frac{\theta y'}{k'} \right] + \frac{\mu}{\omega} = 1, \\
b : \quad & (1 + \mu\xi)REm' \frac{\omega'}{\omega} \left(1 + \frac{\mu'}{\omega'} \right) - \frac{\mu}{\omega} = 1.
\end{aligned}$$

8.30 v13: $\xi Em'V' \geq wn + \varphi + b - \frac{b'}{R}$

The borrowing constraint is

$$\xi Em'V' \geq wn + \varphi + b - \frac{b'}{R}$$

The FOCs of a firm:

$$\begin{aligned}
d : \quad & \omega = \frac{1}{\varphi_d} - \mu, \\
n : \quad & w = \frac{1 - \theta}{1 + \frac{\mu}{\omega} n} y \\
k : \quad & Em' \frac{\omega'}{\omega} \left[1 - \delta + \frac{\theta y'}{k'} \right] + \frac{\mu}{\omega} \xi = 1, \\
b : \quad & REm' \frac{\omega'}{\omega} \left(1 + \frac{\mu'}{\omega'} \right) + \frac{\mu}{\omega} \left(\frac{\xi R}{1 + r} - 1 \right) = 1.
\end{aligned}$$

8.31 v14: $\xi Em'V' \geq k' - (1 - \delta)k + \varphi + b - \frac{b'}{R}$

The borrowing constraint is

$$\xi Em'V' \geq i + \varphi + b - \frac{b'}{R}.$$

The FOCs of a firm:

$$\begin{aligned}
d : \quad & \omega = \frac{1}{\varphi_d}, \\
n : \quad & w = (1 - \theta) \frac{y}{n}, \\
k : \quad & (1 + \mu\xi)Em' \frac{\omega'}{\omega} \left[(1 - \delta) \left(1 + \frac{\mu'}{\varphi_d} \right) + \frac{\theta y'}{k'} \right] + \frac{\mu}{\omega} = 1, \\
b : \quad & (1 + \mu\xi)REm' \frac{\omega'}{\omega} = 1.
\end{aligned}$$

8.32 v15: $\xi Em'V' \geq y$

The borrowing constraint is

$$\xi Em'V' \geq y.$$

The FOCs of a firm:

$$\begin{aligned} d : \quad & \omega = \frac{1}{\varphi_d(d)}, \\ n : \quad & w = \frac{(1-\theta)(1-\frac{\mu}{\omega})y}{n}, \\ k : \quad & (1+\mu\xi)Em'\frac{\omega'}{\omega} \left\{ 1-\delta + \left[1-\frac{\mu'}{\omega'} \right] \frac{\theta y'}{k'} \right\} = 1, \\ b : \quad & (1+\xi\mu)REm'\frac{\omega'}{\omega} = 1. \end{aligned}$$

8.33 v16: $\xi Em'V' \geq \frac{b'}{1+r}$

The borrowing constraint is

$$\xi Em'V' \geq \frac{b'}{1+r}.$$

The FOCs of a firm:

$$\begin{aligned} d : \quad & \omega = \frac{1}{\varphi_d(d)}, \\ n : \quad & w = \frac{(1-\theta)y}{n}, \\ k : \quad & (1+\mu\xi)Em'\frac{\omega'}{\omega} \left\{ 1-\delta + \frac{\theta y'}{k'} \right\} = 1, \\ b : \quad & (1+\xi\mu)REm'\frac{\omega'}{\omega} + \frac{\mu R}{(1+r)\omega} = 1. \end{aligned}$$

9 Appendix: Business cycle properties

In this section, we investigate the models by their ability to replicate the main statistical features of US business cycles. First, we construct the TFP and financial conditions ξ . ξ vary among models because ξ is constructed by each borrowing constraint. Second, we calibrate the productivity shocks and financial shocks of every model. Third, using the constructed series of shocks, we calculate the stochastic simulation to compare the ability to replicate actual data. Thus, it becomes clear that some models can replicate the usual business cycle properties; we focus on these models in Section 4.

9.1 Constructing TFP z and financial conditions ξ

TFP is constructed as the standard Solow residuals. The production function is assumed to be the Cobb–Douglas function, and we log-linearize around the steady state. The log-linearized production function is

$$\tilde{y} = \tilde{z} + \theta\tilde{k} + (1 - \theta)\tilde{n},$$

where $\tilde{x} = (\ln x - \ln x^*)/\ln x^*$ and x^* is the steady-state value of the corresponding variable. The Solow residuals can be taken from

$$\tilde{z} = \tilde{y} - \theta\tilde{k} - (1 - \theta)\tilde{n}.$$

When constructing the Solow residuals, \tilde{y} , \tilde{k} and \tilde{n} are the used data in logs linearly detrended over the period 1984:I-2010:II. Data definition depends on Jermann and Quadrini (2012b)¹³. All data are in real terms and linearly detrended over the period 1984:I-2010:II. The calibration target is also 1984:I-2010:II.

Financial conditions ξ are also constructed similarly with the Solow residuals using borrowing constraints. In this paper, we consider various forms of borrowing constraints; this means that the observed financial conditions and financial shocks vary among models. To understand this clearly, we construct financial conditions using the m15 model. The first-order log-linear approximation on the borrowing constraint of m15 around the steady state is

$$\tilde{\xi} = \tilde{y} - \frac{\xi^*k^*}{y^*}\tilde{k}' + \frac{\beta\xi^*b^*}{y^*}\tilde{b}^e,$$

where $b^e \equiv b'/(1 + r)$. The series of $\tilde{\xi}$ are also taken as residuals. As for all m-models, the log-linearized borrowing constraints are listed in Appendix 10.1, and we obtain $\tilde{\xi}$ in the same way. Following the working paper version of JQ,¹⁴ the v-models are also the equations used in Appendix 10.1 for each corresponding model between the m-models and v-models, because it is difficult to choose data for \tilde{V} in the aggregate level in v-models. For example, when we conduct numerical simulation for v05, we use the financial shocks of m05. Note that the steady-state values of the variables are different between the m-models and v-models. Thus, the constructed series of financial conditions in the m05 model are different from v05.

Figures 5 and 6 plot the constructed series of financial shocks e_ξ for each model. The figures also plot the ‘‘Credit-Standard.’’ Credit-Standard represents the net tightening of credit standards for the United States; a positive value represents the ratio of banks that tightened their credit standards over the past three months. We compare the financial shocks

¹³The dataset is available on JQ’s online appendix. URL: <http://dx.doi.org/10.1257/aer.102.1.238>.

¹⁴See Jermann and Quadrini (2009) p.16 footnote 4.

Parameter	Economic interpretation	value
β	the subjective discount factor	0.9825
δ	the depreciation rate of capital	0.025
τ	the tax advantage for debt	0.35
θ	the share of capital in production	0.36
ρ_z	the persistence of productivity	0.9378
σ_z	the standard deviation of productivity	0.0045

	m01	m02	m04	m05	m06	m07	m08	m09	m10	m11	m12	m13	m14	m15	m16
α	1.86	1.89	2.08	1.87	1.88	1.86	1.81	2.08	1.92	1.78	1.87	1.77	1.92	1.88	1.92
ξ^*	0.09	0.04	0.01	0.14	0.11	0.10	0.09	0.01	0.12	0.25	0.14	0.20	0.06	0.16	0.37
κ	0.07	0.11	100	0.13	0.00	0.00	0.00	100	100	0.04	0.00	0.07	0.29	0.05	0.00
ρ_ξ	0.98	0.93	0.99	0.97	0.97	0.91	0.95	0.99	0.98	0.98	0.97	0.96	0.88	0.97	0.99
σ_ξ	0.01	0.05	0.71	0.02	0.01	0.02	0.03	0.22	0.10	0.01	0.01	0.01	0.02	0.01	0.01

	v01	v02	v04	v05	v06	v07	v08	v09	v10	v11	v12	v13	v14	v15	v16
α	1.86	1.88	1.98	1.86	1.88	1.87	1.91	1.90	1.98	1.88	1.87	1.88	1.93	1.88	1.92
ξ^*	0.09	0.03	0.01	0.13	0.12	0.10	0.06	0.04	0.03	0.15	0.14	0.13	0.07	0.16	0.59
κ	0.06	0.09	4.95	0.09	0.00	0.00	0.00	0.00	0.79	0.06	0.00	2.64	0.09	0.05	0.00
ρ_ξ	0.98	0.93	0.90	0.97	0.97	0.91	0.93	0.86	0.91	0.97	0.98	0.92	0.88	0.97	0.99
σ_ξ	0.01	0.06	0.19	0.02	0.01	0.02	0.03	0.03	0.10	0.01	0.01	0.01	0.02	0.01	0.01

Table 7: Parameter settings

and the actual credit standards data to investigate each financial shocks and indicate the credit tightness reasonably well. We also calculate the correlation between (sign reversed) financial shocks and the Credit-Standard. The financial shocks of m04, m07, m09, m10, m13, m16, v04, v07, v10, v13, and v16 are negatively correlated with actual data and do not replicate the actuals. Other financial shocks indicate fairly good results to explain actual data.

9.2 Calibration

In this section, we recalibrate five parameters, utility parameter α , enforcement parameter ξ^* , adjustment cost of equity payout κ , and the parameters for the stochastic process of financial conditions ξ , ρ_ξ and σ_ξ . Parameters κ , ρ_ξ , and σ_ξ depend on the constructed series of financial shocks, which are different for each model. α is chosen to match the steady-state working

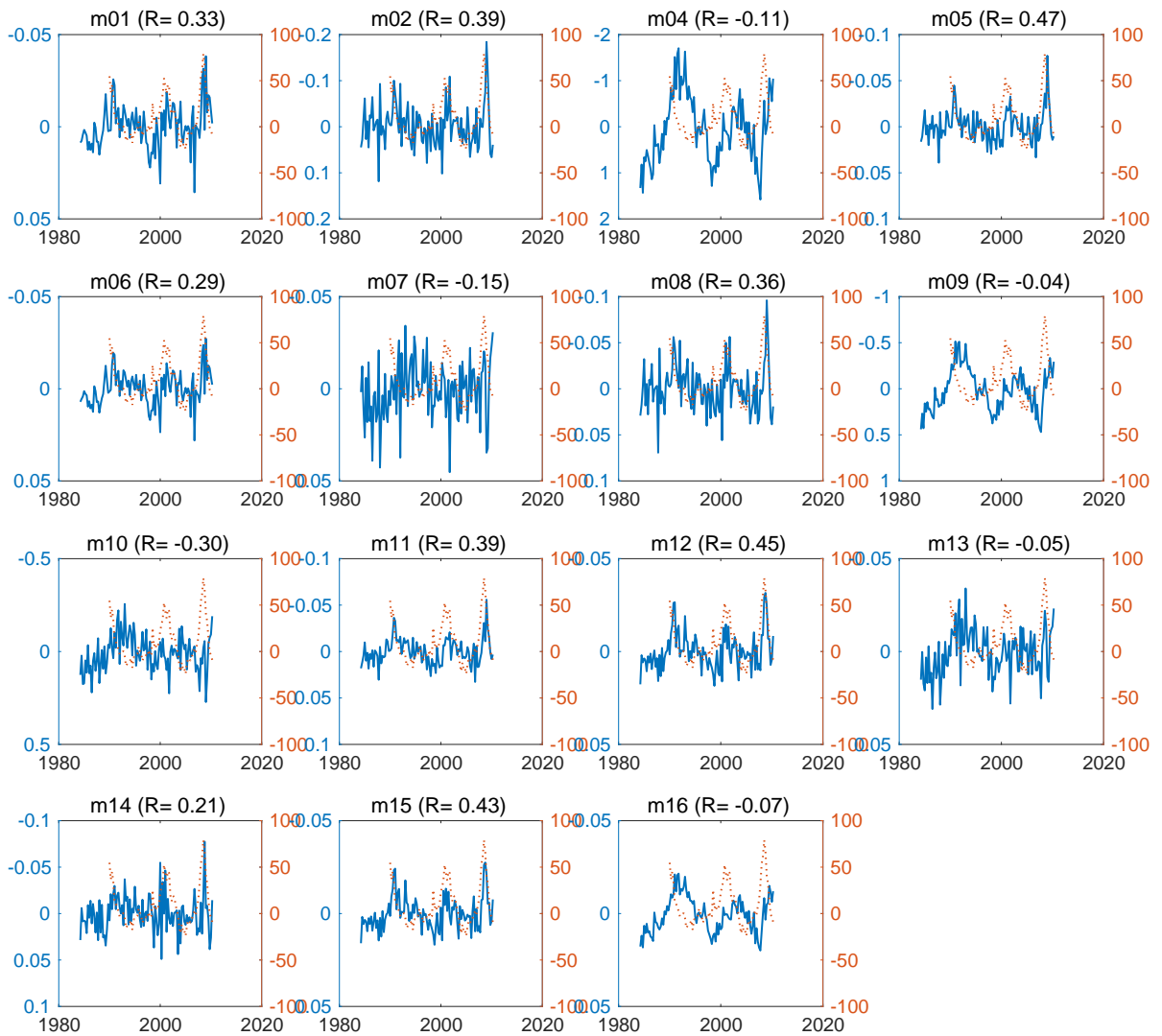


Figure 5: Financial conditions of m-models

Note: $R \equiv \text{corr}(-e_\xi, \text{Credit-Standard})$. Credit-Standard represents the net tightening of credit standards for the United States (Firm size: large and medium).

Source: Credit-Standard: Federal Reserve Board, *Senior Loan Officer Opinion Survey on Bank Lending Practices*.

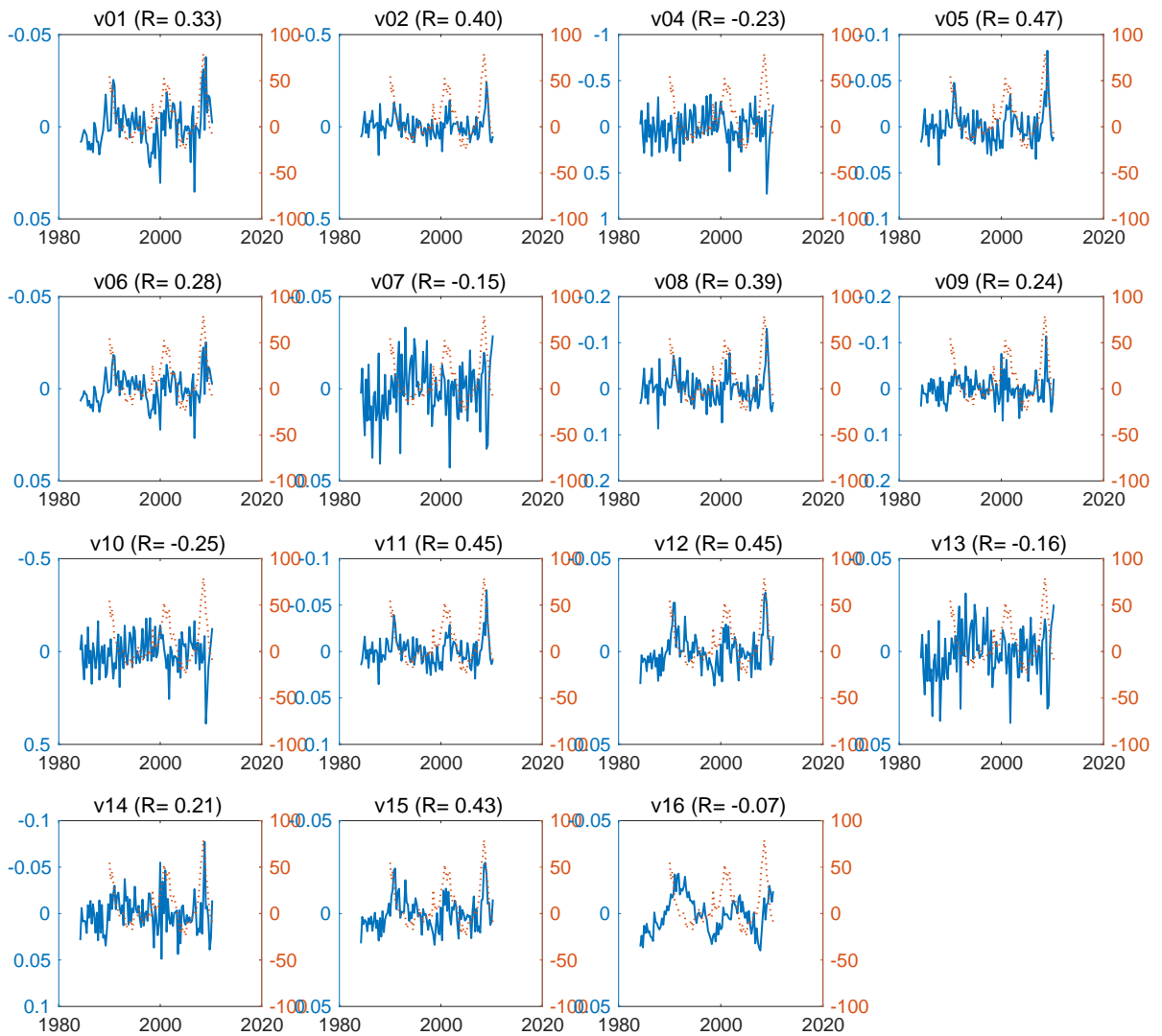


Figure 6: Financial conditions of v-models

Note: $R \equiv \text{corr}(-e_{\xi}, \text{Credit-Standard})$. Credit-Standard represents the net tightening of credit standards for the United States (Firm size: large and medium).

Source: Credit-Standard: Federal Reserve Board, *Senior Loan Officer Opinion Survey on Bank Lending Practices*.

	c^*	w^*	k^*	μ^*	v^*	y^*	$\frac{b^*}{y^*} \left(1 - \frac{1}{R^*}\right)$	d^*/y^*	b^*/y^*	i^*	LW^*	IW^*
m01	0.82	2.17	10.70	0.07	7.18	1.09	0.04	0.12	3.36	0.27	0.94	1.17
m02	0.78	2.12	8.32	0.15	6.35	0.99	0.04	0.11	3.36	0.21	1.00	1.00
m04	0.84	2.49	13.12	0.71	0.41	1.17	0.07	0.01	6.40	0.33	1.00	1.33
m05	0.81	2.16	9.89	0.04	6.90	1.06	0.04	0.11	3.36	0.25	0.96	1.11
m06	0.82	2.19	10.70	0.05	6.79	1.09	0.04	0.11	3.36	0.27	0.95	1.17
m07	0.82	2.18	10.90	0.07	7.10	1.09	0.04	0.11	3.36	0.27	0.94	1.19
m08	0.74	1.90	6.16	0.06	7.56	0.89	0.04	0.15	3.36	0.15	1.00	0.82
m09	0.84	2.49	13.12	0.71	0.41	1.17	0.07	0.01	6.40	0.33	1.00	1.33
m10	0.80	2.19	9.15	0.01	0.74	1.03	0.12	0.01	7.89	0.23	1.00	1.06
m11	0.75	1.89	6.53	0.02	8.19	0.91	0.04	0.16	3.36	0.16	0.98	0.85
m12	0.81	2.17	10.00	0.04	6.84	1.06	0.04	0.11	3.36	0.25	0.96	1.12
m13	0.75	1.90	6.71	0.03	8.30	0.92	0.04	0.16	3.36	0.17	0.97	0.87
m14	0.80	2.19	9.14	0.10	5.80	1.03	0.04	0.10	3.36	0.23	1.00	1.06
m15	0.81	2.18	10.07	0.04	6.62	1.06	0.04	0.11	3.36	0.25	0.96	1.13
m16	0.80	2.19	9.12	0.01	5.82	1.03	0.04	0.10	3.36	0.23	1.00	1.06

Table 8: Steady-state values in m-models

hours equal to 0.3. ξ^* is the steady-state level of financial conditions ξ and targets the debt-to-output ratio (nonfinancial business sector) equal to 3.36 on average between 1984:I-2010:II. κ does not appear in the steady state and only appear in the transition dynamics. Only κ cannot be set with a steady-state target. Therefore, κ is chosen to match the standard deviation of equity payout-to-output ratio d/y between data and model. κ is discussed more in Section 9.3.

Next, we construct productivity shocks and financial shocks. We assume that z and ξ follow the independent AR(1) process, respectively:¹⁵

$$\begin{aligned}\ln z' &= \rho_z \ln z + (1 - \rho_z) \ln \bar{z} + e_z, \\ \ln \xi' &= \rho_\xi \ln \xi + (1 - \rho_\xi) \ln \bar{\xi} + e_\xi.\end{aligned}$$

ρ_ξ are estimated by least squares for each model. ρ_z are common for all models. Thus, the

¹⁵JQ assumes that z and ξ follow the VAR(1)

$$\begin{bmatrix} z' \\ \xi' \end{bmatrix} = \mathbf{A} \begin{bmatrix} z \\ \xi \end{bmatrix} + \begin{bmatrix} e'_z \\ e'_\xi \end{bmatrix}$$

process and this setting implies that z and ξ mutually affect each other. The structural model does not show why the two shocks mutually affect each other.

	c^*	w^*	k^*	μ^*	v^*	y^*	$\frac{b^*}{y^*} \left(1 - \frac{1}{R^*}\right)$	d^*/y^*	b^*/y^*	i^*	LW^*	IW^*	
v01	0.82	2.17	10.69	0.07	7.16	1.09		0.04	0.12	3.36	0.27	0.94	1.17
v02	0.78	2.10	8.10	0.19	6.48	0.98		0.04	0.12	3.36	0.20	1.00	0.98
v04	0.82	2.32	10.69	0.68	4.68	1.09		0.04	0.08	3.36	0.27	1.00	1.17
v05	0.81	2.16	9.94	0.05	6.98	1.06		0.04	0.12	3.36	0.25	0.95	1.12
v06	0.82	2.19	10.69	0.05	6.78	1.09		0.04	0.11	3.36	0.27	0.95	1.17
v07	0.82	2.18	10.69	0.06	6.94	1.09		0.04	0.11	3.36	0.27	0.94	1.17
v08	0.79	2.17	8.90	0.11	5.96	1.02		0.04	0.10	3.36	0.22	1.00	1.04
v09	0.79	2.15	8.64	0.15	6.14	1.01		0.04	0.11	3.36	0.22	1.00	1.02
v10	0.82	2.32	10.69	0.23	4.68	1.09		0.04	0.08	3.36	0.27	1.00	1.17
v11	0.81	2.17	10.03	0.04	6.72	1.06		0.04	0.11	3.36	0.25	0.96	1.12
v12	0.81	2.17	9.99	0.04	6.83	1.06		0.04	0.11	3.36	0.25	0.96	1.12
v13	0.82	2.20	10.69	0.05	6.63	1.09		0.04	0.11	3.36	0.27	0.95	1.17
v14	0.80	2.20	9.31	0.09	5.69	1.03		0.04	0.10	3.36	0.23	1.00	1.07
v15	0.81	2.18	10.06	0.04	6.61	1.06		0.04	0.11	3.36	0.25	0.96	1.13
v16	0.80	2.19	9.12	0.00	5.82	1.03		0.04	0.10	3.36	0.23	1.00	1.06

Table 9: Steady-state values in v-models

series of productivity shocks e_z and financial shocks e_ξ are constructed. In the following, the simulation, e_z , and e_ξ are treated as exogenously given.

Table 7 reports the values of the calibrated parameters.

Tables 8–9 report the steady-state values for all models. Table 8 shows that the debt-to-output ratio b^*/y^* of m04, m09, and m10 do not match the observed ratio. For m04 and m09, when calibrating the steady-state financial condition by trying to match the observed debt-to-output ratio, the steady-state equity payout becomes negative. We assume that the equity payout is positive in the steady state, and so ξ^* is chosen to have a positive value for equity payout in the steady state. In model m10, $\xi^* = 0.12$ is the argument of the minima of b^*/y^* , that is, $\xi^* = \operatorname{argmin}_{\xi^*} f\left(\frac{b^*}{y^*}\right) = 0.12$, and b^*/y^* cannot decrease any more.

On the other hand, Table 9 shows that all v-models match the observed debt-to-output ratio.

9.3 Simulation results

In this subsection, we again conduct the stochastic simulation already explained in Section 4.2. Instead of using the shocks generated by normal distribution, we use the shocks constructed

in Section 9.1.

Tables 10–11 report the standard deviation for actual data and model-generated series.¹⁶ The adjustment cost of equity payout κ is calibrated to match the standard deviation of the equity payout/output ratio between the actual data and generated data by each model. However, this cannot match models m04, m08–m10, m12, m14, m16, v07–v10, v12, and v16.

Tables 10–11 show that the models including credit-constrained debt repurchase m04, m07, m09, m10, v10, v13, and v16 generate too volatile macro aggregate output, consumption, and investment. Note that the standard deviation of investment for models v04, v08, v10, v11, and v16 are almost zero. Other models show roughly similar results with the standard RBC model. Most models do not generate volatility of working hours. McGrattan and Prescott (2010) emphasize that the basic neoclassical growth model greatly understates the 1990s boom. To explain this, we need to include some kind of mechanism to explain the 1990s boom. For example, Jermann and Quadrini (2007) propose that the 1990s boom can be explained by the stock market boom.

Tables 12–13 report the correlation between output and various other variables. In models m04, m09, and m10, the correlation of consumption with output is negative. In models v01, v04, v07, v10, and v11, the correlation of investment is negative. In model m07, the correlation of consumption and investment is very weak and working hours are negatively correlated with output. In addition, in models m02, m05, v02, v04, v10, and v13, consumption is too weakly correlated with output. These features are undesirable for a model to replicate business cycle fluctuations.

To summarize these calibration and simulation results, we conclude that m04, v04, m07, v07, m09, m10, v10, m13, v13, m16, and v16 are irrelevant to replicate actual business cycles. However, the Kiyotaki–Moore borrowing constraints (m16, v16) are also irrelevant, but we do not exclude them when comparing other models. Therefore, in Section 4, we exclude the m04, v04, m07, v07, m09, m10, v10, m13, and v13 models.

¹⁶Note that the equity payout/output ratio and debt repurchase/output ratio represent percentages. To represent the percentage *point* deviation from their steady-state values, these two variables are defined as follows:

$$\begin{aligned} \text{Equity-payout} &\equiv \text{EquPay} = (\tilde{d} - \tilde{y}) \frac{d^*}{y^*}, \\ \text{Debt-repurchase} &\equiv \text{DebtRep} = \left\{ \frac{R^*}{R^* - 1} \left[\tilde{b} - \frac{1}{R^*} (\tilde{b}' - \tilde{R}) \right] - \tilde{y} \right\} \frac{b^*}{y^*} \left(1 - \frac{1}{R^*} \right). \end{aligned}$$

These expressions denote the percentage point deviation from the steady state and the financial flows of tables are calculated using the above equations.

Constrained var.		y	c	i	n	$EquPay$	$DebtRep$	TFP	w
Data		0.022	0.023	0.120	0.033	0.029	0.043	0.013	0.024
RBC		0.021	0.011	0.067	0.009	0.010	NaN	0.013	0.014
m01	wn	0.019	0.010	0.055	0.013	0.029	0.039	0.013	0.013
m02	i	0.019	0.021	0.097	0.016	0.029	0.044	0.013	0.018
m04	$b - \frac{b'}{R}$	0.845	1.014	4.921	1.142	0.742	0.445	0.013	0.613
m05	$wn + i$	0.018	0.014	0.077	0.018	0.029	0.046	0.013	0.012
m06	$wn + \varphi$	0.021	0.012	0.062	0.009	0.022	0.021	0.013	0.014
m07	$wn + b - \frac{b'}{R}$	0.022	0.033	0.080	0.013	0.008	0.011	0.013	0.036
m08	$i + \varphi$	0.021	0.012	0.077	0.009	0.025	0.020	0.013	0.014
m09	$i + b - \frac{b'}{R}$	0.269	0.319	1.562	0.362	0.237	0.139	0.013	0.192
m10	$\varphi + b - \frac{b'}{R}$	0.054	0.065	0.353	0.063	0.034	0.040	0.013	0.047
m11	$wn + i + \varphi$	0.020	0.012	0.066	0.007	0.029	0.030	0.013	0.014
m12	$wn + i + b - \frac{b'}{R}$	0.027	0.015	0.103	0.034	0.009	0.032	0.013	0.020
m13	$wn + \varphi + b - \frac{b'}{R}$	0.022	0.014	0.073	0.016	0.029	0.041	0.013	0.020
m14	$i + \varphi + b - \frac{b'}{R}$	0.020	0.013	0.077	0.012	0.110	0.109	0.013	0.014
m15	$wn + i + \varphi + b - \frac{b'}{R}$	0.020	0.011	0.065	0.013	0.029	0.040	0.013	0.013
m16	$b'/(1+r)$	0.021	0.012	0.068	0.010	0.033	0.031	0.013	0.014

Table 10: Standard deviation

Constrained var.		y	c	i	n	$EquPay$	$DebtRep$	TFP	w
Data		0.022	0.023	0.120	0.033	0.029	0.043	0.013	0.024
RBC		0.021	0.011	0.067	0.009	0.010	NaN	0.013	0.014
v01	wn	0.018	0.010	0.012	0.012	0.029	0.038	0.013	0.013
v02	i	0.022	0.023	0.107	0.017	0.029	0.044	0.013	0.019
v04	$b - \frac{b'}{R}$	0.022	0.022	0.000	0.021	0.029	0.011	0.013	0.017
v05	$wn + i$	0.018	0.013	0.059	0.014	0.029	0.040	0.013	0.013
v06	$wn + \varphi$	0.021	0.012	0.062	0.009	0.022	0.021	0.013	0.014
v07	$wn + b - \frac{b'}{R}$	0.024	0.062	0.145	0.026	0.011	0.013	0.013	0.069
v08	$i + \varphi$	0.021	0.012	0.000	0.009	0.026	0.024	0.013	0.014
v09	$i + b - \frac{b'}{R}$	0.022	0.013	0.079	0.011	0.008	0.007	0.013	0.014
v10	$\varphi + b - \frac{b'}{R}$	0.049	0.052	0.000	0.049	0.040	0.025	0.013	0.039
v11	$wn + i + \varphi$	0.019	0.012	0.003	0.008	0.029	0.031	0.013	0.013
v12	$wn + i + b - \frac{b'}{R}$	0.022	0.012	0.075	0.022	0.007	0.026	0.013	0.015
v13	$wn + \varphi + b - \frac{b'}{R}$	0.180	0.040	0.692	0.279	0.029	0.291	0.013	0.135
v14	$i + \varphi + b - \frac{b'}{R}$	0.023	0.013	0.087	0.014	0.029	0.041	0.013	0.014
v15	$wn + i + \varphi + b - \frac{b'}{R}$	0.020	0.011	0.063	0.012	0.029	0.039	0.013	0.013
v16	$b'/(1+r)$	0.021	0.012	0.000	0.010	0.022	0.019	0.013	0.014

Table 11: Standard deviation

	Constrained var.	y	c	i	n	$EquPay$	$DebtRep$	TFP	w
Data		1.000	0.886	0.643	0.721	0.199	-0.439	0.205	0.302
RBC		1.000	0.830	0.933	0.867	-0.867	NaN	0.993	0.944
m01	wn	1.000	0.765	0.927	0.585	0.146	-0.318	0.865	0.878
m02	i	1.000	0.361	0.648	0.509	-0.060	-0.171	0.681	0.628
m04	$b - \frac{b'}{R}$	1.000	-0.536	0.895	0.851	-0.905	-0.738	-0.196	-0.208
m05	$wn + i$	1.000	0.347	0.821	0.619	0.307	-0.452	0.670	0.796
m06	$wn + \varphi$	1.000	0.761	0.928	0.867	-0.411	0.006	0.995	0.911
m07	$wn + b - \frac{b'}{R}$	1.000	0.628	0.302	0.679	-0.687	-0.200	0.993	0.672
m08	$i + \varphi$	1.000	0.857	0.925	0.865	-0.490	0.173	0.985	0.952
m09	$i + b - \frac{b'}{R}$	1.000	-0.544	0.898	0.855	-0.907	-0.743	-0.169	-0.214
m10	$\varphi + b - \frac{b'}{R}$	1.000	-0.127	0.767	0.692	-0.860	-0.471	0.086	0.222
m11	$wn + i + \varphi$	1.000	0.888	0.922	0.834	-0.230	-0.024	0.961	0.959
m12	$wn + i + b - \frac{b'}{R}$	1.000	0.453	0.911	0.788	-0.336	-0.748	0.479	0.930
m13	$wn + \varphi + b - \frac{b'}{R}$	1.000	0.877	0.905	0.739	0.168	-0.421	0.896	0.910
m14	$i + \varphi + b - \frac{b'}{R}$	1.000	0.523	0.858	0.759	0.081	-0.177	0.944	0.809
m15	$wn + i + \varphi + b - \frac{b'}{R}$	1.000	0.742	0.932	0.723	0.309	-0.480	0.900	0.905
m16	$b'/(1+r)$	1.000	0.784	0.925	0.849	-0.384	0.106	0.988	0.928

Table 12: Correlation

Constrained var.		y	c	i	n	$EquPay$	$DebtRep$	TFP	w
Data		1.000	0.886	0.643	0.721	0.199	-0.439	0.205	0.302
RBC		1.000	0.830	0.933	0.867	-0.867	NaN	0.993	0.944
v01	wn	1.000	0.773	-0.034	0.575	0.081	-0.259	0.890	0.877
v02	i	1.000	0.392	0.657	0.517	-0.181	-0.111	0.678	0.655
v04	$b - \frac{b'}{R}$	1.000	0.055	-0.721	0.689	-0.572	0.048	0.405	0.437
v05	$wn + i$	1.000	0.617	0.823	0.533	0.060	-0.235	0.810	0.888
v06	$wn + \varphi$	1.000	0.762	0.928	0.869	-0.397	-0.006	0.995	0.911
v07	$wn + b - \frac{b'}{R}$	1.000	0.659	-0.198	0.581	-0.689	-0.219	0.969	0.676
v08	$i + \varphi$	1.000	0.818	0.829	0.824	-0.520	0.217	0.982	0.936
v09	$i + b - \frac{b'}{R}$	1.000	0.677	0.898	0.815	-0.554	-0.770	0.943	0.887
v10	$\varphi + b - \frac{b'}{R}$	1.000	0.022	-0.288	0.683	-0.911	0.014	0.304	0.397
v11	$wn + i + \varphi$	1.000	0.836	-0.159	0.751	-0.351	0.110	0.963	0.942
v12	$wn + i + b - \frac{b'}{R}$	1.000	0.605	0.906	0.677	0.200	-0.638	0.720	0.926
v13	$wn + \varphi + b - \frac{b'}{R}$	1.000	0.390	0.987	0.987	0.711	-0.976	0.115	0.990
v14	$i + \varphi + b - \frac{b'}{R}$	1.000	0.491	0.894	0.817	0.242	-0.479	0.945	0.816
v15	$wn + i + \varphi + b - \frac{b'}{R}$	1.000	0.754	0.931	0.720	0.252	-0.433	0.914	0.904
v16	$b'/(1+r)$	1.000	0.797	0.793	0.856	-0.450	0.029	0.992	0.933

Table 13: Correlation

10 Appendix: Data construction

10.1 Constructing financial shocks

We construct financial shocks with respect to each model using the following log-linearized borrowing constraints:

$$\begin{aligned}
\text{m1:} \quad \tilde{\xi} &= \frac{1}{\frac{\xi^*(k^* - \beta b^*)}{y^*}} \left\{ \frac{1 - \theta}{1 + \mu^*} (\tilde{w} + \tilde{n}) - \frac{\xi^* k^*}{y^*} \tilde{k}' + \frac{\beta \xi^* b^*}{y^*} \tilde{b}^e \right\}, \\
\text{m2:} \quad \tilde{\xi} &= \frac{1}{\frac{\xi^*(k^* - \beta b^*)}{y^*}} \left\{ \frac{\delta k^*}{y^*} \tilde{i} - \frac{\xi^* k^*}{y^*} \tilde{k}' + \frac{\beta \xi^* b^*}{y^*} \tilde{b}^e \right\}, \\
\text{m4:} \quad \tilde{\xi} &= \frac{1}{\frac{\xi^*(k^* - \beta b^*)}{y^*}} \left\{ \tilde{y} - (1 - \theta) (\tilde{w} + \tilde{n}) - \frac{\delta k^*}{y^*} \tilde{i} - \frac{d^*}{y^*} \tilde{d} - \frac{\xi^* k^*}{y^*} \tilde{k}' + \frac{\beta \xi^* b^*}{y^*} \tilde{b}^e \right\}, \\
\text{m5:} \quad \tilde{\xi} &= \frac{1}{\frac{\xi^*(k^* - \beta b^*)}{y^*}} \left\{ \frac{1 - \theta}{1 + \mu^*} (\tilde{w} + \tilde{n}) + \frac{\delta k^*}{y^*} \tilde{i} - \frac{\xi^* k^*}{y^*} \tilde{k}' + \frac{\beta \xi^* b^*}{y^*} \tilde{b}^e \right\}, \\
\text{m6:} \quad \tilde{\xi} &= \frac{1}{\frac{\xi^*(k^* - \beta b^*)}{y^*}} \left\{ (1 - \mu^*) (1 - \theta) (\tilde{w} + \tilde{n}) + \frac{d^*}{y^*} \tilde{d} - \frac{\xi^* k^*}{y^*} \tilde{k}' + \frac{\beta \xi^* b^*}{y^*} \tilde{b}^e \right\}, \\
\text{m7:} \quad \tilde{\xi} &= \frac{1}{\frac{\xi^*(k^* - \beta b^*)}{y^*}} \left\{ \tilde{y} - \frac{\delta k^*}{y^*} \tilde{i} - \frac{d^*}{y^*} \tilde{d} - \frac{\xi^* k^*}{y^*} \tilde{k}' + \frac{\beta \xi^* b^*}{y^*} \tilde{b}^e \right\}, \\
\text{m8:} \quad \tilde{\xi} &= \frac{1}{\frac{\xi^*(k^* - \beta b^*)}{y^*}} \left\{ \frac{\delta k^*}{y^*} \tilde{i} + \frac{d^*}{y^*} \tilde{d} - \frac{\xi^* k^*}{y^*} \tilde{k}' + \frac{\beta \xi^* b^*}{y^*} \tilde{b}^e \right\}, \\
\text{m9:} \quad \tilde{\xi} &= \frac{1}{\frac{\xi^*(k^* - \beta b^*)}{y^*}} \left\{ \tilde{y} - (1 - \theta) (\tilde{w} + \tilde{n}) - \frac{d^*}{y^*} \tilde{d} - \frac{\xi^* k^*}{y^*} \tilde{k}' + \frac{\beta \xi^* b^*}{y^*} \tilde{b}^e \right\}, \\
\text{m10:} \quad \tilde{\xi} &= \frac{1}{\theta - \frac{\delta k^*}{y^*}} \left\{ \tilde{y} - (1 - \theta) (\tilde{w} + \tilde{n}) - \frac{\delta k^*}{y^*} \tilde{i} - \frac{\xi^* k^*}{y^*} \tilde{k}' + \frac{\beta \xi^* b^*}{y^*} \tilde{b}^e \right\}, \\
\text{m11:} \quad \tilde{\xi} &= \frac{1}{\frac{\xi^*(k^* - \beta b^*)}{y^*}} \left\{ (1 - \mu^*) (1 - \theta) (\tilde{w} + \tilde{n}) + \frac{d^*}{y^*} \tilde{d} + \frac{\delta k^*}{y^*} \tilde{i} - \frac{\xi^* k^*}{y^*} \tilde{k}' + \frac{\beta \xi^* b^*}{y^*} \tilde{b}^e \right\}, \\
\text{m12:} \quad \tilde{\xi} &= \frac{1}{1 - \frac{d^*}{y^*}} \left\{ \tilde{y} - \frac{d^*}{y^*} \tilde{d} - \frac{\xi^* k^*}{y^*} \tilde{k}' + \frac{\beta \xi^* b^*}{y^*} \tilde{b}^e \right\} \\
\text{m13:} \quad \tilde{\xi} &= \frac{1}{1 - \frac{\delta k^*}{y^*}} \left\{ \tilde{y} - \frac{\delta k^*}{y^*} \tilde{i} - \frac{\xi^* k^*}{y^*} \tilde{k}' + \frac{\beta \xi^* b^*}{y^*} \tilde{b}^e \right\}, \\
\text{m14:} \quad \tilde{\xi} &= \frac{1}{\theta} \left\{ \tilde{y} - (1 - \theta) (\tilde{w} + \tilde{n}) - \frac{\xi^* k^*}{y^*} \tilde{k}' + \frac{\beta \xi^* b^*}{y^*} \tilde{b}^e \right\}, \\
\text{m15:} \quad \tilde{\xi} &= \tilde{y} - \frac{\xi^* k^*}{y^*} \tilde{k}' + \frac{\beta \xi^* b^*}{y^*} \tilde{b}^e.
\end{aligned}$$

where $b^e \equiv b'/(1 + r)$.

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