# Stationary Growth and the Impossibility of Capital Efficiency Gains

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21 May 2016

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**Abstract:** Improving the efficiency either in the process of factor accumulation or in the process of production of final output is often considered as a main driving force for the sustainable growth of modern economies. However, this article proves that for the most important input, physical capital, total efficiency, i.e. the total efficiency gained in the process of accumulation and in the production process, must be zero along a stationary growth path.

**Keywords:** Neoclassical Growth Model; Uzawa's Theorem; Improving Efficiency; Technical Change; Stationary Growth Path

**JEL Classifications**: E13, O30, O41

* Li gratefully acknowledges support from the Social Science Foundation of China (Grant No. 10CJL012) and the Fundamental Research Funds for the Central Universities of Tongji University (2016.1-2018.1). Huang acknowledges the financial support from National Social Science Foundation (13BJL050) and Program for New Century Excellent Talents in University of Ministry of Education of China (NCET-13-0298). The authors take sole responsibility for their views.

## 1. Introduction

Increasing the efficiency of inputs either in the process of factor accumulation or in the process of production of final products is considered as a main driving force for the sustainable growth of modern economies. Specifically, improving the efficiency of physical capital seems to be an important source of continued growth. Theoretically, reaching a stationary growth path is a basic requirement for economic growth models (Kaldor, 1961; Jones and Romer, 2010, p.225). However, by using a simple neoclassical growth model, this paper demonstrates that changes in the marginal product of capital and marginal efficiency of investment must sum to zero along a stationary growth path. That is, if the efficiency of physical capital accumulation is rising then it must be the case that capital becomes less efficient in the production process. In this sense, improving the overall efficiency of physical capital is unlikely to be the driving force for sustainable growth of modern economies!

While this conclusion seems a bit surprising, it is an inherent implication if the modern economy is viewed as a dynamic circulatory system. On the one hand, physical capital is used as an input factor in the production of the final product; on the other hand, it is part of that final product which is used as investment to accumulate physical capital. To guarantee dynamic stability, the system must be characterized by negative feedback. Therefore, the change of marginal efficiencies of factor accumulation and the final production must be completely offset each other. Otherwise, if the marginal efficiency of capital in final production increases (decreases), the output will grow faster (slower) than capital. At the same time, if the marginal efficiency of investment increases (decreases), the capital will grow faster (slower) than output. This would form a positive feedback. That is, following production output grows at a rate that is greater (smaller) than that of capital. The growth rate of capital becomes even further greater (smaller) than that of output through the capital accumulation process. The positive feedback will lead to an infinite increase (decrease) of the growth rates of both the final product and capital, which contradicts stability.

Although the existing literature does provide this conclusion from the perspective of the stability of dynamic circulation systems, it has arrived at this conclusion from other viewpoints. For example, Uzawa (1961) proved that when economic growth is on a stationary path and if the marginal efficiency of capital accumulation remains constant, then at the production stage there can be no capital-augmentation, ensuring that the marginal efficiency of capital remains unchanged.[①] Irmen (2013a) showed

[①] Because Uzawa did not give economic interpretation as to why technological progress cannot be capital-promoting along a stationary growth path, it induced further discussion of the issue that if the marginal efficiency of capital accumulation declines due to investment adjustment costs, then at the production stage must be characterized by capital-augmenting technological progress which improves the marginal efficiency of capital. Maliar and Maliar (2011) proved that if embodied technological progress causes the marginal efficiency of capital accumulation to increase, then at the production stage capital-augmenting technological progress must be negative.[②]

The rest of the paper is organized as follows: Section 2 of this paper presents the neoclassical growth model; Section 3 discusses the constraints a steady-state equilibrium imposes on the change of efficiency in neoclassical economy; Section 4 turns to the constraints a steady-state equilibrium imposes on the technological progress; Section 5 concludes.

## 2. The Neoclassical Growth Model

### 2.1. Technology and preferences

The production function is：

$$\mathrm{Y}_t = \mathrm{F}[\mathrm{K}_t, \mathrm{L}_t, \mathrm{t}], \tag{1}$$

Where $\mathrm{Y}_t, \mathrm{K}_t, \mathrm{L}_t$ respectively represent output, the capital stock and labor, while t represents time. As usual, $F_K \equiv \partial \mathrm{Y}_t / \partial \mathrm{K}_t > 0$ , $F_L \equiv \partial \mathrm{Y}_t / \partial \mathrm{L}_t > 0$ respectively represent the marginal product of capital and labor. Labor grows at an exogenous rate n, so that $\dot{\mathrm{L}}_t = nL_t$. F(•,•) meets all other the standards of the neo-classical properties[③].

Capital accumulates according to:

$$\dot{\mathrm{K}}_t = \mathrm{G}[\mathrm{I}_t] - \delta \mathrm{K}_t \tag{2}$$

Where $\delta$ represents the depreciation rate, $\mathrm{I}_t$ is investment and G(•) describes how investment transforms into new capital, where $\mathrm{G_I} \equiv \partial G / \partial I > 0$ is the marginal efficiency of capital accumulation is greater than zero. However, $\partial \mathrm{G_I} / \partial t$ may be equal to zero, but also may be either less than or greater than zero. This makes

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involving many economists, ranging from the induced innovation literatures in the 1960s which assumed that technological progress is exogenous (Fellner, 1961; von Weizsäcker, 1962; Kennedy, 1964; Samuelson, 1965; Drandakis and Phelps, 1966）to the recent discussion of endogenous technological progress (Acemoglu, 2003, 2009; Barro and Sala-i-Martin, 2004; Jones, 2005; Jones and Scrimgeour, 2008; Irmen 2013a, 2013b, 2015, 2017).

② Other literatures discussing embodied technological progress when steady-state growth are Sheshinski (1967), Krusell et al. (2000). He and Liu (2008) and Grossman et al. (2016), but these literatures did not point out that embodied technological progress and capital-promoting technological progress must be equal to zero.

③ That is, in addition to the above, constant returns to scale, diminishing marginal product, Inada conditions and each factor is essential (Barro and Sala-i-Martin, 2004, chapter 1).

equation (2) includes three common capital accumulation functions in the existing literature. For example, $\dot{K}_t = I_t^{\emptyset} - \delta K_t$ is a simple form of equation (2) (Irmen, 2013), and there are $G_I = \emptyset I_t^{\emptyset-1}$,$\partial G_I/\partial t = \emptyset(\emptyset-1)I_t^{\emptyset-2}dI_t/dt$. When $\emptyset = 1$, it is the standard neoclassical model of capital accumulation function, and $\partial G_I/\partial t = 0$; $\emptyset < 1$, may be interpreted as there is investment adjustment costs, and then $\partial G_I/\partial t < 0$ for $dI_t/dt > 0$; $\emptyset > 1$, it becomes the vintages of capital model, and the embodied technological progress is $I_t^{\emptyset-1}$, with $\partial G_I/\partial t > 0$ when $dI_t/dt > 0$.

The representative consumers have concave preferences over consumption, where the degree of concavity, as measured by the relative risk aversion, is constant. Their lifetime utility can be expressed as

$$\int_{t=0}^{\infty} \frac{C_t^{1-\theta}}{1-\theta} e^{-\rho t} dt, \tag{3}$$

where $C_t$ represents consumption at time t, θ represents the relative risk coefficient, ρ represents the discount rate.

The budget constraint of the representative consumer is:

$$C_t + I_t = r_t K_t + w_t L_t \tag{4}$$

Here $r_t$ represents the market price of capital, $w_t$ is the market wage of labor, and the following constraints apply: $C_t > 0, I_t > 0$.

### 2.2 Market Equilibrium

The Euler equation of the model may be obtained by using the optimal control method[4]

$$\frac{\dot{C}}{C} = [G_I r - \frac{\dot{G}_I}{G_I} - \rho - \delta]/\theta. \tag{5}$$

Equation (5) states that the marginal efficiency of investment $G_I$ influences the Euler equation of consumer. The familiar form, $\dot{C}/C = [r - \rho - \delta]/\theta$ , is obtained only when $G_I = 1$, and $\partial G_I/\partial t = 0$.[5]

In the competitive market, $r = \partial Y/\partial K$ is implied by the profit maximization. Using this relationship in equation (5) we obtain:

$$\theta\frac{\dot{C}}{C} = G_I\frac{\partial Y}{\partial K} - \frac{\dot{G}_I}{G_I} - \rho - \delta \tag{6}$$

## 3 Efficiency changes and the Existence of a Stationary Growth Path

[4] The derivation procedure is shown in the Appendix.

[5] Therefore $\dot{C}/C = [r - \rho - \delta]/\theta$ cannot be used to argue that the direction of technical change must be Harold neutral in a steady-state equilibrium (Acemoglu, 2009, ch15).

Definition 1: Along a *stationary equilibrium growth path* the growth rates of Y, K, L, C, I as well as $\dot{G}_I/G_I$ are all constant.

By Definition 1, if a stationary equilibrium growth path exists, by taking the time-derivative of both sides of equation (6), we get:

$$\partial\left(G_I \frac{\partial Y}{\partial K}\right)/\partial t = 0 \tag{7}$$

Using $F_K \equiv \partial Y_t / \partial K_t$, equation (7) implies:

$$\frac{\dot{G}_I}{G_I} + \frac{\dot{F}_K}{F_K} = 0 \tag{8}$$

Since equation (8) is derived under the assumption that a stationary equilibrium growth path exists, it is a necessary condition for that existence. Since $\dot{G}_I/G_I$ is the rate at which the marginal efficiency of investment in capital accumulation changes, and $\dot{F}_K/F_K$ is the rate at which the marginal product of capital changes, equation (8) reflects the constraint imposed on the two aspects of efficiency change of physical capital. This condition can be summarized as Proposition 1.

**Proposition 1:** For the neoclassical economy**,** along a stationary equilibrium growth path the rates at which the marginal efficiency of investment in capital accumulation and the marginal production of capital in products change cancel each other out**.**

Why can a neoclassical economy's stationary equilibrium growth path be realized only when the changes of the marginal efficiency of investment and capital just cancel each other out? The reason is that neoclassical economy is a dynamic circulatory system which consists of two links of factor accumulation and production. The size of $\dot{G}_I/G_I$ determines the change of $\dot{K}_t/K_t$ relative to $\dot{Y}_t/Y_t$ in the factor accumulation link, and the size of $\dot{F}_K/F_K$ determines the change of $\dot{Y}_t/Y_t$ relative to $\dot{K}_t/K_t$ in the production link. If $\dot{F}_K/F_K$ is greater (smaller) than 0, the production link makes $\dot{Y}_t/Y_t$ greater (smaller) than $\dot{K}_t/K_t$. Moreover, if at the same time $\dot{G}_I/G_I$ is also greater (smaller) than 0, then, together with the capital accumulation link, $\dot{K}_t/K_t$ will be greater (smaller) than $\dot{Y}_t/Y_t$. Thus, a positive feedback is formed in the dynamic circulatory system which makes $\dot{Y}_t/Y_t$ and $\dot{K}_t/K_t$ infinitely rise or fall, and no stationary path can emerge. A stationary path requires that the dynamic circulatory system be a negative feedback one, that is $\dot{F}_K/F_K$ and $\dot{G}_I/G_I$ must cancel each other out.

## 4 Technical changes and the Existence of a Stationary Path

Neoclassical economic technical change may occur at two links: in the production of final output, called factor-augmented technical change, or in the accumulation of capital, called embodied technical change.

The production function including factor-augmentation can be expressed as

$$\mathrm{Y}_t = \mathrm{H}[\mathrm{B}_t\mathrm{K}_t, \mathrm{A}_t\mathrm{L}_t], \tag{9}$$

where $\mathrm{B_t}$ and $\mathrm{A_t}$ expresses capital and labor augmentation respectively.

The capital accumulation function of embodied technical change can be expressed as [⑥]

$$\dot{\mathrm{K}}_t = \mathrm{q}_t\mathrm{I}_t - \delta\mathrm{K}_t \tag{10}$$

where $\mathrm{I}_t = \mathrm{Y}_t - \mathrm{C}_t$, and $\mathrm{q}_t$ represents embodied technology.

From the functions (9) and (10), we can get

$$\begin{cases} \mathrm{F}_K = \mathrm{B}_t\mathrm{H}_1[\mathrm{B}_t\mathrm{K}_t, \mathrm{A}_t\mathrm{L}_t] = \mathrm{B}_t h'(k_t) \\ G_I = \mathrm{q}_t \end{cases} \tag{11}$$

where $k_t \equiv \mathrm{B}_t\mathrm{K}_t/\mathrm{A}_t\mathrm{L}_t$, $h(k_t) = \mathrm{H}[\mathrm{B}_t\mathrm{K}_t/\mathrm{A}_t\mathrm{L}_t, 1]$.

Along a stationary path BK/AL is a constant. From equations (8) and (11) we can now obtain the technology-related condition required by the existence of a stationary equilibrium growth path：

$$\frac{\dot{\mathrm{B}}}{\mathrm{B}} + \frac{\dot{\mathrm{q}}}{\mathrm{q}} = 0 \tag{12}$$

The implication of equation (12) can be summarized by Proposition 2.

**Proposition 2:** The existence of a stationary equilibrium growth path requires that the capital-augmentation in the production of final goods and the embodied technical change in factor accumulation cancel each other out**.**

Proposition 2 indicates that along a stationary equilibrium growth path there cannot be technical progress at both the production and accumulation links of the economy. Specifically, if technology can only progress and cannot regress, then both $\frac{\dot{B}}{\mathrm{B}}$ and $\frac{\dot{q}}{\mathrm{q}}$ must be zero.

Existing papers do not explicitly take equation (12) as a constraint on the characteristics of stationary growth paths. However, Uzawa's (1961) Theorem pointed out that with $\frac{\dot{\mathrm{q}}}{\mathrm{q}}=0$, the stationary growth path must have $\frac{\dot{\mathrm{B}}}{\mathrm{B}} = 0$. Irmen (2013) assumes $\mathrm{q_t} = \mathrm{I_t}^{\emptyset-1}$ and $\emptyset < 1$ to certify that when $\frac{\dot{\mathrm{q}}}{\mathrm{q}} < 0$, steady-state growth must have $\frac{\dot{B}}{\mathrm{B}} > 0$; papers on embodied technical change indicate that when $\frac{\dot{q}}{\mathrm{q}} > 0$, steady-state growth must have $\frac{\dot{B}}{\mathrm{B}} < 0$ (Maliar and Maliar, 2011; Grossman et al., 2016).

## 5 Conclusion

[⑥]In some papers, the embodied technical change is formulated as $\mathrm{Y}_t = \mathrm{C}_t + \mathrm{I}_t/q_t$, $\dot{\mathrm{K}}_t = \mathrm{I}_t - \delta\mathrm{K}_t$. This formulation is equivalent to the one above.

Although increased efficiency or technical change are usually considered as the engines of sustained economic growth, the existing literature rarely pays attention to the constraints the existence of stationary growth paths imposes on the total efficiency increases or technical change. These constraints emerge because the neoclassical economy is a dynamic circulatory system. Using a simple neoclassical growth model, this paper demonstrates that the existence of a stationary growth path implies that efficiency gains or technical change in the two links of capital accumulation and final output production must cancel each other out. Consequently, one of the most important inputs of a modern economy, physical capital, may not realize a total efficiency gain.

In contrast, labor efficiency may increase or labor-augmented technical change may exist along a stationary growth path. This is the case since labor does not constitute a closed dynamic loop in the neoclassical economy. Even though labor is an input in the production process, the growth of labor does not require produced resources as an input.

Is it the case that if labor growth also requires investment of produced resources, any gain of labor efficiency in the production process must come at the expense of the efficiency of labor accumulation? And can the whole economy realize a total efficiency increase in two links of production and factor accumulation? The impact constraints imposed by stationary growth on efficiency gains and technical progress have on long-term development and short-term stabilization are a valuable topic for further research.

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**Appendix: Derivation of Euler equation (5)**

Construct a Hamiltonian as follows:

$$H(C,K,\lambda)=\frac{{C_t}^{1-\theta}}{1-\theta}e^{-\rho t}+\lambda_t\{G[rK_t+wL_t-C_t,t]-\delta K_t\}. \quad \text{(A1)}$$

$\lambda_t$ is the covariant, and the general transversality condition is :

$$\lim_{t\to\infty}\lambda_t K_t=0. \quad \text{(A2)}$$

The first-order conditions of (A1) are：

$$\begin{cases}\dfrac{\partial H}{\partial C}=C^{-\theta}e^{-\rho t}-\lambda G_I=0\\ \dot{\lambda}=-\dfrac{\partial H}{\partial K}=-\lambda(G_I r-\delta)\end{cases}. \quad \text{(A3)}$$

Using the first equation of (A3) to obtain the consumption, together with the second equation, we obtain the Euler equation (5).