Strategic Voting with Almost Perfect Signals

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Abstract

A standard assumption in the literature of strategic voting is the independence of signals. Each juror observes a signal at the *interim* stage of the game. Then she votes according to her private information in order to maximize her expected utility. This work introduces a dependency between signals, reflecting a more realistic situation, in which evidences can be incontrovertible. We give a full characterization of the symmetric equilibria in non-weakly dominated strategies and we provide a benchmark between the classical approach and this new one.

Jel Classification: C72, D72

1 Introduction

The problem of the jury, how jurors vote and when a collective choice is better than an individual one has been object of study in political science and in statistics for a long time. Starting from Condorcet's famous Theorem (1785), that can be stated as "*under the majority rule, groups of people make better decisions than single individuals and large electorates adopt the correct decision with very high probability*", many scholars extended this result using more relaxed statistical assumptions. The main argument is that each juror has a probability $p \in \left(\frac{1}{2}, 1\right)$ of choosing the correct option and she wants to do it. Through a proper aggregation method of the votes the statement holds. The value of p depends on the qualities of the individuals, i.e. their knowledge of the debated matter or the understanding of the consequences. The aggregation method depends on the type of institution considered. Despite the efforts to enrich the model, i.e. introducing the dependency of the probabilities (see, among others, Boland, 1989; Berg, 1993; Ladha, 1995, 1997; Berend and Sapir, 2007) or adding heterogeneity in the chance of making the correct choice (Boland, 1989), all these approaches consider jurors as if they were committed to vote informatively. There is no room for any other type of evaluation, rather than the statistical structure. So the implicit assumption is that each juror behaves as if she was alone. Even if she knows that the final outcome does not depend uniquely by her choice. This contradicts the notion of rationality. For this reason this subject has been analyzed also from the game theoretic point of view. The basic assumption is that, even if jurors share the same objective, they vote strategically in order to maximize their expected utility. The probabilities of making the correct choice, which incorporates information acquisition, competence and understanding of the debated matter are replaced with the observation of noisy signals. The framework becomes a classical setting of games with private information. The incentive to look for the maximization of the expected utility comes from the information-based heterogeneity of individuals at the *interim* stage of the game. This approach gives some insights about the jurors' behavior (Austen-Smith and Banks, 1996), explains the participation rate,

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the roll-off effect, the information disclosure in large elections (Feddersen and Pesendorfer, 1996, 1997) and the robustness of the aggregation methods (Feddersen and Pesendorfer, 1998). Moreover, Myerson (1998) shows that the Condorcet's Jury Theorem holds also with strategic individuals when the number of participants and the correct option are uncertain. This article analyzes the strategic behavior of a jury that must vote to acquit or convict a defendant during a trial. The situation is similar to a group of experts who must choose through a poll, to approve or reject a project. The main issue is that the truth is unknown. Before making her choice, each juror observes a signal that gives an indication about the innocence or the guiltiness of the defendant or the quality of the project. In the statistical approach, as in the game theoretic one, signals are assumed to be independent. The underlying hypothesis is that jurors interpret evidences differently, because of their different life experiences and competencies. This assumption is convenient because it makes computation easier. But, if we interpret signals as evidences or as a technical report, there are at least two possible implications: jurors are allowed to interpret them in opposite ways or during the trial no decisive evidence is produced at all. For this reason in our analysis we assume that, with a positive probability $\alpha \in (0,1)$ evidences are so strong to leave no chance to interpretation. In fact, if we consider the modern investigation techniques, i.e. fingerprints, DNA or some particular circumstances, such as digital recordings or being caught in the act, the independence of signals does not seem reasonable. Following the results in Feddersen and Pesendorfer (1998) we will restrict our attention to simple majority decision rules with no abstention. The remainder of the chapter is organized as follows: in section 2.2 we present the model. In section 2.3 we compute the symmetric equilibria. In section 2.4 we summarize the classical model with independent signals and we compute the corresponding symmetric equilibria. In section 2.5 we compare the two models trying to find a benchmark. Section 2.6 concludes. All the proofs are in the Appendix (Section 2.7).

2 The Model

Let J be the set of jurors with odd cardinality, each $j \in J$ must choose to acquit or convict the defendant. There is no abstention so the action set for each juror is $S_j = \{a, c\}$, moreover jurors share the same payoff function $v_j = v : \Omega_1 \times \mathcal{S} \to \mathbb{R}$, defined as

$$v \triangleq u \circ f$$

where $\Omega_1 = \{\mathrm{I}, \mathrm{G}\}$ is the set of states of Nature, innocent or guilty, concerning the defendant. The set $\mathcal{S} = S_j \times S_{-j}$ is the set of all possible action profiles. The function v is the composition of the aggregation rule $f : S_j \times S_{-j} \to \{a, c\}$ of the action profiles with the utility function $u : \{a, c\} \times \Omega_1 \to \mathbb{R}$, defined as

$$u(a|\omega_1 = \mathrm{I}) = u(c|\omega_1 = \mathrm{G}) = 0$$
$$u(a|\omega_1 = \mathrm{G}) = -(1-q)$$
$$u(c|\omega_1 = \mathrm{I}) = -q$$

with $q \in (0,1)$. The parameter q can be viewed as a threshold of reasonable doubt. The true state in Ω_1 is unknown and the jury members share the same prior probability distribution $\pi_j = \pi = \Pr(\mathrm{I}) \in (0,1)$. Before choosing an action in S_j each juror $j \in J$ observes a private signal t_j that can assume values in $T_j = \{i, g\}$ with state dependent distribution. In particular, with probability $\alpha \in (0,1)$ nature reveals the true state in Ω_1 with degenerate independent signals

$$\Pr(t_j = i|\mathrm{I}, \mathrm{R}) = 1 \quad \text{and} \quad \Pr(t_j = g|\mathrm{G}, \mathrm{R}) = 1$$
$$\Pr(t_j = g|\mathrm{I}, \mathrm{R}) = 0 \quad \text{and} \quad \Pr(t_j = i|\mathrm{G}, \mathrm{R}) = 0$$

where R is the event "degenerate signals" and with probability $(1-\alpha)$ signals are still independent but not so accurate

$$\Pr(t_j = i|\mathrm{I}, \neg\mathrm{R}) = z \quad \text{and} \quad \Pr(t_j = g|\mathrm{G}, \neg\mathrm{R}) = z$$
$$\Pr(t_j = g|\mathrm{I}, \neg\mathrm{R}) = 1-z \quad \text{and} \quad \Pr(t_j = i|\mathrm{G}, \neg\mathrm{R}) = 1-z$$

with $z \in (\frac{1}{2}, 1)$. To summarize the set of states of nature is defined as $\Omega = \Omega_1 \times \Omega_2$ where $\Omega_2 = \{\mathrm{R}, \neg\mathrm{R}\}$ contains the event "degenerate signals" and its complement. The event R is not directly observable and so

$$\Pr(t_j = g|\mathrm{G}) = \alpha + (1-\alpha)z$$
$$\Pr(t_j = i|\mathrm{G}) = (1-\alpha)(1-z)$$
$$\Pr(t_j = i|\mathrm{I}) = \alpha + (1-\alpha)z$$
$$\Pr(t_j = g|\mathrm{I}) = (1-\alpha)(1-z)$$

After getting her signal, each juror $j \in J$ updates her belief using Bayes' rule and then

$$\Pr(\mathrm{I}|t_j = i) = \frac{\alpha\pi + (1-\alpha)\pi z}{\alpha\pi + (1-\alpha)[\pi z + (1-\pi)(1-z)]}$$
$$\Pr(\mathrm{I}|t_j = g) = \frac{(1-\alpha)\pi(1-z)}{\alpha(1-\pi) + (1-\alpha)[\pi(1-z) + (1-\pi)z]}$$
$$\Pr(\mathrm{G}|t_j = i) = \frac{(1-\alpha)(1-\pi)(1-z)}{\alpha\pi + (1-\alpha)[\pi z + (1-\pi)(1-z)]}$$
$$\Pr(\mathrm{G}|t_j = g) = \frac{\alpha(1-\pi) + (1-\alpha)(1-\pi)z}{\alpha(1-\pi) + (1-\alpha)[\pi(1-z) + (1-\pi)z]}$$

the sets $(S_j, T_j)_{j\in J}$ and the values of $|J|, \alpha, z, \pi, q$ are common knowledge. Each juror votes simultaneously and before the poll they cannot communicate. This assumption could seem too strong but the analysis of the deliberation mechanism is beyond the purpose of this work. Let $0 \le k(s) \le |J|$ be the number of acquit votes in an action profile $s \in \mathcal{S}$ and let $\hat{m}$ be equal to $\hat{m} = (|J|-1)/2$[1]. The aggregating rule f used is defined as

$$f(s) \triangleq \begin{cases} a & \text{if} \quad k(s) > \hat{m} \\ c & \text{if} \quad k(s) \le \hat{m} \end{cases}$$

and it corresponds to simple majority. This type of function has two important properties, it is *anonymous* and *monotonic*: it treats all votes equally and if a decision is taken with n votes, it will not change with $n+1$ votes. The presence of private signals and the interdependence of the payoff functions, induce each juror to vote strategically.

Definition 2.1. *A voting strategy for juror $j \in J$ is a map*

$$\sigma_j : T_j \to \Delta(S_j)$$

where $\Delta(S_j) \supset S_j$ is the set of all possible pure and mixed actions.

Following the terminology in Austen-Smith and Banks (1996) each juror can behave in three different ways

Definition 2.2. *A voting strategy $\sigma_j : T_j \to \Delta(S_j)$ is*

i. informative, if $\sigma_j(s_j|t_j = i) = a$ and $\sigma_j(s_j|t_j = g) = c$.

ii. sincere, if given the observed signal, it maximizes the expected utility of juror $j \in J$.

Let $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_{2\hat{m}+1})$ be the voting strategy profile of the jury, then

[1] Since $|J|$ is assumed to be an odd number, $\hat{m}$ is always an integer.

Definition 2.3. *A voting strategy profile* $\sigma : \{i, g\}^{2\hat{m}+1} \to \{\Delta(S_j)\}_{j=1}^{2\hat{m}+1}$ *is rational, if it is a bayesian Nash equilibrium of the game* $\Gamma^b = \langle J, (T_j, S_j)_{j\in J}, (\alpha, z, \pi), v\rangle$.

Definition 2.4. *The voting strategy profile* $\sigma^* = (\sigma_1^*, \dots, \sigma_{2\hat{m}+1}^*)$ *is a bayesian Nash equilibrium of the game* $\Gamma^b = \langle J, (T_j, S_j)_{j\in J}, (\alpha, z, \pi), v\rangle$ *if for all* $j \in J$, $t_j \in T_j$ *and* $\sigma_j \in \Delta(S_j)$

$$\sum_{t_{-j}\in T_{-j}} v(\sigma_j^*|\sigma_{-j}^*, t_j, t_{-j}) \Pr(t_{-j}|t_j) \geq \sum_{t_{-j}\in T_{-j}} v(\sigma_j|\sigma_{-j}^*, t_j, t_{-j}) \Pr(t_{-j}|t_j)$$

Corollary 2.5. *Whenever a voting strategy profile* σ^* *is rational, it is also sincere.*

It is well known that a rational juror is concerned only when her vote is pivotal. In particular for this model, after observing $t_j = g$ juror $j \in J$ prefers to acquit if $q > \Pr(\mathrm{G}|piv, t_j = g)$ or convict if the inequality is reversed. In the same way, after observing $t_j = i$ she prefers to acquit if $q > \Pr(\mathrm{G}|piv, t_j = i)$ or convict if the inequality is reversed. The probability that a defendant is guilty, given that juror $j \in J$ is pivotal and given $t_j = g$ can be computed as

$$\Pr(\mathrm{G}|piv, t_j = g) = \frac{\Pr(\mathrm{G}|t_j = g)\Pr(piv|\mathrm{G}, t_j = g)}{\Pr(\mathrm{G}|t_j = g)\Pr(piv|\mathrm{G}, t_j = g) + \Pr(\mathrm{I}|t_j = g)\Pr(piv|\mathrm{I}, t_j = g)}$$

where

$$\Pr(piv|\mathrm{G}, t_j = g) = \sum_{t_{-j}\in T_{-j}} \Pr(t_{-j}|\mathrm{G}, t_j = g)\Pr(piv|\mathrm{G}, t_{-j}, t_j = g)$$
$$\Pr(piv|\mathrm{I}, t_j = g) = \sum_{t_{-j}\in T_{-j}} \Pr(t_{-j}|\mathrm{I}, t_j = g)\Pr(piv|\mathrm{I}, t_{-j}, t_j = g)$$

and it depends on the distribution of the signals profile and on the strategies adopted by the jurors. The probabilities $\Pr(\mathrm{G}|piv, t_j = i)$, $\Pr(\mathrm{I}|piv, t_j = g)$ and $\Pr(\mathrm{I}|piv, t_j = i)$ are similarly defined.

3 Equilibria

In this paper we will analyze only the symmetric bayesian Nash equilibria in which players do not use weakly dominated strategies. From now on assume $|J| = 3$. To avoid trivial cases the parameter $q \in (0, 1)$ must be restricted. In fact, let us assume that a single juror can observe all signals and she observes the signals profile $t = (g, g, g)$. In this case, if she chooses to acquit irrespectively of the observed profile, it means that she is not responsive as if she chooses to convict after observing the signals profile $t = (i, i, i)$.

Definition 3.1. *A juror* $j \in J$ *is said to be responsive, if there exist at least two signals profiles* $t', t'' \in T$ *such that* $\sigma_j(s_j|t = t') \neq \sigma_j(s_j|t = t'')$ *for some* $s_j \in S_j$*, where* $T = \{T_j\}_{j=1}^{2\hat{m}+1}$.

This definition is similar to the one in Feddersen and Pesendorfer (1998). Before observing the signal, each juror has a prior probability distribution $\pi \in (0, 1)$ and it represents the common sentiment about the innocence or the guiltiness of the defendant. It seems natural to study the behavior of equilibrated jurors who have not a biased opinion.

Definition 3.2. *A juror* $j \in J$ *is said to be unbiased, if* $\pi = \frac{1}{2}$.

The above definition characterizes an impartial juror.

Proposition 3.3. *For given* $\alpha \in (0, 1)$, $z \in (\frac{1}{2}, 1)$ *and* $|T| = 3$*, an unbiased juror* $j \in J$ *is responsive, if and only if*

$$q \in (q_\alpha^{\min}, q_\alpha^{\max}) = \left(\frac{(1-\alpha)(1-z)^3}{\alpha + (1-\alpha)[z^3 + (1-z)^3]}, \frac{\alpha + (1-\alpha)z^3}{\alpha + (1-\alpha)[z^3 + (1-z)^3]}\right)$$

The above proposition gives the boundaries within the parameter q must lie, with a little abuse of notation it is possible to consider $\sigma_j(s_j|t_j = g)$ as the probability of acquit given the signal $t_j = g$ and $\sigma_j(s_j|t_j = i)$ as the probability of acquit given the signal $t_j = i$. As it will be clear below, the value of q determines the feasible strategies and the equilibria. The following proposition describes when the informative strategy is also rational.

Proposition 3.4. *Let J be a set of unbiased and responsive jurors with $\hat{m} \in \mathbb{N} < \infty$ for given $\alpha \in (0,1)$ and $z \in (\frac{1}{2}, 1)$, if $q \in (1-z, z)$ then the informative voting strategy (profile) is rational. That is, for any $q \in (1-z, z)$ the informative voting strategy profile σ^* is a bayesian Nash equilibrium of Γ^b.*

Notice that the above proposition holds for any finite $\hat{m}$. When $q \notin (1-z, z)$ the threshold of reasonable doubt is shifted toward a behavior more concerned about the risk of make a mistake. In particular, for $q \in (q_\alpha^{\min}, 1-z)$ jurors are more afraid of acquit a guilty, rather than convict an innocent defendant. The following proposition describes the symmetric bayesian Nash equilibrium played by this type of jurors.

Proposition 3.5. *Let J be a set of unbiased and responsive jurors with $|J| = 3$, for given $\alpha \in (0,1)$, $z \in (\frac{1}{2}, 1)$ and $q \in (\hat{q}_\alpha^{\min}, 1-z)$ the voting strategy*

$$\Pr(a|t_j = g) = 0$$

$$\Pr(a|t_j = i) = \frac{q[\alpha + (1-\alpha)[z^2 + (1-z)^2]] - (1-\alpha)(1-z)^2}{q[\alpha + (1-\alpha)[z^3 + (1-z)^3]] - (1-\alpha)(1-z)^3}$$

where

$$\hat{q}_\alpha^{\min} = \frac{(1-\alpha)(1-z)^2}{\alpha + (1-\alpha)[z^2 + (1-z)^2]} > q_\alpha^{\min}$$

is rational. That is, for any $q \in (\hat{q}_\alpha^{\min}, 1-z)$ the above strategy is a bayesian Nash equilibrium of Γ^b.

Notice that the lower bound in order to have an unbiased, responsive and rational juror is higher than the one when there is only one decision maker. In a certain sense, it is as if the presence of other jurors narrows the responsiveness interval. Moreover, jurors choose to "accept" the observed signal only when it is consistent with their aptitude, and so they truthfully reveal it through the vote. In the other case they prefer to randomize their vote. When $q \in (z, q_\alpha^{\max})$ the situation is the opposite, jurors are more afraid of convict a possible innocent rather than to acquit a guilty defendant.

Proposition 3.6. *Let J be a set of unbiased and responsive jurors with $|J| = 3$, for given $\alpha \in (0,1)$, $z \in (\frac{1}{2}, 1)$ and $q \in (z, \hat{q}_\alpha^{\max})$ the voting strategy*

$$\Pr(a|t_j = g) = \frac{q(1-\alpha)z(1-z) - (1-\alpha)z^2(1-z)}{\alpha + (1-\alpha)z^3 - q[\alpha + (1-\alpha)[z^3 + (1-z)^3]]}$$

$$\Pr(a|t_j = i) = 1$$

where

$$\hat{q}_\alpha^{\max} = \frac{\alpha + (1-\alpha)z^2}{\alpha + (1-\alpha)[z^2 + (1-z)^2]} < q_\alpha^{\max}$$

is rational. That is, for any $q \in (z, \hat{q}_\alpha^{\max})$ the above strategy is a bayesian Nash equilibrium of Γ^b.

As in the previous proposition, the upper bound changes $\hat{q}_\alpha^{\max} < q_\alpha^{\max}$ and jurors "accept" to reveal truthfully the signal only when it is coherent with their concern, otherwise they choose to randomize.

Asymptotic Behavior

The Condorcet's Jury Theorem states that majority are more likely to select the correct alternative with respect to the single one. Not only, when the number of jurors grows, the probability of making the correct choice increases. Under the informative strategy a defendant is acquitted if and only if $k(t) > \hat{m}$, where $k(t)$ is the number of i signals in the profile $t \in T$. The distribution of $k(t)$ conditioned to one of the two states in Ω_1 is

$$\begin{aligned}
\Pr(k(t) = 2\hat{m}+1|\omega_1 = \mathrm{I}) &= \alpha + (1-\alpha)z^{2\hat{m}+1} \\
\Pr(k(t) = x|\omega_1 = \mathrm{I}) &= \binom{2\hat{m}+1}{x}(1-\alpha)z^x(1-z)^{(2\hat{m}+1)-x}
\end{aligned}$$

for $0 \le x < |J| = 2\hat{m}+1$ and

$$\begin{aligned}
\Pr(k(t) = 0|\omega_1 = \mathrm{G}) &= \alpha + (1-\alpha)z^{2\hat{m}+1} \\
\Pr(k(t) = x|\omega_1 = \mathrm{G}) &= \binom{2\hat{m}+1}{x}(1-\alpha)z^{(2\hat{m}+1)-x}(1-z)^x
\end{aligned}$$

for $0 < x \le |J| = 2\hat{m}+1$. So the probability of convict an innocent is

$$\Pr(conv|\mathrm{I}) = (1-\alpha)\sum_{x=0}^{\hat{m}}\binom{2\hat{m}+1}{x}z^x(1-z)^{(2\hat{m}+1)-x}$$

and the probability of acquit a guilty is

$$\Pr(acq|\mathrm{G}) = (1-\alpha)\sum_{x=0}^{\hat{m}}\binom{2\hat{m}+1}{x}z^x(1-z)^{(2\hat{m}+1)-x}$$

The following proposition shows that with the informative strategy the Condorcet's Theorem still holds.

Proposition 3.7. *Let J be a set of unbiased, rational and responsive jurors with $\hat{m} \in \mathbb{N}$, for given $\alpha \in (0,1)$, $z \in (\frac{1}{2}, 1)$ and $q \in (1-z, z)$ the probability of convict an innocent and the probability of acquit a guilty go to zero as the jury size goes to infinity.*

Single Judge vs. Multiple Jurors

In this part we analyze the situation in which there is only one judge, who must choose to acquit or convict the defendant. The basic assumption made is that, since a judge can interact with the involved parties he can get a more clear exposition of the evidences. For this reason we assume that he can extract more information with respect to a single juror. Another possible justification to this assumption is that judges usually are more competent, than a single juror drawn from a list of common people. In order to keep some analogies with the case of three jurors, we assume that the number of observed signals is equal to 3. We compare the probabilities of making the correct choice for the same value of q. We maintain the same utility function u, but in this case the aggregation rule f is simply the identity from the judge's decision to the final outcome for the defendant. Obviously, in this case, the probability of being pivotal is one. The judge's behavior is the same as the single juror. He chooses to acquit if and only if $q > \Pr(\mathrm{G}|\hat{t})$, where $\hat{t} \in T$ is the observed signals profile, otherwise he chooses to convict the defendant. Assuming that the judge is not biased, let us compute the probabilities of being guilty given the different signals profiles

$$\Pr(\mathrm{G}|g,g,g) = \frac{\alpha + (1-\alpha)z^3}{\alpha + (1-\alpha)[z^3 + (1-z)^3]} = \theta_1 \qquad \Pr(\mathrm{G}|i,g,g) = z = \theta_2$$

$$\Pr(\mathrm{G}|i,i,g) = 1 - z = \theta_3 \qquad \Pr(\mathrm{G}|i,i,i) = \frac{(1-\alpha)(1-z)^3}{\alpha + (1-\alpha)[z^3 + (1-z)^3]} = \theta_4$$

notice that θ_2 and θ_3 are the same independently of the permutation of the signals. Proposition 3.3 gives us the interval of responsiveness for a single decision maker, we can see that $\theta_1 = q_\alpha^{\max}$ and $\theta_4 = q_\alpha^{\min}$. Let $\Pr(\mathrm{G}|conv)$ be the probability that a guilty is convicted by a judge and let $\underline{\Pr}(\mathrm{G}|conv)$ the corresponding probability when the deliberation is made by a group of jurors. When $q \in (0, q_\alpha^{\min})$ both judge and jurors are outside the responsiveness interval, and they always choose to convict the defendant.

$$\Pr(\mathrm{G}|conv) = \frac{1}{2} = \underline{\Pr}(\mathrm{G}|conv)$$

It is easy to see that in this case, the probability of acquitting an innocent defendant is $\Pr(\mathrm{I}|acq) = 0 = \underline{\Pr}(\mathrm{I}|acq)$. For $q \in (q_\alpha^{\min}, \hat{q}_\alpha^{\min})$ jurors always choose to convict the defendant, while the single judge will acquit the defendant if and only if he observers the signal profile $t = (i, i, i)$. Hence

$$\Pr(\mathrm{G}|conv) = \frac{\alpha + (1-\alpha)[z^3 + 3z(1-z)]}{1 + 3(1-\alpha)z(1-z)} > \frac{1}{2} = \underline{\Pr}(\mathrm{G}|conv)$$

since $\alpha + (1-\alpha)[z^3 - (1-z)^3] > 0$ holds for any $\alpha > 0$ and $z \in \left(\frac{1}{2}, 1\right)$ the above inequality is always satisfied. The probability of acquitting an innocent defendant, in this case is

$$\Pr(\mathrm{I}|acq) = \frac{\alpha + (1-\alpha)z^3}{\alpha + (1-\alpha)[z^3 + (1-z)^3]} > 0 = \underline{\Pr}(\mathrm{I}|acq)$$

when $q \in (1-z, z)$ the probabilities of convict a guilty defendant are the same

$$\Pr(\mathrm{G}|conv) = \alpha + (1-\alpha)[z^3 + 3z^2(1-z)] = \underline{\Pr}(\mathrm{G}|conv)$$

as the probabilities of acquitting an innocent one

$$\Pr(\mathrm{I}|acq) = \alpha + (1-\alpha)[z^3 + 3z^2(1-z)] = \underline{\Pr}(\mathrm{I}|acq)$$

For $q \in (\hat{q}_\alpha^{\max}, q_\alpha^{\max})$ jury members always choose to acquit a defendant, while a single juror will convict when he observes the signal profile $t = (g, g, g)$. The corresponding probabilities are

$$\Pr(\mathrm{G}|conv) = \frac{\alpha + (1-\alpha)z^3}{\alpha + (1-alpha)[z^3 + (1-z)^3} > 0 = \underline{\Pr}(\mathrm{G}|conv)$$

while for the probabilities of acquitting an innocent defendant the result is similar as the first case analyzed

$$\Pr(\mathrm{I}|acq) = \frac{\alpha + (1-\alpha)[z^3 + 3z(1-z)]}{1 + 3(1-\alpha)z(1-z)} > \frac{1}{2} = \underline{\Pr}(\mathrm{I}|acq)$$

For $q \in (q_\alpha^{\max}, 1)$ both judge and jurors choose to acquit the defendant, the probabilities of convicting a guilty defendant are zero and the probabilities of acquitting an innocent one are equal to one half.

4 The Classical Model

In this section we will replicate the analysis as in Section 3 when the classical assumption is used. To do this, it is enough to change the structure of the signals distribution as

$$\Pr(t_j = i|\mathrm{I}) = p \quad \text{and} \quad \Pr(t_j = g|\mathrm{G}) = p$$
$$\Pr(t_j = g|\mathrm{I}) = 1-p \quad \text{and} \quad \Pr(t_j = i|\mathrm{G}) = 1-p$$

with $p \in (\frac{1}{2}, 1)$ and we assume the independence of the signals. As in Proposition 3.3 it is possible to define the interval within a juror is responsive, when she can observe all the signals profile

Proposition 4.1. *For given $p \in (\frac{1}{2}, 1)$ and $|T| = 3$ an unbiased juror $j \in J$ is responsive, if and only if*

$$q \in \left(q_0^{\min}, q_0^{\max}\right) = \left(\frac{(1-p)^3}{p^3 + (1-p)^3}, \frac{p^3}{p^3 + (1-p)^3}\right)$$

Notice that letting p equal to z, this implies that signals have the same quality, the responsiveness interval in Section 3 is always wider than this one $\left(q_0^{\min}, q_0^{\max}\right) \subset \left(q_\alpha^{\min}, q_\alpha^{\max}\right)$. It is sufficient to know that with a positive probability, no matter of its value, Nature reveals the truth to convince more concerned jurors to react to the information.

Proposition 4.2. *Let J be a set of unbiased and responsive jurors with $\hat{m} \in \mathbb{N} < \infty$ for given $p \in (\frac{1}{2}, 1)$, if $q \in (1-p, p)$ then the informative voting strategy (profile) is rational. That is, for any $q \in (1-p, p)$ the informative voting strategy profile σ^* is a bayesian Nash equilibrium of Γ_0^b.*

where $\Gamma_0^b = \langle J, (T_j, S_j)_{j \in J}, (p, \pi), v \rangle$ is the modified bayesian game. When $q \notin (1-p, p)$ the threshold of reasonable doubt is shifted and so

Proposition 4.3. *Let J be a set of unbiased and responsive jurors with $|J| = 3$, for given $p \in (\frac{1}{2}, 1)$ and $q \in (\hat{q}_0^{\min}, 1-p)$ the voting strategy*

$$\Pr(a|t_j = g) = 0$$

$$\Pr(a|t_j = i) = \frac{q[p^2 + (1-p)^2] - (1-p)^2}{q[p^3 + (1-p)^3] - (1-p)^3}$$

where

$$\hat{q}_0^{\min} = \frac{(1-p)^2}{p^2 + (1-p)^2} > q_0^{\min}$$

is rational. That is, for any $q \in (\hat{q}_0^{\min}, 1-p)$ the above strategy is a bayesian Nash equilibrium of Γ_0^b.

Similarly to the Proposition 3.5 the lower bound $\hat{q}_0^{\min}$ is higher than $q_0^{\min}$ and the signal t_j is "revealed" only when it is consistent with the aptitude of the jurors.

Proposition 4.4. *Let J be a set of unbiased and responsive jurors with $|J| = 3$, for given $p \in (\frac{1}{2}, 1)$ and $q \in (p, \hat{q}_0^{\max})$ the voting strategy*

$$\Pr(a|t_j = g) = \frac{qp(1-p) - p^2(1-p)}{p^3 - q[p^3 + (1-p)^3]}$$

$$\Pr(a|t_j = i) = 1$$

where

$$\hat{q}_0^{\max} = \frac{p^2}{p^2 + (1-p)^2} < q_0^{\max}$$

is rational. That is, for any $q \in (p, \hat{q}_0^{\max})$ the above strategy is a bayesian Nash equilibrium of Γ_0^b.

as Proposition 3.6 the upper bound $\hat{q}_0^{\max}$ is lower than $q_0^{\max}$ and t_j is "revealed" only when $t_j = i$.

5 Benchmark

Denote as α-model the one described in Section 2 and as 0-model the classic one in Section 4. Now the two models will be compared under the assumption that the *ex ante* utilities, obtained adopting the informative strategy profile, are the same. To do this it is necessary to find the relation, if exists, between the different parameters of the models. Before observing the signal

$$\mathrm{EU}[v] = -\frac{1}{2}[\Pr(conv|\mathrm{I})\, q + \Pr(acq|\mathrm{G})(1-q)]$$

where the conditioned probabilities of acquit and convict depend also by the voting strategy profile

$$\Pr(conv|\mathrm{I}) = \sum_{(t_{-i},t_i)\in T} \Pr(conv|t_{-i},t_i,\mathrm{I})\Pr(t_{-i},t_i|\mathrm{I})$$

$$\Pr(acq|\mathrm{G}) = \sum_{(t_{-i},t_i)\in T} \Pr(acq|t_{-i},t_i,\mathrm{G})\Pr(t_{-i},t_i|\mathrm{G})$$

In both models, under the informative strategy, the *ex ante* expected utilities does not depend on q

$$\mathrm{EU}_\alpha[v] = -\frac{1}{2}(1-\alpha)[(1-z)^3 + 3z(1-z)^2]$$

$$\mathrm{EU}_0[v] = -\frac{1}{2}[(1-p)^3 + 3p(1-p)^2]$$

In this case $\mathrm{EU}_\alpha[v]$ is equal to $\mathrm{EU}_0[v]$ if and only if between α, z and p holds this relation

$$-\frac{1}{2}(1-\alpha)[(1-z)^3 + 3z(1-z)^2] = -\frac{1}{2}[(1-p)^3 + 3p(1-p)^2] \tag{$\star$}$$

derived in the Appendix. When $\alpha = 0$ both models coincide and $p = z$ as when $\alpha = 1$ or $z = 1$, in all other cases $p > z$.

Example 5.1. For $\alpha = 0.5$ and $z = 0.65$ the *ex ante* expected utility is $\mathrm{EU}_\alpha[v] = -0.0704$ and the probability p that satisfies equation ($\star$) is 0.7639.

Let $\hat{q}^{\min} = \min\{\hat{q}_0^{\min}, \hat{q}_\alpha^{\min}\}$ and $\hat{q}^{\max} = \max\{\hat{q}_0^{\max}, \hat{q}_\alpha^{\max}\}$. When $q < \hat{q}^{\min}$ in both models jurors always chooses to convict and the *ex ante* expected utilities are

$$\mathrm{EU}_\alpha[v] = \mathrm{EU}_0[v] = -\frac{1}{2}\,q$$

when $q > \hat{q}^{\max}$ in both models jurors always chooses to acquit and the *ex ante* expected utilities are

$$\mathrm{EU}_\alpha[v] = \mathrm{EU}_0[v] = -\frac{1}{2}\,(1-q)$$

The following proposition determines the intervals of the two models.

Proposition 5.2. *For given $\alpha \in (\frac{1}{2}, 1)$, $z \in (\frac{1}{2}, 1)$ and $p \in (\frac{1}{2}, 1)$ such that equation ($\star$) holds,*

$$\hat{q}^{\min} = \hat{q}_\alpha^{\min}(\hat{q}^{\min} = \hat{q}_0^{\min}) \quad \textit{and} \quad \hat{q}^{\max} = \hat{q}_\alpha^{\max}(\hat{q}^{\max} = \hat{q}_0^{\max})$$

if and only if

$$\alpha > (<)\ \hat{\alpha}$$

with

$$\hat{\alpha} = \frac{p^2(1-z)^2 - (1-p)^2 z^2}{(1-p)^2 + p^2(1-z)^2 - (1-p)^2 z^2}$$

Since $\alpha \in (0, 1)$ can be viewed as the probability of observing a straightforward evidence that leaves no doubt, it is natural that higher values of this parameter implies a wider interval of responsiveness of the jurors. Now it is time to compare the ex ante expected utilities for some values of q.

Proposition 5.3. *Let J be a set of unbiased and responsive jurors with $|J| = 3$, for given $\alpha \in (0, 1)$, $z \in (\frac{1}{2}, 1)$ and $p \in (\frac{1}{2}, 1)$ such that equation ($\star$) holds,*

i) if $q \in (0, \hat{q}^{\min})$ in both models jurors always choose to convict and $EU_\alpha[v] = EU_0[v]$.

ii) if $q \in (1-p, 1-z)$ in the 0−model the informative strategy is rational, while in the α-model is rational the strategy described in proposition 3.5 and $EU_\alpha[v] > EU_0[v]$.

iii) if $q \in (1-z, z)$ in both models the informative strategy is rational and $EU_\alpha[v] = EU_0[v]$.

iv) if $q \in (z, p)$ in the 0−model the informative strategy is rational, while in the α-model is rational the strategy described in proposition 3.6 and $EU_\alpha[v] > EU_0[v]$.

v) if $q \in (\hat{q}^{\max}, 1)$ in both models jurors always choose to acquit and $EU_\alpha[v] = EU_0[v]$.

6 Conclusions

The presence of a positive probability with which Nature reveals the true state of the world, makes the responsiveness interval wider. It is easy to see that when the number of jurors increases the values of $q_\alpha^{\min}$ and $q_0^{\min}$ approach to zero and the values of $q_\alpha^{\max}$ and $q_0^{\max}$ approach to one. The reason is very simple: the more information a single decision maker can observe and the higher is her inclination to change her judgment about the defendant. On the other hand, when the number of jury members increases the probability of being pivotal decreases. This effect should narrow the intervals $(\hat{q}_\alpha^{\min}, 1-z)$, $(\hat{q}_0^{\min}, 1-z)$, $(z, \hat{q}_\alpha^{\max})$ and $(p, \hat{q}_0^{\max})$. A possible extension of the model described in Section 2 is the introduction of a public observable signal with state dependent distribution. It can be interpreted as the role played by "opinion leaders" or by the media, i.e. television, newspapers, Internet forums, etc. In this case, it seems reasonable that the quality of the signal should be lower with respect to the signals produced during the trial. Another possible extension is the introduction of different thresholds of reasonable doubt (as in Gerardi, 2000). In this framework does a heterogeneous jury performs better than a homogeneous one?

7 Appendix

Probabilistic Structure

Let $N = |J|$, the distribution of the signals profiles $t = (t_1, \ldots, t_N) \in \{i, g\}^N$ is defined as

$$f(t) = \mathbf{1}_0(t) \cdot p(0, N) + \mathbf{1}_k(t) \cdot p(k, N) + \mathbf{1}_N(t) \cdot p(N, N)$$

where $\mathbf{1}_0(t)$ is the indicator function of the zero vector,

$$\mathbf{1}_k(t) = \begin{cases} 1 & \text{if } \sum_{j=1}^N t_j = k \in \{1, 2, \ldots, N-1\} \\ 0 & \text{otherwise} \end{cases}$$

and $\mathbf{1}_N(t)$ is the indicator function of the vector with all components equal to 1. The corresponding probabilities are

$$p(0, N) = (1-\pi)\alpha + (1-\alpha)[\pi(1-z)^N + (1-\pi)z^N]$$

$$p(k, N) = (1-\alpha)[\pi z^k (1-z)^{N-k} + (1-\pi) z^{N-k} (1-z)^k]$$

$$p(N, N) = \alpha\pi + (1-\alpha)[\pi z^N + (1-\pi)(1-z)^N]$$

and the sum of the signals for $k \in \{1, 2, \ldots, N-1\}$ is distribuited as

$$g(k) = \binom{N}{j} \sum_{j=k}^{N-1} \pi(1-\alpha) z^j (1-z)^{N-j} + (1-\pi)(1-\alpha) z^{N-j} (1-z)^j$$

for $k = 0$

$$g(0) = \pi(1-\alpha)(1-z)^N + (1-\pi)[\alpha + (1-\alpha)z^N]$$

and for $k = N$

$$g(N) = \pi[\alpha + (1-\alpha)z^N] + (1-\pi)(1-\alpha)(1-z)^N$$

Explicitation of Condition $(\star)$

Conditions $(\star)$ holds if and only if between $\alpha \in (0, 1)$, $z \in (\frac{1}{2}, 1)$ and $p \in (\frac{1}{2}, 1)$ holds this relation

$$p = \frac{1}{2} - \sin\left(\frac{\arcsin(2(1-\alpha)[(1-z)^3 + 3z(1-z)^2] - 1)}{3}\right) \in \left(\frac{1}{2}, 1\right)$$

where

$$\arcsin(\cdot) : [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

is the inverse function of the $\sin(\cdot)$. This is the only admissible solution of the third degree equation derived by condition ($\star$). Table 1 reports some values of p for given α and z.

		α						
		0.000	0.100	0.300	0.500	0.700	0.900	1.000
	0.550	0.550	0.579	0.638	0.703	0.776	0.876	1.000
	0.600	0.600	0.625	0.676	0.733	0.798	0.887	1.000
	0.650	0.650	0.671	0.715	0.764	0.821	0.900	1.000
	0.700	0.700	0.717	0.755	0.796	0.845	0.913	1.000
	0.750	0.750	0.764	0.794	0.829	0.869	0.926	1.000
z	0.800	0.800	0.811	0.835	0.862	0.894	0.940	1.000
	0.850	0.850	0.858	0.876	0.896	0.920	0.954	1.000
	0.900	0.900	0.905	0.917	0.930	0.946	0.969	1.000
	0.950	0.950	0.953	0.958	0.965	0.973	0.984	1.000
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 1: Values of p for fixed z and α.

Proofs

Proof of Corollary 2.5

Trivial. $\square$

Proof of Proposition 3.3

(if part) Fix $\alpha \in (0, 1)$, $z \in (\frac{1}{2}, 1)$ and take an unbiased juror with $q \in (q_\alpha^{\min}, q_\alpha^{\max})$. Without loss of generality take the signals profiles $t_1 = (g, g, g)$ and $t_2 = (i, i, i)$. Since she observes all the signals the probability of being pivotal is one and so she compares her q with these two probabilities

$$\Pr(\mathrm{G}|\, t_1 = (g, g, g)) = \frac{\alpha + (1-\alpha)z^3}{\alpha + (1-\alpha)[z^3 + (1-z)^3]}$$

$$\Pr(\mathrm{G}|\, t_2 = (i, i, i)) = \frac{(1-\alpha)(1-z)^3}{\alpha + (1-\alpha)[z^3 + (1-z)^3]}$$

it results that $\Pr(\mathrm{G}|\, t_2) < q < \Pr(\mathrm{G}|\, t_1)$ and so she chooses to convict the defendant after observing t_1 and acquit him after observing t_2. If there exist a signal profile $\hat{t} \in T$ such that $\Pr(\mathrm{G}|\hat{t}) = q$ then the juror chooses to randomize her choice.

(only if) Let's assume that $q < q_\alpha^{\min}$ and suppose that the juror is responsive. It is easy to see that for any signals profiles $t \in T$ the parameter q is smaller than $\Pr(\mathrm{G}|t)$ and so the juror always chooses to convict the defendant, independently by the signals profile and so she is not responsive. For $q > q_\alpha^{\max}$ the reasoning is similar. $\square$

Proof of Proposition 3.4

Under the assumptions of the proposition and after making simple calculations it results that

$$\Pr(\mathrm{G}|piv, t_j = g) = z \quad \text{and} \quad \Pr(\mathrm{G}|piv, t_j = i) = 1 - z$$

for any $\hat{m} < \infty$. This means that for $q \in (1-z, z)$

$$\Pr(\mathrm{G}|piv, t_j = g) = z > q > 1 - z = \Pr(\mathrm{G}|piv, t_j = i)$$

and so for any $q \in (1-z, z)$ the informative strategy is a symmetric bayesian Nash equilibrium. $\square$

Proof of Proposition 3.5

The proof is by construction, for each $j \in J$ let's consider the symmetric strategy

$$\begin{cases} \text{if} \quad t_j = g \quad \text{then} \quad c \\ \text{if} \quad t_j = i \quad \text{then} \quad \sigma_j \in \Delta(S_j) \end{cases} \tag{1}$$

where $\sigma_j = \Pr(a|t_j = i)$. After observing $t_j = i$ the juror chooses to randomize if and only if she is indifferent, that is, if $\Pr(\mathrm{G}|piv, t_j = i) = q$. So if

$$\Pr(\mathrm{G}|piv, t_j = g) > q = \Pr(\mathrm{G}|piv, t_j = i)$$

this strategy is an equilibrium. Under the strategy described in (1)

$$\Pr(\mathrm{G}|piv, t_j = g) = \frac{1 - \sigma_j(1-z)}{2 - \sigma_j}$$

and

$$\Pr(\mathrm{G}|piv, t_j = i) = \frac{(1-\alpha)(1-z)^2 - \sigma_j(1-\alpha)(1-z)^3}{\alpha + (1-\alpha)[z^2 + (1-z)^2] - \sigma_j[\alpha + (1-\alpha)[z^3 + (1-z)^3]]}$$

since $\sigma_j \in (0,1)$, it results that

$$\Pr(\mathrm{G}|piv, t_j = g) \in \left(\frac{1}{2}, z\right) \tag{2}$$

Moreover it must be

$$\Pr(\mathrm{G}|piv, t_i = i) = \frac{(1-\alpha)(1-z)^2 - \sigma_j(1-\alpha)(1-z)^3}{\alpha + (1-\alpha)[z^2 + (1-z)^2] - \sigma_j[\alpha + (1-\alpha)[z^3 + (1-z)^3]]} = q$$

Now consider the equality

$$\sigma_j = \frac{(1-\alpha)(1-z)^2 - q[\alpha + (1-\alpha)[z^2 + (1-z)^2]]}{(1-\alpha)(1-z)^3 - q[\alpha + (1-\alpha)[z^3 + (1-z)^3]]} \tag{3}$$

since σ_j is positive, it could be

$$\begin{cases} (1-\alpha)(1-z)^2 - q[\alpha + (1-\alpha)[z^2 + (1-z)^2]] & > 0 \\ (1-\alpha)(1-z)^3 - q[\alpha + (1-\alpha)[z^3 + (1-z)^3]] & > 0 \end{cases} \tag{A}$$

or

$$\begin{cases} (1-\alpha)(1-z)^2 - q[\alpha + (1-\alpha)[z^2 + (1-z)^2]] & < 0 \\ (1-\alpha)(1-z)^3 - q[\alpha + (1-\alpha)[z^3 + (1-z)^3]] & < 0 \end{cases} \tag{B}$$

Case A

Let's consider case (A), it means that q must satisfy both these inequalities

$$q < \frac{(1-\alpha)(1-z)^2}{\alpha + (1-\alpha)[z^2 + (1-z)^2]}$$
$$q < \frac{(1-\alpha)(1-z)^3}{\alpha + (1-\alpha)[z^3 + (1-z)^3]}$$

but since for any $\alpha \in (0,1)$

$$1 > \frac{(1-\alpha)(1-z)^2}{\alpha + (1-\alpha)[z^2 + (1-z)^2]} > \frac{(1-\alpha)(1-z)^3}{\alpha + (1-\alpha)[z^3 + (1-z)^3]} > 0$$

we can restrict our attention to

$$q < \frac{(1-\alpha)(1-z)^3}{\alpha + (1-\alpha)[z^3 + (1-z)^3]} \tag{4}$$

moreover it must be

$$\sigma_j = \frac{(1-\alpha)(1-z)^2 - q[\alpha + (1-\alpha)[z^2 + (1-z)^2]]}{(1-\alpha)(1-z)^3 - q[\alpha + (1-\alpha)[z^3 + (1-z)^3]]} < 1$$

and this inequality is satisfied for

$$q > (1-z) \tag{5}$$

So q must satisfy both (4) and (5) in order to have $\sigma_j \in (0,1)$

$$(1-z) < q < \frac{(1-\alpha)(1-z)^3}{\alpha + (1-\alpha)[z^3 + (1-z)^3]} \tag{6}$$

and this is possibile if and only if

$$\alpha + (1-\alpha)z(2z-1) < 0$$

but $\alpha \in (0,1)$ and $(2z-1) > 0$ so case (A) must be discarded.

Case B

Let's consider case (B), it means that q must satisfy both these inequalities

$$q > \frac{(1-\alpha)(1-z)^2}{\alpha + (1-\alpha)[z^2 + (1-z)^2]}$$
$$q > \frac{(1-\alpha)(1-z)^3}{\alpha + (1-\alpha)[z^3 + (1-z)^3]}$$

but since for any $\alpha \in (0,1)$

$$1 > \frac{(1-\alpha)(1-z)^2}{\alpha + (1-\alpha)[z^2 + (1-z)^2]} > \frac{(1-\alpha)(1-z)^3}{\alpha + (1-\alpha)[z^3 + (1-z)^3]} > 0$$

we can restrict out attention to

$$q > \frac{(1-\alpha)(1-z)^2}{\alpha + (1-\alpha)[z^2 + (1-z)^2]} \tag{7}$$

moreover it must be

$$\sigma_j = \frac{q[\alpha + (1-\alpha)[z^2 + (1-z)^2]] - (1-\alpha)(1-z)^2}{q[\alpha + (1-\alpha)[z^3 + (1-z)^3]] - (1-\alpha)(1-z)^3} < 1$$

and this inequality is satisfied for

$$q < (1-z) \tag{8}$$

So q must satisfy both (7) and (8) in order to have $\sigma_j \in (0,1)$

$$\frac{(1-\alpha)(1-z)^2}{\alpha + (1-\alpha)[z^2 + (1-z)^2]} < q < (1-z) \tag{9}$$

and this is possibile if and only if

$$\alpha + (1-\alpha)z(2z-1) > 0$$

since $\alpha \in (0,1)$ and $(2z-1) > 0$, it is always true. From (2) and (8) the strategy (1) is feasible for any

$$q \in \left(\frac{(1-\alpha)(1-z)^2}{\alpha + (1-\alpha)[z^2 + (1-z)^2]}, 1-z \right)$$

and for any q in the interval

$$\Pr(\mathrm{G}|piv, t_j = g) > q$$

and

$$\Pr(\mathrm{G}|piv, t_j = i) = q$$

$\square$

Proof of Proposition 3.6
The proof is by construction, for each $j \in J$ let's consider the symmetric strategy

$$\begin{cases} \text{if} \quad t_j = g \quad \text{then} \quad \sigma_j \in \Delta(S_j) \\ \text{if} \quad t_j = i \quad \text{then} \quad a \end{cases} \tag{10}$$

where $\sigma_j = \Pr(a|t_j = g)$. After observing $t_j = g$ the juror chooses to randomize if and only if she is indifferent, that is, $\Pr(\mathrm{G}|piv, t_j = g) = q$. So if

$$\Pr(\mathrm{G}|piv, t_j = g) = q > \Pr(\mathrm{G}|piv, t_j = i)$$

the strategy is an equilibrium. Under the strategy described in (10)

$$\Pr(\mathrm{G}|piv, t_j = g) = \frac{(1-\alpha)z^2(1-z) + \sigma_j[\alpha + (1-\alpha)z^3]}{(1-\alpha)z(1-z) + \sigma_j[\alpha + (1-\alpha)[z^3 + (1-z)^3]]}$$

and

$$\Pr(\mathrm{G}|piv, t_j = i) = \frac{(1-z) + \sigma_j z}{1 + \sigma_j}$$

since $\sigma_j \in (0,1)$, it results that

$$\Pr(\mathrm{G}|piv, t_j = i) \in \left(1 - z, \frac{1}{2}\right) \tag{11}$$

Moreover it must be

$$\Pr(\mathrm{G}|piv, t_j = g) = \frac{(1-\alpha)z^2(1-z) + \sigma_j[\alpha + (1-\alpha)z^3]}{(1-\alpha)z(1-z) + \sigma_j[\alpha + (1-\alpha)[z^3 + (1-z)^3]]} = q$$

Now consider the equality

$$\sigma_j = \frac{(1-\alpha)z^2(1-z) - q(1-\alpha)z(1-z)}{q[\alpha + (1-\alpha)[z^3 + (1-z)^3]] - [\alpha + (1-\alpha)z^3]} \tag{12}$$

since σ_j is positive, it could be

$$\begin{cases} (1-\alpha)z^2(1-z) - q(1-\alpha)z(1-z) & > 0 \\ q[\alpha + (1-\alpha)[z^3 + (1-z)^3]] - [\alpha + (1-\alpha)z^3] & > 0 \end{cases} \tag{A}$$

or

$$\begin{cases} (1-\alpha)z^2(1-z) - q(1-\alpha)z(1-z) & < 0 \\ q[\alpha + (1-\alpha)[z^3 + (1-z)^3]] - [\alpha + (1-\alpha)z^3] & < 0 \end{cases} \tag{B}$$

Case A

Let's consider case (A), it means that q must satisfy both these inequalities

$$\frac{\alpha + (1-\alpha)z^3}{\alpha + (1-\alpha)[z^3 + (1-z)^3]} < q < z$$

but since for any $\alpha \in (0,1)$

$$z < \frac{\alpha + (1-\alpha)z^3}{\alpha + (1-\alpha)[z^3 + (1-z)^3]}$$

case (A) must be discarded.

Case B

Let's consider case (B), it means that q must satisfy both these inequalities

$$z < q < \frac{\alpha + (1-\alpha)z^3}{\alpha + (1-\alpha)[z^3 + (1-z)^3]}$$

and for any value of $\alpha \in (0,1)$

$$z < \frac{\alpha + (1-\alpha)z^3}{\alpha + (1-\alpha)[z^3 + (1-z)^3]}$$

moreover it must be

$$\sigma_j = \frac{q(1-\alpha)z(1-z) - (1-\alpha)z^2(1-z)^2}{\alpha + (1-\alpha)z^3 - q[\alpha + (1-\alpha)[z^3 + (1-z)^3]]} < 1$$

and this means that

$$z < q < \frac{\alpha + (1-\alpha)z^2}{\alpha + (1-\alpha)[z^2 + (1-z)^2} \tag{13}$$

for all $\alpha \in (0,1)$ it results

$$z < \frac{\alpha + (1-\alpha)z^2}{\alpha + (1-\alpha)[z^2 + (1-z)^2]} < \frac{\alpha + (1-\alpha)z^3}{\alpha + (1-\alpha)[z^3 + (1-z)^3]}$$

from (11) and (13) the strategy (10) is feasible for any

$$q \in \left(z, \frac{\alpha + (1-\alpha)z^2}{\alpha + (1-\alpha)[z^2 + (1-z)^2]} \right)$$

and for any q in the interval

$$\Pr(\mathrm{G}|piv, t_j = g) = q$$

and

$$\Pr(\mathrm{G}|piv, t_j = i) < q$$

$\square$

Proof of Proposition 3.7

Under the informative strategy the probability of convict an innocent is

$$\Pr(conv|\mathrm{I}) = \underbrace{(1-\alpha)\sum_{x=0}^{\hat{m}} \binom{2\hat{m}+1}{x} z^x (1-z)^{(2\hat{m}+1)-x}}_{\text{the minority of signals are } i} \tag{14}$$

and the probability of acquit a guilty is

$$\Pr(acq|\mathrm{G}) = \underbrace{(1-\alpha)\sum_{x=0}^{\hat{m}} \binom{2\hat{m}+1}{x} z^x (1-z)^{(2\hat{m}+1)-x}}_{\text{the minority of signals are } g} \tag{15}$$

Notice that

$$\sum_{x=0}^{\hat{m}} \binom{2\hat{m}+1}{x} z^x (1-z)^{(2\hat{m}+1)-x} = 1 - \sum_{x=\hat{m}+1}^{2\hat{m}+1} \binom{2\hat{m}+1}{x} z^x (1-z)^{(2\hat{m}+1)-x}$$

so

$$\Pr(conv|\mathrm{I}) = (1-\alpha)\left[1 - \sum_{x=\hat{m}+1}^{2\hat{m}+1} \binom{2\hat{m}+1}{x} z^x (1-z)^{(2\hat{m}+1)-x}\right] \tag{16}$$

and

$$\Pr(acq|\mathrm{G}) = (1-\alpha)\left[1 - \sum_{x=\hat{m}+1}^{2\hat{m}+1} \binom{2\hat{m}+1}{x} z^x (1-z)^{(2\hat{m}+1)-x}\right] \tag{17}$$

from Theorem 1 in Boland (1989) it results that

$$\lim_{|J|\to\infty} \Pr(conv|\mathrm{I}) = 0 \quad \text{and} \quad \lim_{|J|\to\infty} \Pr(acq|\mathrm{G}) = 0$$

□

Proof of Proposition 4.1

(if part) Fix $p \in (\frac{1}{2}, 1)$ and take an unbiased juror with $q \in (q_0^{\min}, q_0^{\max})$. Without loss of generality take the signals profiles $t_1 = (g, g, g)$ and $t_2 = (i, i, i)$. Since she observes all the signals the probability of being pivotal is one and so she compares her q with these two probabilities

$$\Pr(\mathrm{G}|\, t_1 = (g,g,g)) = \frac{p^3}{p^3 + (1-p)^3}$$

$$\Pr(\mathrm{G}|\, t_2 = (i,i,i)) = \frac{(1-p)^3}{p^3 + (1-p)^3}$$

it results that $\Pr(\mathrm{G}|\, t_2) < q < \Pr(\mathrm{G}|\, t_1)$ and so she chooses to convict the defendant after observing t_1 and acquit him after observing t_2. If there exist a signal profile $\hat{t} \in T$ such that $\Pr(\mathrm{G}|\hat{t}) = q$ then the juror chooses to randomize her choice.

(only if) Let's assume that $q < q_0^{\min}$ and suppose that the juror is responsive. It is easy to see that for any signals profiles $t \in T$ the parameter q is smaller than $\Pr(\mathrm{G}|t)$ and so the juror always chooses to convict the defendant, independently by the signals profile and so she is not responsive. For $q > q_0^{\max}$ the reasoning is similar. □

Proof of Proposition 4.2

Under the assumptions of the proposition and making simple calculations it results that

$$\Pr(\mathrm{G}|piv, t_j = g) = p \quad \text{and} \quad \Pr(\mathrm{G}|piv, t_j = i) = 1-p$$

for any $\hat{m} < \infty$. This means that for $q \in (1-p, p)$

$$\Pr(\mathrm{G}|piv, t_j = g) = p > q > 1-p = \Pr(\mathrm{G}|piv, t_j = i)$$

so for any $q \in (1-p, p)$ the informative strategy is a symmetric bayesian Nash equilibrium. □

Proof of Proposition 4.3

The proof is by construction, for each $j \in J$ let's consider the strategy

$$\begin{cases} \text{if} \quad t_j = g \quad \text{then} \quad c \\ \text{if} \quad t_j = i \quad \text{then} \quad \sigma_j \in \Delta(S_j) \end{cases} \tag{18}$$

where $\sigma_j = \Pr(a|t_j = i)$. After observing $t_j = i$ the juror chooses to randomize if and only if she is indifferent, that is, $\Pr(\mathrm{G}|piv, t_j = i) = q$. So if

$$Pr(\mathrm{G}|piv, t_j = g) > q = \Pr(\mathrm{G}|piv, t_j = i)$$

this strategy is an equilibrium. Under the strategy described in (18)

$$\Pr(\mathrm{G}|piv, t_j = g) = \frac{1-(1-p)\sigma_j}{2-\sigma_j}$$

and

$$\Pr(\mathrm{G}|piv, t_j = i) = \frac{(1-p)^2 - \sigma_j(1-p)^3}{[p^2+(1-p)^2] - \sigma_j[p^3+(1-p)^3]}$$

since $\sigma_j \in (0,1)$, it results that

$$\Pr(\mathrm{G}|piv, t_j = g) \in \left(\frac{1}{2}, p\right) \tag{19}$$

Moreover it must be

$$\Pr(\mathrm{G}|piv, t_j = i) = \frac{(1-p)^2 - \sigma_j(1-p)^3}{[p^2+(1-p)^2] - \sigma_j[p^3+(1-p)^3]} = q$$

Now consider the equality

$$\sigma_j = \frac{(1-p)^2 - q[p^2+(1-p)^2]}{(1-p)^3 - q[p^3+(1-p)^3]} \tag{20}$$

since σ_j is positive, it could be

$$\begin{cases} (1-p)^2 - q[p^2+(1-p)^2] & > 0 \\ (1-p)^3 - q[p^3+(1-p)^3] & > 0 \end{cases} \tag{A}$$

or

$$\begin{cases} (1-p)^2 - q[p^2+(1-p)^2] & < 0 \\ (1-p)^3 - q[p^3+(1-p)^3] & < 0 \end{cases} \tag{B}$$

Case A

Let's consider case (A), it means that q must satisfy both these inequalities

$$q < \frac{(1-p)^3}{p^3+(1-p)^3}$$
$$q < \frac{(1-p)^2}{p^2+(1-p)^2}$$

but since for any $p \in (\frac{1}{2}, 1)$

$$1 > \frac{(1-p)^2}{p^2+(1-p)^2} > \frac{(1-p)^3}{p^3+(1-p)^3} > 0$$

we can restrict our attention to

$$q < \frac{(1-p)^3}{p^3+(1-p)^3} \tag{21}$$

moreover it must be

$$\sigma_j = \frac{(1-p)^2 - q[p^2+(1-p)^2]}{(1-p)^3 - q[p^3+(1-p)^3]} < 1$$

and this inequality is satisfied for

$$q > (1-p) \tag{22}$$

So q must satisfy both (21) and (22) in order to have $\sigma_j \in (0,1)$ and this is possibile if and only if

$$p(1-2p) > 0$$

but $(1-2p) < 0$ so case (A) must be discarded.

Case B

Let's consider case (B), it means that q must satisfy both these inequalities

$$q > \frac{(1-p)^3}{p^3+(1-p)^3}$$
$$q > \frac{(1-p)^2}{p^2+(1-p)^2}$$

but since for any $p \in (\frac{1}{2}, 1)$

$$1 > \frac{(1-p)^2}{p^2+(1-p)^2} > \frac{(1-p)^3}{p^3+(1-p)^3} > 0$$

we can restrict our attention to

$$q > \frac{(1-p)^2}{p^2+(1-p)^2} \tag{23}$$

moreover it must be

$$\sigma_j = \frac{q[p^2+(1-p)^2]-(1-p)^2}{q[p^3+(1-p)^3]-(1-p)^3} < 1$$

and this inequality is satisfied for

$$q < (1-p) \tag{24}$$

So q must satisfy both (23) and (24) in order to have $\sigma_j \in (0,1)$

$$\frac{(1-p)^2}{p^2+(1-p)^2} < q < (1-p) \tag{25}$$

and this is possible if and only if

$$p(2p-1) > 0$$

since $(2p-1) > 0$, it is always true. From (19) and (24) the strategy (18) is feasible for any

$$q \in \left(\frac{(1-p)^2}{p^2+(1-p)^2}, 1-p\right)$$

and for any q in the interval

$$\Pr(\mathrm{G}|piv, t_j = g) > q$$

and

$$\Pr(\mathrm{G}|piv, t_j = i) = q$$

□

Proof of Proposition 4.4

The proof is by construction, for each $j \in J$ let's consider the symmetric strategy

$$\begin{cases} \text{if} \quad t_j = g \quad \text{then} \quad \sigma_j \in \Delta(S_j) \\ \text{if} \quad t_j = i \quad \text{then} \quad a \end{cases} \tag{26}$$

where $\sigma_j = \Pr(a|t_j = g)$. After observing $t_j = g$ the juror chooses to randomize if and only if she is indifferent, that is, $\Pr(\mathrm{G}|piv, t_j = g) = q$. So if

$$\Pr(\mathrm{G}|piv, t_j = g) = q > \Pr(\mathrm{G}|piv, t_j = i)$$

the strategy is an equilibrium. Under the strategy described in (26)

$$\Pr(\mathrm{G}|piv, t_j = g) = \frac{\sigma_j p^3 + p^2(1-p)}{\sigma_j[p^3+(1-p)^3] + p(1-p)}$$

and

$$\Pr(\mathrm{G}|piv, t_j = i) = \frac{(1-p) + \sigma_j p}{1 + \sigma_j}$$

since $\sigma_j \in (0,1)$, it results that

$$\Pr(\mathrm{G}|piv, t_j = i) \in \left(1 - p, \frac{1}{2}\right) \tag{27}$$

Moreover it must be

$$\Pr(\mathrm{G}|piv, t_j = g) = \frac{\sigma_j p^3 + p^2(1-p)}{\sigma_j[p^3 + (1-p)^3] + p(1-p)} = q$$

Now consider the equality

$$\sigma_j = \frac{p^2(1-p) - qp(1-p)}{q[p^3 + (1-p)^3] - p^3} \tag{28}$$

since σ_j is positive, it could be

$$\begin{cases} p^2(1-p) - qp(1-p) & > 0 \\ q[p^3 + (1-p)^3] - p^3 & > 0 \end{cases} \tag{A}$$

or

$$\begin{cases} p^2(1-p) - qp(1-p) & < 0 \\ q[p^3 + (1-p)^3] - p^3 & < 0 \end{cases} \tag{B}$$

Case A

Let's consider case (A), it means that q must satisfy both these inequalities

$$\frac{p^3}{p^3 + (1-p)^3} < q < p$$

but since for any $p \in (\frac{1}{2}, 1)$

$$p < \frac{p^3}{p^3 + (1-p)^3}$$

case (A) must be discarded.

Case B

Let's consider case (B), it means that q must satisfy both these inequalities

$$p < q < \frac{p^3}{p^3 + (1-p)^3} \tag{29}$$

and for any value of $p \in (\frac{1}{2}, 1)$

$$p < \frac{p^3}{p^3 + (1-p)^3}$$

moreover it must be

$$\sigma_j = \frac{qp(1-p) - p^2(1-p)}{p^3 - q[p^3 + (1-p)^3]} < 1$$

and this means that

$$p < q < \frac{p^2}{p^2 + (1-p)^2} \tag{30}$$

for all $p \in (\frac{1}{2}, 1)$ it results

$$p < \frac{p^2}{p^2 + (1-p)^2} < \frac{p^3}{p^3 + (1-p)^3}$$

from (27) and (30) the strategy (26) is feasible for any

$$q \in \left(p, \frac{p^2}{p^2 + (1-p)^2}\right)$$

and for any q in the interval

$$\Pr(\mathrm{G}|piv, t_j = g) = q$$

and

$$\Pr(\mathrm{G}|piv, t_j = i) < q$$

$\square$

Proof of Proposition 5.2

Take in consideration this system of inequalities

$$\begin{cases} \hat{q}_\alpha^{\min} & < \hat{q}_0^{\min} \\ \hat{q}_\alpha^{\max} & > \hat{q}_0^{\max} \end{cases}$$

that is

$$\frac{(1-\alpha)(1-z)^2}{\alpha + (1-\alpha)[z^2 + (1-z)^2]} < \frac{(1-p)^2}{p^2 + (1-p)^2} \tag{31}$$

and

$$\frac{\alpha + (1-\alpha)z^2}{\alpha + (1-\alpha)[z^2 + (1-z)^2]} > \frac{p^2}{p^2 + (1-p)^2} \tag{32}$$

notice that

$$(1-p)^2 z^2 < (1-z)^2 z^2 < (1-z)^2 p^2$$

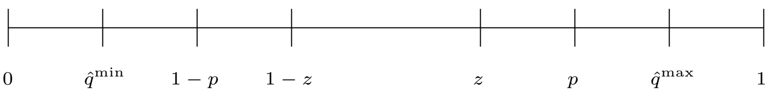
and so for

$$\hat{\alpha} > \frac{p^2(1-z)^2 - (1-p)^2 z^2}{(1-p)^2 + p^2(1-z)^2 - (1-p)^2 z^2}$$

both (31) and (32) hold. $\square$

Proof of Proposition 5.3

Consider the following intervals



i) for $q \in (0, \hat{q}^{\min})$, trivial.

ii) for $q \in (1-p, 1-z)$, in the α-model jurors use the equilibrium stategy described in proposition 3.5 with

$$\sigma_\alpha^* = \Pr(a|t_j = i) = \frac{q[\alpha + (1-\alpha)[z^2 + (1-z)^2]] - (1-\alpha)(1-z)^2}{q[\alpha + (1-\alpha)[z^3 + (1-z)^3]] - (1-\alpha)(1-z)^3} < 1$$

while in the 0-model the informative strategy is still an equilibrium. So

$$\begin{aligned} \mathrm{EU}_\alpha[v] &= -\frac{1}{2}(1-\alpha)[(1-z)^3 + 3z(1-z)^2] \\ &\quad -\frac{1}{2}[(1-\sigma_\alpha^*)^3 + 3\sigma_\alpha^*(1-\sigma_\alpha^*)^2][q[\alpha + (1-\alpha)[z^3 + (1-z)^3]] - (1-\alpha)(1-z)^3] \\ &\quad +\frac{3}{2}[(1-\sigma_\alpha^*)^2 + 2\sigma_\alpha^*(1-\sigma_\alpha^*)](1-\alpha)[(1-z) - q]z(1-z) \end{aligned}$$

$$\mathrm{EU}_0[v] = -\frac{1}{2}[(1-p)^3 + 3p(1-p)^2] = -\frac{1}{2}(1-\alpha)[(1-z)^3 + 3z(1-z)^2]$$

notice that

$$(1-\sigma_\alpha^*) = \frac{(1-\alpha)[(1-z)-q]z(1-z)}{q[\alpha+(1-\alpha)[z^3+(1-z)^3]]-(1-\alpha)(1-z)^3}$$

let's call

$$\begin{aligned}\theta_2 &= (1-\alpha)[(1-z)-q]z(1-z) > 0\\ \theta_1 &= q[\alpha+(1-\alpha)[z^3+(1-z)^3]]-(1-\alpha)(1-z)^3 > 0\end{aligned}$$

then

$$(1-\sigma_\alpha^*) = \frac{\theta_2}{\theta_1} \quad \text{and} \quad \theta_2 = (1-\sigma_\alpha^*)\,\theta_1$$

$$\begin{aligned}\mathrm{EU}_\alpha[v] &= -\frac{1}{2}(1-\alpha)[(1-z)^3+3z(1-z)^2]\\ &\quad -\frac{1}{2}[(1-\sigma_\alpha^*)^3+3\sigma_\alpha^*(1-\sigma_\alpha^*)^2]\,\theta_1\\ &\quad +\frac{3}{2}[(1-\sigma_\alpha^*)^2+2\sigma_\alpha^*(1-\sigma_\alpha^*)](1-\sigma_\alpha^*)\,\theta_1\end{aligned}$$

and so

$$\mathrm{EU}_\alpha[v] = -\frac{1}{2}(1-\alpha)\left[(1-z)^3+3z(1-z)^2\right]+\left[(1-\sigma_\alpha^*)^3+\frac{3}{2}\sigma_\alpha^*(1-\sigma_\alpha^*)^2\right]\theta_1$$

since condition $(\star)$ holds, it results

$$\begin{aligned}\mathrm{EU}_\alpha[v] &= -\frac{1}{2}(1-\alpha)\left[(1-z)^3+3z(1-z)^2\right]+\left[(1-\sigma_\alpha^*)^3+\frac{3}{2}\sigma_\alpha^*(1-\sigma_\alpha^*)^2\right]\theta_1\\ &= -\frac{1}{2}[(1-p)^3+3p(1-p)^2]+\left[(1-\sigma_\alpha^*)^3+\frac{3}{2}\sigma_\alpha^*(1-\sigma_\alpha^*)^2\right]\theta_1\\ &= \mathrm{EU}_0[v]+\left[(1-\sigma_\alpha^*)^3+\frac{3}{2}\sigma_\alpha^*(1-\sigma_\alpha^*)^2\right]\theta_1\end{aligned}$$

and then $\mathrm{EU}_\alpha[v] > \mathrm{EU}_\alpha[0]$.

iii) for $q \in (1-z, z)$, since $1-p < 1-z < q < z < p$ in both models the informative strategy is an equilibrium, since condition $(\star)$ holds, this equality also holds

$$\mathrm{EU}_\alpha[v] = -\frac{1}{2}(1-\alpha)[(1-z)^3+3z(1-z)^2] = -\frac{1}{2}[(1-p)^3+3p(1-p)^2] = \mathrm{EU}_0[v]$$

iv) for $q \in (z, p)$, in the α-model jurors use the equilibrium strategy described in proposition 3.6 with

$$\sigma_\alpha^* = \Pr(a|t_j = g) = \frac{(1-\alpha)(q-z)z(1-z)}{\alpha+(1-\alpha)z^3-q[\alpha+(1-\alpha)[z^3+(1-z)^3]]} < 1$$

while in the 0-model the informative strategy is still an equilibrium. So

$$\begin{aligned}\mathrm{EU}_\alpha[v] &= -\frac{1}{2}(1-\alpha)[(1-z)^3+3z(1-z)^2]\\ &\quad -\frac{1}{2}[(1-\sigma_\alpha^*)^3+3\sigma_\alpha^*(1-\sigma_\alpha^*)^2][\alpha+(1-\alpha)z^3-q[\alpha+(1-\alpha)[z^3+(1-z)^3]]\\ &\quad +\frac{3}{2}[\sigma_\alpha^{*2}+2\sigma_\alpha^*(1-\sigma_\alpha^*)](1-\alpha)(q-z)z(1-z)\end{aligned}$$

$$\mathrm{EU}_0[v] = -\frac{1}{2}[(1-p)^3+3p(1-p)^2] = -\frac{1}{2}(1-\alpha)[(1-z)^3+3z(1-z)^2]$$

let's call

$$\begin{aligned}\theta_4 &= (1-\alpha)(q-z)z(1-z) > 0\\ \theta_3 &= \alpha+(1-\alpha)z^3-q[\alpha+(1-\alpha)[z^3+(1-z)^3]] > 0\end{aligned}$$

then

$$\sigma^*_\alpha = \frac{\theta_4}{\theta_3} \quad \text{and} \quad \theta_4 = \sigma^*_\alpha\,\theta_3$$

$$\begin{aligned}
\mathrm{EU}_\alpha[v] &= -\frac{1}{2}(1-\alpha)[(1-z)^3 + 3z(1-z)^2] \\
&\quad -\frac{1}{2}[(1-\sigma^*_\alpha)^3 + 3\sigma^*_\alpha(1-\sigma^*_\alpha)^2]\,\theta_3 \\
&\quad +\frac{3}{2}[\sigma^{*2}_\alpha + 2\sigma^*_\alpha(1-\sigma^*_\alpha)]\sigma^*_\alpha\,\theta_3
\end{aligned}$$

and so

$$\mathrm{EU}_\alpha[v] = -\frac{1}{2}(1-\alpha)\left[(1-z)^3 + 3z(1-z)^2\right] + \left[\sigma^{*3}_\alpha + \frac{3}{2}\sigma^{*2}_\alpha(1-\sigma^*_\alpha)\right]\theta_3$$

since condition ($\star$) holds, it results

$$\begin{aligned}
\mathrm{EU}_\alpha[v] &= -\frac{1}{2}(1-\alpha)\left[(1-z)^3 + 3z(1-z)^2\right] + \left[\sigma^{*3}_\alpha + \frac{3}{2}\sigma^{*2}_\alpha(1-\sigma^*_\alpha)\right]\theta_3 \\
&= -\frac{1}{2}\left[(1-p)^3 + 3p(1-p)^2\right] + \left[\sigma^{*3}_\alpha + \frac{3}{2}\sigma^{*2}_\alpha(1-\sigma^*_\alpha)\right]\theta_3 \\
&= \mathrm{EU}_0[v] + \left[\sigma^{*3}_\alpha + \frac{3}{2}\sigma^{*2}_\alpha(1-\sigma^*_\alpha)\right]\theta_3
\end{aligned}$$

v) for $q \in (\hat{q}^{\max}, 1)$, trivial. □

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