Coalitional Extreme Desirability in Finitely Additive Economies with Asymmetric Information

Anuj Bhowmik and Francesca Centrone and Anna Martellotti

Indian Statistical Institute, Kolkata, Università del Piemonte Orientale, Università di Perugia

4 May 2016

COALITIONAL EXTREME DESIRABILITY IN FINITELY ADDITIVE ECONOMIES WITH ASYMMETRIC INFORMATION

ANUJ BHOWMIK, FRANCESCA CENTRONE, AND ANNA MARTELLOTTI

Abstract. We prove a coalitional core-Walras equivalence theorem for an asymmetric information exchange economy with a finitely additive measure space of agents, finitely many states of nature, and an infinite dimensional commodity space having the Radon-Nikodym property and whose positive cone has possibly empty interior. The result is based on a new cone condition, firstly developed in Centrone and Martellotti (2015), called *coalitional extreme desirability*. As a consequence, we also derive a new individualistic core-Walras equivalence result.

1. Introduction

Since the seminal paper of Radner (1968) a huge literature has grown in the area of Equilibrium Theory under Asymmetric Information, which allows for the possibility of having differently informed agents. From the mathematical point of view, the classical Arrow-Debreu exchange economy representation is thus enriched to take into account the informational aspects; namely, if Ω is a set of states of the world, each agent is endowed with a probability measure on Ω representing the agent's prior beliefs, an *ex-ante* utility function which depends on the possible states of the world, an initial endowment which specifies the agent's resources in each state, and a partition of Ω which represents the agent's initial information. The notion of Walras equilibrium, called *Walras expectation equilibrium*, is adapted to include the aforesaid informational aspects. The second notion of our paper, the *core*, allows for the possibility of cooperation among agents and is usually associated with Edgeworth. It is well recognized that the asymmetric context gives rise to different possibilities of sharing information among members of coalitions and thus, accordingly, different notions of core have been developed ([29, 30]).

In individual models, both the cases of a finite and an infinite dimensional commodity space have been treated, with various degrees of generality; most of these models assume anyway a countably additive measure space of agents, and a finite dimensional commodity space or a commodity space whose positive cone has nonempty interior, in order to apply classical separation theorems to support optimal allocations with nonnegative prices, refer to [4, 17, 20]. Only recently, Bhowmik

JEL classification: D41; D51; D82.
Keywords. and Phrases: Asymmetric information; Coalitional economies; Core-Walras equivalence; Extremely desirable commodity; Finitely additive measure; Walrasian expectation equilibria; Private core; Radon-Nikodym property.

([12]) has adapted Rustichini and Yannelis's ([27]) additivity condition and extremely desirable commodity assumption (which is very well known in the literature, together with Mas Colell's properness ([24]) and Chichilnisky and Kalman's cone condition ([14]), and is a widely used condition which allows a separation argument; see also [1] for a complete survey) to the asymmetric information framework, in a way to obtain a countably additive individualistic core-Walras equivalence theorem with an infinite dimensional commodity space, without assumptions on the positive cone.

Anyway, at this point, we must note that, also for the asymmetric information framework, Vind's ([28]) motivations to the use of coalitional models instead of individualistic ones, and Armstrong and Richter's ([5]) ones to the extension to a finitely additive context, are very applicable (for a complete overview, refer to [7, 13]). Given these frameworks, Forges et al. ([19]) criticised on the intrinsical coalitional nature of various core notions under asymmetry of information system, which then lead Basile et al. [7] to develop a notion of the core for coalitional economies, based on Yannelis's ([30]) *private information sharing rule.* As pointed out by these authors themselves, the just mentioned criticism can be overcome when private information of the coalitions is defined regardless of individuals contribution.

Despite these facts, to our knowledge, up to now there are no results in the framework of coalitional models with asymmetric information, to cover both the cases of a finitely additive space of agents and an infinite dimensional commodity space whose positive cone has possibly empty interior. Indeed, Basile et al. [7] work with a finitely additive Boolean algebra of agents, but with an Euclidean space of commodities. The aim of this work is to try to fill this gap, by introducing in the asymmetric context with a Banach lattice as the commodity space, the notion of *coalitional extremely desirable commodity*, which is the extension of that in [13] given for complete information. We obtain a coalitional asymmetric core-Walras equivalence result in a framework whose commodity space is X_+, the positive cone of a Banach lattice X having the Radon-Nikodym property (see [16]) and feasibility is defined as free disposal; note that this allows for a great variety of infinite dimensional commodity spaces interesting in economics and finance, for example, all the L_p spaces for $p > 1$. Since we are in asymmetric information framework, this result cannot be considered as a core-Walras equivalence theorem in a finitely additive asymmetric information economy with exact feasibility condition. In fact, the result in the exact feasibility case becomes more difficult to be obtained and it requires a new properness-like assumption. Consequently, this result and the corresponding individualistic result are the first infinite dimensional extensions to an asymmetric information framework with exact feasibility. We also point out that our results are not mere adaptations of the original definitions and results in [13], as the introduction of asymmetry and informational constraints makes it necessary to adopt new assumptions and techniques.

The rest of the paper is organized as follows: Section 2 deals with the description of our model, some assumptions and the necessary concepts. In Section 3, we introduce the notion of *coalitional extremely desirable commodity* in the asymmetric information framework and prove some technical lemmas that play central roles in the proofs of our main results. In Section 4, we present two alternative core-Walras equivalence theorems in coalitional models under the free disposal feasibility condition. Section 5 is devoted to some asymmetric individualistic results, deriving from our coalitional ones in the spirit of comprehensiveness of Armstrong and Richter ([5]). Lastly, we summarize and compare our results in Section 6. Along with this, all of our results in the case of exact feasibility are also studied in this section.

2. Description of the coalitional model

A coalitional model of pure exchange economy $\mathscr{E}_C$ with asymmetric information is presented. The exogenous uncertainty is described by a measurable space $(\Omega, \mathscr{F})$, where $\Omega = \{\omega_1, \cdots, \omega_n\}$ is the set of states of nature containing n elements and $\mathscr{F}$ denotes the power set of Ω. The economy extends over two time periods $\tau = 0, 1$. Consumption takes place at $\tau = 1$. At $\tau = 0$, there is uncertainty over the states and agents make contracts that are contingent on the realized state at $\tau = 1$. Let X be a Banach lattice having the Radon-Nikodym property (RNP) and a quasi-interior point. The partial order on X is denoted by $\leqslant$ and the positive cone of X, given by $X_+ = \{x \in X : 0 \leqslant x\}$, represents the commodity space of $\mathscr{E}_C$. The symbol $0 \ll x$ (resp. $0 < x$) means that x is a quasi-interior (resp. non-zero) point of X_+. Put $X_{++} = \{x \in X_+ : 0 \ll x\}$.

Let the space of agents be a space $(I, \Sigma, \mathbb{P})$, where I is the set of agents with Σ an the algebra on I and $\mathbb{P}$ a strongly non-atomic finitely additive (f.a.) probability measure on Σ, that is, for every $A \in \Sigma$ and $\varepsilon \in (0, 1)$ there is some $B \in \Sigma$ such that $B \subseteq A$ and $\mathbb{P}(B) = \varepsilon \mathbb{P}(A)$. Each element in Σ with positive probability is termed as a *coalition*, whose economic weight on the market is given by $\mathbb{P}$. If E and F are two coalitions, and $E \subseteq F$ then E is called a *sub-coalition* of F.

Analogously to Radner [26], we assume that assignment of resources are state-contingent. By an *assignment*, we mean a function $\alpha : \Sigma \times \Omega \to X_+$ such that $\alpha(\cdot, \omega)$ is a f.a. measure on Σ, for each $\omega \in \Omega$. Moreover, each assignment α can be associated with the function $\bar{\alpha} : \Sigma \to (X_+)^n$ by letting $\bar{\alpha}(E) = (\alpha(E, \omega_1), \cdots, \alpha(E, \omega_n))$, where $(X_+)^n$ is the positive cone of the Banach lattice X^n, which is endowed with the point-wise algebraic operations, the point-wise order and the product norm. We denote by $\leqslant^n$ the point-wise order on X^n. The only admissible assignments in our model are connected with some absolute continuity property. Recall that, given a Banach lattice Y and two vector measures $\mu : \Sigma \to X^k$ and $\nu : \Sigma \to Y$, μ is called *absolutely continuous with respect to* ν if for every $\varepsilon > 0$ there is some $\delta > 0$ such that each $F \in \Sigma$ with $\|\nu(F)\|_Y < \delta$ implies $\|\mu(F)\|_{X^k} < \varepsilon$. Let

$$\mathscr{M} = \{\alpha : \Sigma \times \Omega \to X_+ : \alpha \text{ is an assignment } \text{ and } \bar{\alpha} \ll \mathbb{P}\}.$$

Thus, an *allocation* is defined to be an element of $\mathscr{M}$. The initial endowment allocation, denoted by $e : \Sigma \times \Omega \to X_+$, is an element of $\mathscr{M}$ such that $e(F, \omega)$ is the initial endowment of the coalition F if the state of nature ω occurs. Similarly to Basile et al. [7], a *preference relation* $\succ_F$ is defined on $\mathscr{M}$ for any coalition F. Intuitively, $\alpha \succ_F \beta$ expresses the idea that the members of the coalition F prefer what they get from α to what they get from β. Each coalition F is also associated with some *private information*, which is described by a $\mathscr{F}$-measurable partition $\mathscr{P}_F$ of Ω. The interpretation is that, if ω is the true state of nature, then coalition F can not discriminate the states in the unique element $\mathscr{P}_F(\omega)$ of $\mathscr{P}_F$ containing ω. Let $\mathscr{F}_F$ be the σ-algebra generated by $\mathscr{P}_F$. The triple $(\mathscr{F}_F, \succ_F, e(F, \cdot))$ is called the *characteristics* of the coalition F. Thus, the economy can be described by

$$\mathscr{E}_C = \{(I, \Sigma, \mathbb{P}); X_+; (\Omega, \mathscr{F}); (\mathscr{F}_F, \succ_F, e(F, \cdot))_{F \in \Sigma}\}.$$

To relate the weight of coalitions to the commodities that they can trade on the market, we assume that e is equivalent to $\mathbb{P}$, that is, e and $\mathbb{P}$ are absolutely continuous with respect to each other. We now impose some restriction on the class of preferences. To this end, given an allocation $\alpha \in \mathscr{M}$ and a coalition F, define a vector measure $\bar{\alpha}_{|_F} : \Sigma \to (X_+)^n$ by letting $\bar{\alpha}_{|_F}(E) = \bar{\alpha}(E \cap F)$ for all $E \in \Sigma$. A *simple allocation* is any allocation s such that, for every $\omega \in \Omega$, $s(\cdot,\omega) = \sum_{i=1}^{q} y_i \mathbb{P}_{|_{H_i}}(\cdot,\omega)$, where $\{H_i\}_i$ is a decomposition of I. The following assumptions on preferences will be assumed implicitly throughout the rest of the paper:

[P.1] $\succ_F$ is irreflexive and transitive, for every $F \in \Sigma$;

[P.2] For any coalition F and $\alpha_1, \alpha_2 \in \mathscr{M}$ with $\alpha_1 \succ_F \alpha_2$, we must have $\alpha_1 \succ_G \alpha_2$ for all sub-coalitions G of F;

[P.3] If $\alpha_1 \succ_F \alpha_2$ and $\alpha_1 \succ_G \alpha_2$ for two coalitions F and G, then $\alpha_1 \succ_{F\cup G} \alpha_2$;

[P.4] For any $\alpha \in \mathscr{M}$ and any element $x \in (X_+)^n \setminus \{0\}$, we have $\alpha + x\mathbb{P} \succ_I \alpha$, where the allocation $x\mathbb{P} : \Sigma \times \Omega \to X_+$ is defined by $x\mathbb{P}(F,\omega) = x(\omega)\mathbb{P}(F)$;

[P.5] If $\alpha_1, \alpha_2, \alpha_3 \in \mathscr{M}$ and F is a coalition satisfying $\bar{\alpha}_{1_{|_F}} = \bar{\alpha}_{2_{|_F}}$, then the following double implications hold:

$$[\alpha_1 \succ_F \alpha_3 \iff \alpha_2 \succ_F \alpha_3] \text{ and } [\alpha_3 \succ_F \alpha_1 \iff \alpha_3 \succ_F \alpha_2].$$

Remark 2.1. Note that the transitivity assumption is very standard in coalitional models, refer to [15, 21]. Assumption [P.2] claims that as almost all member of F prefer what they get from α than what they get from β (refer to [15] for a deterministic economy), they do the same under G. Assumption [P.3] is similar to Assumption (VI) in [15]. The monotonicity assumption is discussed in [P.4], which is analogous to the assumption (WM) in [13]. It is worth pointing out that our monotonicity assumption is weaker than the one in [5] for a deterministic economy. Lastly, Assumption [P.5] is termed as *selfish property* in the literature, and can be found in [5, 6, 7].

Let $\mathfrak{P}$ denote the family of partitions of Ω such that for each $\mathscr{Q} \in \mathfrak{P}$ there is some non-empty set $F \subseteq I$ satisfying $\mathscr{P}_F = \mathscr{Q}$. Put, $I_{\mathscr{Q}} = \bigcup\{F : F \subseteq I \text{ and } \mathscr{P}_F = \mathscr{Q}\}$ for all $\mathscr{Q} \in \mathfrak{P}$. Thus, I is decomposed in the sets $I_{\mathscr{Q}}$, $\mathscr{Q} \in \mathfrak{P}$, and for every $\mathscr{Q} \in \mathfrak{P}$ and $F \subseteq I_{\mathscr{Q}}$, we have $\mathscr{P}_F = \mathscr{Q}$.

[A.1] $I_{\mathscr{Q}} \in \Sigma$ for all $\mathscr{Q} \in \mathfrak{P}$.

The following assumption is referred to as *nested condition* in the literature for information sharing rules in individualistic economies, refer to [2, 11, 22]. It also appeared in a coalitional model of Basile et al. [7], representing the intuitive idea that the state of information can never decrease if coalitions share their private information.

[A.2] For each coalition F and for each sub-coalition E of F, $\mathscr{F}_E \subseteq \mathscr{F}_F$.

Remark 2.2. To support [A.2], we assume that the information $\mathscr{F}_F$ of a coalition F is given by a rule that depends on the private information of each of its members. It is well known that the information of an agent can be different for different coalitions and it is captured by an information sharing rule, refer to [2, 11, 22]. Thus, the information of an agent t in F may be different from their initial private information. We can now define $\mathscr{F}_F$ to be the σ-algebra generated by common refinement of information partitions of members of F given by any information sharing rule. If the information sharing rule is nested (see, for example, Definition 5.3 in [2] or the assumption (P_2) in [11]) then [A.2] is trivially satisfied. For various information sharing rules satisfying the nested property, we refer to Table 1 in [11, p. 481].

Similarly to Basile et al. [7], we now restrict the set of consumption bundles that are informationally attainable for any coalition F, that is, the coalition F can not consume different amounts on events that it can not distinguish. Thus, the *consumption set* of a coalition F is the set of such restricted consumption bundles, which can be formally defined as

$$\mathscr{X}_F = \left\{x \in X_+^n : x \text{ is } \mathscr{F}_F\text{-measurable}\right\}.$$

An allocation α is said to be *privately feasible for a coalition* F whenever $\alpha(E, \cdot) \in \mathscr{X}_E$ for each coalition $E \subseteq F$. It means that any privately feasible allocation for a coalition F requires not only that the coalition F is able to distinguish what it consumes but also requires all sub-coalitions of it do the same thing. We denote the set of privately feasible allocations for a coalition F by $\mathscr{M}_F$. In the case when $F = I$, then we simply say $\mathscr{M}_I$ as the set of privately feasible allocations. We assume that e is privately feasible. An allocation α is termed as *physically feasible for a coalition* F if $\alpha(F, \omega) \underline{\ll} e(F, \omega)$ for all $\omega \in \Omega$. In particular, physically feasible allocations for I are simply referred to as physically feasible allocations. Finally, we say that an allocation is *feasible for a coalition* F if it is privately as well as physically feasible for F, and the set of such allocations is denoted by $\mathscr{Y}_F$. Without any confusion, feasibility for I will be termed as *feasibility*.

Definition 2.3. An allocation α is *privately blocked by a coalition* F if there is an allocation $\beta \in \mathscr{Y}_F$ such that $\beta \succ_F \alpha$. The *private core* of $\mathscr{E}_C$, denoted by $\mathscr{PC}(\mathscr{E}_C)$, is the set of feasible allocations which are not privately blocked by any coalition.

A *price system* is a non-zero function $\pi : \Omega \to X_+^*$, where X_+^* is the positive cone of the norm-dual X^* of X. The *budget set* of a coalition F with respect to a price system π is defined by

$$\mathscr{B}(F, \pi) = \left\{\alpha \in \mathscr{M}_F : \sum_{i=1}^{n} \pi(\omega_i)\alpha(F, \omega_i) \leq \sum_{i=1}^{n} \pi(\omega_i)e(F, \omega_i)\right\}.$$

Analogously to the private core, the definition of Walras equilibrium also takes into account the information structure.

Definition 2.4. A *Walrasian expectations equilibrium* of $\mathscr{E}_C$ is a pair (α, π) where α is a feasible allocation and π is a price system such that

(i) $\alpha \in \mathscr{B}(F, \pi)$ for each coalition $F \in \Sigma$;
(ii) $\displaystyle\sum_{i=1}^{n} \pi(\omega_i)\alpha(I, \omega_i) = \sum_{i=1}^{n} \pi(\omega_i)e(I, \omega_i)$.
(iii) for every coalition F and $\beta \in \mathscr{M}_F$, $\beta \succ_F \alpha \Longrightarrow \beta \notin \mathscr{B}(F, \pi)$;

In this case, α is termed as *Walrasian expectations allocation* and the set of such allocations is denoted by $\mathscr{W}(\mathscr{E}_C)$.

3. Some technical results

In this section, we establish some technical lemmas for the later use.

Lemma 3.1. *Under assumptions* [A.1]-[A.2], *if* $\alpha \in \mathscr{M}_F$ *for some coalition* F, *then for each* $\varepsilon > 0$ *there exists a simple allocation* $s \in \mathscr{M}_F$ *such that* $\|\bar{\alpha} - \bar{s}\| < \varepsilon$.

Proof. Choose the element $A_1 \in \mathscr{P}_F$ such that $\omega_1 \in A_1$. Suppose that i_2 is the smallest element in $\{1, \cdots, n\}$ such that $\omega_{i_2} \notin A_1$. Let A_2 be the element in $\mathscr{P}_F$ containing ω_{i_2}. Assume that i_3 is the smallest element in $\{1, \cdots, n\}$ such that $\omega_{i_3} \notin A_1 \cup A_2$. Applying this argument for finitely many times, we obtain a set $\Omega_0 := \{\omega_{i_1}, \cdots, \omega_{i_m}\}$, where $1 = i_1 < i_2 < \cdots < i_m$, such that $\mathscr{P}_F = \{A_1, \cdots, A_m\}$. Consequently, we have a bijective mapping $\varphi : \Omega_0 \to \mathscr{P}_F$ defined by $\varphi(\omega_{i_j}) = A_j$, for all $1 \le j \le m$. Consider an information partition $\mathscr{Q} = \{B_1, \cdots, B_l\}$ such that $\mathscr{P}_F$ is a refinement of $\mathscr{Q}$. Thus, for each $B_j \in \mathscr{Q}$ there is some $A_k \in \mathscr{P}_F$ such that $A_k \subseteq B_j$ and thus, $\omega_{i_k} \in \Omega_0 \cap B_j$. Furthermore, $l \le m$ and it is possible that two different elements of Ω_0 belong to the same element of $\mathscr{Q}$. Define a partition $\bar{\mathscr{Q}} = \{\bar{B}_1, \cdots, \bar{B}_l\}$ of Ω_0 by letting $\bar{B}_i = B_i \cap \Omega_0$, for all $1 \le i \le l$. Conversely, for every partition $\bar{\mathscr{R}} = \{\bar{C}_1, \cdots, \bar{C}_s\}$ of Ω_0, we can associate a partition $\mathscr{R} = \{C_1, \cdots, C_s\}$ of Ω by letting

$$C_k = \bigcup \{A_j : \omega_{i_j} \in \bar{C}_k\}.$$

Let $\{\mathscr{Q}_1, \cdots, \mathscr{Q}_r\} \subseteq \mathfrak{P}$ be the set of all information partitions such that $\mathbb{P}(F \cap I_{\mathscr{Q}_k}) > 0$ for all $1 \le k \le r$. Analogous to above, we can again obtain a subset $\Omega_k := \{\omega_{1_k}, \cdots, \omega_{l_k}\} \subseteq \Omega_0$ with $1 = 1_k < 2_k < \cdots < l_k$ and a bijective mapping $\psi_k : \Omega_k \to \bar{\mathscr{Q}}_k$, for all $1 \le k \le r$. According to the approximate Radon-Nikodym Theorem [16], we can choose a set $\{s_1, \cdots, s_n\}$ of n simple functions such that

$$|\alpha(\cdot, \omega_i) - s_i| < \frac{\varepsilon}{2nr},$$

for all $1 \le i \le n$. Define $s_k^1 : \Sigma \times \Omega_k \to X_+$ and $s_k^2 : \Sigma \times \Omega \to X_+$ by letting

$$s_k^1(E, \omega) = s_{i_k}(\omega_{i_k}) \text{ if } \omega \in \psi_k(\omega_{i_k})$$

and

$$s_k^2(E, \omega) = s_k^1(E, \omega') \text{ if } \omega \in \varphi(\omega').$$

Consider an allocation $s : \Sigma \times \Omega \to X_+$ defined by

$$s(E, \omega_i) = \sum_{k=1}^{r} s_k^2(E \cap F \cap I_{\mathscr{Q}_i}, \omega_i) + s_i(E \cap (I \setminus F)).$$

Using $\|\bar{\alpha} - \bar{s}\| = \sum_{i=1}^{n} \|\alpha(\cdot, \omega_i) - s(\cdot, \omega_i)\|$, we obtain

$$\begin{aligned}
\|\bar{\alpha} - \bar{s}\| &\le \sum_{i=1}^{n} \sum_{k=1}^{r} |\alpha(\cdot, \omega_i)|_{F \cap I_{\mathscr{Q}_i}} - s(\cdot, \omega_i)|_{F \cap I_{\mathscr{Q}_i}}| + n\frac{\varepsilon}{2nr} \\
&\le \sum_{i=1}^{n} r\frac{\varepsilon}{2nr} + \frac{\varepsilon}{2r} \\
&< \varepsilon,
\end{aligned}$$

which completes the proof. ☐

For a fixed allocation $\alpha \in \mathscr{M}_F$ and a coalition F, let

$$\mathscr{K} = \bigcup_{F \in \Sigma,\ \mathbb{P}(F)>0} \{\bar{\gamma}(F) - \bar{e}(F) : \gamma \in \mathscr{M}_F, \gamma \succ_F \alpha\}.$$

Our next technical results and main theorems require some continuity-like assumptions. Here, we employ assumptions similar to those in [13]. Given these assumptions, our proof for the next lemma exactly follows analogous arguments of the final step of Lemma 3.3 in [13], taking into account Lemma 3.1. Thus, we plan to skip the formal proof for this result.

[A.3] For every choice of a coalition F, a number $\tau > 0$ and allocations $\alpha, \beta \in \mathscr{M}_F$ with $\beta \succ_F \alpha$, there exists a coalition $F_0 \subseteq F$ with $\mathbb{P}(F \setminus F_0) < \tau$, and a number $\rho(\tau) > 0$ such that $s \succ_{F_0} \alpha$ for every simple allocation $s \in \mathscr{M}_F$ satisfying $\|\bar{s} - \bar{\beta}\| < \rho(\tau)$.

[A*.3] Let F be a coalition and $\alpha, \beta \in \mathscr{M}_F$ be such that $\beta \succ_F \alpha$. For every $\tau > 0$,

(i) we can find some $\rho(\tau) > 0$ such that for every simple allocation $s \in \mathscr{M}_F$ with $\|\bar{s} - \bar{\beta}\| < \rho$ there exists a coalition $F_0 = F_0(s, \tau) \subseteq F$ with $\mathbb{P}(F \setminus F_0) < \tau$ and $s \succ_{F_0} \alpha$;

(ii) there exist an $\varepsilon \in (0, 1)$ and a coalition $F_0 = F_0(\tau) \subset F$ such that $\mathbb{P}(F \setminus F_o) < \tau$ and $\varepsilon\beta \succ_{F_0} \alpha$.

Lemma 3.2. *Suppose that $\mathscr{E}_C$ satisfies* [A.1] *and* [A.3]*. Then the set $\overline{\mathscr{K}}$ is convex, where $\overline{\mathscr{K}}$ denotes the norm-closure of $\mathscr{K}$ in X^n.*

It is well-known that an affirmative answer to the classical core-Walras equiva lence result in a framework of a Banach lattice as the commodity space can not be obtained without any "properness-like"assumption (refer to [27]). In our model, we suitably extend the *extremely desirable commodity* assumption of [13]. To this aim, consider the auxiliary economy (compare with [20])

$$\mathscr{E}^n = \{(I, \Sigma, \mathbb{P}); (X_+)^n, (\succ_F^n, \bar{e}(F))_{F \in \Sigma}\}$$

where, for each coalition F, the preference relation $\succ_F^n$ is defined as $\bar{\alpha} \succ_F^n \bar{\beta} \iff \alpha \succ_F \beta$ for $\alpha, \beta \in \mathscr{M}$. Thus, we are ready to give an extended version of extremely desirable commodity assumption.

[A.4] There exist some $u \in (X_+)^n$ and an open, convex, solid neighborhood U of 0 in X^n such that the following two conditions are satisfied: (i) $U^c \cap (X_+)^n$ is convex, where U^c is the complement of U in X^n; and (ii) If $y \in X_+^n$ and $z \in \overline{(y + C_u)} \cap X_+^n$, then $z\mathbb{P} \succ_I^n y\mathbb{P}$, where

$$C_u = \bigcup \{t(u + U) : t > 0\}.$$

In other words, we are requiring that u is an extremely desirable commodity for the coalitional preferences in the economy $\mathscr{E}^n$, in the sense of [13]. In the rest of the paper, we shall refer to (u, U) as a *properness pair.* To prove our next result, given (u, U) is a properness pair, we now find other possible properness pairs (w, W). Observe first that, if $\hat{u} \geqq^n u$ then $(\hat{u}, U)$ is a properness pair as well. Indeed, let $y \in X_+^n$ and $z \in \overline{(y + C_{\hat{u}})} \cap X_+^n$. Pick an $\varepsilon > 0$. It follows that $B(z, \varepsilon) \cap (y + t(\hat{u} + U)) \neq \emptyset$ for some $t > 0$, where $B(z, \varepsilon)$ denotes the open ball in X^n centered at z with radius ε. Thus,

$$B(z, \varepsilon) \cap (y + t(\hat{u} - u) + t(u + U)) \neq \emptyset.$$

Consequently, $z \in \overline{(y+t(\hat{u}-u)+C_u)} \cap X^n_+$. So, $z\mathbb{P} \succ^n_I (y+t(\hat{u}-u))\mathbb{P}$. By [P.1] and [P.4], we conclude $z\mathbb{P} \succ^n_I y\mathbb{P}$. As a result, we can replace the original extremely desirable commodity $u=(u_1,\cdots,u_n)$ with $w=(w_o,\cdots,w_o)$, where

$$w_o = \left(\bigvee\{u_i : 1 \le i \le n\}\right) \vee \left(\bigvee\{|\mathfrak{P}|e(I_{\mathscr{Q}},\omega_i) : \mathscr{Q} \in \mathfrak{P}, 1 \le i \le n\}\right),$$

so that the allocation $w\mathbb{P} \in \mathscr{M}_I$. Henceforth, the vector w will be used instead of u in the extreme desirability assumption. Define

$$K = \bigcup \left\{ t\left(w + \frac{1}{n}U\right) : t > 0 \right\}.$$

As $\frac{1}{n}U \subseteq U$, we must have $K \subseteq C_w$. Let $y \in X^n_+$ and $z \in \overline{(y+K)} \cap X^n_+$. It follows that $z \in \overline{(y+C_w)} \cap X^n_+$. Consequently, $z\mathbb{P} \succ^n_I y\mathbb{P}$.

Lemma 3.3. *Assume that* [A.1]-[A.4] *hold. If α is a private core allocation, then $\overline{\mathscr{K}} \cap (-K) = \emptyset$.*

Proof. Since $-K$ is open, it is enough to prove that $\mathscr{K} \cap (-K)$ is empty. Assume $\mathscr{K} \cap (-K) \neq \emptyset$ and that

$$\zeta = \bar{\gamma}(F) - \bar{e}(F) \in -K$$

for some coalition F. Pick an $\varepsilon > 0$ such that $\zeta + B(0,\varepsilon) \subset -K$, where $B(0,\varepsilon)$ is the open ball in X^n centered at the origin and radius ε. By the absolute continuity of γ and e with respect to $\mathbb{P}$, there exists some $\delta > 0$ such that for all $E \in \Sigma$, $\|\gamma(E)\|, \|e(E)\| < \frac{\varepsilon}{7}$ whenever $\mathbb{P}(E) < \delta$. In the light of [A.3], we can find some coalition $F_0 \subseteq F$ and a number $\rho > 0$ such that $\mathbb{P}(F \setminus F_0) < \delta$, and $s \succ_{F_0} \alpha$ for every simple allocation $s \in \mathscr{M}_F$ satisfying $\|\bar{s} - \bar{\gamma}\| < \rho$. By Lemma 3.1, there exists a simple allocation $\bar{s}_0 = \sum_{i=1}^m y_i \mathbb{P}_{|F_i}$ whose values lie in X^n, where $\{F_i : 1 \le i \le m\}$ is a decomposition of F_0, such that

(i) $\bar{s}_0 \in \mathscr{M}_{F_0}$;
(ii) $\|\bar{\gamma} - \bar{s}_0\| < \min\left\{\frac{\varepsilon}{7}, \rho\right\}$;
(iii) for each $1 \le i \le m$, there is some $\mathscr{Q} \in \mathfrak{P}$ such that $F_i \subseteq I_{\mathscr{Q}}$.

Hence, $\bar{s}_0 \succ^n_{F_0} \bar{\alpha}$. Put $\zeta_0 = \bar{s}_0(F_0) - \bar{e}(F_0)$, then

$$\begin{aligned} \|\zeta_0 - \zeta\| &< \|\bar{s}_0(F_0) - \bar{\gamma}(F_0)\| + \|\bar{\gamma}(F \setminus F_0)\| + \|\bar{e}(F \setminus F_0)\| \\ &< \frac{3\varepsilon}{7}. \end{aligned}$$

We assume that $\mathbb{P}(F_i) = \xi$ for all $1 \le i \le m$[1]. Since $\zeta_0 \in -K$, there exists some $t > 0$ such that

$$\bar{s_0}(F_0) - \bar{e}(F_0) \in -t\left(w + \frac{1}{n}U\right),$$

[1]Otherwise, it follows from Lemma 3.1 in [13] that there are a subset $E_0 \subseteq F_0$ with $\mathbb{P}(F_0 \setminus E_0) < \delta$ and a decomposition $\{E_1,\cdots,E_k\}$ of E_0 with $\mathbb{P}(E_i) = \xi$ for all $1 \le i \le k$. If $\zeta' = \bar{s}_0(E_0) - \bar{e}(E_0)$, then

$$\begin{aligned} \|\zeta - \zeta'\| &< \|\zeta - \zeta_0\| + \|\zeta_0 - \zeta'\| \\ &< \frac{3\varepsilon}{7} + \|\bar{s}_0(F_0) - \bar{\gamma}(F_0)\| + \|\bar{\gamma}(F_0 \setminus E_0)\| + \|\bar{s}_0(E_0) - \bar{\gamma}(E_0)\| + \|\bar{e}(F_0 \setminus E_0)\| \\ &< \varepsilon. \end{aligned}$$

As a result, $\zeta' = \bar{s}_0(E_0) - \bar{e}(E_0) \in -K$ with $\bar{s}_0 \succ^n_{E_0} \bar{\alpha}$.

whence $\bar{s}_0(F_0) - \bar{e}(F_0) = -t(w + v_0)$ for some $v_0 \in \frac{1}{n}U$. By setting $z = \frac{t}{\xi}w$ and $v = -\frac{t}{\xi}v_o \in \frac{t}{\xi n}U$, we have

$$\sum_{i=1}^{m} y_i + z - v = \frac{\bar{e}(F_0)}{\xi} \in X_+^n.$$

Since $\sum_{i=1}^m y_i + z \gg^n 0$ and $\frac{t}{\xi n}U$ is solid, we have $v^+ \in \frac{t}{\xi n}U$ and $\sum_{i=1}^m y_i + z \gg^n v^+$. For any m-tuple $\sigma = (\sigma_1, \cdots, \sigma_m)$ of positive real numbers with $\sum_{i=1}^m \sigma_i = 1$, we have

$$v^+ \ll^n \sum_{i=1}^{m} (y_i + \sigma_i z).$$

By the Riesz decomposition property, we obtain a finite set $\{v_1^\sigma, \cdots, v_m^\sigma\} \subseteq X_+^n$ such that

$$v^+ = \sum_{i=1}^{m} v_i^\sigma \text{ and } v_i^\sigma \ll^n y_i + \sigma_i z$$

for all $1 \le i \le m$. Define $\Lambda_{\mathscr{Q}} = \{i : F_i \subseteq I_{\mathscr{Q}}\}$ for all $\mathscr{Q} \in \mathfrak{P}$. Thus, $y_i + \sigma_i z$ is $\mathscr{Q}$-measurable if $i \in \Lambda_{\mathscr{Q}}$. For $i \in \Lambda_{\mathscr{Q}}$, define the function $d_i^\sigma : \Omega \to X_+$ by letting

$$d_i^\sigma(\omega) = \max\{v_i(\omega') : \omega' \in \mathscr{Q}(\omega)\},$$

for all $\omega \in \Omega$. So, d_i^σ is $\mathscr{Q}$-measurable and $d_i^\sigma \ll y_i + \sigma_i z$ for all $i \in \Lambda_{\mathscr{Q}}$. Given any $1 \le i \le m$, put

$$\delta_i^\sigma = \text{dist}\left(y_i + \sigma_i z - d_i^\sigma, (y_i + C_w) \cap X_+^n\right),$$

and consider the continuous function $f : \Delta^m \to \Delta^m$ defined by

$$f(\sigma) = \left(\frac{\sigma_1 + \delta_1^\sigma}{1 + \sum_{j=1}^m \delta_j^\sigma}, \cdots, \frac{\sigma_m + \delta_m^\sigma}{1 + \sum_{j=1}^m \delta_j^\sigma} \right),$$

where Δ^m denotes the $(m-1)$-dimensional simplex. By Brouwer's fixed point theorem, one obtains a $\sigma^* = (\sigma_1^*, \cdots, \sigma_m^*) \in \Delta^m$ satisfying $\delta_i^{\sigma^*} = \sigma_i^* \sum_{j=1}^m \delta_j^{\sigma^*}$ for all $1 \le i \le m$. The rest of the proof is decomposed into two sub-cases.

Sub-case 1. $\delta_i^{\sigma^*} = 0$ *for all* $1 \le i \le m$. In this sub-case,

$$y_i + \sigma_i^* z - d_i^{\sigma^*} \in \overline{(y_i + C_w) \cap X_+^n} \subseteq \overline{(y_i + C_w)} \cap X_+^n$$

for all $1 \le i \le m$. By (ii) of [A.4], we obtain $(y_i + \sigma_i^* z - d_i^{\sigma^*})\mathbb{P} \succ_I^n y_i\mathbb{P}$ for all $1 \le i \le m$. Thus, it follows from [P.2] that $(y_i + \sigma_i^* z - d_i^{\sigma^*})\mathbb{P} \succ_{F_i}^n y_i\mathbb{P}$ for all $1 \le i \le m$. Consequently, applying [P.3], we have

$$\sum_{i=1}^{m} (y_i + \sigma_i^* z - d_i^{\sigma^*})\mathbb{P}_{|F_i} \succ_{F_0}^n \sum_{i=1}^{m} y_i \mathbb{P}_{|F_i} = \bar{s_0} \succ_{F_0}^n \bar{\alpha}.$$

In other words, setting $\bar{s}_1 = \sum_{i=1}^m (y_i + \sigma_i^* z - d_i^{\sigma^*})\mathbb{P}_{|F_i} + \bar{\alpha}_{|I \setminus F_0}$, then $\bar{s}_1 \succ_{F_0}^n \bar{\alpha}$ and

$$\bar{s}_1(F_0) \ll^n \xi \left(\sum_{i=1}^{m} y_i + z - v^+ \right) \ll^n \xi \left(\sum_{i=1}^{m} y_i + z - v \right) = \bar{e}(F_0).$$

Hence, $\alpha \notin \mathscr{PC}(\mathscr{E}_C)$, which is a contradiction.

Sub-case 2. $\sum_{j=1}^{m} \delta_j^{\sigma^*} > 0$. In this sub-case, $\delta_i^{\sigma^*} = 0$ if and only if $\sigma_i^* = 0$. Define $J = \{i : \delta_i^{\sigma^*} = 0\}$. Pick an $i \in J$, then

$$y_i - d_i^{\sigma^*} \in \overline{(y_i + C_w)} \cap X_+^n.$$

From (ii) of [A.4], we conclude

$$(y_i - d_i^{\sigma^*})\mathbb{P} \succ_I^n y_i\mathbb{P}.$$

If $d_i^{\sigma^*} >^n 0$, then $(y_i - d_i^{\sigma^*})\mathbb{P} \succ_I^n y_i\mathbb{P}$ yields a contradiction. Thus, $d_i^{\sigma^*} = 0$ for all $i \in J$. Pick an $i \notin J$, then

$$y_i + \sigma_i^* z - d_i^{\sigma^*} \notin (y_i + C_w) \cap X_+^n.$$

Consequently,

$$y_i + \frac{\sigma_i^* t}{\xi} w - d_i^{\sigma^*} \notin y_i + C_w,$$

which further implies that $d_i^{\sigma^*} \notin \dfrac{\sigma_i^* t}{\xi} U$ and so, by (i) of [A.4], $\sum_{i=1}^{m} d_i^{\sigma^*} \notin \dfrac{t}{\xi} U$. Note that $d_i^{\sigma^*} \leqslant^n \sum_{\omega\in\Omega} v_i^{\sigma^*}(\omega)\mathbf{1}_\Omega$, where $\mathbf{1}_\Omega = (1, \cdots, 1)$, and so, by (i) of [A.4],

$$\sum_{i=1}^{m} d_i^{\sigma^*} \leqslant^n \sum_{\omega\in\Omega} v^+(\omega)\mathbf{1}_\Omega.$$

Since $\sum_{\omega\in\Omega} v^+(\omega)\mathbf{1}_\Omega \in \dfrac{t}{\xi} U$ and $\dfrac{t}{\xi} U$ is solid, we must have $\sum_{i=1}^{m} d_i^{\sigma^*} \in \dfrac{t}{\xi} U$, which is a contradiction. □

Remark 3.4. It can be easily checked that assumption [A.3] can be replaced with (i) of [A*.3] in Lemma 3.2 and Lemma 3.3.

4. Coalitional core-Walras equivalence

In this section, we provide core-Walras equivalence theorems for the model described in Section 2. To obtain the first main result, we use the following assumption on the initial endowment.

[A.5] $e(I_{\mathscr{Q}}, \omega) \gg 0$ for all $\mathscr{Q} \in \mathfrak{P}$ and $\omega \in \Omega$.

We are now ready to state our first core-Walras equivalence Theorem.

Theorem 4.1. *Suppose that $\mathscr{E}_C$ satisfies* [A.1]-[A.5]*, and let α be a private core allocation. Then there exists an equilibrium price for α.*

Proof. Applying Lemma 3.2 and Lemma 3.3 together with the separation theorem, we can find an n-tuple $p = (p_1, \cdots, p_n) \in (X_+^*)^n$ that separates $\overline{\mathscr{K}}$ and $-K$. As usual, this would yield that $px \geq 0$ for every $x \in \mathscr{K}$. Define $\pi : \Omega \to X_+^*$ by letting $\pi(\omega_i) = p_i$ for all $1 \leq i \leq n$. To show that (α, π) is a competitive equilibrium of $\mathscr{E}_C$, we need to verify conditions (i)-(iii) of Definition 2.4. By invoking arguments similar to those of [7], items (i) and (ii) of Definition 2.4 can be proved. Thus, we now turn to prove assertion (iii). Observe first that (i) and (ii) together imply $p\bar{\alpha} = p\bar{e}$ on Σ. Suppose (iii) is not true and that there are a coalition E and an allocation $\beta \in \mathscr{M}_E$ such that $\beta \succ_E \alpha$ and $p[\bar{\beta}(E)] = p[\bar{e}(E)]$. The rest of the proof is decomposed in the following two cases:

Case 1. $p[\bar{\beta}(E)] > 0$. Then there exists some $\tau > 0$ such that for each subcoalition F of E with $\mathbb{P}(E \setminus F) < \tau$, we have $p[\bar{\beta}(F)] > 0$. It follows from [P.2] that

$\beta \succ_F \alpha$. Moreover, [A.3] suggests that there exist some coalition $F_0 \subseteq E$ and a number $\rho > 0$ such that $\mathbb{P}(F \setminus F_0) < \tau$, and $s_0 \succ_{F_0} \alpha$ for every simple allocation $s_0 \in \mathscr{M}_E$ satisfying $\|\bar{s}_0 - \bar{\beta}\| < \frac{\rho}{3}$. Let s be a simple allocation such that

$$\bar{s} = \sum_{i=1}^{\ell} x_i \mathbb{P}_{|F_i} \text{ and } \|\bar{s} - \bar{\beta}\| < \frac{\rho}{3},$$

where $\{F_i : 1 \leq i \leq \ell\}$ is a decomposition of F_0 such that for every $1 \leq i \leq \ell$ there is some $\mathscr{Q} \in \mathfrak{P}$ with $F_i \subseteq I_{\mathscr{Q}}$. As $\mathbb{P}(F_0) > 0$, at least one of the F_i's has strictly positive $p\bar{\beta}$-measure. For the sake of simplicity, let it be F_1. Without loss of generality, we assume that $0 < \mathbb{P}(F_1) < 1$[2]. As the range of the two-dimensional f.a. measure $(\mathbb{P}, p\bar{\beta})$ has convex closure, or else its closure is a zonoid in $\mathbb{R}^2_+$, we can choose a sub-coalition G_1 of F_1 (for a complete explanation see the Appendix in [13]) such that

$$\mathbb{P}(G_1)\|x_1\| \leq \frac{\rho}{3} \text{ and } \frac{p[\bar{\beta}(G_1)]}{\mathbb{P}(G_1)} \geq \frac{p[\bar{\beta}(F_1)]}{\mathbb{P}(F_1)}.$$

Recall that $F_1 \subseteq I_{\mathscr{Q}_0}$ for some $\mathscr{Q}_0 \in \mathfrak{P}$, and define

$$\bar{s}_* = \bar{\beta}(F_1)\mathbf{1}_{G_1} + \frac{d\bar{s}}{d\mathbb{P}}\mathbf{1}_{I \setminus G_1} \text{ and } \bar{\sigma} = \int \bar{s}_* d\mathbb{P}.$$

Since $\mathscr{P}_G = \mathscr{Q}_0$ for any sub-coalition G of F_1, we must have $\sigma \in \mathscr{M}_{F_1}$. Using $\|s - \bar{\sigma}\| = \|\bar{\beta}(F_1) - x_1\|\mathbb{P}(G_1)$ and $\mathbb{P}(G_1) < 1$, we derive

$$\begin{aligned} \|\bar{s} - \bar{\sigma}\| &< \|\bar{\beta}(F_1) - x_1\mathbb{P}(F_1)\| + \|x_1\| \left(1 - \mathbb{P}(F_1)\right) \mathbb{P}(G_1) \\ &< \frac{\rho}{3} + \|x_1\|(1 - \mathbb{P}(F_1))\mathbb{P}(G_1) < \frac{2}{3}\rho. \end{aligned}$$

This implies that $\|\bar{\sigma} - \bar{\beta}\| < \rho$ and hence, $\sigma \succ_{F_0} \alpha$. Consider an allocation γ defined by $\bar{\gamma} = \bar{\sigma}_{|G_1} + \bar{\beta}_{|I \setminus G_1}$. It follows that $\gamma \in \mathscr{M}_E$. Since $\gamma \succ_E \alpha$, we obtain $\bar{\gamma}(E) - \bar{e}(E) \in \mathscr{K}$. Consequently, $p[\bar{e}(E)] \leq p[\bar{\gamma}(E)]$. Hence,

$$\begin{aligned} p[\bar{e}(E)] &\leq p[\bar{\sigma}(G_1)] + p[\bar{\beta}(E \setminus G_1)] \\ &= p[\bar{\beta}(F_1)]\mathbb{P}(G_1) + p[\bar{\beta}(E)] - p[\bar{\beta}(G_1)] \\ &= p[\bar{\beta}(E)] + p[\bar{\beta}(F_1)\mathbb{P}(G_1) - \bar{\beta}(G_1)]. \end{aligned}$$

Furtherly, $\mathbb{P}(F_1) < 1$ yields

$$\frac{p[\bar{\beta}(G_1)]}{\mathbb{P}(G_1)} \geq \frac{p[\bar{\beta}(F_1)]}{\mathbb{P}(F_1)} > p[\bar{\beta}(F_1)],$$

and thus,

$$p[\bar{e}(E)] < p[\bar{\beta}(E)] = p[\bar{e}(E)],$$

which is impossible.

Case 2. $p[\bar{\beta}(E)] = 0$. In this case, $p[\bar{e}(E)] = 0$. As $\mathbb{P}(E) > 0$, we must have $\mathbb{P}(E \cap I_{\mathscr{Q}_0}) > 0$ for some $\mathscr{Q}_0 \in \mathfrak{P}$. Let $F = E \cap I_{\mathscr{Q}_0}$ and define $\gamma = \alpha + \bar{e}(F)\mathbb{P}$. Since e and $\mathbb{P}$ are equivalent, $\bar{e}(F) \in (X_+)^n \setminus \{0\}$. It then follows from [P.4] that

[2]Otherwise, by the nonatomicity of $\mathbb{P}$, we can split F_1 into F_1^1 and F_1^2 with

$$\mathbb{P}(F_1^1) = \mathbb{P}(F_1^2) = \frac{1}{2}\mathbb{P}(F_1),$$

and substitute $x_1\mathbf{1}_{F_1}$ with $x_1\mathbf{1}_{F_1^1} + x_1\mathbf{1}_{F_1^2}$.

$\gamma \succ_I \alpha$, and $\gamma \in \mathscr{M}_F$. Furthermore, $0 \leq p[\bar{e}(F)] \leq p[\bar{e}(E)] = 0$. Hence, $p[\bar{e}(F)] = 0$. Finally,

$$p[\bar{\gamma}(I_{\mathscr{Q}_0})] = p[\bar{\alpha}(I_{\mathscr{Q}_0})] + \mathbb{P}(I_{\mathscr{Q}_0}) \cdot 0 = p[\bar{e}(I_{\mathscr{Q}_0})] > 0,$$

since we have already noticed that $p\bar{\alpha}$ and $p\bar{e}$ coincide on Σ. Hence, by *Case 1*, we can again reach a contradiction. The proof is thus completed. □

We now formulate some alternative versions of Theorem 4.1 assuming different form of availability assumption and [A*.3]. In fact, we introduce *irreducibility* of the economy along with a very mild form of availability, to replace the strong availability condition, refer to [4, 7, 17].

[A*.5] The following two conditions are satisfied for the initial endowment allocation:

(i) $e(I, \omega) \gg 0$ for all $\omega \in \Omega$;
(ii) For every privately feasible allocation α and every partition $\{F_1, F_2\}$ of I, where F_1 and F_2 are coalitions, there exists some $\beta \in \mathscr{M}_{F_2}$ such that $\beta \succ_{F_2} \alpha$ and

$$e(F_1, \omega) + \alpha(F_2, \omega) \gg \beta(F_2, \omega)$$

for all $\omega \in \Omega$.

The second condition is known as *irreducibility assumption* of [7]. Moreover, availability assumption in (i) of [A*.5] is weaker than that in [A.5], which is again weaker than the strong availability assumption, that is, $e(F, \omega) \gg 0$ for all $F \in \Sigma^+$ and $\omega \in \Omega$.

Theorem 4.2. *Assume that* [A.1], [A.2], [A*.3], [A.4], [A*.5] *are satisfied for* $\mathscr{E}_C$. *If α is a private core allocation, then there exists an equilibrium price for α.*

Proof. Note that (ii) of [A*.3] replaces [A.3] in the final part of the proof of Theorem 4.1. We now turn to the alternative formulation [A*.5] and see how [A.5] can be replaced with [A*.5]. Note first that [A.5] is only used in *Case 2* of the proof of Theorem 4.1. Thus, our aim is to show that *Case 2* in the proof of Theorem 4.1 is also true in the light of [A*.5]. So, we assume that $p[\bar{e}(E)] = 0$. The rest of the proof of Case 2 is decomposed into following two sub-cases:

Sub-case 1. There exists an allocation γ such that $\gamma \in \mathscr{M}_{I \setminus E}$, $\gamma \succ_{I \setminus E} \alpha$ and $p[\bar{\gamma}(I \setminus E)] = p[\bar{e}(I \setminus E)]$. Applying (i) of [A*.5], we have $p[\bar{e}(I \setminus E)] = p[\bar{e}(I)] > 0$. Thus, we would again fall in the contradiction determined by occurrence of *Case 1* in Theorem 4.1 with $I \setminus E$ in the role of E.

Sub-case 2. There does not exist any allocation γ such that $\gamma \in \mathscr{M}_{I \setminus E}$, $\gamma \succ_{I \setminus E} \alpha$ and $p[\bar{\gamma}(I \setminus E)] = p[\bar{e}(I \setminus E)]$. In this sub-case, setting $F_1 = E, F_2 = I \setminus E$, by (ii) of [A*.5], there should be some $\gamma_* \in \mathscr{M}_{F_2}$ with $\gamma_* \succ_{F_2} \alpha$ and

$$\bar{e}(F_1) + \bar{\alpha}(F_2) \gg^n \bar{\gamma}_*(F_2).$$

This, along with the fact that $px \geq 0$ for all $x \in \mathscr{K}$, yields $p[\bar{e}(F_1)] + p[\bar{\alpha}(F_2)] \geq p[\bar{\gamma}_*(F_2)]$ and $p[\bar{\gamma}_*(F_2)] > p[\bar{e}(F_2)]$. But, $p[\bar{e}(F_2)] = p[\bar{\alpha}(F_2)]$. Thus,

$$p[\bar{e}(F_1)] + p[\bar{e}(F_2)] = p[\bar{e}(F_1)] + p[\bar{\alpha}(F_2)] \geq p[\bar{\gamma}_*(F_2)] > p[\bar{e}(F_2)]$$

whence $p[\bar{e}(F_1)] = p[\bar{e}(E)] > 0$, which leads to a contradiction. □

5. Individual core-Walras results

In this section, we derive individualistic core-Walras results in an economy with asymmetric information, from the equivalences stated in Theorem 4.1 and Theorem 4.2. We can express an individualistic economic model as follows

$$\mathscr{E}_I = \{(I,\Sigma,\mathbb{P}); X_+; (\Omega,\mathscr{F}); (\mathscr{F}_t, U_t, \eta(t,\cdot), \mathbb{P}_t)_{t\in I}\},$$

where

- $(I,\Sigma,\mathbb{P})$ is a measure space of agents where $\mathbb{P}$ is a non-atomic countably additive measure on the σ-algebra Σ.
- X_+ and $(\Omega,\mathscr{F})$ are the same as in $\mathscr{E}_C$.
- $\mathscr{F}_t$ is the σ-algebra generated by a partition $\mathscr{P}_t \subseteq \mathscr{F}$ of Ω representing the *private information* of agent t. It is interpreted as follows: if $\omega \in \Omega$ is the state of nature that is going to be realized, agent t observes $\mathscr{P}_t(\omega)$, the unique element of $\mathscr{P}_t$ that contains ω.
- $U_t : \Omega \times X_+ \to \mathbb{R}$ is the *state-dependent utility function* of agent t, representing their (ex post) preference.
- $\eta(t,\cdot) : \Omega \to X_+$ is the *initial endowment density* of agent t.
- $\mathbb{P}_t$ is a probability measure on $\mathscr{F}$, representing the *prior belief* of agent t.

The *ex ante expected utility* of an agent t for $x : \Omega \to X_+$ is defined by

$$\mathbb{E}^{\mathbb{P}_t}(U_t(\cdot,x(\cdot))) = \sum_{\omega\in\Omega} U_t(\omega,x(\omega))\mathbb{P}_t(\omega).$$

As in Radner [26], the *consumption set* of an agent t is defined by

$$\mathscr{X}_t = \left\{x \in X_+^n : x \text{ is } \mathscr{F}_t\text{-measurable}\right\}.$$

An *allocation* in $\mathscr{E}_I$ is a function $f : I \times \Omega \to X_+$ such that $f(\cdot,\omega)$ is Bochner integrable for all $\omega \in \Omega$. It is said to *privately feasible* whenever $f(t,\cdot) \in \mathscr{X}_t$ $\mathbb{P}$-a.e., and *physically feasible* if

$$\int_I f(\cdot,\omega)d\mathbb{P} \leqslant \int_I \eta(\cdot,\omega)d\mathbb{P}$$

for all $\omega \in \Omega$. Furthermore, we say that an allocation is *feasible* if it is privately as well as physically feasible. An element of Σ of positive measure is termed as a *coalition* of $\mathscr{E}_I$. An allocation α is *privately blocked by a coalition* F if there is an allocation g such that $g(t,\cdot) \in \mathscr{X}_t$ and $\mathbb{E}^{\mathbb{P}_t}(U_t(\cdot,g(t,\cdot))) > \mathbb{E}^{\mathbb{P}_t}(U_t(\cdot,f(t,\cdot)))$ for all $t \in S$, and

$$\int_S f(\cdot,\omega)d\mathbb{P} \leqslant \int_S \eta(\cdot,\omega)d\mathbb{P}$$

for all $\omega \in \Omega$. The *private core* of $\mathscr{E}_I$, denoted by $\mathscr{P}\mathscr{C}(\mathscr{E}_I)$, is the set of feasible allocations which are not privately blocked by any coalition. Similar to Section 2, a *price system* is a non-zero function $\pi : \Omega \to X_+^*$. Given a price system π, the *budget set* of an agent t is defined by

$$\mathscr{B}(t,\pi) = \left\{x \in \mathscr{X}_t : \sum_{i=1}^n \pi(\omega_i)x(\omega_i) \le \sum_{i=1}^n \pi(\omega_i)\eta(t,\omega_i)\right\}.$$

A *Walrasian expectations equilibrium* of $\mathscr{E}_I$ is a pair (f, π) where f is a feasible allocation and π is a price system such that $f(t, \cdot)$ maximizes $\mathscr{B}(t, \pi)$ $\mathbb{P}$-a.e. and

$$\sum_{i=1}^{n} \pi(\omega_i) \int_I f(\cdot, \omega_i) d\mathbb{P} = \sum_{i=1}^{n} \pi(\omega_i) \int_I \eta(\cdot, \omega_i) d\mathbb{P}.$$

We assume that X is separable. Suppose now that the collection $\{\mathscr{P}_1, \cdots, \mathscr{P}_k\}$ of partitions of Ω such that $I_i = \{t \in I : \mathscr{P}_t = \mathscr{P}_i\}$ is measurable and $\mathbb{P}(I_i) > 0$ is non empty. We assume that $I = \bigcup\{I_i : 1 \leq i \leq k\}$. For any $m \geq 1$, the $(m-1)$-simplex of $\mathbb{R}^m$ is defined as

$$\Delta^m = \left\{ x = (x_1, \cdots, x_m) \in \mathbb{R}^m_+ : \sum_{i=1}^{m} x_i = 1 \right\}.$$

Consider a function $\varphi : (I, \Sigma, \mathbb{P}) \to \Delta^n$ defined by $\varphi(t) = \mathbb{P}_t$ for all $t \in I$. For each $\omega \in \Omega$, define a function $\psi_\omega : I \times X_+ \to \mathbb{R}$ by $\psi_\omega(t, x) = U_t(\omega, x)$. We now impose the following classical assumptions on the factors of this economy:

[B.1] The function φ is measurable, where Δ^n is endowed with the Borel structure.

[B.2] For each $\omega \in \Omega$, the function ψ_ω is Carathéodory, that is, $\psi_\omega(\cdot, x)$ is measurable for all $x \in X_+$ and $\psi_\omega(t, \cdot)$ is norm-continuous for all $t \in I$.

[B.3] For all $(t, \omega) \in I \times \Omega$, $U_t(\omega, x + y) > U_t(\omega, x)$ for all $x, y \in X_+$ with $y > 0$.

[B.4] There exist some $u \in (X_+)^n$ and an open, convex, solid neighbourhood U of 0 in X^n such that (i) $U^c \cap (X_+)^n$ is convex; and (ii) $y \in (X_+)^n$ and $z \in \overline{(y + C_u)} \cap (X_+)^n$ implies

$$\mathbb{E}^{\mathbb{P}_t}(U_t(\cdot, z(\cdot))) > \mathbb{E}^{\mathbb{P}_t}(U_t(\cdot, y(\cdot)))$$

$\mathbb{P}$-a.e., where

$$C = \bigcup\{t(u + U) : t > 0\}.$$

[B.5] η is a privately feasible allocation and $\bigcap\{t \in I_j : \eta(t, \omega_i) \gg 0, 1 \leq i \leq n\} \in \Sigma^+$ for each $1 \leq j \leq k$.

[B*.5] η is a privately feasible allocation such that

(i) $\displaystyle\int_I \eta(\cdot, \omega) d\mathbb{P} \gg 0$ for all $\omega \in \Omega$;

(ii) for every privately feasible allocation f and every partition $\{F_1, F_2\}$ of I, with $F_i \in \Sigma^+$, there exists a private allocation g such that $g(t, \cdot) \in \mathscr{X}_t$ and

$$\mathbb{E}^{\mathbb{P}_t}(U_t(\cdot, g(t, \cdot))) > \mathbb{E}^{\mathbb{P}_t}(U_t(\cdot, f(\cdot)))$$

$\mathbb{P}$-a.e. on F_2, and

$$\int_{F_1} \eta(\cdot, \omega) d\mathbb{P} + \int_{F_2} f(\cdot, \omega) d\mathbb{P} \geqq \int_{F_2} g(\cdot, \omega) d\mathbb{P}$$

for all $\omega \in \Omega$.

Remark 5.1. The first three assumptions are similar to those in [9, 10, 18]. Under [B.1] and [B.2], $t \mapsto \mathbb{E}^{\mathbb{P}_t}(U_t(\cdot, x(\cdot)))$ is Σ-measurable for all $x : \Omega \to X_+$. Continuity and monotonicity of $E^{\mathbb{P}_t}(U_t(\cdot, x(\cdot)))$ follows from [B.2] and [B.3], respectively. Assumptions [B.4] is a properness-like assumption, which is similar to [A.4]. The assumption [B.5] is weaker than the assumption $a(t, \omega_i) \gg 0$ for all $t \in I$ and

$1 \leq i \leq n$. Lastly, the assumption [B*.5] is the combination of irreducibility with a mild availability condition and it will be used to replace [B.5] to obtain an alternative core-Walras equivalence theorem. Since allocations take values in the positive cone of the commodity space, the condition (ii) in [B*.5] is implied by the following condition from [4]:

[C] For every privately feasible allocation f and every partition $\{F_1, F_2\}$ of I, with $F_i \in \Sigma^+$ for $i = 1, 2$, there exist two allocations g, h such that $g(t, \cdot), h(t, \cdot) \in \mathscr{X}_t$ and

$$\mathbb{E}^{\mathbb{P}_t}(U_t(\cdot, g(t, \cdot))) > \mathbb{E}^{\mathbb{P}_t}(U_t(\cdot, f(t, \cdot)))$$

$\mathbb{P}$-a.e. on F_2, and

$$\int_{F_1} \eta(\cdot, \omega) d\mathbb{P} + \int_{F_2} f(\cdot, \omega) d\mathbb{P} = \int_{F_1} h(\cdot, \omega) d\mathbb{P} + \int_{F_2} g(\cdot, \omega) d\mathbb{P}$$

for all $\omega \in \Omega$. □

For each allocation f in $\mathscr{E}_I$, we are associating an allocation $\Xi[f]$ in $\mathscr{E}_C$ by letting $\Xi[f](E, \omega) = \int_E f(\cdot, \omega) d\mathbb{P}$. For each $F \in \Sigma$, we define $\mathscr{P}_F$ to be the smallest partition that refines each $\mathscr{P}_t$ for all $t \in F$. Thus, the individualistic economy $\mathscr{E}_I$ corresponds to the coalitional economy $\mathscr{E}_C$ given by

$$\mathscr{E}_C = \{(I, \Sigma, \mathbb{P}); X_+; (\Omega, \mathscr{F}); (\mathscr{F}_F, \succ_F, e(F, \cdot))_{F \in \Sigma}\},$$

where $e(F, \cdot) = \Xi[\eta](F, \cdot)$; $\mathscr{F}_F$ is the σ-algebra generated by $\mathscr{P}_F$; and the coalitional preference $\succ_F$ is defined by letting $\alpha \succ_F \beta$ if and only if

$$\mathbb{E}^{\mathbb{P}_t}(U_t(\cdot, a(\cdot))) > \mathbb{E}^{\mathbb{P}_t}(U_t(\cdot, b(\cdot)))$$

$\mathbb{P}$-a.e. on F, where $a(\cdot, \omega)$ and $b(\cdot, \omega)$ are Radon-Nikodym derivatives of $\alpha(\cdot, \omega)$ and $\beta(\cdot, \omega)$, respectively, for each $\omega \in \Omega$ with respect to $\mathbb{P}$.

Remark 5.2. It follows from the above definition of $\succ_F$ that [P.1]-[P.3] and [P.5] are satisfied. Assumption [P.4] is implied by [B.3]. Given the structure of information $\{\mathscr{P}_1, \cdots, \mathscr{P}_k\}$, [A.1] is verified trivially. Obviously, the above definition of $\mathscr{F}_F$ for all $F \in \Sigma$ yields [A.2]. By [B.2], $\mathbb{E}^{\mathbb{P}_t}(U_t(\cdot, x(\cdot)))$ is continuous with respect to $x : \Omega \to X_+$. Since $t \mapsto \mathbb{E}^{\mathbb{P}_t}(U_t(\cdot, x(\cdot)))$ is also Σ-measurable, by Propostion 4.1 in [13], [A*.3] is satisfied. Clearly, [B.4] implies [A.4]. Lastly, [A.5] and [A*.5] can be easily derived from [B.5] and [B*.5], respectively.

Since Proposition 4.2 of [7] can be easily extended to this new framework, we have the following core-Walras equivalence theorem.

Theorem 5.3. *Let* [B.1]-[B.5] *hold for the economy* $\mathscr{E}_I$. *A feasible private allocation* f *belongs to the private core of* $\mathscr{E}_I$ *if and only if it is a Walrasian expectations allocation of* $\mathscr{E}_I$.

The first alternative version of the core-Walras equivalence (that is, Theorem 4.2) provides also an individualistic result where availability condition [B.5] is replaced by [B*.5], which is given below.

Theorem 5.4. *Suppose that* $\mathscr{E}_I$ *satisfies* [B.1]-[B.4] *and* [B*.5]. *A feasible private allocation* f *belongs to the private core of* $\mathscr{E}_I$ *if and only if it is a Walrasian expectations allocation of* $\mathscr{E}_I$.

6. Concluding remarks

In this section, we discuss the core-Walras equivalence theorem in the case of exact feasibility and compare our main results with some existing results in the literature.

The physical feasibility of an allocation for any coalition in most of asymmetric information frameworks in the literature (also in our model) is expressed in terms of an inequality while the feasibility of an allocation in a complete information economy is expressed by means of an equality. Towards this direction, the question has been raised by some authors (for instance, [4, 17]) whether free disposal is necessary in the definition of physical feasibility in order to obtain core-Walras equivalence theorems. We now show that a core-Walras equivalence theorem can be established under the exact feasibility condition in the presence of an additional assumption. To introduce this assumption, we first recall Proposition 3.1 in [23].

Proposition 6.1. *If* (i) *of* [A.4] *holds, then there exist a positive functional* $x^* \in (X_+^*)^n$ *and* $c > 0$ *such that* $U \cap X_+^n = G^-(x^*, c) \cap X_+^n$ *and* $U^c \cap X_+^n = F^+(x^*, c) \cap X_+^n$*, where* $G^-(x^*, c)$ (respectively $F^+(x^*, c)$) *denotes the open lower* (resp. closed upper) *half space determined by the hyperplane* $\{x \in X^n : x^*(x) = c\}$.

Since (w, U) is a properness pair, $w \notin U$. As a consequence of Proposition 6.1, we have $x^*(w) \geq c$. In the light of this, we state the following additional properness-like assumption:

[A*.4] The pair $\left(w, \dfrac{x^*(w)}{c} U\right)$ is a properness pair in the sense of [A.4].

Remark 6.2. As the extremely desirable commodity bundle w is larger than the extremely desirable bundle u, the assumption [A*.4] says that w is extremely desirable bundle in the sense that it remains desirable when added to a bundle y even if one subtracts something relatively "large", namely almost at the level of hyperplane. This assumption is employed to demonstrate that our results can be obtained in a framework without free disposal assumption. However, in the absence of this assumption, a slightly different approach has been used to establish the core-Walras equivalence theorem under free disposal assumption. Note that such an approach is not applicable for the case when feasibility is defined to be exact (without free disposal).

Let $\lambda = \dfrac{x^*(w)}{c}$. It follows from Proposition 6.1 that $z \in \lambda U \cap X_+^n$ implies $x^*(z) < \lambda c = x^*(w)$. Define

$$C = \bigcup \{t(w + \lambda U) : t > 0\}.$$

Lemma 6.3. *Suppose that* [A.1]-[A.3] *and* [A*.4] *are satisfied for* $\mathscr{E}_C$*. If* α *is a private core allocation, then* $\overline{\mathscr{K}} \cap (-C) = \emptyset$*.*

Proof. Similarly to Lemma 3.3, it is enough to prove that $\mathscr{K} \cap (-C)$ is empty. For each $\mathscr{Q} \in \mathfrak{P}$, let

$$\mathscr{K}_{\mathscr{Q}} = \bigcup_{F \in \Sigma,\ \mathbb{P}(F) > 0} \{\bar{\gamma}(F \cap I_{\mathscr{Q}}) - \bar{e}(F \cap I_{\mathscr{Q}}) : \gamma \in \mathscr{M}_F, \gamma \succ_F \alpha\}.$$

Case 1. $\mathscr{K}_{\mathscr{Q}} \cap (-C) = \emptyset$ *for all* $\mathscr{Q} \in \mathfrak{P}$ *implies* $\mathscr{K} \cap (-C) = \emptyset$. The claim can be done if we show that $\bar{\gamma}(E) - \bar{e}(E) \in \mathscr{K} \cap (-C)$ for some coalition E implies

$$\bar{\gamma}(E \cap I_{\mathscr{Q}_0}) - \bar{e}(E \cap I_{\mathscr{Q}_0}) \in -C$$

for at least one partition $\mathscr{Q}_0 \in \mathfrak{P}$. Thus, we assume that $\bar{\gamma}(E) - \bar{e}(E) = -t(w+v)$ for some $v \in W, t > 0$. By putting $\bar{\beta} = \bar{\gamma} + \frac{tv^+}{\mathbb{P}(E)}\mathbb{P}$, we have

$$\bar{\beta}(E) - \bar{e}(E) + tw = tv^-,$$

which is equivalent to

$$\bar{\beta}(E) - \bar{e}(E) + w = tv^- + (1-t)w.$$

Clearly, $\bar{\beta}(E) - \bar{e}(E) \in \mathscr{K}$ and $v^- \in \lambda U$. Note that $tv^- + (1-t)w \in X^n_+$ and $x^*(tv^- + (1-t)w) < \lambda c$. As a result, $\bar{\beta}(E) - \bar{e}(E) + w \in \lambda U \cap X^n_+$. If $\mathscr{K}_{\mathscr{Q}} \cap (-C) = \emptyset$ for all $\mathscr{Q} \in \mathfrak{P}$, then we have

$$\bar{\beta}(E \cap I_{\mathscr{Q}}) - \bar{e}(E \cap I_{\mathscr{Q}}) + \frac{1}{|\mathfrak{P}|}w \in \left(\frac{\lambda U}{|\mathfrak{P}|}\right)^c \cap X^n_+.$$

It follows from the convexity of $\left(\frac{\lambda U}{|\mathfrak{P}|}\right)^c \cap X^n_+$ that

$$\bar{\beta}(E) - \bar{e}(E) + w = \sum_{\mathscr{Q} \in \mathfrak{P}} \left[\bar{\beta}(E \cap I_{\mathscr{Q}}) - \bar{e}(E \cap I_{\mathscr{Q}}) + \frac{1}{|\mathfrak{P}|}w\right] \notin \lambda U,$$

which is a contradiction.

Case 2. $\mathscr{K}_{\mathscr{Q}} \cap (-C) = \emptyset$ *for all* $\mathscr{Q} \in \mathfrak{P}$. By invoking the arguments of the proof of Lemma 3.3 and monotonicity of $\succ^n_I$, we can verify the *Claim 2*[3]. □

We now state a properness assumption for the individualistic model.

[B*.4] The pair $\left(w, \frac{x^*(w)}{c}U\right)$ is a properness pair in the sense of [B.4].

In the light of the above lemma, all main results in Section 4 can be easily extended to the case of exact feasibility. As a consequence of [B*.4], analogously to Theorem 5.3 and Theorem 5.4, one can obtain similar individual core-Walras equivalence theorems without free disposal, in the sense of [4]. The first one would then contain Theorem 5.1 in [4] as a corollary when preferences are represented by continuous and monotone utilities. Indeed, all the hypotheses of Theorem 5.3 are either explicitly assumed, or stated as part of the standard definition for the economy in [4] in a framework of an Euclidean space as the commodity space. Note that our core-Walras equivalence theorems for the case of exact feasibility and an infinite dimensional commodity space are the first attempts in the literature for coalitional as well as individualistic models.

We conclude this section with the following two remarks.

[3]The monotonicity assumption will be used to obtain exact physically feasible allocation from $\bar{s}_1$ to block a private core allocation.

Remark 6.4. Our next concern will be that of comparing Theorem 5.3 with some of the core-Walras equivalence result for economies with asymmetric information already existing in the literature. Most of the results are given under the assumption that the commodity space coincides with the Euclidean space $\mathbb{R}^\ell$ for any given $\ell \geq 1$. In this case, clearly X is a separable Banach lattice having the RNP, and [B.4] is default. We begin our overview with [7], where the commodity space is $\mathbb{R}^\ell$. The coalitional results in [7] (i.e. Theorem 3.2 and Theorem 3.7) can not be derived from our coalitional equivalences for two main reasons: (a) we are assuming a different form of continuity, and (b) we need assumption [A.1]. Nevertheless, when one turns to the individual formulation, Theorem 4.3 in [7] can be proven via Theorem 4.2. In fact, the private feasibility of η in [B.5], although not explicitly stated, is mentioned as an implicit assumption (and needed to have condition (A.4) of [7] fulfilled). All the other conditions in Theorem 4.3 of the aforementioned paper either coincide or imply those of Theorem 5.3. Our individualistic results (i.e. Theorem 5.3 and Theorem 5.4) are not the direct extensions of the core-Walras equivalence results in [12, 18] as the commodity spaces in [12, 18] are not necessarily satisfying the RNP. □

Remark 6.5. We conclude this paper with a list of possible directions of further investigations, and problems where the setting that we propose here (X has the RNP and preferences satisfies the properness-like assumption) could enlarge the class of economies in which previous results can be extended:

- Different types of core are considered by several authors, both in the finite dimensional ([2, 17]) and in infinite dimensional ([11, 20]) commodity spaces; it would be interesting to investigate whether the results obtained for these cores in the mentioned papers can be extended under properness-like assumption to a Banach lattice X having the RNP.

- A huge variety of papers focus on the *existence* results in the framework of differential information ([3, 17, 25]). Do the assumptions proposed in our model provide extra tools to prove existence of an equilibrium?

- A final problem to mention is the necessity of assumption [A.1] in the coalitional setting. We have not been able to provide a counterexample in this direction so far; and it could be in fact true that one could move from an economy where [A.1] does not hold to the finer economy where the σ-algebra of coalitions is enlarged somehow to the one generated by Σ and $\{I_{\mathscr{Q}} : \mathscr{Q} \in \mathfrak{P}\}$. Perhaps a suitable extension of the probability $\mathbb{P}$ would provide a way to derive equivalence results in more general situations than those proved in this paper.

References

[1] C.D. Aliprantis, R. Tourky, N.C. Yannelis, Cone Conditions in General Equilibrium Theory, J. Econ. Theory **92** (2000), 96–121.

[2] B. Allen, Market games with asymmetric information: the core, Econ. Theory **29** (2006), 465–487.

[3] L. Angeloni, A. Martellotti, A separation result with applications to Edgeworth equivalence in some infinite dimensional setting, Commentationes Math. **44** (2004) 227-243.

[4] L. Angeloni , V. F. Martins-da-Rocha, Large economies with asymmetric information and without free disposal, Econ. Theory **38** (2009), 263–286.

[5] T.E. Armstrong, M.K. Richter, The Core-Walras equivalence , J. Econ. Theory **33** (1984) 116-151.

[6] A. Basile, Finitely additive nonatomic coalition production economies: Core-Walras equivalence, Internat. Econ. Rev. **34** (1993) 983-995.
[7] A. Basile, C. Donnini, M.G. Graziano, Core and equilibria in coalitional asymmetric information economies, J. Math. Econ. **45** (2009), 293-307.
[8] K.P.S. Bhaskara Rao, M. Bhaskara Rao, Theory of Charges, Academic Press, 1983.
[9] A. Bhowmik, J. Cao, Blocking efficiency in an economy with asymmetric information, J. Math. Econ. **48** (2012), 396–403.
[10] A. Bhowmik, J. Cao, Robust efficiency in mixed economies with asymmetric information, J. Math. Econ. **49** (2013), 49–57.
[11] A. Bhowmik, Core and coalitional fairness: the case of information sharing rules, Econ Theory **60** (2015), 461-494.
[12] A. Bhowmik, Edgeworth equilibria: separable and non separable commodity space, MPRA report 46796 (2013), http://mpra.ub.uni-muenchen.de/46796/
[13] F. Centrone, A. Martellotti, Coalitional extreme desirability in finitely additive exchange economies, Econ. Theory Bull. **4** (2016), 17–34.
[14] G. Chichilnisky, P.J. Kalman, Application of functional analysis to models of efficient allocation of economic resources, J. Optimization Theory Appl., **30** (1980), 19-32.
[15] G. Debreu, Preference functions on measure spaces of economic agents, Econometrica, **35** (1967), 111–122.
[16] J. Diestel, J.J. Jr. Uhl, Vector Measures, American Mathematical Society, Providence (1977)
[17] E. Einy, D. Moreno, B.Shitovitz, Competitive and core allocations in large economies with differentiated information, Econ. Theory **18** (2001), 321–332.
[18] Ö. Evren, F. Hüsseinov, Theorems on the core of an economy with infinitely many commodities and consumers, J. Math. Econ., **44** (2008), 1180–1196.
[19] F. Forges, E. Minelli, R. Vohra, Incentives and the core of an exchange economy: a survey, J. Math. Econ. **38** (2002), 1–41.
[20] M.G. Graziano, C. Meo, The Aubin private core of differential information economies, Decis. Econ. Finance **28** (2005), 9–38.
[21] M. Greinecker, K., Podczeck, Liapounov's vector measure theorems in Banach spaces and applications to general equilibrium theory, Econ. Theory Bull. **1** (2013), 157–173.
[22] C. Hervés-Beloso, C. Meo, E. Moreno-García, Information and size of coalitions, Econ. Thoery **55** (2014), 545-563.
[23] A. Martellotti, Finitely additive economies with free extremely desirable commodities, J. Math. Econ. **44** (2007) 535-549.
[24] A. Mas-Colell, The equilibrium existence problem in topological vector lattices, Econometrica **54** (1986), 1039-1053.
[25] K. Podczek, N. Yannelis, Equilibrium theory with asymmetric information and with infinitely many commodities, J. Econ. Theory **1414** (2008), 152-183.
[26] R. Radner, Competitive equilibrium under uncertainty, Econometrica **36** (1968), 31–58.
[27] A. Rustichini, N.C. Yannelis, Edgeworth's conjecture in economies with a continuum of agents and commodities, J. Math. Econ. **20** (1991), 307–326.
[28] K. Vind, Edgeworth allocations in an exchange economy with many traders, Internat. Econ. Review **5** (1964) 165-177.
[29] R. Wilson, Information, efficiency, and the core of an economy, Econometrica **46** (1978), 807–816.
[30] N. C. Yannelis, The core of an economy with asymmetric information, Econ Theory **1**, 183–197 (1991)

Corresponding Author, Economic Research Unit, Indian Statistical Institute, 203 Barackpore Trunk Road, Kolkata 700108, India, Tel.: ++64 33 2575 2620
E-mail address: anuj.bhowmik@isical.ac.in

Dipartimento di Studi per l'Economia e l'Impresa, Università del Piemonte Orientale, Via Perrone 18, 28100 Novara, Italy
E-mail address: francesca.centrone@uniupo.it

Dipartimento di Matematica e Informatica, Università di Perugia, Via Vanvitelli 1, 06123 Perugia, Italy
E-mail address: anna.martellotti@unipg.it