Box-Jenkins modelling and forecasting of Brent crude oil price

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Abstract

The volatility in the crude oil price in the international market has risen much interest into the investigation of its price swing. In this project, we examine the dynamics of the monthly Brent oil price for the last two decades using the Box Jenkins ARIMA techniques and show that such model is not able to capture the volatility inherent in the crude oil price for an accurate forecast. We first divided the data into two. The first seventeen years used for the model construction and the last three years validating forecasting accuracy. The data is first differenced for stationarity and autocorrelation and residuals techniques used to select different ARIMA models for analysis. The performance of different models were compared and the result shows that a non-parsimonious ARIMA (1,1,1) model was the best forecasting model amidst the volatilities in the oil price.

Keywords: Brent crude oil, ARIMA, stationarity, forecasting

JEL: C22; C52; E3; E37

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Note: This project also demonstrates fundamentally, how to model a univariate series using the Box Jenkins (ARIMA) model for beginners in time series analysis. The graphical demonstrations are meant to serve as a pictorial guide for readers.

1. Introduction

Crude oil is undoubtedly one of the important commodities in the world. Since the discovery of oil in the 1800's, there have not been much alternative to the use of the product. Products derived from crude oil such as diesel fuel, motor gasoline, jet fuel, and heating oil provides about 33% of the energy needs of household, businesses and manufacturers globally (Energy Information Administration, 2013). However, its price behave like any other commodity with price swings in times of shortage and surplus. Such price swings have multiplier effect on our daily life ranging from diesel and gasoline to detergent and medicines and household appliances. Though crude oil is non-renewable, it is consumed every day in the world. Because of its multifaceted usefulness, there is a broad consensus that oil price volatility can have significant impact on the financial market and economy. For instance, an increase in oil price induces higher cost of production and changes capacity utilization of firms. Such higher cost of production are usually passed on to consumers through soaring prices of consumer goods.

Modelling and forecasting crude oil price have therefore attracted the interest of energy researchers, business moguls and policy makers. Different methodologies have been used to ascertain an accurate forecast of the oil price. In particular, the GARCH model and its variants (See Sadorsky, 2006; Narayan and Narayan, 2007; Gileva, 2010) have been used to model the volatility in the oil price. Recently, the support vector machine (SVM) model has been employed to compare forecast efficiency with the traditional time series models. An SVM method proposed by Xie et al (2006) showed that the SVM forecast better than the ARIMA model and the back-propagation neural network (BPNN). Similar analysis based on the genetic algorithm (GA) SVM proved a better forecasting performance than the traditional SVM (Guo and Zhang, 2012).

Over the past two decades, oil price has been experiencing ups and downs. The recent sharp decline in the global oil price has left many industry players much concern on the future deterministic price. As many research in this area goes on unabated, we explore the strength of the Box Jenkins ARIMA models in modelling and forecasting the crude oil price. In particular, with the crude oil price prone to volatilities in the financial market, we try to find a non-parsimonious ARIMA model that has the best forecasting amidst the volatilities in the oil price.

2. BRENT Crude Oil Spot Price (USD$)

The Brent crude oil (North Sea-Europe) spot price is used in the design of the model. The Brent crude oil also classified as light crude oil together with the West Texas Intermediate crude oil are widely considered as the benchmark for world oil pricing and trading. The data is obtained from the U.S Energy Information Department. Monthly price data from November, 1994 - November, 2014 comprising 241 observations with mean 54.30 is available and considered. The observation shows a record highest price of $132.72 recorded in July 2008 mainly due to the economic crises in that period and the lowest price of $9.82 in December 1998. The data is further divided into two time frames. The first seventeen years of the data used for the model construction and the last three years validating forecasting accuracy.

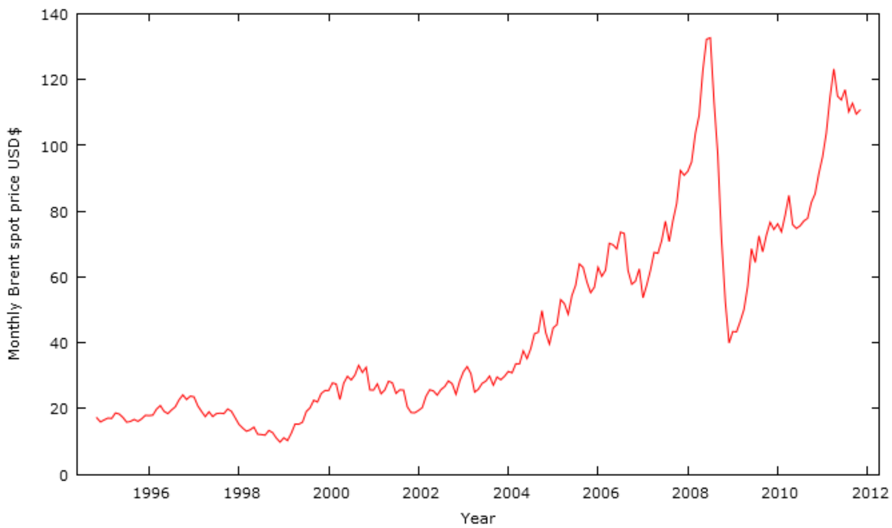

Figure 1. Monthly Brent crude oil spot price from November, 1994 - November, 2011

Figure 1. shows a Time Series plot of the Brent crude oil price which indicate a non-stationary series. Clearly there is seemingly increasing trend coupled with fluctuations from 2002 after a leverage up and down price observations from 1994 to 2002.

3. Methodology

3.1. Stationarity

Time series data such as the crude oil price may exhibit non stationarity at their levels. For the estimation of its model, it becomes imperative to detrend the data before certain statistical inference can be made. A stationary series can be said to be a flat looking series without trend, a constant variance over time, a constant autocorrelation over time and no periodic fluctuations (See Brockwell and Davis, 2002). The plot in figure 1 shows a non-constant mean and variance. A technique for making series of non-constant mean and variance stationary is the differencing. We work with the log return instead for our data because of the non-constant variance. The log return approach is considered as

$$y_t \quad = \quad \triangle\, log(x_t) = log(x_t) - log(x_{t-1}) = log(\frac{p_t}{p_{t-1}})$$

where $x_t = p_t$ represent the price of the crude oil and y_t is it's differenced series.

We used the Augmented Dickey-Fully (ADF) test for stationarity which returns a p-value of 0.0111 for lag 14. We can therefore reject the null hypothesis of non-stationarity and claim that the series with log differencing is a realization of stationary process.

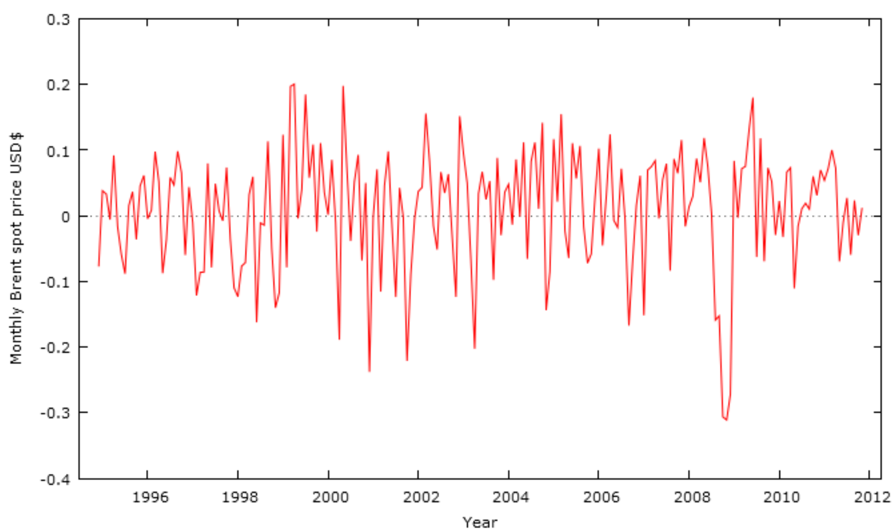

Figure 2. Stationary monthly Brent crude oil spot price from Nov. 1994 to Nov. 2011

3.2. ARIMA Model

The ARIMA(*p,d,q*)-Box Jenkins Model proposed by Box and Jenkins is one of the common methods for building univariate time series forecasting model. Once our series is stationary, we begin to explore the different ways we can have a fitting model for the Brent data. The ARIMA(*p, d, q*) Model is the differenced series of the ARMA(*p, q*) model wherein the difference 'd' corresponds to y_t. The ARMA(*p, q*) Model has the general form

$$x_t = \theta_0 + \sum_{i=1}^{p} \phi_i x_{t-i} + \sum_{j=1}^{q} \theta_j u_{t-j} + u_t$$

where p and q refer to the order of the autoregressive terms x_t and moving average terms u_t respectively and ϕ, and θ are their respective coefficients.

We start the model identification by plotting the ACF and PACF against different lags to determine the appropriate order of p and q for our model. The general ACF and PACF has the theoretical behavior as summarized in this table 1

	AR(p)	MA(q)	ARMA(p,q)
ACF	Tail off	Cut off after lag q	Tail off
PACF	Cut off after lag p	Tail off	Tail off

Sometimes, for some mixed model series, when it is not sufficient to identify the model using the table above, the information based criteria such as the BIC (Bayes Information Criterion) and AIC (Akaike Information Criterion) among others are used in determining the order of p and q. The AIC is defined by the formula

$$AIC(p,q) = ln(\bar{\sigma}^2) + \frac{2(p+q)}{T}$$

where $\bar{\sigma}^2$ is the maximum likelihood estimate of the white noise variance, T is the sample size and ($p+q$) is the total number of parameters found in the ARMA (*p, q*) model. The appropriate model is then found by selecting the set of values of p and q that minimizes the AIC (*p, q*). Intuitively, we can consider the term $\frac{2(p+q)}{T}$ as the penalty term to avoid over parametrization. We can similarly employ the BIC which has large penalty to avoid overfitting of over parametrization. The BIC is defined by the formula:

$$BIC(p,q) = ln(\bar{\sigma}^2) + \frac{ln(T)(p+q)}{T}$$

with penalty term $\frac{ln(T)}{T} > \frac{2}{T}$ for all $T \geq 8$

The SACF and SPACF plot in figure 3 does not give much information to establish the order of the lags. We resort to the AIC and BIC criteria for more information. Comparing values of AIC and BIC obtained by fitting the different p and q ranging from 0 to 2 in Table 2, the AIC and BIC criteria both suggest an ARMA(1,0) model.

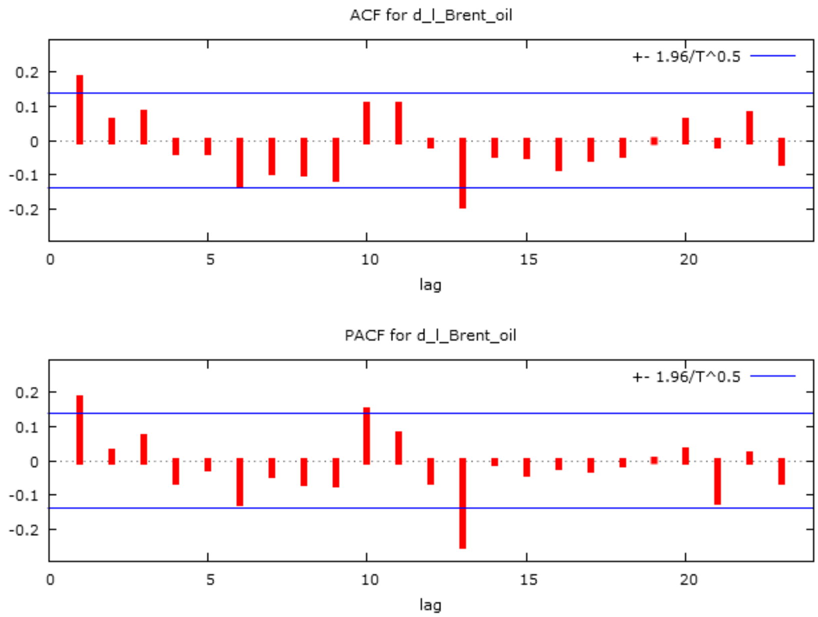

Figure 3. ACF and PACF plot for Brent oil spot price US$

According to Box and Draper (1987), 'All models are wrong but some are useful'. We select the models which according to Table 2 may also be useful. We select the ARMA (0,1), ARMA (1,1), ARMA(1,0) and ARMA (2,0) model which were considered by inspection for the monthly log return of the Brent crude oil price and analyse their forecasting power. Also, by inspection we get AIC values of -400.34 and -401.49 if we consider the ARMA (3,2) and ARMA (3,3) model, however may produce parsimonious model.

Order	0,1	0,2	1,0	1,1	1,2	2,0	2,1	2,2
AIC	-400.94	-399.12	-401.41	-399.83	-397.86	-399.63	-397.85	-396.72
BIC	-394.23	-389.23	-394.78	-389.89	-384.59	-389.67	-384.58	-380.13

Table 2: BIC and AIC information criteria.

3.3. Residual Analysis

If the Box - Jenkins models selected is good enough for the Brent crude oil price data, we expect the residual to be a realization of white noise. That is residual must be independent following its normal distribution. We analyse this graphically using the time series residual plot, the residual correlogram and the Q-Q plot. We also perform the Ljung-Box test which is based on the autocorrelation being different from zero. The Ljung-Box test with test statistic Q has this hypothesis:

H_0 : The residuals are independently distributed

H_1 : The residuals are not independently distributed.

$$Q = n(n+2)\sum_{k=1}^{h}\frac{\hat{\rho_k}^2}{n-k}$$

where n is the sample size, $\hat{\rho_k}$ is the sample autocorrelation at lag k and h is the number of lags being tested. The Null hypothesis are rejected at α% significance level if $Q(k) > \chi^2_{1-\alpha,k}$ where $\chi^2_{1-\alpha,k}$ is the α quatile of the chi-square distribution with k degrees of freedom.

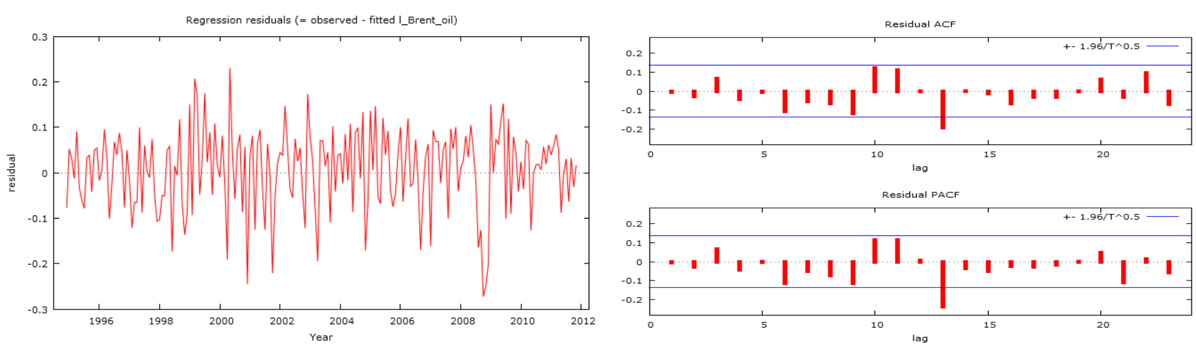

Figure 4: Residual plot and residual correlogram of ARMA(1,1)

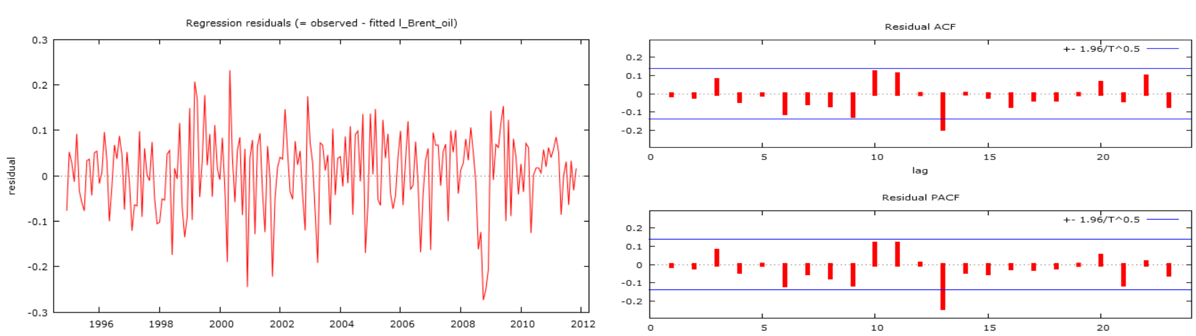

Figure 5: Residual plot and residual correlogram of AR(2)

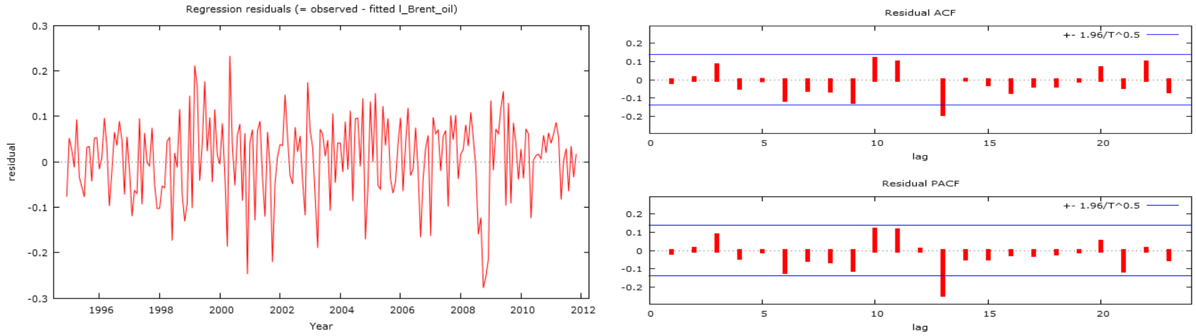

Figure 6: Residual plot and residual correlogram of AR(1)

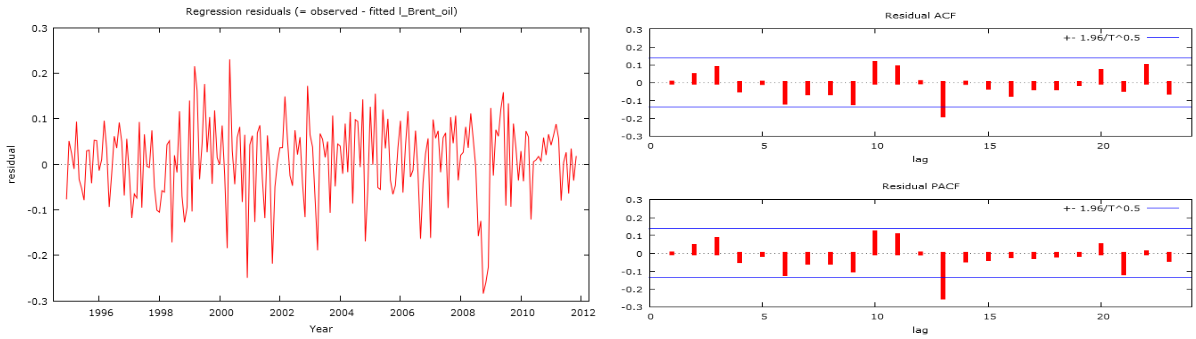

Figure 7: Residual plot and residual correlogram of MA(1)

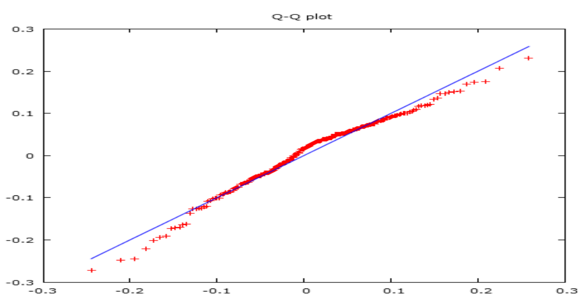

Figure 8: Q-Q plot of ARMA(1,1)

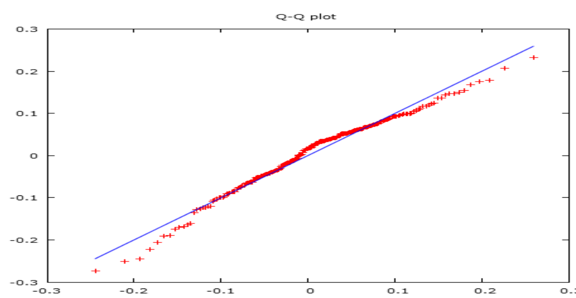

Figure 9: Q-Q plot of AR(2)

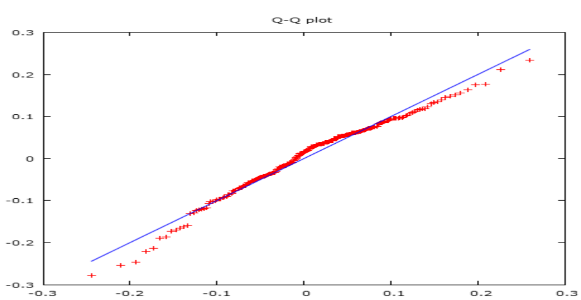

Figure 10: Q-Q plot of AR(1)

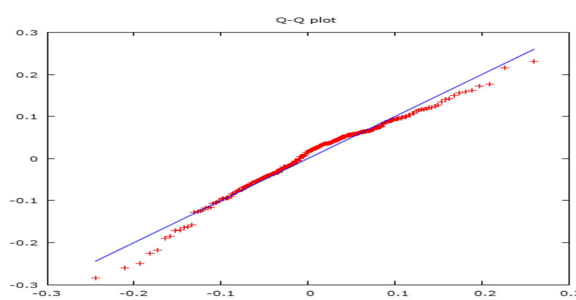

Figure 11: Q-Q plot of MA(1)

It is clear from the diagrams that the models residuals are stationary, however with a spike at lag 13 which is due to the financial crises in 2008. For instance, points which deviate from the normal distribution in the Q-Q plot are the prices for crude oil in the crises period. This is also shown by the Q-Q plot which are not strongly normally distributed. However the Ljung-Box test all suggest p-value which is more than 0.05, validating the normality of the residuals.

4. Result and Forecasting

Once the residuals of our feasible models are accepted to be normally distributed, we compare the forecasting accuracies of these models. We generate the Mean Square Error (MSE) and Mean Absolute Error (MAE) result and use similar approach in Gileva (2010) where multiple n-step ahead (n = 1, 2, 3, years in our case) are used to predict the future by the different models as time evolves. Here, the models with the lowest MSE and MAE are selected as the best model for forecasting of the Brent log return crude oil spot price.

First year	AR(1)	MA(1)	ARMA(1,1)	ARMA(2,0)
MSE	0.0048102	0.0048106	0.0048184	0.0048132
MAE	0.05013	0.050176	0.050123	0.050104

Second year	AR(1)	MA(1)	ARMA(1,1)	ARMA(2,0)
MSE	0.0034038	0.0034174	0.003378	0.0033874
MAE	0.042573	0.042689	0.042353	0.042431

Third year	AR(1)	MA(1)	ARMA(1,1)	ARMA(2,0)
MSE	0.0078316	0.0078705	0.0077459	0.0077799
MAE	0.055297	0.05553	0.05479	0.054989

Table 3: Analysis of forecasting accuracy

We compare the result of the MSE and MAE on different models and realize that ARIMA(1,1,1) model out performs the other models as it has the minimum MSE and MAE in the second and third year forecast. In the first year forecast however, the AR(1) model and the MAE has the minimum MSE and MAE respectively. The ARIMA(1,1,1) in this case produce errors which are not significant different from these two models. The analysis suppose that the ARIMA(1,1,1) has the best forecasting power to forecast the Brent log return crude oil price. We estimate the parameters of this model using the maximum likelihood estimation. This is shown in Table 4.

Variable	Coefficient	Standard Error	z-Statistics	p-value
ϕ	0.437833	0.296988	1.474	0.1404
θ	-0.261620	0.318373	-0.8217	0.4112

Table 4: MLE for the ARIMA(1,1,1) of monthly log return of Brent crude oil price.

The ARIMA(1,1,1) model has the estimated representation:

$$y_t = 0.437y_{t-1} - 0.261u_{t-1} + u_t$$

$$(1 - 0.437L)y_t = (1 - 0.26L)u_t$$

with $y_t = (1 - L)log(x_t)$ where x_t is the original series. The series y_t is stationary with $u_t = WN(0, 0.0080)$.

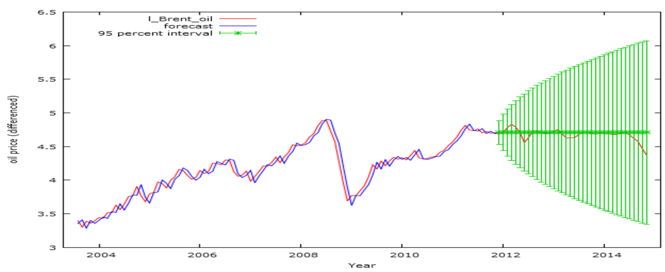

Figure 12: Forecasted plot of Brent crude oil spot price

5. Conclusion

An elementary and popular tool to modelling and forecasting in time series is the ARIMA model. While the model can predict well in some series, its forecasting performance can be woefully bad in the presence of outliers, measurement errors and volatilities in the series. In this project, the Box Jenkins method is used to model the Brent crude oil price to examine a best model and its forecastability. The forecast accuracy was considered using the MSE and MAE technique. The result showed that the proposed ARIMA(1,1,1) model has the best forecasting model. However, a review of volatility in the price of the oil price suggest that the proposed model may not forecast well in period of high volatilities. The ARCH model proposed by Engle (1982) and its variants (GARCH, EGARCH, APARCH etc.) and recent models such as the SVM proposed in literature may provide best forecasting accuracy for such volatile models.

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