Publication Bias in Meta-Analysis: Confidence Intervals for Rosenthal's Fail-Safe Number

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Abstract

The purpose of the present paper is to assess the efficacy of confidence intervals for Rosenthal's fail-safe number. Although Rosenthal's estimator is highly used by researchers, its statistical properties are largely unexplored. First of all we developed statistical theory allowing us to produce confidence intervals for Rosenthal's fail-safe number. This was produced by discerning whether the number of studies analysed in a meta-analysis is fixed or random. Each case produces different variance estimators. For a given number of studies and a given distribution, we provided five variance estimators. Confidence intervals are examined with a normal approximation and a non-parametric bootstrap. The accuracy of the different confidence interval estimates was then tested by methods of simulation under different distributional assumptions. The half normal distribution variance estimator has the best probability coverage.. Finally, we provide a table of lower confidence intervals for Rosenthal's estimator.

Keywords: Rosenthal's fail-safe number; publication bias; confidence intervals; distribution; meta-analysis

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1 Introduction

Meta-analysis refers to methods focused on contrasting and combining results from different studies, in the hope of identifying patterns among study results, sources of disagreement among those results, or other interesting relationships that may come to light in the context of multiple studies [1]. In its simplest form, this is normally done by identification of a common measure of effect size, of which a weighted average might be the output of a meta-analysis. The weighting might be related to sample sizes within the individual studies [2, 3]. More generally there are other differences between the studies that need to be allowed for, but the general aim of a meta-analysis is to more powerfully estimate the true effect size as opposed to a less precise effect size derived in a single study under a given single set of assumptions and conditions [4]. For reviews on meta-analysis models, see [2], [5] and [6]. Meta-analysis can be applied to various effect sizes collected from individual studies. These include odds ratios and relative risks; standardized mean difference, Cohen's *d*, Hedges' *g*, Glass's Δ; correlation coefficient and relative metrics; sensitivity and specificity from diagnostic accuracy studies; and *p*-values. For more comprehensive reviews see Rosenthal [7], Hedges and Olkin [8] and Cooper et al. [9].

2 Publication bias

Publication bias is a threat to any research that attempts to use the published literature, and its potential presence is perhaps the greatest threat to the validity of a meta-analysis [10]. Publication bias exists because research with statistically significant results is more likely to be submitted and published than work with null or non-significant results. This very issue was memorably termed as the *file-drawer problem* by Rosenthal [11]; non-significant results are stored in file drawers without ever being published. In addition to publication bias, several other related biases exist including pipeline bias, subjective reporting bias, duplicate reporting bias or language bias (see [12-15] for definitions and examples).

The implication of these various biases is that combining only the identified published studies uncritically may lead to an incorrect, usually over optimistic, conclusion [10, 16]. The ability to detect publication bias in a given field is a key strength of meta-analysis, because identification of publication bias will challenge the validity of common views in that area, and will spur further investigations [17]. There are two types of statistical procedures for dealing with publication bias in meta-analysis: methods for identifying the existence of publication bias and methods for assessing the impact of publications bias [16]. The first includes the funnel plot (and other visualisation methods such as the normal quantile plot) and regression/correlation-based tests; while the second includes the fail-safe (also called file-drawer) number, the trim and fill method and selection model approaches [10, 14, 18]. Recent approaches include the Test for Excess Significance [19] and the *p*-curve [20].

The most commonly used method is the visual inspection of a funnel plot. This assumes that the results from smaller studies will be more widely spread around the mean effect because of larger random error. The next most frequent method used to assess publication bias is Rosenthal's fail-safe number[21]. Two recent reviews examining the assessment of publication bias in psychology and ecology reported that funnel plots were the most frequently used (24% and 40 % respectively), followed by Rosenthal's fail-safe number (22% and 30% respectively).

Assessing publication bias by computing the number of unpublished studies

Assessing publication bias can be performed by trying to estimate the number of unpublished studies in the given area a meta-analysis is studying. The fail-safe number represents the number of studies required to refute significant meta-analytic means. Although apparently intuitive, it is in reality difficult to interpret not only because the number of data points (i.e. sample size) for each of *k* studies is not defined, but also because no benchmarks regarding the fail-safe number exist, unlike Cohen's benchmarks for effect size statistics [22]. However, these versions have been heavily criticised, mainly because such numbers are often misused and misinterpreted [23]. The main reason for the criticism is that depending on which method is used to estimate the fail-safe N, the number of studies can greatly vary.

Rosenthal's fail-safe number

Although Rosenthal's fail-safe number of publication bias was proposed as early as 1979 and is frequently cited in the literature [11] (over 2000 citations), little attention has been given to the statistical properties of this estimator. This is the aim of the present paper, which is discussed in further detail in Section 3.

Rosenthal [11] introduced what he called the *file drawer problem*. His concern was that some statistically non-significant studies may be missing from an analysis (i.e., placed in a file drawer) and that these studies, if included, would nullify the observed effect. By *nullify*, he meant to reduce the effect to a level not statistically significantly different from zero. Rosenthal suggested that rather than speculate on whether the file drawer problem existed, the actual number of studies that would be required to nullify the effect could be calculated [24]. This method calculates the significance of multiple studies by calculating the significance of the mean of the standard normal deviates of each study. Rosenthal's method calculates the number of additional studies N_R, with the mean null result necessary to reduce the combined significance to a desired α level (usually 0.05).

The necessary prerequisites is that each study examines a directional null hypothesis such that the effect sizes θ_i from each study are examined under $\theta_i \leq 0$ or ($\theta_i \geq 0$). Then the null hypothesis of Stouffer's [25] test is:

$$H_0 : \theta_1 = \cdots = \theta_k = 0 .$$

The test statistic for this is:

$$Z_S = \frac{\sum_{i=1}^{k} Z_i}{\sqrt{k}} , \tag{1}$$

with $z_i = \frac{\theta_i}{s_i}$, where s_i are the standard errors of θ_i. Under the null hypothesis we have $Z_S \sim N(0,1)$ [7].

So we get that the number of additional studies N_R, with mean null result necessary to reduce the combined significance to a desired α level (usually 0.05 [7, 11]), is found after solving

$$Z_\alpha = \frac{\sum_{i=1}^{k} Z_i}{\sqrt{N_R + k}} . \tag{2}$$

So, N_R is calculated as

$$N_R = \frac{\left(\sum_{i=1}^{k} Z_i\right)^2}{Z_\alpha^2} - k , \tag{3}$$

where k is the number of studies and Z_α is the one-tailed Z score associated with the desired α level of significance. Rosenthal further suggested that if $N_R > 5k + 10$, the likelihood of publication bias would be minimal.

Cooper [26, 27] called this number the fail-safe sample size or fail-safe number. If this number is relatively small, then there is cause for concern. If this number is large, one might be more confident that the effect, although possibly inflated by the exclusion of some studies, is, nevertheless, not zero [28]. This approach is limited in two important ways [24, 29]. First, it assumes that the association in the hidden studies is zero, rather than considering the possibility that some of the studies could have an effect in the reverse direction or an effect that is small but not zero. Therefore, the number of studies required to nullify the effect may be different than the fail-safe number, either larger or smaller. Second, this approach focuses on statistical significance rather than practical or substantive significance (effect sizes). That is, it may allow one to assert that the mean correlation is not zero, but it does not provide an estimate of what the correlation might be (how it has changed in size) after the missing studies are included [23, 30-32]. However, for many fields it remains the gold standard to assess publication bias, since its presentation is conceptually simple and eloquent. In addition, it is computationally easy to perform.

Iyengar and Greenhouse [12] proposed an alternative formula for Rosenthal's fail-safe number, in which the sum of the unpublished studies' standard variates is not zero. In this case the number of unpublished studies n_α is approached through the following equation

$$Z_\alpha = \frac{\sum_{i=1}^{k} Z_i + n_\alpha M(\alpha)}{\sqrt{n_\alpha + k}}, \tag{4}$$

where $M(\alpha) = -\frac{\phi(z_\alpha)}{\Phi(z_\alpha)}$ [1] and α is the desired level of significance. This is justified by the author that the unpublished studies follow a truncated normal distribution with $x \le z_\alpha$.[2] $\Phi(\cdot)$ and $\phi(\cdot)$

[1] This results immediately from the definition of truncated normal distribution.

[2] There are certain other fail-safe numbers which have been described, but their explanation goes beyond the scope of the present article [33]. Duval and Tweedie [34, 35] present three different estimators for the number of missing studies and the method to calculate this has been named Trim and Fill Method. Orwin's [36] approach is very similar to Rosenthal's [11] without considering the normal variates but taking Cohen's *d* [22] to compute a fail-safe number. Rosenberg's fail-safe number is very similar to Rosenthal's and Orwin fail-safe number [37]. Its difference is that it takes into account the meta-analytic estimate under investigation by incorporating individual weights per study. Gleser and Olkin [38] proposed a model under which the number of unpublished studies in a field where a meta-analysis is undertaken could be estimated. The maximum likelihood estimator of their fail-safe number only needs the number of studies and the maximum *p* value of the studies. Finally, the Eberly-Casella fail-safe number assumes a Bayesian methodology which aims to estimate the number of unpublished studies in a certain field where a meta-analysis is undertaken [39].

denote the cumulative distribution function (CDF) and probability distribution function (PDF) respectively of a standard normal distribution.

The aim of the present paper is to study the statistical properties of Rosenthal's [11] fail-safe number. In the next section we introduce the statistical theory for computing confidence intervals for Rosenthal's [11] fail-safe number. We initially compute the probability distribution function of $\hat{N}_R$, which gives formulas for variance and expectation; next we suggest distributional assumptions for the standard normal variates used in Rosenthal's fail-safe number and finally suggest confidence intervals.

3 Statistical Theory

The estimator $\hat{N}_R$ of unpublished studies is approached through Rosenthal's formula

$$\hat{N}_R = \frac{\left(\sum_{i=1}^{k} Z_i\right)^2}{Z_\alpha^2} - k \, . \tag{5}$$

Let Z_i, $i = 1,2,\ldots,i,\ldots,k$ be i.i.d. random variables with $E[Z_i] = \mu$ and $Var[Z_i] = \sigma^2$. We discern two cases:

a) k is fixed or,

b) k is random, assuming additionally assume that $k \sim Pois(\lambda)$. This is reasonable since the number of studies included in a meta-analysis is like observing counts.[3]

In both cases, estimators of μ, σ^2 and λ can be calculated without distributional assumptions for the Z_i with the method of moments or with distributional assumptions regarding the Z_i.

Probability Distribution Function of $\hat{N}_R$

Fixed k

We compute the PDF of $\hat{N}_R$ by following the next steps

[3] Other distributions might be assumed, such as the Gamma distribution, but this would require more information or assumptions to compute the parameters of the distribution.

Step 1. $Z_1, Z_2, \ldots, Z_i, \ldots, Z_k$ in the formula of the estimator $\hat{N}_R$ (5) are i.i.d. distributed with $E[Z_i] = \mu$ and $Var[Z_i] = \sigma^2$. Let $S = \sum_{i=1}^{k} Z_i$ and according to the Lindeberg-Lévy Central Limit Theorem [40], we have

$$\sqrt{k}\left(\frac{S}{k} - \mu\right) \xrightarrow{d} N(0, \sigma^2) \Rightarrow$$

$$S \xrightarrow{d} N(k\mu, k\sigma^2). \tag{9}$$

So the PDF of S is

$$f_S(s) = \frac{1}{\sqrt{2\pi k\sigma^2}} \exp\left[-\frac{(s - k\mu)^2}{2k\sigma^2}\right]. \tag{10}$$

Step 2. The PDF of Rosenthal's $\hat{N}_R$ can be retrieved from a truncated version of (10). From (2) we get that:

$$S = Z_\alpha \sqrt{\hat{N}_R + k}\,. \tag{11}$$

We advocate that Rosenthal's equation (2) and equation (11) implicitly impose two conditions which must be taken into account when we seek to estimate the distribution of N_R:

$$S \geq 0, \tag{12}$$

$$\hat{N}_R \geq 0. \tag{13}$$

Expression (12) is justified by the fact that the right hand side of (11) is positive, so then $S \geq 0$. Expression (13) is justified by the fact that N_R expresses the number of studies, so it must be at least 0. Hence, expression (12) and (13) are satisfied when S is a truncated normal random variable, let it be S^*, such that $S^* \geq Z_\alpha \sqrt{k}$. So the PDF of S^* then becomes:

$$f_{S^*}\left(s^*\right)=\frac{1}{\Phi\left(\lambda^*\right)\sqrt{2\pi k\sigma^2}}\exp\left[-\frac{\left(s^*-k\mu\right)^2}{2k\sigma^2}\right],\ s^*\geq Z_\alpha\sqrt{k}\,, \tag{14}$$

where $\lambda^*=\dfrac{\sqrt{k}\mu-Z_\alpha}{\sigma}$.[4]

Then, we have:

$$f_{\hat{N}_R}\left(n_R\right)=f_{S^*}\left(s^*\right)\left|\frac{dS^*}{dN_R}\right|\overset{(11),(14)}{\Rightarrow}$$

$$f_{\hat{N}_R}\left(n_R\right)=\frac{Z_\alpha}{2\Phi\left(\lambda^*\right)\sqrt{2\pi k\sigma^2\left(n_R+k\right)}}\exp\left[-\frac{\left(Z_\alpha\sqrt{n_R+k}-k\mu\right)^2}{2k\sigma^2}\right],\ n_R\geq 0\,. \tag{15}$$

The characteristic function is:

$$\psi_{\hat{N}_R}\left(t\right)=E\left[\exp\left(itN_R\right)\right]=\frac{\Phi\left(\dfrac{\mu_1+\lambda^*}{\sigma_1}\right)}{\Phi\left(\lambda^*\right)}\cdot\frac{Z_\alpha\exp\left(\dfrac{k^2\mu^2 it}{Z_\alpha^2\text{-}2k\sigma^2 it}-kit\right)}{\left(Z_\alpha^2\text{-}2k\sigma^2 it\right)^{1/2}}\,, \tag{16}$$

where $i=\sqrt{-1}\,,\ \mu_1=\dfrac{2k\sqrt{k}\mu\sigma it}{Z_\alpha^2-2k\sigma^2 it},\ \sigma_1^2=\dfrac{Z_\alpha^2}{Z_\alpha^2-2k\sigma^2 it}\,.$

From (16) we get: $$E\left[\hat{N}_R\right]=\frac{k^2\mu^2+k\sigma^2}{Z_\alpha^2}-k+\varepsilon\,, \tag{17}$$

where $\varepsilon=\dfrac{\phi\left(\lambda^*\right)}{\Phi\left(\lambda^*\right)}\cdot\dfrac{k\sigma\left(\sqrt{k}\mu+Z_\alpha\right)}{Z_\alpha^2}\,.$

Also,

[4] The *truncated normal distribution* is a probability distribution related to the normal distribution. Given a normally distributed random variable X with mean μ_t and variance σ_t^2, let it be that $X\in\left(a,b\right),-\infty\leq a\leq b\leq\infty$. Then X conditional on $a<X<b$ has a truncated normal distribution with PDF: $f_X\left(x\right)=\dfrac{\frac{1}{\sigma}\phi\left(\frac{x\text{-}\mu_t}{\sigma_t}\right)}{\Phi\left(\frac{b\text{-}\mu_t}{\sigma_t}\right)-\Phi\left(\frac{a\text{-}\mu_t}{\sigma_t}\right)}$, for $a\leq x\leq b$ and $f_X\left(x\right)=0$ otherwise [41].

$$Var\left[\hat{N}_R\right]=\frac{2k^2\sigma^2\left(2k\mu^2+\sigma^2\right)}{Z_\alpha^4}+\delta^*, \tag{18}$$

where $\delta^*=\dfrac{\phi\left(\lambda^*\right)}{\Phi\left(\lambda^*\right)}\left[\dfrac{k^2\sigma^3\left(5\sqrt{k}\mu+Z_a\right)^2}{Z_\alpha^4}-\left(\dfrac{\phi\left(\lambda^*\right)}{\Phi\left(\lambda^*\right)}+\lambda^*\right)\dfrac{k^{3/2}\sigma^2\left(\sqrt{k}\mu+Z_a\right)^2}{Z_\alpha^4}\right].$

Proofs for expressions (16-18) are given in the Appendix.

Comments:

1. For a significantly large k we have that $\Phi\left(\lambda^*\right)\approx 1$. So (15) becomes

$$f_{\hat{N}_R}\left(n_R\right)=\frac{Z_\alpha}{2\sqrt{2\pi k\sigma^2\left(n_R+k\right)}}\exp\left[-\frac{\left(Z_\alpha\sqrt{n_R+k}-k\mu\right)^2}{2k\sigma^2}\right],\ n_R\geq 0. \tag{19}$$

Also we get:

$$E\left[\hat{N}_R\right]=\frac{k^2\mu^2+k\sigma^2}{Z_\alpha^2}-k, \tag{20}$$

$$Var\left[\hat{N}_R\right]=\frac{2k^2\sigma^2\left(2k\mu^2+\sigma^2\right)}{Z_\alpha^4}. \tag{21}$$

2. A limiting element of this computation is that $\hat{N}_R$ takes discrete values because it describes number of studies, but it has been described by a continuous distribution.

Random k

It is assumed that $k\sim Pois(\lambda)$. So taking into account the result from the distribution of $\hat{N}_R$ for a fixed k we get that the joint distribution of k and $\hat{N}_R$ is

$$f_{\hat{N}_R,n}\left(n_R,k\right)=f_{\hat{N}_R}\left(n_R|k=k\right)\cdot p\left(k=k\right)\Rightarrow$$

$$f_{\hat{N}_R,n}\left(n_R,k\right)=\frac{Z_\alpha}{2\Phi\left(\lambda^*\right)\sqrt{2\pi k\sigma^2\left(n_R+k\right)}}\exp\left[-\frac{\left(Z_\alpha\sqrt{n_R+k}-k\mu\right)^2}{2k\sigma^2}-\lambda\right]\cdot\frac{\lambda^k}{k!},\ n_R\geq 0,\ \ k=0,1,2,\ldots \tag{22}$$

Expectation and Variance for Rosenthal's estimator $\hat{N}_R$

a) When *k* is fixed, expressions (20) and (21) denote the expectation and variance respectively for $\hat{N}_R$. This is derived from the PDF of $\hat{N}_R$; an additional proof without reference to the PDF is given in the Appendix.

b) When *k* is random with $k \sim Pois(\lambda)$, the expectation and variance of $\hat{N}_R$ are :

$$E\left[\hat{N}_R\right] = \frac{\lambda^2 \mu^2 + \lambda\left(\mu^2 + \sigma^2\right)}{Z_\alpha^2} - \lambda \,, \tag{23}$$

$$Var\left[\hat{N}_R\right] = \frac{\left(4\lambda^3 + 6\lambda^2 + \lambda\right)\mu^4 + \left(4\lambda^3 + 16\lambda^2 + 6\lambda\right)\mu^2\sigma^2 + \left(2\lambda^2 + 3\lambda\right)\sigma^4}{Z_\alpha^4} - 2 \cdot \frac{\left(2\lambda^2 + \lambda\right)\mu^2 + \lambda\sigma^2}{Z_\alpha^2} + \lambda \,. \tag{24}$$

Proofs are given in the Appendix.

Estimators for μ, σ² and λ

Having now computed a formula for the variance which is necessary for a confidence interval, we need to estimate μ, σ^2 and λ. In both cases, estimators of μ, σ^2 and λ can be calculated without distributional assumptions for the Z_i with the method of moments or with distributional assumptions regarding the Z_i.

Method of moments [42]

When *k* is fixed, we have:

$$\hat{\mu} = \frac{\sum_{i=1}^{k} \hat{Z}_i}{k} \,,\ \hat{\sigma}^2 = \frac{\sum_{i=1}^{k} \hat{Z}_i^2}{k} - \left(\frac{\sum_{i=1}^{k} \hat{Z}_i}{k}\right)^2 . \tag{25}$$

When *k* is random, we have:

$$\hat{\lambda} = k \,,\ \hat{\mu} = \frac{\sum_{i=1}^{k} \hat{Z}_i}{k} \,,\ \hat{\sigma}^2 = \frac{\sum_{i=1}^{k} \hat{Z}_i^2}{k} - \left(\frac{\sum_{i=1}^{k} \hat{Z}_i}{k}\right)^2 . \tag{26}$$

Distributional assumptions for the $\mathrm{Z_i}$

If we suppose that the Z_i follow a distribution we would replace the values of μ and σ^2 with their distributional values. Below we consider special cases.

Standard Normal Distribution

The Z_i follow a standard normal distribution i.e. $Z_i \sim N(0,1)$. This is the original assumption for the Z_i [11]. In this case we have:

$$\hat{\lambda} = k, \ \mu = 0, \ \sigma^2 = 1. \tag{27}$$

Although the origin of the Z_i is from the standard normal distribution, the studies in a meta-analysis are a selected sample of published studies. For this reason, the next distribution is suggested as better.

Half Normal Distribution

Here we propose that the Z_i follow a half normal distribution $HN(0,1)$, which is special case of folded normal distribution. Before we explain the rational of this distribution, a definition of this type of distribution is provided. A half normal distribution is also a special case of a truncated normal distribution.

Definition 1: The *folded normal distribution* is a probability distribution related to the normal distribution. Given a normally distributed random variable X with mean μ_f and variance σ_f^2, the random variable $Y = |X|$ has a folded normal distribution [41, 43, 44].

Remark 1: The folded normal distribution has the following properties:

a) Probability density function (PDF):

$$f_Y(y) = \frac{1}{\sigma_f \sqrt{2\pi}} \exp\left[-\frac{(-y-\mu_f)^2}{2\sigma_f^2}\right] + \frac{1}{\sigma_f \sqrt{2\pi}} \exp\left[-\frac{(y-\mu_f)^2}{2\sigma_f^2}\right], \text{ for } y \geq 0.$$

b)
$$E[Y] = \sigma_f \sqrt{2/\pi}\exp\left(-\mu_f^2/2\sigma_f^2\right) + \mu_f\left[1 - 2\Phi\left(-\mu_f/\sigma_f\right)\right],$$

$$Var[Y] = \mu_f^2 + \sigma_f^2 - \left\{\sigma_f \sqrt{2/\pi}\exp\left(-\mu_f^2/2\sigma_f^2\right) + \mu_f\left[1 - 2\Phi\left(-\mu_f/\sigma_f\right)\right]\right\}^2.$$

Remark 2: When $\mu_f = 0$, the distribution of Y is a *half-normal distribution*. This distribution is identical to the *truncated normal distribution*, with left truncation point 0 and no right truncation point. For this distribution we have

a) $f_Y(y) = \frac{\sqrt{2}}{\sigma_f \sqrt{\pi}} \exp\left(-\frac{y^2}{2\sigma_f^2}\right)$, for $y \geq 0$.

b) $E[Y] = \sigma_f \sqrt{2/\pi}$, $Var[Y] = \sigma_f^2 (1 - 2/\pi)$.

Assumption: The Z_i in Rosenthal's estimator N_R are derived from a half normal distribution, based on a normal distribution $N(0,1)$.

Support: When a researcher begins to perform a meta-analysis, the sample of studies is drawn from those studies that are already published. So his sample is most likely biased by some sort of selection bias, produced via a specific selection process [45]. Thus, although when we study Rosenthal's N_R assuming that all Z_i are drawn from the normal distribution, they are in essence drawn from a truncated normal distribution. This has been commented on by Iyengar and Greenhouse [12] and Schonemann and Scargle [46]. But at which point is this distribution truncated? We would like to advocate that the half normal distribution, based on a normal distribution $N(0,1)$, is the one best representing the Z_i Rosenthal uses to compute his fail-safe N_R. The reasons for this are:

1. Firstly, to assume that all Z_i are of the same sign does not impede the significance of the results from each study. That is the test is significant when either $Z_i > Z_{\alpha/2}$ or $Z_i < Z_{1-\alpha/2}$ occurs.

2. However, when a researcher begins to perform a meta-analysis of studies, many times Z_i can be either positive or negative. Although this is true, when the researcher is interested in doing a meta-analysis, usually the Z_i that have been published are indicative of a significant effect of the same direction (thus Z_i have the same sign) or are at least indicative of such an association without being statistically significant; hence, producing Z_i of the same sign but not producing significance (e.g. the confidence interval of the effect might include the null value).

3. There will definitely be studies that produce a totally opposite effect, thus producing an effect of opposite direction; but these will definitely be a minority of the studies. Also there is the case that these other signed Z_i are not significant.

Hence, in this case

$$\hat{\lambda} = k\,,\ \mu = \sqrt{2/\pi}\,,\ \sigma^2 = 1 - 2/\pi\,. \qquad (28)$$

Skew Normal Distribution

Here we propose that the Z_i follow a skew normal distribution i.e. $Z_i \sim SN(\xi,\omega,\alpha)$:

Definition 3: The skew normal distribution is a continuous probability distribution that generalises the normal distribution to allow for non-zero skewness. A random variable X follows a univariate skew normal distribution with location parameter $\xi \in R$, scale parameter $\omega \in R^+$ and skewness parameter $\alpha \in R$ [47], if it has the density

$$f_X(x) = \frac{2}{\omega}\phi\left(\frac{x-\xi}{\omega}\right)\Phi\left(\alpha\frac{x-\xi}{\omega}\right) \qquad x \in R.$$

Note that if $\alpha = 0$, the density of X reduces to the $N(\xi,\omega^2)$

Remark 1: The expectation and variance of X are [47]:

$$E[X] = \xi + \omega\delta\sqrt{\frac{2}{\pi}}\text{, where } \delta = \frac{\alpha}{\sqrt{1+\alpha^2}}\,,\ Var(X) = \omega^2\left(1 - \frac{2\delta^2}{\pi}\right).$$

Remark 2: The method of moments estimators for ξ,ω,δ are [48, 49]:

$$\tilde{\xi} = m_1 - a_1\left(\frac{m_3}{b_1}\right)^{\frac{1}{3}},\ \tilde{\omega}^2 = m_2 - a_1^2\left(\frac{m_3}{b_1}\right)^{\frac{2}{3}},\ \tilde{\delta} = \left[a_1^2 + m_2\left(\frac{b_1}{m_3}\right)^{\frac{2}{3}}\right]^{-\frac{1}{2}},$$

where $a_1 = \sqrt{2/\pi}$, $b_1 = (4/\pi - 1)a_1$, $m_1 = n^{-1}\sum_{i=1}^{n} X_i$, $m_2 = n^{-1}\sum_{i=1}^{n}(X_i - m_1)^2$, $m_3 = n^{-1}\sum_{i=1}^{n}(X_i - m_1)^3$. The sign of $\tilde{\delta}$ is taken to be the sign of m_3.

Explanation: The skew normal distribution allows for a dynamic way to fit the available *Z*-scores. The fact that there is ambiguity on the derivation of the standard deviates from each study from a normal or a truncated normal distribution, creates the possibility that the distribution could be a skew-normal, with the skewness being attributed that we are including only the published Z-scores in the estimation of Rosenthal's [11] estimator.

Hence, in this case and taking the method of moments estimators of ξ, ω, δ, we get:

$$\hat{\lambda} = n\,,\ \hat{\mu} = \tilde{\xi} + \tilde{\omega}\tilde{\delta}\sqrt{\frac{2}{\pi}}\ ,\ \hat{\sigma}^2 = \tilde{\omega}^2\left(1 - \frac{2\tilde{\delta}^2}{\pi}\right), \qquad (29)$$

where $a_1 = \sqrt{2/\pi}$, $b_1 = (4/\pi - 1)a_1$, $m_1 = n^{-1}\sum_{i=1}^{n} Z_i$, $m_2 = n^{-1}\sum_{i=1}^{n}(Z_i - m_1)^2$, $m_3 = n^{-1}\sum_{i=1}^{n}(Z_i - m_1)^3$.

Methods for confidence intervals

Normal Approximation

In the previous section formulas for computing the variance of $\hat{N}_R$ were derived. We compute asymptotic $(1-\alpha/2)\%$ confidence intervals for N_R as:

$$\left(\hat{N}_{R\ low}\ ,\hat{N}_{R\ up}\right) = \left(\hat{N}_R - Z_{1-\alpha/2}\sqrt{Va\hat{r}\left[\hat{N}_R\right]}\ ,\ \hat{N}_R + Z_{1-\alpha/2}\sqrt{Va\hat{r}\left[\hat{N}_R\right]}\right), \qquad (33)$$

where $Z_{1-\alpha/2}$ is the $(1-\alpha/2)th$ quantile of the standard normal distribution.

The variance of $\hat{N}_R$ for a given set of values Z_i depends firstly on whether the number of studies k is fixed or random and secondly whether the estimators of μ, σ^2 and λ are derived from the method of moments or from the distributional assumptions.

Nonparametric Bootstrap

Bootstrap is a well-known resampling methodology for obtaining nonparametric confidence intervals of a parameter [50, 51]. In most statistical problems one needs an estimator of a parameter of interest as well as some assessment of its variability. In many such problems the estimators are complicated functionals of the empirical distribution function and it is difficult to derive trustworthy analytical variance estimates for them. The primary objective of this technique is to estimate the sampling distribution of a statistic. Essentially, bootstrap is a method that mimics the process of sampling from a population, like one does in Monte Carlo simulations, but instead drawing samples from the observed sampling data. The tool of this mimic process is the Monte Carlo algorithm of Efron [52]. This process is explained properly by Efron and Tibshirani [53] and Davison and Hinkley [54], who also noted that bootstrap confidence intervals are approximate, yet better than the standard ones. Nevertheless, they do not try to replace the theoretical ones and neither is bootstrap a substitute for precise parametric results, but rather a way to reasonably proceed when such results are unavailable.

Non-parametric resampling makes no assumptions concerning the distribution of, or model for, the data [55]. Our data is assumed to be a vector $\mathbf{Z}_{obs}$ of *k* independent observations, and we are interested in a confidence interval for $\hat{\theta}(\mathbf{Z}_{obs})$. The general algorithm for a non-parametric bootstrap is as follows:

1. Sample *k* observations randomly with replacement from $\mathbf{Z}_{obs}$ to obtain a bootstrap data set, denoted $\mathbf{Z}^*$.
2. Calculate the bootstrap version of the statistic of interest $\hat{\theta}^* = \hat{\theta}(\mathbf{Z}^*)$.
3. Repeat steps 1 and 2 a large number of times, say *B*, to obtain an estimate of the bootstrap distribution.

In our case:

1. Compute a random sample from the initial sample of Z_i, size *k*.
2. Compute N_R^* from this sample.
3. Repeat these process *b* times.

Then the bootstrap estimator of N_R is:

$$N_{R\_bootstrap} = \frac{\sum N_R^*}{b}.$$

From this we can compute also confidence intervals for $N_{R\_bootstrap}$.

In the next section, we investigate these theoretical aspects with simulations and examples.

4 Simulations and Results

The method for simulations is as follows:

1. Initially we draw random numbers from the following distributions
    a. Standard Normal Distribution

b. Half Normal Distribution (0,1)
c. Skew Normal Distribution with negative skewness $SN(\delta = -0.5, \xi = 0, \omega = 1)$
d. Skew Normal Distribution with positive skewness $SN(\delta = 0.5, \xi = 0, \omega = 1)$

2. The numbers we draw from each distribution represent the number of studies in a meta-analysis and we have chosen k = 5, 15, 30, and 50. When k is assumed to be random, then the parameter λ is equal to the values chosen for the simulation, i.e. 5, 15, 30, and 50 respectively.
3. We compute the normal approximation confidence interval with the formulas described in Section 3 and the bootstrap confidence interval. We also discern whether the number of studies is fixed or random. For the computation of the bootstrap confidence interval, we generate 1,000 bootstrap samples each time. We also study the performance of the different distributional estimators in cases where the distributional assumption is not met, hence comparing each of the six confidence interval estimators under all four distributions.
4. We compute the coverage probability comparing with the true value of Rosenthal's fail-safe number. When the number of studies is fixed the true value of Rosenthal's number is:

$$E\left[\hat{N}_R\right] = \frac{k^2\mu^2 + k\sigma^2}{Z_\alpha^2} - k .$$

When the number of studies is random [from a Poisson(λ) distribution] the true value of Rosenthal's number is:

$$E\left[\hat{N}_R\right] = \frac{\lambda^2\mu^2 + \lambda\left(\mu^2 + \sigma^2\right)}{Z_\alpha^2} - \lambda .$$

We execute the above procedure 10,000 times each time. Our alpha-level is considered 5%.

This process is shown schematically in Table 1. All simulations were performed in R and the code is shown in the Supplementary Materials.

We observe from Table 2 and Figure 1 that the bootstrap confidence intervals perform the poorest both when the number of studies are considered fixed or random. The only case in which they perform acceptably is when the distribution is half normal and the number of studies is fixed. The moment estimators of variance either perform poorly or too efficiently in all cases, with coverages being under 90% or near 100%. The most acceptable confidence intervals for Rosenthal's estimator appear to be in the distribution based method, and much better for a fixed number of studies than for random number of studies. We also observe that for the distribution based confidence intervals in the fixed category, the half normal distribution *HN*(0,1) produces coverages which are all 95%. This is also stable for all number of studies in a meta-analysis. When the distributional assumption is not met the coverage is poor except for the cases of the positive and negative skewness skew normal distributions which perform similarly, possibly due to symmetry.

In the next sections we give certain examples and we present the lower limits of confidence intervals for testing whether $N_R > 5k + 10$, according to the suggested rule of thumb by [11]. We choose only the variance from a fixed number of studies when the Z_i are drawn from a half normal distribution *HN*(0,1).

(Tables 1 and 2 and Figure 1 here)

5 Examples

In this section we present two examples of meta-analyses from the literature. The first study is a meta-analysis of the effect of probiotics for preventing antibiotic associated diarrhoea and included 63 studies [56]. The second meta-analysis comes from the psychological literature and is a meta-analysis examining reward, cooperation and punishment, including analysis of 148 effect sizes [57]. For each meta-analysis we computed Rosenthal's fail-safe number and the respective confidence interval with the methods described above.

We observe that both fail-safe numbers exceed Rosenthal's rule of thumb, but some lower confidence intervals, especially in the first example go as low as 369 which only slightly surpasses the rule of thumb (5*63 + 10 = 325 in this case). This is not the case with the second example. Hence the confidence interval and especially the lower confidence interval value is important to establish whether the fail-safe number surpasses the rule of thumb.

In the next section we present a table with values according to which future researchers can get advice on whether their value truly supersedes the rule of thumb.

(Table 3 here)

6 Suggested Confidence Limits for N_R

We wish to answer the question whether $N_R > 5k + 10$ for a given level of significance and the estimate $\hat{N}_R$, which is the rule of thumb suggested by Rosenthal. We formulate a hypothesis test according to which

$$H_0 : N_R \le 5k + 10$$

$$H_1 : N_R > 5k + 10$$

An asymptotic test statistic for this is:

$$T = \frac{\hat{N}_R - 5k - 10}{\sqrt{Var[\hat{N}_R]}} \xrightarrow{d} N(0,1), \text{ under the null hypothesis}$$

So we reject the null hypothesis if $\frac{\hat{N}_R - 5k - 10}{\sqrt{Var[\hat{N}_R]}} > Z_\alpha \Rightarrow \hat{N}_R > Z_\alpha \sqrt{Var[\hat{N}_R]} + 5k + 10$.

In Table 3.7 we give the limits of N_R above which we are 95% confident that $N_R > 5k + 10$. For example if a researcher performs a meta-analysis of 25 studies, the rule of thumb suggests that over $5 \cdot 25 + 10 = 135$ studies there is no publication bias. The present approach and the values of Table 7 suggest that we are 95% confident for this when N_R exceeds 209 studies. So this approach allows for inferences about Rosenthal's $\hat{N}_R$ and is also slightly more conservative especially when Rosenthal's fail-safe number is characterised from overestimating the number of published studies.

(Table 4 here)

7 Discussion and Conclusion

The purpose of the present paper was to assess the efficacy of confidence intervals for Rosenthal's fail-safe number. We initially defined publication bias and described an overview of the available literature on fail-safe calculations in meta-analysis. Although Rosenthal's estimator is highly used by researchers, its properties and usefulness have been questioned [46, 58].

The original contributions of the present paper are its theoretical and empirical results. First, we developed statistical theory allowing us to produce confidence intervals for Rosenthal's fail-safe number. This was produced by discerning whether the number of studies analysed in a meta-analysis is fixed or random. Each case produces different variance estimators. For a given number of studies and a given distribution, we provided five variance estimators: moment and distribution based estimators based on whether the number of studies is fixed or random and bootstrap confidence intervals. Secondly, we examined four distributions by which we can simulate and test our hypotheses of variance, namely standard normal distribution, half normal distribution a positive skew normal distribution and a negative skew normal distribution. These four distributions were chosen as closest to the nature of the Z_is. The half normal distribution variance estimator appears to present the best coverage for the confidence intervals. Hence this might support the hypothesis that the Z_is are derived from a half normal distribution. Thirdly, we provide a table of lower confidence intervals for Rosenthal's estimator.

The limitations of the study initially stem from the flaws associated with Rosenthal's estimator. This usually means that the number of negative studies needed to disprove the result is highly overestimated. However its magnitude can give an indication for no publication bias. Another possible flaw could come from the simulation planning. We could try more values for the skew normal distribution, for which we tried only two values in present paper.

The implications of this research for applied researchers in psychology, medicine and social sciences, which are the fields that predominantly use Rosenthal's fail-safe number, are immediate. Table 4 provides an accessible reference for researchers to consult and apply this more conservative rule for Rosenthal's number. Secondly, the formulas for the variance estimator are all available to researchers so they can compute normal approximation confidence intervals on their own. The future step that needs to be attempted is to develop an R-package program or a Stata program to execute this quickly and efficiently and make it available to the public domain. This will allow widespread use of these techniques.

In conclusion, the present study is the first in the literature to study the statistical properties of Rosenthal's fail-safe number. Statistical theory and simulations were presented and tables for applied researchers were also provided. Despite the limitations of Rosenthal's fail-safe number, it can be a trustworthy way to assess publication bias, especially under the more conservative nature of the present paper.

Appendix

Proofs for expressions (20, 21, 23, 24)

a) Fixed k

$Z_1, Z_2, \ldots, Z_i, \ldots, Z_k$ in the formula of the estimator $\hat{N}_R$ (11) are i.i.d. distributed with $E[Z_i] = \mu$ and $Var[Z_i] = \sigma^2$. Let $S = \sum_{i=1}^{k} Z_i$; then, according to the Lindeberg-Lévy Central Limit Theorem [40], we have

$$\sqrt{k}\left(\frac{S}{k} - \mu\right) \xrightarrow{d} N\left(0, \sigma^2\right) \Rightarrow \qquad S \xrightarrow{d} N\left(k\mu, k\sigma^2\right)$$

So we have

$$E[S] = k\mu$$

$$Var[S] = k\sigma^2$$

$$E\left[S^2\right] = \left(E[S]\right)^2 + Var[S] = k^2\mu^2 + k\sigma^2$$

Then, from (5) we get:

$$E\left[\hat{N}_R\right] = \frac{E\left[S^2\right]}{Z_\alpha^2} - k = \frac{k^2\mu^2 + k\sigma^2}{Z_\alpha^2} - k$$

$$Var\left[\hat{N}_R\right] = \frac{Var\left[S^2\right]}{Z_\alpha^4} = \frac{E\left[S^4\right] - \left(E\left[S^2\right]\right)^2}{Z_\alpha^4}$$

Now we seek to compute $E\left[S^4\right], E\left[S^2\right]$. For this we need the moments of the normal distribution, which are given below [59]:

Order	Non-central moment	Central moment
1	μ	0
2	$\mu^2 + \sigma^2$	σ^2
3	$\mu^3 + 3\mu\sigma^2$	0
4	$\mu^4 + 6\mu^2\sigma^2 + 3\sigma^4$	$3\sigma^4$
5	$\mu^5 + 10\mu^3\sigma^2 + 15\mu\sigma^4$	0

So:

$$Var\left[\hat{N}_R\right]=\frac{k^4\mu^4+6k^3\mu^2\sigma^2+3k^2\sigma^4-\left(k^2\mu^2+k\sigma^2\right)^2}{Z_\alpha^4}=\frac{4k^3\mu^2\sigma^2+2k^2\sigma^4}{Z_\alpha^4}\Rightarrow$$

$$Var\left[\hat{N}_R\right]=\frac{2k^2\sigma^2\left(2k\mu^2+\sigma^2\right)}{Z_\alpha^4}$$

b) Random k

In this approach we additionally assume that $k \sim Pois(\lambda)$. So then S is a Compound Poisson distributed variable [60]. Hence, from the law of total expectation and the law of total variance [42], we get:

$$E[S]=E[k]E[Z_i]=\lambda\mu$$

$$Var[S]=E[k]E\left[Z_i^2\right]=\lambda\left(\mu^2+\sigma^2\right)$$

Thus, from (5) we get:

$$E\left[\hat{N}_R\right]=\frac{E\left[S^2\right]}{Z_\alpha^2}-E[k]=\frac{(E[S])^2+Var[S]}{Z_\alpha^2}-\lambda\Rightarrow$$

$$E\left[\hat{N}_R\right]=\frac{\lambda^2\mu^2+\lambda\left(\mu^2+\sigma^2\right)}{Z_\alpha^2}-\lambda$$

and

$$Var\left[\hat{N}_R\right]=\frac{Var\left[S^2\right]}{Z_\alpha^4}+Var[k]-2\cdot\frac{Cov\left[S^2,k\right]}{Z_\alpha^2}=$$

$$=\frac{E\left[S^4\right]-\left(E\left[S^2\right]\right)^2}{Z_\alpha^4}+\lambda-2\cdot\frac{E\left[kS^2\right]-E[k]E\left[S^2\right]}{Z_\alpha^2}$$

To compute the final variance, it is more convenient to compute each component separately. We will need the moments of a Poisson distribution [60], which are given below:

Order	**Non-central moment**	**Central moment**
1	λ	λ
2	$\lambda+\lambda^2$	λ
3	$\lambda+3\lambda^2+\lambda^3$	λ
4	$\lambda+7\lambda^2+6\lambda^3+\lambda^4$	$\lambda+3\lambda^2$
5	$\lambda+15\lambda^2+25\lambda^3+10\lambda^4+\lambda^5$	$\lambda+10\lambda^2$

We then have:

$$E\left[S^4\right] = E\left[E\left[S^4 | k\right]\right] = E\left[k^4\mu^4 + 6k^3\mu^2\sigma^2 + 3k^2\sigma^4\right] =$$
$$= \mu^4 E\left[k^4\right] + 6\mu^2\sigma^2 E\left[k^3\right] + 3\sigma^4 E\left[k^2\right] \Rightarrow$$
$$E\left[S^4\right] = \left(\lambda^4 + 6\lambda^3 + 7\lambda^2 + \lambda\right)\mu^4 + 6\left(\lambda^3 + 3\lambda^2 + \lambda\right)\mu^2\sigma^2 + 3\left(\lambda^2 + \lambda\right)\sigma^4$$

$$E\left[S^2\right] = \left(E[S]\right)^2 + Var[S] = \lambda^2\mu^2 + \lambda\left(\mu^2 + \sigma^2\right) = \left(\lambda^2 + \lambda\right)\mu^2 + \lambda\sigma^2$$

$$\left(E\left[S^2\right]\right)^2 = \left(\lambda^4 + 2\lambda^3 + \lambda^2\right)\mu^4 + 2\left(\lambda^3 + \lambda^2\right)\mu^2\sigma^2 + \lambda^2\sigma^4$$

So:

$$E\left[S^4\right] - \left(E\left[S^2\right]\right)^2 = \left(4\lambda^3 + 6\lambda^2 + \lambda\right)\mu^4 + \left(4\lambda^3 + 16\lambda^2 + 6\lambda\right)\mu^2\sigma^2 + \left(2\lambda^2 + 3\lambda\right)\sigma^4$$

Also:

$$E\left[kS^2\right] = E\left[E\left[kS^2 | k\right]\right] = E\left[kE\left[S^2 | k\right]\right] = E\left[k\left(k^2\mu^2 + k\sigma^2\right)\right] =$$
$$= \mu^2 E\left[k^3\right] + \sigma^2 E\left[k^2\right] = \left(\lambda^3 + 3\lambda^2 + \lambda\right)\mu^2 + \left(\lambda^2 + \lambda\right)\sigma^2$$

$$E\left[kS^2\right] - E[k]E\left[S^2\right] = \left(2\lambda^2 + \lambda\right)\mu^2 + \lambda\sigma^2$$

Hence we finally have:

$$Var\left[\hat{N}_R\right] = \frac{\left(4\lambda^3 + 6\lambda^2 + \lambda\right)\mu^4 + \left(4\lambda^3 + 16\lambda^2 + 6\lambda\right)\mu^2\sigma^2 + \left(2\lambda^2 + 3\lambda\right)\sigma^4}{Z_\alpha^4} - 2 \cdot \frac{\left(2\lambda^2 + \lambda\right)\mu^2 + \lambda\sigma^2}{Z_\alpha^2} + \lambda$$

Proof of expression (16): the characteristic function

From (15) we have that

$$\psi_{N_R}(t) = E[\exp(itN_R)] = \int_0^{+\infty} \exp(itn_R) f_{N_R}(n_R) dn_R =$$

$$= \int_0^{+\infty} \frac{Z_\alpha}{2\Phi(\lambda^*)\sqrt{2\pi k\sigma^2 (n_R + k)}} \exp\left[itn_R - \frac{\left(Z_\alpha \sqrt{n_R + k} - k\mu\right)^2}{2k\sigma^2} \right] dn_R = \qquad \left(let\ \ w = Z_\alpha \sqrt{n_R + k} \right)$$

$$= \frac{1}{\Phi(\lambda^*)} \int_{Z_\alpha \sqrt{k}}^{+\infty} \frac{1}{\sqrt{2\pi k\sigma^2}} \exp\left[it\left(\frac{w^2}{Z_\alpha^2} - k \right) - \frac{(w - k\mu)^2}{2\sigma^2} \right] dw = \qquad \left(let\ \ y = \frac{w\text{-}k\mu}{\sqrt{n}\sigma} \right)$$

$$= \frac{1}{\Phi(\lambda^*)} \int_{-\lambda^*}^{+\infty} \frac{1}{\sqrt{2\pi}} \exp\left[it\left(\frac{k\sigma^2 y^2 + 2n\sqrt{k}\mu\sigma y + k^2\mu^2}{Z_\alpha^2} - k \right) - \frac{y^2}{2} \right] dy =$$

$$= \frac{\exp(-kit)}{\Phi(\lambda^*)} \int_{-\lambda^*}^{+\infty} \frac{1}{\sqrt{2\pi}} \exp\left[\frac{2k\sigma^2 it - Z_\alpha^2}{2Z_\alpha^2} y^2 + \frac{2k\sqrt{k}\mu\sigma it}{Z_\alpha^2} y + \frac{k^2\mu^2 it}{Z_\alpha^2} \right] dy = \qquad \left(\mu_1 = \frac{2k\sqrt{k}\mu\sigma it}{Z_\alpha^2 - 2k\sigma^2 it}, \sigma_1^2 = \frac{Z_\alpha^2}{Z_\alpha^2 - 2k\sigma^2 it} \right)$$

$$= \frac{\exp(-kit)}{\Phi(\lambda^*)} \int_{-\lambda^*}^{+\infty} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{(y\text{-}\mu_1)^2}{2\sigma_1^2} + \frac{k^2\mu^2 it}{Z_\alpha^2\text{-}2k\sigma^2 it} \right] dy = \qquad \left(let\ \ x = \frac{y\text{-}\mu_1}{\sigma_1} \right)$$

$$= \frac{\sigma_1 \exp\left(\frac{k^2\mu^2 it}{Z_\alpha^2\text{-}2k\sigma^2 it} - kit \right)}{\Phi(\lambda^*)} \int_{\frac{-\mu_1 - \lambda^*}{\sigma_1}}^{+\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2} \right) dx =$$

$$= \frac{Z_\alpha \exp\left(\frac{k^2\mu^2 it}{Z_\alpha^2\text{-}2k\sigma^2 it} - kit \right)}{\Phi(\lambda^*)\left(Z_\alpha^2\text{-}2n\sigma^2 it\right)^{1/2}} \left[\Phi(+\infty) - \Phi\left(\frac{-\mu_1 - \lambda^*}{\sigma_1} \right) \right] \Rightarrow$$

$$\psi_{N_R}(t) = \frac{\Phi\left(\frac{\mu_1 + \lambda^*}{\sigma_1} \right)}{\Phi(\lambda^*)} \cdot \frac{Z_\alpha \exp\left(\frac{k^2\mu^2 it}{Z_\alpha^2\text{-}2\sigma^2 it} - kit \right)}{\left(Z_\alpha^2\text{-}2k\sigma^2 it\right)^{1/2}}$$

Because $\Phi(+\infty) - \Phi\left(\frac{-\mu_1 - \lambda^*}{\sigma_1} \right) = 1 - \Phi\left(-\frac{\mu_1 + \lambda^*}{\sigma_1} \right) = \Phi\left(\frac{\mu_1 + \lambda^*}{\sigma_1} \right)$

Proof of expressions (17) and (18)

The cumulant generating function is

$$g(t)=\ln\left[\psi_{N_R}(-it)\right]=\frac{k^2\mu^2 t}{Z_\alpha^2\text{-}2k\sigma^2 t}-kt-\frac{1}{2}\ln\left(Z_\alpha^2\text{-}2k\sigma^2 t\right)+\ln\left[\Phi\left(\frac{\mu_1+\lambda^*}{\sigma_1}\right)\right]+\ln\frac{Z_\alpha}{\Phi\left(\lambda^*\right)},\ t<\frac{Z_\alpha^2}{2k\sigma^2}$$

For simplicity let $\Delta=\frac{\mu_1+\lambda^*}{\sigma_1}$. Then

$$g'(t)=\frac{k^2\mu^2 Z_\alpha^2}{\left(Z_\alpha^2\text{-}2k\sigma^2 t\right)^2}-k+\frac{k\sigma^2}{Z_\alpha^2\text{-}2k\sigma^2 t}+\frac{\phi(\Delta)}{\Phi(\Delta)}\Delta'$$

with $\Delta'=\frac{2k\sqrt{k}\mu\sigma Z_\alpha}{\left(Z_\alpha^2\text{-}2k\sigma^2 t\right)^{3/2}}-\frac{k\sigma^2\lambda^*}{Z_\alpha\left(Z_\alpha^2\text{-}2k\sigma^2 t\right)^{1/2}}$

Then, $g'(0)$ leads to (17).

Next:

$$g''(t)=\frac{4k^3\mu^2\sigma^2 Z_\alpha^2}{\left(Z_\alpha^2\text{-}2k\sigma^2 t\right)^3}+\frac{2k^2\sigma^4}{\left(Z_\alpha^2\text{-}2k\sigma^2 t\right)^2}+\frac{\phi(\Delta)}{\Phi(\Delta)}\left[-\frac{\phi(\Delta)}{\Phi(\Delta)}\Delta'^2-\Delta\Delta'^2+\Delta''\right]$$

Then $g''(0)$ leads to (18).

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Tables

Table 1 Schematic table for simulation plan

	Variance formula for Normal Approximation confidence intervals		**Bootstrap confidence interval**	**Real Value of N_R**
	Distributional	**Moments**		
k =5, 15, 30, 50 studies Draw Z_i from $N(0,1)$, $HN(0,1)$, $SN(\delta$=-0.5, ξ=0, ω=1), $SN(\delta$=0.5, ξ=0,ω=1)	Fixed k (Standard Normal Distribution Values μ = 0, σ^2 = 1; Half normal Distribution $\mu=\sqrt{2/\pi}$, $\sigma^2=1-2/\pi$; Skew normal with negative skewness $\mu=-\sqrt{1/2\pi}$, $\sigma^2=1-1/2\pi$; Skew normal with positive skewness $\mu=\sqrt{1/2\pi}$, $\sigma^2=1-1/2\pi$) $Var\left[\hat{N}_R\right]=\frac{2k^2\sigma^2\left(2k\mu^2+\sigma^2\right)}{Z_\alpha^4}$	Fixed k ($\hat{\mu}=\frac{\sum_{i=1}^{k}\hat{Z}_i}{k}$, $\hat{\sigma}^2=\frac{\sum_{i=1}^{k}\hat{Z}_i^2}{k}-\left(\frac{\sum_{i=1}^{k}\hat{Z}_i}{k}\right)^2$) $Var\left[\hat{N}_R\right]=\frac{2k^2\sigma^2\left(2k\mu^2+\sigma^2\right)}{Z_\alpha^4}$	Using the N_R^* we compute the $N_{R\_bootstrap}$ and resepctgively the standard error needed to compute the confidence interval	Fixed k (Standard Normal Distribution Values μ = 0, σ^2 = 1; Half normal Distribution $\mu=\sqrt{2/\pi}$, $\sigma^2=1-2/\pi$; Skew normal with negative skewness $\mu=-\sqrt{1/2\pi}$, $\sigma^2=1-1/2\pi$; Skew normal with positive skewness $\mu=\sqrt{1/2\pi}$, $\sigma^2=1-1/2\pi$) $E\left[\hat{N}_R\right]=\frac{k^2\mu^2+k\sigma^2}{Z_\alpha^2}-k$
	Random k (Standard Normal Distribution Values μ = 0, σ^2 = 1; Half normal Distribution $\mu=\sqrt{2/\pi}$, $\sigma^2=1-2/\pi$; Skew normal with negative skewness $\mu=-\sqrt{1/2\pi}$, $\sigma^2=1-1/2\pi$; Skew normal with positive skewness $\mu=\sqrt{1/2\pi}$, $\sigma^2=1-1/2\pi$; λ = 5, 15, 30, 50) $Var\left[\hat{N}_R\right]=\frac{\left(4\lambda^3+6\lambda^2+\lambda\right)\mu^4+\left(4\lambda^3+16\lambda^2+6\lambda\right)\mu^2\sigma^2+\left(2\lambda^2+3\lambda\right)\sigma^4}{Z_\alpha^4}-$ $-2\cdot\frac{\left(2\lambda^2+\lambda\right)\mu^2+\lambda\sigma^2}{Z_\alpha^2}+\lambda$	Random k ($\hat{\mu}=\frac{\sum_{i=1}^{k}\hat{Z}_i}{k}$, $\hat{\sigma}^2=\frac{\sum_{i=1}^{k}\hat{Z}_i^2}{k}-\left(\frac{\sum_{i=1}^{k}\hat{Z}_i}{k}\right)^2$, λ = 5, 15, 30, 50) $Var\left[\hat{N}_R\right]=\frac{\left(4\lambda^3+6\lambda^2+\lambda\right)\mu^4+\left(4\lambda^3+16\lambda^2+6\lambda\right)\mu^2\sigma^2+\left(2\lambda^2+3\lambda\right)\sigma^4}{Z_\alpha^4}-$ $-2\cdot\frac{\left(2\lambda^2+\lambda\right)\mu^2+\lambda\sigma^2}{Z_\alpha^2}+\lambda$	Using the N_R^* we compute the $N_{R\_bootstrap}$ and resepctgively the standard error needed to compute the confidence interval	Random k (Standard Normal Distribution Values μ = 0, σ^2 = 1; Half normal Distribution $\mu=\sqrt{2/\pi}$, $\sigma^2=1-2/\pi$; Skew normal with negative skewness $\mu=-\sqrt{1/2\pi}$, $\sigma^2=1-1/2\pi$; Skew normal with positive skewness $\mu=\sqrt{1/2\pi}$, $\sigma^2=1-1/2\pi$; λ = 5, 15, 30, 50) $E\left[\hat{N}_R\right]=\frac{\lambda^2\mu^2+\lambda\left(\mu^2+\sigma^2\right)}{Z_\alpha^2}-\lambda$

Table 2. Probability coverage of the different methods for confidence intervals (CI) according to the number of studies k. The figure is organised as follows: the Zi are drawn from four different distributions (Standard Normal Distribution, Half normal Distribution, Skew normal with negative skewness, and Skew normal with positive skewness).

			values of μ and σ^2 from the Standard Normal Distribution				**values of μ and σ^2 from the Half Normal Distribution HN(0,1)**				**values of μ and σ^2 from the Skew normal Distribution with negative skewness SN(δ = -0.5, ξ = 0, ω = 1)**				**values of μ and σ^2 from the Skew normal Distribution with positive skewness SN(δ = 0.5, ξ = 0, ω = 1)**			
draw Z_i from			*k=5*	*k=15*	*k=30*	*k=50*	*k=5*	*k=15*	*k=30*	*k=50*	*k=5*	*k=15*	*k=30*	*k=50*	*k=5*	*k=15*	*k=30*	*k=50*
Standard Normal Distribution	*Fixed k*	*Distribution Based CI*	0.948	0.950	0.948	0.952	0.994	0.110	0.002	0.000	0.985	0.999	1.000	1.000	0.982	0.998	1.000	1.000
		Moments Based CI	0.933	0.996	1.000	1.000	0.529	0.088	0.005	0.000	0.842	0.686	0.337	0.120	0.842	0.686	0.337	0.120
		Bootstrap CI	0.929	0.996	1.000	1.000	0.514	0.089	0.005	0.000	0.830	0.680	0.337	0.120	0.830	0.680	0.337	0.120
	Random k	*Distribution Based CI*	0.966	0.956	0.951	0.955	0.999	1.000	0.084	0.001	0.998	1.000	1.000	1.000	0.990	0.999	1.000	1.000
		Moments Based CI	1.000	1.000	1.000	1.000	0.535	0.094	0.006	0.000	1.000	0.702	0.338	0.122	1.000	0.702	0.338	0.122
		Bootstrap CI	0.929	0.996	1.000	1.000	0.429	0.074	0.004	0.000	0.804	0.649	0.322	0.115	0.804	0.649	0.322	0.115
Half Normal Distribution HN(0,1)	*Fixed k*	*Distribution Based CI*	0.635	0.021	0.000	0.000	0.945	0.952	0.951	0.948	0.864	0.624	0.279	0.053	0.841	0.483	0.142	0.014
		Moments Based CI	0.861	0.187	0.000	0.000	0.771	0.880	0.911	0.927	0.885	0.657	0.126	0.003	0.885	0.657	0.126	0.003
		Bootstrap CI	0.858	0.217	0.000	0.000	0.775	0.884	0.913	0.929	0.887	0.672	0.138	0.003	0.887	0.672	0.138	0.003
	Random k	*Distribution Based CI*	0.720	0.027	0.000	0.000	0.989	0.995	0.996	0.997	0.966	0.915	0.762	0.459	0.901	0.578	0.198	0.027
		Moments Based CI	1.000	1.000	0.130	0.000	0.806	0.937	0.971	0.984	1.000	1.000	0.995	0.358	1.000	1.000	0.995	0.358
		Bootstrap CI	0.858	0.217	0.000	0.000	0.715	0.859	0.899	0.920	0.885	0.698	0.152	0.004	0.885	0.698	0.152	0.004
Skew normal Distribution with negative skewness SN(δ = -0.5, ξ = 0, ω = 1)	*Fixed k*	*Distribution Based CI*	0.872	0.666	0.399	0.174	0.980	0.472	0.184	0.048	0.953	0.970	0.977	0.981	0.944	0.948	0.949	0.957
		Moments Based CI	0.917	0.979	0.944	0.858	0.597	0.375	0.200	0.074	0.845	0.860	0.882	0.895	0.845	0.860	0.882	0.895
		Bootstrap CI	0.912	0.978	0.945	0.857	0.586	0.377	0.199	0.074	0.840	0.857	0.881	0.894	0.840	0.857	0.881	0.894
	Random k	*Distribution Based CI*	0.903	0.688	0.409	0.178	0.996	1.000	0.687	0.306	0.987	0.997	0.998	0.999	0.965	0.964	0.965	0.968
		Moments Based CI	1.000	1.000	0.999	0.968	0.609	0.399	0.237	0.103	1.000	0.872	0.886	0.902	1.000	0.872	0.886	0.902
		Bootstrap CI	0.912	0.978	0.945	0.857	0.514	0.342	0.181	0.066	0.818	0.845	0.874	0.889	0.818	0.845	0.874	0.889
Skew normal Distribution with positive skewness SN(δ = 0.5, ξ = 0, ω = 1)	*Fixed k*	*Distribution Based CI*	0.880	0.673	0.402	0.164	0.982	0.471	0.186	0.050	0.956	0.972	0.976	0.979	0.948	0.952	0.951	0.955
		Moments Based CI	0.923	0.980	0.947	0.852	0.596	0.372	0.201	0.076	0.850	0.865	0.874	0.896	0.850	0.865	0.874	0.896
		Bootstrap CI	0.918	0.978	0.946	0.846	0.583	0.372	0.200	0.077	0.841	0.862	0.873	0.896	0.841	0.862	0.873	0.896
	Random k	*Distribution Based CI*	0.911	0.696	0.415	0.169	0.996	1.000	0.683	0.314	0.989	0.996	0.998	0.999	0.967	0.967	0.964	0.966
		Moments Based CI	1.000	1.000	0.999	0.964	0.606	0.399	0.236	0.105	1.000	0.875	0.880	0.905	1.000	0.875	0.880	0.905
		Bootstrap CI	0.918	0.978	0.946	0.846	0.514	0.335	0.180	0.068	0.819	0.850	0.868	0.893	0.819	0.850	0.868	0.893

Table 3 Confidence intervals for example meta-analyses.

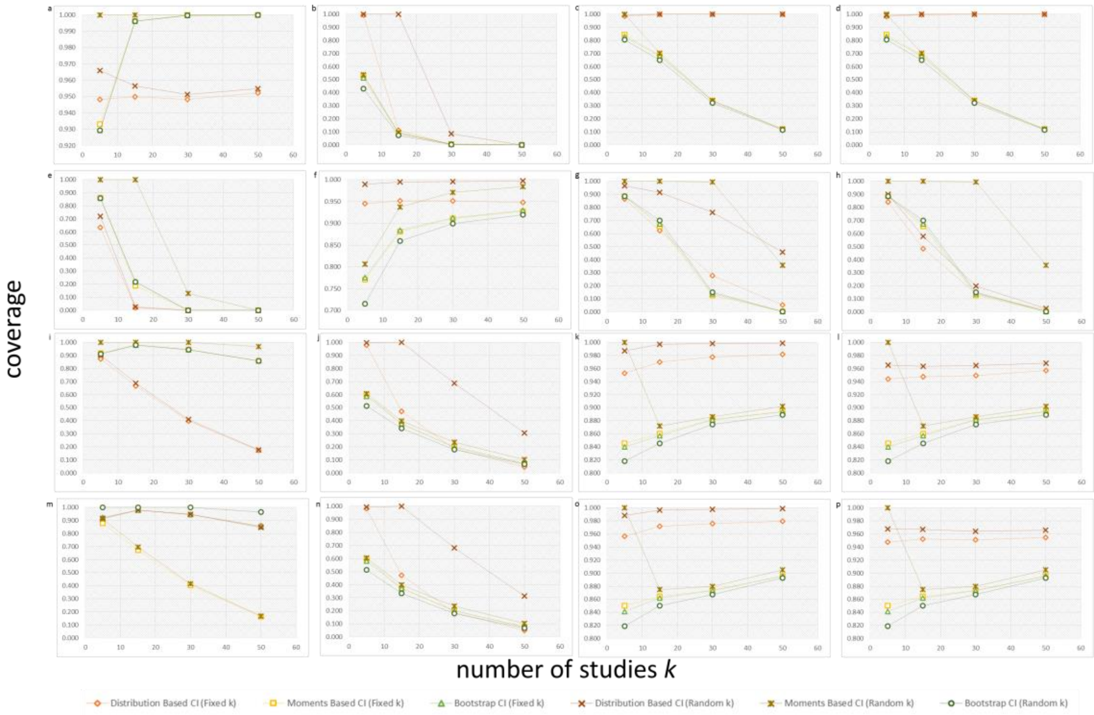
	Fixed number of studies		Random number of studies		Bootstrap based CI
	Distribution based CI	Moment based CI	Distribution based CI	Moment based CI	
Study 1 [56] Rosenthal's N_R = 2124	(2060, 2188)	(788, 3460)	(2059, 2189)	(369, 3879)	(740, 3508)
Study 2 [57] Rosenthal's N_R = 73860	(73709, 74012)	(51618, 96102)	(73707, 74013)	(40976, 106745)	(51662, 96059)

Table 4. 95% one-sided confidence limits above which the estimated N_R is significantly higher than $5k+10$, which is the rule of thumb suggested by Rosenthal [11]. k represents the number of studies included in a meta-analysis. We choose the variance from a fixed number of studies when the Z_i are drawn from a half normal distribution $HN(0,1)$, as these performed the best the simulations

k	Cut offpoint	*k*	Cut off point	*k*	Cut off point	*k*	Cut off point
1	17	**41**	369	**81**	842	**121**	1394
2	26	**42**	380	**82**	855	**122**	1409
3	35	**43**	390	**83**	868	**123**	1424
4	45	**44**	401	**84**	881	**124**	1438
5	54	**45**	412	**85**	894	**125**	1453
6	63	**46**	423	**86**	907	**126**	1468
7	71	**47**	434	**87**	920	**127**	1483
8	79	**48**	445	**88**	934	**128**	1498
9	86	**49**	456	**89**	947	**129**	1513
10	93	**50**	467	**90**	960	**130**	1528
11	99	**51**	479	**91**	973	**131**	1543
12	106	**52**	490	**92**	987	**132**	1558
13	112	**53**	501	**93**	1000	**133**	1573
14	118	**54**	513	**94**	1014	**134**	1588
15	125	**55**	524	**95**	1027	**135**	1603
16	132	**56**	536	**96**	1041	**136**	1619
17	140	**57**	547	**97**	1055	**137**	1634
18	147	**58**	559	**98**	1068	**138**	1649
19	155	**59**	571	**99**	1082	**139**	1664
20	164	**60**	582	**100**	1096	**140**	1680
21	172	**61**	594	**101**	1109	**141**	1695
22	181	**62**	606	**102**	1123	**142**	1711
23	190	**63**	618	**103**	1137	**143**	1726
24	199	**64**	630	**104**	1151	**144**	1742
25	209	**65**	642	**105**	1165	**145**	1757
26	218	**66**	654	**106**	1179	**146**	1773
27	228	**67**	666	**107**	1193	**147**	1788
28	237	**68**	679	**108**	1207	**148**	1804
29	247	**69**	691	**109**	1221	**149**	1820
30	257	**70**	703	**110**	1236	**150**	1835
31	266	**71**	716	**111**	1250	**151**	1851
32	276	**72**	728	**112**	1264	**152**	1867
33	286	**73**	740	**113**	1278	**153**	1883
34	296	**74**	753	**114**	1293	**154**	1899
35	307	**75**	766	**115**	1307	**155**	1915
36	317	**76**	778	**116**	1322	**156**	1931
37	327	**77**	791	**117**	1336	**157**	1947
38	338	**78**	804	**118**	1351	**158**	1963
39	348	**79**	816	**119**	1365	**159**	1979
40	358	**80**	829	**120**	1380	**160**	1995

Figures

Figure 1. This figures shows the probability coverage of the different methods for confidence intervals (CI) according to the number of studies *k*. The figure is organised as follows: the Z_i are drawn from four different distributions (Standard Normal Distribution, Half normal Distribution, Skew normal with negative skewness, and Skew normal with positive skewness) which are depicted in each row respectively (a-d, e-h, i-l, m-p). Each column shows the different values of μ and σ^2 for the variance according to the Standard Normal Distribution (a, e, i, m), Half normal Distribution (b, f, j, n), Skew normal with negative skewness (c, g, k, o), and Skew normal with positive skewness (d, h, l, p).



Appendix

Codes for Simulations in R

```
### Examining the Standard Normal distribution mu and sigma-square values
and drawing Zi from all four distributions###
#######################


R  =  10000 ; B  =  1000
## R is the number of simulations, B is the number of bootstrap resamples

rosent1 = matrix(nrow = R,ncol = 4)
## here we will store the Rosenthal's values for the 4 sample sizes

v1 = array(dim = c(R,5,4))
## the variances will be stored here

mat1 = mat2 = mat3 = mat4 = mat5 = array(dim = c(R,2,4))
## the confidence intervals will be stored here

za = qnorm(0.95) ; n = lam = c(5, 15, 30, 50) ; f = (n-1)/n
## f is used to get the unbiased variance estimator

m = 0 ; s = 1
## parameters of the normal distribution

fixed1 = ( n^2 * m^2 + n * s )/za^2-n
## real values of Rosenthal's as fixed

random1 = ( lam^2 * m^2 + lam * (m^2 + s) )/za^2-lam
## real values of Rosenthal's as random

coverage1 = matrix(nrow = 4,ncol = 6)
## the coverages will be stored here

set.seed(123456)

## seed number
for (k in 1:4) {
for (i in 1:R) {

z = rnorm(n[k])
## random values of z-statistics are generated
## When we draw from the half normal we use z = abs(rnorm(n[k],0,1)); when
we draw from the skew normal distribution we use z = rsn(n[k],xi = 0,omega =
1,alpha = -0.5773503) or z = rsn(n[k],xi = 0,omega = 1,alpha = 0.5773503)
for negative and positive skewness respectively ###
###############

rosent1[i, k] = ( sum(z)/za )^2 - n[k]
## Rosenthal's value

m1 = 0 ; s1 = 1
v1[i, 1, k] = 2 * n[k]^2 * s1 * ( 2*n[k] * m1^2 + s1 )/za^4
## distributional variance of the fixed studies

m2 = mean(z) ; s2 = f[k] * var(z)
v1[i, 2, k] = 2 * n[k]^2 * s2 * ( 2 * n[k] * m2^2 + s2 )/za^4
## moments variance of the fixed studies

v1[i, 3, k] = ( (4 * lam[k]^3 + 6 * lam[k]^2 + lam[k]) * m1^4 + (4 *
lam[k]^3 + 16 * lam[k]^2 + 6 * lam[k]) * m1^2 * s1 +
( 2 * lam[k]^2 + 3 * lam[k] ) * s1^2 )/za^4 – 2 *( (2 * lam[k]^2 + lam[k]) *
m1^2 + lam[k] * s1 )/za^2 + lam[k]
```

```
## distributional variance of the random studies

v1[i, 4, k] = ( (4 * lam[k]^3 + 6 * lam[k]^2 + lam[k]) * m2^4 + (4 *
lam[k]^3 + 16 * lam[k]^2 + 6 * lam[k]) * m2^2 * s2 +
(2 * lam[k]^2 + 3 * lam[k]) * s2^2 )/za^4 – 2 * ( (2 * lam[k]^2 + lam[k]) *
m2^2 + lam[k] * s2 )/za^2 + lam[k]
## moments variance of the random studies

## then is the bootstrap case
t = rep(0, B)
for (j in 1:B) {
nu = sample(1:n[k], n[k], replace = T)
t[j] = (sum(z[nu])/za)^2-n[k] }
v1[i, 5, k] = var(t)
mat1[i, , k] = c(rosent1[i, k] - 1.96 * sqrt(v1[i, 1, k]), rosent1[i, k] +
1.96 * sqrt(v1[i, 1, k]))
mat2[i, , k] = c(rosent1[i, k] - 1.96 * sqrt(v1[i, 2, k]), rosent1[i, k] +
1.96 * sqrt(v1[i, 2, k]))
mat3[i, , k] = c(rosent1[i, k] - 1.96 * sqrt(v1[i, 3, k]), rosent1[i, k] +
1.96 * sqrt(v1[i, 3, k]))
mat4[i, , k] = c(rosent1[i, k] - 1.96 * sqrt(v1[i, 4, k]), rosent1[i, k] +
1.96 * sqrt(v1[i, 4, k]))
mat5[i, , k] = c(rosent1[i, k] - 1.96 * sqrt(v1[i, 5, k]), rosent1[i, k] +
1.96 * sqrt(v1[i, 5, k])) } }
for (l in 1:4) coverage1[l, 1] = 1 - (sum(mat1[, 1, l]>fixed1[l])/R +
sum(mat1[, 2, l]<fixed1[l])/R)
for (l in 1:4) coverage1[l, 2] = 1 -(sum(mat2[, 1, l]>fixed1[l])/R +
sum(mat2[,2,l]<fixed1[l])/R)
for (l in 1:4) coverage1[l, 3] = 1 - (sum(mat3[, 1, l]>random1[l])/R +
sum(mat3[, 2, l]<random1[l])/R)
for (l in 1:4) coverage1[l, 4] = 1 - (sum(mat4[, 1, l]>random1[l])/R +
sum(mat4[, 2, l]<random1[l])/R)
for (l in 1:4) coverage1[l, 5] = 1 - (sum(mat5[, 1, l]>fixed1[l])/R +
sum(mat5[, 2, l]<fixed1[l])/R)
for (l in 1:4) coverage1[l, 6] = 1 - (sum(mat5[, 1, l]>random1[l])/R +
sum(mat5[, 2, l]<random1[l])/R)
colnames(coverage1) = c('Dist_fixed', 'Mom_fixed', 'Dist_random',
'Mom_random', 'Boot_fixed', 'Boot_random')
rownames(coverage1) = c('n = 5', 'n = 15', 'n = 30', 'n = 50')
```

```
### Examining the Half Normal distribution mu and sigma-square values and
drawing Zi from all four distributions###
#####################

R = 10000 ; B = 1000
## R is the number of simulations, B is the number of bootstrap resamples

rosent2 = matrix(nrow = R,ncol = 4)
## here we will store the Rosenthal's values for the 4 sample sizes

v2 = array(dim = c(R, 5, 4))
## the variances will be stored here

mat6 = mat7 = mat8 = mat9 = mat10 = array( dim = c(R, 2, 4) )
## the confidence intervals will be stored here
za = qnorm(0.95) ; n = lam = c(5, 15, 30, 50) ; f = (n - 1)/n
## f is used to get the unbiased variance estimator

m = sqrt(2/pi) ; s = 1 - 2/pi
## parameters of the half normal distribution

fixed2 = (n^2 * m^2 + n * s)/za^2 - n
## real values of Rosenthal's as fixed

random2 = (lam^2 * m^2 + lam * (m^2 + s))/za^2 - lam
## real values of Rosenthal's as random

coverage2 = matrix(nrow = 4,ncol = 6)
## the coverages will be stored here

set.seed(123456)
## seed number

for (k in 1:4) {
for (i in 1:R) {

z = rnorm(n[k])
## random values of z-statistics are generated
## When we draw from the half normal we use z = abs(rnorm(n[k],0,1)); when
we draw from the skew normal distribution we use z = rsn(n[k],xi = 0,omega =
1,alpha = -0.5773503) or z = rsn(n[k],xi = 0,omega = 1,alpha = 0.5773503)
for negative and positive skewness respectively ###
###############

rosent2[i, k] = (sum(z)/za)^2 - n[k]
## Rosenthal's value

m1 = sqrt(2/pi) ; s1 = 1 - 2/pi
v2[i, 1, k] = 2 * n[k]^2 * s1 *( 2 * n[k] * m1^2 + s1 )/za^4
## distributional variance of the fixed studies

m2 = mean(z) ; s2 = f[k] * var(z)
v2[i, 2, k] = 2 * n[k]^2 * s2 * (2 * n[k] * m2^2 + s2)/za^4
## moments variance of the fixed studies

v2[i, 3, k] = ( (4 * lam[k]^3 + 6 * lam[k]^2 + lam[k]) * m1^4 + (4 *
lam[k]^3 + 16 * lam[k]^2 + 6 * lam[k]) * m1^2 * s1 +
(2 * lam[k]^2 + 3 * lam[k]) * s1^2 )/za^4 – 2 * ( (2 * lam[k]^2 + lam[k]) *
m1^2 + lam[k] * s1 )/za^2 + lam[k]
## distributional variance of the random studies

v2[i, 4, k] = ( (4 * lam[k]^3 + 6 * lam[k]^2 + lam[k]) * m2^4 + (4 *
lam[k]^3 + 16 * lam[k]^2 + 6 * lam[k]) * m2^2 * s2 +
(2 * lam[k]^2 + 3 * lam[k]) * s2^2 )/za^4 – 2 * ( (2 * lam[k]^2 + lam[k]) *
m2^2 + lam[k] * s2 )/za^2 + lam[k]
## moments variance of the random studies

## then is the bootstrap case
```

```
t = rep(0, B)
for (j in 1:B) {
nu = sample(1:n[k], n[k], replace = T)
t[j] = (sum(z[nu])/za)^2 - n[k] }
v2[i, 5, k] = var(t)
mat6[i, , k] = c(rosent2[i, k] - 1.96 * sqrt(v2[i, 1, k]), rosent2[i, k] +
1.96 * sqrt(v2[i,1,k]))
mat7[i, , k] = c(rosent2[i, k]- 1.96 * sqrt(v2[i, 2, k]), rosent2[i, k] +
1.96 * sqrt(v2[i, 2, k]))
mat8[i, , k] = c(rosent2[i, k] - 1.96 * sqrt(v2[i, 3, k]), rosent2[i, k] +
1.96 * sqrt(v2[i, 3, k]))
mat9[i, , k] = c(rosent2[i, k] - 1.96 * sqrt(v2[i, 4, k]), rosent2[i, k] +
1.96 * sqrt(v2[i, 4, k]))
mat10[i, , k] = c(rosent2[i, k] - 1.96 * sqrt(v2[i, 5, k]), rosent2[i, k] +
1.96 * sqrt(v2[i, 5, k])) } }
for (l in 1:4) coverage2[l, 1] = 1 - (sum(mat6[, 1, l]>fixed2[l])/R +
sum(mat6[, 2, l]<fixed2[l])/R)
for (l in 1:4) coverage2[l, 2] = 1 - (sum(mat7[, 1, l]>fixed2[l])/R +
sum(mat7[, 2, l]<fixed2[l])/R)
for (l in 1:4) coverage2[l, 3] = 1 - (sum(mat8[, 1, l]>random2[l])/R +
sum(mat8[, 2, l]<random2[l])/R)
for (l in 1:4) coverage2[l, 4] = 1 - (sum(mat9[, 1, l]>random2[l])/R +
sum(mat9[, 2, l]<random2[l])/R)
for (l in 1:4) coverage2[l, 5] = 1 - (sum(mat10[, 1, l]>fixed2[l])/R +
sum(mat10[, 2, l]<fixed2[l])/R)
for (l in 1:4) coverage2[l, 6] = 1 - (sum(mat10[, 1, l]>random2[l])/R +
sum(mat10[, 2, l]<random2[l])/R)
colnames(coverage2) = c('Dist_fixed', 'Mom_fixed', 'Dist_random',
'Mom_random', 'Boot_fixed', 'Boot_random')
rownames(coverage2) = c('n = 5', 'n = 15', 'n = 30', 'n = 50')
```

Examining the Skew Normal distribution (negative skewness) mu and sigma-square values and drawing Zi from all four distributions###
########################

```
R = 10000 ; B = 1000
## R is the number of simulations, B is the number of bootstrap resamples

rosent3 = matrix(nrow = R,ncol = 4)
## here we will store the Rosenthal's values for the 4 sample sizes

v3 = array(dim = c(R, 5, 4))
## the variances will be stored here

mat11 = mat12 = mat13 = mat14 = mat15 = array(dim = c(R, 2, 4))
## the confidence intervals will be stored here

za = qnorm(0.95) ; n = lam = c(5, 15, 30, 50) ; f = (n - 1)/n
## f is used to get the unbiased variance estimator
m = - sqrt( 1/(2 * pi) ) ; s = 1 - 1/(2 * pi)

fixed3 = (n^2 * m^2 + n * s)/za^2 - n
## real values of Rosenthal's as fixed

random3 = (lam^2 * m^2 + lam * (m^2 + s))/za^2 - lam
## real values of Rosenthal's as random

coverage3 = matrix(nrow = 4, ncol = 6)

set.seed(123456)
## seed number
for (k in 1:4) {
for (i in 1:R) {

z = rnorm(n[k])
## random values of z-statistics are generated
## When we draw from the half normal we use z = abs(rnorm(n[k],0,1)); when
we draw from the skew normal distribution we use z = rsn(n[k],xi = 0,omega =
1,alpha = -0.5773503) or z = rsn(n[k],xi = 0,omega = 1,alpha = 0.5773503)
for negative and positive skewness respectively ###
###############

rosent3[i, k] = (sum(z)/za)^2 - n[k]
## Rosenthal's value

m1 = - sqrt(2/pi) ; s1 = 1 - 2/pi

v3[i, 1, k] = 2 * n[k]^2 * s1 * ( 2 * n[k] * m1^2 + s1 )/za^4
## distributional variance of the fixed studies

m2 = mean(z) ; s2 = f[k] * var(z)
v3[i, 2, k] = 2 * n[k]^2 * s2 * ( 2 * n[k] * m2^2 + s2 )/za^4
## distributional variance of the random studies
v3[i, 3, k] = ( (4 * lam[k]^3 + 6 * lam[k]^2 + lam[k]) * m1^4 + (4 *
lam[k]^3 + 16 * lam[k]^2 + 6 * lam[k]) * m1^2 * s1 +
(2 * lam[k]^2 + 3 * lam[k]) * s1^2 )/za^4 – 2 *( (2 * lam[k]^2 + lam[k]) *
m1^2 + lam[k] * s1 )/za^2 + lam[k]
## moments variance of the fixed studies

v3[i, 4, k] = ( (4 * lam[k]^3 + 6 * lam[k]^2 + lam[k]) * m2^4 + (4 *
lam[k]^3 + 16 * lam[k]^2 + 6 * lam[k]) * m2^2 * s2 +
(2 * lam[k]^2 + 3 * lam[k]) * s2^2 )/za^4 – 2 * ( (2 * lam[k]^2 + lam[k]) *
m2^2 + lam[k] * s2 )/za^2 + lam[k]
## moments variance of the random studies

## then is the bootstrap case
t = rep(0, B)
for (j in 1:B) {
nu = sample(1:n[k], n[k], replace = T)
```

```
t[j] = (sum(z[nu])/za)^2 - n[k] }
v3[i, 5, k] = var(t)
mat11[i, , k] = c(rosent3[i, k] - 1.96 * sqrt(v3[i, 1, k]), rosent3[i, k] +
1.96 * sqrt(v3[i, 1, k]))
mat12[i, , k] = c(rosent3[i, k] - 1.96 * sqrt(v3[i, 2, k]), rosent3[i, k] +
1.96 * sqrt(v3[i, 2, k]))
mat13[i, , k] = c(rosent3[i, k] - 1.96 * sqrt(v3[i, 3, k]), rosent3[i, k] +
1.96 * sqrt(v3[i, 3, k]))
mat14[i, ,k] = c(rosent3[i, k] - 1.96 * sqrt(v3[i, 4, k]), rosent3[i, k] +
1.96 * sqrt(v3[i, 4, k]))
mat15[i, , k] = c(rosent3[i, k] - 1.96 * sqrt(v3[i, 5, k]), rosent3[i,k] +
1.96 * sqrt(v3[i, 5, k])) } }
for (l in 1:4) coverage3[l,1] = 1 -(sum(mat11[, 1,
l]>fixed3[l])/R+sum(mat11[, 2, l]<fixed3[l])/R)
for (l in 1:4) coverage3[l, 2] = 1 - (sum(mat12[, 1,
l]>fixed3[l])/R+sum(mat12[, 2, l]<fixed3[l])/R)
for (l in 1:4) coverage3[l, 3] = 1- (sum(mat13[, 1, l]>random3[l])/R +
sum(mat13[, 2, l]<random3[l])/R)
for (l in 1:4) coverage3[l, 4] = 1-(sum(mat14[, 1, l]>random3[l])/R +
sum(mat14[, 2, l]<random3[l])/R)
for (l in 1:4) coverage3[l, 5] = 1 - (sum(mat15[, 1, l]>fixed3[l])/R +
sum(mat15[, 2, l]<fixed3[l])/R)
for (l in 1:4) coverage3[l, 6] = 1 - (sum(mat15[, 1, l]>random3[l])/R +
sum(mat15[, 2, l]<random3[l])/R)
colnames(coverage3) = c('Dist_fixed', 'Mom_fixed', 'Dist_random',
'Mom_random', 'Boot_fixed', 'Boot_random')
rownames(coverage3) = c('n = 5', 'n = 15', 'n = 30', 'n = 50')
```

```
### Examining the Skew Normal distribution (positive skewness) mu and sigma-
square values and drawing Zi from all four distributions###
#######################

R = 10000 ; B = 1000
## R is the number of simulations, B is the number of bootstrap resamples

rosent4 = matrix(nrow = R, ncol = 4)
## here we will store the Rosenthal's values for the 4 sample sizes

v4 = array(dim = c(R, 5, 4))
## the variances will be stored here

mat16 = mat17 = mat18 = mat19 = mat20 = array(dim = c(R, 2, 4))
## the confidence intervals will be stored here

za = qnorm(0.95) ; n = lam = c(5, 15, 30, 50) ; f = (n - 1)/n
## f is used to get the unbiased variance estimator

m = sqrt( 1/(2 * pi) ) ; s = 1 - 1/( 2 * pi )

fixed4 = ( n^2 * m^2 + n * s )/za^2 - n
## real values of Rosenthal's as fixed

random4 = ( lam^2 * m^2 + lam * (m^2 + s) )/za^2 - lam
## real values of Rosenthal's as random

coverage4 = matrix(nrow = 4,ncol = 6)

set.seed(123456)
## seed number

for (k in 1:4) {
for (i in 1:R) {

z = rnorm(n[k])
## random values of z-statistics are generated
## When we draw from the half normal we use z = abs(rnorm(n[k],0,1)); when
we draw from the skew normal distribution we use z = rsn(n[k],xi = 0,omega =
1,alpha = -0.5773503) or z = rsn(n[k],xi = 0,omega = 1,alpha = 0.5773503)
for negative and positive skewness resepctively ###
###############

rosent4[i, k] = (sum(z)/za)^2-n[k]
## Rosenthal's value

m1 = sqrt(1/(2 * pi)) ; s1 = 1 - 1/(2 * pi)
v4[i, 1, k] = 2 * n[k]^2 * s1 * (2 * n[k] * m1^2 + s1)/za^4
## distributional variance of the fixed studies

m2 = mean(z) ; s2 = f[k] * var(z)

v4[i, 2, k] = 2 * n[k]^2 * s2 * (2 * n[k] * m2^2 + s2)/za^4
## distributional variance of the random studies
V
4[i,3,k] = ( (4 * lam[k]^3 + 6 * lam[k]^2 + lam[k]) * m1^4 + (4 * lam[k]^3 +
16 * lam[k]^2 + 6 * lam[k]) * m1^2 * s1 +
(2 * lam[k]^2 + 3 * lam[k]) * s1^2 )/za^4 - 2 * ( (2 * lam[k]^2 + lam[k]) *
m1^2 + lam[k] * s1 )/za^2 + lam[k]
## moments variance of the fixed studies

v4[i, 4, k] = ( (4 * lam[k]^3 + 6 * lam[k]^2 + lam[k]) * m2^4 + (4 *
lam[k]^3 + 16 * lam[k]^2 + 6 * lam[k]) * m2^2 * s2 +
(2 * lam[k]^2 + 3 * lam[k]) * s2^2 )/za^4 - 2 * ( (2 * lam[k]^2 + lam[k]) *
m2^2 + lam[k] * s2 )/za^2 + lam[k]
## moments variance of the random studies

## then is the bootstrap case
```

```
t = rep(0, B)
for (j in 1:B) {
nu = sample(1:n[k], n[k], replace = T)
t[j] = (sum(z[nu])/za)^2 - n[k] }
v4[i, 5, k] = var(t)
mat16[i, , k] = c(rosent4[i, k] - 1.96 * sqrt(v4[i, 1, k]), rosent4[i, k] +
1.96 * sqrt(v4[i, 1, k]))
mat17[i, , k] = c(rosent4[i, k] - 1.96 * sqrt(v4[i, 2, k]), rosent4[i, k] +
1.96 * sqrt(v4[i, 2, k]))
mat18[i, , k] = c(rosent4[i, k] - 1.96 * sqrt(v4[i, 3, k]), rosent4[i, k] +
1.96 * sqrt(v4[i, 3, k]))
mat19[i, , k] = c(rosent4[i, k] - 1.96 * sqrt(v4[i, 4, k]), rosent4[i, k] +
1.96 * sqrt(v4[i, 4, k]))
mat20[i, , k] = c(rosent4[i, k] - 1.96 * sqrt(v4[i, 5, k]), rosent4[i, k] +
1.96 * sqrt(v4[i, 5, k])) } }
for (l in 1:4) coverage4[l, 1] = 1-(sum(mat16[, 1, l]>fixed4[l])/R +
sum(mat16[, 2, l]<fixed4[l])/R)
for (l in 1:4) coverage4[l, 2] = 1-(sum(mat17[, 1, l]>fixed4[l])/R +
sum(mat17[, 2, l]<fixed4[l])/R)
for (l in 1:4) coverage4[l, 3] = 1-(sum(mat18[, 1, l]>random4[l])/R +
sum(mat18[, 2, l]<random4[l])/R)
for (l in 1:4) coverage4[l, 4] = 1-(sum(mat19[, 1, l]>random4[l])/R +
sum(mat19[, 2, l]<random4[l])/R)
for (l in 1:4) coverage4[l, 5] = 1-(sum(mat20[, 1, l]>fixed4[l])/R +
sum(mat20[, 2, l]<fixed4[l])/R)
for (l in 1:4) coverage4[l, 6] = 1 - (sum(mat20[, 1, l]>random4[l])/R +
sum(mat20[, 2, l]<random4[l])/R)
colnames(coverage4) = c('Dist_fixed', 'Mom_fixed', 'Dist_random',
'Mom_random',
'Boot_fixed', 'Boot_random')
rownames(coverage4) = c('n = 5', 'n = 15', 'n = 30', 'n = 50')
```

Meta-analyses Example Data

Meta-analysis study by [1]

Authors	Year	z values
Ligny	1976	2.962859
Adam	1977	3.757467
Gotz	1979	1.650607
Monteiro	1981	2.166813
Borgia	1982	1.581611
Frigerio	1986	4.059215
Surawicz	1989	2.27334
Wunderlich	1989	1.487448
Tankanow	1990	0.17785
Reid	1992	-0.0474
McFarland	1995	1.627997
Lewis	1998	-0.79902
Arvola	1999	1.891838
Benhamou	1999	-1.44082
Vanderhoof	1999	3.074713
Felley	2001	-0.70246
Thomas	2001	0.210804
Jirapinyo	2002	1.553301
Sheu	2002	2.199867
La Rosa	2003	3.024602
Sullivan	2003	-0.68994
Erdeve	2004	1.620978
Erdeve	2004	3.755796
Lighthouse	2004	-0.70038
Plummer	2004	0
Schrezenmeir	2004	-0.94973
Tursi	2004	1.864488
Corrêa	2005	2.198868
Duman	2005	2.608535
Kotowska	2005	3.186323
Myllyluoma	2005	-0.90258
Can	2006	1.812663
Beausoleil	2007	1.969595
Cindoruk	2007	2.071372
Conway	2007	1.778388
De Bortoli	2007	2.914381
Hickson	2007	2.597848
Park	2007	2.95422
Stein	2007	-0.98815

Authors	Year	z values
Bravo	2008	0.598887
Kim	2008	-0.56565
Koning	2008	2.171299
Ruszczynski	2008	-2.10516
Safdar	2008	1.256991
Szymanski	2008	0.595156
Wenus	2008	1.987948
Engelbrektson	2009	-0.98909
Merenstein	2009	0.553913
Szajewska	2009	1.505209
Gao	2010	3.849341
Koning	2010	-0.4195
Li	2010	2.816526
Lönnermark	2010	-0.35165
Sampalis	2010	1.776439
Song	2010	0.426735
Song	2010	1.63705
Yasar	2010	0.249171
Cimperman	2011	1.533658
de Vrese	2011	-0.35655
Saneeyan	2011	2.486708
Selinger	2011	1.630008
Yoon	2011	-1.19603

Meta-analysis study by [2]

Authors	Year	Z
Bell	1989	0.829231
Bochet	2006	3.330265
Bornstein	2010a	4.24916
Sample b		2.08
Bornstein	2010b	4.55493
Sample b		2.702136
Caldwell	1976	2.688
Camera	2009	1.855267
Carpenter	2007b	2.59759
Sample b		2.078072
Sample c		0.860488
Carpenter	2009	2.891
Carpenter	2004	3.72202
Casari	2009	6.465979
Chen (Study 2)	2009	1.803894
Cinyabuguma	2005	2.24
Dickinson	2001	5.145
Sample b		5.2675
Dreber	2008	4.131892
Sample b		0.824091
Eek	2002	7.115294
Egas	2008	0.696889
Sample b		4.355556
Sample c		1.829333
Sample d		-0.80182
Etran	2009	3.223111
Fehr	2000	2.989831
Sample b		6.40396
Fehr	2002	14.7
Fuster	2010	3.477419
Sample b		3.411852
Gachter	2009	3.574118
Sample b		3.015385
Sample c		-0.67586
Sample d		-1.08138
Gachter	2011	-0.98
Sample b		0.70359
Sample c		2.016
Sample d		-2.0825
Gachter	2008	3.479
Sample b		8.131915

Authors	Year	Z
Gachter	2005	5.535878
Sample a		1.104225
Sample b		2.229725
Herrmann	2008	7.404444
Study 2		7.410411
Study 3		7.197377
Study 4		8.563692
Study 5		7.798298
Study 6		8.949434
Study 7		4.227451
Study 8		0.522667
Study 9		4.632727
Study 10		-1.04533
Study 11		2.5872
Study 12		-1.03158
Study 13		-0.07127
Study 14		9.740606
Study 15		9.097358
Study 16		7.423717
Hopfensitz	2009	3.441951
Kieruj	2008	7.466667
Kocher	2008	2.330811
Komorita	1985	-0.70966
Kroll	2007	4.422564
Martichuski	1991	2.230345
McCusker	1995	4.174545
Study 2		2.103415
Mulder	2005	3.5525
Mulder	2008	2.024615
Mulder	2001	2.232911
Sample b		2.184427
Mulder	2002	2.698182
Mulder	2003	1.905556
Mulder	2006a	4.581818
Study 2		3.705205
Sample b		1.737273
Study 3		2.597
Mulder	2005	0.276056
Mulder	2006	2.94
Myers	2009	4.434747
Nelissen	2010	7.466667

Authors	Year	Z
Nikiforakis	2008	5.530959
Sample b		7.888395
Sample c		1.176
Sample d		6.055724
Nikiforakis	2010	3.364409
Sample b		3.82439
Sample c		5.115461
Sample d		5.371852
O'Gorman	2008	2.24918
Sample b		-0.392
O'Gorman	2010	4.611765
Sample b		2.441967
Page	2005	6.653684
Patel	2010	0.658824
Sample b		0.897349
Sample c		0.722105
Sample d		-1.68304
Rand	2009	3.486851
Rapoport	2001	1.448153
Reuben	2009	7.459578
Sample b		2.672727
Sample c		5.90481
Sample d		4.505455
Riedl	2009	1.26359
Sample b		1.180372
Sample c		1.801081
Sample d		0.969309
Sato	1987	4.856855
Sefton	2007	1.334468
Sell	1999	4.157576
Shaw	1976	2.655484
Shinada	2007	4.459
Sample b		2.512821
Study 2		3.250732
Sample b		1.666
Sutter	2010	6.135652
Sample b		2.94
Sutter	2009	2.189091
Tan	2008	4.17088
Tenbrunsel	1999	-1.93747
Study 2		-2.43185
Study 3		1.583077
Sample b		-0.96946

Authors	Year	Z
Tyran	2004	0
van Prooijen	2008	-0.89091
Sample b		-3.8357
Study 2		0.598473
Sample b		-2.24824
Van Vugt	1999	3.464186
Walker	2004	-0.27509
Sample b		0.756491
Wit	1990	1.905047
Study 2		0.368941
Study 3		0
Study 4		2.255849
Xiao	2010	1.2152
Sample b		3.01
Sample c		1.588276
Sample d		4.957647
Sample e		1.8424
Sample f		4.971707
Sample g		1.158957
Sample h		4.878222
Yamagishi	1986	3.275616
Yamagishi	1988	7.454426
Yamagishi	1992	5.377436

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