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The Inflation Target at the Zero Lower Bound

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Abstract

We propose that the monetary authority adopt the inflation target as a time varying policy instrument at the zero lower bound (ZLB) with the same zeal with which they have adopted a fixed inflation target away from the ZLB. After an extreme adverse shock reduces demand, the monetary authority promises future inflation by raising the inflation target in the Taylor Rule. Time paths for inflation and output closely approximate those under optimal policy with the advantages that it is communicable using the language of the inflation target and implementable using the Taylor Rule.

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Keywords: New-Keynesian Model, Inflation Target, Liquidity Trap

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1 Introduction

Once the nominal interest rate reaches the zero lower bound (ZLB), monetary policy loses the ability to stimulate the economy by further reducing the nominal interest rate. Yet, the monetary authority retains the ability to stimulate by promising future inflation, thereby reducing the current real interest rate. The zeal with which many monetary authorities have adopted inflation targeting could be extended to using the inflation target as a policy instrument at the ZLB. An increase in the inflation target raises inflationary expectations, thereby reducing the real interest rate, even at the ZLB, and stimulating the economy.

Monetary policy in the standard New Keynesian model is characterized by a Taylor Rule, whereby the nominal interest rate is set to equal a target, comprised of the sum of targets for the real interest rate and inflation, and to respond strongly to deviations of inflation and output from their respective targets. Woodford (2003, p. 287) has shown that the real interest rate target should be time varying and follow the natural interest rate when the economy is away from the ZLB. We argue that the inflation rate target should also be time varying after the economy reaches the ZLB.

Woodford's argument is that a Taylor Rule with a time-varying interest rate target implements optimal monetary policy away from the ZLB. Optimal policy is a series of values for the interest rate target such that the interest rate follows the natural rate. However, an interest rate rule, which sets the interest rate at the natural rate, admits multiple equilibria, and therefore does not implement optimal policy. In contrast, the Taylor Rule adds a strong response of the interest rate to deviations of inflation and output from their targets assuring that the equilibrium is locally unique.

We show that a simple extension of Woodford's Taylor Rule, which implements optimal policy away from the ZLB, is able to implement a policy which closely approximates optimal policy, even around the ZLB. We propose that the monetary authority introduce time-variation to the inflation target in a truncated Taylor Rule. In normal times, the inflation target takes on a value of zero, consistent with optimal policy under both discretion and commitment. In the event of an adverse demand shock, severe enough to send the economy to the ZLB, the monetary authority announces and commits to a positive path for the inflation target. The inflation target rises to a positive value and retains this value until the period after exit from the ZLB, whereupon it falls at a preannounced rate. This simple characterization of the path for the inflation target assures that it is easily communicable. The inflation target and its rate of decline are both chosen to minimize expected loss.

The positive inflation target postpones the date of exit from the ZLB, compared with optimal discretion, and creates the expectation that exit will occur with positive values for inflation and the output gap. Both the postponed exit date and the positive inflation upon exit stimulate current inflation and output. Inflation and the output gap overshoot their long-run equilibrium values of zero, but magnitudes are small with inflation never larger than 0.1% at an annual rate. Additionally, the equilibrium time paths for inflation and the output gap closely approximate those under optimal policy at the ZLB, implying that our policy yields almost identical loss with optimal policy. In contrast, loss under discretion is much larger and is increasing in both the magnitude and the persistence of the adverse shock.

These results imply that the Taylor Rule with a time-varying inflation target can be a close approximation to optimal policy at the ZLB. Our policy provides a way to implement and communicate a monetary policy which closely approximates optimal monetary policy once the economy reaches the ZLB. Additionally our policy yields much lower loss than optimal discretion.¹

We also consider the possibility of using the time-varying inflation target to stimulate the economy sufficiently to completely avoid the ZLB in the event of the extreme shock, something like the proposals by Krugman (1998) and Svensson (2001, 2003) to exit a liquidity trap by promising higher inflation. The key here is the persistence of the inflation target. With sufficient persistence, an increase in the target can actually increase the nominal interest rate because the increases in output and inflation, due to the higher inflationary expectations, can be large enough to offset the effect of the larger target itself on the nominal interest rate.² Therefore, we find that we can avoid the ZLB completely, even after the extreme adverse event. However, the welfare costs are large. The policy of avoiding the ZLB yields adjustment paths with large and falling positive deviations, whereas optimal paths have small positive and negative deviations. Since welfare loss is measured by squared deviations, the policy yielding positive and falling deviations has larger loss. Welfare costs after hitting the ZLB are necessarily lower than those in Coibion, Gorodnichenko, and Wieland (2012), who study monetary policy with an inflation target fixed high enough to virtually eliminate the possibility of ever hitting the ZLB.

Our paper is related to other papers which address monetary policy at the ZLB. Adam

¹Our policy does require commitment, but arguably not more than required by the Taylor Rule promise to "blow up" (Cochrane 2011) the economy in the event of a sunspot shock.

²The coefficient on inflation target is the sum of the positive coefficient on the target itself plus the negative coefficients on the terms representing responsiveness of the nominal interest rate to deviations of inflation and the output gap from targets. The Taylor Principle yields a negative coefficient on the inflation target.

and Billi (2006, 2007) and Nakov (2008) have analyzed optimal policy under discretion and under commitment when autoregressive demand shocks yield the possibility of the ZLB. They do not explicitly consider implementation. Cochrane (2013) shows that the discretionary commitment to exit the ZLB with zero values for inflation and the output gap yields a unique equilibrium at the ZLB. But, he also argues that if the policy maker could commit to exit the ZLB at different values for inflation and the output gap, this could yield a preferable equilibrium during the ZLB. Krugman (1998), Eggertson and Woodford (2003), Adam and Billi (2006), and Nakov (2008) demonstrate that optimal monetary policy with commitment relies on an increase in inflationary expectations to leave the ZLB.

These policies work within the confines of a simple New Keynesian model, in which the effects of monetary policy are transmitted through the real interest rate. Much of the literature on monetary policy in a liquidity trap expands policy to unconventional methods, which are effective to the extent that financial-market arbitrage is imperfect, that the monetary authority assumes risk on its balance sheet, and/or the quantity of money has an effect on the economy independent of its effect on the real interest rate. These policies are interesting and potentially useful, but the simple New Keynesian model is not complex enough to provide a role for them.³ In a similar context, Williamson (2010) argues that there is no ZLB, in the sense that the monetary authority can always find some stimulative instrument. This instrument can be unconventional monetary policy, but we argue that it can also be a time varying inflation target.

Additionally, Werning (2012) and others have proposed that when conventional monetary policy loses its effectiveness, government can turn to fiscal policy.⁴ However, the fiscal response following the financial crisis which began in 2007 has been highly political and unreliable. The unreliability of a fiscal response, together with the uncertainty over the magnitude of fiscal multipliers, implies that governments cannot rely on fiscal policy as a stabilization tool.

This paper is organized as follows. The next section presents optimal monetary policy in the simple three-equation New Keynesian model. We begin with a demonstration that the Taylor Rule with a time-varying intercept can be used to implement optimal policy as long as the implementation does not imply that the nominal interest rate falls below zero. Section 3 presents our proposal that the monetary authority adopt the time-varying

³Examples of unconventional monetary policy include Auerbach and Obstfeld (2004), Blinder (2000, 2010), Bernanke (2002), Bernanke and Reinhart (2004), Bernanke, Reinhart and Sack (2004), Clouse *et.al.* (2003) and Gurkaynak, Sack and Swanson (2004,2005).

⁴Some unconventional monetary policies are arguably fiscal policies.

inflation target after a severe adverse demand shock that sends the economy to the ZLB. Section 4 uses our policy to avoid the ZLB, and Section 5 concludes.

2 Monetary Policy in the Simple New Keynesian DSGE Model

2.1 Simple New Keynesian Model

Following Woodford (2003) and Walsh (2010), we represent the simple standard linearized New Keynesian model as an IS curve, derived from the Euler Equation of the representative agent, and a Phillips Curve, derived from a model of Calvo pricing (Calvo, 1983). The linearization is about an equilibrium with a long-run inflation rate of zero.⁵

$$y_t = E_t(y_{t+1}) - \sigma [i_t - \bar{i} - E_t\pi_{t+1}] - u_t \quad (1)$$

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa y_t. \quad (2)$$

In these equations y_t denotes the output gap; inflation (π_t) is the deviation about a long-run value of zero; i_t denotes the nominal interest rate, with a long-run equilibrium value of $\bar{i} = r = \frac{1-\beta}{\beta}$, with r defined as the long-run real interest rate; σ represents the intertemporal elasticity of substitution with $\sigma \geq 1$, κ represents the degree of price stickiness;⁶ $\beta \in (0, 1)$ denotes the discount factor; and u_t represents the combination of shocks associated with preferences, technology, fiscal policy, etc. Following Woodford (2003, Chapter 4), we do not add an independent shock to inflation in the Phillips Curve.⁷ This restricts the analysis to the case where monetary policy faces no trade-off between inflation and the output gap.

⁵This does not require that the inflation rate be zero in the long run, only that it not be so far from zero to make the linearization inappropriate (Woodford 2003, p. 79).

⁶ $\kappa = \frac{(1-s)(1-\beta s)}{s} \frac{\sigma^{-1} + \omega}{1 + \omega \varepsilon}$, where $s \in (0, 1)$ represents the fraction of randomly selected firms that cannot adjust their price optimally in a given period. Therefore, $s = 0 \Rightarrow \kappa \rightarrow \infty \Rightarrow$ complete flexibility and $s = 1 \Rightarrow \kappa = 0 \Rightarrow$ complete stickiness. Hence, $\kappa \in (0, \infty) \Rightarrow$ incomplete flexibility. $\omega > 0$ is the elasticity of firm's real marginal cost with respect to its own output, $\varepsilon > 0$ is the price elasticity of demand of the goods produced by monopolistic firms. See, Adam and Billi (2006) and Woodford (2003) for details.

⁷Adam and Billi (2006) demonstrate that calibrated supply shocks are not large enough to send the economy to the zero lower bound.

2.2 Policy to Choose Nominal Interest Rate

2.2.1 Optimal Policy

The model is completed with determination of the nominal interest rate. We consider two alternative methods to specify the nominal interest rate. The first follows Woodford (2003), and chooses values for the time paths of inflation and the output gap to minimize the loss function,

$$L_t = \frac{1}{2} E_t \sum_{j=0}^{\infty} \beta^j (\pi_{t+j}^2 + \lambda y_{t+j}^2), \quad \lambda \in [0, \infty). \quad (3)$$

Woodford derives equation (3) as a linear approximation to the utility function of the representative agent when equilibrium inflation is zero and the flexible-price value for output is efficient.⁸ When the only shock is to the Euler equation, it is optimal to set $\pi_t = y_t = 0$. Given these values, it is straightforward to show that the optimal value for the nominal interest rate is

$$i_t = \bar{i} - \sigma^{-1} u_t = r_t^n, \quad (4)$$

where r_t^n is defined as the natural rate of interest.

According to equation (4), a reduction in the demand for current output (a rise in u_t) reduces the natural interest rate and should be offset by a reduction in the nominal interest rate. The nominal interest rate should remain lower as long as demand and the natural rate are lower. An interest rate which fully offsets demand shocks keeps inflation and the output gap both at their target values of zero. A nominal interest rate, set according to equation (4), is compatible with the target values of zero for inflation and the output gap.

However, if equation (4) is used as the interest rate rule, then there are also many other equilibrium values for inflation and the output gap in addition to the target values. An interest rate rule like equation (4) leaves the price level indeterminate. Sargent and Wallace (1981) were the first to raise the issue of indeterminacy in the context of a policy which fixes the nominal interest rate. Hence, the monetary authority cannot implement optimal policy using equation (4) as an interest rate rule. Equation (4) determines the equilibrium value of the optimal interest rate, but it does not explain how the monetary authority can achieve it.

⁸The government can subsidize firms to increase production to the perfectly competitive level.

2.2.2 Taylor Rule

The method, typically employed in New Keynesian models for determining the nominal interest rate, is to assume that the monetary authority follows a Taylor Rule. In Taylor's original rule, the nominal interest rate is set to equal a fixed real rate plus a fixed inflation target and to respond positively to deviations of inflation and output from fixed target values. The Taylor Rule can be expressed as

$$i_t = r^* + \pi^* + \varphi_\pi (\pi_t - \pi^*) + \varphi_y (y_t - y^*), \quad \varphi_\pi > 0, \quad \varphi_y \geq 0. \quad (5)$$

Allowing the interest rate to respond strongly to endogenous variables solves the problem of indeterminacy which arises if equation (4) is treated as an interest rate rule. Specifically, Bullard and Mitra (2002) demonstrate that if φ_π and φ_y are large enough such that equations (1) and (2), with equation (5) for the interest rate, yields a dynamic system with two unstable roots, corresponding to the two forward-looking variables, then the equilibrium is unique. This condition has been labeled the Taylor Principle.⁹

Woodford (2003) demonstrates that it is possible to use the Taylor Rule to implement¹⁰ optimal monetary policy by following a Taylor Rule with a time-varying intercept ($r_t^* + \pi^*$). Erceg, Henderson, and Levine (2000) and Woodford (1993, 246) also use Taylor Rules in which a time-varying intercept can be chosen by the monetary authority. Woodford sets $\pi^* = 0$, and lets r_t^* be time-varying. Optimal policy can be implemented with

$$r_t^* = r_t^n. \quad (6)$$

Substituting equation (6) into equation (5), setting $\pi^* = 0$, and substituting the Taylor Rule with this optimal policy into equations (1) and (2) sets inflation and the output gap at their target values of zero.¹¹ At equilibrium values for the output gap and inflation of zero, the interest rate equals the optimal interest rate in equation (4), Woodford's (2003) natural rate of interest.

The equilibrium solution is independent of the values for φ_π and φ_y as long as they are large enough to assure two unstable roots.¹² Therefore, it is important to understand the role of these policy parameters. The promise to respond strongly to any sunspot shocks that raise inflation and/or output, in Cochrane's (2011) words, "to blow up the economy"

⁹The Taylor Principle originally referred to requiring $\varphi_\pi > 1$, but has been generalized to allow the nominal interest rate to respond to both inflation and the output gap.

¹⁰Implementability requires local uniqueness of the rational expectations equilibrium.

¹¹Any other values yield an explosive equilibrium, which we rule out.

¹²The criteria for two unstable roots is: $\kappa(\varphi_\pi - 1) + (1 - \beta)\varphi_y > 0$.

in the event of sunspot shocks, serves to rule out sunspot equilibria and to assure a unique equilibrium. Therefore, we can obtain a unique equilibrium in which the interest rate is given by equation (4) only if the monetary authority follows an interest rate rule like (5), which differs from equation (4) by this extraordinary promise. And it must have the ability to commit to this threat. This requires that the monetary authority be completely transparent, communicating the intention to "blow up the economy" and that this threat be completely credible. This is because φ_π and φ_y do not show up in the equilibrium solution and therefore cannot be inferred from any observable evidence.¹³

3 Zero Lower Bound

The above policy is feasible only if the demand shock is never large enough to send the nominal interest rate below zero. We are interested in policy which is effective in the event of those severe adverse demand shocks. We propose allowing the inflation target in the Taylor Rule to be time-varying.

There is empirical evidence supporting the hypothesis that actual monetary policy has operated with a time-varying inflation target in the Taylor Rule. Ireland (2007) argues that US inflation can be explained by a New Keynesian model with a Taylor Rule only if the inflation target is allowed to vary over time. Additionally, Kozicki and Tinsley (2001), Rudebusch and Wu (2004), Gurkaynak, Sack and Swanson (2005) and Dewachter and Lyrio (2006) provide evidence of a time-varying short-run inflation target for the US. Krugman (1998), Svensson (2003), Eggertson and Woodford (2003), Adam and Billi (2006), and Nakov (2008) all suggest policies which increase expected inflation at the ZLB.

3.1 Taylor Rule with Time-Varying Real Interest Rate and Inflation Targets

The Taylor Rule with the real interest rate target equal to the time-varying natural rate and with a time subscript on the inflation target is given by

$$i_t = r_t^n + \pi_t^* + \varphi_\pi (\pi_t - \pi_t^*) + \varphi_y (y_t - y_t^*), \quad (7)$$

¹³Cochrane (2011) emphasizes that at the optimal equilibrium, values for φ_π and φ_y do not affect the equilibrium. Woodford (2003, p. 288) makes the same point. If there were shocks to the Phillips Curve, or if the intercept to the Taylor Rule did not vary optimally, then we would have evidence on the values of φ_π and φ_y . However, we would not have evidence that the monetary authority would actually "blow up" the economy in the event of a sunspot shock.

where, following Woodford (2003), we interpret y_t^* as the value for the output gap from equation (2), when inflation takes on its target value, yielding

$$y_t^* = \frac{\pi_t^* - \beta E_t(\pi_{t+1}^*)}{\kappa} = \frac{\pi_t^*(1 - \rho_\pi/\beta)}{\kappa}. \quad (8)$$

3.1.1 Demand Shocks

The policy we propose is invoked only in the event of a severe adverse demand shock. We choose to model shocks which send the economy to the ZLB differently from the standard literature. An example of standard modeling is Adam and Billi (2006). They calibrate stationary AR(1) shocks, using quarterly data over the period 1983-2002, and find that under policy with commitment, the economy would experience a ZLB one quarter every 17 years and that the ZLB would last between one and two quarters. With discretionary policy, Adam and Billi (2007) find that the more aggressive lowering of the nominal interest rate, as the natural rate moves toward zero, increases the frequency with which the economy hits the ZLB, but they provide no adjustment to their frequency and duration calculations in Adam and Billi (2006).¹⁴

The problem with modeling shocks creating the ZLB based on a sample with no ZLB is that the economy seems to hit the ZLB less frequently than predicted, and once there, seems to remain much longer than predicted. The US has experienced two periods of very low nominal interest rates in the 153 years between 1860 and 2013 (Clouse et al 2003 for earlier data), and both of these have been extremely protracted. Therefore, it is arguable that the ZLB is the result of a gigantic shock – a rare event – a Great Depression or a Financial Crisis – something like falling off a cliff instead of slipping slowly down a hill. Additionally, the two episodes of the ZLB in the US have yielded deep and long-lived recessions in contrast to the predictions in Adam and Billi (2006).

Therefore, we model demand shocks (u_t) as comprised of two components, the normal AR(1) component (v_t), like that observed in the Adam and Billi (2006) sample, and a large shock which represents a rare event (w_t), yielding

$$u_t = v_t + w_t.$$

Since we have experienced only two instances of the ZLB in the US over 153 years, we deviate from Adam and Billi (2006) and assume that the distribution of innovations to

¹⁴Since Jung et al (2005) find that commitment requires exit from the ZLB at a latter date than discretion, perhaps with discretionary policy, the economy would hit the ZLB more frequently, but remain there for shorter periods of time.

the normal AR(1) shock is such that even if we received the worst realization forever, we would never breach the ZLB. We are assuming that economies do not slip down the hill toward the ZLB. This assumption seems consistent with the data and implies that the probability of hitting the ZLB is independent of the economy's current state. Therefore, we do not compare the effect of alternative policies on the frequency of hitting the ZLB since our assumption implies that the frequency is independent of policy. We model small AR(1) shocks by assuming that the innovations (\tilde{v}_t) are drawn from a symmetric bounded normal with bounds $(-\bar{v}, \bar{v})$ tight enough that even if the economy received the worst shock forever, it would never breach the ZLB. The normal AR(1) disturbance is modeled as

$$v_t = \rho_v v_{t-1} + \tilde{v}_t \quad 0 < \rho_v < 1$$

with

$$\frac{\bar{v}}{1 - \rho_v} \leq \sigma \bar{v}. \quad (9)$$

In contrast, the rare event is drawn from the time-varying W_t distribution, which has only large and symmetric elements. A draw from the W_t distribution puts the economy in either the best or worst possible state, unconditional on the current state. There are two equally probable elements at any point in time, given by

$$w_t \in \{w - \rho_v v_{t-1}, -w - \rho_v v_{t-1}\}.$$

The time varying component allows the shock from the W_t distribution to put the economy in the worst or best possible state, unconditional on its prior state. If the shock is negative, it sends the economy to the ZLB

$$w > \sigma \bar{v}.$$

We allow the stochastic behavior of the economy to change after receiving the extreme shock, given by $\pm w$. First, we assume that the shock deteriorates at rate ρ_w , where we explicitly allow persistence to be higher than that of the ordinary shock in order to model the long durations of the ZLB. This requires

$$\rho_w \geq \rho_v.$$

Second, to simplify the solution of the model, we rule out the possibility that either another draw from the W distribution or AR(1) shocks to v_t could send the economy back to or beyond the "worst possible state." Therefore, we assume that the economy

cannot receive another shock from the W distribution until after it has recovered from the current one. Additionally, we assume that variance for the V distribution is small enough that the value of the demand shock is expected to fall over time as the value of the extreme shock decays. This requires at a minimum the additional assumption

$$w - \left(\frac{\bar{v}}{1 - \rho_w} \right) = \omega > 0. \quad (10)$$

Assume that in period one the economy receives the adverse draw from the W distribution putting it in the worst possible state. Given these assumptions, the demand disturbance for periods $t \geq 1$, can be represented by

$$u_t = v_t + \rho_w^{t-1} w,$$

where $v_1 = 0$.

3.1.2 Inflation Target at the ZLB

Our policy for more aggressive use of the inflation target at the ZLB is the following. In normal times, when shocks are drawn from the V distribution, the inflation target is fixed at zero. However, following a large adverse shock in period 1, the inflation target is reset away from zero to

$$\pi_1^* = \bar{\pi}^* > 0.$$

The inflation target retains this value into period $T + 1$, the period in which the economy emerges from the ZLB. Thereafter, the inflation target evolves as

$$\pi_t^* = \rho_\pi^{-(T+1-t)} \bar{\pi}^* \quad t \geq T + 1. \quad (11)$$

Both $\bar{\pi}^*$ and ρ_π are policy variables chosen by the monetary authority to minimize loss, given by equation (3). In a regime of certainty, choices for $\bar{\pi}^*$ and ρ_π yield a unique value for T , the final period at the ZLB.¹⁵ With uncertainty, they yield a unique expected exit time.

Using equation (8) for y_t^* to substitute into the interest rate equation (7), and collecting

¹⁵Raising the inflation target for an adverse shock imparts a small permanent inflation bias to the economy in normal times. This is due to the small probability that the inflation target will be raised. However, we will show that the inflation target is so small, that multiplied by the probability of the rare event, the inflation bias is miniscule.

terms on π_t^* yields

$$i_t = r_t^n - z\pi_t^* + \varphi_\pi\pi_t + \varphi_y y_t, \quad (12)$$

where r_t^n is given by

$$r_t^n = \bar{r} - \sigma^{-1}u_t, \quad (13)$$

and z is a constant given by

$$z = \varphi_\pi + \varphi_y \left(\frac{1 - \rho_\pi\beta}{\kappa} \right) - 1 > 0, \quad (14)$$

with the inequality implied by the Taylor Principle. Since the interest rate cannot be negative, it follows a truncated Taylor Rule such that

$$i_t = \max \left(0, r_t^n + \varphi_\pi\pi_t + \varphi_y y_t - z\pi_t^* \right), \quad (15)$$

where the value for r_t^n is stochastic since the economy can continue to receive shocks from the V distribution after receiving the extreme negative shock.

Our policy of raising the inflation target in the event of an extreme adverse shock requires no current action for implementation, but does affect expectations about future policy actions. Are these announced future actions credible?

Any policy contains implicit or explicit promises for future action. Credibility is an issue when the promise is dynamically inconsistent, and ours is. However, the dynamic inconsistency is not related to whether or not the promise of future action is accompanied by current action or not. The fact that a particular policy requires no current action for implementation would not seem to make announcements about future actions any less credible than if the policy also required current action. In either case, if the authority can commit to a policy, and commitment is required even to follow a Taylor Rule, then the authority will not reoptimize each period. If a policy maker has credibility, then a rational expectations equilibrium requires that agents expect the policy maker to act as he has announced, whether the policy-maker is currently acting or not. The fact that monetary authorities are using forward guidance on interest rates as a policy instrument suggests that they have credibility.

3.2 Model Solution

3.2.1 Certainty

To solve the model, we initially assume that there is no additional uncertainty following the large adverse shock and set the variance of innovations from the V distribution to zero. This assumption permits analytical solution and facilitates comparison with the results to those of optimal policy under commitment and under discretion. Subsequently, we add a moderate amount of uncertainty, consistent with our assumptions above, and compare simulations of time paths under certainty with expected time paths under uncertainty.

Our solution under certainty follows Jung et al (2005) who separate time at T , defined as the final period in which the nominal interest rate is zero. We solve for time paths after the extreme adverse shock.

Periods $t = 1, 2, \dots, T$ For $t \leq T$, the value for the nominal interest rate is zero. Write equations (1) and (2) with $i_t = 0$ as

$$Z_{t+1} = AZ_t - ar_t^n \quad (16)$$

where,

$$Z_t = \begin{bmatrix} y_t \\ \pi_t \end{bmatrix}$$

and

$$A = \begin{bmatrix} \left(1 + \frac{\kappa\sigma}{\beta}\right) & \left(-\frac{\sigma}{\beta}\right) \\ \left(-\frac{\kappa}{\beta}\right) & \left(\frac{1}{\beta}\right) \end{bmatrix}, \quad a = \begin{bmatrix} \sigma \\ 0 \end{bmatrix}.$$

A forward looking solution of equation (16) yields

$$Z_t = \Gamma_t + A^{-(T-t+1)}Z_{T+1} \quad (17)$$

where,

$$\Gamma_t = \sum_{k=t}^T A^{-(k-t+1)}ar_k^n.$$

Equation (17) implies that values for deviations of inflation and the output gap prior to exit from the ZLB depend on their expected values on the date of exit from the ZLB. The promise to exit the ZLB with positive values for inflation and the output gap stimulate the economy while at the ZLB. Additionally, postponement of the exit date with a larger

value for T , stimulates since the coefficients in the $A^{-(T-t+1)}$ matrix are increasing in T . The Taylor Principle upon exit implies a unique value for Z_{T+1} , thereby assuring uniqueness prior to period $T + 1$.

Periods $t = T + 1, T + 2, \dots$ In the period in which the economy exits the ZLB, the nominal interest rate becomes positive and remains positive. Substituting the nominal interest rate from equation (15) where the inflation target is given by equation (11), into equations (1) and (2) yields

$$Z_{t+1} = \Omega Z_t - az\pi_t^* \quad (18)$$

where

$$\Omega = \begin{bmatrix} 1 + \sigma \left(\varphi_y + \frac{\kappa}{\beta} \right) & \sigma \left(\varphi_\pi - \frac{1}{\beta} \right) \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{bmatrix}.$$

When the Taylor Principle is satisfied, $z > 0$ and both characteristic roots of Ω exceed unity. Therefore, initial values must be determined to set the coefficients on both roots equal to zero. Letting the characteristic roots be denoted by λ_1 and λ_2 , the solution of equation (18) yields unique non-explosive solutions for the output gap and inflation after the nominal interest rate becomes positive as

$$Z_t = b\pi_t^* \quad (19)$$

where

$$b = \begin{bmatrix} \frac{(1-\rho_\pi\beta)\sigma z}{\beta(\lambda_1-\rho_\pi)(\lambda_2-\rho_\pi)} \\ \frac{\kappa\sigma z}{\beta(\lambda_1-\rho_\pi)(\lambda_2-\rho_\pi)} \end{bmatrix}$$

for $t \geq T + 1$.

Optimal Value for Inflation Target The monetary authority chooses the value for the inflation target ($\bar{\pi}^*$) and its persistence (ρ_π) on the date of the shock and commits to both. Both are chosen to minimize loss in equation (3).

The nominal interest rate in the exit period is determined by substituting for y_t and π_t , from equation (19), into equation (15) to yield

$$i_t = r_t^n + qz\bar{\pi}^* \quad (20)$$

where

$$q = \frac{\varphi_\pi\kappa + \varphi_y(1 - \rho_\pi\beta)}{\beta(\lambda_1 - \rho_\pi)(\lambda_2 - \rho_\pi)}\sigma - 1,$$

and z is given by equation (14). The value for $T + 1$ is the first period that i_t in equation (20) becomes positive.

Increases in $\bar{\pi}^*$ and ρ_π both raise inflationary expectations, stimulating output and inflation. However, they have opposite effects on exit time. Exit time is the first time that the natural rate (r_t^n) is large enough to offset the negative value for $qz\bar{\pi}^*$. The natural rate is expected to rise over time as its shock component decays at rate ρ_w . For small values of ρ_π , the value for $q < 0$, and qz is increasing in ρ_π .

Consider an increase in $\bar{\pi}^*$ first. Since $qz < 0$, an increase in $\bar{\pi}^*$ reduces the value of the nominal interest rate at each point in time, implying that $T + 1$ must be larger to allow for a larger natural rate. The postponed exit time serves as further stimulus to output and inflation. In contrast, an increase in ρ_π raises qz , thereby raising the value for the nominal interest rate at each point in time. This implies a sooner exit time, since the higher value for $qz\bar{\pi}^*$ can offset an earlier lower value for the natural rate of interest. The earlier exit time mitigates the stimulus associated with the increase in ρ_π .

Calibration and Impulse Response We illustrate the quantitative effects of our policy proposal using the RBC parameterization from Adam and Billi (2006),

$$\sigma = 1, \beta = 0.99, \kappa = 0.057, \varphi_\pi = 1.5, \varphi_y = 0.5.$$

All values are expressed at quarterly rates. The values for the elasticity of substitution and the discount factor are standard. The value of κ is consistent with 44% of firms adjusting their price each period.

We compare three alternative values for the initial state following the extreme adverse shock (w), measured at quarterly rates, $w \in \{0.018, 0.021, 0.024\}$. With $\bar{i} = 0.01$, each of these shocks sends the natural rate below zero. We allow the extreme shock to exhibit three different values for persistence (ρ_w), including $\rho_w = 0.80$, the value Adam and Billi (2006) estimate as the variance of the real rate shock, and higher values of 0.85 and 0.90. For purposes of comparison, we also compute impulse response functions under discretion and optimal policy for the same shocks. The solution under discretion¹⁶ is equivalent to our policy with $\bar{\pi}^* = 0$, and we characterize the solution under commitment in the appendix.

We use a numerical algorithm to choose values for $\bar{\pi}^*$, T , and ρ_π to minimize loss. We choose a value for ρ_π and find the loss-minimizing value for $\bar{\pi}^*$ and the associated T . We

¹⁶With certainty, discretion is a Truncated Taylor Rule with an inflation target of zero.

allow ρ_π to take on alternative values and find associated values for $\bar{\pi}^*$ and T . We choose the loss-minimizing value for ρ_π and the associated values for $\bar{\pi}^*$ and T as the global minimum.

We plot impulse response functions for the highest persistence (0.90) and largest shock case (0.024) in Figure 1 below. The shock is so large and persistent that the natural rate of interest does not become positive until the tenth quarter after the shock. We chose this large value for the shock to come close to replicating the long period of the ZLB following the financial crisis in the US.

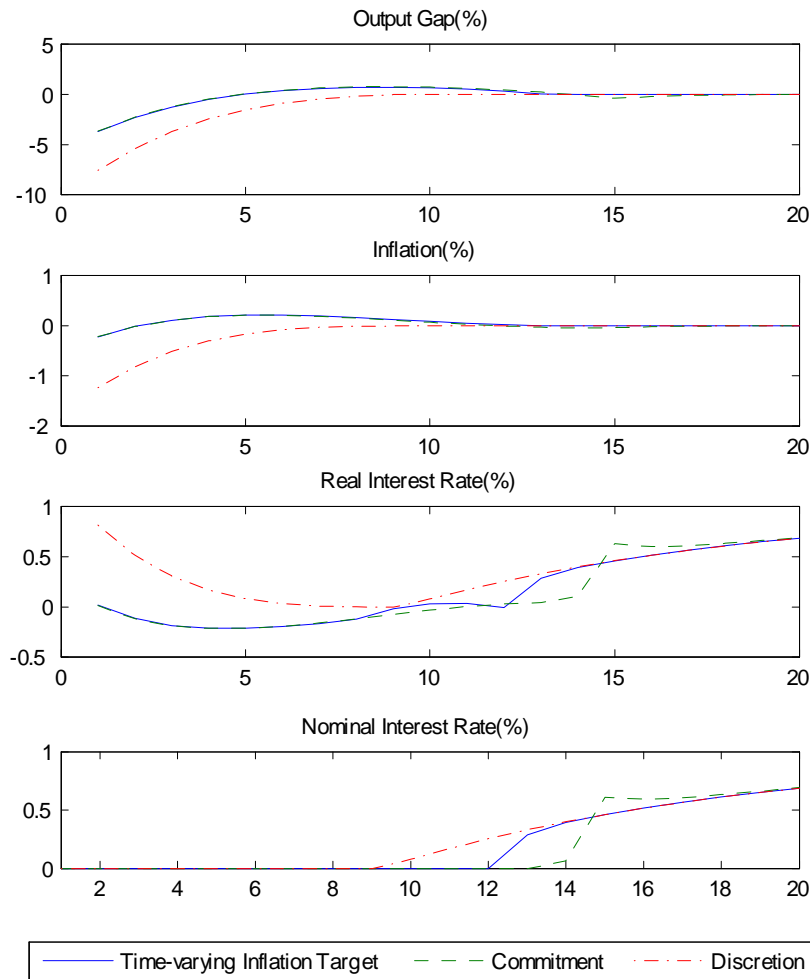


Figure 1: Impulse Response

Our policy generates time paths for inflation and the output gap which are almost indistinguishable from those of optimal policy. Both policies stimulate compared to dis-

cretionary policy because both make two promises which are not made under discretion. The first promise is to keep the nominal interest rate low for a longer period of time than absolutely necessary, and the second is to exit the ZLB with positive values for inflation and the output gap. Both raise inflationary expectations and stimulate compared to discretion.

Under discretionary policy, the nominal interest rate exactly follows the natural interest rate once it becomes positive. In contrast, our policy does not allow the nominal interest rate to become positive until the twelfth quarter after the shock, while under optimal policy it takes until the fourteenth. Upon exiting the zero lower bound, discretionary policy immediately sets inflation and the output gap at their optimal values of zero, while both our policy and commitment retain non-zero values of both. At the point of exit from the ZLB, our policy sets the inflation target to 0.057% at a quarterly rate, allowing it to fall at rate $\rho_\pi = 0.17$ over time. Actual inflation is lower at 0.02%, and both the inflation target and actual inflation fall to zero after two quarters. For smaller shocks and/or smaller persistence, both the magnitudes of the deviations from long-run equilibrium and the length of the adjustment period are shorter.

Table 1 compares values for loss across all three policies: time-varying inflation target, discretion, and commitment. As expected, loss for any given policy is increasing in both the magnitude of the shock and its persistence. Holding both the magnitude and persistence of the shock constant, expected loss is considerably greater under discretion than with the time-varying inflation target. Loss under discretion ranges from 2.7 to 7.7 times as large as loss under our inflation-target policy. Additionally, the relative size of the loss under discretion, compared with the inflation-target policy, is increasing in both the magnitude of the shock and its persistence. In contrast, expected loss under our policy is only slightly larger (between 3 and 5 percent larger) than expected loss under commitment, and there is no relationship between either the magnitude of shock or its persistence and relative loss.

Table 1 also compares optimal values for the inflation target and its persistence for shocks of different magnitudes and persistence. The inflation target is always small, ranging from 0.104% to 0.348% at annual rates. Optimal persistence is often zero, and is always small, implying that our policy returns to discretion quickly after exiting the ZLB (but we exit the ZLB one to two periods later). The optimal exit time ($T + 1$) is increasing in both the magnitude of the shock and in its persistence.

The discrete nature of T in the calibrated solution seems to play a role in the determination of optimal values for the inflation target and its persistence. A discrete change in

the value for T provides a large change in stimulus. To understand, we begin by constraining persistence to be zero, and determine the optimal inflation target and exit time. The value for the inflation target is often constrained by the largest value possible without triggering an increase in the value for exit time. When this occurs, an increase in persistence allows an increase in the inflation target without changing exit time, thereby providing a small amount of additional stimulus. If exit time were not discrete, we conjecture that the persistence variable would not be necessary.

Table 1: Alternative Policies under Different Shocks

u_1	ρ_u	$\% \bar{\pi}^*$	ρ_π	T	$\% \text{loss}$	T^D	T^C	$\frac{\text{loss}^D}{\text{loss}}$	$\frac{\text{loss}}{\text{loss}^C}$
0.018	0.8	.035	0.0	4	0.012	3	5	2.73	1.04
0.018	0.85	.043	0.11	5	0.023	4	6	3.20	1.04
0.018	0.9	.026	0.0	8	0.060	6	9	4.53	1.04
0.021	0.8	.042	0.0	5	0.035	4	6	2.95	1.04
0.021	0.85	.067	0.33	6	0.066	5	8	3.64	1.05
0.021	0.9	.058	0.38	9	0.194	7	11	5.81	1.04
0.024	0.8	.069	0.0	6	0.078	4	7	3.12	1.04
0.024	0.85	.087	0.39	7	0.158	6	9	4.13	1.05
0.024	0.9	.057	0.17	11	0.434	9	13	7.68	1.03

3.2.2 Uncertainty

In this section, we add a moderate amount of uncertainty and simulate the solution of the model under uncertainty. We again present the simulations for the largest adverse shock with the greatest persistence. Our policy proposal for the inflation target is almost identical under certainty as under uncertainty.

Solution Algorithm

Discretize AR(1) Innovation The first step is to discretize the AR(1) process for v_t into a Markov chain with a discrete number of states. We let the autoregressive coefficient be $\rho_v = 0.80$, as in Adam and Billi (2006), but we cannot let variance be as high as theirs because in their simulations, innovations from this disturbance alone send the economy to the ZLB about every five years. Our assumption is that these innovations alone never breach the ZLB.

We restrict the uncertainty with the assumption that the worst state in the V distribution brings the economy to a quarterly nominal interest rate of $i = 0.0025$, the lowest value for the federal funds rate over the period in Adam and Billi (2006). This sets the worst state at

$$V_N = \bar{v} - 0.0025 = 0.0076.$$

We set the standard deviation according to Tauchen's formulation such that we obtain V_N as the worst state in an approximation including 2.5 standard deviations and nine states. This implies that the standard deviation of our process is 0.1824%. We further reduce the variance and the probability of returning to the ZLB by assuming that the economy can move at most one state in one period by placing the probabilities for all states more distant on the one-period-distant state.

Finally, uncertainty affects the authority's choice of persistence in the inflation target on two counts. First, we will demonstrate that uncertainty increases the expected persistence of positive deviations for inflation and the output gap, suggesting that even more persistence would not reduce loss. Second, uncertainty reduces the discreteness associated with the implied exit time since we replace a discrete known exit time with probabilities. Both reduce the desirability of positive persistence in the inflation target. Since optimal persistence is low under certainty and is most likely even lower under uncertainty, we simplify by setting persistence to zero under uncertainty.

Critical Dates The economy exits the ZLB once the nominal interest rate in equation (20) becomes positive. This requires that the natural rate of interest reaches a critical value defined by the inflation target and given by

$$r_t^c = -qz\bar{\pi}^*.$$

Writing the natural rate in terms of critical times ($T_c(j)$), where j indexes the state, critical times are determined implicitly for each of the N possible values of v by

$$v(j) = \sigma(\bar{v} + qz\bar{\pi}^*) - \rho_w^{T_c(j)-1} w \quad j \in \{1, N\}.$$

At time $T_c(j)$, the economy exits the ZLB if it has not already exited and if it is in state j or lower.

Forward Solution The next step is forward solution of the equation

$$Z_t = A^{-1}E_t Z_{t+1} + A^{-1}ar_t^n,$$

where we begin in period $t = 1$ with the extreme shock from the W distribution and with a zero value for the shock from the V distribution. We solve this equation for the value of Z_t , conditional on either remaining at the ZLB in period t or exiting the ZLB in period t . At each iteration, we use the probability of being in each state, and therefore the probability of either continuing at the ZLB or exiting, to compute the expectation. For the expectation upon exit, we need to allow for the possibility that the economy could return to the ZLB.

We solve $Z_{T^x}^x$, the vector containing the output gap and inflation conditional upon exit in time T^x , in the appendix to yield

$$Z_{T^x}^x = b\pi^* + BE_{T^x} Z_{T^x+1}^x$$

where

$$b = [I + A^{-1}a\varphi]^{-1} A^{-1}az \quad B = [I + A^{-1}a\varphi]^{-1} A^{-1} \quad \varphi = [\varphi_y, \varphi_\pi].$$

The value conditional on starting away from the ZLB is simply

$$Z_t^x = BE_t Z_{t+1}^x, \tag{21}$$

since the inflation target is zero one period after exit from the ZLB. We obtain numerical solutions for Z_t^x backwards by picking a time far enough in the future such that the natural rate is positive in all states. At this point, inflation and the output gap in all states are zero and the expectation is zero.

Define $E(R_{t+1}|1, \bar{N})$ as the expected value of the natural interest rate across all eligible states, conditional on remaining at the ZLB and $(\psi_{t+1,s}|1, \bar{N})$ as the probability of exit in state s , both at time $t + 1$ and conditional on starting in period 1 and state \bar{N} as

$$\begin{aligned} E(R_{t+1}|1, \bar{N}) &= \sum_{v_2^c}^{\bar{v}} \left\{ \dots \sum_{v_t^c}^{\bar{v}} \left[\sum_{-v_{t+1}^c}^{\bar{v}} r_{t+1}^n p(v_{t+1}|v_t) \right] p(v_t|v_{t-1}) \dots \right\} p(v_2|v_1), \tag{22} \\ (\psi_{t+1,s}|1, \bar{N}) &= \sum_{v_2^c}^{\bar{v}} \left\{ \dots \sum_{v_t^c}^{\bar{v}} \left[p(v_{t+1}^{c(s)}|v_t) \right] p(v_t|v_{t-1}) \dots \right\} p(v_2|v_1). \end{aligned}$$

The term $v_{t+1}^{c(s)}$ is used to denote the critical value of v at time $t + 1$ for which exit would occur in state s . In general, the value for $Z_{t,j}$, the vector with the output gap and inflation at time t , conditional on starting in state j and period t at the ZLB, is given by

$$Z_{t,j} = \Omega_{t,j} + \Psi_{t,j}, \quad (23)$$

where

$$\Omega_{t,j} = \sum_{i=t+1}^{\bar{T}} A^{t-i-1} a E(R_i | t, j) + A^{-1} a r_t^n$$

and

$$\Psi_{t,j} = \sum_{i=t+1}^{\bar{T}+1} \sum_{s=1}^N A^{t-i} (b\bar{\pi}^* + B E_{i,s} Z_{i+1}) (\psi_{i,s} | t, j).$$

Expected loss is computed by weighing the squared time- t , state- j values of the output gap and inflation from equation (23) by their probabilities beginning from time 1, state \bar{N} , together with the squared time- t , state- j values of the output gap and inflation if the economy has exited the ZLB from Z_t^x in equation (21), multiplied by their probabilities beginning from time 1, state \bar{N} . Expected time paths are computed analogously.

Impulse Response under Uncertainty The optimal value for the inflation target upon exit under uncertainty is 0.046%. Expected loss increases to 0.585% due to variance of inflation and the output gap along the time path. Figure 2 contains impulse-response functions for the expected paths for the output gap, inflation rate, and nominal interest rate in the two cases. Under uncertainty, there is more persistence in inflation and the output gap than under certainty, implying that the optimal amount of stimulus should be slightly less. This explains the slightly lower initial values for inflation and the output gap under uncertainty.¹⁷ The expected path for the nominal interest rate does rise earlier due to the possibility that with favorable shocks, the economy could exit the ZLB sooner than under a zero inflation target. However, our overall assessment is that the expected time paths under uncertainty are very close to the time paths under certainty.

¹⁷We confirm that this is not due to restricting persistence to be zero by comparing paths for a case in which optimal persistence is zero under certainty and obtain identical patterns.

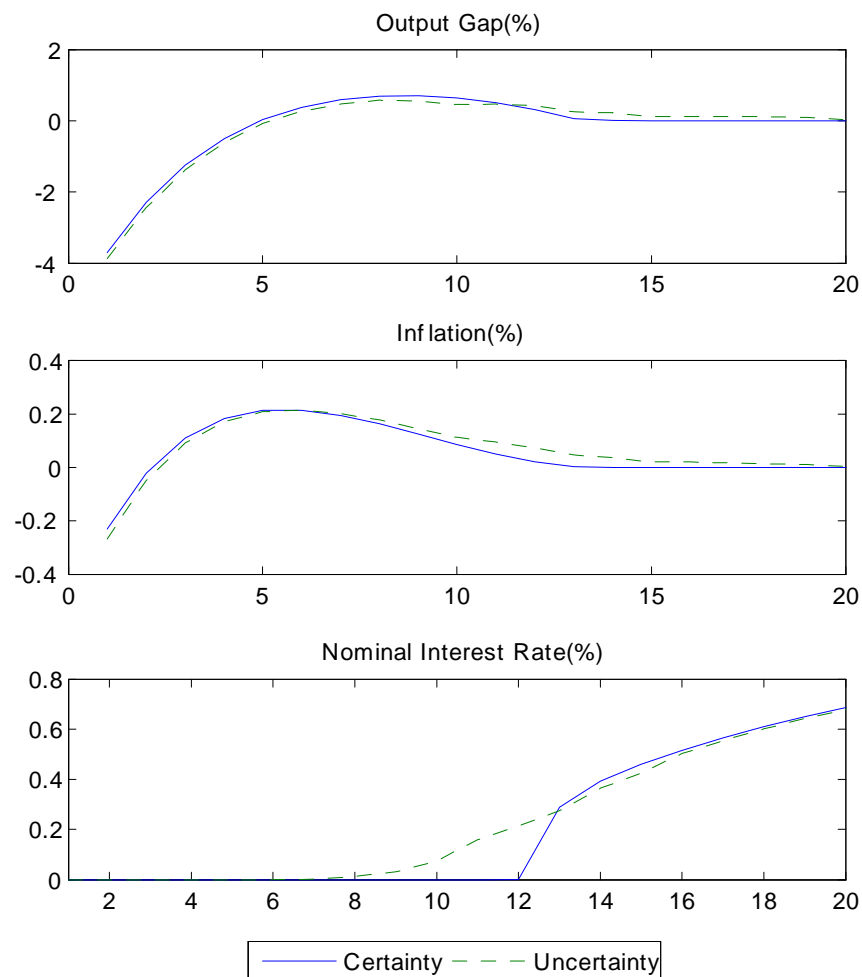


Figure 2: Effects of Uncertainty

In contrast, consideration of uncertainty substantially raises the relative loss of following a Taylor Rule with a zero inflation target¹⁸ compared to following one with the optimal inflation target. Expected loss under the zero inflation target is eleven and a half times that under the optimal inflation target. Therefore, consideration of uncertainty strengthens the argument for adding the time-varying inflation target to the Taylor Rule.

¹⁸With uncertainty the truncated Taylor Rule with a zero inflation target differs from optimal policy under discretion. Under optimal discretion, the monetary authority is choosing the nominal interest rate to minimize loss, knowing that they will rechoose in the future. The possibility of returning to the ZLB in the future will reduce output and inflation leading to the choice of a lower interest rate. Under the truncated Taylor Rule, they are choosing the nominal interest rate according to the Taylor Rule.

4 Using the Inflation Target to Avoid the ZLB

Consider the possibility of raising the inflation target and its persistence in the period of the shock by a large enough amount that the economy exits the ZLB immediately in period 1. Equivalently, the economy receives a shock in period 1, sending it to the worst possible state, and monetary policy is so stimulative that inflation and the output gap rise sufficiently to keep the nominal interest rate from ever hitting the ZLB. The recession is reversed into a boom.

In this case, the first period when the nominal interest rate becomes positive is the period of the shock. This implies that the solutions for output and inflation are given by equation (19) with the nominal interest rate given by equation (20).

Equation (20) reveals that for an increase in the inflation target to raise the nominal interest rate, ρ_π must be large enough to make q positive. In the New Keynesian model, the direct effect of an increase in the inflation target is a reduction in the nominal interest rate, and this stimulates demand and inflation. However, the increase in the inflation target also raises expectations of inflation, further stimulating demand, and through the Taylor Rule responses to inflation and the output gap, leads to an increase in the interest rate. For large enough persistence of the short-run inflation target, this indirect effect dominates, implying that an increase in the inflation target raises the nominal interest rate.¹⁹

With ρ_π set large enough to assure $q > 0$, we can choose $\bar{\pi}^*$ such that the nominal interest rate is always above zero, thereby avoiding the ZLB and validating the above solutions for output and inflation.

Assuming that the economy receives a shock from the W distribution at time $t = 1$, the value for the nominal interest rate, conditional on not receiving another shock from the W distribution, can be expressed as

$$i_t = \bar{i} - \sigma^{-1} \left[\rho_w^{t-1} \left(\omega + \frac{\bar{v}}{1 - \rho_w} \right) + \sum_{h=2}^t \rho_v^{t-h} \tilde{v}_h \right] + qz\rho_\pi^{t-1}\bar{\pi}^*.$$

We must choose $\bar{\pi}^*$ and ρ_w to assure that $i_t > 0$, allowing the economy to avoid the liquidity trap. Note that if the economy continues to receive the worst shock possible

¹⁹This is why calibrated models fail to find a liquidity effect of a negative interest rate shock when persistence is high.

from the V distribution from $t = 2$ going forward, then

$$u_t = \rho_w^{t-1} \left(\omega + \frac{\bar{v}}{1 - \rho_w} \right) + \frac{\bar{v}}{1 - \rho_v} (1 - \rho_v^{t-1}).$$

The nominal interest rate in this case becomes

$$i_t = \bar{i} - \frac{\sigma^{-1}\bar{v}}{1 - \rho_v} - \sigma^{-1} \left[\rho_w^{t-1}\omega + \bar{v} \left(\frac{\rho_w^{t-1}}{1 - \rho_w} - \frac{\rho_v^{t-1}}{1 - \rho_v} \right) \right] + qz\rho_\pi^{t-1}\bar{\pi} > 0. \quad (24)$$

The values for $\bar{\pi}$ and ρ_π must be chosen such that the nominal interest rate is positive for all t . Note, first, that this requires $\rho_\pi \geq \rho_w$. To understand this, divide the right-hand side of equation (24) by ρ_π^t to yield

$$\frac{1}{\rho_\pi^t} \left(\bar{i} - \frac{\sigma^{-1}\bar{v}}{1 - \rho_v} \right) - \sigma^{-1} \left(\frac{\rho_w}{\rho_\pi} \right)^t \left[\omega + \bar{v} \left(\frac{1}{1 - \rho_w} - \left(\frac{\rho_v}{\rho_w} \right)^t \frac{1}{1 - \rho_v} \right) \right] + \frac{qz\bar{\pi}}{\rho_\pi} > 0.$$

As t increases, the negative term explodes unless $\rho_\pi \geq \rho_w$.

Given $\rho_\pi \geq \rho_w$, the constraint in equation (24) is most binding for $t = 1$. Therefore, setting $t = 1$ yields a lower bound on the inflation target as

$$\begin{aligned} \bar{\pi}^* &\geq \frac{-\left(\bar{i} - \frac{\sigma^{-1}\bar{v}}{1 - \rho_v}\right) + \sigma^{-1} \left[\omega + \bar{v} \left(\frac{1}{1 - \rho_w} - \frac{1}{1 - \rho_v} \right) \right]}{qz} = \frac{-\left(\bar{i} - \frac{\sigma^{-1}\bar{v}}{1 - \rho_v}\right) + \sigma^{-1}\omega}{qz} \\ &= \frac{-\bar{i} + \sigma^{-1}\omega}{qz} > 0. \end{aligned} \quad (25)$$

To satisfy the lower bound, we assume that the inflation target is set at

$$\bar{\pi}^* = \frac{-\bar{i} + \sigma^{-1}\omega + \epsilon}{qz}, \quad (26)$$

where ϵ is a small positive number. The nominal interest rate with the worst draw from W followed by the worst draws from V thereafter, is given by

$$i_t = \left(\bar{i} - \frac{\sigma^{-1}\bar{v}}{1 - \rho_v} \right) (1 - \rho_\pi^{t-1}) + \sigma^{-1}\omega (\rho_\pi^{t-1} - \rho_w^{t-1}) + \sigma^{-1}\bar{v} \left(\frac{\rho_\pi^{t-1} - \rho_w^{t-1}}{1 - \rho_w} - \frac{\rho_\pi^{t-1} - \rho_v^{t-1}}{1 - \rho_v} \right) + \rho_\pi^{t-1}\epsilon > 0. \quad (27)$$

Setting $t = 1$ reveals that the interest rate in the period of the shock is given by ϵ , implying that $\epsilon > 0$ is sufficient to avoid the ZLB. In subsequent periods, the interest rate rises even if the economy receives the worst possible shock from the V distribution.²⁰

²⁰This assumption keeps us from having to model the effect on the expected inflation target if the

Therefore, the policy keeps the nominal interest rate positive. The actual interest rate evolves as

$$i_t = \bar{i} (1 - \rho_\pi^{t-1}) + \sigma^{-1} \left[\left(\omega + \frac{\sigma^{-1} \bar{v}}{1 - \rho_w} \right) (\rho_\pi^{t-1} - \rho_w^{t-1}) - \sum_{h=2}^t \rho_v^{t-h} \tilde{v}_h \right] + \rho_\pi^{t-1} \epsilon > 0. \quad (28)$$

4.1 Impulse Response

To compute impulse response functions with $\bar{\pi}^*$ and ρ_π chosen to exit the ZLB immediately, we consider the three alternative values for the extreme adverse shock together with the three different degrees of persistence. For each case, we set $\rho_\pi \geq \rho_w$ and large enough to assure $q > 0$. Then, $\bar{\pi}^*$ is set according to equation (26) with $\epsilon = 0.025$. We iterate over alternative values for ρ_π , and choose the loss-minimizing value of 0.92, for all shocks and persistences. Impulse response functions for the largest-shock, highest-persistence case are plotted in Figure 3.

The inflation target rises to 2.1% at a quarterly rate stimulating inflation to rise to 1.8% and the output gap to 2.9%, both at quarterly rates. The nominal interest rate rises slightly above zero. The increase in the inflation target together with its strong persistence reverses the recession into a sustained boom. This boom has costs, with welfare loss over six times as high as under discretion. And in the case of the least extreme and least persistent shock, welfare costs are over 200 times as large as those under discretion. Billi (2011) argues that the cost of slipping to the unfavorable stable equilibrium following an extreme adverse shock are so high that even the high welfare costs might be justifiable. Therefore, raising the inflation target and its persistence sufficiently to avoid the ZLB is a feasible policy, but one with high welfare costs.

economy could return to the "worst possible state," thereby returning the inflation target to $\bar{\pi}$.

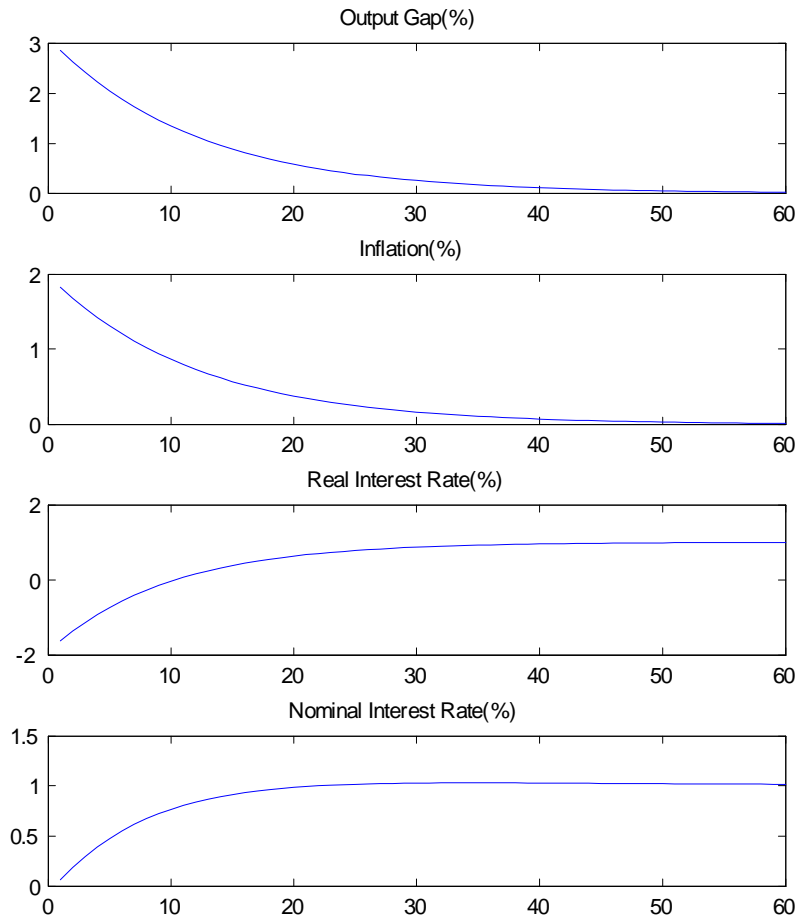


Figure 3: Avoid ZLB

5 Conclusions

The nominal interest rate cannot fall below zero. The economy enters a liquidity trap when a large adverse demand shock sends the nominal interest rate to zero as policy-makers try to stimulate the economy. We propose that the monetary authority adopt a time-varying inflation target at the ZLB with the same zeal with which they have adopted a fixed inflation target away from the ZLB.

In the event of a large adverse demand shock, which sends the economy into the liquidity trap under the conventional Taylor Rule, the monetary authority raises the inflation target and promises to retain that target until after the economy exits the ZLB.

After exiting the ZLB the inflation target rapidly returns to zero. The increase in the inflation target postpones the date on which the monetary authority will raise the interest rate, the exit date from the ZLB. Inflationary expectations rise, stimulating the economy. Inflation and the output gap are higher along an adjustment path characterized by small negative and positive deviations compared with the large and falling negative deviations under discretionary policy.

The reductions in loss under the proposed policy compared with discretion are substantial and are increasing in the magnitude and persistence of the shock. Additionally, the policy achieves almost all the gains of optimal commitment. The commitment to a time-varying inflation target with low persistence is a commitment to exit the ZLB with both a slightly positive inflation and output gap, and at a later date than required under discretion. The exit date is one to two quarters greater than under discretion with the date increasing in the severity of the shock creating the ZLB and its persistence. The inflation target upon exit is less than one half of one percent at an annual rate. The policy is implementable because it relies on the Taylor Rule with the Taylor Principle after exit from the ZLB. Consideration of uncertainty further raises the gains to policy with an optimal inflation target compared to a Taylor Rule with a zero inflation target.

The policy does require commitment because it is dynamically inconsistent. The monetary authority must maintain its commitment to the inflation target and the implied lower nominal interest rate beyond the date on which the natural rate of interest becomes positive. But, given the assumption embodied in the Taylor Principle, that the monetary authority can commit to "blow up the economy" (Cochrane 2011) in the event of a sunspot shock, this seems a small additional commitment.

Our policy produces an outcome similar to that of commitment, implying that it provides a way to implement a policy which closely approximates optimal policy. Implementability is important because it ensures local uniqueness of the equilibrium. Additionally, communicating commitment to an inflation target, which rapidly returns to zero after exit from the ZLB, seems relatively straight-forward compared with complicated paths for interest rates under optimal commitment.

The policy shares one attribute with a policy recently suggested by Cochrane (2013), where he proposes exiting the ZLB at a positive rate of inflation, but on the same date as implied by discretion. Our policy allows higher welfare by proposing a later exit date.

Our analysis does not support a policy to raise the inflation target and its persistence, and thereby inflationary expectations, sufficiently to immediately escape the liquidity trap, unless the expected loss generated by the possibility that the economy could transit

to the unfavorable equilibrium is very large (Billi 2011). Welfare costs to immediate exit from the ZLB are very large, even though they are substantially smaller than they would be with policies to create a permanent increase in the inflation target.

The financial crisis which began in 2007 created a growth industry for papers dealing with liquidity traps. Most of them developed unconventional monetary policies, many of which were implemented. Yet, in the United States and Japan, we remain in liquidity traps. Our paper is about conventional monetary policy under a Taylor Rule. There is no role for unconventional monetary policy in simple New Keynesian models. It is also noteworthy that our policy of promising an increase in short-run inflation has not been adopted by countries in liquidity traps. The US policy of keeping nominal interest rates at zero for a substantial period of time could be interpreted as an increase in the inflation target if it were not accompanied by concerns about "exit strategies" to keep inflation low once the economy recovers. Our analysis implies that some positive inflation can be part of an optimal policy response to a severely adverse demand shock which sends the economy to the ZLB.

6 Appendix: Solution under Optimal Policy with Commitment

This appendix follows Jung, Teranishi, and Watanabe (2005) with a few exceptions and can be omitted from the published version.

The relevant Lagrangian is,

$$\mathcal{L} = \frac{1}{2} E_1 \sum_{t=1}^{\infty} \beta^{t-1} \{ (\pi_t^2 + \lambda y_t^2) + 2\phi_{1,t} [y_{t+1} - y_t - \sigma (i_t - \pi_{t+1} - r_t^n)] + 2\phi_{2,t} [\pi_t - \beta\pi_{t+1} - \kappa y_t] \}$$

where, $\phi_{1,t}$ and $\phi_{2,t}$ are Lagrange multipliers. All values should be interpreted as expectations conditional on information in period 1. First order conditions with respect to $\pi_t, y_t, i_t, \phi_{1,t}$ and $\phi_{2,t}$ are,

$$\pi_t - \frac{\sigma}{\beta} \phi_{1,t-1} + \phi_{2,t} - \phi_{2,t-1} = 0 \quad (29)$$

$$\lambda y_t + \phi_{1,t} - \frac{1}{\beta} \phi_{1,t-1} - \kappa \phi_{2,t} = 0 \quad (30)$$

$$i_t \phi_{1,t} = 0 \quad (31)$$

$$i_t \geq 0 \quad (32)$$

$$\phi_{1,t} \geq 0 \quad (33)$$

$$y_{t+1} - y_t - \sigma(i_t - \pi_{t+1} - r_t^n) = 0 \quad (34)$$

$$\pi_t - \beta\pi_{t+1} - \kappa y_t = 0 \quad (35)$$

Following Jung et al (2005), we assume that, $i_t = 0$ and $\phi_{1,t} > 0$ for $t = 1, 2, \dots, T^c$ and $i_t > 0$ and $\phi_{1,t} = 0$ for $t = T^c + 1, T^c + 2, \dots$. We divide the entire time period in three parts to solve the dynamics under commitment.

6.1 Period $t = 1, 2, \dots, T^c$

We write equations (34) and (35) as,

$$Z_{t+1} = AZ_t - ar_t^n, \quad (36)$$

and equations (29) and (30) as,

$$\Phi_t = C\Phi_{t-1} - DZ_t, \quad (37)$$

where

$$Z_t = \begin{bmatrix} y_t \\ \pi_t \end{bmatrix}, \quad \Phi_t = \begin{bmatrix} \phi_{1,t} \\ \phi_{2,t} \end{bmatrix},$$

$$A = \begin{bmatrix} \left(1 + \frac{\kappa\sigma}{\beta}\right) & \left(-\frac{\sigma}{\beta}\right) \\ \left(-\frac{\kappa}{\beta}\right) & \left(\frac{1}{\beta}\right) \end{bmatrix}, \quad a = \begin{bmatrix} \sigma \\ 0 \end{bmatrix},$$

$$C = \begin{bmatrix} \left(\frac{1+\kappa\sigma}{\beta}\right) & \kappa \\ \left(\frac{\sigma}{\beta}\right) & 1 \end{bmatrix}, \quad D = \begin{bmatrix} \lambda & \kappa \\ 0 & 1 \end{bmatrix}.$$

A forward looking solution of equation (36) yields,

$$Z_t = \Gamma_t + A^{-(T^c-t+1)}Z_{T^c+1} \quad (38)$$

where,

$$\Gamma_t = \sum_{k=t}^{T^c} A^{-(k-t+1)} ar_k^n.$$

A forward solution of equation (37) yields

$$\begin{aligned}\Phi_t &= [C^{-1}DZ_{t+1} + C^{-2}DZ_{t+2} + C^{-3}DZ_{t+3} + \dots] \\ &= \sum_{k=t+1}^{\infty} C^{(t-k)}DZ_k\end{aligned}\tag{39}$$

Solving equation (39) as a function of the initial condition $\Phi_0 = 0$ yields,

$$\begin{aligned}\Phi_t &= C^t\Phi_0 - \sum_{k=1}^t C^{t-k}DZ_k \\ &\quad - \sum_{k=1}^t C^{(t-k)}DZ_k.\end{aligned}\tag{40}$$

Therefore from equation (40) we have,

$$\Phi_T = - \sum_{t=1}^T C^{(T-t)}DZ_t\tag{41}$$

6.2 Period $t = T^c + 1$

Equation (2), (29) and (30) at $t = T^c + 1$ with $\phi_{1,T^c+1} = 0$ yield,

$$-\kappa y_{T^c+1} + \pi_{T^c+1} - \beta\pi_{T^c+2} = 0\tag{42}$$

$$\pi_{T^c+1} + \phi_{2,T^c+1} = \frac{\sigma}{\beta}\phi_{1,T^c} + \phi_{2,T^c}\tag{43}$$

and,

$$\lambda y_{T^c+1} - \kappa\phi_{2,T^c+1} = \frac{1}{\beta}\phi_{1,T^c}\tag{44}$$

6.3 Period $t = T^c + 2, T^c + 3, \dots$

Equations (29) and (30) with $\phi_{1,T^c+1} = \phi_{1,T^c+2} = \dots = 0$ yield,

$$\begin{bmatrix} \pi_{t+1} \\ \phi_{2,t} \end{bmatrix} = M \begin{bmatrix} \pi_t \\ \phi_{2,t-1} \end{bmatrix}\tag{45}$$

where,

$$M = \begin{bmatrix} \left(\frac{1+\kappa^2}{\beta} \right) & \left(-\frac{\kappa^2}{\beta\lambda} \right) \\ -1 & 1 \end{bmatrix}$$

Matrix M has two characteristic roots, $\mu_1 > 1$ and $\mu_2 \in (0, 1)$. Imposing initial values to eliminate the unstable root yields the solution of equation (45) as

$$\pi_{t+1} = (1 - \mu_2) \phi_{2,T^c+1} \mu_2^{t-T^c-1}, \quad (46)$$

$$\phi_{2,t} = \phi_{2,T^c+1} \mu_2^{t-T^c-1}. \quad (47)$$

Equation (46) implies

$$\pi_{T^c+2} = (1 - \mu_2) \phi_{2,T^c+1} \quad (48)$$

Equations (42), (43), (44) along with equation (48) can be written as,

$$\begin{bmatrix} Z_{T^c+1} \\ \phi_{2,T^c+1} \end{bmatrix} = F^{-1} H \Phi_{T^c} \quad (49)$$

where

$$F = \begin{bmatrix} -\kappa & 1 & \{-\beta(1 - \mu_2)\} \\ 0 & 1 & 1 \\ \lambda & 0 & -\kappa \end{bmatrix} \quad H = \begin{bmatrix} 0 & 0 \\ \left(\frac{\sigma}{\beta} \right) & 1 \\ \left(\frac{1}{\beta} \right) & 0 \end{bmatrix}$$

Solution of equation (49) yields

$$Z_{T+1} = h \Theta \Phi_T \quad (50)$$

$$\phi_{2,T+1} = h \left\{ \frac{1}{\beta} \left(\frac{\lambda}{\sigma} - \kappa \right) \phi_{1,T} + \lambda \phi_{2,T} \right\}, \quad (51)$$

where

$$\Theta = \begin{bmatrix} \frac{1}{\beta} \left[\frac{\kappa}{\sigma} + (1 + \beta(1 - \mu_2)) \right] & \kappa \\ \frac{1}{\beta} \left[\frac{\kappa^2 + \lambda\beta(1 - \mu_2)}{\sigma} + \kappa \right] & \kappa^2 + \lambda\beta(1 - \mu_2) \end{bmatrix},$$

and

$$h = \frac{1}{\lambda(1 + \beta(1 - \mu_2)) + \kappa^2}.$$

6.4 Algorithm

Begin with equation (41). Substitute equation (17) for Z_t and equation (50) for Z_{T^c+1} to yield

$$\begin{aligned}
\Phi_{T^c} &= - \sum_{t=1}^{T^c} C^{(T^c-t)} D Z_t \\
&= - \sum_{t=1}^{T^c} C^{(T^c-t)} D (\Gamma_t + A^{-(T^c-t+1)} Z_{T^c+1}) \\
&= - \sum_{t=1}^{T^c} C^{(T^c-t)} D (\Gamma_t + A^{-(T^c-t+1)} h \Theta \Phi_{T^c}) \\
&= - \sum_{t=1}^{T^c} C^{(T^c-t)} D \Gamma_t - h \sum_{t=1}^{T^c} C^{(T^c-t)} D A^{-(T^c-t+1)} \Theta \Phi_{T^c} \\
\Phi_{T^c} &= - \left[I + h \sum_{t=1}^{T^c} C^{(T^c-t)} D A^{-(T^c-t+1)} \Theta \right]^{-1} \sum_{t=1}^{T^c} C^{(T^c-t)} D \Gamma_t \quad (52)
\end{aligned}$$

Note, equation (52) determines $t = T^c$ as the time period such that $\phi_{1,t} > 0$ for $t = 1, 2, \dots, T^c$, i.e., $t = T^c$ is the last period when $\phi_{1,t} > 0$. Once, Φ_{T^c} is known, equation (49) solves Z_{T^c+1} and ϕ_{2,T^c+1} . Using, Z_{T^c+1} , equation (17) solves Z_t for $t = 1, 2, \dots, T^c$. Equation (46) and (47) then solve π_t and $\phi_{2,t}$ respectively for $t = T^c + 2, T^c + 3, \dots$. Then from equation (30) we have y_t for $t = T^c + 2, T^c + 3, \dots$ as,

$$y_t = \frac{\kappa}{\lambda} \phi_{2,t} \quad (53)$$

and i_t for $t = T^c + 2, T^c + 3, \dots$ from equation (1) as,

$$i_t = r_t^n + \pi_{t+1} + \sigma^{-1} (y_{t+1} - y_t). \quad (54)$$

7 Appendix: Solution under Uncertainty after Exiting ZLB

We proceed with the solution by first solving for Z_t^x , the vector containing values for the output gap and inflation after exit. The vector evolves as

$$Z_t^x = A^{-1} E_t Z_{t+1}^x + A^{-1} a (r_t^n - i_t), \quad (55)$$

where

$$i_t = \max(0, r_t^n + \varphi Z_t^x),$$

with

$$\varphi = [\varphi_y, \varphi_\pi].$$

We solve equation (55) forward to yield

$$\begin{aligned} Z_t^x &= A^{-1}a(r_t^n - i_t) + A^{-1}E_t[A^{-1}a(r_{t+1}^n - i_{t+1}) + E_{t+1}Z_{t+2}^x] \\ &= \sum_{h=0}^{TN-t} A^{-h}E_t[A^{-1}a(r_{t+h}^n - i_{t+h})], \end{aligned}$$

$$\varphi = [\varphi_y, \varphi_\pi].$$

If the nominal interest rate is positive, we can substitute to yield

$$Z_t^x = -A^{-1}a\varphi Z_t^x + A^{-1}E_t Z_{t+1}^x.$$

Solving for Z_t^x yields

$$Z_t^x = [I + A^{-1}a\varphi]^{-1} A^{-1}E_t Z_{t+1}^x.$$

Substituting into the Taylor Rule for the nominal interest rate yields

$$i_t = \max(0, r_t^n + \varphi [I + A^{-1}a\varphi]^{-1} A^{-1}E_t Z_{t+1}^x), \quad (56)$$

and

$$Z_t^x = A^{-1}a(r_t^n - i_t) + A^{-1}E_t Z_{t+1}^x.$$

We solve the problem backwards. Begin with a time period for which the natural rate of interest is positive even in the worst state and call this period T^n .

$$E_{T^n} Z_{T^n+1}^x = 0$$

The monetary authority will always be able to set the nominal interest rate equal to the positive natural rate and set the output gap and inflation each to zero from T^n forward. From equation (56), the nominal interest rate in period T^n equals the maximum of zero and natural rate,

$$i_{T^n} = \max(0, r_{T^n}^n)$$

$$Z_{T^n}^x = A^{-1}a(r_{T^n}^n - i_{T^n}).$$

Back up one period to yield the expression for the vector Z^x one period earlier as

$$Z_{T^{n-1}}^x = A^{-1}a (r_{T^{n-1}}^n - i_{T^{n-1}}) + A^{-1}E_{T^{n-1}}Z_{T^n}^x$$

with the nominal interest rate given by

$$i_{T^{n-1}} = \max(0, r_{T^{n-1}}^n + \varphi [I + A^{-1}a\varphi]^{-1} A^{-1}E_{T^{n-1}}Z_{T^n}^x).$$

We continue backwards until we reach the period at which the economy exits from the ZLB.

In the period in which the economy exits the ZLB, the nominal interest rate is adjusted downwards by the inflation target such that

$$i_{T^x} = r_{T^x}^n - z\pi^* + \varphi Z_{T^x}^x$$

with

$$Z_{T^x}^x = A^{-1}a (r_{T^x}^n - i_{T^x}) + A^{-1}E_{T^x}Z_{T^{x+1}}^x.$$

Substituting the interest rate yields

$$Z_{T^x}^x = A^{-1}a (z\pi^* - \varphi Z_{T^x}^x) + A^{-1}E_{T^x}Z_{T^{x+1}}^x.$$

Solving for $Z_{T^x}^x$ yields

$$\begin{aligned} Z_{T^x}^x &= [I + A^{-1}a\varphi]^{-1} [A^{-1}az\pi^* + A^{-1}E_{T^x}Z_{T^{x+1}}^x] \\ &= b\pi^* + BE_{T^x}Z_{T^{x+1}}^x \end{aligned}$$

where

$$b = [I + A^{-1}a\varphi]^{-1} A^{-1}az,$$

T^n a 2 x 1 vector and

$$B = [I + A^{-1}a\varphi]^{-1} A^{-1},$$

a 2 x 2 matrix. Therefore, the nominal interest rate in the exit period (T^x) is given by

$$\begin{aligned} i_{T^x} &= r_{T^x}^n - z\pi^* + \varphi Z_{T^x}^x \\ &= r_{T^x}^n - z\pi^* + \varphi (b\pi^* + BE_{T^x}Z_{T^{x+1}}^x) \\ &= r_{T^x}^n + qz\pi^* + \varphi BE_{T^x}Z_{T^{x+1}}^x. \end{aligned}$$

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