ITQs, Firm Dynamics and Wealth Distribution: Does full tradability increase inequality?

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Abstract

Concerns over the re-distributive effects of ITQ's lead to restrictions on their tradability. We consider a general equilibrium model with firm dynamics. In contrast with the standard framework, the distribution of firms is not exogenous, but is instead determined endogenously by entry/exit decisions made by firms. We show that the stationary wealth distribution depends on whether the ITQs are fully tradable or not. We calibrate our model to match the observed increase in revenue inequality in the Northeast Multispecies (ground-fish) U.S. Fishery. We show that although observed revenue inequality increases, wealth inequality is reduced by 40%.

Keywords: ITQ, wealth distribution, firm dynamics, inequality, permit markets

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1 Introduction

A crucial question in environmental and resource economics is why tradable output permits are not more widely adopted as a solution for environmental problems. The consensus appears to be that equity concerns provide an important reason for the resistance to the wider use of individual transferable quotas. In particular, the literature argues that the efficiency gains associated with tradable quotas will not be captured by small firms.

In fisheries, for instance, transferable and divisible catch quotas are usually referred to as individual transferable quotas (or ITQs). If ITQs are also permanent they constitute a complete property righ[1] that could be fully tradable. In this industry the idea is widespread that full tradability of ITQs will gradually squeeze out small vessel owners[2] and that efficiency gains will be captured only by larger producers[3]. These distributional concerns lead to restrictions in tradability (implying incompleteness of the right) of ITQ's in many fisheries.

A key economic question is whether these concerns are properly grounded in economic theory. To answer this question formally requires the characterisation of wealth distribution in a model of a natural resource industry that is managed with output permits and, in particular, the analysis of the consequences of different degrees of tradability of permits. That is what this paper sets out to do.

We consider a fishery regulated with ITQs. In this type of industry, different levels of tradability of ITQs arise for many reasons, e.g. depending on whether the vessel or its owner is the ultimate holder of the property right. If the property right is associated with ownership of an active qualifying vessel, the permit can be leased in each period but it is not possible to exit the fishery and keep the right. If instead, the property right is assigned to the owner

[1] See, for instance Arnason (2002) .

[2] There is an extensive literature on the relationship between ITQs and the consolidation of the fisheries. See, for instance, Grafton et al. (2000), Fox et al. (2003), Kompas and Nu (2005) among others.

[3] See, for example Libecap (2007) or Olson (2011). Distributional implications, associated with interest group and rent-seeking behavior are studied by Joskow and Schmalensee (1998).

of the vessel and divorced from the ownership of an active vessel, it could actually be traded as a separate asset and constitute a complete property right.

As Brandt (2005) reports, the change in the holder of the property right was a critical change that took palce in the Atlantic Surf Clam and Ocean Quahog Fishery (and many other US fisheries). In the initial implementation of the regulation the claim to ITQs was tied to each currently active vessel. However, Amendment Eight to the *"Fishery management plan for the Atlantic Surf Clam and Ocean Quahog Fishery"* allowed a firm to retain ownership of the ITQ even if the vessel was sold[4]. Obviously, in this last case, the property right is assigned to the owner of the vessel and divorced from the ownership of an active vessel, and could actually be traded as a separate asset.

We consider a model of firm dynamics that builds on Hopenhayn and Rogerson (1993) and Da-Rocha et al. (2014a). Firms are heterogeneous with respect to their production opportunities, but in contrast with the standard framework their distribution is not exogenous but rather determined endogenously by entry/exit decisions made by firms. That is, the definition of stationary equilibrium in a general equilibrium model with heterogeneous agents requires an invariant distribution of firms to be found which is determined by agents' optimal policies, and also determines the agents' optimal choices. We use the Kolmogorov-Fokker-Planck equation to find that distribution[5] .

We use the model to investigate the impact of changing the transferability of output permits on wealth distributions. The change in the transferability of the permits will affect the entry/exit decisions and also the wealth distribution. We calibrate our model to match the observed increase in revenue inequality in the Northeast Multispecies (groundfish) U.S. Fishery after the introduction of ITQs reported by Kitts et al. (2011).

[4]To that end a confirmation of permit history (CPH) was required, showing that a person who does not currently own a fishing vessel once owned a qualifying vessel that had sunk, been destroyed, or been transferred to another owner (this shows that the fishing and permit history of such vessels has been retained lawfully by the applicant).

[5]The use of Kolmogorov-Fokker-Planck equation to characterize the distribution of firms was suggested by Dixit and Pindyck (1994).

Our simulations show that, taking the innovation rate as given, total transferability of ITQs raises the Value function and always decreases inequality for active vessels. Transferability decreases inequality because allowing vessel owners to sell ITQs is equivalent to a lump sum transfer delivered to all initial ITQ holders in the industry. This is the first direct effect of tradability of ITQs on wealth distribution. However, more complete and transferable property rights could also spur innovation rates and we show that increases in innovation rates increase the wealth inequality of active firms.[6] This would be a second (indirect) effect of increasing tradability of ITQs on wealth distribution. Which of the two effects dominates is an empirical fact. In our calibration, although the innovation rate increased by 6%, wealth inequality decreased by 40%.

Moreover, when the property right is assigned to the owner and divorced from the ownership of an active vessel, a new class of fishermen appears that no longer actually fishes but participates in the fishery only by leasing their ITQs. Therefore transferability of ITQs squeezes out small *vessels* but, by leasing the total quota, efficiency gains can also be captured by small *owners* through increases in the lease price. Therefore, no segment of the (former) fishermen is adversely affected by the tradability of ITQs.

Our paper is closely related to Weninger and Just (2002) in the sense that they both use an option model in continuous time. In addition, we extend their model to a world where quotas are a continuous variable and firms can lease part of their quota, and extend the analysis to a General Equilibrium environment for computing endogenous firm distribution, prices and wealth.

Our paper is also related to the literature on the distributional implications of alternative market-based control mechanisms. The "mechanism" that generates redistribution in our model is supported by the empirical findings of Brandt (2007). Furthermore, we are also able to show that wealth distribution among fishermen would actually improve with tradability.

[6] We are indebted to an anonymous referee for this suggestion.

Because it computes how differences in property rights affect market outcomes, the paper is also related to Grainger and Costello (2014) and Grainger and Costello (2015). A key difference between these papers and ours is that we compute the full wealth distribution, which is an endogenous object in our model.

Finally, our paper is also related to the growing literature on general equilibrium models with heterogeneous firms that uses the Kolmogorov-Fokker-Planck equation to characterise the equilibrium invariant distribution of firms as in Luttmer (2007), Da-Rocha and Pujolas (2011a), Luttmer (2011), Luttmer (2012), Impullitti et al. (2013) , Gourio and Roys (2014), Da-Rocha et al. (2014a), Da-Rocha et al. (2014b) and Da-Rocha et al. (2015), among others.[7]

The rest of the paper is organised as follows: Section 2 d describes the economic environment. In Section 3 we characterise the equilibrium of this model and solve the closed form for the stationary distribution of firms' wealth. In section 4 calibrates the model with data from the US Northeast Multispecies Fishery, and finally Section 5 assesses the impacts of introducing free transferability into wealth distribution.

2 The Economic Environment

There is a natural resource industry that is managed with tradable output permits q, where firms must own permits to exploit the resource legally. There are four markets in the economy: final goods, labour, an output permit lease market where trade takes place between incumbent firms, and a permit market where trade takes place between entrants and exiting firms. Taking output price as the numeraire, we denote as w, r_q and p_q,the labour, quota lease and quota ownership prices, respectively.

[7] The Kolmogorov-Fokker-Planck equation is widely used to describe population dynamics in ecology, biology, and finance, among other sciences. It has been used in economics by Merton (1975) in neoclassical growth models, by Dixit and Pindyck (1984) in a renewable resources model and by Da-Rocha and Pujolas (2011b) in fisheries. Two good surveys are Gabaix (2009) and Luttmer (2010).

We assume that all firms are identical before entry takes place. Potential entrants decide whether to enter knowing that they face a distribution $g(c)$ on potential draws, where c is a firm-specific shock to production opportunities[8]. The entry problem produces two decision rules: one for the optimal choice of the number of quotas, and,the other for the optimal entry decision. That is, firms choose how many quotas to hold and at the same time decide whether to enter the fishery. After entry, entrants become incumbents.

Although we are interested in the stationary competitive equilibrium distribution of firms, note that individual firms will change over time. Some of them expand production, hiring staff and borrowing quotas, others contract production, firing staff and leasing out quotas, and others exit the industry and sell their quotas. Therefore, the incumbent firms' decision problem produces two types of decision rule. On one hand, there are continuous decision rules for the optimal choice of output $y(c)$, labour $l(c)$ and the number of quotas leased $y(c)|q$, and on the other hand there is a discrete decision rule $d(c)$ for the optimal stay/exit decision at the beginning of the next period.

We also assume that there is a fixed operating cost of c_f. If a firm wants to remain active then it must pay the fixed cost (and, conversely, if a firm chooses to exit, then it does not pay the fixed cost). The decision to exit depends on this period's employment $l(c)$, output $y(c)$, and permit leasing decisions $q^d(c) - q$. Conditional on this period's choices ($l(c)$, $y(c)$ and $y(c) - q$), the firm must evaluate the expected value of remaining in the industry, and must compare this with the present discounted value of profits associated with exiting the industry $p_q q$.

A stationary equilibrium in a model with heterogeneous firms requires an invariant distribution of producing firms over production opportunities c. Given an initial guess for the exit decision rule c_*, potential entrants and incumbent firms can calculate the value of entry and market prices and solve their individual problem. Note that the distribution of firms

[8]This is a standard assumption in models with firm dynamics. See Hopenhayn (1992), Hopenhayn and Rogerson (1993) or Restuccia and Rogerson (2008).

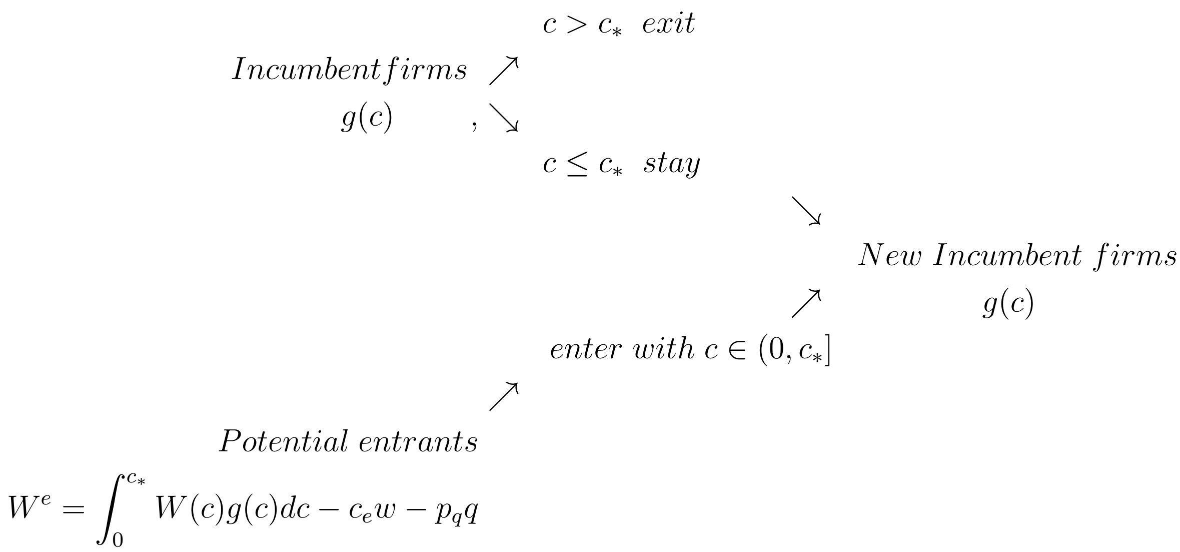


Figure 1: entrants and incumbents decision

for characteristic c depends on the support $[0, c*)$. Therefore a (stationary) competitive equilibrium is a fixed point in the distribution $g(c) \in [0, c_*)$ of active firms over production opportunities. This sequence of decisions by entrants and incumbent firms is graphically explained in Figure 1.

3 Equilibrium

Below, we consider a standard general equilibrium model with heterogeneous firms. First we solve the model when ITQs are permanent and fully tradable. That is, quotas can be leased for the current period but also permanently transferred (this could be interpreted as the ITQ being dissociated from the ownership of an active vessel without losing its privileges). Later, we analyse the case in which ITQs can be leased for a given period but are not permanent (this could be interpreted as the ITQ having to be associated with an active vessel to keep its privileges, so that exiting the market implies the loss of privileges).

3.1 Incumbent firms' problem

Firms maximise profits subject to their available technology, $y = \sqrt{\frac{l}{c}}$. Note that we extend the model in Weninger and Just (2002) to a world in which quotas are a continuous variable and firms can lease part of their quota[9]. Moreover, our technology is in accordance with the fifty-fifty rule, i.e. 50% of net revenues are accounted for by payments to crew members.

Intra-temporal profits are given by

$$\begin{aligned} \Pi &= \max_{y,l,q^d} \; y - r_q q^d - wl + r_q(q^d - q) - c_f, \\ & \quad s.t. \begin{cases} y = \sqrt{\frac{l}{c}}, \\ q^d \geq y. \end{cases} \end{aligned}$$

From the f.o.c. we have, $l(w, r_q, c) = \left(\frac{1 - r_q}{2w}\right)^2 c^{-1}$, $y(w, r_q, c) = \left(\frac{1 - r_q}{2w}\right) c^{-1}$, and profits are given by

$$\Pi(w, r_q, c, q) = \frac{(1 - r_q)^2 c^{-1}}{4w} + r_q q - c_f = \pi(w, r_q) c^{-1} + r_q q - c_f.$$

Now we can evaluate the inter-temporal decision making of firms. As in Weninger and Just (2002), we assume that the shock follows a geometric Brownian motion stochastic process with a positive expected growth rate, μ, i.e.

$$\frac{dc}{c} = \mu dt + \sigma_c dz,$$

where σ_c is the per-unit time volatility, and dz is the random increment to a Weiner process. Each firm has to weigh up its current and future potential profits against the benefits of

[9] That is, we are assuming that c characterizes the firm marginal cost function, $MC = cwy^2$. When firms are restricted to produce a single unit of output in each period, and w is given, then c is equal to the unit cost.

selling its quota. Formally

$$\begin{aligned} W(c) &= \max_{d\in\{stay,exit\}} \left\{\pi(w,r_q)c^{-1} + (r_q q - c_f) + (1+\rho dt)^{-1} EW(c+dc), \quad p_q q\right\} \\ & s.t. \ \frac{dc}{c} = \mu dt + \sigma dz, \end{aligned}$$

where $(1+\rho dt)^{-1}$ is the discount factor, $EW(c+dc)$ are the expected future profits, and p_q q are the benefits of selling and exiting the market. Note that the value matching and the smooth pasting conditions at the switching point c_* where firms choose to exit are equal to $W(c_*) = p_q q$ and $W'(c_*) = 0$, respectively.

It is important to notice that in the competitive equilibrium all firms, revgardless of their cost or productivity, sell their quotas at the same competitive price. That is, the price of quotas is independent of idiosyncratic characteristics. In Proposition 1 we characterise the value function and the switching point.

Proposition 1. *Assume that $p_q > 0$, so c_* and $W(c)$ are given by*

$$c_* = \frac{(1+\beta)}{\beta}\frac{\rho}{(\rho+\mu-\sigma^2)}\left(\frac{\pi(w,r_q)}{\rho p_q q + c_f - r_q q}\right),$$

and

$$W(c) = \left(p_q q - \frac{(r_q q - c_f)}{\rho}\right)\frac{\beta}{1+\beta}\left(\frac{c}{c_*}\right)^{\beta} + \frac{\pi(w,r_q)c^{-1}}{\rho+\mu-\sigma^2} - \left(\frac{c_f - r_q q}{\rho}\right).$$

where $\beta = \frac{1}{2} + \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2\rho}{\sigma^2}} > 1$ is the root of the standard quadratic equation associated with the geometric Brownian motion.

Proof See Appendix A.1.

3.2 Entrants problem

If the value function $W(c)$ is known, the gross value of entry W^e can be computed by using $g(c)$. That is

$$W^e = \int_0^{c_*} W(c)g(c)dc - wc_e - p_q q.$$

Potential entrants choose the number of quotas by solving

$$q^* \in arg \max_q \int_0^{c_*} W(c)g(c)dc - wc_e - p_q q.$$

The result below provides a non arbitrage condition relating the price of permanently selling an ITQ to the leasing price. It implies equivalence between selling the permit and leasing it for an infinite number of periods.

Proposition 2. *In an equilibrium with exit, the no-arbitrage condition* $p_q = \frac{r_q}{\rho}$ *holds.*

Proof See Appendix A.2.

Finally, notice that in an equilibrium with entry W^e must be zero, since otherwise additional firms would enter.

3.3 Invariant distribution of firms

In this model the distribution of firms is determined endogenously by entry/exit decisions made by firms, so the definition of the stationary equilibrium requires that an invariant distribution of firms be found. To find this distribution. we start by rewriting the model in logarithms, i.e. $x = \log(c/c_*)$, and apply the the Fokker-Planck-Kolmogorov equation of the stochastic process $dx = \hat{\mu}dt + \sigma dz$

$$\frac{\partial M(x,t)}{\partial t} = -\hat{\mu}\frac{\partial M(x,t)}{\partial x} + \frac{\sigma^2}{2}\frac{\partial^2 M(x,t)}{\partial x^2} + \varepsilon M(x,t).$$

where $M(x,t)$ is the mass of firms over the variable x, and ε are the new firms that entry with productivity x at time t and $\hat{\mu} = \mu - \frac{\sigma^2}{2}$. The partial differential equation is supplemented by the boundary conditions

$$\begin{aligned} M(0,t) &= 0. \\ \lim_{x\to-\infty} M(x,t) &= 0, \\ \lim_{x\to-\infty} \frac{\partial M(x,t)}{\partial x} &= 0, \end{aligned}$$

The first two boundary conditions guarantee that the mass of firms at the boundary is zero, and the last boundary condition guarantees no exit of firms at the upper limit of the distribution. We are interested in the steady state distribution for the mass of firms. Therefore, $M(x,t) = M(t)f(x)$ implies

$$\frac{M'(t)}{M(t)}f(x) = \eta f(x) = -\hat{\mu}f'(x) + \frac{\sigma^2}{2}f''(x) + \varepsilon f(x),$$

with boundary conditions:

$$\begin{aligned} \lim_{x\to-\infty} f(x) &= 0, \\ \lim_{x\to-\infty} f'(x) &= 0, \\ f(0) &= 0; \end{aligned}$$

and the additional requirement that $f(x)$ must be a probability distribution function leads to the conditions

$$\begin{aligned} f(x) &\geq 0, & (1) \\ \int_{+\infty}^{0} f(x)dx &= 1. & (2) \end{aligned}$$

Note that $\frac{M'(t)}{M(t)}$ is the separation rate denoted by η, i.e. $M(t) = e^{\eta t}M(0)$. We restrict the separation rate η. By integrating the Kolmogorov-Fokker-Planck equation we find:

$$\eta \int_0^\infty f(x)dx = \left(-\hat{\mu}f(x) + \frac{\sigma^2}{2}f'(x)\right)\Big|_{-\infty}^{x=0} + \varepsilon \int_0^\infty f(x)dx \frac{\sigma^2}{2}f'(0) + \varepsilon = 0.$$

The expression for η has a very intuitive interpretation. It states that the growth rate of the mass of firms η is equal to the entry rate ε minus the rate at which firms decide to sell the ITQ and leave the distribution $-\frac{\sigma^2}{2}f_2'(0)$. If this number is zero, the number of firms does not grow over time. Therefore, we can rewrite the Kolmogorov-Fokker-Planck equation as

$$-\varepsilon f(x) = -\hat{\mu}f'(x) + \frac{\sigma^2}{2}f_2''(x).$$

We solve this equation with a guess and verify method. Consider the steady state distribution $f(x) = -xe^{\xi x}$. Then, $f'(x) = -e^{\xi x} + \xi f(x)$ and $f''(x) = -2\xi e^{\xi x} + \xi^2 f(x)$. Therefore

$$-\varepsilon f(x) = -\hat{\mu}\left[-e^{\xi x} + \xi f(x)\right] + \frac{\sigma^2}{2}\left[-2\xi e^{\xi x} + \xi^2 f(x)\right].$$

Proposition 3 summarizes our findings.

Proposition 3. *The invariant distribution of firms is equal to* $f(x) = -xe^{\xi x}$, *where* $\xi = \frac{\hat{\mu}}{\sigma^2}$ *and the entry rate* $\varepsilon = \frac{\hat{\mu}^2}{2\sigma^2}$.

Finally, given the distribution over logs $f(x)$, we can compute the stationary cost distribution as $g(c) = -\frac{(1+\xi)^2}{c_*}\log(c/c_*)\left(\frac{c}{c_*}\right)^{\xi}$.

3.4 Feasibility conditions

To close the model we need to define feasibility conditions. Feasibility in the model requires resource balance in the output market, the leasing quota market and the labour market. By normalising the Total Allowable Catch (TAC) in 1 (given that ITQs are shares of the total quota allowed in each period), we have that feasibility in the output market is given by

$$Mq = 1.$$

Note that we are assuming that the TAC is determined exogenously in order to maximise a biological reference point. Therefore, changes in productivity do not affect the aggregate quota. Feasibility in the leasing ITQ market implies that the aggregate excess demand function $z_q = \int_0^{c_*} y(c)g(c)dc - q$ is equal to zero. That is

$$q = \int_0^{c_*} y(c)g(c)dc$$

Finally, equilibrium in the labor market implies that

$$1 - \varepsilon M c_e = M\int_0^{c_*} l(c)g(c)dc,$$

where we normalise the total labour supply to 1, and $\varepsilon M c_e$, the entry cost multiplied by the mass of entrants, is the labour force allocated to produce the entry cost.

3.5 Definition of equilibrium

A stationary equilibrium is an invariant cost distribution $g(c)$, a mass of firms M, a number of permits q, permit prices p_q and r_q, wage rate w, incumbents and entrants value functions $W(c)$, W^e, and individual decision rules $l(c)$, $y(c)$, $\pi(c)$ and c_*, such that:

i) (Firm optimisation) Given prices (r_q, p_q, w), the functions *entry*, and $W(c)$ and W^e solve incumbent and entrant problems and $l(c)$, $y(c)$, $\pi(c)$, and c_* are optimal policy functions.

ii) (Free-entry and optimal quota) Potential entrants choose quotas q and make zero profits $W^e = 0$.

iii) (non-arbitrage condition) $p_q = \dfrac{r_q}{\rho}$.

iv) (Market clearing-feasibility) Given individual decision rules, prices (r_q, p_q, w) solve

$$\begin{aligned} 1 - \varepsilon M c_e &= M \int_0^{c_*} l(c) g(c) dc, \\ q &= \int_0^{c_*} y(c) g(c) dc, \\ Mq &= 1. \end{aligned}$$

v) (Invariant distribution) $g(c)$ satisfies the Kolmogorov-Fokker-Planck equation.

Note that the definition of equilibrium is similar to the standard definition in Hopenhayn and Rogerson (1993) or Restuccia and Rogerson (2008). The main difference is that Hopenhayn and Rogerson (1993) and Restuccia and Rogerson (2008) consider a discrete time model. However, obvious equivalences appear. In fact, assuming Brownian motion is equivalent to assuming an AR(1) stochastic process, and the Kolmogorov-Fokker-Planck equation is the continuous time version of the (endogenous) discrete Markovian chain [10].

[10] The Kolmogorov-Fokker-Planck equation is obtained by applying a simple Markov principle to the transition density function of the continuous stochastic process. Kolmogorov in the 1930's and Feller at

3.6 Equilibrium when quotas are not fully tradable

In some fisheries distribution and other concerns have led to limits being placed on the trading of quotas. For example, Arnason (2002) reports that in most Canadian quota-managed fisheries ITQs are only transferable within the year[11] and are therefore not permanent. In other fisheries, ITQs are distributed on an active vessel basis, and not directly to vessel owners[12]. In this section we analyse the related case in which quotas can be leased only for a set time. That is, we assume that if a vessel leaves the market it loses the property right. In consequence, quotas are not permanent (and are not a fully tradable asset).

To model this situation we assume that quotas are allocated free to entrants at the beginning of the period[13]. Quotas can be used or leased. However, if the firm exits the market, it has to give the quota back to the regulator. That is, the only way of keeping the right to hold a quota is by holding an active vessel.

If ITQs are originally grandfathered and if they have to be given back to the regulator for free when firms exit the industry, this implies that in our model $p_q = 0$ and therefore the lease price will not be obtained by solving the non-arbitrage condition (which would disappear from our equilibrium condition). Thus firms will solve the following optimisation problem.

$$\begin{aligned} W(c) &= \max_{d \in \{stay, exit\}} \left\{ \pi(w, r_q)c^{-1} + (r_q q - c_f) + (1+\rho dt)^{-1} EW(c+dc), \quad 0 \right\} \\ & \quad s.t. \ \frac{dc}{c} = \mu dt + \sigma dz. \end{aligned}$$

the end of the 40's characterised the Kolmogorov-Fokker-Planck equation in such a way. For a formal characterisation of the forward Kolmogorov equation and its relationship with the Markov stochastic process see Mangel (2006).

[11] That is, from a legal standpoint an individual fishing quota is simply a fishing licence with a certain tuple of stipulations.

[12] In Iceland, all quotas must be associated with an active vessel. This means that only those individuals or firms that own qualifying active vessels can hold quotas. This restricts the set of possible quota holders and the degree of tradability of the quotas.

[13] This type of grandfathering is widely used when regulation based on output markets is used, and is defended, for instance by Libecap (2007) and Anderson et al. (2011)

Therefore, there is a cost value c_*, for which owners will find it optimal to exit the industry.

Corollary 1. *When $p_q = 0$, c_* and $W(c)$ are given by*

$$c_{p_q=0}* = \frac{(1+\beta)}{\beta}\frac{\rho}{(\rho+\mu-\sigma^2)}\left(\frac{\pi(w,r_q)}{c_f - r_q q}\right),$$

and

$$W(c) = \frac{(c_f - r_q q)}{\rho}\frac{1}{1+\beta}\left(\frac{c}{c_*}\right)^{\beta} + \frac{\pi(w,r_q)c^{-1}}{\rho+\mu-\sigma^2} - \left(\frac{c_f - r_q q}{\rho}\right).$$

Note that, as when ITQs are fully tradable, incumbent firms can choose to perpetually lease their ITQs without doing any fishing (quasi-exiting the market). However, in contrast with the previous case, if a firm wants to remain activeI on the ITQ market it must pay the fixed cost associated with keeping a vessel active (given that ITQs are associated with an active vessel). Notice that in this case there could be equilibria without entry. This is the equilibrium with $qr_q - c_f \geq 0$, so that producing zero, paying the fixed cost and leasing all permits is at least as profitable as exiting the market (implying that $c_{p_q=0}*$ goes to ∞). That is, all incumbent firms would stay in the industry and some could lease their whole quota without exiting. If no firm exits, no firm will enter, as no permits would be supplied in the market.

4 Calibration

We calibrate the model in order to match the stylised facts observed in the New England groundfish fishery. On 1 May 2010 a new management programme consisting of a hard quota of annual catch limits was implemented for the said fishery[14]. This programme was designed to comply with the new requirements for catches and stock rebuilding laid down by the Magnuson-Stevens Fishery Conservation and Management Reauthorization Act of

[14]Amendment 16 to the Northeast Multispecies Fishery Management Plan.

Table 1: Model Parameter Calibration

Discount factor, ($\rho = 0.05$)					
Parameter			Target		
Stochastic process with permanent permit output markets					
μ	drift	0.0568	Lorenz_{2010}		
σ^2	volatility	0.0247	Entry rate	ε	1/25
Stochastic process without permanent permit output markets $p_q = 0$					
$\mu_{p_q=0}$	drift	0.0630	Lorenz_{2007}		
Costs					
c_f	fix cost	0.3565	$\Delta\text{fleet}_{2010}$	$1 - M$	0.3164
c_e	entry cost	3.0108	margin with entry	$(p - r_q)$	0.6273

2006. The 2010 Final Report implied that, after the introduction of ITQs, there was an improvement in economic performance, as indicated by gross nominal revenue per unit effort, and by vessel owners' share of nominal net revenue per day, see Kitts et al. (2011).

There was also a decrease in the number of active vessels and effort became more concentrated. There were 31% fewer active vessels in 2010 than in 2007 (a reduction from 658 to 450). There was also an increase in concentration of groundfish gross nominal revenues among top earning vessels and vessel affiliations, as gross nominal revenues became consolidated among fewer individual vessels and fewer vessel affiliations. Kitts et al. (2011) report that the Gini coefficient of total revenue increased from 0.66 in 2007 to 0.76 in 2010 for the active vessels.[15]

We need to calibrate six parameters: $\mu_{p_q=0}$, μ, σ^2, c_f, c_e and ρ. The discount factor is taken from the literature ($\rho = 0.05$). Note that CDF of revenues and vessels are in functions of the ξ_t parameter for $t = 2007, 2010$. That is [16]

$$F_t(y) = \int_{y_*}^{y} f(y)dy = 1 - \left(\frac{y_*}{y}\right)^{\xi_t+1} \left[1 - (1+\xi_t)\ln\left(\frac{y_*}{y}\right)\right],$$

[15] See Kitts et al. (2011) , Table 36, page 63 and Figure 21 page 96.
[16] CDF of revenue are characterized in Appendix A.3.

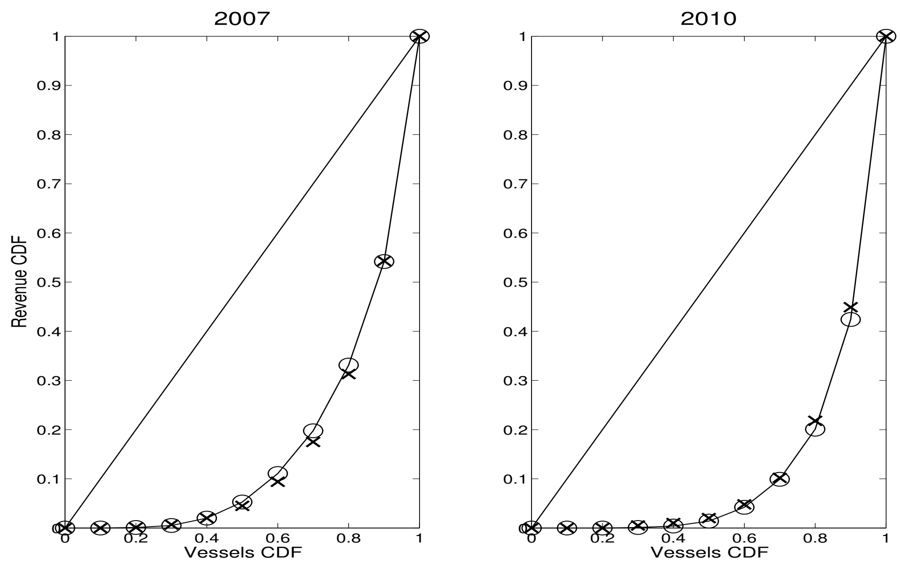


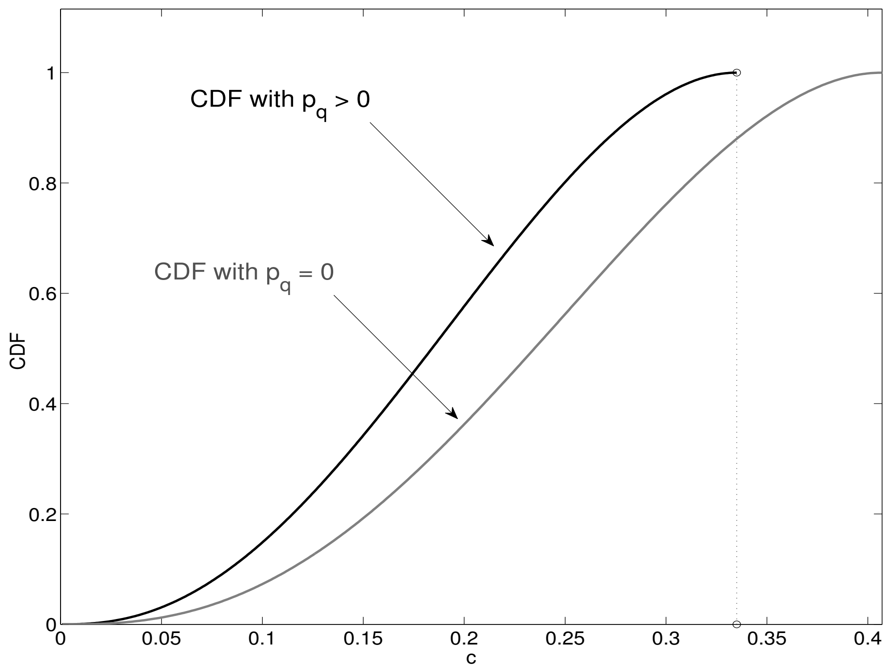
Figure 2: Lorenz Curve calibration. The Figure shows the calibration of the changes observed in revenue distribution. The circles represent the observed data and the crosses the prediction of the model. The curve shows what percentage (y%) of the total revenues the bottom x% of vessels have. The percentage of vessels is plotted on the x-axis, the percentage of revenues on the y-axis. As is well known, the area between the Lorenz curve and the (equal income) straight line is the Gini coefficient. The higher the coefficient, the more unequal the distribution is.

and vessels

$$F_t(c) = \int_{c_*}^{c} f(c)dc = 1 - \left(\frac{c_*}{c}\right)^{\xi_t - 1} \left[(1 - \xi_t)\ \ln\left(\frac{c_*}{c}\right) + 1\right].$$

We calibrate the model to match the Lorenz curve in 2007 and 2010, and the reduction of the industry size observed after the introduction of ITQs. [17] Table 1 summarises targets and parameters (see Appendix A.4) and Table 2 summarizes how the model matches the observed changes in the revenue distribution. Figure 2 shows the calibration of Lorenz curves of nominal revenues from groundfish among vessels for 2007 and 2010. The circles represent the observed data and the crosses the prediction of the model. As it is well known, the area between the Lorenz curve and the (equal income) straight line is the Gini coefficient.

[17] Appendix A.6 shows with a simple example the relationship between Lorenz curves and CDF's.



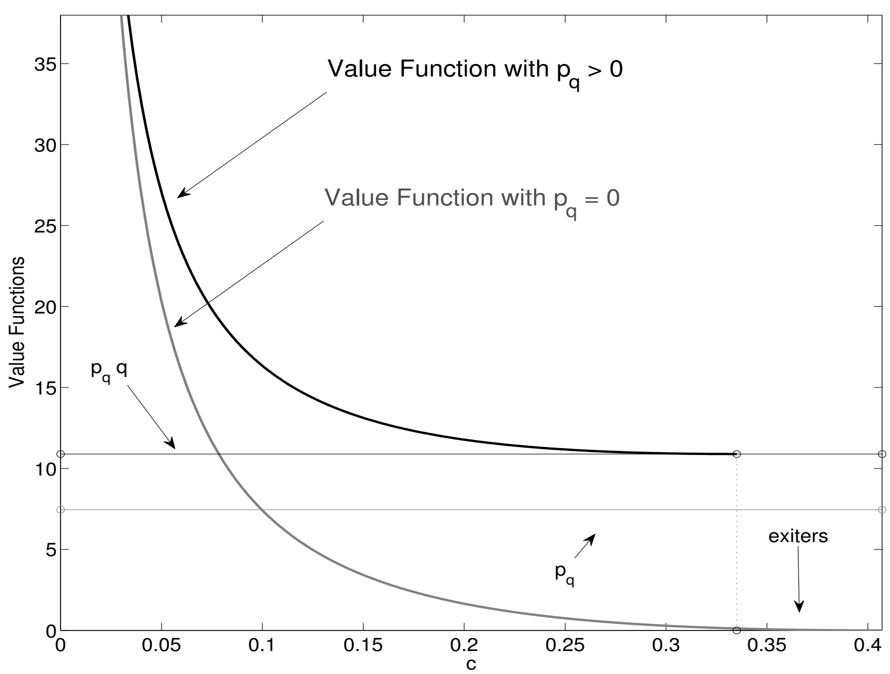


Figure 3: Changes in cost distribution and value functions under both market structures. A decrease in maximum cost is shown (a fleet squeeze) that generates an increase in productivity (the efficiency gain). Moreover, by allowing total transferability, the exit constraint t $W(c) \geq p_q q$, raises the Value function.

Table 2: Revenue Distribution (Lorenz curves)

	bottom 10	20	30	40	50	60	80	90	top 10
2007									
Data	0.00	0.10	0.30	1.00	2.80	5.70	10.20	22.30	57.60
Model	0.00	0.50	0.50	1.00	2.80	5.40	11.60	23.10	55.10
2010									
	bottom 10	20	30	40	50	60	80	90	top 10
Data	0.10	0.40	1.50	3.29	5.79	8.68	13.37	21.06	45.81
Model	0.10	0.60	1.30	2.50	4.90	8.10	13.80	23.00	45.70

Table 3: Model

		Transferability	
		partial ($p_q = 0$)	total ($p_q > 0$)
		Efficiency Gains	
$\pi(w, r_q)$	profit (per unit of productivity)	0.1190	0.0929
$\overline{\pi}(w, r_q)$	average profits	0.6344	0.6709
		Fleet squeeze	
c_*	maximum cost	0.4071	0.3350
M	industry size	1.0000	0.6839
q	effective quota utilized by active firms	1.0000	1.4622
		Endogenous prices	
w	effort cost	0.8273	1.0599
p_q	permanent sell price	—	7.4535

Table 3 shows that the empirical results from the Groundfish Final Report 2010 can be replicated by the theoretical model. For example, it shows a decrease in maximum cost (a fleet squeeze) that generates an increase in productivity (the efficiency gain). Figure 3 shows the changes in cost distribution and in the value functions under the two market structures. Note that, by allowing total transferability, the exit constraint $W(c) \geq p_q q$, raises the Value function. Moreover, notice that active firms are more productive and demand more than one permit (as $q = 1.4622$).

In the model low-productivity firms can sell their permits and exit the market, but the non-arbitrage condition implies that this is equivalent to ceasing activities and leasing their full quotas permanently. Therefore, the model can be read in the following way. After full transferability of the ITQs is allowed, a new sector appears in the industry, made up of the owners of small-scale (or less productive) *vessels* that permanently cease harvesting activities and lease their quota perpetually . This is consistent with the empirical findings of Brandt (2005).

5 Wealth Distribution

Table 4: Wealth Distribution with $p_q = 0$

	bottom	quartiles				top		
	5%	q1	q2	q3	q4	5%	mean	Gini
wealth (%)	0.0151	0.4003	1.7818	6.4910	91.3269	65.5455	100	0.85
mean	0.0030	0.0160	0.0713	0.2596	3.6531	13.1091	1	

The baseline economy (the industry with restricted tradeability of ITQs) generates more inequality in wealth than in income. That is, the Gini of the wealth distribution (0.86) is higher than the Gini of income distribution (0.70)[18],This is a stylised fact of the US Economy (see Diaz-Gimenez et al. (2011)). The top 5% over the mean wealth ratio is 12.58[19]. Note also that firms at the bottom are very poor. This is due to the fact that they are obtaining negative profits and waiting for better times[20].

Table 5 shows wealth distribution with fully tradable ITQs after the exit of the less efficient firms. Note that the possibility of trading ITQs permanently reduces wealth concentration. This reduction in inequality comes from the exit condition that implies $W(c) \geq p_q q$, which implies that with permanent transferable quotas the marginal firm has a positive value. That is, grandfathering permanent, fully transferable fishing rights is equivalent to giving a (same)

[18]Appendix A.6 describes the Brown Formula used to compute Gini coefficient.
[19]This ratio for the total US Economy (i.e., including households) is 8.1631
[20]This is a well know result. See Weninger and Just (2002).

Table 5: Wealth Distribution with fully tradable ITQs (Active firms)

	bottom	quartiles				top		
	5%	q1	q2	q3	q4	5%	mean	Gini
wealth (%)	0.1612	43.6991	10.8531	20.1120	65.3358	32.4628	100	0.5615
mean	0.0322	0.1480	0.4341	0.8045	2.6134	6.4926	1	

lump sum transfer to all firms in the market which is independent of their wealth level. This reduces inequality as the hypothetical transfer to the poorer firms is larger in proportion to their original wealth than that given to the richer ones. In Appendix A.6 we consider an example of equal distribution of a lump sum transfer and present in detail the calculation of the Gini coefficient and the Lorenz curve. The example shows how this redistributive mechanism reduces the Gini coefficient.

Tables 4 and 5 are not directly comparable as only the wealth of active firms is considered, and the proportion of the total firms accounted for by active firms may be different for the different market structures. For the sake of comparison of the two economies we construct Table 6, which compares wealth levels for different hypotheses of transferability of ITQs for the different parts of the wealth distribution, including exiters in both market structures. 31.64% of firms exit the fishery and their owners permanently lease out their ITQs at a price p_q (7.45). The wealth of these small owners (we associate small owners with less efficient firms) is multiplied by 122, as when they could not sell the right their wealth was approximately zero (the marginal wealth was zero, given that the firm was indifferent between exiting and staying).

5.1 Wealth and innovation rate

The conventional wisdom would imply that better functioning markets, with more complete and transferable property rights would spur innovation. The following experiments explore

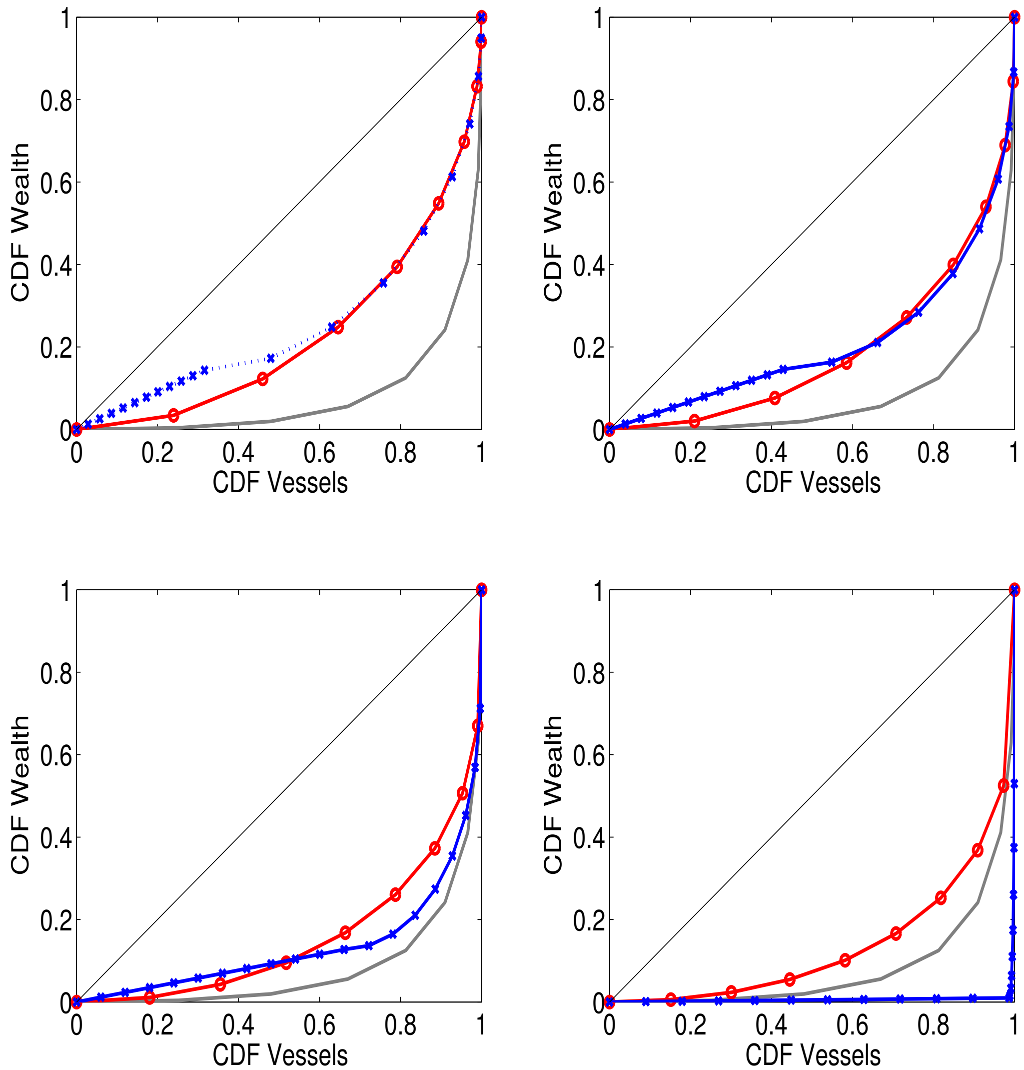


Figure 4: Wealth Lorenz Curves. Each subplot represents the Lorenz curves associated with the Gini coefficients of Table 7. In each case, the line with crosses corresponds to the wealth distribution of active vessels, the line with bullets corresponds to the wealth distribution of all owners, and the other line represents the wealth distribution in the baseline economy. The figures are in the same order as the us with the first figure corresponding to the basic μ and the last one (bottom right) corresponding to $1/4\mu$. It is shown that only in the bottom right figure is the Lorenz curve with crosses Lorenz dominated by the Lorenz curve of the baseline economy.

Table 6: Wealth levels

	small owners	active vessels					
		bottom 5%	quartiles q1	q2	q3	q4	top 5%
mass with $p_q = 0$	31.64	3.42	17.09	17.09	17.09	17.09	3.42
wealth							
$p_q = 0$	0.06	0.15	0.28	1.01	3.34	30.70	98.07
$p_q > 0$	7.45	10.88	49.96	146.58	271.64	888.45	12192.3

Table 7: Wealth and innovation rate

	active vessels	owners		
innovation rate	Gini	exit	p_q	Gini
μ	0.5615	31.64	7.45	0.5018
$3/4\mu$	0.6307	42.87	8.94	0.5907
$1/2\mu$	0.7014	66.05	13.25	0.7410
$1/4\mu$	0.7413	98.68	19.74	0.9866

the impact of higher innovation rates on wealth inequality[21].

Table 7 reports Gini coefficient, percentage of exiters, and sale ITQ price for four different levels of μ.Notice that if more tradability of ITQs generates an increase in the innovation rate, this would be a force for increasing the level of inequality. In our model, in order to generate more inequality than in the case with restricted tradability (remember that in that case the Gini coefficient was 0.86), the innovation rate must increase by 50 % (however the data only suggest an innovation rate of 6%), which would be associated with a fleet shrinkage of around 90% and an increase in ITQ prices of 2.5 times the observed price.

Figure 4 represents the Lorenz curves associated with the Gini coefficients of Table 7. In each case, the line with crosses corresponds to the wealth distribution of active vessels, the line with bullets corresponds to the wealth distribution of all owners, and the other line

[21]We are indebted to an anonymous referee for this suggestion.

represents the wealth distribution in the baseline economy. The figures are ordered in the same order as the us with the first figure corresponding to the basic μ and the last one (bottom right) corresponding to $1/4\mu$. It is shown that only in the bottom right figure is the Lorenz curve with crosses Lorenz dominated by the Lorenz curve of the baseline economy.

6 Conclusions

Much of the resistance to the use of individual transferable quotas in the US centres on the concern that ITQs will change participants' relative positions in the fishery—in particular the fear that small-scale fishermen will be disadvantaged relative to larger producers. However Brandt (2005) shows that in the mid-Atlantic clam US fishery no segment of the industry was disproportionately adversely affected by the regulatory change. In this paper we build a formal model that supporst this findings. Moreover, we found that grandfathering the permanent and fully transferrable fishing rights is equivalent to giving a (same) lump sum transfer to all firms in the market which is independent of their wealth level. This reduces inequality as the transfer to the poorer is larger in proportion to their original wealth than the one given to the richer.

Finally, in our model heteregeneity was generated by firm-specific shocks to production opportunities. However, the same result be achieved with other firm-specific shocks, e.g. differences in prices and demands driven by the composition of catches and/or quality[22] If so, then perhaps a more precise statement of the results would say that if agent heterogeneity is high enough then trade of permits does not necessarily increase wealth inequality.

[22]For instance, in Da-Rocha and Pujolas (2011a) heterogeneity comes by differences in the species composition of vessel catches.

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A Appendix

A.1 Proof of Proposition 1

The problem is standard[23]. The proof obtained is by guessing and verifying, we guess the following functional form for the value function $W(c) = Ac^{\beta} + Bc^{-1} - C$, and by solving the Hamilton-Jacobi-Bellman equation we find that $B = \frac{\pi}{\rho+\mu-\sigma^2}$ and $C = \frac{(c_f - r_q q}{\rho}$ and $\beta = \frac{1}{2} + \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2\rho}{\sigma^2}}$. From the value matching and the smooth pasting conditions we find A and c_* by solving

$$\begin{aligned} W(c_*) &= Ac_*^{\beta} + \frac{ac^{-1}}{\rho+\mu-\sigma^2} + \frac{b}{\rho} = p_q q, \\ cW'(c_*) &= \beta Ac_*^{\beta} + \frac{ac^{-1}}{\rho+\mu-\sigma^2} = 0. \end{aligned}$$

A.2 Proof of Proposition 2

First note that $\int_0^{c_*} c^a g(c)dc = \left(\frac{1+\xi}{1+\xi+a}\right)^2 c_*^a$. Taking expectations, and using the value of c_*, we have that

$$\begin{aligned} W^e &= \int_0^{c_*} W(c)g(c)dc - wc_e - p_q q \\ &= \frac{(\xi+1)^2}{(1+\beta)(\xi+1+\beta)^2}\left(p_q q - \frac{(r_q q - c_f)}{\rho}\right) + \frac{(\xi+1)^2}{\xi^2}\left(\frac{(p-r_q)^2}{4w}\frac{c_*^{-1}}{\rho+\mu-\sigma^2}\right) + \frac{(r_q q - c_f)}{\rho} - wc_e - p_q q \\ &= \left(p_q q - \frac{(r_q q - c_f)}{\rho}\right)\left[\frac{(1+\xi)^2}{(1+\beta+\xi^2)(1+\beta)} + \frac{\beta(1+\xi)^2}{(1+\beta)\xi^2} - 1\right] - wc_e. \end{aligned}$$

Then, from the f.o.c. of the entering firm's problem we have

$$\left(p_q - \frac{r_q}{\rho}\right)\left[\frac{(1+\xi)^2}{(1+\beta+\xi^2)(1+\beta)} + \frac{\beta(1+\xi)^2}{(1+\beta)\xi^2} - 1\right] = 0 \Rightarrow p_q = \frac{r_q}{\rho}.$$

[23]See Dixit and Pindyck (1994).

A.3 Cumulative distribution function

Revenue $y(w, r_q, c) = \left(\frac{1-r_q}{2w}\right) c^{-1}$ is non linear in c. However, the invariant distribution of revenue is a simple change in the power of the invariant cost distribution. That is,

$$f(y) = \frac{(\alpha-1)^2}{y_*}\left(\frac{y_*}{y}\right)^{\alpha} \ln(y_*/y)$$

where $\alpha = \xi + 2$. We calculate

$$F(y) = \int_{y_*}^{y} f(y)dy = \int_{y_*}^{y} \frac{(\alpha-1)^2}{y_*}\left(\frac{y_*}{y}\right)^{\alpha} \ln(y_*/y)dy.$$

Trivial manipulation implies that the cumulative distribution function is

$$F(y) = \int_{y_*}^{y} f(y)dy = 1 - \left(\frac{y_*}{y}\right)^{\alpha-1}\left[(1-\alpha)\ln\left(\frac{y_*}{y}\right)+1\right].$$

A.4 Calibration

We proceed in the following way. First we calibrate the process μ and σ match the entry with the the Northeast Multispecies (groundfish) Fishery Pareto right tail index of nominal revenues (from active firms), in 2010. That is, $\text{Gini}_{2010} = \frac{2}{\xi}$ and Entry rate $= \frac{\sigma^2\xi}{2}$, where $\xi = \frac{\mu}{\sigma^2} - \frac{1}{2}$. Second, we set $M = 1 - \Delta\text{fleet}_{2010}$ and $p - r_q = \text{margin}_{2010}$, and we compute

$$c_f = \frac{\text{margin}_{2010}}{2(1-\Delta\text{fleet}_{2010})^2}\left(\frac{\xi}{\xi+1}\right)^2 \frac{(1+\beta)}{\beta}\left(\frac{\rho}{(\rho+\mu-\sigma^2)}\right),$$

$$\begin{aligned}
c_e &= \frac{1}{\text{entry}}\left[\frac{1}{(1-\Delta\text{fleet}_{2010})} - 2c_f\left(\frac{\xi+1}{\xi}\right)^2 \frac{\beta}{(1+\beta)}\left(\frac{(\rho+\mu-\sigma^2)}{\rho}\right)\right], \\
w &= \frac{c_f}{\rho c_e}\left[\frac{(1+\xi)^2}{(1+\beta+\xi)^2(1+\beta)} + \frac{\beta(1+\xi)^2}{(1+\beta)\xi^2} - 1\right], \\
c_* &= \frac{(\text{margin}_{2010})^2}{4w}\frac{1}{c_f}\frac{(1+\beta)}{\beta}\left(\frac{\rho}{(\rho+\mu-\sigma^2)}\right).
\end{aligned}$$

Third, $\mu_{p_q=0}$ match the Gini index in 2007

$$\text{Gini}_{2007} \quad = \quad \frac{2}{\xi_{p_q=0}},$$

where $\xi_{p_q=0} = \frac{\mu_{p_q=0}}{\sigma^2} - \frac{1}{2}$, and $c_*^{p_q=0}$ match $M^{p_q=0} - M = \Delta\text{fleet}_{2010}$. That is

$$M^{p_q=0} - M = \Delta\text{fleet}_{2010} = \int_{c_*}^{c_*^{p_q=0}} -\frac{(1+\xi_{p_q=0})^2}{c_*^{p_q=0}} \log(x/c_*^{p_q=0}) \left(\frac{x}{c_*^{p_q=0}}\right)^{\xi_{p_q=0}} dx.$$

Finally, to compute profits in 2007 we use

$$\begin{aligned} c_*^{p_q=0} &= \frac{(\text{margin}_{2010})^2}{4w} \frac{1}{K_2^{p_q=0}(c_f - r_q)} \left(\frac{\xi^{p_q=0}+1}{\xi^{p_q=0}}\right)^2 \left(\frac{(\rho + \mu^{p_q=0} - \sigma^2)}{\rho}\right), \\ w^{p_q=0} &= \frac{(c_f - r_q^{p_q=0})EW^{p_q=0}}{\rho c_e}. \end{aligned}$$

A.5 Solving for the Equilibrium

Given μ and σ, the equilibrium, w, r_q p_q, M, and c_*, are given by the following set of five equations. First, entry condition

$$w = \frac{1}{c_e}\left(\frac{c_f}{\rho}\right)\left[\frac{(1+\xi)^2}{(1+\beta+\xi)^2(1+\beta)} + \frac{\beta(1+\xi)^2}{(1+\beta)\xi^2} - 1\right].$$

From the labour market condition, we can obtain the mass of firms M,

$$1 - M\varepsilon \times c_e = M\int_0^{c_*} l(c)g(c)dc = M\left(\frac{(p-r_q)}{2w}\right)^2\left(\frac{\xi+1}{\xi}\right)^2 c_*^{-1}.$$

From the output market, we have

$$1 = M\overline{q} = M^2\int_0^{c_*} y(c)g(c)dc = M^2\frac{(p-r_q)}{2w}\left(\frac{\xi+1}{\xi}\right)^2 c_*^{-1}.$$

and the maximum cost c_*, is

$$c_* = \frac{(1+\beta)}{\beta}\frac{(p-r_q)^2}{4w}\frac{1}{(\rho+\mu-\sigma^2)}\left(\frac{\rho}{c_f}\right),$$

and p_q is such that $p_q = \frac{r_q}{\rho}$. Simple manipulation allows us to find the close-form solution:

$$\begin{aligned}
w &= \frac{c_f}{\rho c_e}\left[\frac{(1+\xi)^2}{(1+\beta+\xi)^2(1+\beta)} + \frac{\beta(1+\xi)^2}{(1+\beta)\xi^2} - 1\right], \\
\frac{1}{M} &= c_e\varepsilon + 2c_f\left(\frac{\xi+1}{\xi}\right)^2\frac{\beta}{(1+\beta)}\left(\frac{(\rho+\mu-\sigma^2)}{\rho}\right), \\
(p-r_q) &= 2c_f M^2\left(\frac{\xi+1}{\xi}\right)^2\frac{\beta}{(1+\beta)}\left(\frac{(\rho+\mu-\sigma^2)}{\rho}\right), \\
c_* &= \frac{(p-r_q)^2}{4c_f w}\frac{(1+\beta)}{\beta}\left(\frac{\rho}{(\rho+\mu-\sigma^2)}\right).
\end{aligned}$$

A.6 The computation of Gini coefficients and Lorenz curve

In order to compute the Gini coefficients in our calibrations we use the approximation by trapezoids known as Brown's formula. Formally, define $p(n)$ as the density and $P(n)$ as the accumulated proportion of the population variable, for$n = 0$, with N being the types of individuals differentiated by wealth (and ordered from lesser to greater wealth), with $P(0) = 0$ and $P(N) = 1$. Define as $w = 0....W$the different wealth levels (where wealth is ordered in a non decreasing fashion) and let $f(w)$ be the density and $F(w)$ be the cumulative proportion of the wealth variable. Then the Gini coefficient can be defined as

$$Gini = 1 - \sum_i (P(i) - P(i-1))(F(i) + F(i-1))$$

An application that measures the effect of a lump sum transfer on the Gini coefficient is presented in table 8. Column 1 is the amount transferred. Column 2 is the proportion of population in each wealth level. Columns 3 and 4 are the wealth levels before and after the transfer, respectively. Column 5 represents the cumulaivte distribution of people and columns 6 and 7 the cumulative distribution of wealth before and after the transfer. The rest of the columns are helpful in computing the Brown's formula. It is immediately apparent by straightforward application of the formula that the Gini coefficient is 0.44 before the transfer and 0.22 after the transfer.

Table 8: Gini Index: Impact of a Transfer

Transfer	p(n)	w0	w1	P(i)	F(i)	F(w1)	A=P(i)-P(i-1)	B=F(i)+F(i-1)	
5.00	0.33	0.00	5.00	0.33	0.00	0.17	0.33	0.00	0.17
5.00	0.33	5.00	10.00	0.67	0.33	0.50	0.33	0.33	0.67
5.00	0.33	10.00	15.00	1.00	1.00	1	0.33	1.33	1.50
Total	1.00	15.00	30.00			1	–	–	–

The Lorenz curve plots the cumulative proportion of wealth as a function of the cumulative proportion of the population. Table 9 shows the calculation and the effect on the Lorenz curve of the transfer discussed in Table 8. As before, Column 1 is the amount transferred. Column 2 is the proportion of population in each wealth level. Columns 3 and 4 are the wealth levels before and after the transfer, respectively. Column 5 and 6 represent the proportion of wealth belonging to each type of agent before and after the transfer. The Lorenz curve corresponding to the case without the subsidy would plot column 7 in the horizontal axis and column 8 in the vertical axis (the Lorenz curve corresponding to the economy with the subsidy would be symmetricaly defined using column 9).

Table 9: Lorenz curve: Impact of a Transfer

Transfer	p(n)	w0	w1	f(i)	f(i)	p(i)+p(i-1)	f(i)+f(i-1)	
5.00	0.33	0.00	5.00	0.00	0.17	0.33	0.00	0.17
5.00	0.33	5.00	10.00	0.33	0.33	0.66	0.33	0.67
5.00	0.33	10.00	15.00	0.67	0.50	1	1	1
Total	1.00	15.00	30.00	1.00	1.00	–	–	–