Fraudulent Democracy: A Dynamic Ordinal Game Approach

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Abstract

We propose a model of political competition and stability in nominally democratic societies characterized by fraudulent elections. In each election, an opposition leader is pitted against the leader in power. If the latter wins, he remains in power, which automatically makes him the incumbent candidate in the next election as there are no term limits. If he loses, there is an exogenously positive probability that he will steal the election. We model voter forward-looking behavior, defining a new solution concept. We then examine the existence, popularity, and welfare properties of equilibrium leaders, these being leaders who would remain in power indefinitely without stealing elections. We find that equilibrium leaders always exist. However, they are generally unpopular, and may be inefficient. We identify three types of conditions under which equilibrium leaders are efficient. First, efficiency is achieved under any constitutional arrangement if and only if there are at most four competing leaders. Second, when there are more than four competing leaders, efficiency is achieved if and only if the prevailing political system is an oligarchy, which means that political power rests with a unique minimal coalition. Third, for a very large class of preferences that strictly includes the class of single-peaked preferences, equilibrium leaders are always efficient and popular regardless of the level of political competition. The analysis implies that an excessive number of competing politicians, perhaps due to a high level of ethnic fragmentation, may lead to political failure by favoring the emergence of a ruling leader who is able to persist in power forever without stealing elections, despite being inefficient and unpopular.

Keywords: Fraudulent democracy, farsightedness, efficiency, popularity, naiveté, political failure, fragmentation.

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1 Introduction

Political rights have expanded to an unprecedented extent over the last century. Measures of democratic political participation show impressive upward trends since 1900 (Acemoglu (2012)). These trends are also observed in countries that achieved independence relatively recently. For instance, between 1960 and 2010, African countries have held over 650 presidential and legislative elections. Yet, despite this remarkable rise in political freedom, it is also recognized that democracy continues to be weak in many parts of the world. In most less-developed regions, the majority of democracies are suffering from the persistent problem of fraudulent elections. According to Freedom House, the number of full electoral democracies in sub-Saharan Africa has fallen significantly since 2005. In addition to manipulating elections, many political regimes have demonstrated a tendency to manipulate their constitution to eliminate presidential term limits, thus making sure that their leader is able to stay in power for "life." Power monopolization has also been observed at the party level. In most African countries, many political parties have not renewed their leadership since their creation following the wave of democratization that swept the continent in the 1990s, which suggests that the leaders of these parties will try to confiscate power if they eventually become their country's ruling leader.

In this paper, we propose a model of political competition and stability in a democracy that is tainted by fraudulent elections. In a society of infinitely lived individuals, elections are held at regular intervals to choose a leader from a finite set of politicians under a fixed constitution. In each election, an opposition leader is pitted against the leader in power. If the latter wins, he remains in power, which automatically makes him the incumbent candidate in the next election as there are no term limits. If the opposition leader wins, there is an exogenously positive probability that the election will be stolen by the incumbent regime. A victory by an opposition leader therefore does not necessarily lead to a political transition.

Our goal in this paper is threefold. First, we model rational citizen behavior in this dynamic environment, introducing a new solution concept for this class of games. Second, we study the existence of equilibrium leaders, and third, we analyze their popularity and welfare implications. Our analyses also have implications for the ways in which *naive* versus *farsighted* voting behavior affects the quality of equilibrium leaders.

In proposing a model of rational behavior, we answer the question of when a citizen will support a challenger over the incumbent leader. In order to answer this question, we assume that each citizen is farsighted or forward-looking, given the dynamic nature of the political process. This means that the decision to support a particular candidate not only should be based on the immediate benefits that would flow from that candidate coming to, or remaining in, power, but also should take into account future possible political transitions that would take place following the outcome of the present election. Farsightedness therefore dictates two basic rules. A citizen will support a challenger over the status quo only if: (1) he or she prefers the former to the latter; and (2) the election of the challenger will not, following subsequent political transitions, lead to the rise to power of another politician who, from the point of view of this particular citizen, is worse than the present-day status quo.

The first rule takes into account the possibility that a newly elected leader might succeed in staying in power indefinitely by, for instance, manipulating future elections. An individual who supports change over the status quo therefore should be motivated to do so. The second rule incorporates two prescriptions. The first postulates that, if there is a possibility that the election of the challenger would lead to the rise of a worse leader following one or more subsequent transitions, then a voter should not support him or her over the status quo. The second prescription is simply the inverse of the first and seeks to optimize individual decision making. It states that an individual should support the challenger if the election of the latter would not lead to a worse leader relative to the status quo. As a result, an individual who follows the two rules always acts optimally when choosing between two candidates.

We model farsightedness as a binary relation over the set of political leaders. An equilibrium leader is therefore a maximal element with respect to that binary relation. Intuitively, an equilibrium leader is a politician who, if elected and propelled to power at some point of the dynamic electoral process, would be able to remain in office indefinitely without stealing elections. If a leader is not an equilibrium, then there exists a rational path away from that leader.

We examine the existence of equilibrium leaders when citizens are farsighted and have common knowledge of rationality. We find that an equilibrium leader always exists if citizens have linear preferences. However, equilibrium leaders can be unpopular in the sense of being less preferred by a majority of the population than another leader. What makes an unpopular leader stable is the fact that the leader who dominates him in terms of popularity is himself unstable. It follows that the stability of an unpopular leader is guaranteed by the legitimate fear of some citizens that there is a positive probability that change, following subsequent political transitions, would lead to an outcome that is worse for them than the current status quo.

We also analyze the welfare properties of equilibrium leaders. We find that an equilibrium leader might be Pareto inefficient. In fact, it might happen that voting against an inefficient leader who is in power today will lead to the election of an inferior alternative in the future, which is the reason why inefficient leaders are sometimes stable. We investigate conditions that guarantee the efficiency of equilibrium leaders. We identify three types of conditions. The first is related to the number of competing politicians, the second is related to the nature of the prevailing political system or constitutional arrangement, and the third is related to the domain of preferences. More precisely, we find that efficiency at any preference profile and under any constitutional arrangement is guaranteed if and only if there are at most four competing politicians. If the number of politicians is greater than four, efficiency at any preference profile is achieved if and only if the prevailing political system is an oligarchy, with decisive power being concentrated by a unique minimal coalition of citizens. This minimal coalition may contain just one person, in which case the political system is a dictatorship, or it may be the entire population, in which case the system is governed by the unanimity rule. Furthermore, we analyze the effect of preference domain restrictions on outcome efficiency. We identify a very large class of preferences for which any equilibrium leader is efficient. This class of preferences was first discovered by Salles (1976). It strictly includes the class of single-peaked preferences and the class of single-carved preferences. Importantly, we also find that, for that class of preferences, any equilibrium leader is popular and any popular leader is an equilibrium leader, which provides an unexpected condition under which leader stability and popularity coincide within our framework.

The finding that the limitation of the number of competing politicians to a maximum of four guarantees the efficiency of equilibrium leaders was unexpected. The intuition underlying this result is that a greater number of competing politicians creates more uncertainty about who will govern the society in the future if the current ruling leader loses power. This uncertainty might sometimes combine with voter prudence or forward-looking behavior to maintain the current leader in power. If the number of competing leaders is sufficiently large, thus creating a high level of uncertainty about the future, the current leader might be maintained in power even if he or she is inefficient. Although this explanation provides some insight into why a high level of political competition might lead to political inefficiency, the minimum level of competition that is necessary to lead to this situation has yet to be determined. In this sense, the sharp threshold provided by our analysis is a mathematical discovery.

Our analysis of the conditions that guarantee the efficiency of equilibrium leaders seems to indicate that efficiency is achieved at the cost of violating well-accepted democratic principles, such as capping the number of competing politicians at four[2], or distributing political rights in an inequitable manner. The analysis therefore has implications for the quality and longevity of ruling leaders in fragmented societies. For example, societies that are organized around ethnic groups and in which the number of political leaders reflects the number of these groups might be more likely to elect a stable but inefficient leader, especially if there are more than four major ethnic groups. Such a leader does not need to manipulate elections in order to remain in power, as his or her stability is guaranteed by the political antagonism between the major ethnic groups in the country. This suggests that the plethora of competing politicians and political parties that generally characterizes ethnically fragmented societies partly explains the quality and political longevity of the ruling leader of those societies. Drawing in part on the vast literature pertaining to the economic impacts of ethnic divisions, our analysis reveals that another channel through which ethnic fragmentation can lead to underdevelopment is its tendency to favor the emergence of bad leaders who are able to persist in power indefinitely without stealing elections.

Our analysis also has implications for how voting behavior affects the quality of elected leaders. In particular, we contrast voting outcomes under conditions of *farsighted behavior* and *naive* (or *myopic*) *behavior*. Citizens vote myopically when they view each election as a one-shot game. Surprisingly, we find that such behavior always leads to equilibria that are efficient and popular. Indeed, a myopically stable leader is also a farsightedly stable leader, but the converse is not true in

[2] Interestingly, our analysis implies that democracies that have only two major parties like the United States always induce efficient leaders, even if, unlike the United States, those democracies are characterized by fraudulent elections.

general. In addition, given the fact that myopically stable leaders are also popular, it follows that myopically stable leaders are the first-best for society, and that farsightedly stable leaders who are not myopically stable are the second-best. Our analysis therefore implies that individually optimal behavior is in general detrimental to society, whereas behavior that leads to good outcomes for society may be detrimental to the individuals who adopt it. Importantly, our identified class of preferences for which equilibrium leaders are efficient resolves this dilemma, as naive and farsighted behaviors lead to the same set of stable leaders under those preferences.

The remainder of this paper is organized as follows. Section 2 situates our study in the literature. Section 3 introduces our model of a dynamic political economy characterized by fraudulent elections. Section 4 models rational behavior and introduces a new solution concept. Section 5 studies the existence of equilibrium leaders, and Section 6 examines their welfare properties and popularity. Section 7 draws the implications of our analyses for the majority rule, which is viewed as the fairest of all political rules (Dasgupta and Maskin (2008)). Implications for the longevity and quality of leaders in ethnic societies are also provided. Section 8 contrasts voting outcomes under farsighted and naive voting behaviors. Section 9 suggests a different application of our model to the selection of *sticky policies* in fully developed democracies and concludes.

2 Closely Related Literature

Instances of electoral fraud across the world have been widely documented. However, we are not aware of any prior theoretical analysis of how electoral fraud affects voter rationality as well as the stability and quality of political leaders in nominally democratic societies. In addition to supplying a simple framework for analyzing this crucial question, our study has yielded testable implications for how the level of political competition and certain social structures might lead to political failure in fraudulent democracies. A key characteristic of such democracies is that, although an "official" constitution exists, it is respected only in case the ruling leader wins the election. If the ruling leader loses, he will steal from the challenger with a positive probability, but this probability is not known. The distinctive features of a fraudulent democracy are therefore its official constitution and the "unknown" probability with which the constitution is respected by the ruling leader. It is these features that distinguish our model from models of dynamic political games in which the rules are clearly "known" to the players.

Our paper shares some features with studies on dynamic political decision-making. In particular, our assumption that voters are farsighted in a "conservative" manner is closely related to previous works by Harsanyi (1974), Greenberg (1990), Chwe (1994), Xue (1998), Chakravorti (1999), and Ray and Vohra (2014).[3] Like our study, most of these studies also assume ordinal preferences.[4] A

[3] A conservative behavior is a behavior that ensures that the voter will never regret his current decision to support change. As remarked by Ray and Vohra (2014), it is a leading standard of behavior in the literature.

[4] An alternative approach to modelling preferences would have been to assume that each voter receives a payoff at each stage of the dynamic game, and adds discounted payoffs over time to obtain his total payoff. But such an approach is not meaningful within our framework because, for this to be feasible, voters should anticipate the exact probability with which a leader will retain power after losing a future election. This probability is not known, as in

distinctive feature of our framework, however, is that a transition from one stage of the political game to another stage can be resisted even if it is supported by a winning coalition, as an incumbent has the ability to steal the election. Such imperfections affect voters' rational behavior, as they care not only about their long-term payoff, but also about their immediate payoff when deciding to support a challenger against the status quo. The aforementioned studies assume that individuals receive their payoffs only after the dynamic process has reached a stable outcome, which implicitly assumes, as remarked by Acemoglu et al. (2012), that voters are sufficiently patient. Our model of rationality is therefore different, which also implies that our set of equilibrium outcomes differs from those previously defined in the literature. Indeed, the solution concept introduced by our analysis is new.

Our assumption that, in each election, the challenger is exogenously chosen from the set of candidates in the opposition has been made in a number of studies. Penn (2009), for instance, argues that, in real-life politics, citizens rarely have any control over which policy will be chosen to be pitted against the status quo, in part due to the fact that a number of the complex factors that govern elections are not under the control of voters. In her view, this argument makes the exogeneity assumption realistic. Our scope however differs from that of Penn in that we are interested in imperfect democracy.

Like our paper, a few other papers have found that certain equilibrium outcomes in dynamic collective decision making may be inefficient (see, e.g., Pongou et al. (2008), Acemoglu et al. (2012)). Our scopes and analyses, however, are different. Pongou et al. (2008) analyze the effect of binding solidarity agreements on outcome efficiency in a political game that lasts at most two periods. Unlike this study, we do not assume that voters cooperate. Acemoglu et al. (2012) analyze collective decision-making in a context in which current decisions determine the future distribution of political power and therefore influence future decisions. In their framework, the competing alternatives are the constitutions. It follows that a constitution that is valid today may be changed to an alternative constitution in the future by a population subgroup that is sufficiently powerful under the current constitution. Our analysis is different. Within our framework, the competing alternatives are politicians, and they compete for political power under a constitution that is fixed over time. We are also interested in a different question. We analyze political competition and stability in a context of fraudulent democracy. Also, we study the conditions under which stable leaders are efficient. We show that inefficiency is possible only when the number of competing politicians is greater than four. We also provide a full characterization of the constitutional arrangements that rule out inefficiency, showing that, if the number of competing politicians is greater than four, efficiency at any preference profile is guaranteed if and only if the prevailing political system is an oligarchy. Furthermore, we identify a large class of preferences for which equilibrium leaders are always efficient and popular regardless of the level of political competition. To the best of our knowledge, these analyses and results are new.

real-life politics, it would depend on factors which are likely to vary over time and which cannot be fully anticipated by ordinary citizens.

3 A Fraudulent Dynamic Political Economy

A *political economy* is a society, $N = \{1, 2, ..., n\}$, populated by a finite number of individuals, and endowed with a constitution, C, and a set of political leaders, A. Each individual has preferences over leaders. We assume that elections are held at regular intervals and that in each election, an opposition leader is pitted against the incumbent leader. If the incumbent wins, he retains power, which makes him the status quo leader of the next election. But if he loses, the election is stolen with an exogenously positive probability. We fully define these notions as well as the fraudulent dynamic electoral process below.

3.1 Political Leaders

A *political leader* is an individual who might run the society. We assume that there is a finite number of competing political leaders, and that there are at least two leaders. In a society organized around ethnic groups, for example, the number of leaders might reflect the number of these groups. Each leader promotes a distinct political platform, and his goal is to gain access to political power in order to implement his political program.

3.2 Constitution

A *constitution or a political rule* is a distribution of political decision-making power among the various subgroups of the society. It is formalized as a function C which maps each subgroup S of the society into either 1 or 0; $C(S) = 1$ means that the members of S have the power to change the status quo to a new social alternative (with a positive probability)[5]; and $C(S) = 0$ means that S does not have such a power. Denote by 2^N the set of all the subsets of N, and by W the set of all the elements of 2^N such that $C(S) = 1$. We assume that W is non-empty. In addition, we impose the following natural conditions on W.

1. For any subgroups S and T such that $S \subset T$, if $S \in W$, then $T \in W$.

2. For any subgroup S, if $S \in W$, then $N \setminus S \notin W$.

Each subgroup in W is called a majority or a winning coalition. Condition (1) means that the enlargement of a winning coalition of voters by adding new members results in another winning coalition. Condition (2) means that the complementary set of a winning coalition is a losing coalition. This condition prevents trivial political instability by avoiding situations in which two non-overlapping winning coalitions have entirely opposing views on how the society should be run.

We say that a winning coalition S is minimal if any proper subset of S is a losing coalition. In other words, a winning coalition is minimal if the coalition that results after one individual withdraws from it is losing. We denote by $W_m(C)$ (or simply W_m) the set of all minimal winning coalitions under a constitution C.

[5] As is made clear in Section 3.4, this precision is important because a transition from an incumbent leader to a challenger might be resisted even if it is supported by a coalition S such that $C(S) = 1$.

Certain familiar constitutions will attract our attention in the paper. These constitutions include the *majority rule*, which is a rule under which a coalition of individuals is winning if and only if it contains more than half of the population. Dasgupta and Maskin (2008) show that the majority rule has some very appealing properties, making it the most democratic of all rules. Another constitutional structure that is of special interest is the *oligarchy*, which is a political rule under which there exists only one minimal winning coalition. If this unique coalition contains only one individual, then the oligarchy is a dictatorship. If, on the other hand, it contains the entire population, then the oligarchy is the unanimity rule. A typical oligarchy contains only a few members of the society, and therefore it is neither a case of dictatorship, nor a case of unanimity rule. However, we do not restrict our definition of an oligarchy to those more typical cases.

3.3 Preferences

Each individual $i \in N$ has a preference relation represented by a binary relation $\succeq_i$ on the set of political leaders A. We assume that each preference relation $\succeq_i$ is:

- reflexive: for any $x \in A$, $x \succcurlyeq_i x$;
- transitive: for any $x, y, z \in A$, if $x \succcurlyeq_i y$ and $y \succcurlyeq_i z$, then $x \succcurlyeq_i z$; and
- complete: for any $x, y \in A$, $x \succcurlyeq_i y$ or $y \succcurlyeq_i x$ or both.

The asymmetric and symmetric components of a preference relation $\succeq_i$, denoted respectively by $\succ_i$ and $\sim_i$, are defined as follows:

- For any $x, y \in A$, $x \succ_i y$ if $x \succcurlyeq_i y$ and *not*$(y \succcurlyeq_i x)$; and
- For any $x, y \in A$, $x \sim_i y$ if $x \succcurlyeq_i y$ and $y \succcurlyeq_i x$.

We will also sometimes assume that each preference relation $\succeq_i$ is:

- anti-symmetric: for any $x, y \in A$, $x \succcurlyeq_i y$ and $y \succcurlyeq_i x$ imply $x = y$.

A preference relation that is reflexive, transitive, antisymmetric, and complete is said to be a linear order, and a preference relation that is reflexive, transitive, and complete is said to be a weak order. We denote the set of linear orders on A by $\mathcal{L}$, and the set of weak orders on A by $\mathcal{U}$. A preference profile is denoted by $(\succcurlyeq_i)_{i \in N}$. Where there is no confusion, $(\succcurlyeq_i)_{i \in N}$ will be denoted by $(\succcurlyeq_i)$. We denote by $\mathcal{L}^N$ the set of all the preference profiles of linear orders on A, and by $\mathcal{U}^N$ the set of all the preference profiles of weak orders on A.

If S is a population subgroup and x and y are two political leaders, we say that S strictly prefers x over y, denoted by $x \succ_S y$, if each individual in S strictly prefers x over y (that is, $x \succ_i y$ for every $i \in S$). Similarly, we say that S prefers x over y, denoted as $x \succcurlyeq_S y$, if each individual in S prefers x over y (that is, $x \succcurlyeq_i y$ for every $i \in S$).

If $B \subseteq A$ is a subset of political leaders, then we denote by $(\succcurlyeq_i |_B)$ the restriction of $(\succcurlyeq_i)_{i \in N}$ to B. If $B = A \setminus \{x\}$ where $x \in A$, then $(\succcurlyeq_i |_B)$ is denoted by $(\succcurlyeq_i^x)$.

3.4 A Fraudulent Dynamic Electoral Competition

We assume that competition for power between political leaders takes place in a dynamic framework under a fixed constitution C as follows:

1. In period $t = 0$, nature chooses a political leader y to rule the society.

2. In period $t = 1$, an electoral contest is organized between the incumbent leader y and an opposition leader x_0 exogenously chosen from the set $A\backslash\{y\}$.

   (a) If y wins (which means that no majority coalition under C chooses x_0 over y), he remains in power and becomes the incumbent in the next election taking place in period $t = 2$.

   (b) If y loses to x_0, y steals the election (and retains power) with exogenous probability $p(x = y, t = 1)$[6], $0 < p < 1$, and concedes defeat to x_0 with probability $1 - p$.

3. In each period $t \geq 2$, an electoral contest is organized between x_{t-1}, the leader in power in period $t - 1$, and a leader y_t exogenously chosen from the set $A\backslash\{x_{t-1}\}$, and the outcome is decided as in stage 2.

The dynamic framework in which the winner of the current election becomes the status quo (or incumbent) leader in the next election is classical (see, e.g., Harsanyi (1974), Chwe (1994), Xue (1998), Penn (2011), Acemoglu et al. (2008), and Ray and Vohra (2014)). We adopt a similar framework, though we differ in assuming that an incumbent leader who loses an election might nonetheless retain power by stealing from the challenger. The probability with which electoral fraud takes place cannot be determined in advance, as no individual or group is able to control all the factors that make fraud a success or a failure. For instance, fraud might be prevented by an active international community or by a revolt of citizens. However, it is not possible to predict whether the international community will intervene or whether an internal revolt will take place in the face of electoral fraud. For this reason, it is not possible to model individual utility by adding discounted payoffs over time to obtain a total payoff, as this approach is feasible only if the probability that a leader who is defeated in an election will retain power is known.

The assumption that, in each election, the challenger is exogenously chosen from the set of opposition leaders has been made in several studies. In the literature, the main argument in support of this exogeneity assumption is that, in real-life politics, voters rarely have any control over the policies that are selected to challenge the status quo (see, e.g., Penn (2009)). Within our framework, this assumption can be relaxed, though this relaxation will not change our main conclusions. We retain it for simplicity and expository purposes.

[6] This probability may depend on the unobserved personal characteristics of the incumbent leader. It may also vary over time depending on the internal and international political climate. It may also depend on the cost to the leader of stealing an election. These factors are in general unknown in real-life politics. For these and other reasons, the probability that a leader will steal an election is unknown.

The assumption that nature chooses a political leader in period $t = 0$ is consistent with the reality of many countries. In many African countries, for instance, the first leader was not chosen through a competitive electoral process, but instead by the former colonial power.

In a democracy, the constitution C that governs elections can be taken to be the majority rule. However, we do not restrict ourselves to the majority rule as most of our results are valid under any constitutional arrangement in the class of constitutions defined in Section 3.2.

4 Farsighted Behavior and the Farsighted Equilibrium Set

In this section, we model the behavior of voters within the dynamic framework described in Section 3.4. In each election, each voter chooses between the *status quo leader* and his *challenger*. Choosing the challenger over the status quo leader is dictated by two basic rules:

1- The challenger should be preferred over the status quo.

2- Future political transitions following the replacement of the status quo leader by the challenger should not possibly lead to a leader who is worse than the incumbent.

The first rule follows from the fact that, if the challenger wins the current election and gains access to power, he may retain power forever, even by stealing future elections. Therefore, a voter who supports the challenger over the status quo leader should be motivated to do so. The second rule incorporates the notion of farsightedness. In fact, if there is a possibility that supporting the challenger would, following subsequent transitions, result in a leader who is worse than the incumbent, then a voter should not support the challenger over the incumbent. This is again because there is a possibility that the leader who would ultimately emerge would retain power indefinitely, even by manipulating future elections.

It follows that a voter within our framework is prudent or risk-averse, as his behavior rules out the possibility that he will ever regret a current decision to support a challenger. Such a behavior is described by Greenberg (1990) as being "conservative." As noted by Ray and Vohra (2014), it is a leading standard of behavior, and has been adopted in several important studies on farsightedness. Examples include Chwe (1994), Xue (1998), and Ray and Vohra (2014) among others.

We now formalize voter rational behavior. Let i be a voter. Denote by $\gg_i$ the rationale by which voter i decides to support or not to support a challenger x_0 over an incumbent y. Following the first rule that guides the behavior of i, i will vote for x_0 against y, which is denoted by $x_0 \gg_i y$, if $x_0 \succcurlyeq_i y$. For x_0 to win the election against y, a winning coalition S_0 should vote for x_0 against y, which is denoted by $x_0 \gg_S y$. If x_0 gains access to power, he might, in a future election, be defeated by another leader x_1 supported by a winning coalition S_1 (that is, $x_1 \gg_{S_1} x_0$). Furthermore, if x_1 gains access to power, he might in turn be defeated by another competing leader x_2 supported by a winning coalition S_2 ($x_2 \gg_{S_2} x_1$). The transition process will continue, possibly stabilizing at a leader x_p supported by a winning coalition S_p against leader x_{p-1} ($x_{p-1} \gg_{S_p} x_p$). Such a transition path is denoted by $[(y, x_0, x_1, x_2, ..., x_p); (S_0, S_1, S_2, ..., S_p)]$. Our individual i who contributed to the defeat of y for x_0 knows that if the transition process reaches any leader x_t $(0 \leq t \leq p)$ along the transition path $[(y, x_0, x_1, x_2, ..., x_p); (S_0, S_1, S_2, ..., S_p)]$, it might stop, as there is a positive probability that x_t will stay in power forever even by manipulating elections. The second rule that guides the behavior of i implies that i should weakly prefer any such leader x_t over the current status quo y. In general, the two basic rules that guide a voter's behavior imply that each member of a winning coalition S_t ($0 \leq t \leq p$) who initiates the (possible) transition from x_{t-1} to x_t (where $y = x_{-1}$) should weakly prefer each of the leaders x_r ($t \leq r \leq p$) along the future transition path over x_{t-1} (that is, $x_r \succcurlyeq_{S_t} x_{t-1}$); he should also strictly prefers the leader x_p over x_{t-1} (that is, $x_r \succ_{S_t} x_p$) if the process is to be stabilized at x_p if reached (the process stabilizes at x_p if reached if x_p will stay in power forever without stealing elections; in other words, if x_p is reached, there will be no winning coalition following the rationale $\gg$ that will be willing to replace x_p by another leader).

The definition of a transition path is formalized below.

Definition 1 *Let $\Gamma = (N, W, A, (\succcurlyeq_i)_{i \in N})$ be a political economy.*

A transition path is a path $[(y, x_0, x_1, x_2, ..., x_p); (S_0, S_1, S_2, ..., S_p)]$ where y, x, x_1, x_2, ..., x_p are distinct political leaders and $S_0, S_1, ..., S_p$ are winning coalitions satisfying:

1. *$x_0 = x$ and $x \succcurlyeq_{S_0} y$;*
2. *$\forall t \in \{1, 2, ..., p\}, \forall r \in \{t, t+1, ..., p\}, x_r \succcurlyeq_{S_t} x_{t-1}$;*
3. *$x_p \succ_{S_t} x_{t-1}$ for all $t \in \{1, 2, ..., p\}$.*

Such a path will be referred to as a (y, x)-path. It is said to be profitable for $S = S_0$ (or to be S-rational) if $x_p \succ_S y$.

Any (y, x)-path $[(y, x_0, x_1, x_2, ..., x_p); (S_0, S_1, S_2, ..., S_p)]$ such that not($x_p \succcurlyeq_S y$) is said to be deterred.

The following definition introduces the farsighted equilibrium set, which is the set of leaders who do not need to steal elections in order to remain in power.

Definition 2 *Let $\Gamma = (N, W, A, (\succcurlyeq_i)_{i \in N})$ be a political economy, S a winning coalition, and y and x two political leaders.*

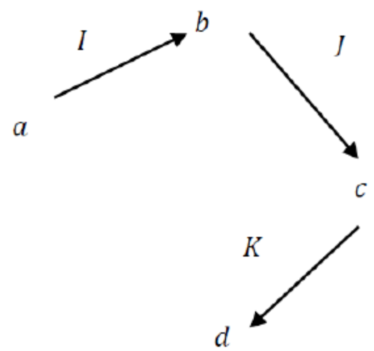
1. *x defeats y thanks to S, denoted $x \gg_S y$, if:*
   - *There exits an S-rational (y, x)-path $[(y, x, x_1, x_2, ..., x_p); (S, S_1, S_2, ..., S_p)]$;*
   - *Any (y, x)-path $[(y, x, z_1, z_2, ..., z_q); (S, T_1, T_2, ..., T_q)]$ satisfies $z_q \succcurlyeq_S y$; in other words, no such path is deterred.*
2. *y is defeated if there exist a leader x and a winning coalition S such that x defeats y thanks to S.*

3. The set of undefeated leaders, also called the farsighted equilibrium set, is denoted $Un(\Gamma)$.

We illustrate the farsighted equilibrium set through the following simple example.

Example 1 *Consider a political economy $\Gamma = (N, W, A, (\succcurlyeq_i)_{i \in N})$ where: $N = \{1, 2, 3, 4, 5, 6\}$, $A = \{a, b, c, d\}$, $W = \{S \subset N : I = 135 \subset S, J = 126 \subset S$, or $K = 234 \subset S\}$, and $(\succcurlyeq_i)_{i \in N}$ is the profile of preferences defined as follows: $c \succ_1 b \succ_1 a \succ_1 d$, $a \succ_2 d \succ_2 c \succ_2 b$, $d \succ_3 b \succ_3 c \sim_3 a$, $b \succ_4 d \succ_4 c \succ_4 a$, $b \succ_5 c \sim_5 a \sim_5 d$, $c \succ_6 a \sim_6 d \succ_6 b$.*

The popularity relationship[7] *among the different leaders is depicted by the following graph:*



If d is the status quo leader, he will stay in power forever without stealing elections since no winning coalition exists for change. It follows that d is an equilibrium leader.

If c is the status quo leader, then winning coalition K will vote for d if the latter is chosen as the challenger, since the members of K prefer d over c and are aware of the fact that if d becomes the new leader, he will stay forever. Thus, c is not an equilibrium leader since, in order to remain in power, he will need to steal the election whenever he is opposed to d.

If the status quo leader is b, no winning coalition will be willing to support an alternative leader. In fact, even though the members of the winning coalition J prefer c over b, some of them will not support c over b if c is chosen to challenge b; voter 1, for instance, knows that, if c defeats b and becomes the new leader, he will lose a future election against d, and so there is a positive probability that a transition from c to d (the worst option of 1) will occur, which precludes 1 from joining J to defeat b for c. It should therefore be noted that the stability of b is a result of the instability of c.

If a is the status quo leader, winning coalition I will support b if the latter is chosen as the challenger, knowing that there will not be any further transition from b as b is an equilibrium leader.

The farsighted equilibrium set is therefore $Un(\Gamma) = \{b, d\}$, which means that only leaders b and d are able to remain in power without manipulating elections.

5 Existence of Equilibrium Leaders

In this section, we study the existence of equilibrium leaders. The following theorem shows that an equilibrium leader always exists if preferences are linear.

[7] We say that a leader x is more popular than another leader y if x is preferred over y by a winning coalition. The formal definition of this notion is given in Section 6.

Theorem 1 *Let $\Gamma = (N, W, A, (\succcurlyeq_i)_{i\in N})$ be a political economy in which individual preferences are linear orders. Then, $Un(\Gamma) \neq \emptyset$.*

Proof. Assume by contradiction that there exists a political economy $\Gamma = (N, W, A, (\succcurlyeq_i)_{i\in N})$ with $(\succcurlyeq_i)_{i\in N} \in \mathcal{L}^N$ for which $Un(\Gamma) = \emptyset$. Since A is finite, the binary relation $\gg$ has a cycle. Then there exists a sequence of leaders $(x_1, x_2, ..., x_q, x_{q+1})$ such that the x_i are distinct except $x_{q+1} = x_1$ and for all $t = 1, 2, ..., q$, x_{t+1} defeats x_t (i.e., $x_{t+1} \gg x_t$). Let S_{t+1}, $t \in \{1, 2, ..., q\}$, be the winning coalition that induces the transition from x_t to x_{t+1}, with $S_{q+1} = S_1$. Given that $(\succcurlyeq_i)_{i\in N}$ is a profile of linear orders, it holds that $x_1 \succ_{S_1} x_q \succ_{S_q} ... \succ_{S_3} x_2 \succ_{S_2} x_1$. Let us now show that $x_q \succ_{S_2} x_1$. In order to do so, we prove by induction that for all $t \in \{2, 3, ...q-1\}$, $x_q \succ_{S_t} x_{t-1}$.

The result is obvious for $t = q - 1$.

Consider $t \in \{3, ...q-1\}$ such that $x_q \succ_{S_t} x_{t-1}$. Consider the path $[(x_{t-2}, x_{t-1}, x_q); (S_{t-1}, S_t)]$. We have $x_q \succcurlyeq_{S_{t-1}} x_{t-1}$ because x_{t-1} defeats x_{t-2} thanks to S_{t-1}. Since preferences are linear orders, $x_q \succcurlyeq_{S_{t-1}} x_{t-1}$ is equivalent to $x_q \succ_{S_{t-1}} x_{t-1}$.

We have just proved that for all $t \in \{2, 3, ...q-1\}$, $x_q \succ_{S_t} x_{t-1}$; taking, for example, $t = 2$ yields $x_q \succ_{S_2} x_1$. Furthermore, we have $x_1 \succ_{S_1} x_q$. Given that S_1 and S_2 are winning coalitions, both coalitions share a voter who therefore strictly prefers x_1 over x_q and x_q over x_1, a contradiction. It follows that, for all political economy $\Gamma = (N, W, A, (\succcurlyeq_i)_{i\in N})$ in which $(\succcurlyeq_i)_{i\in N} \in \mathcal{L}^N$, $Un(\Gamma) \neq \emptyset$.
■

According to Theorem 1, a leader who would remain in power indefinitely without stealing elections always exists if citizens have linear preferences. This is a nice property for the class of political economies we are analyzing, as this ensures that the society will not experience a situation of complete destabilization. However, stability is not the only criterium on which to judge a society. The quality of its ruling leaders also matters. Quality can be measured by whether a leader is popular and/or efficient. The question now is whether all equilibrium leaders are efficient and/or popular. In Example 1, for example, we saw that an unpopular leader can be stable, which is a bad property. In the next section, we will also see that an equilibrium leader can be inefficient, which is another bad property. What are then the conditions that guarantee that a stable leader is efficient and/or popular? The goal of the next section is precisely to answer this crucial question.

6 Welfare Properties and Popularity of Equilibrium Leaders

In this section, we examine the welfare properties and popularity of equilibrium leaders. We show that an equilibrium leader may be Pareto inefficient, and that equilibrium leaders are unpopular in general. We also investigate the conditions under which equilibrium leaders are Pareto efficient and/or popular. The formal definitions of the notions of Pareto efficiency and popularity are given below.

Definition 3 *Let $\Gamma = (N, W, A, (\succcurlyeq_i)_{i\in N})$ be a political economy, and a and b two political leaders.*

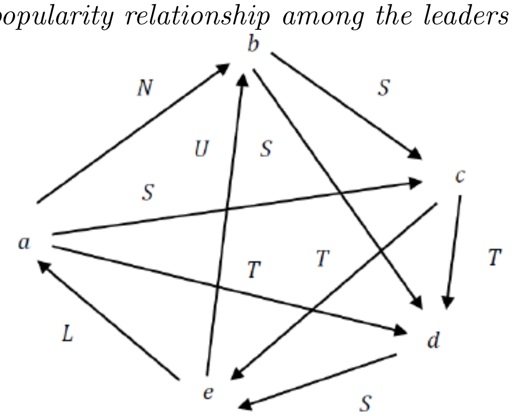
(i) b is said to Pareto-dominate a if all the voters strictly prefer b over a, denoted by $b \succ_N a$. A Pareto-dominated leader is also said to be Pareto inefficient.

(ii) a is said to be Pareto efficient if no leader Pareto-dominates a.

(iii) a is said to be unpopular if he is less preferred than another leader, say c, by a constitutional majority $S \in W$ (that is, $c \succ_S a$). The set of leaders who are not unpopular is denoted by $C(\Gamma)$.[8]

In order to show that equilibrium leaders may be inefficient and unpopular, consider the following simple example.

Example 2 *Consider a political economy $\Gamma = (N, W, A, (\succcurlyeq_i)_{i\in N})$ where: $N = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{a, b, c, d, e\}$, W is the majority rule (a coalition S is winning if and only if $|S| \geq 4$), and $(\succcurlyeq_i)_{i\in N}$ is the profile of preferences defined as follows: $d \succ_1 b \succ_1 e \succ_1 a \succ_1 c$, $d \succ_2 b \succ_2 a \succ_2 e \succ_2 c$, $d \succ_3 b \succ_3 a \succ_3 e \succ_3 c$, $e \succ_4 d \succ_4 c \succ_4 b \succ_4 a$, $c \succ_5 b \succ_5 a \succ_5 e \succ_5 d$, $c \succ_6 b \succ_6 a \succ_6 e \succ_6 d$, $c \succ_7 b \succ_7 a \succ_7 e \succ_7 d$. Let $L = \{2, 3, 5, 6\}$, $S = \{4, 5, 6, 7\}$, $T = \{1, 2, 3, 4\}$, and $U = \{1, 2, 3, 5\}$. The popularity relationship among the leaders is depicted by the following graph.*



We now identify the equilibrium leaders.

- If a is the status quo leader, he will not be defeated by b, even though the entire population strictly prefers b over a. Indeed, assume by contradiction that a is defeated by b thanks to a coalition S. Since $|S| \geq 4$, then either $S \cap \{1, 2, 3\} \neq \emptyset$ or $S \cap \{5, 6, 7\} \neq \emptyset$. If $S \cap \{1, 2, 3\} \neq \emptyset$, then we obtain a contradiction because $c \succ_{4567} b$ but not$(c \succ_S a)$ (every member of 123 prefers a to c). If $S \cap \{5, 6, 7\} \neq \emptyset$, then we obtain another contradiction because $d \succ_{1234} b$ but not$(d \succ_S a)$.

-Neither does c defeat a. Indeed, if c defeats a, it is only thanks to the support of the coalition $\{4, 5, 6, 7\}$. This is clearly impossible because $d \succ_{1234} c$ and $a \succ_{567} d$.

- Likewise, d does not defeat a.

It follows that $a \in Un(\Gamma)$. It can also be verified that b, c and d are not defeated either. Hence, $Un(\Gamma) = \{a, b, c, d, e\}$.

We emphasize that, in Example 2, even though a is an equilibrium leader, he is unpopular and Pareto inefficient. This implies that inefficient leaders can arise and persist in power forever. We also remark that all leaders are unpopular in Example 2. This implies that, even though stable

[8] The popularity relation is a very popular notion in the literature, and the set $C(\Gamma)$ is also known as the equilibrium set of Γ.

leaders exist in this economy, the society is not satisfied with any of them. In the next section, we will see that such a situation is partially caused by a plethora of competing leaders. In general, we will investigate the conditions under which such a situation can be avoided.

6.1 Welfare Properties of Equilibrium Leaders

We now seek to investigate the conditions under which equilibrium leaders are Pareto efficient. A few preliminary results are needed (Lemmas 1-2). The first states that, if one excludes from a political economy a leader who is Pareto-dominated by all the other leaders, that will not change its set of equilibrium leaders.

Lemma 1 *Let $\Gamma = (N, W, A, (\succcurlyeq_i)_{i\in N})$ be a political economy in which individual preferences are weak orders, and $y \in A$ a political leader. Let $\Gamma_y = (N, W, A\backslash\{y\}, (\succcurlyeq^y))$ be the political economy resulting from the exclusion of y. Suppose that $x \succ_N y$ for all $x \in A\backslash\{y\}$. Then,*

(i) *$y \notin Un(\Gamma)$; and*

(ii) *$Un(\Gamma) = Un(\Gamma_y)$.*

Proof. Suppose that $x \succ_N y$ for all $x \in A\backslash\{y\}$.

(i) Consider any $x \in A\backslash\{y\}$. Then $[(x, y); N]$ is an N-rational (x, y)-path. Let $[(y, x_0, x_1, x_2, ..., x_p); (S_0, S_1, S_2, ..., S_p)]$ be a (y, x)-path with $S_0 = N$ and $x_0 = x$. By definition of a (x, y)-path, $x_p \neq y$ and therefore $x_p \succ_N y$. Such a (y, x)-path is not deterred. Thus x defeats y thanks to N and $y \notin Un(\Gamma)$.

(ii) Let $z \in Un(\Gamma)$. Since $y \notin Un(\Gamma)$, it follows that $z \in A\backslash\{y\}$. Recall that $\succcurlyeq_i^y$ is the restriction of $\succcurlyeq_i$ on $A\backslash\{y\}$. Thus z is still undefeated in Γ_y and $z \in Un(\Gamma_y)$. Conversely, let $z \in Un(\Gamma_y)$. Consider $x \in A\backslash\{z, y\}$ and $S \in W$. Given a (y, x)-path $[(z, x_0, x_1, x_2, ..., x_p); (S_0, S_1, S_2, ..., S_p)]$ in Γ with $S_0 = S$ and $x_0 = x$, note that $x_0 = x \succcurlyeq_S y$ and, for each $t \in \{1, 2, ..., p\}$, $x_t \succcurlyeq_S x_{t-1}$. Thus, $\{x_0, x_1, x_2, ..., x_p\} \subseteq A\backslash\{y\}$ and $[(z, x_0, x_1, x_2, ..., x_p); (S_0, S_1, S_2, ..., S_p)]$ is also a (y, x)-path in Γ_y. Therefore, z is undefeated in Γ, otherwise z will also be defeated in Γ_y; which is a contradiction. In other words, we have $z \in Un(\Gamma)$ and hence $Un(\Gamma) = Un(\Gamma_y)$. ■

Our second preliminary result gives an equivalent definition of the farsighted rationale $\gg$.

Lemma 2 *Let $\Gamma = (N, W, A, (\succcurlyeq_i)_{i\in N})$ be a political economy in which individual preferences are weak orders, $x, y \in A$ two political leaders, and $S \in W$ a winning coalition. Then, $x \gg_S y$ if and only if: (i) $x \succcurlyeq_S y$; (ii) $x \succ_S y$ or ($z \succ x$ and $z \succ_S y$) for some $z \in A$; and (iii) for all $z \in A$, $z \succcurlyeq_S y$ holds whenever $z \succ x$.*

Proof. Let $\Gamma = (N, W, A, (\succcurlyeq_i)_{i\in N})$ be a political economy in which individual preferences are weak orders, $x, y \in A$ two political leaders, and $S \in W$ a winning coalition. Assume that x defeats y thanks to S, that is, $x \gg_S y$. Then Γ admits an S-rational (y, x)-path

$[(y,x_0,x_1,x_2,...,x_p);(S_0,S_1,S_2,...,S_p)]$. (i) Clearly $S_0 = S$ and $x \not\succcurlyeq_S y$ by the definition of a (y,x)-path. (ii) Note that if $p = 0$, then $x \succ_S y$. If $p \geq 1$, we have $z \succ_{S_1} x$ and $z \succ_S y$ for $z = x_p$. (iii) Let $z \in A$ such that $z \succ x$. Then there exists $T \in W$ such that $z \succ_T x$. Therefore $[(y,x,z);(S,T)]$ is a (y,x)-path and since x defeats y thanks to S, $z \not\succcurlyeq_S y$ obtains.

Conversely, assume that: (i) $x \not\succcurlyeq_S y$; (ii) $x \succ_S y$ or ($z \succ x$ and $z \succ_S y$) for some $z \in A$; and (iii) for all $z \in A$ for which $z \succ x$, $z \not\succcurlyeq_S y$. We first prove that Γ admits an S-rational (y,x)-path. If $x \succ_S y$, then $[(y,x);S]$ is an S-rational (y,x)-path. Otherwise, there exists $z \in A$ such that $z \succ x$ and $z \succ_S y$. Then $z \succ_T x$ for some $T \in W$. Since, by assumption, $x \not\succcurlyeq_S y$, it follows that $[(y,x,z);(S,T)]$ is an S-rational (y,x)-path. In both cases, Γ admits an S-rational (y,x)-path.

Now we prove that Γ does not admit a deterred (y,x)-path. Consider any (y,x)-path

$[(y,x_0,x_1,x_2,...,x_p);(S_0,S_1,S_2,...,S_p)]$ with $S_0 = S$ and $x_0 = x$. If $p = 0$, then by definition, $x \succ_S y$. Otherwise $x_p \succ_{S_1} x$, and by assumption, $x_p \not\succcurlyeq_S y$. In both cases, this path is not deterred. In conclusion, x defeats y thanks to S. ■

The following result is an immediate consequence of Lemma 2.

Corollary 1 *Let $\Gamma = (N,W,A,(\succcurlyeq_i)_{i\in N})$ be a political economy, $x,y \in A$ two political leaders, and $S \in W$ a winning coalition.*

1. *If individual preferences are weak orders and $x \succ_S y$, then $x \gg_S y$ if and only if for all $z \in A$, $z \not\succcurlyeq_S y$ holds whenever $z \succ x$.*
2. *If individual preferences are linear orders, then $x \gg_S y$ if and only if $x \succ_S y$ and for all $z \in A$, $z \succ_S y$ holds whenever $z \succ x$.*

The following result provides a sufficient condition for each equilibrium leader to be Pareto efficient. It states that if the number of competing leaders is not greater than four, then every equilibrium leader is Pareto efficient.

Proposition 1 *Let $\Gamma = (N,W,A,(\succcurlyeq_i)_{i\in N})$ be a political economy in which individual preferences are weak orders. If $|A| \leq 4$, then every equilibrium leader $x \in Un(\Gamma)$ is Pareto efficient.*

Proof. It is sufficient to prove that if two leaders x and y are such that $x \succ_N y$, then $y \notin Un(\Gamma)$. Let x and y be two leaders. Suppose that $x \succ_N y$.

First assume that $|A| = 3$ and let $A = \{x,y,u\}$. If $u \succ x$, then there exists $S \in W$ such that $u \succ_S x$. By transitivity of individual preferences, it holds that $u \succ_S y$. Since $N \backslash S \notin W$, neither $x \succ u$ nor $y \succ u$ holds. Therefore $u \gg_S x$ and $u \gg_S y$. It follows that $y \notin Un(\Gamma)$. If *not* $(u \succ x)$, then neither $u \succ x$ nor $y \succ x$ holds. Therefore $x \gg_N y$ and thus $y \notin Un(\Gamma)$.

Now assume that $|A| = 4$ and let $A = \{x,y,u,v\}$.

Let us remark that given two political leaders a and b, if $a \succ_L b$ and *not*$(a \gg_L b)$, then there exists another political leader c such that $c \succ a$.

Now let us consider the following four possible cases:

(i) Suppose that *not* $(u \succ x)$ and *not* $(v \succ x)$. By the remark above, $x \gg_N y$.

(ii) Suppose that $u \succ_S x$ for some $S \in W$ and *not* $(v \succ x)$. Note that $u \succ_S y$ holds by transivity. By Lemma 2, $x \gg_S y$.

(iii) Suppose that $v \succ_S x$ for some $S \in W$ and *not* $(u \succ x)$. Clearly as in case (ii), $x \gg_S y$.

(iv) Suppose that $u \succ_S x$ and $v \succ_T x$ for some $S, T \in W$. Note that $u \succ_S y$ and $v \succ_T y$ hold by transitivity.

- If *not* $(u \succ v)$, then *not* $(z \succ v)$ for all $z \in A \setminus \{v\}$, and by the remark above, $v \gg_T y$.
- If $u \succ v$, then *not* $(z \succ u)$ for all $z \in A \setminus \{u\}$. Clearly, $u \gg_S y$.

In each of these four cases, y is defeated and hence, $y \notin Un(\Gamma)$. ■

The following result provides another sufficient condition for equilibrium leaders to be Pareto efficient. It states that, under an oligarchic constitution, all equilibrium leaders are Pareto efficient.

Proposition 2 *Let* $\Gamma = (N, W, A, (\succcurlyeq_i)_{i \in N})$ *be a political economy in which individual preferences are weak orders. If* W *is an oligarchy, then every equilibrium leader* $x \in Un(\Gamma)$ *is Pareto efficient.*

Proof. Assume that there exists a unique minimal winning coalition S in W. Let x and y be two political leaders such that $x \succ_N y$. The relation $x \succ_S y$ obviously holds. Let $z \in A$ be a political leader and suppose that $z \succ x$. Then $z \succ_T x$ for some $T \in W$. By the fact that S is the unique minimal winning coalition, $S \subseteq T$, and thus $z \succ_S y$. By Lemma 2, $x \gg_S y$, and hence $y \notin Un(\Gamma)$.

■

The next result shows that an equilibrium leader may be inefficient if the constitution is non-oligarchic and there are five competing leaders.

Proposition 3 *Let* N *be a society endowed with a non-oligarchic constitution* W *and a set of five competing political leaders* A*. There exists a linear preference profile* $(\succcurlyeq_i)_{i \in N}$ *such that the political economy* $\Gamma = (N, W, A, (\succcurlyeq_i)_{i \in N})$ *contains a Pareto inefficient equilibrium leader.*

Proof. Let N be a society under a non-oligarchic constitution W. Assume that $A = \{a_1, a_2, a_3, a_4, a_5\}$. Since W is not an oligarchy, there exist at least two distinct minimal winning coalitions S_1 and S_2. Note that $S_1 \cap \overline{S_2} \neq \emptyset$ ($\overline{S_2}$ is the complementary set of the set S_2) and $S_2 \cap \overline{S_1} \neq \emptyset$ as S_1 and S_2 are minimal winning coalitions. For any $i \in N$, define $\succcurlyeq_i$ as follows :

$$a_5 \succ_i a_4 \succ_i a_3 \succ_i a_2 \succ_i a_1 \text{ if } i \in S_1 \cap S_2$$

$$a_4 \succ_i a_2 \succ_i a_1 \succ_i a_5 \succ_i a_3 \text{ if } i \in S_1 \cap \overline{S_2}$$

$$a_3 \succ_i a_2 \succ_i a_1 \succ_i a_5 \succ_i a_4 \text{ if } i \in \overline{S_1}$$

We have $a_2 \succ_N a_1$. To prove that $a_1 \in Un(\Gamma)$, we show that for each $a_j \in \{a_2, a_3, a_4, a_5\}$, a_1 is not defeated by a_j (that is, $not(a_j \gg a_1)$).

Case $a_j = a_5$. First note that $\{i \in N : a_5 \succ_i a_1\} = S_1 \cap S_2$. But S_1 and S_2 are distinct minimal winning coalitions. Then $S_1 \cap S_2 \notin W$. Therefore, by Lemma 2, a_1 is not defeated by a_j.

Case $a_j = a_4$. First observe that $\{i \in N : a_4 \succ_i a_1\} = S_1$. Now suppose there exists $S \in W$ such that $a_4 \succ_S a_1$. Then $S \subseteq S_1$. But S_1 is a minimal winning coalition. Thus $S = S_1$. Since $S_1 \cap \overline{S_2} \neq \emptyset$, $a_5 \succ_{S_2} a_4$ and $a_1 \succ_{S_1 \cap \overline{S_2}} a_5$, then by Lemma 2, a_5 does not defeats a_1 thanks to $S = S_1$.

Case $a_j = a_3$. Note that $\{i \in N : a_3 \succ_i a_1\} = (S_1 \cap S_2) \cup \overline{S_1}$. Suppose there exists $S \in W$ such that $a_3 \succ_S a_1$. Then $S \subseteq (S_1 \cap S_2) \cup \overline{S_1}$. But S_1 is a minimal winning coalition. Thus $S \cap \overline{S_1} \neq \emptyset$; otherwise $S \subseteq S_1 \cap S_2$ yields a contradiction. Since $S \cap \overline{S_1} \neq \emptyset$, $a_4 \succ_{S_1} a_3$ and $a_1 \succ_{S \cap \overline{S_1}} a_4$, by Lemma 2, a_3 does not defeat a_1 thanks to S.

Case $a_j = a_2$. Let $S \in W$. Then S can be rewritten as:

$$S = (S \cap S_1 \cap S_2) \cup \left(S \cap S_1 \cap \overline{S_2}\right) \cup \left(S \cap \overline{S_1}\right)$$

Therefore $S \cap S_1 \cap \overline{S_2} \neq \emptyset$ or $S \cap \overline{S_1} \neq \emptyset$ holds; otherwise $S \subseteq S_1 \cap S_2$ yields a contradiction. If $S \cap S_1 \cap \overline{S_2} \neq \emptyset$, then a_2 does not defeat a_1 thanks to S since $a_3 \succ_{S_2} a_2$ and $a_1 \succ_{S \cap S_1 \cap \overline{S_2}} a_3$. If $S \cap \overline{S_1} \neq \emptyset$, then a_2 does not defeat a_1 thanks to S since $a_4 \succ_{S_1} a_2$ and $a_1 \succ_{S_1 \cap \overline{S_2}} a_4$. In both situations, a_2 does not defeat a_1 thanks to S.

In summary, a_1 is Pareto-dominated and $a_1 \in Un(\Gamma)$. ■

Our second main result provides a complete characterization of political economies for which equilibrium leaders are always Pareto efficient. It states that all equilibrium leaders are Pareto efficient under any constitution if and only if the economy has at most four competing political leaders. However, if there are more than four competing political leaders, all equilibrium leaders are Pareto efficient if and only if the constitution is oligarchic.

Theorem 2 *Let N be a society endowed with a constitution W. The following two assertions are equivalent:*

1) For all political economy $\Gamma = (N, W, A, (\succcurlyeq_i)_{i \in N})$ where the preferences $(\succcurlyeq_i)_{i \in N}$ are weak orders, every equilibrium leader $x \in Un(\Gamma)$ is Pareto efficient.

2) W is oligarchic or $|A| \leq 4$.

Proof. Suppose that for all political economy $\Gamma = (N, W, A, (\succcurlyeq_i)_{i \in N})$, every $x \in Un(\Gamma)$ is Pareto efficient. Suppose that $|A| \geq 5$ and let $A = A_1 \cup A_2$ with $A_1 = \{a_1, a_2, a_3, a_4, a_5\}$, $A_2 = \{a_6, a_7, ..., a_m\}$ and $A_1 \cap A_2 = \emptyset$. Suppose that W is not oligarchic. Then by Proposition 3, there exists a political economy $\Gamma' = (N, A_1, W, (\succcurlyeq'))$ such that a_1 is Pareto-dominated and $a_1 \in Un(\Gamma')$. Now consider a political economy $\Gamma = (N, W, A, (\succcurlyeq_i)_{i\in N})$ where the profile $(\succcurlyeq)$ is such that for all $i \in N$: (i) the restriction of $\succcurlyeq_i$ on A_1 coincides with $\succcurlyeq'_i$; (ii) $a \succ_i b$ for all $a \in A_1$ and $b \in A_2$; and (iii) $a_{t-1} \succ_i a_t$ for all $t \in \{5, 6, ..., m\}$. By Proposition 1, $Un(\Gamma) = Un(\Gamma')$ and thus $a_1 \in Un(\Gamma)$. This is a contradiction. Therefore W is oligarchic.

Conversely, suppose that W is oligarchic or $|A| \leq 4$. If W is oligarchic, then by Proposition 2, every $x \in Un(\Gamma)$ is Pareto efficient. If $|A| \leq 4$, then by Proposition 1, every $x \in Un(\Gamma)$ is Pareto efficient, which completes our proof. ■

The finding that an oligarchy always induces an inefficient leader is not very hard to imagine. However, the finding that the limitation of the number of competing politicians to a maximum of four ensures that all the equilibrium leaders are efficient was unexpected. It therefore deserves an explanation. The intuition underlying this result is that a greater number of competing politicians creates more uncertainty about who will govern the society in the future if the current ruling leader loses power. As shown in Example 2, this uncertainty might sometimes combine with voter prudence or forward-looking behavior to maintain the current leader in power, even if he or she is inefficient. Our findings therefore show that an excessively high level of political competition is not necessarily desirable, as it might lead to political failure or inefficiency. Our analysis has identified the minimum level of political competition that is necessary to lead to this situation, and therefore has practical implications for the level of compromise that political leaders should achieve in order to rescue their citizens from an eventually bad and persistent equilibrium. Political competition is clearly desirable, but our analysis implies that there should be no more than four political parties. This implies that political leaders should strive to form coalitions, especially in highly fragmented societies where the number of competing political leaders generally reflects the number of factions.

6.2 Popularity of Equilibrium Leaders

In Example 2, we saw that equilibrium leaders may be unpopular and inefficient. The requirement that an equilibrium leader be popular is more stringent than the requirement that he be Pareto efficient. In fact, Pareto inefficient leaders are always unpopular by definition, but an unpopular leader is not always Pareto inefficient. In this section, we provide a sufficient condition on preferences for equilibrium leaders to not be unpopular. We find that, if the popularity relation $\succ$ is transitive, then all equilibrium leaders are popular and hence Pareto efficient, and each popular leader is an equilibrium.

Theorem 3 *Let $\Gamma = (N, W, A, (\succcurlyeq_i)_{i\in N})$ be a political economy in which individual preferences are weak orders. If the popularity relation $\succ$ is transitive, then every equilibrium leader $x \in Un(\Gamma)$ is popular and hence Pareto efficient, and every popular leader $x \in C(\Gamma)$ is an equilibrium leader: $Un(\Gamma) = C(\Gamma)$.*

Proof. Note that by Lemma 2, $C(\Gamma) \subseteq Un(\Gamma)$ for any political economy Γ. Now let $\Gamma = (N, W, A, (\succcurlyeq_i)_{i\in N})$ be a political economy. Suppose that $\succ$ is transitive. Assume an equilibrium leader $y \in Un(\Gamma)$ and suppose that $y \notin C(\Gamma)$. Then $t \succ y$ for some $t \in A\backslash\{y\}$. In other words,

y is not a maximal element of the relation $\succ$. Since A is finite and the dominance relation $\succ$ is transitive, there exists a maximal element x for $\succ$ such that $x \succ y$. Therefore there exists $S \in W$ such that $x \succ_S y$. Since there is no $z \in A \backslash \{x\}$ such that $z \succ x$, then by Lemma 2, x defeats y thanks to S. This is a contradiction since $y \in Un(\Gamma)$. Thus $C(\Gamma) \supseteq Un(\Gamma)$. In conclusion, $C(\Gamma) = Un(\Gamma)$. ■

The popularity relation has been widely studied in the literature. The main focus has been to uncover conditions on the structure of preferences under which this relation is transitive (see, e.g., Sen (1966), Inada (1964, 1969), and Salles (1976)). In the class of constitutions considered in this paper, Salles (1976) provides three conditions that, when taken disjunctively, are necessary and sufficient for the popularity relation to be transitive. These conditions are *value restriction* (VR), *dichotomous preferences* (DP), and *cyclical dependence* (CD) (see Theorems 1-4 in Salles (1976)).[9] The value-restriction property was identified by Sen (1966), the dichotomous-preferences property was identified by Inada (1969), and the cyclical-dependence property was identified by Salles (1976). Each of these properties identifies a very wide class of preferences. Given the analysis of Salles (1976), the following result immediately follows from Theorem 3.

Corollary 2 *Let $\Gamma = (N, W, A, (\succcurlyeq_i)_{i \in N})$ be a political economy in which preferences are weak orders. If the preference profile $(\succcurlyeq_i)_{i \in N}$ satisfies value restriction, the dichotomous-preferences property, or cyclical dependence, then every equilibrium leader $x \in Un(\Gamma)$ is popular, and every popular leader $x \in C(\Gamma)$ is an equilibrium leader: $Un(\Gamma) = C(\Gamma)$.*

Our analysis therefore identifies a wide class of preferences for which every equilibrium leader is popular and thus Pareto efficient. In particular, it is well-known that the class of preference profiles that satisfy value restriction includes the popular class of single-peaked preference profiles identified by Black (1948), as well as the class of single-carved preference profiles identified by Inada (1964). It follows that when voters have single-peaked or single-carved preferences, not only does an equilibrium leader exist, but all equilibrium leaders are popular are efficient.

We also note that the class of preferences for which our set of equilibrium leaders coincides with the set of popular leaders is in general larger than the class of preferences for which the popularity relation is transitive. For example, consider the following political economy $\Gamma = (N, W, A, (\succcurlyeq_i)_{i \in N})$ where $N = \{1, 2, 3, 4, 5\}$, $W = \{S \subseteq N : |S| \geq 0.75 |N|\}$, $t \succ_1 y \succ_1 x \succ_1 z$, $t \succ_2 x \succ_2 z \succ_2 y$ and $t \succ_i z \succ_i y \succ_i x$ for $i \in \{3, 4, 5\}$. It can be easily verified that the popularity relation $\succ$ is not transitive, but that $C(\Gamma) = Un(\Gamma) = \{t\}$.

[9] A preference profile $(\succcurlyeq_i)_{i \in N}$ is said to satisfy VR if, in any three-alternative subset $\{x, y, z\} \subset A$, there is an alternative such that all concerned individuals for this subset agree that it is not the worst, or the best, or the medium-ranked alternative (a concerned individual for a set X is an individual who is not indifferent between every pair of elements in X). A preference profile $(\succcurlyeq_i)_{i \in N}$ satisfies DP if, for any three-alternative subset $\{x, y, z\} \subset A$, each individual is indifferent over some pair of alternatives $\{a, b\} \subset \{x, y, z\}$. A preference profile $(\succcurlyeq_i)_{i \in N}$ satisfies CD if, for any three-alternative subset $\{x, y, z\} \subset A$, if an individual i has preferences $a \succ_i b \succ_i c$, then there exist no individuals j and k such that $b \succ_j c \succ_j a$ and $c \succcurlyeq_k a \succcurlyeq_k b$, and there exist no individuals j' and k' such that $b \succ_{j'} c \succ_{j'} a$ and $c \succ_{k'} a \succ_{k'} b$, and there exist no individuals j'' and k'' such that $b \succ_{j''} c \sim_{j''} a$ and $c \sim_{k''} a \succ_{k''} b$.

7 Implications for the Majority Rule and Ethnic Societies

In this section, we draw some lessons about the majority rule from the analysis carried out thus far. Our focus on the majority rule is justified by the fact that it is widely regarded as the fairest and the most democratic of all political rules. Maskin and Dasgupta (2008) show that the simple majority rule satisfies five appealing properties (the Pareto property, neutrality, anonymity, decisiveness, and independence of irrelevant alternatives) over a larger domain of preferences than any other rule. The majority rule is also used to select policies in most societies. Therefore, deriving the implications of our findings for this rule is likely to shed light on some of the factors that determine the longevity and quality of political leaders in these societies.

Under the majority rule, there always exists at least one equilibrium leader when preferences are linear (Theorem 1). However, such a leader might be inefficient and unpopular, as shown in Example 2 for leader a. From Theorem 2, we also note that, under the majority rule, an equilibrium leader will be efficient at any preference profile if and only if there are at most four competing leaders. This has implications for the quality and longevity of leaders in countries that are organised around ethnic groups and in which the number of leaders often reflects the number of these groups. When such countries have several competing ethnic groups, our analysis implies that they can be trapped under an inefficient and hence unpopular leader who does not even need to manipulate elections to retain political power. This result suggests another channel through which more ethnically fragmented societies often have leaders who, despite being amazingly incompetent and unpopular, persist in power for long periods even when elections are fair. A testable implication implied by our analysis is therefore that, under the majority rule, ethnic fragmentation induces bad economic policies by favoring the election of bad leaders.

8 Naiveté versus Farsightedness

Our analysis has clear implications for how voting behavior affects political outcomes. In particular, we contrast voting outcomes under *farsighted behavior* on the one hand and *naive* or *myopic behavior* on the other hand. Scholars have expressed doubt over whether agents are really farsighted when making real-life economic decisions. In real-life politics, voters might view each election as a one-shot game, voting as if the next election is an entirely different game. In this context, voters only consider their immediate gains. How would the political outcome that results from such naive behavior differ from the outcome generated by farsighted behavior?

If voters are naive, it is obvious that they will support the challenger over the status quo leader whenever they prefer the former over the latter, thus destabilizing or ejecting the status quo leader with a positive probability (based on the assumption that the incumbent might steal the election). This implies that only popular leaders, if they gain power at some point, will be able to remain in office forever without stealing elections. In other words, the equilibrium set of a political economy Γ is $C(\Gamma)$ under naive voting behavior.

We note that the set $C(\Gamma)$ is included in the farsighted equilibrium set $Un(\Gamma)$, which means that a myopically stable leader is also a farsightedly stable leader. In addition, by definition, each myopically stable leader is popular. This implies that myopically stable leaders in general have more desirable properties than farsightedly stable leaders in that farsightedly stable leaders who are not myopically stable are unpopular and possibly are inefficient as well. In that sense, myopically stable leaders are the first-best for society, whereas leaders who are only farsightedly stable are the second-best. Note however that the set $C(\Gamma)$ may be empty as shown in Example 2, which depicts a situation in which all farsightedly stable leaders are unpopular.

Our analysis implies that naive voting behavior is more likely to lead to leaders that are better for the society than sophisticated behavior, which seems counterintuitive. However, naive behavior might cause an individual to regret his political choice, whereas farsighted behavior, by definition, prevents any such disappointment. It follows that individually optimal behavior is in general detrimental to the society, whereas behavior that is optimal for the society might be detrimental to the individuals who adopt it. However, as our characterization result obtained in Section 7 implies, this dilemma is resolved for a large class of preferences which strictly includes the popular class of single-peaked preferences, as naive and farsighted behaviors lead to the same set of equilibrium leaders for this class.

9 Conclusions

We have proposed a tractable framework for studying political competition and stability in nominally democratic societies characterized by fraudulent elections. In these democracies, an incumbent leader can retain power indefinitely with a strictly positive probability by manipulating elections. We have modeled the behavior of forward-looking citizens, thus defining a new solution concept. An equilibrium leader is a leader who is able to remain in power forever without stealing elections. We show that such a leader always exists when citizens have linear preferences. However, he may be unpopular and inefficient. We have uncovered three types of conditions under which an equilibrium leader is never inefficient. Two of these conditions clearly show that efficiency is achieved at the cost of restricting basic democratic principles, such as limiting the number of competing leaders to four or distributing political rights in an inequitable manner. The third condition identifies a wide class of preferences for which equilibrium leaders are never inefficient, with this class containing the popular class of single-peaked preferences. We also have demonstrated that, under this class of preferences, the set of equilibrium leaders coincides with the set of popular leaders, which was an unexpected finding.

The analysis suggests a new testable channel through which a high level of ethnic fragmentation can lead to underdevelopment. Societies that are highly fragmented along ethnic lines generally have a large number of competing political leaders. Our analysis implies that this plethora of leaders might lead to political failure by favoring the emergence of an inefficient and hence unpopular leader who is able to persist in power indefinitely without needing to steal elections.

The finding that the limitation of the number of competing politicians to a maximum of four guarantees political efficiency therefore has clear implications for the level of political compromise that leaders should achieve in highly fragmented societies in order to prevent political failure. Even though political competition is clearly desirable, our analysis suggests that there should only exist a limited number of political parties, like, for instance, in the United States. A way to achieve a smaller number of political parties could be to provide incentives for political leaders who are not too distant ideologically to merge their political platforms. This could prevent the society from being trapped under a bad equilibrium. An excessively large number of political leaders creates a high level of uncertainty about who will govern the society in the future if the current ruling leader loses power. This uncertainty sometimes combines with voter prudence to maintain the current leader in power, even if he or she is inefficient and hence highly unpopular. An excessively large number of competing political leaders only helps to create a form of political inertia that only benefits the status quo leader, even when the latter should clearly be ejected from power.

We conclude by discussing a different application of our model. It might also be applied to study the dynamic selection of *sticky policies* in fully developed democracies. Sticky policies are policies that, once enacted, remain in force for an indefinite period of time until they are voted out in favor of new legislation. At the time that such policies are voted, lawyers and other analysts *a priori* cannot foretell how long they will remain in effect. Sticky policies are therefore characterized by uncertainty about their durability. In general, such policies include, but are not limited to, redistributive programs (e.g., fiscal policies, minimum-wage laws, and social-welfare programs), health-care programs, land property rights, environmental policies, and eligibility requirements for political participation and competition. There is no certainty over when an election will be organized to challenge the extant policy. Even when there is enough political support for change, it is not clear when the next election will take place as this often depends on the willingness of a political leader to bring the issue to the attention of the public and the main political decision-makers. There is a number of complex factors that affect elections and that are not under the control of voters. Sticky policies are therefore unlike political leadership which, in a fully functioning democracy, is renewed on a regular basis through presidential, legislative or mayoral elections that take place at regular intervals. When deciding to support change over the status quo, voters might behave as if the new policy program, if chosen, will remain in place forever with a positive probability, just like a new leader who retains power indefinitely by manipulating elections. In this context, all our results are valid. In particular, our analysis implies that an unpopular and inefficient policy may remain in place forever in fully democratic societies.

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